

20 Years'
CHAPTERWISE TOPICWISE
(2021-2002)

JEE Main
SOLVED PAPERS

Physics

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(2021-2002)

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SOLVED PAPERS

Physics

Complete Collection of all Questions asked
in ONLINE & OFFLINE JEE Main & AIEEE



ARIHANT PRAKASHAN (Series), MEERUT



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Agra, Ahmedabad, Bengaluru, Bareilly, Chennai, Delhi, Guwahati,
Hyderabad, Jaipur, Jhansi, Kolkata, Lucknow, Nagpur & Pune.

⌘ **ISBN** 978-93-25796-23-2

PO No : TXT-XX-XXXXXXX-X-XX

Published by Arihant Publications (India) Ltd.

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PREFACE

JEE Main is a gateway examination for candidates expecting to seek admission in Bachelor in Engineering (BE), Bachelor of Technology (B.Tech) and Bachelor of Architecture (B.Arch) at Indian Institutes of Information Technology (IIITs), National Institutes of Technology (NITs), Delhi Technological University and other Centrally Funded Technical Institutes (CFTIs).

JEE Main is also an examination which is like screening examination for JEE Advanced (The gateway examination to India's most reputed Technical Institutes, Indian Institutes of Technology— IITs).

To make the students well-versed with the pattern as well as the level of the questions asked in the exam, this book contains **Chapterwise Topicwise Solutions of Questions asked in Last 20 Years' Examinations of JEE Main** (formerly known as AIEEE). Solutions to all the questions have been kept very detailed and accurate. Along with the indication of level of the exam, this book will also teach you to solve the questions objectively in the examination.

We hope this book would be highly beneficial for the students. We would be grateful if any discrepancy or mistake in the questions or answers is brought to our notice so that these could be rectified in subsequent editions.

Publisher

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SYLLABUS

NOTE The syllabus contains two Sections - A & B. Section A pertains to the Theory Part, having 80% weightage, while Section B contains Practical Component (Experimental Skills) having 20% weightage.

SECTION- A

UNIT 1 Physics and Measurement

Physics, technology and society, SI units, Fundamental and derived units. Least count, accuracy and precision of measuring instruments, Errors in measurement, Significant figures. Dimensions of Physical quantities, dimensional analysis and its applications.

UNIT 2 Kinematics

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position time graphs, relations for uniformly accelerated motion.

Scalars and Vectors, Vector addition and Subtraction, Zero Vector, Scalar and Vector products, Unit Vector, Resolution of a Vector. Relative Velocity, Motion in a plane, Projectile Motion, Uniform Circular Motion.

UNIT 3 Laws of Motion

Force and Inertia, Newton's First Law of motion; Momentum, Newton's Second Law of motion; Impulse; Newton's Third Law of motion. Law of conservation of linear momentum and its applications, Equilibrium of concurrent forces.

Static and Kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: Centripetal force and its applications.

UNIT 4 Work, Energy and Power

Work done by a constant force and a variable force; kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and nonconservative forces; Elastic and inelastic collisions in one and two dimensions.

UNIT 5 Rotational Motion

Centre of mass of a two-particle system, Centre of mass of a rigid body; Basic concepts of rotational motion; moment of a force, torque, angular momentum, conservation of angular momentum and its applications; moment of inertia, radius of gyration. Values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

UNIT 6 Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Kepler's laws of planetary motion.

Gravitational potential energy; gravitational potential.

Escape velocity. Orbital velocity of a satellite.

Geo-stationary satellites.

UNIT 7 Properties of Solids & Liquids

Elastic behaviour, Stress-strain relationship, Hooke's Law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column; Pascal's law and its applications.

Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, Reynolds number. Bernoulli's principle and its applications.

Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

Heat, temperature, thermal expansion; specific heat capacity, calorimetry; change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

UNIT 8 Thermodynamics

Thermal equilibrium, zeroth law of thermodynamics, concept of temperature. Heat, work and internal energy. First law of thermodynamics.

Second law of thermodynamics: reversible and irreversible processes. Carnot engine and its efficiency.

UNIT 9 Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic energy and temperature: rms speed of gas molecules; Degrees of freedom, Law of equipartition of energy, applications to specific heat capacities of gases; Mean free path, Avogadro's number.

UNIT 10 Oscillations And Waves

Periodic motion - period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M.) and its equation; phase; oscillations of a spring - restoring force and force constant; energy in S.H.M. - kinetic and potential energies; Simple pendulum - derivation of expression for its time period; Free, forced and damped oscillations, resonance.

Wave motion Longitudinal and transverse waves, speed of a wave. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, Standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect in sound.

UNIT 11 Electrostatics

Electric charges Conservation of charge, Coulomb's law-forces between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field Electric field due to a point charge, Electric field lines, Electric dipole, Electric field due to a dipole, Torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy stored in a capacitor.

UNIT 12 Current Electricity

Electric current, Drift velocity, Ohm's law, Electrical resistance, Resistances of different materials, V-I characteristics of Ohmic and nonohmic conductors, Electrical energy and power, Electrical resistivity, Colour code for resistors; Series and parallel combinations of resistors; Temperature dependence of resistance.

Electric Cell and its Internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and their applications. Wheatstone bridge, Metre bridge.

Potentiometer - principle and its applications.

UNIT 13 Magnetic Effects of Current and Magnetism

Biot-Savart law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field, Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter.

Current loop as a magnetic dipole and its magnetic dipole moment. Bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements. Para, dia and ferro-magnetic substances

Magnetic susceptibility and permeability, Hysteresis, Electromagnets and permanent magnets.

UNIT 14 Electromagnetic Induction and Alternating Currents

Electromagnetic induction; Faraday's law, induced emf and current; Lenz's Law, Eddy currents. Self and mutual inductance.

Alternating currents, peak and rms value of alternating current/ voltage; reactance and impedance; LCR series circuit, resonance; Quality factor, power in AC circuits, wattless current. AC generator and transformer.

UNIT 15 Electromagnetic Waves

Electromagnetic waves and their characteristics. Transverse nature of electromagnetic waves.

Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of e.m. waves.

UNIT 16 Optics

Reflection and refraction of light at plane and spherical surfaces, mirror formula, Total internal reflection and its applications, Deviation and Dispersion of light by a prism, Lens Formula, Magnification, Power of a Lens, Combination of thin lenses in contact, Microscope and Astronomical Telescope (reflecting and refracting) and their magnifying powers.

Wave optics wave front and Huygens' principle, Laws of reflection and refraction using Huygen's principle.

Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes, Polarisation, plane polarized light; Brewster's law, uses of plane polarized light and Polaroids.

UNIT 17 Dual Nature of Matter and Radiation

Dual nature of radiation. Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation; particle nature of light. Matter waves-wave nature of particle, de Broglie relation. Davisson-Germer experiment.

UNIT 18 Atoms and Nuclei

Alpha-particle scattering experiment; Rutherford's model of atom; Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars; isotones. Radioactivity-alpha, beta and gamma particles/rays and their properties; radioactive decay law. Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

UNIT 19 Electronic Devices

Semiconductors; semiconductor diode: I-V characteristics in forward and reverse bias; diode as a rectifier; I-V characteristics of LED, photodiode, solar cell, and Zener diode; Zener diode as a voltage regulator. Junction transistor, transistor action, characteristics of a transistor transistor as an amplifier (common emitter configuration) and oscillator. Logic gates (OR, AND, NOT, NAND & NOR). Transistor as a switch.

UNIT 20 Communication Systems

Propagation of electromagnetic waves in the atmosphere; Sky and space wave propagation, Need for modulation, Amplitude and Frequency Modulation, Bandwidth of signals, Bandwidth of Transmission medium, Basic Elements of a Communication System (Block Diagram only)

SECTION- B

UNIT 21 Experimental Skills

Familiarity with the basic approach and observations of the experiments and activities

1. Vernier callipers - its use to measure internal and external diameter and depth of a vessel.
2. Screw gauge - its use to determine thickness/ diameter of thin sheet/wire.
3. Simple Pendulum - dissipation of energy by plotting a graph between square of amplitude and time.
4. Metre Scale - mass of a given object by principle of moments.
5. Young's modulus of elasticity of the material of a metallic wire.
6. Surface tension of water by capillary rise and effect of detergents.
7. Coefficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
8. Plotting a cooling curve for the relationship between the temperature of a hot body and time.
9. Speed of sound in air at room temperature using a resonance tube.
10. Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures.
11. Resistivity of the material of a given wire using metre bridge.
12. Resistance of a given wire using Ohm's law.
13. Potentiometer
 - (i) Comparison of emf of two primary cells.
 - (ii) Internal resistance of a cell.
14. Resistance and figure of merit of a galvanometer by half deflection method.
15. Focal length of
 - (i) Convex mirror
 - (ii) Concave mirror
 - (iii) Convex lens
16. Using parallax method. Plot of angle of deviation vs angle of incidence for a triangular prism.
17. Refractive index of a glass slab using a travelling microscope.
18. Characteristic curves of a p-n junction diode in forward and reverse bias.
19. Characteristic curves of a Zener diode and finding reverse break down voltage.
20. Characteristic curves of a transistor and finding current gain and voltage gain.
21. Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
22. Using multimeter to
 - (i) Identify base of a transistor.
 - (ii) Distinguish between npn and pnp type transistor.
 - (iii) See the unidirectional flow of current in case of a diode and an LED.
 - (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).



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01

Units and Measurements

TOPIC 1

Units

01 If E and H represent the intensity of electric field and magnetising field respectively, then the unit of E/H will be [2021, 27 Aug Shift-I]

- (a) ohm (b) mho
(c) joule (d) newton

Ans. (a)

Unit of intensity of electric field E is Vm^{-1} .

Unit of intensity of magnetising field H is Am^{-1} .

Unit of E/H can be calculated as

$$\frac{\text{Unit of } E}{\text{Unit of } H} = \frac{\text{Vm}^{-1}}{\text{Am}^{-1}} = \frac{\text{V}}{\text{A}} = \text{ohm.}$$

Thus, the unit of E/H will be ohm.

02 Match List-I with List-II.

List-I	List-II
A. R_H (Rydberg constant)	1. $\text{kg m}^{-1} \text{s}^{-1}$
B. h (Planck's constant)	2. $\text{kg m}^2 \text{s}^{-1}$
C. μ_B (Magnetic field energy density)	3. m^{-1}
D. η (Coefficient of viscosity)	4. $\text{kg m}^{-1} \text{s}^{-2}$

Choose the most appropriate answer from the options given below. [2021, 27 Aug Shift-II]

- | | | | |
|-------|---|---|---|
| A | B | C | D |
| (a) 2 | 3 | 4 | 1 |
| (b) 3 | 2 | 4 | 1 |
| (c) 4 | 2 | 1 | 3 |
| (d) 3 | 2 | 1 | 4 |

Ans. (b)

As we know that, SI unit of following terms are

(a) R_H (Rydberg constant) = m^{-1}

(b) h (Planck's constant)

$$= \text{J} \cdot \text{s} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{s} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$$

(c) Magnetic field energy density

$$\begin{aligned} (\mu_B) &= \frac{\text{Energy}}{\text{Volume}} \\ &= \frac{\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}}{\text{m}^3} \\ &= \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2} \end{aligned}$$

(d) η (coefficient of viscosity)

$$\therefore F = \eta A \frac{dv}{dx}$$

$$\therefore \eta = \frac{F dx}{A dv} = \frac{\text{kg} \cdot \text{ms}^{-2} \cdot \text{m}}{\text{m}^2 \cdot \text{ms}^{-1}} = \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$$

So, the correct match is

A-3, B-2, C-4 and D-1.

03 The density of a material in SI units is 128 kg m^{-3} . In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is [2019, 10 Jan Shift-I]

- (a) 40 (b) 16
(c) 640 (d) 410

Ans. (a)

To convert a measured value from one system to another system, we use

$$N_1 u_1 = N_2 u_2$$

where, N is numeric value and u is unit.

We get

$$\begin{aligned} 128 \cdot \frac{\text{kg}}{\text{m}^3} &= N_2 \cdot \frac{50 \text{ g}}{(25 \text{ cm})^3} \\ \left[\therefore \text{density} = \frac{\text{mass}}{\text{volume}} \right] \\ \Rightarrow \frac{128 \times 1000 \text{ g}}{100 \times 100 \times 100 \text{ cm}^3} &= N_2 \cdot \frac{50 \text{ g}}{(25 \text{ cm})^3} \end{aligned}$$

$$\begin{aligned} &= \frac{N_2 \times 50 \text{ g}}{25 \times 25 \times 25 \text{ cm}^3} \\ \Rightarrow N_2 &= \frac{128 \times 1000 \times 25 \times 25 \times 25}{50 \times 100 \times 100 \times 100} = 40 \end{aligned}$$

04 The 'rad' is the correct unit used to report the measurement of [AIEEE 2006]

- (a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

Ans. (c)

'Rad' is used to measure biological effect of radiation.

TOPIC 2

Errors in Measurement and Significant Figures

05 Two resistors $R_1 = (4 \pm 0.8) \Omega$ and $R_2 = (4 \pm 0.4) \Omega$ are connected in parallel. The equivalent resistance of their parallel combination will be [2021, 1 Sep Shift-II]

- (a) $(4 \pm 0.4) \Omega$ (b) $(2 \pm 0.4) \Omega$
(c) $(2 \pm 0.3) \Omega$ (d) $(4 \pm 0.3) \Omega$

Ans. (c)

Given, $R_1 = (4 \pm 0.8) \Omega \Rightarrow R_2 = (4 \pm 0.4) \Omega$

Equivalent resistance when the resistors are connected in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{4}$$

$$R_{eq} = 2 \Omega$$

$$\text{Now, } \frac{\Delta R_{eq}}{R_{eq}^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

Substituting the values in the above equation, we get

$$\frac{\Delta R_{eq}}{4} = \frac{0.8}{16} + \frac{0.4}{16} \Rightarrow \Delta R_{eq} = 0.3 \Omega$$

$\therefore$ The equivalent resistance in parallel combination is $R_{eq} = (2 \pm 0.3) \Omega$.

- 06** A student determined Young's modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2 , without any significant error, his observations are as following.

Physical quantity	Least count of the equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then, the fractional error in the measurement of Y is

[2021, 1 Sep Shift-II]

- (a) 0.0083 (b) 0.0155
(c) 0.155 (d) 0.083

Ans. (b)

The given formula of Young's modulus of elasticity,

$$Y = \frac{mgL^3}{4bd^3\delta}$$

where, Y = Young's modulus of elasticity,

m = mass of the bar,

L = length of the bar,

b = breadth of the bar,

d = thickness of the bar

and δ = depression of the bar.

There is no error in the value of the g .

The fractional error in the measurement of Y ,

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + 3 \frac{\Delta L}{L} + \frac{\Delta b}{b} + 3 \frac{\Delta d}{d} + \frac{\Delta \delta}{\delta}$$

Substituting the values in the above expression, we get

$$\frac{\Delta Y}{Y} = \frac{10^{-3}}{2} + 3 \frac{(1 \times 10^{-3})}{1} + \frac{0.1 \times 10^{-3}}{4 \times 10^{-2}} + 3 \frac{(0.01 \times 10^{-3})}{0.4 \times 10^{-2}} + \frac{(0.01 \times 10^{-3})}{5 \times 10^{-3}}$$

$$\frac{\Delta Y}{Y} = 0.0155$$

The fractional error in the measurement of the Young's modulus is 0.0155.

- 07** The diameter of a spherical bob is measured using a Vernier callipers. 9 divisions of the main scale, in the vernier calipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8th division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.04 cm, then the radius of the bob is $\times 10^{-2}$ cm.

[2021, 31 Aug Shift-II]

Ans. (52)

Given, 9 divisions of main scale are equal to 10 divisions of Vernier scale.

i.e. 9 MSD = 10 VSD

$$\Rightarrow \text{VSD} = \frac{9}{10} \text{ MSD} \quad \dots(i)$$

Size of 1 main scale division,

1 MSD = 1 mm

Now, least count, LC = 1MSD – 1VSD

....(ii)

Using Eqs. (i) and (ii), we get

$$\text{LC} = 1\text{MSD} - \frac{9}{10} \text{MSD} = \frac{1}{10} \text{MSD} = \frac{1}{10} \text{mm}$$

While measuring the diameter of bob.

Main Scale Reading, MSR = 10 mm

Vernier Scale Reading, VSR = 8

Zero error, $e = 0.04 \text{ cm}$

$$\text{Now, diameter, } d = [\text{MSR} + \text{LC} \times \text{VSR}] - e = \left(10 \text{ mm} + \frac{1}{10} \times 8 \text{ mm}\right) - 0.04 \text{ cm}$$

$$= (10.8) \text{ mm} - 0.04 \text{ cm}$$

$$= 1.08 \text{ cm} - 0.04 \text{ cm} = 1.04 \text{ cm}$$

$$\text{Radius, } r = \frac{d}{2} = \frac{1.04}{2} \text{ cm} = 0.52 \text{ cm}$$

$$= 52 \times 10^{-2} \text{ cm}$$

$\therefore$ Correct answer is 52.

- 08** If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is

[2021, 26 Aug Shift-II]

- (a) 86.4 s (b) 4.32 s
(c) 43.2 s (d) 8.64 s

Ans. (c)

Increase in length of pendulum is 0.1%.

$$\text{i.e. } \frac{\Delta L}{L} \times 100 = 0.1$$

Time period of pendulum is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Here, 2π and g are constant.

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \frac{\Delta L}{L} \times 100$$

$$\Rightarrow \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times 0.1$$

$$\Rightarrow \Delta T = 0.05 \times \frac{T}{100}$$

In one single day, the time in seconds is

$$T = 24 \times 60 \times 60 = 86400 \text{ s}$$

$$\therefore \Delta T = 0.05 \times \frac{86400}{100} = 43.2 \text{ s}$$

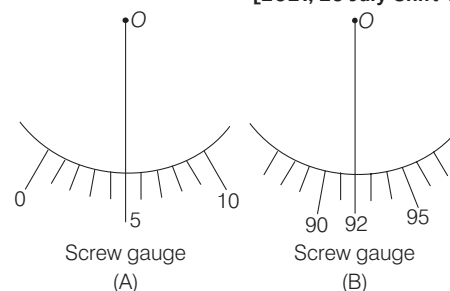
Thus, the error in time per day is 43.2 s.

- 09** Student A and student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is

[Figure shows position of reference O when jaws of screw gauge are closed]

Given, pitch = 0.1 cm.

[2021, 25 July Shift-I]



Ans. (13)

Given,

Number of circular scale division = 100

True value of radius, $R = 0.322 \text{ cm}$

Least count (LC) = $0.1 \text{ cm}/100 = 0.001 \text{ cm}$

Now,

As we know that,

True value (TV) = Main scale reading (MSR) + Circular scale reading (CSR) + Error

where, Error = $n \text{th division} \times \text{LC}$

Now,

For student A,

$$0.322 = 0.300 + \text{CSR}_A + 5 \times 0.001$$

$$CSR_A = 0.322 - 0.300 - 0.005 = 0.017$$

and for student B,

$$0.322 = 0.2 + CSR_B + 92 \times 0.001$$

$$CSR_B = 0.322 - 0.2 - 0.092 = 0.030$$

$$\therefore \text{Difference} = CSR_B - CSR_A \\ = 0.030 - 0.017 = 0.013 \text{ cm}$$

Now, division on circular scale

$$= \frac{0.013}{0.001} = 13$$

10 Three students S_1 , S_2 and S_3

perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of pendulum (cm)	No. of oscillations (n)	Total time for n oscillations (s)	Time period (s)
1.	64.0	8	128.0	16.0
2.	64.0	4	64.0	16.0
3.	20.0	4	36.0	9.0

(Least count of length = 0.1m, least count for time = 0.1s)

If E_1 , E_2 and E_3 are the percentage errors in g for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student number

[2021, 22 July Shift-II]

Ans. (1)

Given, observation of three students named S_1 , S_2 and S_3 .

Let $l_1, l_2, l_3, T_1, T_2, T_3$ be the measured length and time period by student S_1, S_2 and S_3 respectively,

and E_1, E_2 and E_3 be the errors in g and $\Delta T = 0.1 \text{ s}, \Delta l = 0.1 \text{ m}$

As we know that,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

On squaring both sides,

$$T^2 = 4\pi^2 \frac{l}{g}$$

By using concept of relative error,

$$\frac{\Delta g}{g} = \frac{2\Delta T}{T} + \frac{\Delta l}{l}$$

$$E_1 = 2 \times \frac{0.1}{16} + \frac{0.1}{64}$$

$$= 0.0125 + 0.0016$$

$$= 0.0141$$

$$E_2 = 2 \times \frac{0.1}{16} + \frac{0.1}{64} = 0.0141$$

$$E_3 = 2 \times \frac{0.1}{9} + \frac{0.1}{20} = 0.027$$

From above error calculations $E_1 = E_2$

But, since number of oscillations in E_1 is more, so more precise observation, less error.

$\therefore$ Student number 1 will have least error.

11 The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement, it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is cm.

[2021, 17 March Shift-I]

- (a) 8.36 (b) 8.54
(c) 8.58 (d) 8.56

Ans. (b)

Given, positive zero error

$$= 0.2 \text{ mm} = 0.02 \text{ cm}$$

$$[\therefore \text{Least count LC} = 0.01 \text{ cm}]$$

Main scale reading = 8.5 cm

Vernier scale reading

$$= \text{Vernier scale coincidence}$$

$$\times \text{Least count}$$

$$= 6 \times 0.01 = 0.06 \text{ cm}$$

Final reading = Main scale reading +

Vernier scale reading - Zero error

$$= 8.5 + 0.06 - 0.02 = 8.54 \text{ cm}$$

12 In order to determine the Young's modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's modulus determined by this experiment?

[2021, 16 March Shift-II]

- (a) 0.14% (b) 0.9% (c) 9% (d) 1.4%

Ans. (d)

$$\text{Young's modulus, } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{\Delta l A}$$

$$\Rightarrow Y = \frac{mgL}{\pi R^2 l} \quad \dots (i)$$

$$[\therefore F = mg \text{ and } A = \pi R^2]$$

To determine the fractional errors, we can write

Eq. (i) as follows

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + \frac{2 \cdot \Delta R}{R} + \frac{\Delta l}{l}$$

$$\Rightarrow \frac{\Delta Y}{Y} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{\Delta L}{L} \times 100 \\ + \frac{2\Delta R}{R} \times 100 + \frac{\Delta l}{l} \times 100$$

$$\Rightarrow \frac{\Delta Y}{Y} \times 100 = 100 \left[\frac{\Delta m}{m} + \frac{\Delta L}{L} + \frac{2\Delta R}{R} + \frac{\Delta l}{l} \right]$$

$$= 100 \left[\frac{1}{1000} + \frac{1}{1000} + 2 \left(\frac{0.001}{0.2} \right) + \frac{0.001}{0.5} \right]$$

$$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 14\%$$

13 One main scale division of a vernier callipers is a cm and n th division of the vernier scale coincide with $(n-1)$ th division of the main scale. The least count of the callipers (in mm) is

[2021, 16 March Shift-I]

$$(a) \frac{10na}{(n-1)} \quad (b) \frac{10a}{(n-1)}$$

$$(c) \left(\frac{n-1}{10n} \right) a \quad (d) \frac{10a}{n}$$

Ans. (d)

According to the question,

One division of main scale reading

$$= a \text{ cm}$$

n th vernier scale division

$$= (n-1) \text{th main scale division}$$

$\therefore$ One division of vernier scale reading

$$= \frac{(n-1) \times a}{n} \quad \dots (i)$$

We know that,

Least count (LC) =

[1 main scale division - 1

vernier scale division] cm

$$= a - \frac{(n-1)a}{n} \quad [\text{using Eq. (i)}]$$

$$= \frac{a(n-n+1)}{n} = \frac{a}{n} \text{ cm} = \frac{a}{n} \times 10 \text{ mm}$$

$$\Rightarrow LC = \frac{10a}{n} \text{ mm}$$

14 The resistance $R = \frac{V}{I}$, where

$V = (50 \pm 2) \text{ V}$ and $I = (20 \pm 0.2) \text{ A}$. The percentage error in R is $x\%$. The value of x to the nearest integer is.....

[2021, 16 March Shift-I]

Ans. (5)

$$\text{Given, } R = \frac{V}{I} \quad \dots(i)$$

$$\text{where, } V = (50 \pm 2) \text{ V}$$

$$I = (20 \pm 0.2) \text{ A}$$

From Eq. (i)

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$\% \text{ error in } R = \left[\frac{2}{50} \times 100 + \frac{0.2}{20} \times 100 \right] \%$$

$$= [2 \times 2 + 0.2 \times 5] \%$$

$$= 5 \%$$

Comparing with the given value in the question i.e., $x\%$, the value of $x = 5$.

- 15** A large number of water drops, each of radius r , combine to have a drop of radius R . If the surface tension is T and mechanical equivalent of heat is J , the rise in heat energy per unit volume will be

[2021, 26 Feb Shift-I]

$$(a) \frac{2T}{J} \left(\frac{1}{r} - \frac{1}{R} \right)$$

$$(b) \frac{2T}{rJ}$$

$$(c) \frac{3T}{rJ}$$

$$(d) \frac{3T}{J} \left(\frac{1}{r} - \frac{1}{R} \right)$$

Ans. (d)Given, radius of small drop = r Radius of big drop = R Surface tension = T and mechanical equivalent of heat = J

As, small drops combine to form big drop.

 $\therefore$ Volume of big drop (V_B) = $n \times$ Volume of small drop (V_S)

$$\Rightarrow \frac{4}{3} \pi R^3 = n \cdot \frac{4}{3} \pi r^3$$

$$\Rightarrow nr^3 = R^3$$

$$\Rightarrow r = \frac{R}{n^{1/3}} \quad \dots(i)$$

Surface energy of small drop

(E_S) = Surface tension (T) $\times$ Area (A)

$$\Rightarrow E_S = n \times 4 \pi r^2 T$$

$$\text{and } E_B = 4 \pi R^2 T$$

Now, change in energy will be

$$\Delta E = E_B - E_S = 4 \pi T (R^2 - R^2)$$

 $\therefore$ Heat energy per unit volume

$$= \frac{\Delta E}{V} = \frac{4 \pi T (nr^2 - R^2)}{J \times \frac{4}{3} \pi R^3}$$

$$= \frac{3T}{J} \left(\frac{nr^2}{R^3} - \frac{1}{R} \right)$$

$$= \frac{3T}{J} \left(n \frac{R^2}{n^{2/3} R^3} - \frac{1}{R} \right)$$

$$= \frac{3T}{J} \left[\frac{n^{1/3}}{R} - \frac{1}{R} \right] \quad [\text{from Eq. (i)}]$$

$$= \frac{3T}{J} \left[\frac{1}{r} - \frac{1}{R} \right]$$

- 16** The period of oscillation of a simple pendulum is $T = 2\pi \sqrt{\frac{L}{g}}$. Measured

value of L is 1.0 m from metre scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of g will be

[2021, 24 Feb Shift-II]

$$(a) 1.13 \%$$

$$(b) 1.03 \%$$

$$(c) 1.33 \%$$

$$(d) 1.30 \%$$

Ans. (a)

Given,

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \dots(i)$$

where, time period, $T = 1.95$ sLength of string, $L = 1$ mAcceleration due to gravity = g Error in time period, $\Delta T = 0.01$ s = 10^{-2} sError in length, $\Delta L = 1$ mm = 1×10^{-3} m

Squaring Eq. (i) on both sides, we get

$$T^2 = 4\pi^2 \frac{L}{g}$$

$$\Rightarrow g = 4\pi^2 \frac{L}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T} = \frac{10^{-3}}{1} + \frac{2 \times 10^{-2}}{1.95}$$

$$= 10^{-3} + 1.025 \times 10^{-2}$$

$$= 10^{-3} + 10.25 \times 10^{-3}$$

$$= 11.25 \times 10^{-3}$$

$$\therefore \Delta g / g \times 100 = 11.25 \times 10^{-3} \times 10^2$$

$$= 1.125 \approx 1.13 \%$$

- 17** A physical quantity z depends on four observables a, b, c and d , as

$$z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}. \text{ The percentages of}$$

error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

[2020, 5 Sep Shift-I]

$$(a) 13.5 \%$$

$$(b) 16.5 \%$$

$$(c) 14.5 \%$$

$$(d) 12.25 \%$$

Ans. (c)

Given,

$$z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$$

According to question,

$$\% \text{ error in } z = (2)\% \text{ error in } a + \left(\frac{2}{3}\right)\% \text{ error}$$

$$\text{in } b + \left(\frac{1}{2}\right)\% \text{ error in } c + (3)\% \text{ error in } d$$

$$\frac{\Delta z}{z} = 2 \frac{\Delta a}{a} + \frac{2}{3} \frac{\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + 3 \frac{\Delta d}{d}$$

$$= 2 \times 2\% + \frac{2}{3} \times 1.5\% + \frac{1}{2} \times 4\% + 3 \times 2.5\%$$

$$= 14.5 \%$$

- 18** The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100}\right)\%$. If the

relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is

[2020, 6 Sep Shift-I]**Ans. (1050)**

Given, relative error in mass,

$$\frac{\Delta m}{m} \times 100 = 6\%$$

Relative error in diameter,

$$\frac{\Delta d}{d} \times 100 = 1.5\%$$

Density of sphere,

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{M}{\frac{4}{3} \pi r^3} = \frac{M}{\frac{4}{3} \pi \left(\frac{d}{2}\right)^3}$$

$$\rho = \frac{6}{\pi} M d^{-3} \text{ or } \rho \propto M d^{-3}$$

For maximum error in density,

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + 3 \times \frac{\Delta d}{d}$$

$$= 6\% + 3 \times 1.5\%$$

$$\frac{\Delta \rho}{\rho} \times 100 = 10.5\%$$

$$\Rightarrow \frac{1050}{100} \% = \frac{x}{100} \% \quad (\text{given})$$

$$\therefore x = 1050$$

- 19** For the four sets of three measured physical quantities as given below. Which of the following options is correct?

[2020, 9 Jan Shift-II]

- (i) $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$
 (ii) $A_2 = 24.44, B_2 = 16.082, C_3 = 240.2$
 (iii) $A_3 = 25.2, B_3 = 19.2812, C_3 = 236.183$

(iv) $A_4 = 25, B_4 = 236.191, C_4 = 19.5$

(a) $A_1 + B_1 + C_1 < A_3 + B_3 + C_3$
 $< A_2 + B_2 + C_2 < A_4 + B_4 + C_4$

(b) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1$

$$= A_2 + B_2 + C_2$$

$$= A_3 + B_3 + C_3$$

(c) $A_4 + B_4 + C_4 < A_1 + B_1 + C_1$

$$= A_3 + B_3 + C_3 < A_2 + B_2 + C_2$$

(d) $A_1 + B_1 + C_1 = A_2 + B_2 + C_2$

$$= A_3 + B_3 + C_3 = A_4 + B_4 + C_4$$

Ans. (*)

Given, $A_1 = 24.36, B_1 = 0.0724, C_1 = 256.2$

$$\therefore A_1 + B_1 + C_1 = 280.6324$$

As sum contains same number of digits after decimal as present in the number having the least number of decimal places.

So, $A_1 + B_1 + C_1 = 280.6$

Similarly,

$$A_2 + B_2 + C_2 = 280.7$$

$$A_3 + B_3 + C_3 = 280.7 \quad (\text{rounded off})$$

$$A_4 + B_4 + C_4 = 280.7 \quad (\text{rounded off})$$

So, $A_1 + B_1 + C_1 < A_2 + B_2 + C_2$

$$= A_3 + B_3 + C_3$$

$$= A_4 + B_4 + C_4$$

None of the options is matching with result.

- 20** In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to

[2019, 8 April Shift-II]

- (a) 0.7% (b) 6.8%
 (c) 3.5% (d) 0.2%

Ans. (b)

Relation used for finding acceleration due to gravity by using a pendulum is

$$g = \frac{4\pi^2 l}{T^2}$$

So, fractional error in value of g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \quad \dots(i)$$

Given, $\Delta l = 0.1 \text{ cm}, l = 55 \text{ cm}, \Delta T = 1 \text{ s}$ and T for 20 oscillations = 30 s

Substituting above values in Eq. (i), we

get $\frac{\Delta g}{g} = \frac{0.1}{55} + 2 \times \frac{1}{30}$

Hence, percentage error in g is

$$= \frac{\Delta g}{g} \times 100 = \frac{10}{55} + \frac{20}{3} = 6.8\%$$

- 21** In the density measurement of a cube, the mass and edge length are measured as $(10.00 \pm 0.10) \text{ kg}$ and $(0.10 \pm 0.01) \text{ m}$, respectively. The error in the measurement of density is

[2019, 9 April Shift-I]

- (a) 0.01 kg/m^3 (b) 0.10 kg/m^3
 (c) 0.07 kg/m^3 (d) 0.31 kg/m^3

Ans. (*)

Given, mass = $(10.00 \pm 0.10) \text{ kg}$

Edge length = $(0.10 \pm 0.01) \text{ m}$

Error in mass, $\frac{\Delta M}{M} = \frac{0.1}{10} \quad \dots(i)$

and error in length, $\frac{\Delta l}{l} = \frac{0.01}{0.1} \quad \dots(ii)$

Density of the cube is given by

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{l^3}$$

$\therefore$ Permissible error in density is

$$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + 3 \frac{\Delta l}{l} \quad \dots(iii)$$

Substituting the value from Eqs. (i) and (ii) in Eq. (iii), we get

$$\frac{\Delta \rho}{\rho} = \frac{0.1}{10} + 3 \times \frac{0.01}{0.1} = \frac{1}{100} + \frac{3}{10} = \frac{31}{100}$$

$$\Rightarrow \frac{\Delta \rho}{\rho} = \frac{31}{100} = 0.31$$

Since, $\frac{\Delta \rho}{\rho}$ should be unitless quantity.

But there is no option with unitless error. Hence, no option is correct.

- 22** The area of a square is 5.29 cm^2 . The area of 7 such squares taking into account the significant figures is

[2019, 9 April Shift-II]

- (a) 37.030 cm^2 (b) 37.0 cm^2
 (c) 37.03 cm^2 (d) 37 cm^2

Ans. (c)

Area of 1 square = 5.29 cm^2

Area of seven such squares

= 7 times addition of area of 1 square

$$= 5.29 + 5.29 + 5.29 \dots 7 \text{ times}$$

$$= 37.03 \text{ cm}^2$$

As we know that, if in the measured values to be added/subtracted the least

number of significant digits after the decimal is n .

Then, in the sum or difference also, the number of significant digits after the decimal should be n .

Here, number of digits after decimal in 5.29 is 2, so our answer also contains only two digits after decimal point.

$$\therefore \text{Area required} = 37.03 \text{ cm}^2$$

- 23** The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100, respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is

[2019, 9 Jan Shift-II]

- (a) 5.950 mm (b) 5.725 mm
 (c) 5.755 mm (d) 5.740 mm

Ans. (c)

For a measuring device, the least count is the smallest value that can be measured by measuring instrument.

Least count

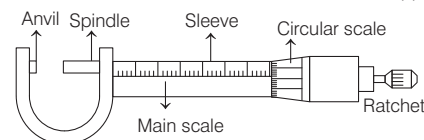
$$= \frac{\text{Minimum reading on main - scale}}{\text{Total divisions on the scale}}$$

Here, screw gauge is used for measurement therefore,

$$LC = \frac{\text{Pitch}}{\text{number of division}}$$

$$LC = \frac{0.5}{100} \text{ mm}$$

$$LC = 5 \times 10^{-3} \text{ mm} \quad \dots(i)$$



According to question, the zero line of its circular scale lies 3 division below the mean line and the readings of main scale = 5.5 mm

The reading of circular scale = 48

then the actual value is given by

actual value of thickness (t) = (main scale reading) + (circular scale reading + number of division below mean line) $\times$ LC

$$\Rightarrow t = 5.5 \text{ mm} + (48 + 3) \times 5 \times 10^{-3} \text{ mm}$$

$$\Rightarrow t = 5.755 \text{ mm}$$

- 24** The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures?

[2019, 10 Jan Shift-II]

- (a) 4300 ± 80 cm³
 (b) 4260 ± 80 cm³
 (c) 4264.4 ± 81.0 cm³
 (d) 4264 ± 81 cm³

Ans. (b)

Volume of a cylinder of radius ' r ' and height ' h ' is given by

$$V = \pi r^2 h$$

or $V = \frac{1}{4} \pi D^2 h$, where D is the diameter of circular surface. Here, $D = 12.6$ cm and $h = 34.2$ cm

$$\Rightarrow V = \frac{\pi}{4} \times (12.6)^2 \times (34.2)$$

$$V = 4262.22 \text{ cm}^3$$

$$V = 4260 \text{ (in three significant numbers)}$$

Now, error calculation can be done as

$$\frac{\Delta V}{V} = 2 \left(\frac{\Delta D}{D} \right) + \frac{\Delta h}{h}$$

$$= \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$$

$$\Rightarrow \frac{\Delta V}{V} = 0.0158 + 0.0029$$

$$\Rightarrow \Delta V = (0.01879) \times (4262.22)$$

$$\Rightarrow \Delta V = 79.7 \approx 80 \text{ cm}^3$$

$\therefore$ For proper significant numbers, volume reading will be

$$V = 4260 \pm 80 \text{ cm}^3$$

- 25** The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure $5 \mu\text{m}$ diameter of a wire is

[2019, 12 Jan Shift-I]

- (a) 50 (b) 200
 (c) 500 (d) 100

Ans. (b)

In a screw gauge,

Least count

$$= \frac{\text{Measure of 1 main scale division (MSD)}}{\text{Number of division on circular scale}}$$

Here, minimum value to be measured/least count is $5 \mu\text{m}$.

$$= 5 \times 10^{-6} \text{ m}$$

$\therefore$ According to the given values,

$$5 \times 10^{-6} = \frac{1 \times 10^{-3}}{N}$$

$$\text{or } N = \frac{10^{-3}}{5 \times 10^{-6}} = \frac{1000}{5}$$

$$= 200 \text{ divisions}$$

- 26** The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

[JEE Main 2018]

- (a) 2.5% (b) 3.5%
 (c) 4.5% (d) 6%

Ans. (c)

$$\therefore \text{Density, } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} \text{ or } \rho = \frac{M}{L^3}$$

$$\Rightarrow \text{Error in density } \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$$

So, maximum % error in measurement of ρ is

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{3\Delta L}{L} \times 100$$

$$\text{or \% error in density} = 1.5 + 3 \times 1$$

$$\% \text{ error} = 4.5\%$$

- 27** The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$ m. Using $g = 9.80 \text{ m/s}^2$ and the simplified relation

$$T = \frac{r h g}{2} \times 10^3 \text{ N/m, the possible error}$$

in surface tension is closest to

[JEE Main 2017]

- (a) 1.5% (b) 2.4%
 (c) 10% (d) 0.15%

Ans. (a)

By ascent formula, we have surface tension,

$$T = \frac{r h g}{2} \times 10^3 \frac{\text{N}}{\text{m}} = \frac{d h g}{4} \times 10^3 \frac{\text{N}}{\text{m}} \left(\because r = \frac{d}{2} \right)$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \text{ [given, } g \text{ is constant]}$$

So, percentage

$$= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h} \right) \times 100$$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}} \right) \times 100$$

$$= 1.5\%$$

$$\therefore \frac{\Delta T}{T} \times 100 = 1.5\%$$

- 28** A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 92 s and 95 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be

[JEE Main 2016]

- (a) (92 ± 2) s (b) (92 ± 5) s
 (c) (92 ± 18) s (d) (92 ± 3) s

Ans. (b)

Arithmetic mean time of an oscillating

$$\text{simple pendulum} = \frac{\sum x_i}{N}$$

$$= \frac{90 + 91 + 92 + 95}{4}$$

$$= 92 \text{ s}$$

Mean deviation of a simple pendulum

$$= \frac{\sum |\bar{x} - x_i|}{N} = \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1 s. Thus, the reported mean time of an oscillating simple pendulum = (92 ± 2) s

- 29** A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

[JEE Main 2014]

- (a) A meter scale
 (b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
 (c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
 (d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Ans. (b)

If student measure 3.50 cm, it means that there is an uncertainty of order 0.01 cm.

For vernier scale with 1 MSD

$$= 1 \text{ mm and } 9 \text{ MSD}$$

$$= 10 \text{ VSD}$$

$$\therefore \text{LC of VC} = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= \frac{1}{10} \left(1 - \frac{9}{10} \right) = \frac{1}{100} \text{ cm}$$

- 30** The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring $\pm 0.01V$ while measuring the current of 5 mA at 300K, what will be the error in the value of current in mA? [JEE Main 2013]

- (a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

Ans. (a)

$$\begin{aligned} \text{Given, } I &= (e^{1000V/T} - 1) \text{ mA} \\ dV &= \pm 0.01V, T = 300K \\ I &= 5 \text{ mA} \Rightarrow I = e^{1000V/T} - 1 \\ I + 1 &= e^{1000V/T} \end{aligned}$$

Taking log on both sides, we get

$$\begin{aligned} \log(I + 1) &= \frac{1000V}{T} \\ \Rightarrow \frac{d(I + 1)}{I + 1} &= \frac{1000}{T} dV \Rightarrow \frac{dI}{I + 1} = \frac{1000}{T} dV \\ \Rightarrow dI &= \frac{1000}{T} \times (I + 1) dV \\ dI &= \frac{1000}{300} \times (5 + 1) \times 0.01 = 0.2 \text{ mA} \end{aligned}$$

So, error in the value of current is 0.2 mA.

- 31** A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is [AIEEE 2012]

- (a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree

Ans. (c)

1 Vernier scale division = $\frac{29}{30}$ main scale division

$$1 \text{ VSD} = \frac{29}{30} \times 0.5^\circ = \left(\frac{29}{60}\right)^\circ$$

Thus, least count = 1 MSD - 1 VSD

$$= \left(\frac{1}{2}\right)^\circ - \left(\frac{29}{60}\right)^\circ = \left(\frac{1}{60}\right)^\circ$$

$$\begin{aligned} \therefore \text{Reading} &= \text{Main scale reading} \\ &\quad + \text{Vernier scale reading} \\ &= \text{MSR} + n \times \text{LC} \\ &= 58.5^\circ + 9 \times (1/60)^\circ \\ &= 58.65^\circ \end{aligned}$$

- 32** Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is [AIEEE 2012]

- (a) 6% (b) zero (c) 1% (d) 3%

Ans. (a)

From Ohm's law, $R = V/I$

$$\begin{aligned} \text{By error method, } \frac{\Delta R}{R} &= \frac{\Delta V}{V} + \frac{\Delta I}{I} \\ &= 3\% + 3\% = 6\% \end{aligned}$$

- 33** The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are [AIEEE 2010]

- (a) 5, 1, 2 (b) 5, 1, 5
(c) 5, 5, 2 (d) 4, 4, 2

Ans. (a)

The reliable digit plus the first uncertain digit is known as significant figures.

For the number 23.023, all the non-zero digits are significant, hence 5.

For the number 0.0003, number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant, hence 1.

For the number 2.1×10^{-3} , significant figures are 2.

- 34** In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half a degree ($= 0.5^\circ$), then the least count of the instrument is [AIEEE 2009]

- (a) one minute
(b) half minute
(c) one degree
(d) half degree

Ans. (a)

Least count

$$\begin{aligned} &= \frac{\text{Value of main scale division}}{\text{Number of divisions on vernier scale}} \\ &= \frac{1}{30} \text{ MSD} = \frac{1}{30} \times \frac{1^\circ}{2} \\ &= \frac{1^\circ}{60} = 1 \text{ min} \end{aligned}$$

- 35** A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 ms^{-1} . The magnitude of its momentum is recorded as

[AIEEE 2008]

- (a) 17.6 kg ms^{-1} (b) $17.565 \text{ kg ms}^{-1}$
(c) 17.56 kg ms^{-1} (d) 17.57 kg ms^{-1}

Ans. (a)

So, momentum, $p = mv = 17.565 \text{ kg ms}^{-1}$ where $m = 3.513 \text{ kg}$ and $v = 5.00 \text{ ms}^{-1}$

As the number of significant digits in m is 4 and in v is 3, so, p must have 3 (minimum) significant digits.

Hence, $p = 17.6 \text{ kg ms}^{-1}$

TOPIC 3 Dimensions

- 36** Which of the following equations is dimensionally incorrect?

Where, t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, V = volume, p = pressure, W = work done, τ = torque, ϵ = permittivity, E = electric field, J = current density, L = length.

[2021, 31 Aug Shift-I]

- (a) $V = \frac{\pi \rho a^4}{8\eta L}$ (b) $h = \frac{2s \cos \theta}{\rho g}$
(c) $J = \epsilon \frac{\partial E}{\partial t}$ (d) $W = \tau \theta$

Ans. (a)

As we know that,

Dimensional formula of volume
 $= [M^0 L^3 T^0]$... (i)

Since, $F = 6\pi\eta rv$

$$\therefore \eta \propto \frac{F}{rv}$$

where, η is viscosity and F is force.

$$\therefore [\eta] = \frac{[M L T^{-2}]}{[L \cdot L T^{-1}]} = [M L^{-1} T^{-1}]$$

$$\begin{aligned} \text{So, } \left[\frac{\pi \rho a^4}{8\eta L} \right] &= \frac{[M L^{-1} T^{-2}][L^4]}{[M L^{-1} T^{-1}][L^1]} \\ &= [M^0 L^3 T^{-1}] \quad \dots (ii) \end{aligned}$$

Since, Eq. (i) is not equal to Eq (ii), so option (a) is wrong.

Now, since formula of capillary rise in tube, $h = \frac{2s \cos \theta}{\rho g r}$

Dimensional formula of LHS part,

$$\therefore [h] = [L]$$

Dimensional formula of RHS part

$$= \frac{[s]}{[p][g][r]} = \frac{[MT^{-2}]}{[ML^{-3}][LT^{-2}][L]} = [L]$$

Hence, $h = \frac{2s \cos \theta}{\rho r g}$ is dimensionally correct.

correct.

So, option (b) will also be dimensionally correct.

In option (c),

$$J = \epsilon \frac{dE}{dt} \quad \dots(iii)$$

$$\Rightarrow J = \epsilon \frac{E}{t}$$

Dimension of current density J is calculated as

$$\text{Since, } J = \frac{I}{A}$$

$$\therefore [J] = \frac{[I]}{[A]} = \frac{[A]}{[L^2]}$$

$$\Rightarrow [J] = [AL^{-2}] \quad \dots(iv)$$

Again, we know that

$$E = \frac{1}{4\pi\epsilon} \cdot \frac{q}{r^2}$$

$$\Rightarrow \epsilon E = \frac{1}{4\pi} \cdot \frac{q}{r^2} \Rightarrow \frac{\epsilon E}{t} = \frac{1}{4\pi} \cdot \frac{q}{tr^2}$$

$$\Rightarrow \left[\frac{\epsilon E}{t} \right] = \frac{[q]}{[t][r^2]} = \frac{[AT]}{[T][L^2]}$$

$$\Rightarrow \left[\frac{\epsilon E}{t} \right] = [AL^{-2}] \quad \dots(v)$$

Form Eqs. (iv) and (v), we see that Eq. (iii) is dimensionally correct.

In option (d) $W = \tau \theta$

and $\tau = r \times F$

So, dimensional formula of

$$[\tau][\theta] = [r][F] = [L][MLT^{-2}]$$

$$= [ML^2T^{-2}] = [W]$$

- 37** If velocity $[v]$, time $[T]$ and force $[F]$ are chosen as the base quantities, the dimensions of the mass will be

[2021, 31 Aug Shift-II]

- (a) $[FT^{-1}v^{-1}]$ (b) $[FTv^{-1}]$
(c) $[FT^2v]$ (d) $[FvT^{-1}]$

Ans. (b)

When the velocity (v), time (T) and force (F) are chosen as base quantities. Then, mass is given by

$$m \propto v^x T^y F^z \quad \dots(i)$$

Using dimensional formula of all quantities,

$$[ML^0T^0] = [LT^{-1}]^x [T]^y [MLT^{-2}]^z$$

$$[M^1L^0T^0] = [M^zL^{x+z}T^{-x+y-2z}]$$

Comparing the powers of dimensions on both sides, we get

$$z = 1$$

$$x + z = 0 \text{ and } -x + y - 2z = 0$$

$$\Rightarrow x + 1 = 0 \Rightarrow x = -1$$

$$\text{and } -(-1) + y - 2(1) = 0$$

$$\Rightarrow 1 + y - 2 = 0 \Rightarrow y = 1$$

$$\text{Substituting these values in Eq. (i), we get}$$

$$m \propto v^{-1} T^1 F^1$$

$$\Rightarrow m = [FTv^{-1}]$$

- 38.** If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density?

[2021, 27 Aug Shift-II]

- (a) $[FL^{-4}T^2]$ (b) $[FL^{-3}T^2]$
(c) $[FL^{-5}T^2]$ (d) $[FL^{-3}T^3]$

Ans. (a)

As we know that, the dimensional formula of

$$\text{density } [D] = [ML^{-3}T^0]$$

Since, dimensional formula of force $[F] = [MLT^{-2}]$

Dimensional formula of length $[L] = [M^0L^1T^0]$

Dimensional formula of time $[T] = [M^0L^0T^1]$

$$\therefore [D] = [F]^a [L]^b [T]^c$$

$$\Rightarrow [ML^{-3}T^0] = [MLT^{-2}]^a [M^0L^1T^0]^b [M^0L^0T^1]^c$$

$$= [M^aL^{a+b}T^{-2a+c}]$$

Comparing powers of dimensions on both sides, we get

$$a = 1$$

$$a + b = -3$$

$$\Rightarrow b = -3 - 1 = -4$$

$$\text{and } -2a + c = 0$$

$$c = 2a$$

$$\Rightarrow c = 2 \times 1 = 2$$

$\therefore$ Dimensional formula of density will be $[F^1L^{-4}T^2]$.

- 39** Which of the following is not a dimensionless quantity?

[2021, 27 Aug Shift-I]

- (a) Relative magnetic permeability (μ_r)
(b) Power factor
(c) Permeability of free space (μ_0)
(d) Quality factor

Ans. (c)

Relative magnetic permeability (μ_r) It is the ratio of permeability of medium to

the permeability of free space i.e.,

$$\mu_r = \frac{\mu_m}{\mu_0}$$

As, it is a ratio of permeabilities. So, it is unitless and dimensionless quantity.

Power factor ($\cos \phi$) It is cosine of phase difference between alternating current and alternating voltage. Hence, it has only a numerical value, so it is also a unitless and dimensionless quantity.

Permeability of free space (μ_0) It is the ratio of magnetising field induction (B) to magnetising field intensity (H)

$$\text{i.e. } \mu_0 = \frac{B}{H}$$

$$\Rightarrow [\mu_0] = \frac{[B]}{[H]} \quad \dots(i)$$

$$\text{Since, } H = nI$$

where, n = number of turns per unit length

and I = current

$$[H] = [n][I] = [L^{-1}][A] = [AL^{-1}]$$

We know that, force on a current carrying conductor in magnetic field B is given as

$$F = BIl$$

where, l = length of the conductor, B = magnetic field

and I = current.

$$\therefore B = \frac{F}{Il}$$

$$\Rightarrow [B] = \frac{[F]}{[I][l]} = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2}A^{-1}]$$

$\therefore$ From Eq. (i), we have

$$[\mu_0] = \frac{[MT^{-2}A^{-1}]}{[AL^{-1}]} = [MLT^{-2}A^{-2}]$$

Hence, permeability of free space is not a dimensionless quantity.

Quality factor It is the ratio of energy stored to the energy dissipated per cycle.

Quality factor

$$= \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$$

As, it is ratio of energies so, it will be unitless and dimensionless quantity.

- 40** Match List-I with List-II.

List-I		List-II
A. Magnetic induction	1.	$[ML^2T^{-2}A^{-1}]$
B. Magnetic flux	2.	$[ML^{-1}A]$
C. Magnetic permeability	3.	$[MT^{-2}A^{-1}]$
D. Magnetisation	4.	$[MLT^{-2}A^{-2}]$

Choose the most appropriate answer from the options given below. **[2021, 26 Aug Shift-II]**

- | | | | | |
|-----|---|---|---|---|
| | A | B | C | D |
| (a) | 2 | 4 | 1 | 3 |
| (b) | 2 | 1 | 4 | 3 |
| (c) | 3 | 2 | 4 | 1 |
| (d) | 3 | 1 | 4 | 2 |

Ans. (*)

Magnetic induction

Magnetic force of induction can be given as

$$F = qvB \Rightarrow B = \frac{F}{qv}$$

$$[B] = \frac{[F]}{[q][v]} = \frac{[MLT^{-2}]}{[AT][LT^{-1}]} = [MT^{-2}A^{-1}]$$

Magnetic Flux Magnetic flux is the product of magnetic induction and area.

$$\begin{aligned}\phi &= B \cdot A \\ [\phi] &= [B][A] = [MT^{-2}A^{-1}][L^2] \\ &= [ML^2T^{-2}A^{-1}]\end{aligned}$$

Magnetic permeability

We know that, magnetic field inside the solenoid.

$$\begin{aligned}B &= \mu_0 nI \\ \Rightarrow \mu_0 &= \frac{B}{nI}\end{aligned}$$

(where, n is number of turns per unit length)

$$\Rightarrow [\mu_0] = \frac{[B]}{[n][I]} = \frac{[MT^{-2}A^{-1}]}{[L^{-1}][A]} = [MLT^{-2}A^{-2}]$$

Magnetisation The magnetic dipole moment acquired per unit volume is known as magnetisation.

$$\begin{aligned}I &= \frac{M}{V} \\ [I] &= \frac{[L^2A]}{[L^3]} = [L^{-1}A]\end{aligned}$$

Hence, no option is correct.

- 41** If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimension of P in the formula $P = EL^2M^{-5}G^{-2}$ is

[2021, 26 Aug Shift-I]

- | | |
|----------------------|-------------------------|
| (a) $[M^0L^1T^0]$ | (b) $[M^{-1}L^{-1}T^2]$ |
| (c) $[M^1L^1T^{-2}]$ | (d) $[M^0L^0T^0]$ |

Ans. (d)

Dimension of energy, $E = [ML^2T^{-2}]$

Angular momentum, $L = [ML^2T^{-1}]$

Mass, $M = [M]$

and gravitational constant,

$$G = [M^{-1}L^3T^{-2}]$$

The dimension of P in formula, $P = EL^2M^{-5}G^{-2}$ is given as follows

$$\begin{aligned}[P] &= \frac{[E][L^2]}{[M^5][G^2]} \\ &= \frac{[ML^2T^{-2}][L^2]}{[M^5][M^{-2}L^6T^{-4}]} \\ &= [M^0L^0T^0]\end{aligned}$$

- 42** The force is given in terms of time t and displacement x by the equation

$$F = A \cos Bx + C \sin Dt$$

The dimensional formula of $\frac{AD}{B}$ is

[2021, 25 July Shift-II]

- | | |
|----------------------|----------------------|
| (a) $[M^0L^1T^{-1}]$ | (b) $[ML^2T^{-3}]$ |
| (c) $[M^1L^1T^{-2}]$ | (d) $[M^2L^2T^{-3}]$ |

Ans. (b)

Given,

Equation of force, $F = A \cos Bx + C \sin Dt$ where, x is displacement and t is time.

Now, Bx should be dimensionless.

$$\begin{aligned}\therefore [Bx] &= [M^0L^0T^0] \\ \Rightarrow [B][L] &= [M^0L^0T^0] \\ \Rightarrow [B] &= [M^0L^{-1}T^0] \quad \dots(i)\end{aligned}$$

$$\begin{aligned}\text{Similarly, } [Dt] &= [M^0L^0T^0] \\ \Rightarrow [D] &= \frac{[M^0L^0T^0]}{[t]} = [M^0L^0T^{-1}] \quad \dots(ii)\end{aligned}$$

$$\text{and } [A] = [F] = [MLT^{-2}] \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\begin{aligned}\frac{[AD]}{[B]} &= \frac{[MLT^{-2}][M^0L^0T^{-1}]}{[M^0L^{-1}T^0]} \\ &= \frac{[MLT^{-3}]}{[L^{-1}]} = [ML^2T^{-3}]\end{aligned}$$

- 43** If time (t), velocity (v) and angular momentum (I) are taken as the fundamental units, then the dimension of mass (m) in terms of t, v and I is **[2021, 20 July Shift-II]**

- | | |
|-------------------------|-------------------------|
| (a) $[t^{-1}v^1I^{-2}]$ | (b) $[t^1v^2I^{-1}]$ |
| (c) $[t^{-2}v^{-1}I^1]$ | (d) $[t^{-1}v^{-2}I^1]$ |

Ans. (d)

Let us suppose,

$$m \propto t^a$$

$$m \propto v^b$$

$$\text{and } m \propto I^c$$

$$\Rightarrow m \propto t^a v^b I^c$$

$$\text{or } m = kt^a v^b I^c \quad \dots(i)$$

where, k is any constant and $k = 1$

$\therefore$ We know that dimensions of

$$\text{Mass, } m = [M^1L^0T^0]$$

$$\text{Time, } t = [M^0L^0T^1]$$

$$\text{Velocity, } v = [M^0L^1T^{-1}]$$

$$\text{Angular momentum, } I = [M^1L^2T^{-1}]$$

Substituting all these dimensions in Eq. (i), we get

$$[M^1L^0T^0] = [M^0L^0T^1]^a [M^0L^1T^{-1}]^b [M^1L^2T^{-1}]^c$$

$$[M^1L^0T^0] = [M^cL^{b+2c}T^{a-b-c}]$$

On comparing powers, we get

$$c = 1,$$

$$b + 2c = 0$$

$$b + 2 = 0$$

$$\Rightarrow b = -2$$

$$a - b - c = 0$$

$$\Rightarrow a - (-2) - 1 = 0$$

$$\Rightarrow a = -1$$

$$\therefore \text{We can write } m \propto t^{-1}v^{-2}I^1$$

- 44** The entropy of any system is given by

$$S = \alpha^2 \beta \ln \left[\frac{\mu k R}{J \beta^2} + 3 \right]$$

where, α and β are the constants; μ, J, k and R are number of moles, mechanical equivalent of heat, Boltzmann constant and gas constant, respectively.

$$\left[\text{Take, } S = \frac{dQ}{T} \right] \quad \text{[2021, 20 July Shift-I]}$$

Choose the incorrect option.

- α and J have the same dimensions.
- S, β, k and μR have the same dimensions.
- S and α have different dimensions.
- α and k have the same dimensions.

Ans. (d)

Since, entropy of the system is given by

$$S = \alpha^2 \beta \ln \left[\frac{\mu k R}{J \beta^2} + 3 \right] \quad \dots(i)$$

$$\text{As, } S = \frac{Q}{\Delta T} \quad \text{[given]}$$

$$\Rightarrow [S] = \frac{[ML^2T^{-2}]}{[K]} \quad \dots(ii)$$

$$\therefore \text{Dimensions of } Q = [ML^2T^{-2}]$$

$$\text{Dimension of } T = [K]$$

$$\text{Boltzmann constant, } k = \frac{\text{energy}}{T}$$

$$[\therefore \text{Dimensions of energy} = [ML^2T^{-2}]]$$

$$\Rightarrow [k] = \frac{[ML^2T^{-2}]}{[K]} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we can write

$$[S] = [k] = \frac{[ML^2T^{-2}]}{[K]} \quad \dots(iv)$$

∴ Gas constant,

$$[R] = \frac{[\text{Energy}]}{[nT]} = \frac{[ML^2T^{-2}]}{[mol K]} \dots(v)$$

and mechanical equivalent of heat

$$[J] = [M^0L^0T^0] \dots(vi)$$

As, $[\mu kR] = [J\beta]^2$

Using Eqs. (iii), (v) and (vi), we get

$$\Rightarrow [mol] \times \frac{[ML^2T^{-2}]}{[K]} \times \frac{[ML^2T^{-2}]}{[mol K]} = [\beta^2]$$

$$\Rightarrow [\beta] = [ML^2T^{-2}K^{-1}] \dots(vii)$$

Using Eq. (i), we can write,

$$[\alpha^2] = \frac{[S]}{[\beta]} = \frac{[ML^2T^{-2}K^{-1}]}{[ML^2T^{-2}K^{-1}]}$$

$$\Rightarrow \alpha = [M^0L^0T^0] \dots(viii)$$

So, from Eqs. (iii) and (viii), we can say that α and k have different dimensions.

- 45** If C and V represent capacity and voltage respectively, then what are the dimensions of λ , where $\frac{C}{V} = \lambda$?

[2021, 26 Feb Shift-II]

- (a) $[M^{-2}L^{-3}T^6]$ (b) $[M^{-3}L^{-4}T^7]$
(c) $[M^{-1}L^{-3}T^{-7}]$ (d) $[M^{-2}L^{-4}T^7]$

Ans. (d)

Given, C and V represent capacity and voltage, respectively.

Dimensional formula of $[C] = [M^{-1}L^{-2}T^4I^2]$

Dimensional formula of $[V] = [ML^2T^{-3}I^{-1}]$

Therefore, dimensional formula of $[C/V]$

$$= \frac{[M^{-1}L^{-2}T^4I^2]}{[ML^2T^{-3}I^{-1}]} \\ = [M^{-2}L^{-4}T^7I^3]$$

- 46** In a typical combustion engine, the work done by a gas molecule is

given $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be

[2021, 26 Feb Shift-I]

- (a) $[MLT^{-2}]$ (b) $[M^0LT^0]$
(c) $[M^2LT^{-2}]$ (d) $[MLT^{-1}]$

Ans. (b)

Given, work done by gas molecule,

$$W = \alpha^2 \beta e^{-\beta x^2 / kT}$$

Here, x is displacement, k is Boltzmann constant, α and β are constants and T is temperature.

Dimensional formula of $[W] = [ML^2T^{-2}]$

∴ Dimensions of $[\alpha^2 \beta] = [ML^2T^{-2}]$

$$\Rightarrow \alpha = \left[\frac{ML^2T^{-2}}{\beta} \right]^{1/2} \dots(i)$$

The term $[e^{-\beta x^2 / kT}]$ should be dimensionless, i.e. $[M^0L^0T^0]$.

$$\Rightarrow \left[\frac{\beta x^2}{kT} \right] = [M^0L^0T^0]$$

$$\Rightarrow [\beta] = \frac{[k][T]}{[x^2]} \dots(ii)$$

Energy of gaseous molecule $(E) = \frac{7}{2} kT$

$$\Rightarrow [k] = [E]/[T] = [ML^2T^{-2}K^{-1}]$$

Substituting the value of k in Eq. (ii), we get

$$[\beta] = \frac{[ML^2T^{-2}K^{-1}][K]}{[L^2]} = [MT^{-2}]$$

Substituting the value of β in Eq. (i), we get

$$[\alpha] = \left[\frac{[ML^2T^{-2}]}{[MT^{-2}]} \right]^{1/2} = [M^0L^0T^0]$$

- 47** If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity

$$\frac{1}{4\pi\epsilon_0} \frac{|e|^2}{hc}$$

[2021, 25 Feb Shift-I]

- (a) $[MLT^0]$ (b) $[MLT^{-1}]$
(c) $[M^0LT^0]$ (d) $[LC^{-1}]$

Ans. (c)

Dimensional formula of $[e] = [IT]$

$$[h] = [ML^2T^{-1}]$$

$$[c] = [M^0LT^{-1}]$$

$$\left[\frac{1}{4\pi\epsilon_0} \right] = [M^1L^3T^{-4}I^{-2}]$$

where, $\frac{1}{4\pi\epsilon_0}$ is Coulomb's constant.

Therefore, dimensional formula of

$$\frac{1}{4\pi\epsilon_0} \frac{|e|^2}{hc} \\ = [M^1L^3T^{-4}I^{-2}] \cdot \frac{[IT]^2}{[M^1L^2T^{-1}][M^0LT^{-1}]} \\ = [M^0L^0T^0I^0]$$

or $[M^0L^0T^0]$

- 48** Match List-I with List-II

List-I	List-II
A. h (Planck's constant)	1. $[MLT^{-1}]$
B. E (kinetic energy)	2. $[ML^2T^{-1}]$
C. V (electric potential)	3. $[ML^2T^{-2}]$
D. P (linear momentum)	4. $[ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below.

[2021, 25 Feb Shift-I]

- A B C D
(a) 3 4 2 1
(b) 2 3 4 1
(c) 1 2 4 3
(d) 3 2 4 1

Ans. (b)

The dimensional formulae of given terms are

Planck's constant $(h) = [ML^2T^{-1}]$

Kinetic energy $(E) = [ML^2T^{-2}]$

Electric potential $(V) = [ML^2I^{-1}T^{-3}]$

Linear momentum $(p) = [MLT^{-1}]$

So, the correct match is

$A \rightarrow 2, B \rightarrow 3, C \rightarrow 4, D \rightarrow 1$.

- 49** The work done by a gas molecule in an isolated system is given by,

$W = \alpha \beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants.

Then, the dimensions of β will be

[2021, 24 Feb Shift-I]

- (a) $[M^2L^2T^2]$
(b) $[M^0LT^0]$
(c) $[MLT^{-2}]$
(d) $[ML^2T^{-2}]$

Ans. (c)

Given, work done, $W = \alpha \beta^2 e^{-\frac{x^2}{\alpha kT}}$

where, k is Boltzmann constant,

T is temperature and x is displacement.

We know that, $\frac{x^2}{\alpha kT}$ is a dimensionless quantity.

$$\therefore \left[\frac{x^2}{\alpha kT} \right] = [M^0L^0T^0] \Rightarrow [\alpha] = \frac{[x^2]}{[k][T]}$$

$$\Rightarrow [\alpha] = \frac{[L^2]}{[k][T]} \dots(i)$$

Since, dimensions of k are

$$[k] = [M^1L^2T^{-2}K^{-1}] \dots(ii)$$

Dimensions of temperature are

$$[T] = [K] \dots(iii)$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$[\alpha] = \frac{[L^2]}{[M^1L^2T^{-2}K^{-1}][K]}$$

$$[\alpha] = [M^{-1}T^2]$$

According to dimensional analysis,

$$[W] = [\alpha\beta^2] \Rightarrow [\beta^2] = \frac{[W]}{[\alpha]}$$

$$\Rightarrow [\beta^2] = \frac{[M^1 L^2 T^{-2}]}{[M^{-1} T^2]} = [M^2 L^2 T^{-4}]$$

$$\Rightarrow [\beta] = [MLT^{-2}]$$

- 50** If speed V , area A and force F are chosen as fundamental units, then the dimensional formula of Young's modulus will be **[2020, 2 Sep Shift-I]**

- (a) $[FA^2V^{-3}]$ (b) $[FA^{-1}V^0]$
(c) $[FA^2V^{-2}]$ (d) $[FA^2V^{-1}]$

Ans. (b)

Let Young's modulus is related to speed, area and force, as

$$Y = F^x A^y V^z$$

Substituting dimensions, we have

$$[ML^{-1}T^{-2}] = [MLT^{-2}]^x [L^2]^y [LT^{-1}]^z$$

Comparing power of similar quantities, we have

$$x = 1, x + 2y + z = -1 \text{ and } -2x - z = -2$$

Solving these, we get

$$x = 1, y = -1, z = 0$$

$$\text{So, } Y = FA^{-1}V^0$$

Hence, correct option is (b).

- 51** If momentum P , area A and time T are taken to be the fundamental quantities, then the dimensional formula for energy is

[2020, 2 Sep Shift-II]

- (a) $[P^2AT^{-2}]$ (b) $[PA^{-1}T^{-2}]$
(c) $[PA^{1/2}T^{-1}]$ (d) $[P^{1/2}AT^{-1}]$

Ans. (c)

Let dimensions of energy E in terms of momentum P , area A and time T are

$$[E] = [P]^x [A]^y [T]^z$$

Substituting dimensions of fundamental quantities for E , P , A and T , we have

$$[ML^2T^{-2}] = [MLT^{-1}]^x [L^2]^y [T]^z$$

$$[ML^2T^{-2}] = [M^x L^{x+2y} T^{-x+z}]$$

Equating powers of same physical quantities on both sides, we have

$$x = 1, x + 2y = 2 \text{ and } -x + z = -2$$

$$\text{so, } x = 1, y = 1/2, z = -1$$

$$\therefore \text{Dimensional formula of } [E] = [PA^{1/2}T^{-1}]$$

Hence, correct option is (c).

- 52** Amount of solar energy received on the earth's surface per unit area per unit time is defined as solar constant. Dimensional formula of solar constant is **[2020, 3 Sep Shift-II]**

- (a) $[MLT^{-2}]$ (b) $[ML^0T^{-3}]$
(c) $[M^2L^0T^{-1}]$ (d) $[ML^2T^{-2}]$

Ans. (b)

$$\text{Solar constant} = \frac{\text{Solar energy}}{\text{Area} \times \text{Time}} = \frac{[ML^2T^{-2}]}{[L^2][T]} = [ML^0T^{-3}]$$

Hence, correct option is (b).

- 53** Dimensional formula for thermal conductivity is (Here, K denotes the temperature)

[2020, 4 Sep Shift-I]

- (a) $[MLT^{-2}K]$ (b) $[MLT^{-2}K^{-2}]$
(c) $[MLT^{-3}K^{-1}]$ (d) $[MLT^{-3}K]$

Ans. (c)

For conduction of heat,

$$\begin{aligned} \frac{dQ}{dt} &= KA \frac{dT}{dx} \\ K &= \frac{\left(\frac{dQ}{dt}\right)}{A \left(\frac{dT}{dx}\right)} = \frac{dQ \times dx}{A \times dt \times dT} \\ &= \frac{\text{joule} \times \text{metre}}{(\text{metre})^2 \times \text{second} \times \text{kelvin}} \\ &= \frac{\text{kilogram} \times (\text{metre})^2 \times \text{metre}}{(\text{second})^2 \times (\text{metre})^2 \times \text{second} \times \text{kelvin}} \\ &= \frac{\text{kg} \cdot \text{m}^2 \cdot \text{m}}{\text{s}^2 \cdot \text{m}^2 \cdot \text{s} \cdot \text{K}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^3 \cdot \text{K}} \end{aligned}$$

$$[K] = \frac{[M^1][L^1]}{[T^3][K^1]}$$

$$\Rightarrow [K] = [MLT^{-3}K^{-1}]$$

Hence, correct option is (c).

- 55** A quantity x is given by $x = \frac{IFv^2}{WL^4}$

where, I is moment of inertia, F is force, v is velocity and L is length. The dimensional formula for x is same as that of **[2020, 4 Sep Shift-II]**

- (a) Planck's constant
(b) force constant
(c) coefficient of viscosity
(d) energy density

Ans. (d)

$$\text{Given that, } x = \frac{IFv^2}{WL^4}$$

Dimensionally,

$$\begin{aligned} [x] &= \frac{[I][F][v]^2}{[W][L]^4} \\ &= \frac{[M^1L^2][M^1L^1T^{-2}][L^1T^{-1}]^2}{[M^1L^2T^{-2}][L^1]^4} \end{aligned}$$

$$\begin{aligned} &= [M^1L^2] \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[M^1L^2T^{-2}][L^4]} \\ &= [M^1L^{-1}T^{-2}] \quad \dots(i) \end{aligned}$$

On checking the alternatives:

- (a) Planck's constant $\Rightarrow [h] = [M^1L^2T^{-1}]$ doesn't match with dimensional formula of x .
(b) Force constant $\Rightarrow [K] = [M^1T^{-2}]$ doesn't match with dimensional formula of x .
(c) Coefficient of viscosity $\Rightarrow [\eta] = [M^1L^{-1}T^{-1}]$ doesn't match with dimensional formula of x .
(d) Energy density $\Rightarrow [E_d] = [M^1L^{-1}T^{-2}]$ matches with dimensional formula of x .

Hence, option (d) is correct.

- 55** The quantities $x = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $y = \frac{E}{B}$ and $z = \frac{l}{CR}$ are defined, where C is

capacitance, R is resistance, l is length, E is electric field, B is magnetic field, ϵ_0 is free space permittivity and μ_0 is permeability, respectively. Then,

[2020, 5 Sep Shift-II]

- (a) x , y and z have the same dimension
(b) Only x and z have the same dimension
(c) Only x and y have the same dimension
(d) Only y and z have the same dimension

Ans. (a)

$$x = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \text{speed of light in vacuum}$$

$$\therefore \text{Dimension of } x, [x] = [M^0L^1T^{-1}]$$

$$y = \frac{E}{B} = \text{speed of EM wave}$$

$$\therefore \text{Dimension of } y, [y] = [M^0L^1T^{-1}]$$

$$z = \frac{l}{RC} = \frac{l}{\tau} = \frac{\text{length}}{\text{time}}$$

$$\therefore \text{Dimension of } z, [z] = [M^0L^1T^{-1}]$$

Thus, all quantities have same dimensions i.e., of velocity.

Hence, correct option is (a).

- 56** The dimension of $\frac{B^2}{2\mu_0}$, where B is

magnetic field and μ_0 is the magnetic permeability of vacuum, is **[2020, 7 Jan Shift-II]**

- (a) $[ML^{-1}T^{-2}]$
(b) $[MLT^{-2}]$
(c) $[ML^2T^{-1}]$
(d) $[ML^2T^{-2}]$

Ans. (a)As, $\frac{B^2}{2\mu_0}$ = energy density of magnetic field

$$= \frac{\text{Energy}}{\text{Volume}}$$

$$\text{So, } \left[\frac{B^2}{2\mu_0} \right] = [\text{Energy/Volume}]$$

$$= \frac{[ML^2T^{-2}]}{[L^3]}$$

$$= [ML^{-1}T^{-2}]$$

- 57** The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant h , speed of light c and gravitational constant G and ampere A is

[2020, 8 Jan Shift-I]

- (a) $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$
 (b) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$
 (c) $h^2G^{3/2}c^{1/3}A^{-1}$
 (d) $h^{2/3}c^{5/3}G^{1/3}A^{-1}$

Ans. (*)Let $V_0 = (h)^a \cdot (c)^b \cdot (G)^c \cdot (A)^d$... (i)Then, $[V_0] = [\text{potential}]$

$$= \left[\frac{\text{potential energy}}{\text{charge}} \right]$$

$$= \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

$$[h] = \left[\frac{\text{Energy}}{\text{Frequency}} \right] = \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

$$= [ML^2T^{-1}]$$

$$[c] = [\text{Speed}] = [LT^{-1}]$$

$$[G] = \left[\frac{\text{Force} \times (\text{Distance})^2}{(\text{Mass})^2} \right]$$

$$= \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$$

Substituting the dimensions of V_0, h, c, G and A in Eq. (i) and equating dimension on both sides, we get

$$[ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^a \times [LT^{-1}]^b$$

$$\times [M^{-1}L^3T^{-2}]^c \times [A]^d$$

$$\Rightarrow a - c = 1 \quad \dots (ii)$$

$$2a + b + 3c = 2 \quad \dots (iii)$$

$$-a - b - 2c = -3 \quad \dots (iv)$$

$$d = -1 \quad \dots (v)$$

On solving above equations, we get

$$a = 0, b = 5, c = -1, d = -1$$

Substituting these values in Eq. (i), we get

$$V_0 = h^0 \cdot c^5 \cdot G^{-1} \cdot A^{-1}$$

None of the given options matches with the result.

- 58** A quantity f is given by $f = \sqrt{hc^5/G}$, where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of **[2020, 9 Jan Shift-I]**

- (a) area
 (b) volume
 (c) momentum
 (d) energy

Ans. (d)Dimensions of quantity f are

$$[f] = \frac{[h]^{1/2} [c]^{5/2}}{[G]^{1/2}} \quad \dots (i)$$

[Note] To produce dimensions of different constants, just remember/recall nearest formulae containing these constants.]

$$\text{As, } h = \frac{E}{\nu}; [h] = [ML^2T^{-2}][T] = [ML^2T^{-1}]$$

$$c = [LT^{-1}] \text{ and } G = \frac{F \cdot r^2}{m^2}$$

$$\Rightarrow [G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$$

So, dimensions of f using Eq. (i),

$$[f] = \frac{[ML^2T^{-1}]^{1/2} [LT^{-1}]^{5/2}}{[M^{-1}L^3T^{-2}]^{1/2}} = \left[M^{\frac{1}{2} + \frac{1}{2} - \frac{1}{2}} L^{\frac{5}{2} - \frac{3}{2} + 1} T^{-\frac{1}{2} - \frac{5}{2} + 2} \right] = [ML^2T^{-2}]$$

Thus, it is the dimensions of energy.

- 59** In SI units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is **[2019, 8 April Shift-I]**

- (a) $[A^{-1}TML^3]$ (b) $[AT^2M^{-1}L^{-1}]$
 (c) $[AT^{-3}ML^{3/2}]$ (d) $[A^2T^3M^{-1}L^{-2}]$

Ans. (d)

Dimensions of ϵ_0 (permittivity of free space) are

$$[\epsilon_0] = M^{-1}L^{-3}T^4A^2$$

As, c = speed of light.

$$\therefore \text{Dimension of } [c] = [LT^{-1}]$$

So, dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ are

$$\left[\sqrt{\frac{\epsilon_0}{\mu_0}} \right] = \left[\sqrt{\frac{\epsilon_0^2}{\epsilon_0 \mu_0}} \right] = [\epsilon_0 c] \left[\because c^2 = \frac{1}{\mu_0 \epsilon_0} \right] = [M^{-1}L^{-3}T^4A^2][LT^{-1}] = [M^{-1}L^{-2}T^3A^2]$$

- 60** If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **[2019, 8 April Shift-II]**

- (a) $S^{1/2}I^{1/2}h^{-1}$ (b) $S^{3/2}I^{1/2}h^0$
 (c) $S^{1/2}I^{1/2}h^0$ (d) $S^{1/2}I^{3/2}h^{-1}$

Ans. (c)

Suppose, linear momentum (p) depends upon the Planck's constant (h) raised to the power (a), surface tension (S) raised to the power (b) and moment of inertia (I) raised to the power (c).

$$\text{Then, } p \propto (h)^a (S)^b (I)^c \text{ or } p = kh^a S^b I^c$$

where, k is a dimensionless proportionality constant.

$$\text{Thus, } [p] = [h]^a [S]^b [I]^c \quad \dots (i)$$

Then, the respective dimensions of the given physical quantities, i.e.

$$[p] = [\text{mass} \times \text{velocity}] = [MLT^{-1}]$$

$$[I] = [\text{mass} \times \text{distance}^2]$$

$$= [ML^2T^0]$$

$$[S] = [\text{force} \times \text{length}] = [ML^0T^{-2}]$$

$$[h] = [ML^2T^{-1}]$$

Then, substituting these dimensions in Eq. (i), we get

$$[MLT^{-1}] = [ML^2T^{-1}]^a [MT^{-2}]^b [ML^2]^c$$

For dimensional balance, the dimensions on both sides should be same.

Thus, equating dimensions, we have

$$a + b + c = 1$$

$$2(a + c) = 1 \text{ or } a + c = \frac{1}{2}$$

$$-a - 2b = -1 \text{ or } a + 2b = 1$$

Solving these three equations, we get

$$a = 0, b = \frac{1}{2}, c = \frac{1}{2}$$

$$\therefore p = h^0 S^{1/2} I^{1/2} \text{ or } p = S^{1/2} I^{1/2} h^0$$

- 61** In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **[2019, 10 April Shift-II]**

- (a) $[M^{-1}L^{-2}T^4A^2]$ (b) $[M^{-2}L^0T^{-4}A^{-2}]$
 (c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^{-2}T^6A^3]$

Ans. (c)

To find dimensions of capacitance in the given relation, we can use formula for energy.

Capacitors energy is $U = \frac{1}{2} CV^2$

So, dimensionally,

$$\Rightarrow [C] = \left[\frac{U}{V^2} \right]$$

As, $V = \text{potential} = \frac{\text{potential energy}}{\text{charge}}$

We have,

$$[C] = \left[\frac{[U]}{[V^2]} \right] = \left[\frac{[q^2]}{[U]} \right] = \left[\frac{A^2 T^2}{ML^2 T^{-2}} \right]$$

$$\therefore X = [M^{-1} L^{-2} A^2 T^4]$$

To get dimensions of magnetic field, we use force on a current carrying conductor in magnetic field,

$$F = BIl \Rightarrow [B] = \frac{[F]}{[I][l]} = \left[\frac{MLT^{-2}}{AL} \right]$$

$$\therefore Z = [M^1 L^0 T^{-2} A^{-1}]$$

Now, using given relation,

$$X = 5YZ^2$$

$$[Y] = \frac{[X]}{[Z^2]} = \left[\frac{M^{-1} L^{-2} A^2 T^4}{(M^1 L^0 T^{-2} A^{-1})^2} \right]$$

$$= \frac{M^{-1} L^{-2} A^2 T^4}{M^2 T^{-4} A^{-2}}$$

$$\therefore [Y] = [M^{-3} L^{-2} T^8 A^4]$$

- 62** Which of the following combinations has the dimension of electrical resistance (ϵ_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)? **[2019, 12 April Shift-I]**

(a) $\sqrt{\frac{\mu_0}{\epsilon_0}}$ (b) $\frac{\mu_0}{\epsilon_0}$ (c) $\sqrt{\frac{\epsilon_0}{\mu_0}}$ (d) $\frac{\epsilon_0}{\mu_0}$

Ans. (a)

Key Idea A formula is valid only, if the dimensions of LHS and RHS are same. So, we need to balance dimensions of given options with the dimension of electrical resistance.

Let dimensions of resistance R , permittivity ϵ_0 and permeability μ_0 are $[R]$, $[\epsilon_0]$ and $[\mu_0]$, respectively.

So, $[R] = [\epsilon_0]^\alpha [\mu_0]^\beta \quad \dots(i)$

$$[R] = [M^1 L^2 T^{-3} A^{-2}],$$

$$[\epsilon_0] = [M^{-1} L^{-3} T^4 A^2],$$

$$[\mu_0] = [M^1 L^1 T^{-2} A^{-2}]$$

Now, from Eq. (i), we get

$$[M^1 L^2 T^{-3} A^{-2}]$$

$$= [M^{-1} L^{-3} T^4 A^2]^\alpha [M^1 L^1 T^{-2} A^{-2}]^\beta$$

$$[M^1 L^2 T^{-3} A^{-2}]$$

$$= M^{-\alpha + \beta} L^{-3\alpha + \beta} T^{4\alpha - 2\beta} A^{2\alpha - 2\beta}$$

On comparing both sides, we get

$$-\alpha + \beta = 1 \quad \dots(ii)$$

$$-3\alpha + \beta = 2 \quad \dots(iii)$$

$$4\alpha - 2\beta = -3 \quad \dots(iv)$$

$$2\alpha - 2\beta = -2 \quad \dots(v)$$

Value of α and β can be found using any two Eqs. from (ii) to (v),

On subtracting Eq. (iii) from Eq. (ii), we get

$$(-\alpha + \beta) - (-3\alpha + \beta) = 1 - 2$$

$$\Rightarrow 2\alpha = -1 \text{ or } \alpha = -\frac{1}{2}$$

Put the value of α in Eq. (ii), we get

$$\beta = +\frac{1}{2}$$

$$\therefore [R] = [\epsilon_0]^{-1/2} [\mu_0]^{1/2} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

- 63** In form of G (universal gravitational constant), h (Planck constant) and c (speed of light), the time period will be proportional to

[2019, 9 Jan Shift-II]

(a) $\sqrt{\frac{Gh}{c^5}}$

(b) $\sqrt{\frac{hc^5}{G}}$

(c) $\sqrt{\frac{c^3}{Gh}}$

(d) $\sqrt{\frac{Gh}{c^3}}$

Ans. (a)

Key Idea According to dimensional analysis, if a physical quantity (let y) depends on another physical quantities (let A, B, C), then the dimension formula is given by

$$y = A^a B^b C^c$$

(where, a, b, c are the power of physical quantity and $k = \text{constant}$)

For the given question, the time (t) depends on the G (gravitational constant), h (Planck's constant) and c (velocity of light), then according to dimensional analysis

$$t = k(G)^a (h)^b (c)^c \quad \dots(i)$$

For calculating the values of a, b and c , compare the dimensional formula for both side.

$$\text{LHS } t = \text{time} = [M^0 L^0 T^1]$$

$$\text{RHS } G = \frac{F \cdot r^2}{m_1 m_2}$$

$$= \frac{\text{kg ms}^{-2} \times \text{m}^2}{(\text{kg})^2} = [M^{-1} L^3 T^{-2}]^a$$

$$h = [M^1 L^2 T^{-1}]^b$$

$$c = \frac{d}{t} = [M^0 L^1 T^{-1}]^c$$

Compare both side for powers of M, L and T ,

$$M \Rightarrow 0 = -a + b \quad \dots(i)$$

$$L \Rightarrow 0 = 3a + 2b + c \quad \dots(ii)$$

$$T \Rightarrow 1 = -2a - b - c \quad \dots(iii)$$

Solving Eqs. (i), (ii), (iii), we get

$$a = \frac{1}{2}, \quad b = \frac{1}{2} \text{ and } c = -\frac{5}{2}$$

So, put these values in Eq. (i), we get

$$t = k G^{1/2} h^{1/2} c^{-5/2}$$

$$t = k \sqrt{\frac{Gh}{c^5}}$$

- 64** The force of interaction between two atoms is given by

$$F = \alpha \beta \exp\left(-\frac{x^2}{\alpha k T}\right); \text{ where } x \text{ is the}$$

distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is

[2019, 11 Jan Shift-I]

(a) $[MLT^{-2}]$

(b) $[M^0 L^2 T^{-4}]$

(c) $[M^2 L T^{-4}]$

(d) $[M^2 L^2 T^{-2}]$

Ans. (c)

Force of interaction between two atoms is given as

$$F = \alpha \beta \exp(-x^2 / \alpha k T)$$

As we know, exponential terms are always dimensionless, so

$$\text{dimensions of } \left(\frac{-x^2}{\alpha k T} \right) = [M^0 L^0 T^0]$$

$$\Rightarrow \text{Dimensions of } \alpha = \text{Dimension of } (x^2 / kT)$$

Now, substituting the dimensions of individual term in the given equation, we

$$\text{get } = \frac{[M^0 L^2 T^0]}{[M^1 L^2 T^{-2}]} \quad \{\therefore \text{Dimensions of } kT\}$$

equivalent to the dimensions of energy

$$= [M^1 L^2 T^{-2}] = [M^{-1} L^0 T^2] \quad \dots(ii)$$

Now from given equation, we have dimensions of

$$F = \text{dimensions of } \alpha \times \text{dimensions of } \beta$$

$$\Rightarrow \text{Dimensions of } \beta = \text{Dimensions of } \left(\frac{F}{\alpha} \right)$$

$$= \frac{[M^1 L T^{-2}]}{[M^{-1} L^0 T^2]} \quad [\therefore \text{using Eq. (i)}]$$

$$= [M^2 L T^{-4}]$$

- 65** If speed (v), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be

[2019, 11 Jan Shift-II]

- (a) $[v^{-4}A^{-2}F]$
 (b) $[v^{-2}A^2F^2]$
 (c) $[v^{-2}A^2F^{-2}]$
 (d) $[v^{-4}A^2F]$

Ans. (d)

Dimensions of speed are, $[v] = [LT^{-1}]$
 Dimensions of acceleration are,
 $[A] = [LT^{-2}]$
 Dimensions of force are, $[F] = [MLT^{-2}]$
 Dimension of Young modulus is,
 $[Y] = [ML^{-1}T^{-2}]$

Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as;

$$[Y] = [v]^\alpha [A]^\beta [F]^\gamma \quad \dots(i)$$

Then substituting dimensions in terms of M, L and T we get,

$$[ML^{-1}T^{-2}] = [LT^{-1}]^\alpha [LT^{-2}]^\beta [MLT^{-2}]^\gamma$$

$$= [M]^\gamma [L]^{\alpha+\beta+\gamma} [T]^{-\alpha-2\beta-2\gamma}$$

Now comparing powers of basic quantities on both sides we get, $\gamma = 1$

$$\alpha + \beta + \gamma = -1$$

$$\text{and } -\alpha - 2\beta - 2\gamma = -2$$

Solving these, we get

$$\alpha = -4, \beta = 2, \gamma = 1$$

Substituting α, β , and γ in Eq. (i) we get;

$$[Y] = [v^{-4}A^2F]$$

- 66** Let l, r, c , and v represent inductance, resistance, capacitance and voltage,

respectively. The dimension of $\frac{l}{rcv}$

in SI units will be

[2019, 12 Jan Shift-II]

- (a) $[LT^2]$ (b) $[LTA]$
 (c) $[A^{-1}]$ (d) $[LA^{-2}]$

Ans. (c)

Dimensions of given quantities are

$$l = \text{inductance} = [M^1L^2T^{-2}A^{-2}]$$

$$r = \text{resistance} = [M^1L^2T^{-3}A^{-2}]$$

$$c = \text{capacitance} = [M^{-1}L^{-2}T^4A^2]$$

$$v = \text{voltage} = [M^1L^2T^{-3}A^{-1}]$$

So, dimensions of $\frac{l}{rcv}$ are

$$\left[\frac{l}{rcv} \right] = \frac{[M^1L^2T^{-2}A^{-2}]}{[M^1L^2T^{-2}A^{-1}]} = [A^{-1}]$$

- 67** Let $[\epsilon_0]$ denotes the dimensional

formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current,

then

[JEE Main 2013]

- (a) $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$
 (b) $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$
 (c) $[\epsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$
 (d) $[\epsilon_0] = [M^{-1}L^2T^{-1}A^2]$

Ans. (b)

Electrostatic force between two charges,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{R^2} \Rightarrow \epsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

Substituting the units.

$$\text{Hence, } \epsilon_0 = \frac{C^2}{N \cdot m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$$

$$= [M^{-1}L^{-3}T^4A^2]$$

- 68** The dimensions of magnetic field in M, L, T and C (coulomb) is given as

[AIEEE 2008]

- (a) $[MLT^{-1}C^{-1}]$ (b) $[MT^2C^{-2}]$
 (c) $[MT^{-1}C^{-1}]$ (d) $[MT^2C^{-1}]$

Ans. (c)

From the relation $F = qvB$

$$\Rightarrow [MLT^{-2}] = [C][LT^{-1}][B]$$

$$\Rightarrow [B] = [MC^{-1}T^{-1}]$$

- 69** Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge?

[AIEEE 2006]

- (a) Wb/m^2 (b) henry (H)
 (c) H/m^2 (d) weber (Wb)

Ans. (b)

$$\text{Magnetic energy} = \frac{1}{2} L I^2 = \frac{L q^2}{2 t^2} \left[\text{as } I = \frac{q}{t} \right]$$

where L = inductance, I = current

Energy has the dimensions $[ML^2T^{-2}]$

Equate the dimensions, we have

$$[ML^2T^{-2}] = [\text{henry}] \times \frac{[Q^2]}{[T^2]}$$

$$\Rightarrow [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

- 70** Out of the following pairs, which one does not have identical dimensions?

[AIEEE 2005]

- (a) Angular momentum and Planck's constant
 (b) Impulse and momentum

- (c) Moment of inertia and moment of a force
 (d) Work and torque

Ans. (c)

Moment of inertia, $I = mr^2 \therefore [I] = [ML^2]$

and τ = Moment of force $= r \times F$

$$\therefore [\tau] = [L][MLT^{-2}] = [ML^2T^{-2}]$$

- 71** Which one of the following represents the correct dimensions of the coefficient of viscosity?

[AIEEE 2004]

- (a) $[ML^{-1}T^{-2}]$
 (b) $[MLT^{-1}]$
 (c) $[ML^{-1}T^{-1}]$
 (d) $[ML^{-2}T^{-2}]$

Ans. (c)

By Newton's formula, $\eta = \frac{F}{A(\Delta v_x / \Delta z)}$

$\therefore$ Dimensions of η

$$= \frac{\text{Dimensions of force}}{\text{Dimensions of area} \times \text{Dimensions of velocity gradient}}$$

$$= \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [ML^{-1}T^{-1}]$$

- 72** Dimensions of $1/\mu_0 \epsilon_0$, where

symbols have their usual meaning, are

[AIEEE 2003]

- (a) $[L^{-1}T]$ (b) $[L^2T^2]$
 (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

Ans. (c)

As we know that formula of velocity is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \epsilon_0} = [LT^{-1}]^2$$

$$\therefore \frac{1}{\mu_0 \epsilon_0} = [L^2T^{-2}]$$

- 73** The physical quantities not having same dimensions are [AIEEE 2003]

- (a) torque and work
 (b) momentum and Planck's constant
 (c) stress and Young's modulus
 (d) speed and $(\mu_0 \epsilon_0)^{-1/2}$

Ans. (b)

Planck's constant (in terms of unit)

$$h = J \cdot s = [ML^2T^{-2}][T] = [ML^2T^{-1}]$$

Momentum (p) = $kg \cdot ms^{-1}$

$$= [M][L][T^{-1}]$$

$$= [MLT^{-1}]$$

02

Motion in a Straight Line

TOPIC 1

Terms Related to Motion

- 01** If the velocity of a body related to displacement x is given by $v = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is m/s². **[2021, 27 Aug Shift-I]**

Ans. (12)

Velocity of body is

$$v = \sqrt{5000 + 24x} \text{ m/s}$$

Acceleration of body can be calculated as

$$\begin{aligned} a &= \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} \\ \Rightarrow a &= v \frac{dv}{dx} \quad \dots(i) \\ \frac{dv}{dx} &= \frac{d}{dx} (\sqrt{5000 + 24x}) \\ \Rightarrow \frac{dv}{dx} &= \frac{1}{2(\sqrt{5000 + 24x})} \times 24 \\ &= \frac{12}{\sqrt{5000 + 24x}} \end{aligned}$$

Substituting the value of $\frac{dv}{dx}$ in Eq. (i), we get

$$\begin{aligned} a &= \sqrt{5000 + 24x} \times \frac{12}{\sqrt{5000 + 24x}} \\ &= 12 \text{ m/s}^2 \end{aligned}$$

Thus, the acceleration of body is 12 m/s².

- 02** The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is **[2021, 25 July Shift-II]**

- (a) $3\alpha + 7\beta$ (b) $\frac{3}{2}\alpha + \frac{7}{3}\beta$
(c) $\frac{\alpha}{2} + \frac{\beta}{3}$ (d) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

Ans. (b)

Given, instantaneous velocity

$$(v) = \alpha t + \beta t^2$$

Let distance travelled by particle be s and t_1 and t_2 be the initial and final time i.e. $t_1 = 1$ s and $t_2 = 2$ s

As,

$$v = \frac{ds}{dt}$$

$$\Rightarrow ds = v dt \Rightarrow ds = (\alpha t + \beta t^2)$$

Integrating both sides, we get

$$\begin{aligned} s &= \left(\alpha \frac{t^2}{2} + \frac{\beta t^3}{3} \right) \Big|_1^2 \\ &= \frac{\alpha}{2} [2^2 - 1^2] + \frac{\beta}{3} [2^3 - 1^3] \\ &= \frac{\alpha}{2} [4 - 1] + \frac{\beta}{3} [8 - 1] \\ &= \frac{3\alpha}{2} + \frac{7\beta}{3} \end{aligned}$$

- 03** The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is (when v stands for velocity) **[2021, 25 July Shift-II]**

- (a) $2mv^3$ (b) $2mnv^3$
(c) $2nv^3$ (d) $2n^2v^3$

Ans. (a)

Given, distance-time relation

$$t = mx^2 + nx \quad \dots(i)$$

Let a be the acceleration.

$$\Rightarrow a = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dx}{dt} \right)$$

where, v is speed and x is distance.

On differentiating both sides of Eq. (i) with respect to time t , we get

$$\begin{aligned} \frac{d}{dt}(t) &= \frac{d}{dt}(mx^2 + nx) \\ \Rightarrow 1 &= m \left[2x \frac{dx}{dt} \right] + n \frac{dx}{dt} \Rightarrow 1 = \frac{dx}{dt} (2mx + n) \\ \Rightarrow v &= \frac{1}{2mx + n} \quad \dots(ii) \end{aligned}$$

Now, on differentiating Eq. (ii) with respect to time t , we get

$$\begin{aligned} \frac{dv}{dt} &= \frac{-1 \left(2m \frac{dx}{dt} \right)}{(2mx + n)^2} \\ \Rightarrow a &= \frac{-2mv}{(2mx + n)^2} \quad \dots(iii) \end{aligned}$$

On squaring Eq. (ii) both sides, we get

$$v^2 = \frac{1}{(2mx + n)^2}$$

Substituting the value of v^2 in Eq. (iii), we get

$$\begin{aligned} a &= -2mv \cdot v^2 = -2mv^3 \\ \therefore \text{Retardation} &= -a = 2mv^3 \end{aligned}$$

- 04** A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1 . If he remains stationary on a moving escalator, then the escalator takes him up in time t_2 . The time taken by him to walk up on the moving escalator will be **[2021, 20 July Shift-II]**

- (a) $\frac{t_1 t_2}{t_2 - t_1}$ (b) $\frac{t_1 + t_2}{2}$
(c) $\frac{t_1 t_2}{t_2 + t_1}$ (d) $t_2 - t_1$

Ans. (c)Let the length of escalator be L .Also, suppose that the velocity of boy with respect to escalator be v_1 and the velocity of escalator be v_2 . $\therefore$ Velocity of boy with respect to stationary escalator can be given as

$$v_1 = \frac{L}{t_1} \quad \dots(i)$$

When escalator is moving and boy is at rest velocity of escalator can be given as

$$v_2 = \frac{L}{t_2} \quad \dots(ii)$$

So, the resultant velocity can be given as when both escalator and boy are moving,

$$v = v_1 + v_2 \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$v = \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow \frac{v}{L} = \frac{1}{t_1} + \frac{1}{t_2}$$

$$\Rightarrow \frac{v}{L} = \frac{t_1 + t_2}{t_1 t_2} \Rightarrow \frac{L}{v} = \frac{t_1 t_2}{t_1 + t_2}$$

$$\Rightarrow t = \frac{t_1 t_2}{t_1 + t_2} \quad [\because t = L/v]$$

- 05** The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$, then its displacement after time ($t = 1$) is [2021, 17 March Shift-II]

(a) $v_0 + g + F$ (b) $v_0 + \frac{g}{2} + \frac{F}{3}$

(c) $v_0 + \frac{g}{2} + F$ (d) $v_0 + 2g + 3F$

Ans. (b)

Given, equation of velocity,

$$v = v_0 + gt + Ft^2$$

$$\text{At } t = 0, x = 0$$

$$\text{At } t = 1 \text{ s } x = ?$$

We know that, $v = \frac{dx}{dt} \Rightarrow dx = v dt$

Integrating the above equation with proper limits, we get

$$\int_0^x dx = \int_0^1 v dt$$

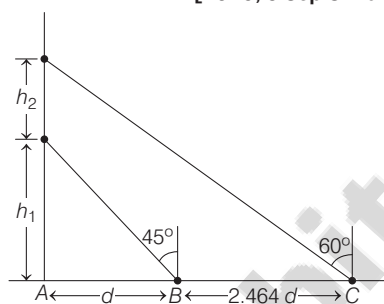
$$x - 0 = \int_0^1 (v_0 + gt + Ft^2) dt$$

$$\Rightarrow x = \left[v_0 t + \frac{gt^2}{2} + \frac{Ft^3}{3} \right]_0^1$$

$$\Rightarrow x = \left[v_0(1) + \frac{g(1)^2}{2} + \frac{F(1)^3}{3} \right] - 0$$

$$\Rightarrow x = \left[v_0 + \frac{g}{2} + \frac{F}{3} \right]$$

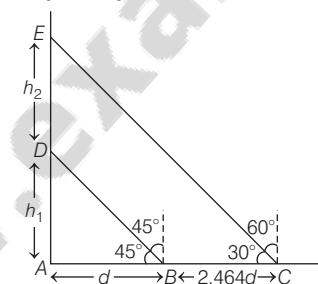
- 06** A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1 , a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2 , it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance $2.464 d$ (point C). Then, the height h_2 is (Given, $\tan 30^\circ = 0.5774$) [2020, 5 Sep Shift-I]



- (a) $1.464 d$ (b) d
(c) $0.464 d$ (d) $0.732 d$

Ans. (b)

The given figure is shown below,



$$\text{In } \triangle ABD, \quad \tan 45^\circ = \frac{h_1}{d}$$

$$\therefore \tan 45^\circ = 1 \Rightarrow 1 = \frac{h_1}{d} \Rightarrow h_1 = d \quad \dots(i)$$

Now, in $\triangle ACE$,

$$\tan 30^\circ = \frac{AD + DE}{AB + BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_1 + h_2}{d + 2.464d}$$

$$0.5774 = \frac{h_1 + h_2}{3.464d}$$

$$\Rightarrow h_1 + h_2 = (0.5774) \times (3.464d)$$

$$h_1 + h_2 = 2d$$

$$d + h_2 = 2d$$

$$h_2 = 2d - d$$

$$\Rightarrow h_2 = d$$

Hence, correct option is (b).

- 07** The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n} , where n is an integer, the value of n is [2020, 9 Jan Shift-I]

Ans. (3)Given, displacement (x) and time (t) relation,

$$x^2 = at^2 + 2bt + c \quad \dots(i)$$

On differentiating Eq. (i) w.r.t. t , we get

$$\Rightarrow 2x \frac{dx}{dt} = 2at + 2b$$

$$\text{or } xv = at + b \quad \dots(ii)$$

Differentiating again, we have

$$x \cdot \frac{dv}{dt} + v \cdot \frac{dx}{dt} = a \left(\because \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = A \right)$$

$$\Rightarrow xA + v^2 = a \Rightarrow xA = a - v^2$$

$$\Rightarrow xA = a - \left(\frac{at + b}{x} \right)^2$$

$$[\text{from Eq. (ii), } v = at + b/x]$$

$$\Rightarrow A = \frac{ac - b^2}{x^3} \Rightarrow A \propto x^{-3}$$

Thus,

$$n = 3$$

- 08** A particle is moving with speed $v = b\sqrt{x}$ along positive X -axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$). [2019, 12 April Shift-II]

(a) $\frac{b^2 \tau}{4}$ (b) $\frac{b^2 \tau}{2}$ (c) $b^2 \tau$ (d) $\frac{b^2 \tau}{\sqrt{2}}$

Ans. (b)Given, speed, $v = b\sqrt{x}$

Now, differentiating it with respect to

$$\text{time, we get } \frac{dv}{dt} = \frac{b}{2\sqrt{x}}$$

Now, acceleration

$$\Rightarrow a = \frac{b}{2\sqrt{x}} \cdot \frac{dx}{dt} \quad \left[\because \frac{dv}{dt} = a \right]$$

$$\Rightarrow a = \frac{b}{2\sqrt{x}} \cdot v = \frac{b}{2\sqrt{x}} \cdot b\sqrt{x} = \frac{b^2}{2}$$

As acceleration is constant, we use

$$v = u + at \quad \dots(i)$$

Now, it is given that $x = 0$ at $t = 0$.

So, initial speed of particle is

$$u = b\sqrt{x} \Big|_{x=0} = b \times 0 = 0$$

Hence, when time $t = \tau$, speed of the particle using Eq. (i) is

$$v = u + at = 0 + \frac{b^2}{2} \cdot \tau = \frac{b^2 \tau}{2}$$

- 09** The position of a particle as a function of time t , is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be

[2019, 9 April Shift-II]

- (a) $a + \frac{b^2}{2c}$ (b) $a + \frac{b^2}{4c}$
(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{c}$

Ans. (c)

Position of particle is,

$$x(t) = at + bt^2 - ct^3$$

So, its velocity is,

$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

and acceleration is,

$$a = \frac{dv}{dt} = 2b - 6ct$$

Acceleration is zero, then $2b - 6ct = 0$

$$\Rightarrow t = \frac{2b}{6c} = \frac{b}{3c}$$

Substituting this 't' in expression of velocity, we get

$$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$$

$$= a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$$

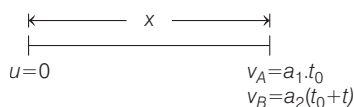
- 10** In a car race on a straight path, car A takes a time t less than car B at the finish and passes finishing point with a speed ' V ' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then ' V ' is equal to

[2019, 9 Jan Shift-II]

- (a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}t$
(c) $\sqrt{a_1a_2}t$ (d) $\frac{a_1+a_2}{2}t$

Ans. (c)

Let car B takes time $(t_0 + t)$ and car A takes time t_0 to finish the race.



Then,

$$\text{Given, } x_A - x_B = v = (a_1 - a_2)t_0 - a_2 t \quad \dots(i)$$

$$s_B = s_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2 (t_0 + t)^2$$

$$\text{or } \sqrt{a_1} t_0 = \sqrt{a_2} (t_0 + t)$$

$$\text{or } \sqrt{a_1} t_0 = \sqrt{a_2} t_0 + \sqrt{a_2} t$$

$$\text{or } (\sqrt{a_1} - \sqrt{a_2}) t_0 = \sqrt{a_2} t$$

$$\text{or } t_0 = \frac{\sqrt{a_2} t}{(\sqrt{a_1} - \sqrt{a_2})} \quad \dots(ii)$$

Substituting the value of t_0 from Eq. (ii) into Eq. (i), we get

$$v = (a_1 - a_2) \frac{\sqrt{a_2} t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= (\sqrt{a_1} - \sqrt{a_2})(\sqrt{a_1} + \sqrt{a_2}) \cdot \frac{\sqrt{a_2} t}{(\sqrt{a_1} - \sqrt{a_2})} - a_2 t$$

$$\text{or } v = (\sqrt{a_1} + \sqrt{a_2}) \cdot \sqrt{a_2} t - a_2 t$$

$$= \sqrt{a_1 a_2} \cdot t + a_2 t - a_2 t$$

$$\text{or } v = \sqrt{a_1 a_2} t$$

- 11** An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $dv/dt = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object to come to rest, would be

[AIEEE 2011]

- (a) 2 s (b) 4 s
(c) 8 s (d) 1 s

Ans. (a)

$$\text{Given, } \frac{dv}{dt} = -2.5\sqrt{v} \Rightarrow \frac{dv}{\sqrt{v}} = -2.5 dt$$

$$\Rightarrow \int_{6.25}^0 v^{-1/2} dv = -2.5 \int_0^t dt$$

$$\Rightarrow -2.5 [t]_0^t = [2v^{1/2}]_{6.25}^0 \Rightarrow t = 2 \text{ s}$$

- 12** The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

[AIEEE 2007]

- (a) $v_0 + 2g + 3f$ (b) $v_0 + \frac{g}{2} + \frac{f}{3}$
(c) $v_0 + g + f$ (d) $v_0 + \frac{g}{2} + f$

Ans. (b)

$$\text{Given, } v = v_0 + gt + ft^2$$

After differentiating with respect to time, we get

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\Rightarrow dx = (v_0 + gt + ft^2) dt$$

$$\text{So, } \int_0^x dx = \int_0^1 (v_0 + gt + ft^2) dt$$

$$\Rightarrow x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Since, vertical component of velocity is zero.

- 13** A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x -direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

[AIEEE 2006]

- (a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

Ans. (a)

$$\text{Given, } v = \alpha\sqrt{x} \text{ or } \frac{dx}{dt} = \alpha\sqrt{x} \quad [\because v = \frac{dx}{dt}]$$

$$\text{or } \frac{dx}{\sqrt{x}} = \alpha dt$$

On integrating, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha dt$$

$$[\because \text{at } t = 0, x = 0 \text{ and let at any time } t, \text{ particle be at } x]$$

$$\Rightarrow \left[\frac{x^{1/2}}{1/2} \right]_0^x = \alpha t \text{ or } x^{1/2} = \frac{\alpha}{2} t$$

$$\text{or } x = \frac{\alpha^2}{4} t^2 \text{ or } x \propto t^2$$

- 14** The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is

[AIEEE 2005]

- (a) $-2abv^2$ (b) $2bv^3$
(c) $-2av^3$ (d) $2av^2$

Ans. (c)

$$\text{Given, } t = ax^2 + bx$$

Differentiating it w.r.t. t , we get

$$\frac{dt}{dt} = 2ax \frac{dx}{dt} + b \frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{(2ax + b)}$$

Again, differentiating w.r.t. t , we get

$$\frac{d^2x}{dt^2} = \frac{-2a}{(2ax + b)^2} \cdot \frac{dx}{dt}$$

$$\therefore f = \frac{d^2x}{dt^2} = \frac{-1}{(2ax + b)^2} \cdot \frac{2a}{(2ax + b)}$$

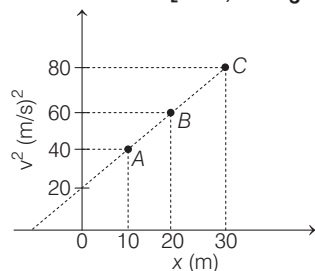
$$\text{or } f = \frac{-2a}{(2ax + b)^3}$$

$$\Rightarrow f = -2av^3$$

TOPIC 2

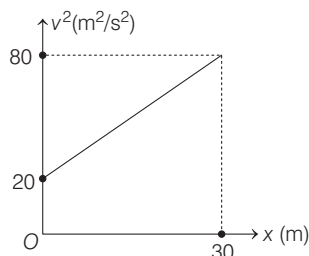
Kinematics Equations of Uniformly Accelerated Motion

- 15** A particle is moving with constant acceleration a . Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is m/s^2 [2021, 31 Aug Shift-II]



Ans. (1)

v^2 versus x graph is shown below.



Using equation of motion,

$$v^2 = u^2 + 2ax \quad \dots(i)$$

We know that, the equation of straight line.

$$y = mx + c \quad \dots(ii)$$

Comparing Eqs. (i) and (ii),

$$\text{Slope, } m = 2a = \frac{80 - 20}{30 - 0} = \frac{60}{30} = 2$$

$$\text{Now, the acceleration, } a = \frac{2}{2} = 1 \text{ m/s}^2$$

- 16** Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

[2021, 27 Aug Shift-II]

- (a) 4.18 m (b) 2.94 m
(c) 2.45 m (d) 7.35 m

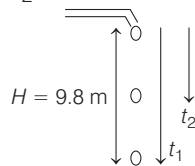
Ans. (d)

Given, height of nozzle from floor, $H = 9.8 \text{ m}$

Let, t_1 = time taken by first drop to reach the ground

and t_2 = time taken by 2nd drop.

$$t_2 = \frac{t_1}{2}$$



g = acceleration due to gravity (9.8 ms^{-2})

u = initial speed = 0

$$\text{Since, } H = ut + \frac{1}{2}gt^2 \quad \dots(i)$$

$$\therefore t_1 = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow t_1 = \sqrt{\frac{2 \times 9.8}{9.8}} = \sqrt{2} \text{ s}$$

$$\therefore t_2 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ s}$$

Again from Eq. (i)

$$\text{Distance covered by 2nd drop, } x = \frac{1}{2}gt_2^2$$

$$\therefore x = \frac{1}{2} \times 9.8 \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 9.8 \times \frac{1}{2} = \frac{9.8}{4} = 2.45 \text{ m}$$

and position of second drop from ground,

$$H_2 = H_1 - x = 9.8 - 2.45 = 7.35 \text{ m}$$

- 17** Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of m. (Take, $g = 10 \text{ m/s}^2$)

[2021, 26 Aug Shift-I]

Ans. (50)

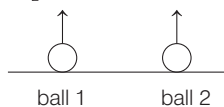
Given, $m_1 = m_2 = m$

$$r_1 = r_2 = r = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$u_1 = u_2 = 35 \text{ m/s}$$

$$t_1 = 0$$

and $t_2 = 3 \text{ s}$



When both ball will collide,

$$(S_1)_{\text{vertical}} = (S_2)_{\text{vertical}} \quad \dots(i)$$

Let after time t both ball will collide.

$$(S_1)_{\text{vertical}} = u_1 t - \frac{1}{2}gt^2$$

$$(S_2)_{\text{vertical}} = u_2(t-3) - \frac{1}{2}g(t-3)^2$$

From Eq. (i), we have

$$\Rightarrow u_1 t - \frac{1}{2}gt^2 = u_2(t-3) - \frac{1}{2}g(t-3)^2$$

$$\Rightarrow 35t - \frac{1}{2} \times 10 \times t^2$$

$$= 35(t-3) - \frac{1}{2} \times 10(t-3)^2$$

$$\Rightarrow 35t - 35(t-3) = 5t^2 - 5(t-3)^2$$

$$\Rightarrow 35(t-t+3) = 5[t^2 - (t-3)^2]$$

$$\Rightarrow 35 \times 3 = 5(t+t-3)(t-t+3)$$

$$\Rightarrow \frac{105}{5} = (2t-3)(3)$$

$$\Rightarrow \frac{21}{3} = 2t-3 \Rightarrow 7 = 2t-3$$

$$\Rightarrow 2t = 10 \Rightarrow t = 5 \text{ s}$$

So, the height at which ball will collide is

$$(S_1)_{\text{vertical}} = u_1 t - \frac{1}{2}gt^2$$

$$= 35 \times 5 - \frac{1}{2} \times 10 \times (5)^2$$

$$= 175 - 125$$

$$= 50 \text{ m}$$

- 18** A ball is thrown up with a certain velocity, so that it reaches a height h . Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both

the directions. [2021, 27 July Shift-I]

$$(a) \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

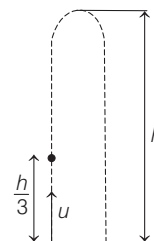
$$(b) \frac{1}{3}$$

$$(c) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$(d) \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Ans. (c)

The given scenario can be represented as follows.



$\therefore$ From Newton's third equation of motion,

When the ball is thrown up, we have

$$v^2 = u^2 + 2as$$

$$\Rightarrow v^2 = u^2 + 2(-g)h$$

$$\therefore a = -g \text{ (against gravity) and } s = h$$

$$\Rightarrow 0 = u^2 + 2(-g)h \quad \{\therefore v = 0\}$$

$$\Rightarrow u^2 = 2gh \quad \dots(i)$$

From Newton's second equation of motion, we have

$$s = ut + \frac{1}{2}at^2$$

When the ball reaches $\frac{h}{3}$ height which

coming downward

$$\Rightarrow \frac{h}{3} = ut - \frac{1}{2}gt^2$$

$$\dots(ii) \{ \therefore a = -g \text{ and } s = \frac{h}{3} \}$$

From Eqs. (i) and (ii), we get

$$\frac{h}{3} = \sqrt{2gh}t - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{1}{2}gt^2 - \sqrt{2gh}t + \frac{h}{3} = 0 \quad \dots(iii)$$

$\therefore$ Eq. (iii) is a quadratic equation, so we have to calculate its roots using formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\therefore$ On solving, we will get

$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{3}h}}{g}$$

$$\therefore \frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}} \Rightarrow \frac{t_1}{t_2} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

- 19** A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around, is (Take, the value of $g = 10 \text{ m/s}^2$)

[2021, 25 July Shift-II]

- (a) 300 m (b) 200 m
(c) 125 m (d) 250 m

Ans. (c)

Given, initial speed of balloon,

$$u_b = 10 \text{ ms}^{-1}$$

Initial height of balloon from ground, $h = 75 \text{ m}$

Initial speed of object, $u_o = -10 \text{ ms}^{-1}$

Let the time taken by object to reach ground be t .

$$\text{As, } h = ut + \frac{1}{2}gt^2$$

For object a ,

$$\Rightarrow 75 = -10t + \frac{1}{2} \times 10 \times t^2 = -10t + 5t^2$$

$$\Rightarrow t^2 - 2t - 15 = 0$$

$$\Rightarrow t^2 - 5t + 3t - 15 = 0$$

$$\Rightarrow t(t - 5) + 3(t - 5) = 0 \Rightarrow t = 5, -3$$

$$\therefore t = 5 \text{ s } (\therefore t \neq -ve)$$

$$\text{Also, } u_b = \frac{s}{t}$$

where, s is distance covered by balloon and t is time taken by object to reach ground

$$\therefore \text{Final height, } h_f = h + s$$

$$= h + u_b \times t$$

$$= 75 + 10 \times 5$$

$$= 75 + 50 = 125 \text{ m}$$

- 20** Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4th second after its fall to the next droplet is 34.3 m. At what rate, the droplets are coming from the tap? (Take, $g = 9.8 \text{ m/s}^2$)

[2021, 25 July Shift-I]

- (a) 3 drops / 2 s (b) 2 drops / s
(c) 1 drop / s (d) 1 drop / 7 s

Ans. (c)

Given, distance travelled in 4th second,

$$s_2 = 34.3 \text{ m,}$$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Let distance covered by 1st drop in time $t (= 4 \text{ s})$ be s_1 .

As we know that,

$$s = ut + \frac{1}{2}gt^2 \quad \dots(i)$$

where, u = initial speed of drop = 0 ms^{-1}

$$\Rightarrow s_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 4^2$$

$$= \frac{1}{2} \times 9.8 \times 16 = 9.8 \times 8 = 78.4 \text{ m}$$

Now, distance covered by 2nd drop will be s .

$$s = s_1 - s_2$$

$$= 78.4 - 34.3 = 44.1 \text{ m}$$

Again, By using Eq. (i)

$$s = \frac{1}{2}gt'^2$$

where, t' is the time taken by 2nd drop to cover 44.1 m.

$$\therefore 44.1 = 4.9t'^2 \Rightarrow t'^2 = \frac{44.1}{4.9}$$

$$\Rightarrow t' = \sqrt{\frac{44.1}{4.9}} = \sqrt{9} = 3 \text{ s} \quad \dots(ii)$$

Since, time taken by 1st drop is 4 s, and time taken by 2nd drop is 3 s, so one drop is falling per second.

- 21** A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be

[2021, 26 Feb Shift-II]

- (a) $\frac{a_1 + a_2}{a_2}$ (b) $\frac{a_2}{a_1}$
(c) $\frac{a_1}{a_2}$ (d) $\frac{a_1 + a_2}{a_1}$

Ans. (b)

According to given data;

Initial speed of scooter, $u_1 = 0 \text{ ms}^{-1}$

and final speed, $v_2 = 0 \text{ ms}^{-1}$

By using first equation of motion,

$$v_1 = u + a_1 t_1$$

$$\Rightarrow v_1 = a_1 t_1 \quad \dots(i)$$

and for second case,

$$v_2 = v_1 - a_2 t_2$$

$$\Rightarrow 0 = v_1 - a_2 t_2$$

$$\Rightarrow v_1 = a_2 t_2 \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$v_1 / v_1 = 1 = \frac{a_1 t_1}{a_2 t_2}$$

$$\Rightarrow a_2 t_2 = a_1 t_1$$

$$\Rightarrow t_1 / t_2 = a_2 / a_1$$

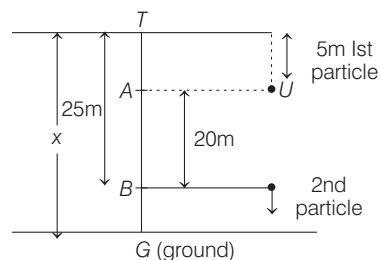
- 22** A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is

[2021, 25 Feb Shift-II]

- (a) 45 m (b) 25 m (c) 35 m (d) 50 m

Ans. (a)

Let the total height of building be x .



$TA = 5 \text{ m}$
 $TB = 25 \text{ m}$
 $\therefore AG = x - 5 \text{ and } BG = x - 25$
 For initial conditions, from second equation of motion under gravity,
 $s = ut + \frac{1}{2}gt^2$
 where, $g = 10 \text{ ms}^{-2}$
 $\therefore 5 = 0 + \frac{1}{2} \times 10t^2 \Rightarrow t = 1 \text{ s}$
 Now, by first equation of motion under gravity,
 $v_A = u + gt$
 $= 0 + 10 = 10 \text{ ms}^{-1}$
 From second equation of motion,
 $x - 5 = v_A t + \frac{1}{2}gt^2 \quad \dots (i)$
 Similarly, $x - 25 = \frac{1}{2}gt^2$
 Put the above value in Eq. (i), we get
 $x - 5 = 10t + x - 25$
 $20 = 10t$
 $\Rightarrow t = 2 \text{ s}$
 Put the value of t in Eq. (i), we get
 $x - 5 = 10 \times 2 + \frac{1}{2} \times 10 \times 4$
 $\Rightarrow x - 5 = 20 + 20$
 $\Rightarrow x = 45 \text{ m}$

- 23** An engine of a train moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v . The velocity with which middle point of the train passes the signal post is **[2021, 25 Feb Shift-I]**

(a) $\sqrt{\frac{v^2 + u^2}{2}}$ (b) $\frac{v - u}{2}$
 (c) $\frac{u + v}{2}$ (d) $\sqrt{\frac{v^2 - u^2}{2}}$

Ans. (a)

Given, initial speed of engine = u
 Speed of engine last compartment = v
 Let the length of train be l
 and the speed of mid-point of train be v' .
 Using third equation of motion,
 $v^2 = u^2 + 2as$
 $\Rightarrow v^2 = u^2 + 2al/2$
 $\Rightarrow v^2 - u^2 = al \quad \dots (i)$
 Also, $v^2 - u^2 = 2al \Rightarrow a = \frac{v^2 - u^2}{2l}$

Substituting the value of a in Eq. (i), we get

$$v^2 - u^2 = \frac{v^2 - u^2}{2l} \cdot l = \frac{v^2 - u^2}{2}$$

$$\Rightarrow v' = \sqrt{\frac{v^2 - u^2}{2}} + u = \sqrt{\frac{v^2 + u^2}{2}}$$

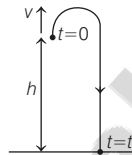
- 24** A helicopter rises from rest on the ground vertically upwards with a constant acceleration g . A food packet is dropped from the helicopter when it is at a height h . The time taken by the packet to reach the ground is close to (Here, g is the acceleration due to gravity). **[2020, 5 Sep Shift-I]**

(a) $t = \frac{2}{3}\sqrt{\frac{h}{g}}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$
 (c) $t = \sqrt{\frac{2h}{3g}}$ (d) $t = 3.4\sqrt{\frac{h}{g}}$

Ans. (d)

Let t be the time taken by the packet to reach the ground. As, the helicopter rises from rest in upward direction, its final velocity is

$$v = \sqrt{0^2 + 2gh} \Rightarrow v = \sqrt{2gh}$$



From second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Here, $s = -h$

$$u \text{ or } v = \sqrt{2gh} \Rightarrow a = g$$

Substituting all these values in above equation, we get

$$-h = \sqrt{2gh}t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \frac{1}{2}gt^2 - \sqrt{2gh}t - h = 0$$

This is a quadratic equation in t .

$$\therefore t = \frac{\sqrt{2gh} \pm \sqrt{(\sqrt{2gh})^2 - 4 \times \frac{g}{2}(-h)}}{2 \times \frac{g}{2}}$$

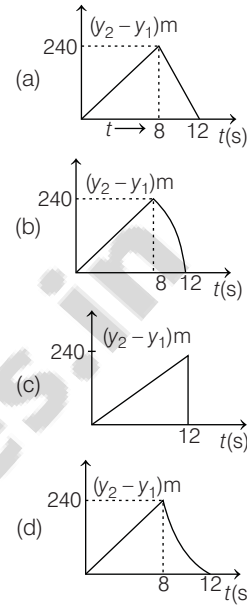
$$= \frac{\sqrt{2gh} \pm \sqrt{2gh + 2gh}}{g} = \frac{\sqrt{2gh}}{g}(1 + \sqrt{2})$$

$$= \sqrt{\frac{2h}{g}}(1 + \sqrt{2}) = (2 + \sqrt{2})\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

Hence, correct option is (d).

- 25** Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second

stone with respect to the first? Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 \text{ m/s}^2$ **[JEE Main 2015]**



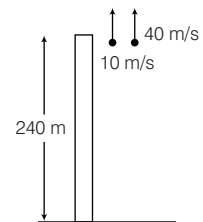
Ans. (b)

Key Idea Concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.

Consider the stones thrown up simultaneously as shown in the diagram below.

Considering motion of the second particle with respect to the first we have relative acceleration

$$|a_{21}| = |a_2 - a_1| = g - g = 0$$



Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$$-240 = 10t - \frac{1}{2} \times 10 \times t^2$$

$$\text{or } t^2 - 2t - 48 = 0$$

$$\text{or } t^2 - 8t + 6t - 48 = 0$$

$$\text{or } t = 8, -6 \quad (\text{not possible})$$

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})t = (40 - 10)(8\text{s}) = 30 \times 8 = 240 \text{ m}$$

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t^2$$

or $-240 = 40t - 5t^2$

or $5t^2 - 40t - 240 = 0$

or $t^2 - 8t - 48 = 0$

$$t^2 - 12t + 4t - 48 = 0$$

or $t(t - 12) + 4(t - 12) = 0$

or $t = 12, -4$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground.

- 26** From a tower of height H , a particle is thrown vertically upwards with a speed u . The time taken by the particle to hit the ground is n times that taken by it to reach the highest point of its path. The relation between H , u and n is [JEE Main 2014]

(a) $2gH = n^2 u^2$

(b) $gH = (n-2)^2 u^2$

(c) $2gH = nu^2 (n-2)$

(d) $gH = (n-2)^2 u^2$

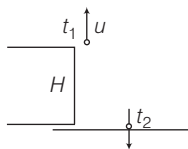
Ans. (c)

Time taken to reach the maximum height

$$t_1 = \frac{u}{g}$$

If t_2 is the time taken to hit ground

i.e., $-H = ut_2 - \frac{1}{2}gt_2^2$



But $t_2 = nt_1$ [given]

So, $-H = u \frac{nu}{g} - \frac{1}{2}g \frac{n^2 u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2} \frac{n^2 u^2}{g}$$

$$2gH = nu^2 (n-2)$$

- 27** A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2} . He reaches the ground with a speed of 3 ms^{-1} . At what height, did he bail out? [AIEEE 2005]

(a) 91 m (b) 182 m

(c) 293 m (d) 111 m

Ans. (c)

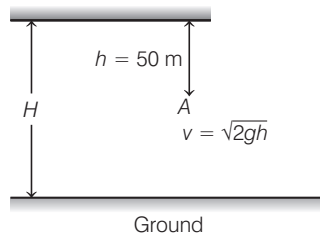
Parachute bails out at height H from ground (d) Velocity at A

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50}$$

$$= \sqrt{980} \text{ ms}^{-1}$$

Velocity at ground, $v_1 = 3 \text{ ms}^{-1}$ [given]

Acceleration $= -2 \text{ ms}^{-2}$ [given]



$$\therefore H - h = \frac{v^2 - v_1^2}{2 \times 2} = \frac{980 - 9}{4}$$

$$= \frac{971}{4} = 242.75$$

$$\Rightarrow H = 242.75 + h = 242.75 + 50 \approx 293 \text{ m}$$

- 28** A ball is released from the top of a tower of height h metre. It takes T second to reach the ground (d) What is the position of the ball in $T/3$ second? [AIEEE 2004]

(a) $h/9$ m from the ground

(b) $7h/9$ m from the ground

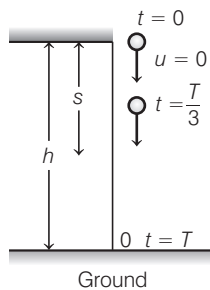
(c) $8h/9$ m from the ground

(d) $17h/18$ m from the ground

Ans. (c)

Second law of motion gives

$$s = ut + \frac{1}{2}gT^2$$



or $h = 0 + \frac{1}{2}gT^2$

$$\Rightarrow T = \sqrt{\left(\frac{2h}{g}\right)} \quad [\because u = 0]$$

$$\text{At } t = \frac{T}{3}, s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$

$$\text{or } s = \frac{1}{2}g \cdot \frac{T^2}{9}$$

$$\Rightarrow s = \frac{g}{18} \times \frac{2h}{g}$$

or $s = \frac{h}{9} \text{ m}$

$$\left[\because T = \sqrt{\frac{2h}{g}} \right]$$

Hence, the position of ball from the

$$\text{ground} = h - \frac{h}{9} = \frac{8h}{9} \text{ m}$$

- 29** An automobile travelling with a speed of 60 km/h , can brake to stop within a distance of 20 m . If the car is going twice as fast, i.e., 120 km/h , the stopping distance will be [AIEEE 2004]

(a) 20 m

(b) 40 m

(c) 60 m

(d) 80 m

Ans. (d)

The braking retardation will remain same and assumed to be constant, let it be a .

From third equation of motion,

$$v^2 = u^2 + 2as$$

$$\text{Case I} \quad 0 = \left(60 \times \frac{5}{18}\right)^2 - 2a \times s_1$$

$$\Rightarrow s_1 = \frac{(60 \times 5/18)^2}{2a}$$

$$\text{Case II} \quad 0 = \left(120 \times \frac{5}{18}\right)^2 - 2a \times s_2$$

$$\Rightarrow s_2 = \frac{(120 \times 5/18)^2}{2a}$$

$$\therefore \frac{s_1}{s_2} = \frac{1}{4} \Rightarrow s_2 = 4s_1 = 4 \times 20 = 80 \text{ m}$$

- 30** A car moving with a speed of 50 km/h , can be stopped by brakes after atleast 6 m . If the same car is moving at a speed of 100 km/h , the minimum stopping distance is [AIEEE 2003]

(a) 12 m

(b) 18 m

(c) 24 m

(d) 6 m

Ans. (c)

As the first equation of motion

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = \left(50 \times \frac{5}{18}\right)^2 + 2a \times 6$$

or $a = -16 \text{ m/s}^2$ [$\because a$ is retardation]

Again $v^2 = u^2 + 2as$

$$\Rightarrow 0 = \left(100 \times \frac{5}{18}\right)^2 - 16 \times 2 \times s$$

$$\text{or } s = \frac{(100 \times 5)^2}{18 \times 18 \times 32} \text{ or } s = 24.1 \approx 24 \text{ m}$$

- 31** Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is [AIEEE 2002]

(a) 1:1 (b) 1:4
(c) 1:8 (d) 1:16

Ans. (d)

In this question, the cars are identical means coefficient of friction between the tyre and the ground is same for both the cars, as a result retardation is same for both the cars equal to μg .

Let first car travel distance s_1 , before stopping while second car travel distance s_2 , then from

$$v^2 = u^2 - 2as$$

$$\Rightarrow 0 = u^2 - 2\mu g \times s_1$$

$$\Rightarrow s_1 = \frac{u^2}{2\mu g}$$

and $0 = (4u)^2 - 2\mu g \times s_2$

$$\Rightarrow s_2 = \frac{16u^2}{2\mu g} = 16s_1$$

$$\therefore \frac{s_1}{s_2} = \frac{1}{16}$$

- 32** From a building, two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then [AIEEE 2002]

(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses

Ans. (b)

From conservation of energy,
Potential energy at height

$$h = \text{Kinetic energy at ground}$$

Therefore, at height h , PE of ball A

$$= m_A gh$$

$$\text{KE at ground} = \frac{1}{2} m_A v_A^2$$

$$\text{So, } m_A gh = \frac{1}{2} m_A v_A^2$$

$$\text{or } v_A = \sqrt{2gh}$$

$$\text{Similarly, } v_B = \sqrt{2gh}$$

$$\text{Therefore, } v_A = v_B$$

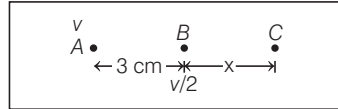
Note In this question, it is not mentioned that magnitude of thrown velocity of both balls are same which is assumed in solution.

- 33** If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? [AIEEE 2002]

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

Ans. (a)

Let initial velocity of body at point A be v , AB is 3 cm.



$$\text{From } v^2 = u^2 - 2as$$

$$\Rightarrow \left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3 \text{ or } a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.

So, for B to C, $u = v/2$, $v = 0$,

$$s = x, a = \frac{v^2}{8} \quad [\text{deceleration}]$$

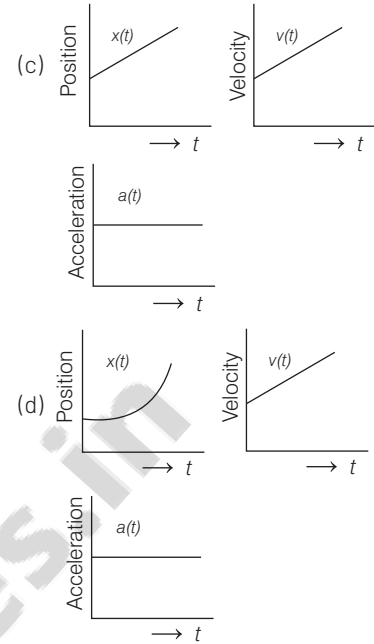
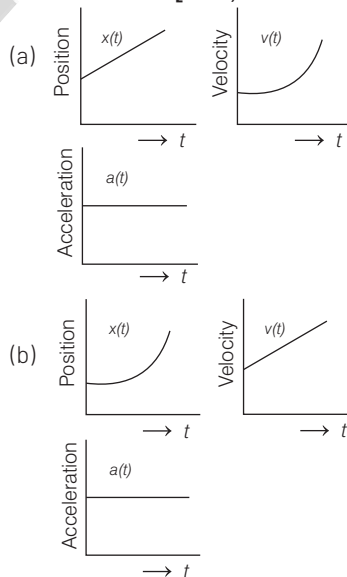
$$\therefore (0)^2 = \left(\frac{v}{2}\right)^2 - 2 \cdot \frac{v^2}{8} \cdot x$$

$$\text{or } x = 1 \text{ cm}$$

Note Here, it is assumed that retardation is uniform.

TOPIC 3 Graphs in Motion

- 34** The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by [2021, 18 March Shift-I]

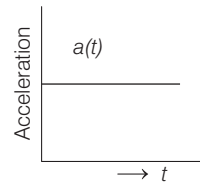


Ans. (b)

Since, the particle is moving with constant acceleration.

Acceleration = Constant, (the acceleration-time graph is parallel to the time axis, and acceleration is positive)

$$a = \frac{dv}{dt}$$



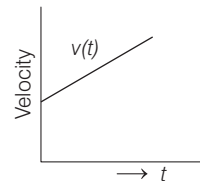
As we know,

$$\frac{dv}{dt} = C \Rightarrow \int dv = C \int dt$$

$$\Rightarrow v = Ct + C_0$$

The velocity-time graph is straight line with positive slope

$$v = \frac{dx}{dt}$$



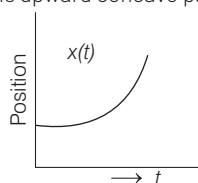
As we know,

$$\frac{dx}{dt} = Ct + C_0$$

$$\Rightarrow \int dx = \int (Ct + C_0) dt$$

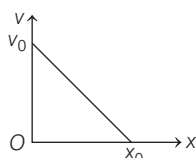
$$\Rightarrow x = \frac{Ct^2}{2} + C_0t + C_1$$

This equation shows that the distance-time graph is upward concave parabola.



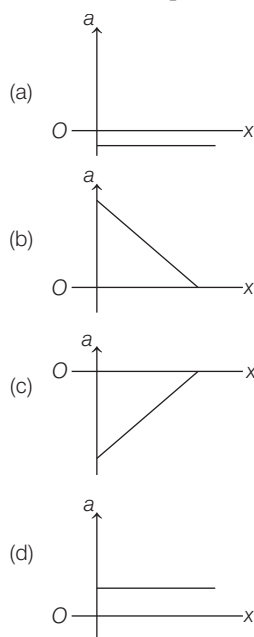
Hence, the correct option is (b).

- 35** The velocity-displacement graph of a particle is shown in the figure.



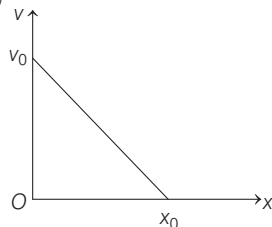
The acceleration-displacement graph of the same particle is represented by

[2021, 18 March Shift-II]



Ans. (c)

For the given velocity-displacement graph,



Intercept, $c = v_0$

$$\text{Slope, } m = -\frac{v_0}{x_0}$$

$$\therefore \text{Slope, } m = \frac{dv}{dx} = -\frac{v_0}{x_0} \quad \dots(i)$$

For given diagram, the equation of line,

$$y = mx + c$$

$$v = -\frac{v_0}{x_0}x + v_0 \quad \dots(ii)$$

$$\text{Acceleration, } a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$\Rightarrow a = v \frac{dv}{dx}$$

$$\Rightarrow a = \left(-\frac{v_0}{x_0}x + v_0 \right) \left(-\frac{v_0}{x_0} \right) \quad [\text{From Eqs. (i) and (ii)}]$$

$$\Rightarrow a = \left(\frac{v_0}{x_0} \right)^2 x - \frac{v_0^2}{x_0}$$

Hence, the intercept is negative and the slope is positive, so the correct graph is given in option (c).

- 36** A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is

[2021, 17 March Shift-I]

- (a) $\frac{4\alpha\beta}{(\alpha+\beta)} t^2$ (b) $\frac{2\alpha\beta}{(\alpha+\beta)} t^2$
(c) $\frac{\alpha\beta}{2(\alpha+\beta)} t^2$ (d) $\frac{\alpha\beta}{4(\alpha+\beta)} t^2$

Ans. (c)

Consider 1st case,

A car starts from rest at a constant acceleration α . It means

Acceleration, $a_1 = \alpha$

initial velocity, $u = 0$

$$v = u + a_1 t \Rightarrow v_0 = 0 + \alpha t_1$$

$$\Rightarrow v_0 = \alpha t_1 \quad \dots(i)$$

Consider 2nd case,

The same car decelerates after some time at a constant acceleration β and comes to rest. It means

Acceleration, $a_2 = \beta$

Final velocity = 0

$$\Rightarrow v_0 = \beta t_2 \quad \dots(ii)$$

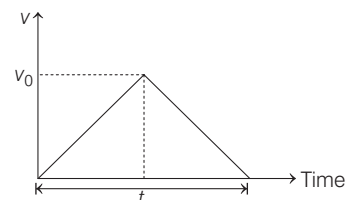
According to question,

$$t_1 + t_2 = t$$

$$\Rightarrow v_0 \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) = t \quad [\text{from Eqs. (i) \& (ii)}]$$

$$\Rightarrow v_0 = \frac{\alpha\beta t}{\alpha + \beta} \quad \dots(iii)$$

If we draw the velocity-time graph for the given situation, it will be as follows

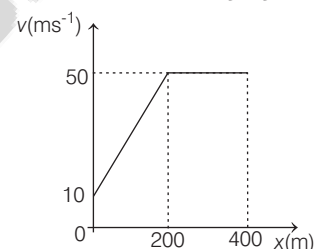


$\therefore$ Distance travelled = Area of $v-t$ graph

$$= \frac{1}{2} \times t \times v_0 = \frac{1}{2} \times t \times \frac{\alpha\beta t}{(\alpha + \beta)} \quad [\text{from Eq. (iii)}]$$

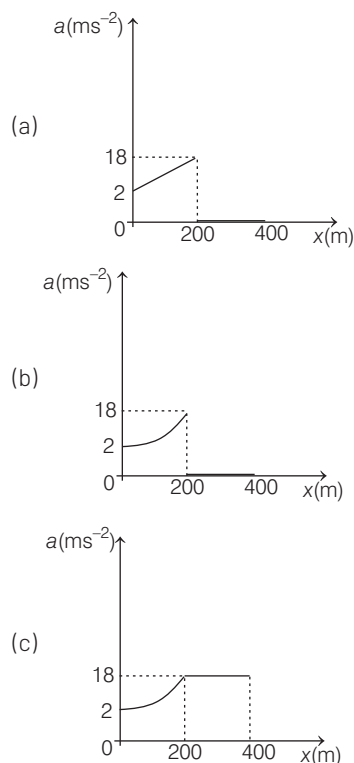
$$= \frac{1}{2} \times \frac{\alpha\beta t^2}{(\alpha + \beta)} = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

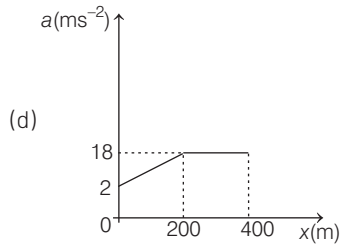
- 37** The velocity-displacement graph describing the motion of a bicycle is shown in the following figure.



The acceleration-displacement graph of the bicycle's motion is best described by

[2021, 16 March Shift-I]





Ans. (a)

From the given velocity-displacement graph, for $0 \leq x \leq 200$, the graph is a straight line i.e., varying linearly.

As, the equation of straight line is

$$y = mx + c$$

Here, it can be given as $v = mx + c$

$$\text{where, } m = \text{slope} = \frac{(50-10)}{200-0} = \frac{1}{5}$$

and $c = 10$

$$\Rightarrow v = \frac{x}{5} + 10 \quad \dots(i)$$

As, acceleration, $a = dv/dt = v dv/dx$

$$a = \left(\frac{x}{5} + 10\right) \frac{d}{dx} \left(\frac{x}{5} + 10\right) \quad [\text{using Eq. (i)}]$$

$$= \left(\frac{x}{5} + 10\right) \left(\frac{1}{5}\right) = 2 + \frac{x}{25}$$

$$\text{For } x=0, a = 2 + \frac{0}{25} = 2 \text{ ms}^{-2}$$

$$\text{and for } x=200, a = 2 + \frac{200}{25} = 10 \text{ ms}^{-2}$$

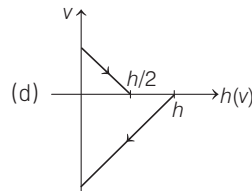
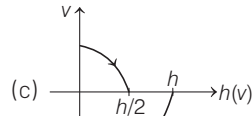
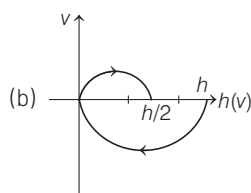
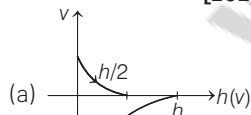
Therefore, the graph will have straight line till $x = 200$ and for $x > 200$;

$v = \text{constant}$ and $a = 0$

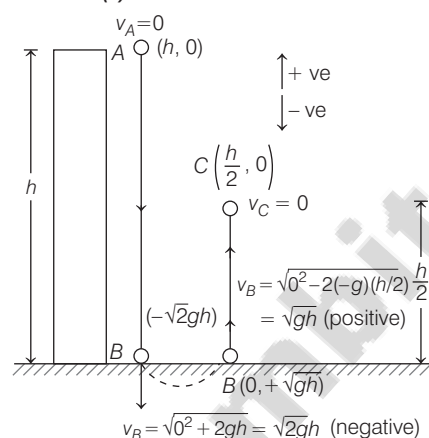
So, from the graphs given in the options, the above conditions are satisfied in option (a) only.

- 38** A tennis ball is released from a height h and after freely falling on a wooden floor, it rebounds and reaches height $h/2$. The velocity versus height of the ball during its motion may be represented graphically by (Graphs are drawn schematically and on not to scale)

[2020, 4 Sep Shift-I]



Ans. (c)



Using third equation of motion, the relation between v and h is given by,

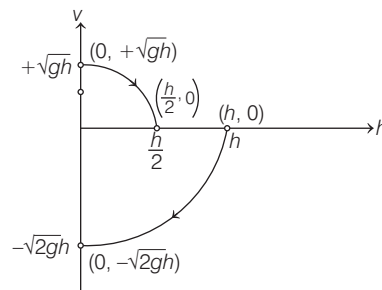
$$v^2 = u^2 + 2gh$$

$$= 0^2 + 2gh = 2gh$$

$$\Rightarrow v^2 \propto h$$

It is the equation of a parabola. So, the shape of v versus h graph will be parabolic.

Now, as we have some co-ordinate points from diagram. On plotting them and tracing with a parabolic locus, we will get the following graph :

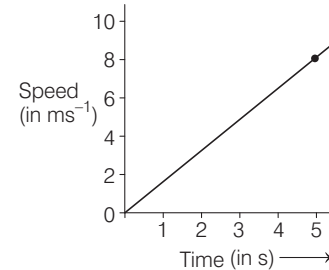


Hence, correct option is (c).

- 39** The speed versus time graph for a particle is shown in the figure. The distance travelled (in metre) by the

particle during the time interval $t = 0$ s to $t = 5$ s will be

[2020, 4 Sep Shift-II]



Ans. (20)

Distance travelled = Area under speed-time graph

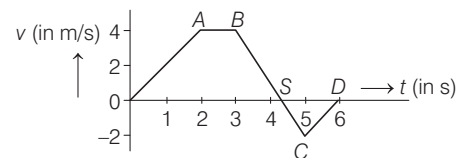
Distance travelled (from $t = 0$ s to $t = 5$ s)

$$= \text{Area of } \triangle OAB = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times (5 \text{ s}) \times (8 \text{ m/s}) = 20 \text{ m}$$

- 40** The v - t graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 s. The total distance covered by the body in 6 s is

[2020, 5 Sep Shift-II]



(a) $\frac{37}{3}$ m

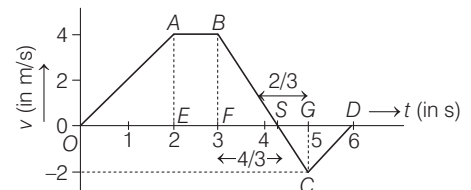
(b) 12 m

(c) 11 m

(d) $\frac{49}{4}$ m

Ans. (a)

The given v - t graph is shown below,



Distance covered by the body in 6 s = Sum of magnitudes of area of each part in v - t curve

$$s = \text{Area of } \triangle AOE + \text{Area of rectangle ABFE} + \text{Area of } \triangle BSF + \text{Area of } \triangle SCG + \text{Area of } \triangle GCD$$

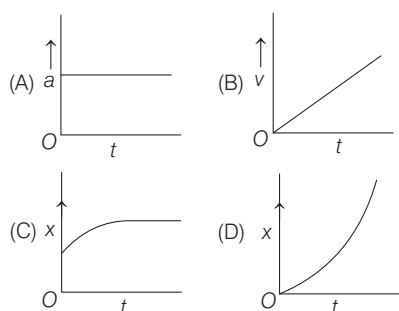
$$= \frac{1}{2}(OE \times AE) + (EF \times AE) + \frac{1}{2}(SF \times BF) + \frac{1}{2}(SG \times GC) + \frac{1}{2}(GD \times GC)$$

$$\begin{aligned}
 &= \frac{1}{2}(2 \times 4) + 1 \times 4 + \frac{1}{2} \times 4 \times \left(\frac{4}{3}\right) \\
 &\quad + \frac{1}{2} \times 2 \times \left(\frac{2}{3}\right) + \frac{1}{2} \times 2 \times 1 \\
 &= 4 + 4 + \frac{8}{3} + \frac{2}{3} + 1 = \frac{37}{3} \text{ m}
 \end{aligned}$$

Hence, correct option is (a).

- 41** A particle starts from origin O from rest and moves with a uniform acceleration along the positive X -axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

[2019, 8 April Shift-II]



- (a) (A) (b) (A), (B), (C)
(c) (B), (C) (d) (A), (B), (D)

Ans. (d)

Since, the particle starts from rest, this means, initial velocity, $u = 0$
Also, it moves with uniform acceleration along positive X -axis. This means, its acceleration (a) is constant.

$\therefore$ Given, $a - t$ graph in (A) is correct.

As we know, for velocity-time graph, slope = acceleration.

Since, the given $v - t$ graph in (B) represents that its slope is constant and non-zero.

$\therefore$ Graph in (B) is also correct.

Also, the displacement of such a particle w.r.t. time is given by

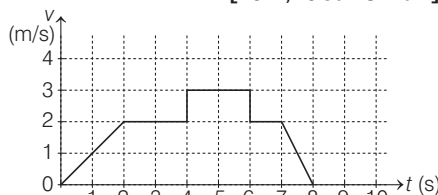
$$\begin{aligned}
 x &= ut + \frac{1}{2}at^2 \\
 &= 0 + \frac{1}{2}at^2 \Rightarrow x \propto t^2
 \end{aligned}$$

So, x versus t graph would be a parabola with starting from origin. This is correctly represented in displacement-time graph given in (D).

- 42** A particle starts from the origin at time $t = 0$ and moves along the positive X -axis. The graph of velocity with respect to time is

shown in figure. What is the position of the particle at time $t = 5$ s?

[2019, 10 Jan Shift-II]

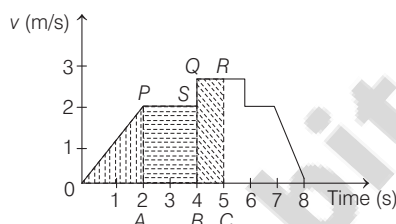


- (a) 6 m (b) 3 m (c) 10 m (d) 9 m

Ans. (d)

Key Idea Area under the velocity-time curve represents displacement.

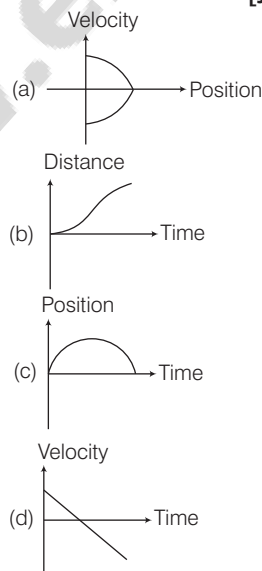
To get exact position at $t = 5$ s, we need to calculate area of the shaded part in the curve as shown below



$$\begin{aligned}
 \therefore \text{Displacement of particle} &= \text{Area of } OPA \\
 &+ \text{Area of } PABSP + \text{Area of } QBCRQ \\
 &= \left(\frac{1}{2} \times 2 \times 2\right) + (2 \times 2) + (3 \times 1) \\
 &= 2 + 4 + 3 = 9 \text{ m}
 \end{aligned}$$

- 43** All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

[JEE Main 2018]



Ans. (b)

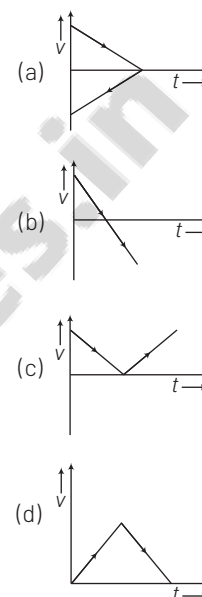
If velocity versus time graph is a straight line with negative slope, then acceleration is constant and negative.

With a negative slope distance-time graph will be parabolic $\left(s = ut - \frac{1}{2}at^2\right)$.

So, option (b) will be incorrect.

- 44** A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

[JEE Main 2017]

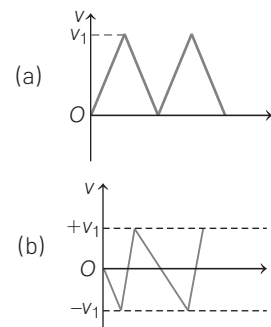


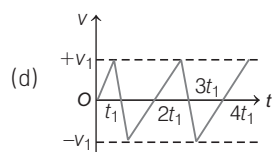
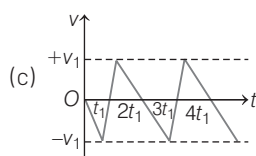
Ans. (b)

Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

- 45** Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then, the velocity as a function of time the height as function of time will be

[AIIEEE 2009]





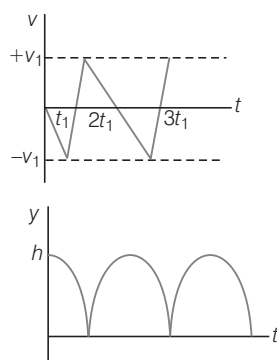
Ans. (c)

As we know that for vertical motion,

$$h = \frac{1}{2}gt^2 \quad [\text{parabolic}]$$

$v = -gt$ and after collision, $v = gt$ (straight line).

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.

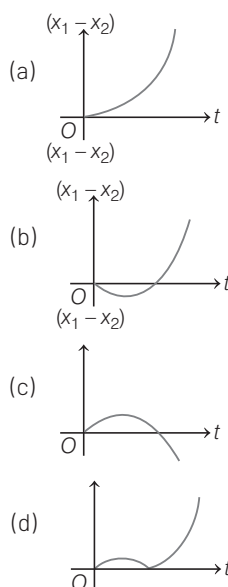


Hence, option (c) is true.

- 46** A body is at rest at $x=0$. At $t=0$, it starts moving in the positive x -direction with a constant acceleration. At the same instant, another body passes through $x=0$ moving in the positive x -direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly

describes $(x_1 - x_2)$ as a function of time?

[AIEEE 2008]



Ans. (b)

Here, $x_2 = vt$ and $x_1 = \frac{at^2}{2}$

$$\therefore x_1 - x_2 = -\left(vt - \frac{at^2}{2}\right)$$

From the above expression, it is clear that at $t=0$, $x_1 - x_2 = 0$ further for increasing values, the graph is as follows.

Hence, option (c) is true.

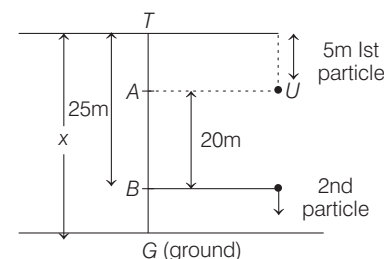
- 47** A car starting from rest, accelerates at the rate f through a distance s , then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance travelled is $15s$, then [AIEEE 2005]

- (a) $s = ft$
 (b) $s = \frac{1}{6}ft^2$
 (c) $s = \frac{1}{2}ft^2$
 (d) None of the above

Ans. (d)

The velocity-time graph for the given situation can be drawn as below.

Magnitudes of slope of $OA = f$



and slope of $BC = \frac{f}{2}$

$$\Rightarrow v = ft_1 = \frac{f}{2}t_2$$

$$\therefore t_2 = 2t_1$$

In graph, area of $\triangle OAD$ gives distance,

$$s = \frac{1}{2}ft_1^2 \quad \dots(i)$$

Area of rectangle $ABED$ gives distance travelled in time t .

$$s_2 = (ft_1)t$$

Distance travelled in time t_2 ,

$$s_3 = \frac{1}{2} \frac{f}{2} (2t_1)^2$$

Thus, $s_1 + s_2 + s_3 = 15s$

$$\Rightarrow s + (ft_1)t + ft_1^2 = 15s$$

$$\text{or } s + (ft_1)t + 2s = 15s \quad \left[\because s = \frac{1}{2}ft_1^2 \right]$$

$$\text{or } (ft_1)t = 12s \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

$$\text{or } t_1 = \frac{t}{6}$$

From Eqs. (i) and (ii), we get

$$s = \frac{1}{2}f(t_1)^2$$

$$\Rightarrow s = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{1}{72}ft^2$$

Hence, none of the given options is correct.

03

Motion in a Plane

TOPIC 1

Vectors

01 Statement I Two forces $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$ where $\mathbf{P} \perp \mathbf{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2 + Q^2)}$, when they act at an angle θ_2 , the magnitude of their resultant becomes $\sqrt{2(P^2 + Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II In the situation given above. $\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$.

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 31 Aug Shift-II]

- (a) Statement I is false but statement II is true.
- (b) Both statement I and statement II are true.
- (c) Statement I is true but statement II is false.
- (d) Both statement I and statement II are false.

Ans. (b)

Given, force vectors $\mathbf{P} \perp \mathbf{Q}$, i.e. $\theta = 90^\circ$

Let resultant of $\mathbf{P} + \mathbf{Q} = \mathbf{x}$

and resultant of $\mathbf{P} - \mathbf{Q} = \mathbf{y}$

$$\therefore x^2 = P^2 + Q^2 + 2PQ \cos 90^\circ = P^2 + Q^2 \quad \dots(i)$$

$$y^2 = P^2 + Q^2 - 2PQ \cos 90^\circ = P^2 + Q^2 \quad \dots(ii)$$

When θ_1 is the angle between $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$, then, their resultant is given by

$$|\mathbf{x} + \mathbf{y}| = \sqrt{3(P^2 + Q^2)}$$

$$\therefore \sqrt{x^2 + y^2 + 2xy \cos \theta_1} = \sqrt{3P^2 + 3Q^2} \quad \dots(iii)$$

Substituting the values of x^2 and y^2 in Eq. (iii), we get

$$P^2 + Q^2 + P^2 + Q^2 + 2\sqrt{P^2 + Q^2} \sqrt{P^2 + Q^2} \cos \theta_1 = 3P^2 + 3Q^2$$

$$2P^2 + 2Q^2 + (2P^2 + 2Q^2) \cos \theta_1 = 3P^2 + 3Q^2$$

$$\Rightarrow \cos \theta_1 = \frac{P^2 + Q^2}{2(P^2 + Q^2)} = \frac{1}{2}$$

$$\therefore \theta_1 = 60^\circ$$

When θ_2 is the angle between $(\mathbf{P} + \mathbf{Q})$ and $(\mathbf{P} - \mathbf{Q})$, then the magnitude of their resultant is given by

$$|\mathbf{x} + \mathbf{y}| = \sqrt{2(P^2 + Q^2)}$$

$$x^2 + y^2 + 2xy \cos \theta_2 = 2P^2 + 2Q^2 \quad \dots(iv)$$

Substituting the values, we get

$$P^2 + Q^2 + P^2 + Q^2 + 2(P^2 + Q^2) \cos \theta_2 = 2P^2 + 2Q^2$$

$$\cos \theta_2 = 0 \Rightarrow \theta_2 = 90^\circ$$

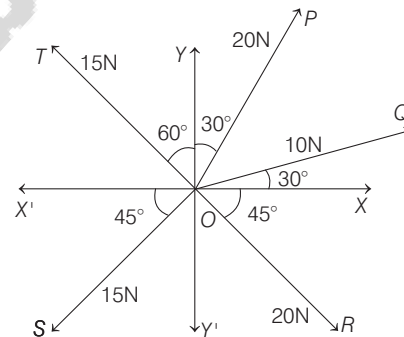
So, $\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$ and $\theta_1 < \theta_2$

Hence, both statement I and statement II are true.

02 The resultant of these forces $\mathbf{OP}$, $\mathbf{OQ}$, $\mathbf{OR}$, $\mathbf{OS}$ and $\mathbf{OT}$ is approximately N.

[Take, $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$ and given $\hat{i}$ and $\hat{j}$ unit vectors along X, Y axis]

[2021, 27 Aug Shift-I]



(a) $9.25\hat{i} + 5\hat{j}$

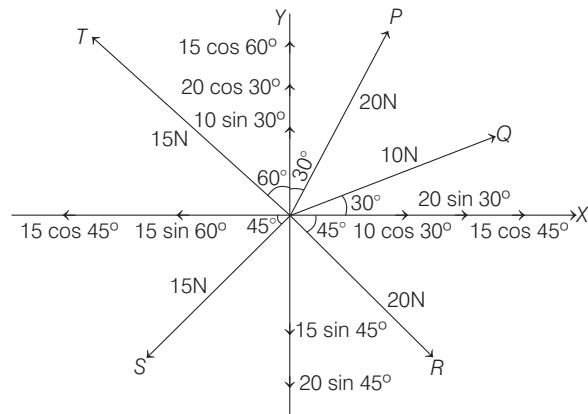
(b) $3\hat{i} + 15\hat{j}$

(c) $2.5\hat{i} - 14.5\hat{j}$

(d) $-1.5\hat{i} - 15.5\hat{j}$

Ans. (a)

From given figure, we can write all the forces in vector notation as



$$\mathbf{OP} = (20 \sin 30^\circ \hat{i} + 20 \cos 30^\circ \hat{j}) \text{ N}$$

$$\begin{aligned}
 &= (10\hat{i} + 17.32\hat{j}) \text{ N} \\
 \mathbf{OQ} &= (10 \cos 30^\circ \hat{i} + 10 \sin 30^\circ \hat{j}) \text{ N} \\
 &= (8.66\hat{i} + 5\hat{j}) \text{ N} \\
 \mathbf{OR} &= (20 \cos 45^\circ \hat{i} - 20 \sin 45^\circ \hat{j}) \text{ N} \\
 &= (14.14\hat{i} - 14.14\hat{j}) \text{ N} \\
 \mathbf{OS} &= (-15 \cos 45^\circ \hat{i} - 15 \sin 45^\circ \hat{j}) \text{ N} \\
 &= (-10.60\hat{i} - 10.60\hat{j}) \text{ N} \\
 \mathbf{OT} &= (-15 \sin 60^\circ \hat{i} + 15 \cos 60^\circ \hat{j}) \text{ N} \\
 &= (-12.99\hat{i} + 7.5\hat{j}) \text{ N}
 \end{aligned}$$

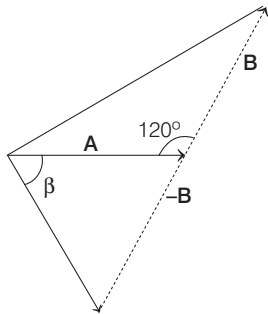
The resultant of these forces is given as

$$\begin{aligned}
 \mathbf{F} &= \mathbf{OP} + \mathbf{OQ} + \mathbf{OR} + \mathbf{OS} + \mathbf{OT} \\
 \mathbf{F} &= [(10\hat{i} + 17.32\hat{j}) + (8.66\hat{i} + 5\hat{j}) \\
 &\quad + (14.14\hat{i} - 14.14\hat{j}) + (-10.60\hat{i} - 10.60\hat{j}) \\
 &\quad + (-12.99\hat{i} + 7.5\hat{j})] \\
 \mathbf{F} &= 9.21\hat{i} + 5.08\hat{j} \text{ N} \\
 \mathbf{F} &\approx 9.25\hat{i} + 5\hat{j} \text{ N}
 \end{aligned}$$

Thus, the resultant force is

$$\mathbf{F} \approx 9.25\hat{i} + 5\hat{j} \text{ N}$$

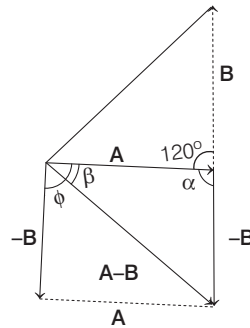
- 03** The angle between vector $\mathbf{A}$ and $(\mathbf{A} - \mathbf{B})$ is [2021, 26 Aug Shift-II]



$$\begin{aligned}
 \text{(a)} & \tan^{-1} \left(\frac{-\frac{B}{2}}{A - B \frac{\sqrt{3}}{2}} \right) \\
 \text{(b)} & \tan^{-1} \left(\frac{A}{0.7B} \right) \\
 \text{(c)} & \tan^{-1} \left(\frac{\sqrt{3}B}{2A - B} \right) \\
 \text{(d)} & \tan^{-1} \left(\frac{B \cos \theta}{A - B \sin \theta} \right)
 \end{aligned}$$

Ans. (c)

Given, angle between $\mathbf{A}$ and $\mathbf{B}$, $\theta = 120^\circ$.
 The angle between $\mathbf{A}$ and $-\mathbf{B}$,
 $\alpha = 180^\circ - \theta = 180^\circ - 120^\circ = 60^\circ$
 Now, from diagram the resultant vector of $\mathbf{A}$ and $-\mathbf{B}$ will be $\mathbf{A} - \mathbf{B}$ and the angle between $\mathbf{A} - \mathbf{B}$ and $\mathbf{A}$ is denoted by β .



So, angle between $\mathbf{A}$ and $-\mathbf{B}$ is calculated as,

$$\phi = 180^\circ - \alpha = 180^\circ - 60^\circ = 120^\circ$$

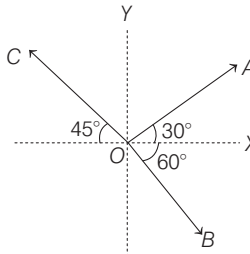
Now, from parallelogram law of vector addition angle β can be calculated as follows

$$\begin{aligned}
 \tan \beta &= \frac{-B \sin \phi}{-A + (-B \cos \phi)} \\
 &= \frac{-B \sin 120^\circ}{-A - B \cos 120^\circ} = \frac{-B \frac{\sqrt{3}}{2}}{-A + \frac{B}{2}} \\
 &= \frac{\sqrt{3}B}{2A - B}
 \end{aligned}$$

$$\Rightarrow \beta = \tan^{-1} \left(\frac{\sqrt{3}B}{2A - B} \right)$$

Hence, the angle between vector $\mathbf{A}$ and $(\mathbf{A} - \mathbf{B})$ is $\tan^{-1} \left(\frac{\sqrt{3}B}{2A - B} \right)$.

- 04** The magnitude of vectors $\mathbf{OA}$, $\mathbf{OB}$, and $\mathbf{OC}$ in the given figure are equal. The direction of $\mathbf{OA} + \mathbf{OB} - \mathbf{OC}$ with X-axis will be [2021, 26 Aug Shift-I]

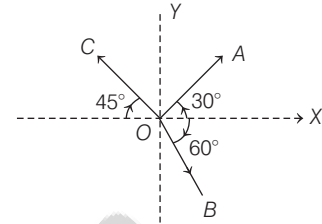


$$\begin{aligned}
 \text{(a)} & \tan^{-1} \frac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})} \\
 \text{(b)} & \tan^{-1} \frac{(\sqrt{3} - 1 + \sqrt{2})}{(1 + \sqrt{3} - \sqrt{2})} \\
 \text{(c)} & \tan^{-1} \frac{(\sqrt{3} - 1 + \sqrt{2})}{(1 - \sqrt{3} + \sqrt{2})} \\
 \text{(d)} & \tan^{-1} \frac{(1 + \sqrt{3} - \sqrt{2})}{(1 - \sqrt{3} - \sqrt{2})}
 \end{aligned}$$

Ans. (a)

In the given figure, the magnitude of vectors $\mathbf{OA}$, $\mathbf{OB}$ and $\mathbf{OC}$ are equal. Let it be m .

From figure, we can write



$$\begin{aligned}
 \mathbf{OA} &= m(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) \\
 &= m \left(\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{OB} &= m(\cos 60^\circ \hat{i} - \sin 60^\circ \hat{j}) \\
 &= m \left(\frac{1}{2} \hat{i} - \frac{\sqrt{3}}{2} \hat{j} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{OC} &= m(\cos 45^\circ (-\hat{i}) + \sin 45^\circ \hat{j}) \\
 &= m \left\{ \frac{-1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right\}
 \end{aligned}$$

According to the question,

$$\begin{aligned}
 \mathbf{OA} + \mathbf{OB} - \mathbf{OC} &= m \left(\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j} \right) \\
 &\quad + m \left(\frac{1}{2} \hat{i} - \frac{\sqrt{3}}{2} \hat{j} \right) - m \left(-\frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right) \\
 &= m \hat{i} \left(\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}} \right) + m \hat{j} \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \right)
 \end{aligned}$$

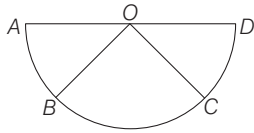
$\therefore$ Angle with X-axis is given by

$$\begin{aligned}
 \tan \theta &= \frac{y}{x} = \frac{m \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \right)}{m \left(\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}} \right)} \\
 \Rightarrow \tan \theta &= \frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2}} \\
 \Rightarrow \tan \theta &= \frac{1 - \sqrt{3} - \sqrt{2}}{\sqrt{3} + 1 + \sqrt{2}} \\
 \Rightarrow \theta &= \tan^{-1} \left(\frac{1 - \sqrt{3} - \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}} \right)
 \end{aligned}$$

- 05 Assertion (A)** If A, B, C, D are four points on a semi-circular arc with centre at O such that $|AB| = |BC| = |CD|$, then $\mathbf{AB} + \mathbf{AC} + \mathbf{AD} = 4\mathbf{AO} + \mathbf{OB} + \mathbf{OC}$
Reason (R) Polygon law of vector addition yields

$$AB + BC + CD + AD = 2AO$$

[2021, 27 July Shift-I]



In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both A and R are correct and R is the correct explanation of A.
 (b) Both A and R are correct but R is not the correct explanation of A.
 (c) A is correct but R is not correct.
 (d) A is not correct but R is correct.

Ans. (c)

Applying triangular law of vector addition in the given figure in question, we can write

$$\begin{aligned} OA + AB &= OB & \dots(i) \\ OB + BC &= OC & \dots(ii) \\ OC + CD &= OD & \dots(iii) \end{aligned}$$

Also,

$$\begin{aligned} AB &= OB - OA & \dots(iv) \\ AC &= OC - OA & \dots(v) \\ AD &= OD - OA & \dots(vi) \\ &= -OA - OA & \dots(vi) \end{aligned}$$

On adding Eqs. (iv), (v) and (vi), we get

$$\begin{aligned} AB + AC + AD &= OB + OC - 4OA \\ &= 4AO + OB + OC & \dots(vii) \end{aligned}$$

According to polygon law of vector addition,

$$\begin{aligned} AB + BC + CD + AD &= (OB - OA) + (OC - OB) \\ &\quad + (OD - OA) + 2OA \\ &= OC + OD = OC + AO \end{aligned}$$

Hence, A is correct but R is not correct.

- 06** Two vectors $\mathbf{x}$ and $\mathbf{y}$ have equal magnitude. The magnitude of $(\mathbf{x} - \mathbf{y})$ is n times the magnitude of $(\mathbf{x} + \mathbf{y})$. The angle between $\mathbf{x}$ and $\mathbf{y}$ is [2021, 25 July Shift-II]

- (a) $\cos^{-1}\left(\frac{-n^2 - 1}{n^2 - 1}\right)$ (b) $\cos^{-1}\left(\frac{n^2 - 1}{-n^2 - 1}\right)$
 (c) $\cos^{-1}\left(\frac{n^2 + 1}{-n^2 - 1}\right)$ (d) $\cos^{-1}\left(\frac{n^2 + 1}{n^2 - 1}\right)$

Ans. (b)

Given,

$\mathbf{x}$ and $\mathbf{y}$ are equal vector.

$$\therefore |\mathbf{x}| = |\mathbf{y}| \quad \dots(i)$$

$$\text{and } |\mathbf{x} - \mathbf{y}| = n|\mathbf{x} + \mathbf{y}| \quad \dots(ii)$$

As we know that,

$$|\mathbf{x} - \mathbf{y}| = \sqrt{x^2 + y^2 - 2xy \cos \theta}$$

$$\text{and } |\mathbf{x} + \mathbf{y}| = \sqrt{x^2 + y^2 + 2xy \cos \theta}$$

Put value of $|\mathbf{x} - \mathbf{y}|$ and $|\mathbf{x} + \mathbf{y}|$ in Eq. (ii), we get

$$\begin{aligned} \sqrt{x^2 + y^2 - 2xy \cos \theta} \\ = n\sqrt{x^2 + y^2 + 2xy \cos \theta} \end{aligned}$$

On squaring both sides, we get

$$\begin{aligned} x^2 + y^2 - 2|\mathbf{x}||\mathbf{y}|\cos \theta \\ = n^2(x^2 + y^2 + 2|\mathbf{x}||\mathbf{y}|\cos \theta) \end{aligned}$$

Now, $|\mathbf{x}| = |\mathbf{y}|$

$$\therefore x^2 + x^2 - 2x^2 \cos \theta$$

$$= n^2(x^2 + x^2 + 2x^2 \cos \theta)$$

$$\Rightarrow 2x^2(1 - \cos \theta) = 2n^2x^2(1 + \cos \theta)$$

$$\Rightarrow (1 - \cos \theta) = n^2(1 + \cos \theta)$$

$$\Rightarrow 1 - \cos \theta = n^2 + n^2 \cos \theta$$

$$\Rightarrow -n^2 \cos \theta - \cos \theta = n^2 - 1$$

$$\Rightarrow \cos \theta(-n^2 - 1) = n^2 - 1$$

$$\Rightarrow \cos \theta = \frac{n^2 - 1}{-n^2 - 1}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{n^2 - 1}{-n^2 - 1}\right)$$

07 Match List I with List II.

[2021, 25 July Shift-1]

List I	List II
(A) $\mathbf{C} - \mathbf{A} - \mathbf{B} = 0$	(i)
(B) $\mathbf{A} - \mathbf{C} - \mathbf{B} = 0$	(ii)
(C) $\mathbf{B} - \mathbf{A} - \mathbf{C} = 0$	(iii)
(D) $\mathbf{A} + \mathbf{B} = -\mathbf{C}$	(iv)

Choose the correct answer from the options given below.

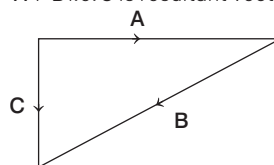
- (a) (A) $\rightarrow$ (iv), (B) $\rightarrow$ (i), (C) $\rightarrow$ (iii), (D) $\rightarrow$ (ii)
 (b) (A) $\rightarrow$ (iv), (B) $\rightarrow$ (iii), (C) $\rightarrow$ (i), (D) $\rightarrow$ (ii)
 (c) (A) $\rightarrow$ (iii), (B) $\rightarrow$ (ii), (C) $\rightarrow$ (iv), (D) $\rightarrow$ (i)
 (d) (A) $\rightarrow$ (i), (B) $\rightarrow$ (iv), (C) $\rightarrow$ (ii), (D) $\rightarrow$ (iii)

Ans. (b)

Using triangular law of vector addition for each case,

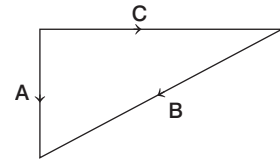
$$(A) \mathbf{C} - \mathbf{A} - \mathbf{B} = 0$$

$$\Rightarrow \mathbf{C} = \mathbf{A} + \mathbf{B} \text{ i.e. } \mathbf{C} \text{ is resultant vector.}$$



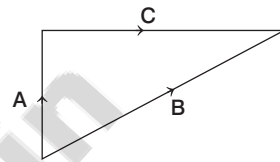
$$(B) \mathbf{A} - \mathbf{C} - \mathbf{B} = 0$$

$$\Rightarrow \mathbf{A} = \mathbf{C} + \mathbf{B} \text{ i.e. } \mathbf{A} \text{ is resultant vector.}$$



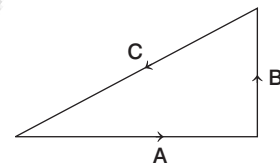
$$(C) \mathbf{B} - \mathbf{A} - \mathbf{C} = 0$$

$$\Rightarrow \mathbf{B} = \mathbf{A} + \mathbf{C} \text{ i.e. } \mathbf{B} \text{ is resultant vector.}$$



$$(D) \mathbf{A} + \mathbf{B} = -\mathbf{C}$$

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = 0 \text{ i.e. these are cyclic vectors.}$$



Hence, the correct match is A - (iv), B - (iii), C - (i) and D - (ii).

- 08** What will be the projection of vector $\mathbf{A} = \hat{i} + \hat{j} + \hat{k}$ on vector

$$\mathbf{B} = \hat{i} + \hat{j}?$$

[2021, 22 July Shift-II]

- (a) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$ (b) $2(\hat{i} + \hat{j} + \hat{k})$
 (c) $\sqrt{2}(\hat{i} + \hat{j})$ (d) $(\hat{i} + \hat{j})$

Ans. (d)

Given,

$$\mathbf{A} = \hat{i} + \hat{j} + \hat{k} \text{ and } \mathbf{B} = \hat{i} + \hat{j}$$

As we know that, vector projection of $\mathbf{A}$ on $\mathbf{B}$ is $\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{B}|} \cdot \frac{\mathbf{B}}{|\mathbf{B}|}$

Therefore, projection of $\mathbf{A}$ on $\mathbf{B}$ will be

$$\begin{aligned} \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}} \cdot \frac{(\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}} \\ = \frac{1 + 1 + 0}{\sqrt{2}} \cdot \frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{2}{2}(\hat{i} + \hat{j}) = (\hat{i} + \hat{j}) \end{aligned}$$

- 09** Two vectors $\mathbf{P}$ and $\mathbf{Q}$ have equal magnitudes. If the magnitude of $\mathbf{P} + \mathbf{Q}$ is n times the magnitude of $\mathbf{P} - \mathbf{Q}$, then angle between $\mathbf{P}$ and $\mathbf{Q}$ is [2021, 20 July Shift-II]

- (a) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$ (b) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$
 (c) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (d) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Ans. (d)

According to question,

$$|P| = |Q| \quad \dots(i)$$

$$\text{Also, } |P+Q| = n|P-Q| \quad \dots(ii)$$

On squaring Eq. (ii) both sides, we get

$$\begin{aligned} |P+Q|^2 &= n^2 |P-Q|^2 \\ \Rightarrow P^2 + Q^2 + 2PQ \cos\theta &= n^2 (P^2 + Q^2 - 2PQ \cos\theta) \end{aligned}$$

$$\Rightarrow 1 + 1 + 2 \cos\theta = n^2 (1 + 1 - 2 \cos\theta)$$

$$\Rightarrow 2 + 2 \cos\theta = n^2 (2 - 2 \cos\theta)$$

$$\Rightarrow 1 + \cos\theta = n^2 (1 - \cos\theta)$$

$$\Rightarrow \frac{1 + \cos\theta}{1 - \cos\theta} = \frac{n^2}{1} \Rightarrow \frac{1}{\cos\theta} = \frac{n^2 + 1}{n^2 - 1}$$

$$\Rightarrow \cos\theta = \frac{n^2 - 1}{n^2 + 1} \Rightarrow \theta = \cos^{-1} \left(\frac{n^2 - 1}{n^2 + 1} \right)$$

- 10** If A and B are two vectors satisfying the relation $A \cdot B = |A \times B|$. Then, the value of $|A - B|$ will be

[2021, 20 July Shift-I]

$$(a) \sqrt{A^2 + B^2}$$

$$(b) \sqrt{A^2 + B^2 + \sqrt{2}AB}$$

$$(c) \sqrt{A^2 + B^2 + 2AB}$$

$$(d) \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

Ans. (d)

$$\text{Given, } |A \cdot B| = |A \times B| \quad \dots(i)$$

Also, we know that

$$A \cdot B = AB \cos\theta \quad \dots(ii)$$

$$\text{and } A \times B = AB \sin\theta \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\begin{aligned} AB \cos\theta &= AB \sin\theta \\ \Rightarrow \cos\theta &= \sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1 \end{aligned}$$

$$\Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

$$\begin{aligned} \therefore |A - B| &= \sqrt{A^2 + B^2 - 2AB \cos\theta} \\ &= \sqrt{A^2 + B^2 - 2AB \cos(45^\circ)} \\ &= \sqrt{A^2 + B^2 - \sqrt{2}AB} \end{aligned}$$

- 11** If $P \times Q = Q \times P$, the angle between P and Q is θ ($0^\circ < \theta < 360^\circ$). The value of θ will be

[2021, 25 Feb Shift-II]**Ans. (180)**

$$\text{Given, } P \times Q = Q \times P$$

$$\Rightarrow P \times Q = -P \times Q \Rightarrow 2(P \times Q) = 0$$

$$\text{or } P \times Q = 0$$

It is possible only, if $P = 0$ or $Q = 0$, angle between them is 180° .

$$\therefore \theta = 180^\circ$$

- 12** The sum of two forces P and Q is R such that $|R| = |P|$. The angle θ (in degrees) that the resultant of $2P$ and Q will make with Q is, ...

[2020, 7 Jan Shift-II]**Ans. (90°)**Given, sum of P and Q is R . Let angle between P and Q is β .Then, resultant of P and Q ,

$$|R| = \sqrt{|P|^2 + |Q|^2 + 2|P||Q|\cos\beta}$$

$$\text{As, } |R| = |P| \quad (\text{given})$$

$$\text{So, } |P|^2 = |P|^2 + |Q|^2 + 2|P||Q|\cos\beta$$

$$\text{or } |P|\cos\beta = -\frac{Q}{2} \quad \dots(i)$$

If resultant of $2P$ and Q makes angle θ with Q , then angle θ is given by

$$\tan\theta = \frac{|2P|\sin\beta}{|Q| + |2P|\cos\beta}$$

Substituting the value of $|P|\cos\beta$ from Eq. (i) in above equation, we get

$$\tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2} = 90^\circ$$

- 13** Let $|A_1| = 3, |B_2| = 5$ and

 $|A_1 + A_2| = 5$. The value of $(2A_1 + 3A_2) \cdot (3A_1 - 2A_2)$ is
[2019, 8 April Shift-II]

$$(a) -106.5 \quad (b) -112.5$$

$$(c) -99.5 \quad (d) -118.5$$

Ans. (d)For vector $A_1 + A_2$, we have

$$|A_1 + A_2|^2 = (A_1 + A_2) \cdot (A_1 + A_2) \quad [\because x \cdot x = |x|^2]$$

$$\Rightarrow |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2A_1 \cdot A_2$$

$$\text{Given, } |A_1| = 3, |A_2| = 5$$

$$\text{and } |A_1 + A_2| = 5$$

$$\text{So, we have, } (5)^2 = 9 + 25 + 2A_1 \cdot A_2$$

$$\Rightarrow A_1 \cdot A_2 = -\frac{9}{2}$$

$$\text{Now, } (2A_1 + 3A_2) \cdot (3A_1 - 2A_2)$$

$$= 6|A_1|^2 - 4A_1 \cdot A_2 + 9A_1 \cdot A_2 - 6|A_2|^2$$

$$= 6|A_1|^2 - 6|A_2|^2 + 5A_1 \cdot A_2$$

Substituting values, we have

$$(2A_1 + 3A_2) \cdot (3A_1 - 2A_2)$$

$$= 6(9) - 6(25) + 5\left(-\frac{9}{2}\right)$$

$$= -118.5$$

Alternate SolutionAs we know, $|A_1 + A_2|$ can also be written as

$$|A_1 + A_2| = \sqrt{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos\theta}$$

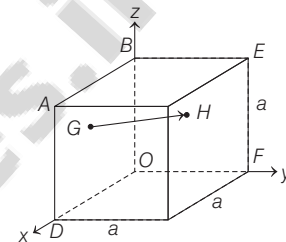
Substituting the given values, we get

$$5 = \sqrt{(3)^2 + (5)^2 + 2 \times 3 \times 5 \cos\theta}$$

$$\text{or } \cos\theta = -\frac{9}{2 \times 3 \times 5} = -\frac{3}{10}$$

$$\begin{aligned} \text{So, } (2A_1 + 3A_2) \cdot (3A_1 - 2A_2) &= 6|A_1|^2 - 6|A_2|^2 + 5A_1 \cdot A_2 \\ &= 6|A_1|^2 - 6|A_2|^2 + 5|A_1||A_2|\cos\theta \\ &= 6 \times 9 - 6 \times 25 + 5 \times 2 \times 3 \times \left(-\frac{3}{10}\right) \\ &= -118.5 \end{aligned}$$

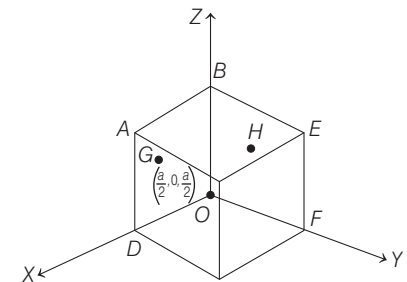
- 14** In the cube of side ' a ' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be

[2019, 10 Jan Shift-I]

- (a) $\frac{1}{2}a(\hat{i} - \hat{k})$ (b) $\frac{1}{2}a(\hat{j} - \hat{i})$
(c) $\frac{1}{2}a(\hat{j} - \hat{k})$ (d) $\frac{1}{2}a(\hat{k} - \hat{i})$

Ans. (b)

In the given cube, coordinates of point G (centre of face $ABOD$) are $x_1 = \frac{a}{2}, y_1 = 0, z_1 = \frac{a}{2}$

where, a = side of cubeand coordinates of point H are

$$x_2 = 0, y_2 = \frac{a}{2}, z_2 = \frac{a}{2}$$

So, vector GH is

$$\begin{aligned} GH &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= -\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} = \frac{a}{2}(\hat{j} - \hat{i}) \end{aligned}$$

- 15** Two vectors A and B have equal magnitudes. The magnitude of $(A+B)$ is ' n ' times the magnitude of $(A-B)$. The angle between A and B is

[2019, 10 Jan Shift-II]

$$(a) \sin^{-1}\left(\frac{n^2-1}{n^2+1}\right) \quad (b) \sin^{-1}\left(\frac{n-1}{n+1}\right)$$

$$(c) \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right) \quad (d) \cos^{-1}\left(\frac{n-1}{n+1}\right)$$

Ans. (c)Given, $|A| = |B|$ or $A = B$... (i)Let magnitude of $(A + B)$ is R and for $(A - B)$ is R' .Now, $R = A + B$ and $R^2 = A^2 + B^2 + 2AB \cos \theta$

$$R^2 = 2A^2 + 2A^2 \cos \theta \quad \dots (ii)$$

Again, $R' = A - B$ [$\because$ using Eq. (i)]

$$\Rightarrow R'^2 = A^2 + B^2 - 2AB \cos \theta$$

$$R'^2 = 2A^2 - 2A^2 \cos \theta \quad \dots (iii)$$

[$\because$ using Eq. (i)]

$$\text{Given, } R = nR' \text{ or } \left(\frac{R}{R'}\right)^2 = n^2$$

Dividing Eq. (ii) with Eq. (iii), we get

$$\frac{n^2}{1} = \frac{1 + \cos \theta}{1 - \cos \theta}$$

$$\text{or } \frac{n^2 - 1}{n^2 + 1} = \frac{(1 + \cos \theta) - (1 - \cos \theta)}{(1 + \cos \theta) + (1 - \cos \theta)}$$

$$\Rightarrow \frac{n^2 - 1}{n^2 + 1} = \frac{2 \cos \theta}{2} = \cos \theta$$

$$\text{or } \theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$$

16 If $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$, then the anglebetween A and B is [AIEEE 2004]

- (a)
- π
- (b)
- $\pi/3$
- (c)
- $\pi/2$
- (d)
- $\pi/4$

Ans. (a)As $(\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{A})$ [given]

$$\Rightarrow (\mathbf{A} \times \mathbf{B}) - (\mathbf{B} \times \mathbf{A}) = 0$$

$$\text{or } (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{B}) = 0$$

$$[\because (\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})]$$

$$\text{or } 2(\mathbf{A} \times \mathbf{B}) = 0$$

$$\Rightarrow 2AB \sin \theta = 0 \text{ or } \sin \theta = 0$$

$$[\because |A| = A \neq 0, |B| = B \neq 0]$$

$$\Rightarrow \theta = 0 \text{ or } \pi$$

TOPIC 2**Motion in a Plane and Projectile Motion**

17 The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 , respectively. Choose the correct option. [2021, 1 Sep Shift-II]

- (a) $R_1 > R_2$ and $H_1 = H_2$
 (b) $R_1 = R_2$ and $H_1 < H_2$
 (c) $R_1 < R_2$ and $H_1 < H_2$
 (d) $R_1 = R_2$ and $H_1 = H_2$

Ans. (b)

Here, two projectiles are projected at angles 42° and 48° with same initial velocity. As we know the expression of range of projectile,

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

At $\theta_1 = 42^\circ$

$$\text{Range, } R_1 = \frac{u^2 \sin 2(42^\circ)}{g} = \frac{0.99u^2}{g}$$

At $\theta_2 = 48^\circ$

$$\text{Range, } R_2 = \frac{u^2 \sin 2(48^\circ)}{g} = \frac{0.99u^2}{g}$$

The range of the projectile is same for the two projectiles.

Therefore, $R_1 = R_2$

Now, as we know the expression of height of the projectile,

$$H_{\max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{At } 42^\circ, H_{\max} = \frac{0.669u^2}{2g}$$

$$\text{At } 48^\circ, H_{\max} = H_2 = \frac{u^2 \sin^2 48^\circ}{2g} = \frac{0.743u^2}{2g}$$

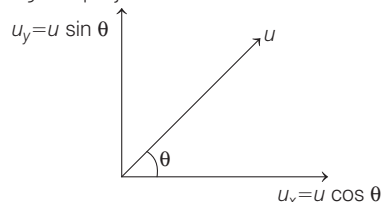
Higher the value of θ higher the value of maximum height.

Therefore, $H_1 < H_2$.

18 A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take, $g = 10 \text{ ms}^{-2}$)

[2021, 27 Aug Shift-II]

- (a) $h_{\max} = 10 \text{ m}, T = 2.5 \text{ s}$
 (b) $h_{\max} = 15.625 \text{ m}, T = 3.54 \text{ s}$
 (c) $h_{\max} = 15.625 \text{ m}, T = 1.77 \text{ s}$
 (d) $h_{\max} = 3.54 \text{ m}, T = 0.125 \text{ s}$

Ans. (c)Given, initial speed, $u = 25 \text{ ms}^{-1}$ Angle of projection, $\theta = 45^\circ$ Let, the maximum height = H Time taken to reach, $H = t$ Acceleration due to gravity $g = 10 \text{ ms}^{-2}$ Velocity at maximum height along Y -axis, $v_y = 0 \text{ ms}^{-1}$ As we know that, for motion along Y -axis,

$$\therefore v_y^2 - u_y^2 = 2gH$$

$$\therefore 0^2 - (25 \sin 45^\circ)^2 = -2 \times 10 \times H$$

$$\Rightarrow -312.5 = -20H$$

$$\Rightarrow H = 15.625 \text{ m}$$

$$\therefore H = \frac{1}{2}gt^2$$

$$\therefore t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 15.625}{10}} = 1.767 \approx 1.77 \text{ s}$$

19 A helicopter is flying horizontally with a speed v at an altitude h has to drop a food packet for a man on the ground. (d) What is the distance of helicopter from the man when the food packet is dropped?

[2021, 31 Aug Shift-I]

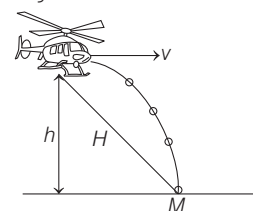
- (a) $\sqrt{\frac{2ghv^2 + 1}{h^2}}$ (b) $\sqrt{2ghv^2 + h^2}$
 (c) $\sqrt{\frac{2v^2h}{g} + h^2}$ (d) $\sqrt{\frac{2gh}{v^2} + h^2}$

Ans. (c)

Given, horizontal velocity of helicopter, $u_x = v$

Height of the helicopter from ground = h

The given condition can be drawn in the figure given below



Here, M is the position of man.

Let, time taken by food packet to reach ground from helicopter = t

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

Initial velocity of food packet along

 Y -axis, $u_y = 0$

$$\text{Since, } h = u_y t + \frac{1}{2}gt^2$$

$$\therefore t = \sqrt{\frac{2h}{g}}$$

and distance travelled by helicopter in time t along horizontal direction

$$\Rightarrow x = u_x t$$

$$= v \sqrt{\frac{2h}{g}}$$

Hence, distance of helicopter from man,

$$H = \sqrt{h^2 + x^2} = \sqrt{h^2 + \left(v \sqrt{\frac{2h}{g}}\right)^2}$$

$$= \sqrt{h^2 + \frac{v^2 2h}{g}}$$

- 20** A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a

[2021, 26 Aug Shift-II]

- hyperbola
- parabola in the direction of motion of plane
- straight line vertically down the plane
- parabola in a direction opposite to the motion of plane

Ans. (c)

The bomb is dropped from a fighter plane flying in horizontal direction only.

Let us assume the velocity of fighter plane, $\mathbf{v}_o = v_x \hat{i}$. As fighter plane is moving in horizontal direction only. Hence, it has only one component.

When fighter plane drops the bomb, the bomb will free fall from the fighter plane and have a horizontal velocity in the direction of plane and vertical velocity due to gravity.

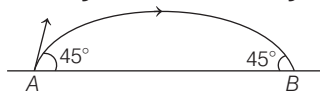
Velocity of bomb can be given as, $\mathbf{v}_b = v_x \hat{i} - v_y \hat{j}$. Velocity in vertical direction will be negative as bomb is moving downwards.

Now, for the person sitting inside the plane, hence the relative velocity of bomb with respect to plane can be calculated as

$$\mathbf{v}_{bo} = \mathbf{v}_b - \mathbf{v}_o = v_x \hat{i} - v_y \hat{j} - v_x \hat{i} = -v_y \hat{j}$$

The velocity of bomb with respect to person sitting in plane is only in vertical direction. Thus, the trajectory of bomb with respect to the person will be a straight line vertically down to the plane.

- 21** The projectile motion of a particle of mass 5 g is shown in the figure.

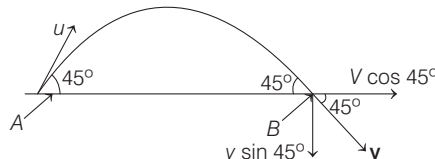


The initial velocity of the particle is $5\sqrt{2} \text{ ms}^{-1}$ and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A

and B is $x \times 10^{-2} \text{ kg}\cdot\text{ms}^{-1}$. The value of x to the nearest integer, is [2021, 18 March Shift-II]

Ans. (5)

The given situation is shown below



Initial velocity of the projectile,
 $\mathbf{u} = u \cos 45^\circ \hat{i} + u \sin 45^\circ \hat{j}$

Final velocity of the projectile,
 $\mathbf{v} = v \cos 45^\circ \hat{i} - v \sin 45^\circ \hat{j}$

At point B, $v \cos 45^\circ = u \cos 45^\circ$
 and $v \sin 45^\circ = u \sin 45^\circ$
 $\mathbf{v} = u \cos 45^\circ \hat{i} - u \sin 45^\circ \hat{j}$

The magnitude of change in the momentum between the points A and B

$$|\Delta \mathbf{p}| = |m(\mathbf{v} - \mathbf{u})|$$

$$\Rightarrow \Delta p = 2mu \sin 45^\circ$$

$$\Rightarrow \Delta p = 2 \times 0.005 \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \Delta p = 5 \times 10^{-2} \text{ kg}\cdot\text{m/s}$$

Hence, the value of the x is 5.

- 22** A mosquito is moving with a velocity $\mathbf{v} = (0.5t^2 \hat{i} + 3t \hat{j} + 9\hat{k}) \text{ m/s}$ and accelerating in uniform conditions. What will be the direction of mosquito after 2s?

[2021, 16 March Shift-II]

$$(a) \tan^{-1}\left(\frac{2}{3}\right) \text{ from X-axis}$$

$$(b) \tan^{-1}\left(\frac{2}{3}\right) \text{ from Y-axis}$$

$$(c) \tan^{-1}\left(\frac{5}{2}\right) \text{ from Y-axis}$$

$$(d) \tan^{-1}\left(\frac{5}{2}\right) \text{ from X-axis}$$

Ans. (*)

Given, velocity of mosquito,

$$\mathbf{v} = (0.5t^2 \hat{i} + 3t \hat{j} + 9\hat{k}) \text{ m/s}$$

$$\text{At } t = 2 \text{ s, } \mathbf{v} = 0.5(2)^2 \hat{i} + 3(2) \hat{j} + 9\hat{k}$$

$$= 2\hat{i} + 6\hat{j} + 9\hat{k}$$

$\therefore$ Angle along X-axis,

$$\cos \alpha = \frac{2}{\sqrt{2^2 + 6^2 + 9^2}} = \frac{2}{\sqrt{121}} = \frac{2}{11}$$

$$\text{or } \alpha = \cos^{-1}\left(\frac{2}{11}\right)$$

$$\text{Similarly, angle along Y-axis, } \cos \beta = \frac{6}{11}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{6}{11}\right)$$

Angle along Z-axis, $\cos \gamma = \frac{9}{11}$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{9}{11}\right)$$

$\therefore$ Angle made by the direction of motion of mosquito will be

$$\cos^{-1}\left(\frac{2}{11}\right) \text{ from X-axis} = \tan^{-1} \frac{\sqrt{117}}{2}$$

$$\cos^{-1}\left(\frac{6}{11}\right) \text{ from Y-axis} = \tan^{-1} \frac{\sqrt{85}}{2}$$

$$\cos^{-1}\left(\frac{9}{11}\right) \text{ from Z-axis} = \tan^{-1}\left(\frac{\sqrt{40}}{9}\right)$$

- 23** The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by

[2021, 26 Feb Shift-II]

$$(a) \tan^{-1} \alpha, \frac{\alpha^2}{4\beta} \quad (b) \tan^{-1} \beta, \frac{\alpha^2}{2\beta}$$

$$(c) \tan^{-1} \alpha, \frac{4\alpha^2}{\beta} \quad (d) \tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$$

Ans. (a)

Given, equation of trajectory of projectile,

$$y = \alpha x - \beta x^2 \quad \dots (i)$$

The standard equation of trajectory of projectile,

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} \quad \dots (ii)$$

Now, from Eqs. (i) and (ii), we get

$$\alpha = \tan \theta \text{ and } \beta = \frac{g}{2u^2 \cos^2 \theta}$$

$$\Rightarrow \alpha^2 = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\therefore \frac{\beta}{\alpha^2} = \frac{g}{2u^2 \cos^2 \theta \times \frac{\sin^2 \theta}{\cos^2 \theta}}$$

$$= \frac{g}{2u^2 \sin^2 \theta} \times \frac{2}{1}$$

$$\Rightarrow \frac{4\beta}{\alpha^2} = \frac{2g}{u^2 \sin^2 \theta} = \frac{1}{H} \Rightarrow H = \alpha^2 / 4\beta$$

$$\therefore \theta = \tan^{-1} \alpha \text{ and } H = \alpha^2 / 4\beta$$

- 24** Starting from the origin at time $t = 0$, with initial velocity $5\hat{j} \text{ ms}^{-1}$, a particle moves in the xy -plane with a constant acceleration of $(10\hat{i} + 4\hat{j}) \text{ ms}^{-2}$. At time t , its

coordinates are
(20 m, y_0 m). The values of t and y_0
respectively, are

[2020, 4 Sep Shift-I]

- (a) 2 s and 18 m (b) 5 s and 25 m
(c) 2 s and 24 m (d) 4 s and 52 m

Ans. (a)

$$\mathbf{u} = (5\hat{j}) \text{ m/s} = (0\hat{i} + 5\hat{j}) \text{ m/s}$$

$$\mathbf{a} = (10\hat{i} + 4\hat{j}) \text{ m/s}^2$$

$$\mathbf{s} = [(20 - 0)\hat{i} + (y_0 - 0)\hat{j}] \text{ m} \\ = (20\hat{i} + y_0\hat{j}) \text{ m}$$

Using second equation of motion in
x-direction,

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$20 = (0)t + \frac{1}{2} (10) (t)^2$$

$$20 = 0 + 5t^2 \Rightarrow t^2 = 4 \Rightarrow t = 2 \text{ s}$$

Using second equation of motion in
y-direction,

$$s_y = u_y t + \frac{1}{2} a_y t^2 = 5 \times 2 + \frac{1}{2} \times 4 \times (2)^2$$

$$= 10 + 8 = 18 \text{ m}$$

Hence, correct option is (a).

- 25 The position vector of particle
changes with time according to the
relation $\mathbf{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$.

What is the magnitude of the
acceleration (in ms^{-2}) at $t = 1$?

[2019, 9 April Shift-II]

- (a) 50 (b) 100
(c) 25 (d) 40

Ans. (a)

Position vector of particle is given as
 $\mathbf{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt} [15t^2\hat{i} + (4 - 20t^2)\hat{j}] \\ = 30t\hat{i} - 40t\hat{j}$$

Acceleration of particle is

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d}{dt} (30t\hat{i} - 40t\hat{j}) \\ = 30\hat{i} - 40\hat{j}$$

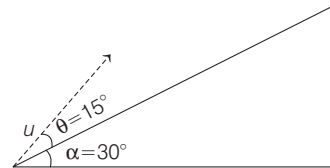
So, magnitude of acceleration at $t = 1$ s is

$$|\mathbf{a}|_{t=1\text{s}} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2} \\ = 50 \text{ ms}^{-2}$$

- 26 A plane is inclined at an angle
 $\alpha = 30^\circ$ with respect to the
horizontal. A particle is projected

with a speed $u = 2 \text{ ms}^{-1}$, from the
base of the plane, making an angle
 $\theta = 15^\circ$ with respect to the plane as
shown in the figure. The distance
from the base, at which the particle
hits the plane is close to
[Take, $g = 10 \text{ ms}^{-2}$]

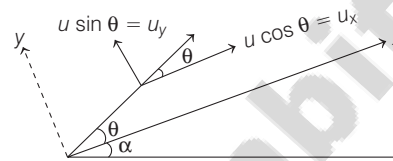
[2019, 10 April Shift-II]



- (a) 26 cm (b) 20 cm
(c) 18 cm (d) 14 cm

Ans. (b)

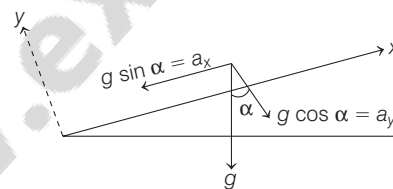
When a projectile is projected at an
angle θ with an inclined plane making
angle α with the horizontal, then



Components of u along and
perpendicular to plane are

$$u_x = u \cos \theta \text{ and } u_y = u \sin \theta$$

We can also resolve acceleration due to
gravity into its components along and
perpendicular to plane as shown below



So, we can now apply formula for range,
i.e. net horizontal displacement of the
particle as

$$R = u_x T + \frac{1}{2} a_x T^2 \quad \dots(i)$$

where, T = time of flight.

Using formula for time of flight, we have

$$T = \frac{2u_y}{a_y} = \frac{2u \sin \theta}{g \cos \alpha} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

Range up the inclined plane is

$$R = u_x T + \frac{1}{2} a_x T^2 \\ = u \cos \theta \left(\frac{2u \sin \theta}{g \cos \alpha} \right) - \frac{1}{2} g \sin \alpha \left(\frac{2u \sin \theta}{g \cos \alpha} \right)^2$$

Here, $u = 2 \text{ ms}^{-1}$, $g = 10 \text{ ms}^{-2}$, $\theta = 15^\circ$,
 $\alpha = 30^\circ$

$$\text{So, } T = \frac{2u \sin \theta}{g \cos \alpha} = \frac{2 \times 2 \sin 15^\circ}{10 \times \cos 30^\circ} \\ = \frac{2 \times 2 \times 0.258 \times 2}{10 \times 1.732} = 0.1191$$

$$\text{Now, } R = 2 \times \cos 15^\circ \times 0.1191 - \frac{1}{2} \\ \times 10 \sin 30^\circ (0.1191)^2 \\ = 2 \times 0.965 \times 0.1191 - \frac{5}{2} (0.1191)^2 \\ = 0.229 - 0.0354 = 0.1936 \text{ m} \\ \approx 0.20 \text{ m} = 20 \text{ cm}$$

- 27 A shell is fired from a fixed artillery
gun with an initial speed u such
that it hits the target on the ground
at a distance R from it. If t_1 and t_2
are the values of the time taken by
it to hit the target in two possible
ways, the product $t_1 t_2$ is

[2019, 12 April Shift-I]

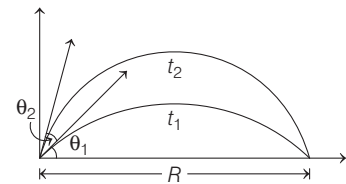
- (a) $\frac{R}{4g}$ (b) $\frac{R}{g}$ (c) $\frac{R}{2g}$ (d) $\frac{2R}{g}$

Ans. (d)

Key Idea Range of a projectile motion
is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Time of flight is given by $t = \frac{2u \sin \theta}{g}$



Given, range of the fired shell,

$$R = R$$

and time of flights are t_1 and t_2 .

Let θ_1 and θ_2 are the two angles at which
shell is fired. As, range in both cases is
same, i.e.

$$R_1 = R_2 = R$$

$$\text{Here, } R_1 = \frac{u^2 \sin 2\theta_1}{g}$$

$$\text{and } R_2 = \frac{u^2 \sin 2\theta_2}{g}$$

$$\Rightarrow R = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2\theta_2}{g} \quad \dots(i)$$

$$\Rightarrow \sin 2\theta_1 = \sin 2\theta_2$$

$$\Rightarrow \sin 2\theta_1 = \sin (180 - 2\theta_2)$$

$$[\because \sin (180 - \theta) = \sin \theta]$$

$$\Rightarrow 2(\theta_1 + \theta_2) = 180 \text{ or } \theta_1 + \theta_2 = 90$$

$$\Rightarrow \theta_2 = 90 - \theta_1 \quad \dots(ii)$$

So, time of flight in first case,

$$t_1 = \frac{2u \sin \theta_1}{g} \quad \dots(iii)$$

and time of flight in second case,

$$t_2 = \frac{2u \sin \theta_2}{g} = \frac{2u \sin(90^\circ - \theta_1)}{g} = \frac{2u \cos \theta_1}{g} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\begin{aligned} t_1 t_2 &= \frac{2u \sin \theta_1}{g} \times \frac{2u \cos \theta_1}{g} \\ \Rightarrow t_1 t_2 &= \frac{4u^2 \sin \theta_1 \cos \theta_1}{g^2} \\ \Rightarrow t_1 t_2 &= \frac{2u^2 \sin 2\theta_1}{g^2} \\ &(\because \sin 2\theta = 2 \sin \theta \cos \theta) \quad \dots(v) \end{aligned}$$

From Eq. (i), we get

$$R = \frac{u^2 \sin 2\theta_1}{g} \therefore t_1 t_2 = \frac{2R}{g}$$

- 28** Two particles are projected from the same point with the same speed u such that they have the same range R , but different maximum heights h_1 and h_2 . Which of the following is correct?

[2019, 12 April Shift-II]

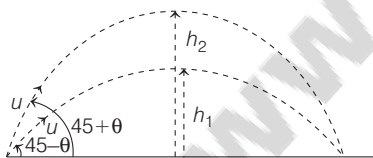
- (a) $R^2 = 4h_1 h_2$ (b) $R^2 = 16h_1 h_2$
(c) $R^2 = 2h_1 h_2$ (d) $R^2 = h_1 h_2$

Ans. (b)

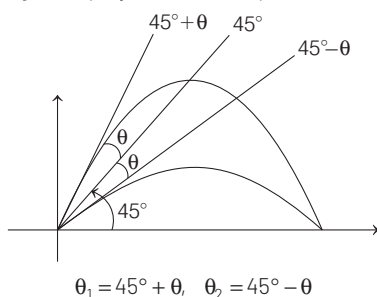
Key Idea Same range for two particles thrown with some initial speed may occur when they are projected at complementary angle.

$$\therefore \theta_1 + \theta_2 = 90^\circ$$

where, θ_1 and θ_2 are angles of projection.



As maximum range occurs at $\theta = 45^\circ$ for a given initial projection speed, we take angles of projection of two particles as



where, θ is angle of projectiles with 45° line.

So, range of projectiles will be

$$\begin{aligned} R &= R_1 = R_2 = \frac{u^2 \sin 2(\theta_1)}{g} \\ \Rightarrow R &= \frac{u^2 \sin 2(45^\circ + \theta)}{g} \\ \Rightarrow R &= \frac{u^2 \sin(90^\circ + 2\theta)}{g} \\ \Rightarrow R &= \frac{u^2 \cos 2\theta}{g} \\ \Rightarrow R^2 &= \frac{u^4 \cos^2 2\theta}{g^2} \quad \dots(i) \end{aligned}$$

Maximum heights achieved in two cases are

$$\begin{aligned} h_1 &= \frac{u^2 \sin^2(45^\circ + \theta)}{2g} \\ \text{and } h_2 &= \frac{u^2 \sin^2(45^\circ - \theta)}{2g} \\ \text{So, } h_1 h_2 &= \frac{u^4 \sin^2(45^\circ + \theta) \sin^2(45^\circ - \theta)}{4g^2} \end{aligned}$$

Using

$$2 \sin A \cdot \sin B = \cos(A - B) - \cos(A + B),$$

we have

$$\begin{aligned} \sin(45^\circ + \theta) \sin(45^\circ - \theta) &= \frac{1}{2} (\cos 2\theta - \cos 90^\circ) \\ \Rightarrow \sin(45^\circ + \theta) \sin(45^\circ - \theta) &= \frac{\cos 2\theta}{2} \\ &[\because \cos 90^\circ = 0] \end{aligned}$$

So, we have

$$\begin{aligned} h_1 h_2 &= \frac{u^4 \left(\frac{\cos 2\theta}{2} \right)^2}{4g^2} \\ \Rightarrow h_1 h_2 &= \frac{u^4 \cos^2 2\theta}{16g^2} \quad \dots(ii) \end{aligned}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow h_1 h_2 = \frac{R^2}{16} \Rightarrow R^2 = 16 h_1 h_2$$

- 29** The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 , then (Take, $g = 10 \text{ ms}^{-2}$)

[2019, 12 April Shift-I]

- (a) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} \text{ ms}^{-1}$
(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} \text{ ms}^{-1}$
(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} \text{ ms}^{-1}$
(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} \text{ ms}^{-1}$

Ans. (c)

Given, $g = 10 \text{ m/s}^2$

Equation of trajectory of the projectile,
 $y = 2x - 9x^2 \quad \dots(i)$

In projectile motion, equation of trajectory is given by

$$y = x \tan \theta_0 - \frac{g x^2}{2v_0^2 \cos^2 \theta_0} \quad \dots(ii)$$

By comparison of Eqs. (i) and (ii), we get

$$\tan \theta_0 = 2 \quad \dots(iii)$$

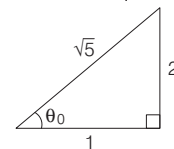
and

$$\frac{g}{2v_0^2 \cos^2 \theta_0} = 9 \quad \dots(iv)$$

or $v_0^2 = \frac{g}{9 \times 2 \cos^2 \theta_0}$

From Eq. (iii), we can get value of $\cos \theta_0$ and $\sin \theta_0$

$$\begin{aligned} \cos \theta_0 &= \frac{1}{\sqrt{5}} \\ \text{and } \sin \theta_0 &= \frac{2}{\sqrt{5}} \quad \dots(v) \end{aligned}$$



Using value of $\cos \theta_0$ from Eq. (v) to Eq. (iv), we get

$$\begin{aligned} v_0^2 &= \frac{10 \times (\sqrt{5})^2}{2 \times (1)^2 \times 9} = \frac{10 \times 5}{2 \times 9} \\ \Rightarrow v_0^2 &= \frac{25}{9} \text{ or } v_0 = \frac{5}{3} \text{ m/s} \quad \dots(vi) \end{aligned}$$

From Eq. (v), we get

$$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

- 30** A particle is moving with a velocity $\mathbf{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is

[2019, 9 Jan Shift-I]

- (a) $y = x^2 + \text{constant}$
(b) $y^2 = x + \text{constant}$
(c) $xy = \text{constant}$
(d) $y^2 = x^2 + \text{constant}$

Ans. (d)

Given, velocity of a particle is

$$\mathbf{v} = k(y\hat{i} + x\hat{j}) \quad \dots(i)$$

Suppose, its position is given as

$$\mathbf{r} = x\hat{i} + y\hat{j}$$

$$\begin{aligned} \therefore \mathbf{v} &= \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j}) \\ &= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \quad \dots(ii) \end{aligned}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{dx}{dt} = y \quad \dots(iii)$$

and $\frac{dy}{dt} = x \dots (iv)$

Dividing Eq. (iii) and Eq. (iv), we get

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{y}{x} \Rightarrow x \frac{dx}{dt} = y \frac{dy}{dt}$$

or $x dx = y dy$

Integrating both sides, we get

$$\int x dx = \int y dy$$

$$\text{or } \frac{x^2}{2} + \frac{c_1}{2} = \frac{y^2}{2} + \frac{c_2}{2}$$

where, c_1 and c_2 are the constants of integration. $\Rightarrow x^2 + c = y^2$

[here, c (constant) = $c_1 - c_2$]

or $y^2 = x^2 + \text{constant}$

- 31** In three dimensional system, the position coordinates of a particle (in motion) are given below
[2019, 9 Jan Shift-II]

$$x = a \cos \omega t, \quad y = a \sin \omega t, \quad z = a \omega t$$

The velocity of particle will be

- (a) $\sqrt{2} a \omega$ (b) $2 a \omega$
(c) $a \omega$ (d) $\sqrt{3} a \omega$

Ans. (a)

Given that the position coordinates of a particle

$$\left. \begin{aligned} x &= a \cos \omega t \\ y &= a \sin \omega t \\ z &= a \omega t \end{aligned} \right\} \dots (i)$$

So, the position vector of the particle is

$$\hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow \hat{r} = a \cos \omega t \hat{i} + a \sin \omega t \hat{j} + a \omega t \hat{k}$$

$$\hat{r} = a [\cos \omega t \hat{i} + \sin \omega t \hat{j} + \omega t \hat{k}]$$

therefore, the velocity of the particle is

$$\therefore \hat{v} = \frac{d\hat{r}}{dt} = \frac{d[a(\cos \omega t \hat{i} + \sin \omega t \hat{j} + \omega t \hat{k})]}{dt}$$

$$\Rightarrow \hat{v} = -a\omega \sin \omega t \hat{i} + a\omega \cos \omega t \hat{j} + a\omega \hat{k}$$

The magnitude of velocity is

$$|\hat{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or

$$|\hat{v}| = \sqrt{(-a\omega \sin \omega t)^2 + (a\omega \cos \omega t)^2 + (a\omega)^2}$$

$$= \omega a \sqrt{(-\sin \omega t)^2 + (\cos \omega t)^2 + (1)^2}$$

$$= \sqrt{2} \omega a$$

- 32** Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s, respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the

bullets on the ground fired by the two guns is [2019, 10 Jan Shift-I]

- (a) 1 : 4 (b) 1 : 16 (c) 1 : 8 (d) 1 : 2

Ans. (b)

Bullets from guns can reach upto a distance of maximum range which occurs when projection is made at angle of 45° .

Maximum range for gun, $A = R_1 = \frac{u_1^2}{g}$

Maximum range for gun, $B = R_2 = \frac{u_2^2}{g}$

So, ratio of covered areas

$$= \frac{\pi(R_1)^2}{\pi(R_2)^2} = \frac{\pi \cdot u_1^4 / g^2}{\pi \cdot u_2^4 / g^2} = \frac{u_1^4}{u_2^4}$$

Here, $u_1 = 1 \text{ km/s}$ and $u_2 = 2 \text{ km/s}$

So, ratio of areas = $\frac{1^4}{2^4} = \frac{1}{16} = 1 : 16$

- 33** A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1 \text{ s}$ is R . Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, the value of R is

[2019, 11 Jan Shift-I]

- (a) 10.3 m (b) 2.8 m
(c) 5.1 m (d) 2.5 m

Ans. (b)

Components of velocity at an instant of time t of a body projected at an angle θ is

$$v_x = u \cos \theta + g_x t \quad \text{and} \quad v_y = u \sin \theta + g_y t$$

Here, components of velocity at $t = 1 \text{ s}$, is

$$v_x = u \cos 60^\circ + 0 \quad [\text{as } g_x = 0]$$

$$= 10 \times \frac{1}{2} = 5 \text{ m/s}$$

and $v_y = u \sin 60^\circ + (-10) \times (1)$

$$= 10 \times \frac{\sqrt{3}}{2} + (-10) \times (1)$$

$$= 5\sqrt{3} - 10$$

$$\Rightarrow |v_y| = |10 - 5\sqrt{3}| \text{ m/s}$$

Now, angle made by the velocity vector at time of $t = 1 \text{ s}$

$$|\tan \alpha| = \left| \frac{v_y}{v_x} \right| = \left| \frac{10 - 5\sqrt{3}}{5} \right|$$

$$\Rightarrow \tan \alpha = |2 - \sqrt{3}| \quad \text{or} \quad \alpha = 15^\circ$$

$\therefore$ Radius of curvature of the trajectory of the projected body $R = \frac{v^2}{g \cos \alpha}$

$$= \frac{(5)^2 + (10 - 5\sqrt{3})^2}{10 \times 0.97}$$

$$[\because v^2 = v_x^2 + v_y^2 \text{ and } \cos 15^\circ = 0.97]$$

$$\Rightarrow R = 2.77 \text{ m} \approx 2.8 \text{ m}$$

- 34** A particle moves from the point $(2.0\hat{i} + 4.0\hat{j}) \text{ m}$ at $t = 0$ with an initial velocity $(5.0\hat{i} + 4.0\hat{j}) \text{ ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j}) \text{ ms}^{-2}$.

What is the distance of the particle from the origin at time 2 s?

[2019, 11 Jan Shift-II]

- (a) 5 m (b) $20\sqrt{2} \text{ m}$
(c) $10\sqrt{2} \text{ m}$ (d) 15 m

Ans. (b)

Given, initial position of particle,

$$r_0 = (2\hat{i} + 4\hat{j}) \text{ m}$$

Initial velocity of particle,

$$u = (5\hat{i} + 4\hat{j}) \text{ m/s}$$

Acceleration of particle,

$$a = (4\hat{i} + 5\hat{j}) \text{ m/s}^2$$

According to second equation of motion, position of particle at time t is,

$$r = r_0 + ut + \frac{1}{2}at^2$$

At $t = 2 \text{ s}$ position of particle is,

$$r = (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2$$

$$+ \frac{1}{2}(4\hat{i} + 5\hat{j}) \times 4$$

$$\text{or } r = (2 + 10 + 8)\hat{i} + (4 + 8 + 8)\hat{j}$$

$$\Rightarrow r = 20\hat{i} + 20\hat{j}$$

$\therefore$ Distance of particle from origin is,

$$|r| = 20\sqrt{2} \text{ m}$$

- 35** A projectile is given an initial velocity of $(\hat{i} + 2\hat{j}) \text{ m/s}$, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10 \text{ m/s}^2$, the equation of its trajectory is [JEE Main 2013]

- (a) $y = x - 5x^2$
(b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$
(d) $4y = 2x - 25x^2$

Ans. (b)

As, initial velocity $v = (\hat{i} + 2\hat{j}) \text{ m/s}$

For projectile

Motion along horizontal direction,

$$x = t \dots (i)$$

Motion along vertical direction,

$$y = ut - \frac{1}{2}gt^2$$

where, u is a vertical component of velocity.

$$y = 2t - \frac{1}{2}10t^2 \dots (ii)$$

From Eqs. (i) and (ii), we get

$$y = 2x - 5x^2$$

- 36** A boy can throw a stone upto a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone upto will be [AIEEE 2012]

- (a) $20\sqrt{2}$ m (b) 10 m
(c) $10\sqrt{2}$ m (d) 20 m

Ans. (d)

Maximum speed with which the boy can throw stone is

$$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2} \text{ m/s}$$

Range is maximum when projectile is thrown at an angle of 45° . Thus,

$$R_{\max} = \frac{u^2}{g} = \frac{(10\sqrt{2})^2}{10} = 20 \text{ m}$$

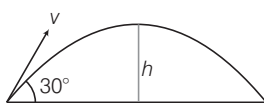
- 37** A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is [AIEEE 2011]

- (a) $\frac{\sqrt{3}}{2} \frac{mv^2}{g}$ (b) zero
(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $\frac{\sqrt{3}}{16} \frac{mv^3}{g}$

Ans. (d)

Angular momentum of the projectile as given by

$$L = mv_h r_{\perp} = m(v \cos \theta) h$$



where, h is the maximum height.

$$L = m(v \cos \theta) \left(\frac{v^2 \sin^2 \theta}{2g} \right) = \frac{mv^3 \sin^2 \theta \cos \theta}{2g} = \frac{\sqrt{3}}{16} \frac{mv^3}{g}$$

- 38** A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v , the total area around the fountain that gets wet is [AIEEE 2011]

- (a) $\pi \frac{v^4}{g^2}$ (b) $\frac{\pi}{2} \frac{v^4}{g^2}$
(c) $\pi \frac{v^2}{g^2}$ (d) $\pi \frac{v^2}{g}$

Ans. (a)

Maximum range of water coming out of the fountain,

$$R_m = v^2 / g$$

$\therefore$ Total area around fountain,

$$A = \pi R_m^2 = \pi \frac{v^4}{g^2}$$

- 39** A particle is moving with velocity $\mathbf{v} = k(y\mathbf{i} + x\mathbf{j})$, where k is a constant. The general equation for its path is [AIEEE 2010]

- (a) $y = x^2 + \text{constant}$
(b) $y^2 = x + \text{constant}$
(c) $xy = \text{constant}$
(d) $y^2 = x^2 + \text{constant}$

Ans. (d)

Given, $\mathbf{v} = ky\mathbf{i} + kx\mathbf{j}$

$$\Rightarrow \frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{kx}{ky}$$

$$\Rightarrow y dy = x dx$$

Integrating and using

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \text{ we have}$$

$$y^2 = x^2 + c$$

- 40** A particle has an initial velocity $3\mathbf{i} + 4\mathbf{j}$ and an acceleration of $0.4\mathbf{i} + 0.3\mathbf{j}$. Its speed after 10 s is [AIEEE 2009]

- (a) 10 units
(b) $7\sqrt{2}$ units
(c) 7 units
(d) 8.5 units

Ans. (b)

Given, $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$; $\mathbf{a} = 0.4\mathbf{i} + 0.3\mathbf{j}$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t = 3\mathbf{i} + 4\mathbf{j} + (0.4\mathbf{i} + 0.3\mathbf{j})10$$

$$= 3\mathbf{i} + 4\mathbf{j} + 4\mathbf{i} + 3\mathbf{j} = 7\mathbf{i} + 7\mathbf{j}$$

So, speed is equal to magnitude of velocity

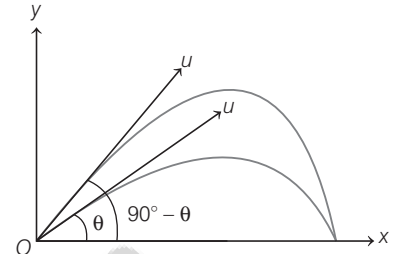
$$= \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ units}$$

- 41** A projectile can have the same range R for two angles of projection. If t_1 and t_2 are the times of flights in the two cases, then the product of the two times of flights is proportional to [AIEEE 2005, 04]

- (a) R^2 (b) $\frac{1}{R^2}$
(c) $\frac{1}{R}$ (d) R

Ans. (d)

A projectile can have same range, if angles of projection are complementary of each other i.e., θ and $(90^\circ - \theta)$. Thus, in both cases,



The time of flights are

$$t_1 = \frac{2u \sin \theta}{g} \quad \dots(i)$$

$$\text{and } t_2 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u \cos \theta}{g} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$t_1 t_2 = \frac{4u^2 \sin \theta \cos \theta}{g^2}$$

$$t_1 t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2}{g} \frac{u^2 \sin 2\theta}{g}$$

$$\therefore t_1 t_2 = \frac{2R}{g} \quad \left[\because R = \frac{u^2 \sin 2\theta}{g} \right]$$

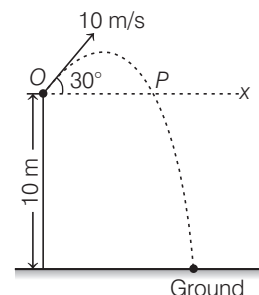
$$\text{or } t_1 t_2 \propto R$$

- 42** A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground? [$g = 10 \text{ m/s}^2$, $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$] [AIEEE 2003]

- (a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m

Ans. (d)

The ball will be at point P when it is at a height of 10 m from the ground. So, we have to find the distance OP , which can be calculated directly by considering it as a projectile on a levelled plane (OX).



$$\begin{aligned}
 OP = R &= \frac{u^2 \sin 2\theta}{g} \\
 &= \frac{10^2 \times \sin(2 \times 30^\circ)}{10} \\
 &= \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.66 \text{ m}
 \end{aligned}$$

- 43** The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

[AIEEE 2003]

- (a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
 (c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

Ans. (b)

The given coordinate

$$x = \alpha t^3, \quad y = \beta t^3$$

$$\text{Then, } v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

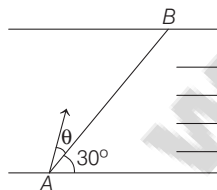
$$\begin{aligned}
 \therefore \text{Resultant velocity, } v &= \sqrt{v_x^2 + v_y^2} \\
 &= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4} \\
 &= 3t^2 \sqrt{\alpha^2 + \beta^2}
 \end{aligned}$$

TOPIC 3

Relative Velocity

- 44** A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be $^\circ$, so that the swimmer reaches point (b)

[2021, 27 July Shift-II]



Ans. (30)

Let v_M be the velocity of swimmer. v_{MR} be the velocity of swimmer with respect to river. v_R be the velocity of river

$$\therefore \mathbf{v}_M = \mathbf{v}_{MR} + \mathbf{v}_R$$

$\mathbf{v}_M$ should be along line AB.

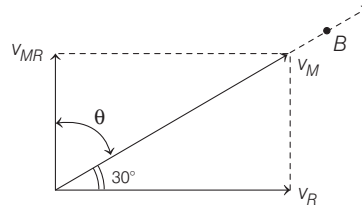
As per question,

Magnitude of velocity of swimmer =

Magnitude of velocity of river

$$\Rightarrow |\mathbf{v}_{MR}| = |\mathbf{v}_R|$$

If we represent the given scenario as follows



$\therefore \mathbf{v}_M$ should be along angle of bisector of angle between $\mathbf{v}_{MR}$ and $\mathbf{v}_R$.

$$\therefore \theta = 30^\circ$$

- 45** Three particles P, Q and R are moving along the vectors $\mathbf{A} = \hat{i} + \hat{j}$, $\mathbf{B} = \hat{j} + \hat{k}$ and $\mathbf{C} = -\hat{i} + \hat{j}$, respectively. They strike on a point and start to move in different directions. Now, particle P is moving normal to the plane which contain vectors A and B. Similarly, particle Q is moving normal to the plane which contain vectors A and C. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then, the value of x

is [2021, 22 July Shift-II]

Ans. (3)

Given, particle P moves normal to plane having A and B.

Particle Q moves normal to plane A and C.

Let the angle between P and Q be θ

According to given information,

$$\mathbf{A} = \hat{i} + \hat{j}$$

$$\mathbf{B} = \hat{j} + \hat{k}$$

$$\mathbf{C} = -\hat{i} + \hat{j}$$

$$\begin{aligned}
 \text{Now, } \mathbf{P} = \mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \\
 &= \hat{i}(1) - \hat{j}(1) + \hat{k}(1) = \hat{i} - \hat{j} + \hat{k}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{Q} = \mathbf{A} \times \mathbf{C} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{vmatrix} \\
 &= \hat{i}(0 - 0) - \hat{j}(0 - 0) + \hat{k}(1 + 1) \\
 &= 2\hat{k}
 \end{aligned}$$

$$\text{Now, } \mathbf{P} \cdot \mathbf{Q} = |\mathbf{P}| |\mathbf{Q}| \cos \theta$$

$$0 + 0 + 2 = 2\sqrt{3} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\begin{aligned}
 \cos^{-1}\left(\frac{1}{\sqrt{x}}\right) &= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \\
 x &= 3
 \end{aligned}$$

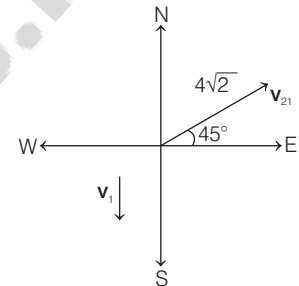
- 46** A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 s is

[2021, 20 July Shift-I]

- (a) 3 m (b) 20 m
 (c) $12\sqrt{2}$ m (d) 15 m

Ans. (d)

The given situation can be represented as



In the above figure, $\mathbf{v}_1$ is the speed of wind and $\mathbf{v}_{21}$ is the speed of butterfly with respect to wind (d)

So, $\mathbf{v}_{21}$ can be given as

$$\begin{aligned}
 \mathbf{v}_{21} &= 4\sqrt{2} \cos 45^\circ \hat{i} + 4\sqrt{2} \sin 45^\circ \hat{j} \\
 &= 4\sqrt{2} \times \frac{1}{\sqrt{2}} \hat{i} + 4\sqrt{2} \times \frac{1}{\sqrt{2}} \hat{j} = 4\hat{i} + 4\hat{j}
 \end{aligned}$$

and $\mathbf{v}_1$ can be given as

$$\mathbf{v}_1 = -\hat{j}$$

$\therefore$ Velocity of butterfly can be given as

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{v}_{21} = 4\hat{i} + 4\hat{j} - \hat{j} = 4\hat{i} + 3\hat{j}$$

$$\begin{aligned}
 \therefore \text{Displacement of butterfly, } \mathbf{D} &= \mathbf{v}_2 \times t \\
 &= (4\hat{i} + 3\hat{j}) \times 3 = 12\hat{i} + 9\hat{j}
 \end{aligned}$$

$\therefore$ Magnitude of displacement,

$$|\mathbf{D}| = \sqrt{12^2 + 9^2} = 15 \text{ m}$$

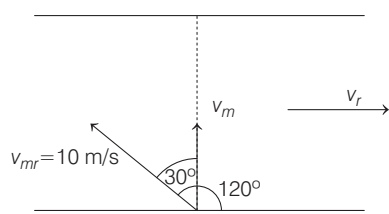
- 47** A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is x m/s. The value of x to the nearest integer is [2021, 18 March Shift-I]

Ans. (5)

Given,

A person's swimming speed with respect to the river, $v_{mr} = 10$ m/s

As shown in the figure,



Let's determine the speed of the river flow, v_r

In X-direction,

$$v_r = v_{mr} \sin 30^\circ$$

$$v_r = 10 \times \sin 30^\circ$$

$$v_r = 5 \text{ m/s}$$

Hence, the speed of the river flow is 5 m/s. So, the value of the x to the nearest integer is 5.

- 48** A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is° (in degree).

(Round off to the nearest integer)

[2021, 16 March Shift-II]

Ans. (120)

The situation is depicted in the following figure.

where, V_{MR} = velocity of man = 12 km/h and v_R = velocity of water flow in river = 6 km/h

As, v_{MR} should be along CD .

$$\Rightarrow v_R - v_{MR} \sin \theta = 0$$

$$\Rightarrow 6 - 12 \sin \theta = 0$$

$$\Rightarrow \sin \theta = \frac{6}{12}$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}(\sin 30^\circ)$$

$$\left[\because \sin 30^\circ = \frac{1}{2} \right]$$

$$\Rightarrow \theta = 30^\circ$$

$$\therefore \alpha = 90^\circ + \theta = 90^\circ + 30^\circ = 120^\circ$$

$$\Rightarrow \alpha = 120^\circ$$

- 49** Trains A and B are running on parallel tracks in the opposite directions with speeds of 36 km/h and 72 km/h, respectively. A person

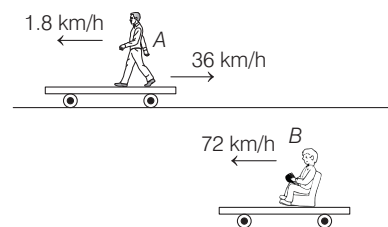
is walking in train A in the opposite direction to its motion with a speed of 1.8 km/h. Speed (in ms^{-1}) of this person as observed from train B will be close to (Take, the distance between the tracks as negligible)

[2020, 2 Sep Shift-I]

- (a) 28.5 (b) 30.5
(c) 29.5 (d) 31.5

Ans. (c)

Condition given in question is as shown,



Speed of person on train A with respect to ground

$$= 36 - 1.8 = 34.2 \text{ km/h in direction of A}$$

So, the situation becomes as shown



Hence, speed of person on train A as observed by an observer on train B is

$$v_{AB} = 34.2 + 72 = 106.2 \text{ km/h}$$

$$= 106.2 \times \frac{5 \text{ m}}{18 \text{ s}} = 29.5 \text{ ms}^{-1}$$

Hence, correct option is (c).

- 50** When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v , he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)v$, this angle changes to 45° . The value of β is close to [2020, 6 Sep Shift-II]

- (a) 0.50 (b) 0.41
(c) 0.37 (d) 0.73

Ans. (d)

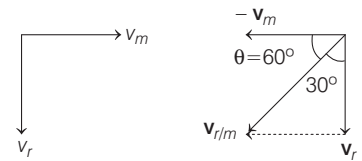
Given, rain drop is falling vertically downwards (w.r.t ground).

So, velocity of rain $\mathbf{v}_r = v_r(-\hat{j})$, velocity of man $\mathbf{v}_m = v\hat{i}$.

Velocity of rain w.r.t to man

$$\mathbf{v}_{r/m} = \mathbf{v}_r - \mathbf{v}_m$$

Drawing velocity vectors



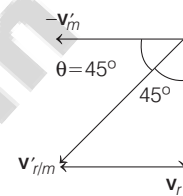
From diagram,

$$\tan 30^\circ = \frac{v_m}{v_r} \Rightarrow \frac{1}{\sqrt{3}} = \frac{v_m}{v_r} \text{ or } v_r = \sqrt{3}v$$

Now, man increases velocity of car to $(1+\beta)v$ in same direction, then

$$\mathbf{v}'_m = (1+\beta)v\hat{i}$$

Drawing velocity vectors



$$\tan 45^\circ = \frac{v'_m}{v_r} \Rightarrow 1 = \frac{(1+\beta)v}{\sqrt{3}v}$$

$$\Rightarrow 1 + \beta = \sqrt{3} \text{ or } \beta = \sqrt{3} - 1 = 0.73$$

Hence, correct option is (d).

- 51** A particle is moving along the x -axis with its coordinate with the t given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y -axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is [2020, 8 Jan Shift-I]

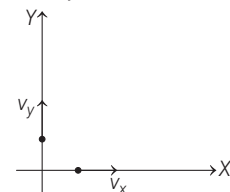
Ans. (580)

Let particle A moves along X -axis such that

$$x = 10 + 8t - 3t^2$$

and let particle B moves over Y -axis such that

$$y = 5 - 8t^3$$



Velocities of particles A and B are

$$v_A = v_x = \frac{dx}{dt} = \frac{d}{dt}(10 + 8t - 3t^2)$$

$$= 8 - 6t$$

or v_A at $t = 1$ s,

$$v_A = 8 - 6 \times 1 = 2 \text{ ms}^{-1}$$

$$\Rightarrow v_A = +2\hat{i} \text{ ms}^{-1}$$

Velocity of particle B,

$$v_B = v_y = \frac{d}{dt}(5 - 8t^3) = -8 \times 3t^2$$

At $t = 1$ s, $v_B = -8 \times 3 \times 1^2 = -24 \text{ ms}^{-1}$

$$\Rightarrow v_B = -24 \hat{j} \text{ ms}^{-1}$$

Speed of particle B w.r.t. particle A,

v_{BA} = velocity of particle B w.r.t. A

$$= v_B - v_A = -24\hat{j} - (2\hat{i}) = -2\hat{i} - 24\hat{j}$$

$\Rightarrow$ Magnitude of velocity,

$$|v_{BA}| = \sqrt{(2)^2 + (24)^2}$$

But given $|v_{BA}| = \sqrt{v}$

$$\text{So, } \sqrt{v} = \sqrt{24^2 + 2^2}$$

$$\text{or } v = 580 \text{ ms}^{-1}$$

- 52** Ship A is sailing towards north-east with velocity $\mathbf{v} = 30\hat{i} + 50\hat{j}$ km/h, where $\hat{i}$ points east and $\hat{j}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/h. A will be at minimum distance from B in

[2019, 8 April Shift-I]

- (a) 4.2 h (b) 2.6 h
(c) 3.2 h (d) 2.2 h

Ans. (b)

Considering the initial position of ship A as origin, so the velocity and position of ship will be

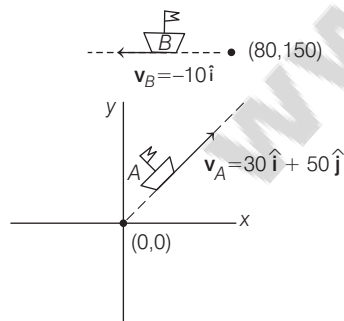
$$v_A = (30\hat{i} + 50\hat{j}) \text{ and } r_A = (0\hat{i} + 0\hat{j})$$

Now, as given in the question, velocity and

position of ship B will be, $v_B = -10\hat{i}$

and $r_B = (80\hat{i} + 150\hat{j})$

Hence, the given situation can be represented graphically as



After time t , coordinates of ships A and B are $(80 - 10t, 150)$ and $(30t, 50t)$.

So, distance between A and B after time t is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(80 - 10t - 30t)^2 + (150 - 50t)^2}$$

$$\Rightarrow d^2 = (80 - 40t)^2 + (150 - 50t)^2$$

Distance is minimum when $\frac{d}{dt}(d^2) = 0$

After differentiating, we get

$$\Rightarrow \frac{d}{dt} [(80 - 40t)^2 + (150 - 50t)^2] = 0$$

$$\Rightarrow 2(80 - 40t)(-40) + 2(150 - 50t)(-50) = 0$$

$$\Rightarrow -3200 + 1600t - 7500 + 2500t = 0$$

$$\Rightarrow 4100t = 10700$$

$$\Rightarrow t = \frac{10700}{4100} = 2.6 \text{ h}$$

Alternate Solution

Time after which the distance is minimum between A and B can be calculated as

$$t = \frac{|r_{BA} \cdot v_{BA}|}{|v_{BA}|^2}$$

where, $r_{BA} = r_B - r_A = 80\hat{i} + 150\hat{j}$

and $v_{BA} = -10\hat{i} - (30\hat{i} + 50\hat{j})$

$$= -40\hat{i} - 50\hat{j}$$

$$\Rightarrow t = \frac{|(80\hat{i} + 150\hat{j}) \cdot (-40\hat{i} - 50\hat{j})|}{|-40\hat{i} - 50\hat{j}|^2}$$

$$= \frac{3200 + 7500}{4100} = \frac{10700}{4100} = 2.6 \text{ h}$$

- 53** The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

[2019, 9 April Shift-I]

- (a) 60° (b) 120° (c) 90° (d) 150°

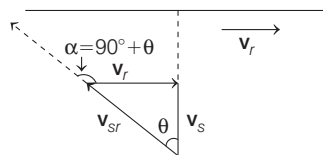
Ans. (b)

Let the velocity of the swimmer is

$$v_s = 4 \text{ km/h}$$

and velocity of river is $v_r = 2 \text{ km/h}$

Also, angle of swimmer with the flow of the river (down stream) is α as shown in the figure below



From diagram, angle θ is

$$\sin \theta = \frac{v_r}{v_{sr}} = \frac{2 \text{ km/h}}{4 \text{ km/h}} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$$\text{Clearly, } \alpha = 90^\circ + 30^\circ = 120^\circ$$

- 54** A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m

travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction and (ii) in the opposite direction is

[2019, 12 Jan Shift-I]

- (a) $\frac{3}{2}$ (b) $\frac{25}{11}$ (c) $\frac{11}{5}$ (d) $\frac{5}{2}$

Ans. (c)

When trains are moving in same direction relative speed $= |v_1 - v_2|$ and in opposite direction relative speed

$$= |v_1 + v_2|$$

Hence, ratio of time when trains move in same direction with time when trains move in opposite direction is

$$\frac{t_1}{t_2} = \frac{\left(\frac{l_1 + l_2}{|v_1 - v_2|} \right)}{\left(\frac{l_1 + l_2}{|v_1 + v_2|} \right)} = \frac{|v_1 + v_2|}{|v_1 - v_2|}$$

where, $l_1 + l_2$ = sum of lengths of trains which is same as distance covered by trains to cross each other

$$\text{So, } \frac{t_1}{t_2} = \frac{80 + 30}{80 - 30} = \frac{110}{50} = \frac{11}{5}$$

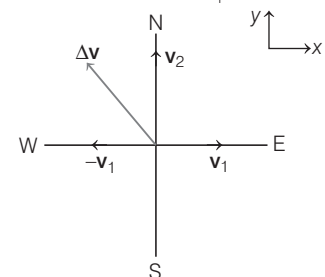
- 55** A particle is moving Eastwards with a velocity of 5 ms^{-1} . In 10 s, the velocity changes to 5 ms^{-1} Northwards. The average acceleration in this time is

[AIEEE 2005]

- (a) $\frac{1}{\sqrt{2}} \text{ ms}^{-2}$ towards North-East
(b) $\frac{1}{2} \text{ ms}^{-2}$ towards North
(c) zero
(d) $\frac{1}{\sqrt{2}} \text{ ms}^{-2}$ towards North-West

Ans. (d)

As, velocity of eastward $v_1 = +5\hat{i}$



and velocity of northward $v_2 = +5\hat{j}$

$$\Delta v = v_2 - v_1 = 5\hat{j} - 5\hat{i}$$

$$|\Delta v| = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$$\therefore a = \frac{|\Delta v|}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} \text{ ms}^{-2}$$

For direction, $\tan \alpha = -\frac{5}{5} = -1$

Average acceleration is $\frac{1}{\sqrt{2}} \text{ ms}^{-2}$ towards North-West.

- 56** A ball is thrown from a point with a speed v_0 at an angle of projection θ . From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

[AIEEE 2004]

- (a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

Ans. (a)

Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man, i.e.,

$$v_0 \cos \theta = \frac{v_0}{2} \text{ or } \cos \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \cos 60^\circ \text{ or } \theta = 60^\circ$$

TOPIC 4

Uniform Circular Motion

- 57** A huge circular arc of length 4.4 ly subtends an angle 4 s at the centre of the circle. How long it would take for a body to complete 4 rev if its speed is 8 AU per sec?

[Given, 1 ly = 9.46×10^{15} m, 1 AU = 1.5×10^{11} m]

[2021, 27 Aug Shift-I]

- (a) 4.1×10^8 s (b) 4.5×10^{10} s
(c) 3.5×10^6 s (d) 7.2×10^8 s

Ans. (b)

$$\text{Length of arc, } l = 4.4 \text{ ly} = 4.4 \times 9.46 \times 10^{15} \text{ m}$$

$$= 4.1624 \times 10^{16} \text{ m}$$

$$[\because 1 \text{ ly} = 9.46 \times 10^{15} \text{ m}]$$

Angle subtended by arc, $\theta = 4 \text{ s}$

Number of revolution, $n = 4$

Speed of body,

$$v = 8 \text{ AU s}^{-1} = 1.5 \times 10^{11} \times 8$$

$$= 1.2 \times 10^{12} \text{ ms}^{-1} \quad [\because 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}]$$

We know that, length of arc subtended by an angle,

$$l = R\theta$$

Here, R = radius of arc

Substituting the values in above expression, we get

$$4.1624 \times 10^{16} = R \times 4 \times \left(\frac{1^\circ}{3600} \right) \times \left(\frac{\pi}{180^\circ} \right)$$

$$\left(\because 1 \text{ s} = \frac{1}{3600} \text{ deg ree} \right)$$

$$R = 2.146 \times 10^{21} \text{ m}$$

We know that, time = distance/speed

$$t = \frac{n \times 2\pi R}{v}$$

($\because$ Distance = $n \times$ length of circular path)

Substituting the values in above expression, we get

$$t = \frac{4 \times 2\pi \times 2.146 \times 10^{21}}{1.2 \times 10^{12}}$$

$$= 4.5 \times 10^{10} \text{ s}$$

Thus, the time required is $4.5 \times 10^{10} \text{ s}$.

- 58** A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in ms^{-2}) is of the order of

[2020, 6 Sep Shift-I]

- (a) 10^{-3} (b) 10^{-4}
(c) 10^{-2} (d) 10^{-1}

Ans. (a)

Given that, radius, $R = 0.1 \text{ m}$

Angular frequency,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105 \text{ rad/s}$$

$\therefore$ Average acceleration,

$$a = \omega^2 R = (0.105)^2 (0.1)$$

$$= 1.102 \times 10^{-3} \text{ m/s}^2$$

and it is of the order of 10^{-3} .

- 59** A particle moves such that its position vector $\mathbf{r}(t) = \cos \omega t \hat{i} + \sin \omega t \hat{j}$, where ω is a constant and t is time. Then, which of the following statements is true for the velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ of the particle?

[2020, 8 Jan Shift-II]

- (a) $\mathbf{v}$ and $\mathbf{a}$ both are parallel to $\mathbf{r}$.
(b) $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed away from the origin.
(c) $\mathbf{v}$ and $\mathbf{a}$ both are perpendicular to $\mathbf{r}$.
(d) $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed towards the origin.

Ans. (d)

Position vector of a particle moving around a unit circle ($r = 1$) in xy -plane is given by

$$\mathbf{r}(t) = \cos \omega t \hat{i} + \sin \omega t \hat{j} \quad \dots(i)$$

Velocity,

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} = \omega(-\sin \omega t \hat{i} + \cos \omega t \hat{j})$$

$$\Rightarrow \mathbf{v}(t) = \omega[\cos \omega t + \pi/2 \hat{i} + \sin \omega t + \pi/2 \hat{j}] \dots(ii)$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = -\omega^2 \cos \omega t \hat{i} - \omega^2 \sin \omega t \hat{j}$$

$$= -\omega^2(\cos \omega t \hat{i} + \sin \omega t \hat{j})$$

$$\mathbf{a}(t) = -\omega^2 \mathbf{r}(t) \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), it is clear that $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and $\mathbf{a}$ is directed towards the origin.

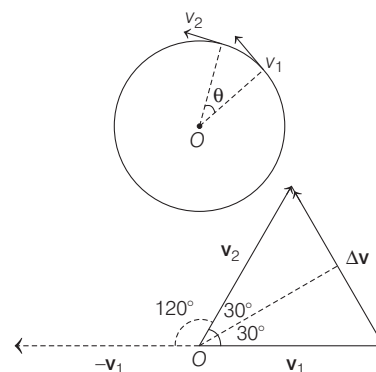
- 60** A particle is moving along a circular path with a constant speed of 10 ms^{-1} . What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

[2019, 11 Jan Shift-I]

- (a) $10\sqrt{2} \text{ m/s}$
(b) 10 m/s
(c) $10\sqrt{3} \text{ m/s}$
(d) Zero

Ans. (b)

Let $\mathbf{v}_1$ be the velocity of the particle moving along the circular path initially, $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocity when it moves through an angle of 60° as shown below.



From the figure, $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$

$$\Rightarrow |\Delta \mathbf{v}| = 2v \sin \frac{\theta}{2} = 2v \sin 30^\circ$$

$$[\because |\mathbf{v}_1| = |\mathbf{v}_2|]$$

$$= 2v \times \frac{1}{2} = v \quad (\text{Given, } v = 10 \text{ m/s})$$

$$\Rightarrow |\Delta \mathbf{v}| = 10 \text{ m/s}$$

Alternate Solution

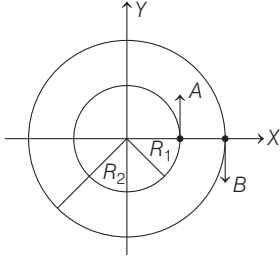
$$\therefore \Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = v_2 + (-\mathbf{v}_1)$$

$$\therefore |\Delta \mathbf{v}|^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos 120^\circ$$

$$= v^2 + v^2 + 2v \times v \times \left(-\frac{1}{2} \right)$$

$$\Rightarrow |\Delta \mathbf{v}| = v = 10 \text{ m/s}$$

- 61** Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω . At $t=0$, their positions and direction of motion are shown in the figure



The relative velocity $\mathbf{v}_A - \mathbf{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by

[2019, 12 Jan Shift-II]

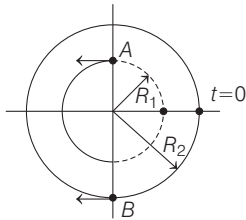
- (a) $\omega(R_1 + R_2)\hat{i}$
 (b) $-\omega(R_1 + R_2)\hat{i}$
 (c) $\omega(R_1 - R_2)\hat{i}$
 (d) $\omega(R_2 - R_1)\hat{i}$

Ans. (d)

Angle covered by each particle in time duration 0 to $\frac{\pi}{2\omega}$ is

$$\theta = \omega \times t = \omega \times \frac{\pi}{2\omega} = \frac{\pi}{2} \text{ rad}$$

So, positions of particles at $t = \frac{\pi}{2\omega}$ is as shown below;



Velocities of particles at $t = \frac{\pi}{2\omega}$ are

$$\mathbf{v}_A = -\omega R_1 \hat{i} \text{ and } \mathbf{v}_B = -\omega R_2 \hat{i}$$

The relative velocity of particles is

$$\begin{aligned} \mathbf{v}_A - \mathbf{v}_B &= -\omega R_1 \hat{i} - (-\omega R_2 \hat{i}) \\ &= -\omega(R_1 - R_2)\hat{i} \\ &= \omega(R_2 - R_1)\hat{i} \end{aligned}$$

- 62** Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 , respectively. Their speeds are such

that they make complete circles in the same time t . The ratio of their centripetal acceleration is

[AIEEE 2012]

- (a) $m_1 r_1 : m_2 r_2$ (b) $m_1 : m_2$
 (c) $r_1 : r_2$ (d) $1 : 1$

Ans. (c)

As their period of revolution is same, so their angular speed is also same. Centripetal acceleration is circular path, $a = \omega^2 r$.

$$\text{Thus, } \frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

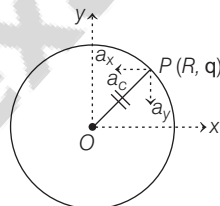
- 63.** For a particle in uniform circular motion the acceleration $\mathbf{a}$ at a point $P(R, \theta)$ on the circle of radius R is (here, θ is measured from the x-axis)

[AIEEE 2010]

- (a) $-\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j}$
 (b) $-\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j}$
 (c) $-\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$
 (d) $\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j}$

Ans. (c)

For a particle in uniform circular motion,



$$\mathbf{a} = \frac{v^2}{R} \text{ towards centre of circle [centripetal acceleration]}$$

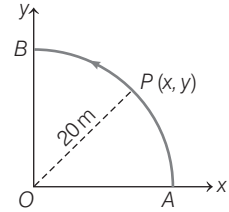
$$\therefore \mathbf{a} = \frac{v^2}{R} (-\cos \theta \hat{i} - \sin \theta \hat{j})$$

$$\text{or } \mathbf{a} = -\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$$

- 64** A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in

second. (d) The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly

[AIEEE 2010]



- (a) 13 ms^{-2} (b) 12 ms^{-2}
 (c) 7.2 ms^{-2} (d) 14 ms^{-2}

Ans. (d)

$$\text{Given, } s = t^3 + 5$$

$$\therefore \text{Speed, } v = \frac{ds}{dt} = 3t^2$$

$$\text{and rate of change of speed, } a_t = \frac{dv}{dt} = 6t$$

$$\therefore \text{Tangential acceleration at } t = 2 \text{ s,}$$

$$a_t = 6 \times 2 = 12 \text{ ms}^{-2}$$

$$\text{and at } t = 2 \text{ s, } v = 3(2)^2 = 12 \text{ ms}^{-1}$$

$$\therefore \text{Centripetal acceleration,}$$

$$a_c = \frac{v^2}{R} = \frac{144}{20} \text{ ms}^{-2}$$

$$\therefore \text{Net acceleration} = \sqrt{a_t^2 + a_c^2} \approx 14 \text{ ms}^{-2}$$

- 65** Which of the following statements is false for a particle moving in a circle with a constant angular speed?

[AIEEE 2004]

- (a) The velocity vector is tangent to the circle
 (b) The acceleration vector is tangent to the circle
 (c) The acceleration vector points to the centre of the circle
 (d) The velocity and acceleration vectors are perpendicular to each other

Ans. (b)

For a particle moving in a circle with constant angular speed, velocity vector is always along the tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle.

Since, tangential vector is perpendicular to radial vector, therefore velocity vector will be perpendicular to the acceleration vector. But in no case, acceleration vector is tangent to the circle.

4

Laws of Motion

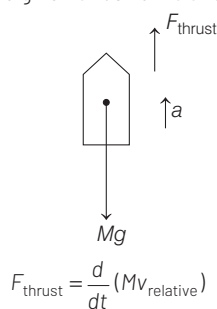
TOPIC 1

Newton's Laws of Motion and Conservation of Momentum

- 01** The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt, so that the rocket is given an acceleration of 20 ms^{-1} . The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket [Use, $g = 10 \text{ m/s}^2$] **[2021, 26 Aug Shift-I]**
- (a) $6.0 \times 10^2 \text{ kg s}^{-1}$ (b) 500 kg s^{-1}
(c) 10 kg s^{-1} (d) 60 kg s^{-1}

Ans. (d)

Given, $M = 1000 \text{ kg}$, $a = 20 \text{ m/s}^2$
 $v_{\text{relative}} = 500 \text{ m/s}$, $g = 10 \text{ m/s}^2$
The given situation is shown below



$$\Rightarrow F_{\text{thrust}} = v_{\text{relative}} \left(\frac{dM}{dt} \right)$$

By Newton's second law of motion,

$$\Rightarrow F_{\text{thrust}} - Mg = Ma$$

$$\Rightarrow v_{\text{relative}} \left(\frac{dM}{dt} \right) - Mg = Ma$$

$$\Rightarrow 500 \left(\frac{dM}{dt} \right) - 1000 \times 10 = 1000 \times 20$$

$$\Rightarrow 500 \left(\frac{dM}{dt} \right) = 1000 (20 + 10)$$

$$\Rightarrow 500 \left(\frac{dM}{dt} \right) = 1000 \times 30$$

$$\Rightarrow \frac{dM}{dt} = \frac{1000 \times 30}{500} = 60$$

$$\Rightarrow \frac{dM}{dt} = 60 \text{ kg/s}$$

- 02** A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0 \left[1 - \left(\frac{t-T}{T} \right)^2 \right]$$

where, F_0 and T are constants. The force acts only for the time interval $2T$. The velocity v of the particle after time $2T$ is

[2021, 27 July Shift-II]

(a) $\frac{2F_0 T}{M}$ (b) $\frac{F_0 T}{2M}$ (c) $\frac{4F_0 T}{3M}$ (d) $\frac{F_0 T}{3M}$

Ans. (c)

Given, force on particle,

$$F = F_0 \left[1 - \left(\frac{t-T}{T} \right)^2 \right] \quad \dots(i)$$

$\therefore$ Acceleration of ball will given as

$$a = \frac{F}{M} \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$a = \frac{F_0}{M} \left[1 - \left(\frac{t-T}{T} \right)^2 \right]$$

$$\Rightarrow \frac{dv}{dt} = \frac{F_0}{M} \left[1 - \left(\frac{t-T}{T} \right)^2 \right]$$

$$\Rightarrow dv = \frac{F_0}{M} \left[1 - \left(\frac{t-T}{T} \right)^2 \right] dt$$

Integrate above equation, we get

$$\Rightarrow \int dv = \frac{F_0}{M} \int_0^{2T} \left[1 - \left(\frac{t-T}{T} \right)^2 \right] dt$$

[$\therefore$ The force acts only for time interval $2T$]

$$\Rightarrow v = \frac{F_0}{M} \left[t - \frac{1}{3T^2} (t-T)^3 \right]_0^{2T}$$

$$\Rightarrow v = \frac{F_0}{M} \left[2T - \frac{1}{3T^2} (2T-T)^3 \right]$$

$$= \frac{F_0 T}{3M} \left[6 - \frac{1}{3} (2-1)^3 \right]$$

$$= \frac{4F_0 T}{3M}$$

- 03** A force $\mathbf{F} = (40\hat{i} + 10\hat{j}) \text{ N}$ acts on a body of mass 5 kg. If the body starts from rest its position vector $\mathbf{r}$ at time $t = 10 \text{ s}$ will be

[2021, 25 July Shift-II]

(a) $(100\hat{i} + 400\hat{j}) \text{ m}$ (b) $(100\hat{i} + 100\hat{j}) \text{ m}$
(c) $(400\hat{i} + 100\hat{j}) \text{ m}$ (d) $(400\hat{i} + 400\hat{j}) \text{ m}$

Ans. (c)

Given, force, $\mathbf{F} = 40\hat{i} + 10\hat{j}$

Mass, $m = 5 \text{ kg}$

Initial speed, $u = 0$

Time taken, $t = 10 \text{ s}$

Let the distance travelled by the body be s .

Since, $F = ma \Rightarrow a = \frac{F}{m}$

where, a is acceleration.

$$\therefore a = \frac{40\hat{i} + 10\hat{j}}{5} = 8\hat{i} + 2\hat{j} \text{ m/s}^2$$

Now, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow s = 0 \times 10 + \frac{1}{2} \times (8\hat{i} + 2\hat{j}) 10^2$$

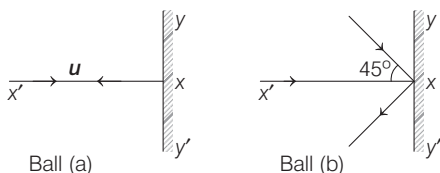
$$= \frac{100}{2} (8\hat{i} + 2\hat{j}) = 50(8\hat{i} + 2\hat{j})$$

$$= (400\hat{i} + 100\hat{j}) \text{ m}$$

- 04** Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 km/h (as shown) but at different angles. If the balls get reflected with the same speed,

then the ratio of the magnitude of impulses imparted to ball *a* and ball *b* by the wall along, *x*. direction is

[2021, 25 July Shift-I]



(a) 1 : 1 (b) $\sqrt{2} : 1$ (c) 2 : 1 (d) $1 : \sqrt{2}$

Ans. (b)

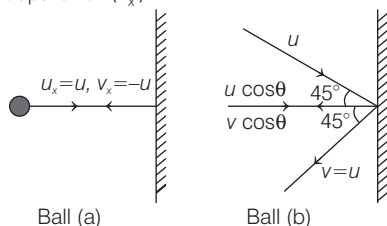
Given, mass of ball,

$$m = 30 \text{ g} = 30 \times 10^{-3} \text{ kg}$$

Speed of approach, $u = 180 \text{ kmph}$

$$= 180 \times \frac{5}{18} = 50 \text{ ms}^{-1}$$

and speed of approach (u_x) = speed of separation (v_x)



As we know that,

Impulse (I) = change in momentum (Δp)

Impulse for ball (a),

$$I_a = m(v_x - u_x)$$

$$I_a = m(u + u) = 2mu$$

$$= 2 \times 30 \times 10^{-3} \times 50 = 3 \text{ kg} \cdot \text{ms}^{-1}$$

and impulse for ball (b),

$$I_b = m(u \cos \theta + v \cos \theta)$$

$$\Rightarrow I_b = m(u \cos \theta + u \cos \theta) \quad (\because v = u)$$

$$= 2mu \cos \theta$$

$$= 2 \times 30 \times 10^{-3} \times 50 \times \cos 45^\circ$$

$$= 3/\sqrt{2} \text{ kg} \cdot \text{ms}^{-1}$$

$$\therefore I_a : I_b = \sqrt{2} : 1$$

- 05** A bullet of 4 g mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1} , the impulse imparted to the gun and velocity of recoil of gun are [2021, 22 July Shift-II]

(a) $0.4 \text{ kg} \cdot \text{ms}^{-1}$, 0.1 ms^{-1}

(b) $0.2 \text{ kg} \cdot \text{ms}^{-1}$, 0.05 ms^{-1}

(c) $0.2 \text{ kg} \cdot \text{ms}^{-1}$, 0.1 ms^{-1}

(d) $0.4 \text{ kg} \cdot \text{ms}^{-1}$, 0.05 ms^{-1}

Ans. (b)

Given, mass of bullet,

$$m_B = 4 \text{ g} = 4 \times 10^{-3} \text{ kg}$$

Mass of gun, $m_G = 4 \text{ kg}$

Speed of bullet, $v_B = 50 \text{ ms}^{-1}$

Let recoiling velocity of gun be v_G .

By using law of conservation of momentum for recoiling of gun

$$m_B v_B = -m_G v_G$$

$$\Rightarrow v_G = -\frac{m_B v_B}{m_G} = -\frac{4 \times 10^{-3} \times 50}{4}$$

$$= -50 \times 10^{-3} = -0.05 \text{ ms}^{-1}$$

Impulse (I) = Change in momentum of gun

$$I = m_G (0 - v_G) = 4 \times 0.05 = 0.2 \text{ kg} \cdot \text{ms}^{-1}$$

- 06** A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is *x* N. The value of *x* to the nearest integer is [2021, 18 March Shift-I]

Ans. (10)

Given,

The mass of the bullet, $m = 0.1 \text{ kg}$

The initial velocity of the bullet before hitting the wooden, $u = 10 \text{ m/s}$

The final velocity of the bullet after hitting the wooden, $v = 0$

The distance travelled by the bullet before coming to the rest, $s = 50 \text{ cm}$

Using the equation of the motion,

$$v^2 - u^2 = 2as$$

$$\Rightarrow 0^2 - (10)^2 = 2a(0.50)$$

$$\Rightarrow a = -100 \text{ m/s}^2$$

The uniform retardation of the bullet is 100 m/s^2 . The magnitude of the effective retarding force on the bullet,

$$F = ma$$

$$F = 0.1(-100)$$

$$F = -10 \text{ N}$$

Hence, the value of *x* to the nearest integer is 10.

- 07** A body of mass 2 kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k}) \text{ N}$. It starts from rest and was at the origin initially. After 4 s, its new coordinates are (8, *b*, 20). The value of *b* is (Round off to the nearest integer) [2021, 16 March Shift-II]

Ans. (12)

Given,

$$\text{Force, } \mathbf{F} = (2\hat{i} + 3\hat{j} + 5\hat{k}) \text{ N}$$

Mass, $m = 2 \text{ kg}$, Time, $t = 4 \text{ s}$

We know that

$$\text{Force} = \text{mass} \times \text{acceleration} \\ \Rightarrow \mathbf{F} = m \times \mathbf{a} \Rightarrow \mathbf{a} = \frac{\mathbf{F}}{m}$$

$$\Rightarrow \mathbf{a} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2} \quad \dots (i)$$

From second equation of motion,

$$\mathbf{s} = \mathbf{ut} + \frac{1}{2} \mathbf{at}^2 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\mathbf{s} = \mathbf{ut} + \frac{1}{2} \frac{(2\hat{i} + 3\hat{j} + 5\hat{k})}{2} t^2$$

$$= 0 + \frac{1}{4} (2\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (4)^2$$

$$[\because \mathbf{u} = 0 \text{ and } t = 4 \text{ s}]$$

$$= 0 + 8\hat{i} + 12\hat{j} + 20\hat{k}$$

$$= 8\hat{i} + 12\hat{j} + 20\hat{k}$$

Let $\mathbf{s} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = 8\hat{i} + 12\hat{j} + 20\hat{k} \quad \dots (iii)$$

According to question, new coordinates are (8, *b*, 20), it means

$$x\hat{i} + y\hat{j} + z\hat{k} = 8\hat{i} + b\hat{j} + 20\hat{k} \quad \dots (iv)$$

Comparing Eqs. (iii) and (iv), we get

$$b = 12$$

- 08** A boy pushes a box of mass 2 kg with a force $\mathbf{F} = (20\hat{i} + 10\hat{j}) \text{ N}$ on a frictionless surface. If the box was initially at rest, then m is displacement along the X-axis after 10 s. [2021, 26 Feb Shift-I]

Ans. (500)

Given, mass of box, $m = 2 \text{ kg}$

Force, $\mathbf{F} = 20\hat{i} + 10\hat{j} \text{ N}$

Initial speed of box, $u = 0 \text{ ms}^{-1}$

Time, $t = 10 \text{ s}$

Let acceleration of box is *a* and

displacement along X-axis after 10 s is s_x .

$$\text{As, } \mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{a} = \mathbf{F}/m$$

$$= \frac{20\hat{i} + 10\hat{j}}{2} = (10\hat{i} + 5\hat{j}) \text{ ms}^{-2}$$

By second equation of motion along X-axis,

$$s_x = u_x t + \frac{1}{2} a_x t^2 = 0 + \frac{1}{2} \times 10 \times (10)^2 = 500 \text{ m}$$

Hence, displacement along X-axis after 10 s is 500 m.

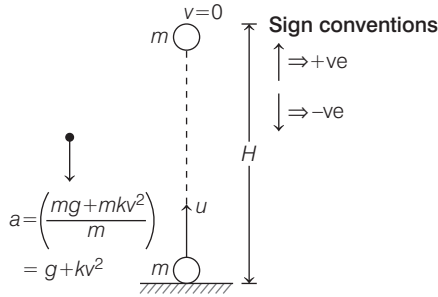
- 09** A small ball of mass *m* is thrown upward with velocity *u* from the ground. The ball experiences a resistive force mkv^2 , where *v* is its

speed) The maximum height attained by the ball is

[2020, 4 Sep Shift-II]

- (a) $\frac{1}{k} \tan^{-1} \left(\frac{ku^2}{2g} \right)$ (b) $\frac{1}{2k} \ln \left(1 + \frac{ku^2}{g} \right)$
 (c) $\frac{1}{k} \ln \left(1 + \frac{ku^2}{2g} \right)$ (d) $\frac{1}{2k} \tan^{-1} \left(\frac{ku^2}{g} \right)$

Ans. (b)



Net force on ball = weight of ball + resistive force

$$\begin{aligned} \text{i.e., } F_{\text{net}} &= W + F_{\text{resistive}} \\ &= (-mg) + (-mkv^2) \\ &= -m(g + kv^2) \end{aligned}$$

So, net acceleration of ball,

$$a = \frac{F_{\text{net}}}{m} = \frac{-m(g + kv^2)}{m} = -(g + kv^2)$$

$$\Rightarrow v \frac{dv}{dy} = -(g + kv^2) \Rightarrow \frac{v dv}{(g + kv^2)} = -dy$$

Integrating both sides,

$$\int_u^0 \frac{v}{(g + kv^2)} dv = - \int_0^H dy \quad \dots(i)$$

$$\begin{aligned} \text{Let } g + kv^2 &= t \Rightarrow 0 + 2kvdv = dt \\ \Rightarrow v dv &= \frac{1}{2k} dt \end{aligned}$$

Lower limit

$$\text{If } v = u, \text{ then } g + ku^2 = t \Rightarrow t = g + ku^2$$

Upper limit

$$\text{If } v = 0, \text{ then } g + k(0)^2 = t \Rightarrow t = g$$

Putting these values in eq. (i), we get

$$\int_{g+ku^2}^g \frac{1}{t} \frac{dt}{2k} = - \int_0^H dy$$

$$\Rightarrow \frac{1}{2k} \int_{g+ku^2}^g \frac{1}{t} dt = -[y]_0^H$$

$$\Rightarrow \frac{1}{2k} [\ln(t)]_{g+ku^2}^g = -[H - 0]$$

$$\Rightarrow \frac{1}{2k} [\ln(g) - \ln(g + ku^2)] = -[H]$$

$$\Rightarrow -\frac{1}{2k} [\ln(g) - \ln(g + ku^2)] = H$$

$$\begin{aligned} \Rightarrow H &= \frac{1}{2k} [\ln(g + ku^2) - \ln(g)] \\ &= \frac{1}{2k} \left[\ln \left(\frac{g + ku^2}{g} \right) \right] \end{aligned}$$

$$= \frac{1}{2k} \ln \left(1 + \frac{ku^2}{g} \right)$$

Hence, option (b) is correct.

10 A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its

instantaneous velocity. The instantaneous acceleration of the satellite is

[2020, 5 Sep Shift-II]

- (a) $-bv^3(t)$ (b) $-\frac{bv^3}{M(t)}$
 (c) $-\frac{2bv^3}{M(t)}$ (d) $-\frac{bv^3}{2M(t)}$

Ans. (b)

Given that, mass increases at the rate,

$$\frac{dM(t)}{dt} = bv^2(t)$$

Now, from Newton's second law,

Force = Rate of change of linear momentum

$$\begin{aligned} \text{i.e., } F &= \frac{dp}{dt} = \frac{d}{dt}(Mv) = M \frac{dv}{dt} + v \frac{dM}{dt} \\ &= M \frac{dv}{dt} + v[bv^2(t)] = M \frac{dv}{dt} + bv^3(t) \end{aligned}$$

Now, force acting on satellite is zero,

i.e., $F = 0$

$$\Rightarrow M \frac{dv}{dt} = -bv^3 \Rightarrow \frac{dv}{dt} = -\frac{bv^3}{M(t)}$$

$$\therefore \text{Acceleration of the satellite, } a = -\frac{bv^3}{M(t)}$$

Hence, correct option is (b).

11 A ball is thrown upward with an initial velocity v_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where, m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is

[2019, 10 April Shift-I]

- (a) $\frac{1}{\sqrt{2\gamma g}} \tan^{-1} \left(\sqrt{\frac{2\gamma}{g}} v_0 \right)$
 (b) $\frac{1}{\sqrt{\gamma g}} \tan^{-1} \left(\sqrt{\frac{\gamma}{g}} v_0 \right)$
 (c) $\frac{1}{\sqrt{\gamma g}} \sin^{-1} \left(\sqrt{\frac{\gamma}{g}} v_0 \right)$
 (d) $\frac{1}{\sqrt{\gamma g}} \ln \left(1 + \sqrt{\frac{\gamma}{g}} v_0 \right)$

Ans. (b)

Given, drag force, $F = m\gamma v^2$... (i)

As we know, general equation of force
 $= ma$... (ii)

Comparing Eqs. (i) and (ii), we get
 $a = \gamma v^2$

$\therefore$ Net retardation of the ball when thrown vertically upward is

$$a_{\text{net}} = -(g + \gamma v^2) = \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{(g + \gamma v^2)} = -dt \quad \dots(iii)$$

By integrating both sides of Eq. (iii) in known limits, i.e.

When the ball thrown upward with velocity v_0 and then reaches to its zenith, i.e. for maximum height at time $t = t, v = 0$

$$\Rightarrow \int_{v_0}^0 \frac{dv}{(\gamma v^2 + g)} = \int_0^t -dt$$

$$\text{or } \frac{1}{\gamma} \int_{v_0}^0 \frac{1}{\left[\left(\sqrt{\frac{g}{\gamma}} \right)^2 + v^2 \right]} dv = - \int_0^t dt$$

$$\Rightarrow \frac{1}{\gamma} \cdot \frac{1}{\sqrt{g/\gamma}} \cdot \left[\tan^{-1} \left(\frac{v}{\sqrt{g/\gamma}} \right) \right]_{v_0}^0 = -t$$

$$\left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right]$$

$$\Rightarrow \frac{1}{\sqrt{\gamma g}} \cdot \tan^{-1} \left(\frac{\sqrt{\gamma} v_0}{\sqrt{g}} \right) = +t$$

12 A bullet of mass 20 g has an initial speed of 1 ms^{-1} , just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of $2.5 \times 10^{-2} \text{ N}$, the speed of the bullet after emerging from the other side of the wall is close to

[2019, 10 April Shift-II]

- (a) 0.3 ms^{-1} (b) 0.4 ms^{-1}
 (c) 0.1 ms^{-1} (d) 0.7 ms^{-1}

Ans. (d)

Given, resistance offered by the wall

$$= F = -2.5 \times 10^{-2} \text{ N}$$

So, deceleration of bullet,

$$a = \frac{F}{m} = \frac{-2.5 \times 10^{-2}}{20 \times 10^{-3}} = -\frac{5}{4} \text{ ms}^{-2}$$

$$(\because m = 20 \text{ g} = 20 \times 10^{-3} \text{ kg})$$

Now, using the equation of motion,

$$v^2 - u^2 = 2as$$

$$\text{We have, } v^2 = 1 + 2 \left(-\frac{5}{4} \right) (20 \times 10^{-2})$$

$$(\because u = 1 \text{ ms}^{-1} \text{ and } s = 20 \text{ cm} = 20 \times 10^{-2} \text{ m})$$

$$\Rightarrow v^2 = \frac{1}{2}$$

$$\therefore v = \frac{1}{\sqrt{2}} \approx 0.7 \text{ ms}^{-1}$$

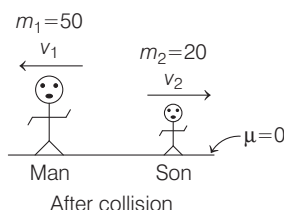
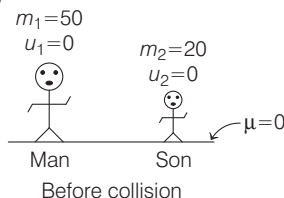
- 13** A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is

[2019, 12 April Shift-I]

- (a) 0.28 ms^{-1} (b) 0.20 ms^{-1}
(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

Ans. (b)

The given situation can be shown as below



Using momentum conservation law,
(Total momentum)_{before collision} = (Total momentum)_{after collision}
 $(m_1 \times 0) + (m_2 \times 0) = m_1 v_1 + m_2 v_2$
 $0 = m_1(-v_1)\hat{i} + m_2 v_2 \hat{i}$

$$\Rightarrow m_1 v_1 = m_2 v_2$$

$$\Rightarrow 50 v_1 = 20 v_2$$

$$\Rightarrow v_2 = 2.5 v_1 \quad \dots(i)$$

Again, relative velocity = 0.70 m/s

But from figure, relative velocity = $v_1 + v_2$
 $\therefore v_1 + v_2 = 0.7 \quad \dots(ii)$

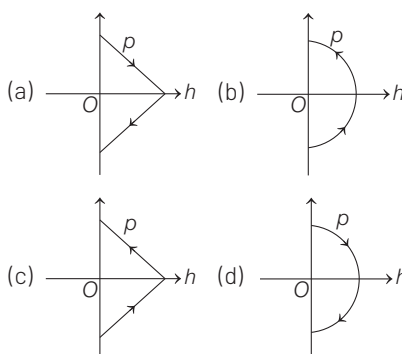
From Eqs. (i) and (ii), we get

$$v_1 + 2.5 v_1 = 0.7$$

$$\Rightarrow v_1(3.5) = 0.7$$

$$v_1 = \frac{0.7}{3.5} = 0.20 \text{ m/s}$$

- 14** A ball is thrown vertically up (taken as +Z-axis) from the ground. The correct momentum-height (p - h) diagram is



[2019, 9 April Shift-I]

Ans. (d)

When a ball is thrown vertically upward, then the acceleration of the ball, a = acceleration due to gravity (g) (acting in the downward direction).

Now, using the equation of motion,

$$v^2 = u^2 - 2gh$$

$$\text{or } h = \frac{-v^2 + u^2}{2g} \quad \dots(i)$$

As we know, momentum, $p = mv$ or $v = p/m$

So, substituting the value of v in Eq. (i), we get

$$h = \frac{u^2 - (p/m)^2}{2g}$$

As we know that, at the maximum height, velocity of the ball thrown would be zero.

So, for the flight when the ball is thrown till it reaches the maximum height (h).

$v \rightarrow$ changes from u to 0

$\Rightarrow p \rightarrow$ changes from mu to 0

Similarly, when it reaches its initial point, then

$h \rightarrow$ changes from $h_{\max}$ to 0

Also, $p \rightarrow$ changes from 0 to some values.

Thus, these conditions are only satisfied in the plot given in option (d).

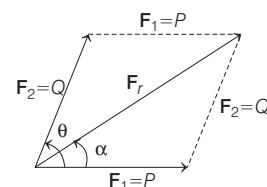
- 15** Two forces P and Q of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets double. Then, the angle θ is

[2019, 10 Jan Shift-II]

- (a) 60° (b) 120°
(c) 30° (d) 90°

Ans. (b)

Resultant force F_r of any two forces F_1 (i.e. P) and F_2 (i.e. Q) with an angle θ between them can be given by vector addition as



$$F_r^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta \quad \dots(i)$$

In first case $F_1 = 2F$ and $F_2 = 3F$

$$\Rightarrow F_r^2 = 4F^2 + 9F^2 + 2 \times 2 \times 3F^2 \cos \theta$$

$$\Rightarrow F_r^2 = 13F^2 + 12F^2 \cos \theta \quad \dots(ii)$$

In second case $F_1 = 2F$ and $F_2 = 6F$

($\because$ Force Q gets doubled)

$$\text{and } F_r' = 2F_r \quad (\text{Given})$$

By putting these values in Eq. (i), we get

$$(2F_r)^2 = (2F)^2 + (6F)^2 + 2 \times 2 \times 6F^2 \cos \theta$$

$$\Rightarrow 4F_r^2 = 40F^2 + 24F^2 \cos \theta \quad \dots(iii)$$

From Eq. (ii) and Eq. (iii), we get;

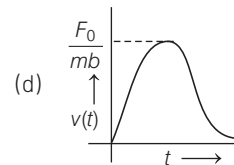
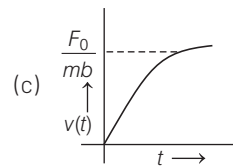
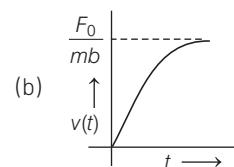
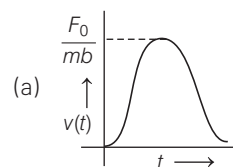
$$52F^2 + 48F^2 \cos \theta = 40F^2 + 24F^2 \cos \theta$$

$$\Rightarrow 12 + 24 \cos \theta = 0 \text{ or } \cos \theta = -1/2$$

$$\text{or } \theta = 120^\circ \quad (\because \cos 120^\circ = -1/2)$$

- 16** A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x -direction. Its speed $v(t)$ is depicted by which of the following curves?

[AIEEE 2012]

**Ans. (c)**

As the force is exponentially decreasing, so its acceleration, i.e., rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curl with decreasing slope with time.

$$\therefore a = \frac{F}{M} = \frac{F_0}{m} e^{-bt} = \frac{dv}{dt}$$

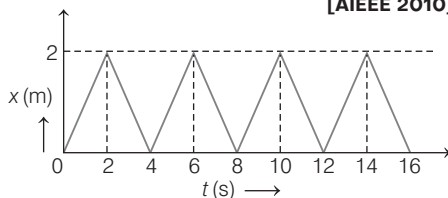
$$\Rightarrow \int_0^v dv = \int_0^t \frac{F_0}{m} e^{-bt} dt$$

$$\Rightarrow v = \frac{F_0}{m} \left[\left(\frac{1}{-b} \right) e^{-bt} \right]_0^t = \frac{F_0}{mb} [e^{-bt}]_t^0$$

$$= \frac{F_0}{mb} (e^0 - e^{-bt}) = \frac{F_0}{mb} (1 - e^{-bt})$$

$$v_{\max} = \frac{F_0}{mb}$$

- 17** The figure shows the position-time ($x-t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is [AIEEE 2010]



- (a) 0.4 N-s (b) 0.8 N-s
(c) 1.6 N-s (d) 0.2 N-s

Ans. (b)

From the graph, it is a straight line, so uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

$$\text{Initial velocity, } v_1 = \frac{2}{2} = 1 \text{ ms}^{-1}$$

$$\text{Final velocity, } v_2 = -\frac{2}{2} = -1 \text{ ms}^{-1}$$

$$p_i = mv_1 = 0.4 \text{ N-s}$$

$$p_f = mv_2 = -0.4 \text{ N-s}$$

$$J = p_f - p_i = -0.4 - 0.4 = -0.8 \text{ N-s}$$

[J = impulse]

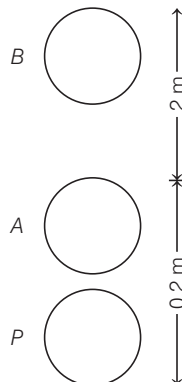
$$\therefore |J| = 0.8 \text{ N-s}$$

- 18** A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10 \text{ m/s}^2$. [AIEEE 2006]
- (a) 4 N (b) 16 N (c) 20 N (d) 22 N

Ans. (d)

The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA be $a \text{ ms}^{-2}$ (assumed to be constant) in upward direction and velocity of ball at be $v \text{ m/s}$.

Then, for PA,



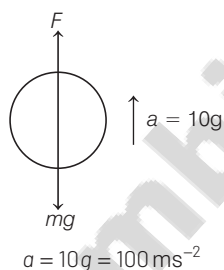
$$v^2 = 0^2 + 2a \times 0.2$$

For AB,

$$0 = v^2 - 2 \times g \times 2$$

$$\Rightarrow v^2 = 2g \times 2$$

From above equations,



Then, for PA, FBD of ball is

$$F - mg = ma$$

[F is the force exerted by hand on ball]

$$\Rightarrow F = m(g + a)$$

$$= 0.2(11g) = 22 \text{ N}$$

Alternate Solution Using work-energy theorem

$$W_{mg} + W_F = 0$$

$$\Rightarrow -mg \times 2.2 + F \times 0.2 = 0$$

$$\text{or } F = 22 \text{ N}$$

- 19** A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to [AIEEE 2006]

- (a) 150 N (b) 3 N
(c) 30 N (d) 300 N

Ans. (c)

This is the question based on impulse-momentum theorem.

$$|F \cdot \Delta t| = |\text{Change in momentum}|$$

$$\Rightarrow F \times 0.1 = |p_f - p_i|$$

As the ball will stop after catching

$$p_f = mv_f = 0.15 \times 20 = 3, p_i = 0$$

$$\Rightarrow F \times 0.1 = 3 \Rightarrow F = 30 \text{ N}$$

- 20** A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15 \text{ N/m}$. What will be its initial acceleration, if it is released from a point 20 cm away from the origin? [AIEEE 2005]

- (a) 3 ms^{-2} (b) 15 ms^{-2}
(c) 5 ms^{-2} (d) 10 ms^{-2}

Ans. (d)

Given, $m = 0.3 \text{ kg}$, $x = 20 \text{ cm}$

and $k = 15 \text{ N/m}$

$$\text{given } F = -kx \quad \dots(i)$$

$$\text{and } F = ma \quad \dots(ii)$$

$$\therefore ma = -kx$$

$$\Rightarrow a = -\frac{15}{0.3} \times 20 \times 10^{-2}$$

$$\Rightarrow a = -\frac{15}{3} \times 2 = -10 \text{ ms}^{-2}$$

Negative sign indicates that acceleration is always towards the mean position.

$\therefore$ Initial acceleration, $a = 10 \text{ ms}^{-2}$

- 21** A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1} . The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? [AIEEE 2004]

- (a) 1 (b) 4
(c) 2 (d) 3

Ans. (d)

Force exerted by machine gun on man's hand in firing a bullet = Change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000} \right) \times 1200 = 48 \text{ N}$$

Force exerted by man on machine gun

= Force exerted on man by machine gun = 144 N

$$\text{Hence, number of bullets fire(d)} = \frac{144}{48} = 3$$

- 22** A rocket with a lift-off mass $3.5 \times 10^4 \text{ kg}$ is blasted upwards with an initial acceleration of 10 ms^{-2} . Then, the initial thrust of the blast is [AIEEE 2003]

- (a) $3.5 \times 10^5 \text{ N}$ (b) $7.0 \times 10^5 \text{ N}$
(c) $14.0 \times 10^5 \text{ N}$ (d) $1.75 \times 10^5 \text{ N}$

Ans. (a)

Here, thrust force is responsible to accelerate the rocket, so initial thrust of the blast

$$= ma = 3.5 \times 10^4 \times 10 = 3.5 \times 10^5 \text{ N}$$

- 23** Two forces are such that the sum of their magnitudes is 18 N and their resultant which has magnitude 12 N, is perpendicular to the smaller force. Then, the magnitudes of the forces are

[AIEEE 2002]

- (a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N

Ans. (b)

The sum of the two forces

$$A + B = 18 \quad \dots(i)$$

$$12 = \sqrt{A^2 + B^2 + 2AB \cos \theta} \quad \dots(ii)$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\Rightarrow \tan 90^\circ = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\Rightarrow \cos \theta = \frac{-A}{B} \quad \dots(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$A = 5 \text{ N}, B = 13 \text{ N}$$

Since, $A + B = 18$ and $\cos \theta = -\frac{A}{B}$

Squaring both sides in Eq. (i), we get

$$144 = A^2 + B^2 + 2AB \cos \theta$$

On putting $\cos \theta = -\frac{A}{B}$ in above equation,

we get

$$144 = A^2 + B^2 + 2AB \left(-\frac{A}{B} \right)$$

$$= A^2 + B^2 - 2A^2$$

$$\Rightarrow 144 = B^2 - A^2$$

$$\Rightarrow 144 = (B - A) \cdot (B + A)$$

On putting $B + A = 18$, we have

$$B - A = \frac{144}{18} = 8$$

$$\text{Solving } B - A = 8$$

$$B + A = 18$$

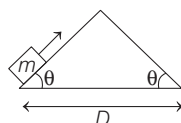
We have, $B = 13 \text{ N}, A = 5 \text{ N}$

TOPIC 2

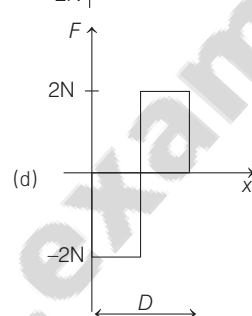
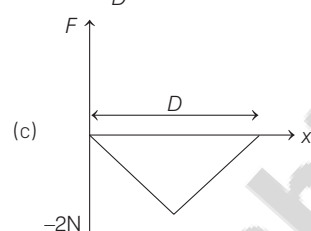
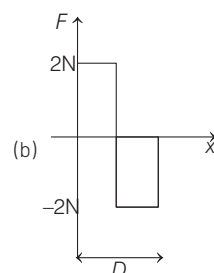
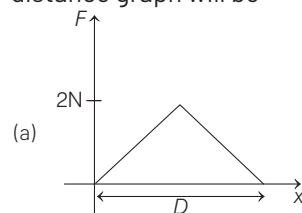
Equilibrium of a Particle and Common Forces in Mechanics

- 24** An object of mass m is being moved with a constant velocity under the action of an applied force of 2 N along a frictionless surface with following surface profile.

[2021, 1 Sep Shift-II]

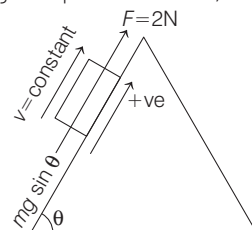


The correct applied force versus distance graph will be



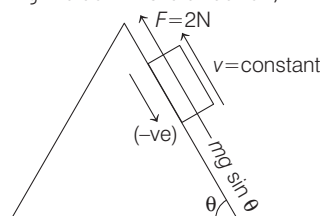
Ans. (b)

Let's draw the free body diagram, (During the upward direction)



$F = 2\text{N} = (+ve)$ constant

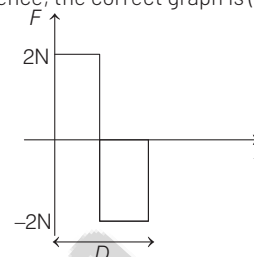
(During the downward direction)



$F = 2\text{N} = (-ve)$ constant

During the upward motion, the force is positive constant and during the downward motion the force is negative constant.

Hence, the correct graph is (b).

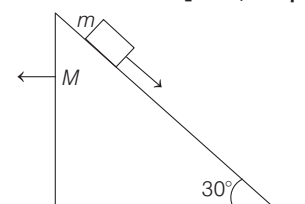


- 25** A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is

[Given, $m = 8 \text{ kg}, M = 16 \text{ kg}$]

Assume all the surfaces shown in the figure to be frictionless.

[2021, 1 Sep Shift-II]



- (a) $\frac{4}{3}g$ (b) $\frac{6}{5}g$ (c) $\frac{3}{5}g$ (d) $\frac{2}{3}g$

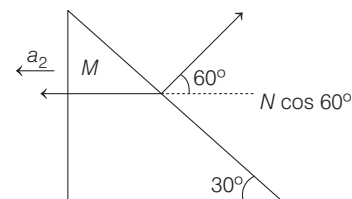
Ans. (d)

Here, both block and wedge are moving.

Consider the acceleration of the block with respect to the wedge is a_1 and the acceleration of the wedge is a_2 .

Given, mass of the wedge, $M = 16 \text{ kg}$ and mass of block, $m = 8 \text{ kg}$

Let's draw the free body diagram of the wedge,



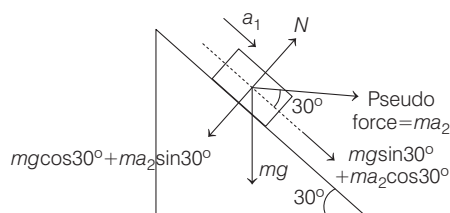
In the x-directions,

$$N \cos 60^\circ = M a_2$$

$$N(0.5) = 16 \times a_2$$

$$\Rightarrow N = 32 a_2$$

Now, draw the free body diagram of the block with respect to the wedge.



Along the perpendicular to the inclined plane,

$$N = 8g \cos 30^\circ - 8a_2 \sin 30^\circ$$

$$32a_2 = 4\sqrt{3}g - 4a_2$$

$$36a_2 = 4\sqrt{3}g \Rightarrow a_2 = \frac{\sqrt{3}}{9}g$$

Along the inclined plane,

$$mg \sin 30^\circ + ma_2 \cos 30^\circ = ma_1$$

$$8g \times \frac{1}{2} + 8 \times \frac{\sqrt{3}}{9}g \times \frac{\sqrt{3}}{2} = 8a_1$$

$$\Rightarrow a_1 = \frac{2g}{3}$$

$\therefore$ The acceleration of the block with respect to the wedge is $2g/3$.

- 26** A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is
(Take, $g = 10 \text{ ms}^{-2}$)

[2021, 31 Aug Shift-I]

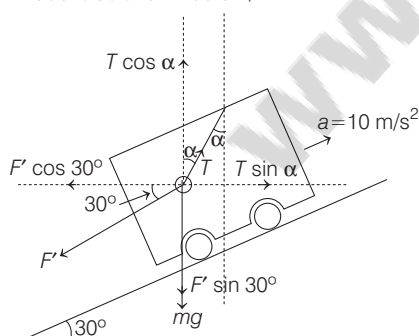
Ans. (30)

Given, Angle of inclination, $\theta = 30^\circ$

Acceleration, $a = 10 \text{ ms}^{-2}$

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

According to the question the car and bob is as shown below,



Here, F' is the pseudo force acting on the bob when we considered it from car's frame and T is the tension on the string.

In equilibrium,

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

$$\Rightarrow \frac{F' \cos 30^\circ}{\sin \alpha} = T \quad \dots(i)$$

where, m is the mass of the bob.

$$F' \sin 30^\circ + mg = T \cos \alpha$$

$$\Rightarrow ma \sin 30^\circ + mg = \frac{ma \cos 30^\circ}{\sin \alpha} (\cos \alpha)$$

[$\because$ using Eq. (i)]

$$\Rightarrow a \sin 30^\circ + g = \frac{a \cos 30^\circ}{\sin \alpha} (\cos \alpha)$$

$$10 \times \frac{1}{2} + 10 = \frac{10 \times \sqrt{3}}{2} \cot \alpha$$

$$\Rightarrow \frac{1}{2} + 1 = \frac{\sqrt{3}}{2} \cot \alpha$$

$$\Rightarrow \frac{3}{2} = \frac{\sqrt{3}}{2} \cot \alpha$$

$$\text{or } \cot \alpha = \sqrt{3}$$

$$\Rightarrow \alpha = 30^\circ$$

27 Statement I If three forces F_1, F_2

and F_3 are represented by three sides of a triangle and $F_1 + F_2 = -F_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II A triangle made up of three forces F_1, F_2 and F_3 as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 31 Aug Shift-II]

- (a) Statement I is false but statement II is true.
- (b) Statement I is true but statement II is false.
- (c) Both statement I and statement II are false.
- (d) Both statement I and statement II are true.

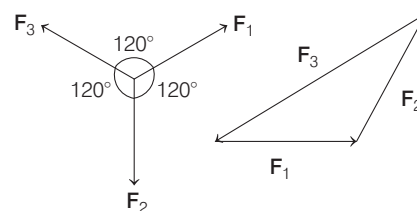
Ans. (d)

Three forces F_1, F_2 and F_3 are acting on a body and if this body is in equilibrium, then resultant of these three forces must be zero i.e. $F_{\text{net}} = F_1 + F_2 + F_3 = 0$

$$\Rightarrow F_1 + F_2 = -F_3$$

This situation can be shown graphically by three concurrent forces at 120° with each others.

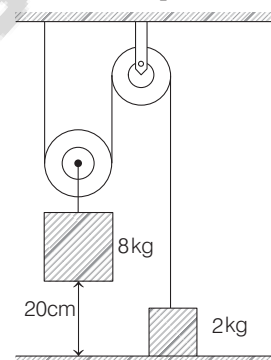
or, by three forces in the same order along three sides of a triangle.



Hence, both statement I and statement II are true.

- 28** The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest.
(Use, $g = 10 \text{ m/s}^2$)

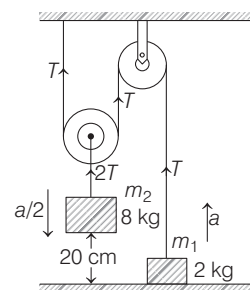
[2021, 27 Aug Shift-II]



- (a) 0.34 s
- (b) 0.2 s
- (c) 0.25 s
- (d) 0.4 s

Ans. (d)

According to the given figure, and free body diagram



masses of two bodies $m_1 = 2 \text{ kg}$ $m_2 = 8 \text{ kg}$,
Acceleration of mass $m_1 = a$

Acceleration of mass $m_2 = \frac{a}{2}$

Tension = T

Distance between m_2 and ground = 20 cm
= 0.2 m

Initial velocity $u = 0$

Equation of motion of 2 kg block,

$$\therefore T - 2g = 2a \quad \dots(i)$$

Equation of motion of 8 kg block,
and $8g - 2T = 8\frac{a}{2} \Rightarrow 4g - T = 2a$... (ii)

From Eqs. (i) and (ii),

$$T - 2g = 4g - T$$

$$\Rightarrow 2T = 6g \Rightarrow T = 3g$$

Substituting the value in Eq. (i) we get

$$3g - 2g = 2a \Rightarrow g = 2a$$

$$\Rightarrow a = \frac{g}{2} = 5 \text{ ms}^{-2}$$

$$\therefore a_1 = 5 \text{ ms}^{-2} \text{ and } a_2 = \frac{5}{2} \text{ ms}^{-2}$$

Since, $s = ut + \frac{1}{2}at^2$

$$\therefore \frac{20}{100} = 0 + \frac{1}{2} \times 5 \times t^2 \Rightarrow t^2 = \frac{20 \times 4}{5 \times 100}$$

$$\Rightarrow t = \frac{2}{5} = 0.4 \text{ s}$$

- 29** A body of mass m is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$. if the

time of ascent is half of the time of descent. The value of x is

[2021, 20 July Shift-II]

Ans. (3)

Let us assume that,

T_A be the time of ascent of body and T_D be the time of descent of body.

From second equation of motion, we have

$$s = ut + \frac{1}{2}at^2$$

During the time of ascent, the distance covered by the body,

$$s = \frac{1}{2}a_A t_A^2 \quad \dots (i)$$

and during the time of descent, the distance covered by the body,

$$s = \frac{1}{2}a_D t_D^2 \quad \dots (ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{1}{2}a_A t_A^2 = \frac{1}{2}a_D t_D^2$$

$$\Rightarrow a_A t_A^2 = a_D t_D^2 \Rightarrow \frac{t_A^2}{t_D^2} = \frac{a_D}{a_A}$$

$$\Rightarrow \frac{t_A^2}{t_D^2} = \frac{g \sin \theta - \mu g \cos \theta}{g \sin \theta + \mu g \cos \theta}$$

$$\Rightarrow \frac{t_A}{t_D} = \sqrt{\frac{g \sin \theta - \mu g \cos \theta}{g \sin \theta + \mu g \cos \theta}}$$

$$= \sqrt{\frac{g \sin 30^\circ - \mu g \cos 30^\circ}{g \sin 30^\circ + \mu g \cos 30^\circ}}$$

$$\Rightarrow \frac{1}{2} = \sqrt{\frac{1 - \sqrt{3}\mu}{1 + \sqrt{3}\mu}} \left\{ \because \frac{t_A}{t_D} = \frac{1}{2} \text{ (given)} \right\}$$

$$\Rightarrow 1 + \sqrt{3}\mu = 4 - 4\sqrt{3}\mu$$

$$\Rightarrow \sqrt{3}\mu + 4\sqrt{3}\mu = 4 - 1$$

$$\Rightarrow \mu(\sqrt{3} + 4\sqrt{3}) = 3 \Rightarrow \mu(5\sqrt{3}) = 3$$

$$\Rightarrow \mu = \frac{\sqrt{3}}{5} \quad \dots (iii)$$

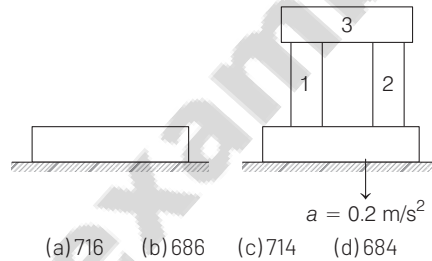
According to question, the coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$, therefore, on comparing it with Eq. (iii), we can write $x = 3$.

- 30** A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2 .

The normal reaction R by the floor, if mass of the iron cylinders are equal and of 20 kg each, is N.

[Take, $g = 10 \text{ m/s}^2$ and $\mu_s = 0.2$]

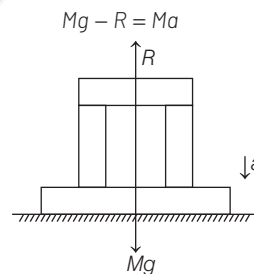
[2021, 20 July Shift-I]



- (a) 716 (b) 686 (c) 714 (d) 684

Ans. (b)

The force equation in vertical direction is



where, M = collective mass of block and all three iron cylinders

$$= 10 + 3 \times 20 = 70 \text{ kg}$$

a = acceleration of block $= 0.2 \text{ ms}^{-2}$

$$g = 10 \text{ ms}^{-2} \text{ and } \mu_s = 0.2$$

and R = normal reaction

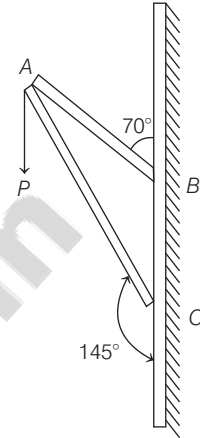
$$\text{Force along vertical axis } Mg - R = Ma$$

$$\therefore 70g - R = 70 \times 0.2$$

$$\Rightarrow R = 70 \times 10 - 14$$

$$= 700 - 14 = 686 \text{ N}$$

- 31** Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force P of magnitude 100 N is applied at point A of the frame.



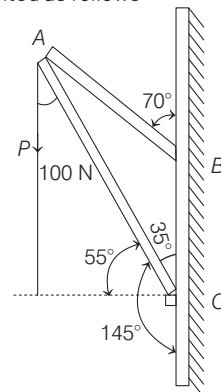
Suppose the force is P resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is x N. The value of x , to the nearest integer, is

[Given, $\sin(35^\circ) = 0.573$, $\cos(35^\circ) = 0.819$, $\sin(110^\circ) = 0.939$, $\cos(110^\circ) = -0.342$]

[2021, 16 March Shift-I]

Ans. (82)

If the force P of magnitude 100 N is resolved parallel to the arms AB and AC of the frame, the above figure will be represented as follows



Component of force along AC $= 100 \cos 35^\circ \text{ N}$

$$= 100 \times 0.819 \text{ N} = 81.9 \text{ N} \approx 82 \text{ N}$$

This is the required magnitude of the resolved component along the arm AC.

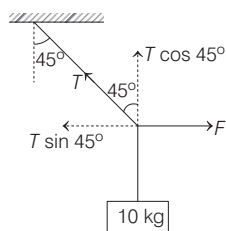
Compare with given in question, $x = 82$.

- 32** A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then, F equals (Take, $g = 10 \text{ ms}^{-2}$ and the rope to be massless) [2020, 7 Jan Shift-II]

(a) 75 N (b) 70 N (c) 100 N (d) 90 N

Ans. (c)

Given situation is as shown below.



We resolve tension T in string into vertical and horizontal components. For equilibrium,

$$F = T \sin 45^\circ \quad \dots (i)$$

$$\text{and } Mg = T \cos 45^\circ \quad \dots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

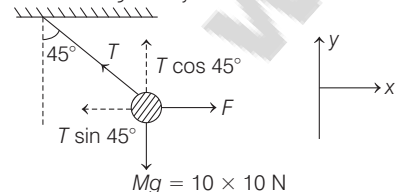
$$\frac{F}{Mg} = \tan 45^\circ \text{ or } F = Mg = 10 \times 10 = 100 \text{ N}$$

- 33** A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the mass, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (Take, $g = 10 \text{ ms}^{-2}$)

[2019, 9 Jan Shift-II]
(a) 70 N (b) 200 N
(c) 100 N (d) 140 N

Ans. (c)

FBD of the given system is follow



Let T = tension in the rope.

For equilibrium condition of the mass,

$$\Sigma F_x = 0 \text{ (force in x-direction)}$$

$$\Sigma F_y = 0 \text{ (force in y-direction)}$$

When $\Sigma F_x = 0$, then

$$\therefore F = T \sin 45^\circ \quad \dots (i)$$

When $\Sigma F_y = 0$, then

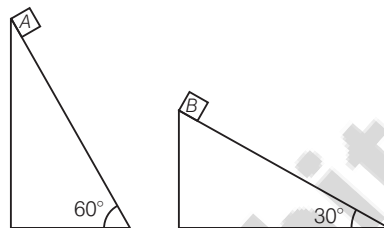
$$Mg = T \cos 45^\circ \quad \dots (ii)$$

Using Eqs. (i) and (ii), we get

$$\Rightarrow \frac{F}{Mg} = \frac{T \sin 45^\circ}{T \cos 45^\circ} \Rightarrow \frac{F}{Mg} = \frac{1}{\frac{1}{\sqrt{2}}} = 1$$

$$\Rightarrow F = Mg = 10 \times 10 = 100 \text{ N}$$

- 34** Two fixed frictionless inclined plane making the angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? [AIEEE 2010]



- (a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

Ans. (d)

Force applying on the block

$$F = mg \sin \theta$$

$$\text{or } mg \sin \theta = ma$$

$$\therefore a = g \sin \theta$$

where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g \sin^2 \theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60^\circ - \sin^2 30^\circ) = \frac{g}{2} = 4.9 \text{ ms}^{-2}$$

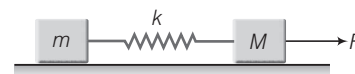
[in vertical direction]

- 35** A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k . The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m . [AIEEE 2007]

- (a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$
(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

Ans. (c)

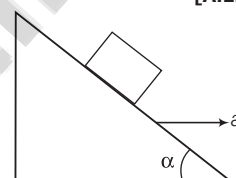
$$\text{Acceleration of system, } a = \frac{F}{m+M}$$



So, force acting on mass,

$$F = ma = \frac{mF}{m+M}$$

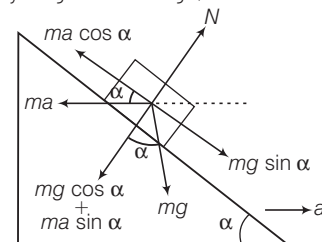
- 36** A block is kept on a frictionless inclined surface with angle of inclination α . The incline is given an acceleration a to keep the block stationary. Then, a is equal to [AIEEE 2005]



- (a) $\frac{g}{\tan \alpha}$ (b) $g \csc \alpha$
(c) g (d) $g \tan \alpha$

Ans. (d)

In the frame of wedge, the force diagram of block is shown in figure. From free body diagram of wedge,



For block to remain stationary,

$$ma \cos \alpha = mg \sin \alpha$$

or

$$a = g \tan \alpha$$

- 37** Two masses $m_1 = 5 \text{ kg}$ and $m_2 = 4.8 \text{ kg}$ tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move? ($g = 9.8 \text{ ms}^{-2}$) [AIEEE 2004]



- (a) 0.2 ms^{-2} (b) 9.8 ms^{-2}
 (c) 5 ms^{-2} (d) 4.8 ms^{-2}

Ans. (d)

On releasing, the motion of the system will be according to figure.

$$m_1 g - T = m_1 a \quad \dots(i)$$

$$\text{and } T - m_2 g = m_2 a \quad \dots(ii)$$

$$\text{On solving, } a = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g \quad \dots(iii)$$

$$a \quad T \quad T \quad a$$

$$m_1 g$$

$$m_2 g$$

$$\text{Here, } m_1 = 5 \text{ kg, } m_2 = 4.8 \text{ kg,}$$

$$g = 9.8 \text{ ms}^{-2}$$

$$\therefore a = \left(\frac{5 - 4.8}{5 + 4.8} \right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2 \text{ ms}^{-2}$$

- 38** A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 ms^{-2} , the reading of the spring balance will be

[AIEEE 2003]

- (a) 24 N (b) 74 N (c) 15 N (d) 49 N

Ans. (a)

In stationary position, spring balance reading = $mg = 49$

$$\text{or } m = \frac{49}{9.8} = 5 \text{ kg}$$

$$T$$

$$a$$

$$mg$$

When lift moves downward,
 $mg - T = ma$

Reading of balance,

$$T = mg - ma$$

$$= 5(9.8 - 5)$$

$$= 5 \times 4.8 = 24.0 \text{ N}$$

- 39** Three forces start acting simultaneously on a particle moving with velocity $\mathbf{v}$. These forces are represented in magnitude and direction by the three sides of a ΔABC (as shown). The particle will now move with velocity **[AIEEE 2003]**

- (a) less than $\mathbf{v}$
 (b) greater than $\mathbf{v}$
 (c) $|\mathbf{v}|$ in the direction of largest force BC
 (d) $\mathbf{v}$, remaining unchanged

Ans. (d)

Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's 2nd law ($\mathbf{F} = m \frac{d\mathbf{v}}{dt}$), the velocity ($\mathbf{v}$) of particle will be same.

- 40** A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m . If a force P is applied at the free end of the rope, the force exerted by the rope on the block is **[AIEEE 2003]**

- (a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$
 (c) P (d) $\frac{PM}{M+m}$

Ans. (d)

Let acceleration of system (rope + block) be a along the direction of applied force. Then,

$$a = \frac{P}{M+m}$$

$$a$$

$$m$$

$$M$$

$$P$$

$$\text{Frictionless surface}$$

Draw the FBD of block and rope as shown in figure.

$$a \quad a$$

$$M \quad T \quad T \quad m \quad P$$

where, T is the required parameter.

$$\text{For block, } T = Ma$$

$$\Rightarrow T = \frac{MP}{M+m}$$

- 41** A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then, the true statement about the scale reading is **[AIEEE 2003]**

- (a) Both the scales read M kg each
 (b) The scale of the lower one reads M kg and of the upper one zero
 (c) The reading of the two scales can be anything but the sum of the readings will be M kg
 (d) Both the scales read $M/2$ kg

Ans. (a)

The arrangement is shown in figure.

Now, draw the free body diagram of the spring balances and block.

S_1 Light spring balance

S_2 Light spring balance

M

For equilibrium of block, $T_1 = Mg$
 where, $T_1 = \text{Reading of } S_2$
 For equilibrium of S_2 , $T_2 = T_1$
 where, $T_2 = \text{Reading of } S_1$
 For equilibrium of S_1 ,

$$T_2 = T_3$$

$$T_3 \quad T_2 \quad T_1$$

$$S_1 \quad S_2 \quad \text{Block}$$

$$T_2 \quad T_1 \quad Mg$$

Hence, $T_1 = T_2 = Mg$
 So, both scales read M kg.

- 42** A lift is moving down with acceleration a . A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively [AIEEE 2002]

- (a) g, g (b) $g - a, g - a$
(c) $g - a, g$ (d) a, g

Ans. (c)

Apparent weight of ball,

$$w' = w - R \text{ [where, } R = \text{normal reaction]}$$

$$R = ma \text{ [acting upward]}$$

$$w' = mg - ma = m(g - a)$$

Hence, apparent acceleration in the lift is $g - a$. Now, if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g .

- 43** When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is [AIEEE 2002]

- (a) $\frac{F_1}{m}$ (b) $\frac{F_2 F_3}{m F_1}$ (c) $\frac{(F_2 - F_3)}{m}$ (d) $\frac{F_2}{m}$

Ans. (a)

The particle remains stationary under the acting of three forces F_1, F_2 and F_3 , it means resultant force is zero,

$$F_1 + F_2 + F_3 = 0$$

Since, in second cases F_1 is removed (in terms of magnitude we are talking now), the forces acting are F_2 and F_3 the resultant of which has the magnitude as F_1 , so acceleration of particle is $\frac{F_1}{m}$ in the

direction opposite to that of F_1 .

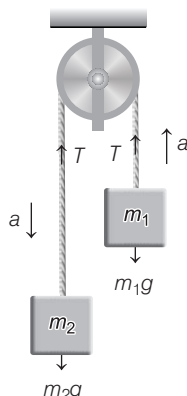
- 44** A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is [AIEEE 2002]

- (a) 8:1 (b) 9:7
(c) 4:3 (d) 5:3

Ans. (b)

As the string is inextensible, both masses have the same acceleration a . Also, the pulley is massless and frictionless, hence the tension at both ends of the string is the same. Suppose,

the mass m_2 is greater than mass m_1 , so the heavier mass m_2 is accelerating downward and the lighter mass m_1 is accelerating upwards.



Therefore, by Newton's 2nd law

$$T - m_1 g = m_1 a \quad \dots(i)$$

$$\text{and} \quad m_2 g - T = m_2 a \quad \dots(ii)$$

After solving Eqs. (i) and (ii), we get

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} \cdot g = \frac{g}{8} \text{ [given]}$$

$$\text{So,} \quad \frac{g}{8} = \frac{m_2(1 - m_1/m_2)}{m_2(1 + m_1/m_2)} \cdot g \quad \dots(iii)$$

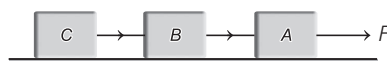
$$\text{Let} \quad \frac{m_1}{m_2} = x$$

Thus, Eq. (iii) becomes

$$\frac{1-x}{1+x} = \frac{1}{8} \text{ or } x = \frac{7}{9} \text{ or } \frac{m_2}{m_1} = \frac{9}{7}$$

So, the ratio of the masses is 9:7.

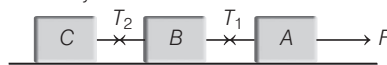
- 45** Three identical blocks of masses $m = 2 \text{ kg}$ are drawn by a force $F = 10.2 \text{ N}$ with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C? [AIEEE 2002]



- (a) 9.2 (b) 7.8
(c) 4 (d) 9.8

Ans. (b)

The system of masses is shown below.



From the figure,

$$F - T_1 = ma \quad \dots(i)$$

$$\text{and} \quad T_1 - T_2 = ma \quad \dots(ii)$$

Eq. (i) gives,

$$10.2 - T_1 = 2 \times 0.6$$

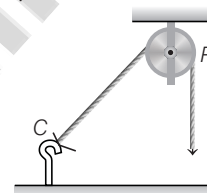
$$\Rightarrow T_1 = 10.2 - 1.2 = 9 \text{ N}$$

Again, from Eq. (ii), we get

$$9 - T_2 = 2 \times 0.6$$

$$\Rightarrow T_2 = 9 - 1.2 = 7.8 \text{ N}$$

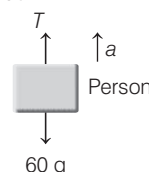
- 46** One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? [AIEEE 2002]



- (a) 16 (b) 6 (c) 4 (d) 80

Ans. (c)

The free body diagram of the person can be drawn as



Let the person move up with an acceleration a , then

$$T - 60g = 60a$$

$$\Rightarrow a_{\max} = \frac{T_{\max} - 60g}{60}$$

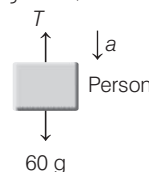
$$\text{or} \quad a_{\max} = \frac{360 - 60g}{60} \rightarrow -ve \text{ value}$$

That means, it is not possible to climb up on the rope.

Even in this problem, it is not possible to remain at rest on rope.

Hence, no option is correct.

But, if they will ask for the acceleration of climbing down, then



$$60g - T = 60a$$

$$\Rightarrow 60g - T_{\max} = 60a_{\min}$$

$$\text{or} \quad a_{\min} = \frac{60g - 360}{60} = 4 \text{ ms}^{-2}$$

TOPIC 3

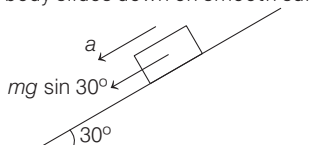
Friction

- 47** When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T . When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT , where α is a constant greater than 1. The coefficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}} \left(\frac{\alpha^2 - 1}{\alpha^2} \right)$, where x is

[2021, 1 Sep Shift-II]

Ans. (3)

Let's draw the free body diagram when body slides down on smooth surface



For smooth surface,

$$ma = mg \sin 30^\circ$$

$$a = g \sin 30^\circ = g/2$$

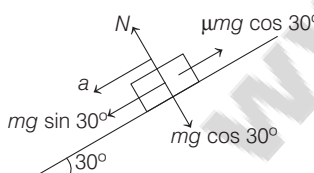
Distance covered by the block on the smooth surface in time T ,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \left(\frac{g}{2} \right) T^2$$

$$\Rightarrow s = \left(\frac{g}{4} \right) T^2 \quad \dots (i)$$

Now, let's draw the free body diagram when body slides down on rough surface



For rough surface,

$$ma = mg \sin 30^\circ - \mu mg \cos 30^\circ$$

$$a = g \sin 30^\circ - \mu g \cos 30^\circ$$

Distance covered by the block on the rough surface in time αT ,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} (g \sin 30^\circ - \mu g \cos 30^\circ) (\alpha T)^2$$

$$s = \frac{g}{4} (1 - \sqrt{3}\mu)(\alpha T)^2 \quad \dots (ii)$$

Distance covered by the block is same for both the case,

$$\Rightarrow \frac{g}{4} (1 - \sqrt{3}\mu)(\alpha T)^2 = \frac{g}{4} T^2$$

[from Eq. (i) & Eq. (ii)]

$$\Rightarrow 1 - \sqrt{3}\mu = \frac{1}{\alpha^2} \Rightarrow \mu = \left(\frac{\alpha^2 - 1}{\alpha^2} \right) \frac{1}{\sqrt{3}}$$

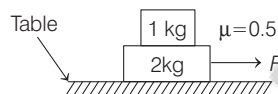
$$\text{Comparing with } \mu = \left(\frac{\alpha^2 - 1}{\alpha^2} \right) \frac{1}{\sqrt{x}}$$

The value of the $x = 3$.

- 48** The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is

(Take, $g = 10 \text{ ms}^{-2}$)

[2021, 26 Aug Shift-II]



Ans. (15)

Given, coefficient of static friction, $\mu = 0.5$

Value of acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

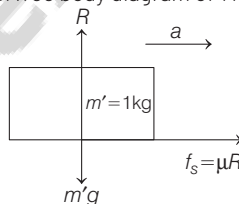
For complete system to move together,

$$F = ma$$

Here, m is total mass.

$$F = (1 + 2)a = 3a \quad \dots (i)$$

From free body diagram of 1 kg block,



Balance forces in horizontal direction,

$$F_s = \mu R = m'a \quad \dots (ii)$$

Balance forces in vertical direction,

$$R = m'g$$

Put value of R in Eq. (ii),

$$\mu m'g = m'a$$

$$\Rightarrow 0.5 \times 1 \times 10 = 1 \times a \Rightarrow a = 5 \text{ ms}^{-2}$$

Put the value of a in Eq. (i), we get

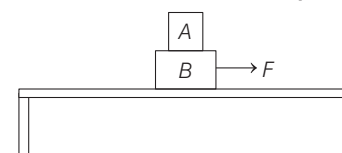
$$F = 3 \times 5 = 15 \text{ N}$$

Thus, the maximum horizontal force required to move block together is 15 N.

- 49** Two blocks A and B of masses $m_A = 1 \text{ kg}$ and $m_B = 3 \text{ kg}$ are kept on the table as shown in figure. The coefficient of friction between A

and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is [Take, $g = 10 \text{ m/s}^2$]

[2019, 10 April Shift-II]



(a) 12 N (b) 16 N (c) 8 N (d) 40 N

Ans. (b)

Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$$= \mu(m_A + m_B)g$$

$$\text{So, } a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \dots (i)$$

Here, $\mu = 0.2$, $m_A = 1 \text{ kg}$, $m_B = 3 \text{ kg}$, $g = 10 \text{ ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1 + 3) \times 10}{1 + 3}$$

$$a = \frac{F - 8}{4} \quad \dots (ii)$$

Due to acceleration of block B, a pseudo force F' acts on A.

This force F' is given by

$$F' = m_A a$$

where, a is acceleration of A and B caused by net force acting on B.

For A to slide over B; pseudo force on A, i.e. F' must be greater than friction between A and B.

$$\Rightarrow m_A a \geq f_2$$

We consider limiting case,

$$m_A a = f_2 \Rightarrow m_A a = \mu(m_A)g$$

$$\Rightarrow a = \mu g = 0.2 \times 10 = 2 \text{ ms}^{-2} \quad \dots (iii)$$

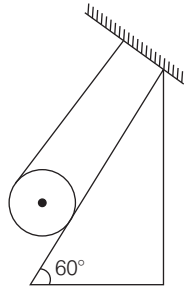
Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F - 8}{4} = 2$$

$$\therefore F = 16 \text{ N}$$

- 50** A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting

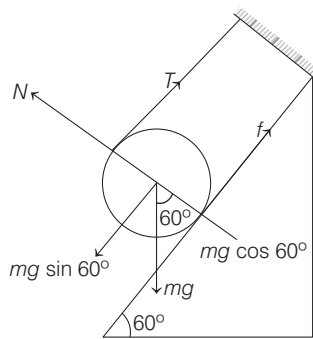
between the cylinder and the inclined plane is



(The coefficient of static friction, μ_s , is 0.4) [2021, 18 March Shift-II]
 (a) $\frac{7}{2}mg$ (b) $5mg$ (c) $\frac{mg}{5}$ (d) 0

Ans. (c)

Let's draw the free body diagram of the solid cylinder.



Using the condition of the equilibrium of the cylinder;

In the direction of inclined plane,

$$T + f - mg \sin 60^\circ = 0$$

In the perpendicular direction of inclined plane,

$$N - mg \cos 60^\circ = 0$$

$$\Rightarrow N = mg \cos 60^\circ$$

The frictional force between the rough surface and cylinder is

$$f = \mu_s N$$

$$\Rightarrow f = 0.4 mg \cos 60^\circ \Rightarrow f = 0.2 mg$$

$$\text{or } f = \frac{mg}{5}$$

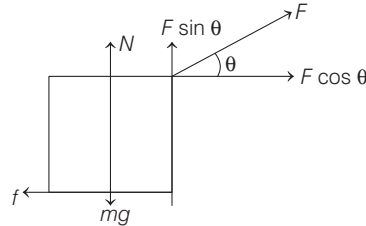
- 51** A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $1/\sqrt{3}$. It is desired to make the body move by applying the minimum possible force F newton. The value of F will be (Round off to the nearest integer) (Take, $g = 10 \text{ ms}^{-2}$) [2021, 17 March Shift-II]

Ans. (5)

Given, mass of the body, $m = 1 \text{ kg}$

Coefficient of static friction, $\mu = 1/\sqrt{3}$

Let's draw the free body diagram of the block



Using the condition of the equilibrium

In x-direction,

$$F \cos \theta = f = \mu N \quad \dots (i)$$

In y-direction,

$$F \sin \theta + N = mg \quad \dots (ii)$$

$$\Rightarrow N = mg - F \sin \theta$$

Substituting the value of N in Eq. (i), we get

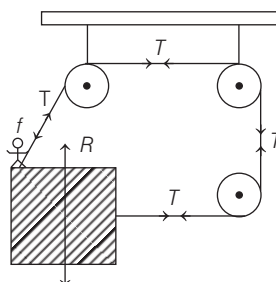
$$F = \frac{\mu mg}{\cos \theta + \mu \sin \theta} \Rightarrow F = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

Substituting the values in the above equation, we get

$$F = \frac{\frac{1}{\sqrt{3}} \times 10}{\sqrt{1 + \left(\frac{1}{\sqrt{3}}\right)^2}} \Rightarrow F = 5 \text{ N}$$

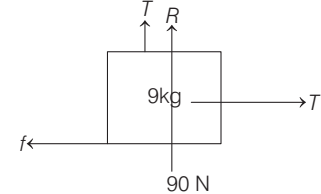
Hence, the body move by applying minimum possible force of 5 N. So, the value of F will be 5.

- 52** A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope, so that the piece of wood does not move from its place is N. (Round off to the nearest integer) (Take, $g = 10 \text{ ms}^{-2}$) [2021, 17 March Shift-II]



Ans. (30)

The free body diagram for the wooden block is shown below



Using the condition of the equilibrium, In the x-direction, the summation of all the forces is to be zero.

$$f - T = 0$$

$$\mu R - T = 0$$

$$T = 0.5 R \quad \dots (i)$$

In the y-direction, the summation of all the forces is to be zero.

$$T + R - 90 = 0$$

$$\Rightarrow 0.5 R + R - 90 = 0 \Rightarrow R = 60 \text{ N}$$

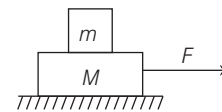
Hence, the normal force on the wooden block is 60 N.

Using the Eq. (i),

$$T = 0.5(60) = 30 \text{ N}$$

Hence, the maximum value of the tension in the rope, so that wooden block will not move is 30 N.

- 53** Two blocks ($m = 0.5 \text{ kg}$ and $M = 4.5 \text{ kg}$) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $3/7$. Then, the maximum horizontal force that can be applied on the larger block so that the blocks move together is N. (Round off to the nearest integer. Take, $g = 9.8 \text{ ms}^{-2}$)



[2021, 17 March Shift-I]

Ans. (21)

When both the blocks move together as a system, then acceleration of this system will be given as

$$a = \frac{F}{m + M} \quad \dots (i)$$

Frictional force on mass,

$$f = ma \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$f = m \left(\frac{F}{m + M} \right)$$

For no slipping, $f \leq \mu mg$ [μ being the coefficient of static friction]

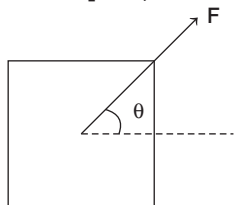
$$\Rightarrow m \left(\frac{F}{m+M} \right) \leq \mu mg \Rightarrow$$

$$F \leq \mu(m+M)g$$

$$\therefore F_{\max} = \frac{3}{7}(0.5 + 4.5) \times 9.8 \Rightarrow F_{\max} = 21 \text{ N}$$

- 54** A block of mass m slides along a floor, while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_k . Then, the block's acceleration a is given by (g is acceleration due to gravity)

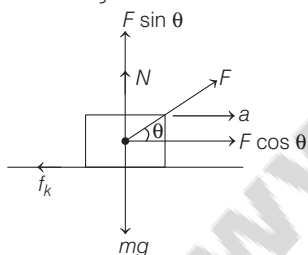
[2021, 16 March Shift-I]



- (a) $-\frac{F}{m} \cos \theta - \mu_k \left(g - \frac{F}{m} \sin \theta \right)$
 (b) $\frac{F}{m} \cos \theta - \mu_k \left(g - \frac{F}{m} \sin \theta \right)$
 (c) $\frac{F}{m} \cos \theta - \mu_k \left(g + \frac{F}{m} \sin \theta \right)$
 (d) $\frac{F}{m} \cos \theta + \mu_k \left(g - \frac{F}{m} \sin \theta \right)$

Ans. (b)

The diagram and the required components of force on given block are shown below



From the above diagram,

$$N = mg - F \sin \theta \quad \dots(i)$$

where, N = normal force

$$\text{and } F \cos \theta - f_k = ma$$

$$\Rightarrow F \cos \theta - \mu_k N = ma \quad \dots(ii)$$

where, f_k = kinetic friction force.

From Eq. (i) and Eq. (ii), we get

$$F \cos \theta - \mu_k (mg - F \sin \theta) = ma$$

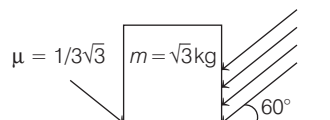
$$\Rightarrow a = \frac{F}{m} \cos \theta - \mu_k \left(g - \frac{F}{m} \sin \theta \right)$$

This is the required acceleration of the block.

- 55** As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $1/3\sqrt{3}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be $3x$. The value of x will be [$g = 10 \text{ ms}^{-2}$; $\sin 60^\circ = \frac{\sqrt{3}}{2}$;

$$\cos 60^\circ = \frac{1}{2}]$$

[2021, 26 Feb Shift-I]



Ans. (3.33)

Given, mass of block, $m = \sqrt{3}$ kg

Coefficient of friction, $\mu = 1/3\sqrt{3}$

According to diagram,

Let F be the force applied on the body, w be the weight ($= mg$),

N be the normal reaction.

Friction force $f = \mu N$

For no movement of body along X-axis, net force along X-axis should be zero.

If, F_y be the net force along y-axis then it will also be zero because body is not accelerating at all.

$$\therefore N = F \sin 60^\circ + mg$$

$$\Rightarrow N = \frac{\sqrt{3}}{2} F + 10\sqrt{3} \quad \dots(i)$$

Similarly, $F_x = F \cos 60^\circ - \mu N = 0$

From Eq. (i), we get

$$\Rightarrow \frac{F}{2} - \frac{1}{3\sqrt{3}} \left(\frac{\sqrt{3}}{2} F + 10\sqrt{3} \right) = 0$$

$$\Rightarrow \frac{F}{2} = \frac{F}{6} + \frac{10}{3} \Rightarrow \frac{F}{2} - \frac{F}{6} = \frac{10}{3}$$

$$\Rightarrow F = 10 \text{ N}$$

Given, $F = 3x$

$$\Rightarrow x = \frac{10}{3} = 3.33$$

- 56** An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where, y is in

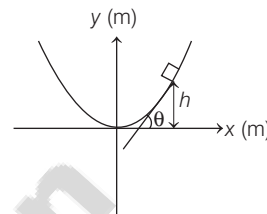
vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the

maximum height in cm at which a stationary block will not slip downward is cm.

[2021, 24 Feb Shift-I]

Ans. (25)

The graph for given equation is shown below



At maximum height, the slope of tangent drawn,

$$\tan \theta = \frac{dy}{dx} = \frac{2x}{4} = \frac{x}{2} \quad \left[\because y = \frac{x^2}{4} \right]$$

$$\Rightarrow 0.5 = \frac{x}{2} \quad (\because \mu = \tan \theta)$$

$$\Rightarrow x = 1 \text{ m}$$

$$\therefore y = \frac{x^2}{4} = \frac{1}{4} = 0.25 \text{ m} = 25 \text{ cm}$$

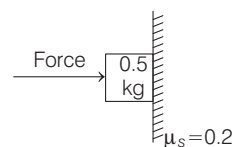
- 57** The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be N.

[Take, $g = 10 \text{ ms}^{-2}$]

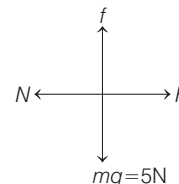
[2021, 24 Feb Shift-I]

Ans. (25)

Given, coefficient of static friction, $\mu_s = 0.2$



Various forces acting on block are shown below



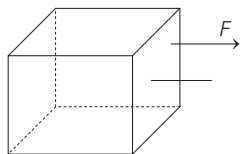
Frictional force $\leq mg$

$$\Rightarrow N \times 0.2 \leq 5$$

$$\Rightarrow N \leq 25$$

$\therefore$ Magnitude of horizontal force, $F = N = 25 \text{ N}$

58



Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is

[2020, 7 Jan Shift-II]

Ans. (75)

According to the given situation; When the minimum force F capable to topple the block is applied, then the block will be on the verge of toppling.

As the block is not moving, we have friction f such that

$$f = F \quad \dots (i)$$

Also, note that reaction N acts from point A as block is at the verge of toppling.

To maintain the equilibrium, net torque about centre of mass C is zero.

$$\Rightarrow F \cdot b + f \left(\frac{a}{2} \right) = N \left(\frac{a}{2} \right)$$

Using result of Eq. (i), we get

$$f \left(\frac{a}{2} + b \right) = N \left(\frac{a}{2} \right)$$

Now, $f = \mu mg$ and $N = mg$

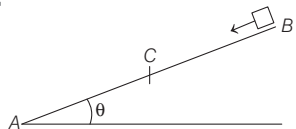
$$\therefore \mu mg \left(\frac{a}{2} + b \right) = mg \left(\frac{a}{2} \right)$$

$$\Rightarrow \mu \left(\frac{a}{2} + b \right) = \frac{a}{2}$$

$$\text{As, } \mu = 0.4 \therefore \frac{b}{a} = \frac{3}{4}$$

$$\text{Hence, } 100 \times \frac{b}{a} = 100 \times \frac{3}{4} = 75$$

59



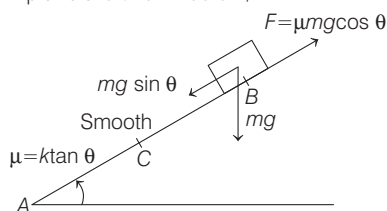
A small block starts slipping down from a point B on an inclined plane AB , which is making an angle θ with the horizontal section BC is smooth and the remaining section

CA is rough with a coefficient of friction μ . It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan \theta$. Then, the value of k is.....

[2020, 2 Sep Shift-I]

Ans. (3)

Different forces acting on the inclined plane are shown below,



As block stops at point A this means work done by component of weight down the plane is dissipated in doing work against friction.

$$\Rightarrow mg \sin \theta (AB) = \mu mg \cos \theta (AC)$$

$$\Rightarrow mg \sin \theta (3AC) = \mu mg \cos \theta (AC)$$

$$\Rightarrow 3 \tan \theta = \mu$$

$$\text{Given, } \mu = k \tan \theta$$

$$\text{Comparing both, we get}$$

$$\Rightarrow k = 3$$

60 A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0 . It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the

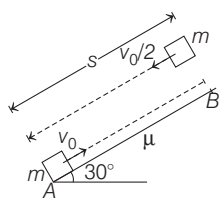
coefficient of kinetic friction between the block and the inclined plane is close to $\frac{l}{1000}$, the nearest integer to l is

[2020, 3 Sep Shift-II]

Ans. (346)

Let μ be the coefficient of kinetic friction between the block and the inclined plane.

The free-body diagram of the given situation is shown below,



While going from A to B , the acceleration of the block is

$$a_1 = g \sin 30^\circ + \mu g \cos 30^\circ$$

$$\Rightarrow a_1 = \frac{g}{2} + \frac{\mu g \sqrt{3}}{2}$$

$$\Rightarrow a_1 = 5 + 5\sqrt{3}\mu \quad \dots (i) (g = 10 \text{ ms}^{-2})$$

Let s be the distance between A and B

From third equation of motion,

$$v_0^2 - 0 = 2a_1 s$$

$$\Rightarrow s = \frac{v_0^2}{2a_1} = \frac{v_0^2}{2(5 + 5\sqrt{3}\mu)} \quad \dots (ii)$$

[Using Eq. (i)]

When the block comes back to its initial position (i.e., from B to A), its velocity is $\frac{v_0}{2}$.

So, the acceleration of the block while coming from B to A is

$$a_2 = g \sin 30^\circ - \mu g \cos 30^\circ$$

$$\Rightarrow a_2 = \frac{g}{2} - \frac{\mu g \sqrt{3}}{2}$$

$$\Rightarrow a_2 = 5 - 5\sqrt{3}\mu \quad \dots (iii)$$

Again, using third equation of motion, we get

$$\left(\frac{v_0}{2} \right)^2 - 0 = 2a_2 s \Rightarrow \frac{v_0^2}{4} = 2a_2 s$$

$$\Rightarrow s = \frac{v_0^2}{8a_2}$$

$$\Rightarrow s = \frac{v_0^2}{8(5 - 5\sqrt{3}\mu)} \quad \dots (iv)$$

[Using Eq. (iii)]

Equating Eqs. (ii) and (iv), we get

$$\frac{v_0^2}{2(5 + 5\sqrt{3}\mu)} = \frac{v_0^2}{8(5 - 5\sqrt{3}\mu)}$$

$$\Rightarrow 4(5 - 5\sqrt{3}\mu) = 5 + 5\sqrt{3}\mu$$

$$\Rightarrow 20 - 20\sqrt{3}\mu = 5 + 5\sqrt{3}\mu$$

$$\Rightarrow 25\sqrt{3}\mu = 15$$

$$\Rightarrow \mu = \frac{15}{25\sqrt{3}} = \frac{3}{5\sqrt{3}} \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

$$\text{So, } \frac{l}{1000} = \frac{\sqrt{3}}{5} = 0.346$$

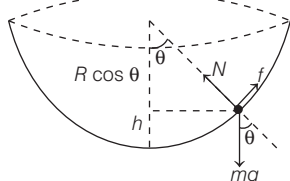
$$\Rightarrow l = 0.346 \times 1000 = 346$$

61 An insect is at the bottom of a hemispherical ditch of radius 1 m . It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75 , then h is (Take, $g = 10 \text{ ms}^{-2}$) [2020, 6 Sep Shift-I]

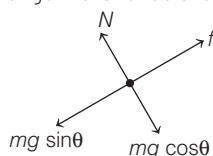
(a) 0.20 m (b) 0.45 m
(c) 0.60 m (d) 0.80 m

Ans. (a)

Let h be maximum height up to which insect crawls up the ditch. The free body diagram is shown,



Resolving the components of force along tangential and radial direction



For balancing, $mg \cos \theta = N$

$$\begin{aligned} mg \sin \theta &= f_{\max} = \mu N \\ \Rightarrow mg \sin \theta &= \mu mg \cos \theta \\ \frac{\sin \theta}{\cos \theta} &= \frac{\mu mg}{mg} \end{aligned}$$

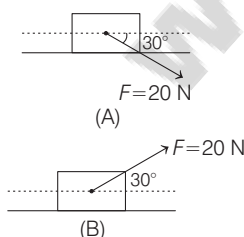
$$\tan \theta = \mu = 0.75 = \frac{3}{4} \Rightarrow \cos \theta = \frac{4}{5}$$

(From Pythagoras' theorem)

$$\begin{aligned} \text{From diagram, } h &= R - R \cos \theta \\ &= R - R \left(\frac{4}{5} \right) = \frac{R}{5} \quad \text{or } h = \frac{1}{5} = 0.20 \text{ m} \end{aligned}$$

- 62** A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20 \text{ N}$, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block, the floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be (Take, $g = 10 \text{ ms}^{-2}$)

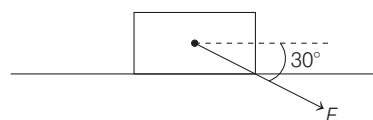
[2019, 12 April Shift-II]



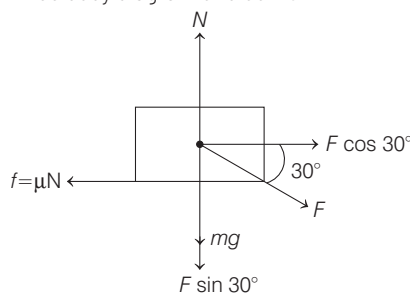
- (a) 0.4 ms^{-2} (b) 3.2 ms^{-2}
(c) 0.8 ms^{-2} (d) 0 ms^{-2}

Ans. (c)

Case I Block is pushed over surface



Free body diagram of block is



In this case, normal reaction,

$$\begin{aligned} N &= mg + F \sin 30^\circ \\ &= 5 \times 10 + 20 \times \frac{1}{2} = 60 \text{ N} \end{aligned}$$

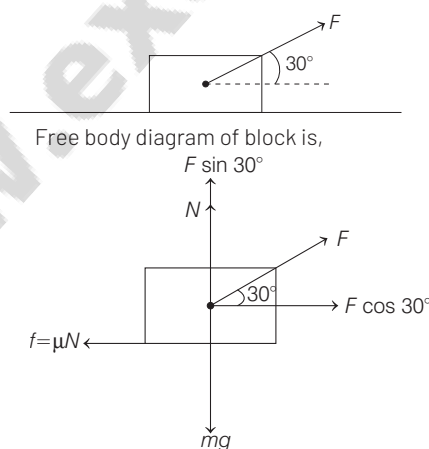
[Given, $m = 5 \text{ kg}$, $F = 20 \text{ N}$]

$$\begin{aligned} \text{Force of friction, } f &= \mu N \\ &= 0.2 \times 60 [\because \mu = 0.2] \\ &= 12 \text{ N} \end{aligned}$$

So, net force causing acceleration (a_1) is

$$\begin{aligned} F_{\text{net}} &= ma_1 = F \cos 30^\circ - f \\ \Rightarrow ma_1 &= 20 \times \frac{\sqrt{3}}{2} - 12 \\ \therefore a_1 &= \frac{10\sqrt{3} - 12}{5} \approx 1 \text{ ms}^{-2} \end{aligned}$$

Case II Block is pulled over the surface



Net force causing acceleration is

$$F_{\text{net}} = F \cos 30^\circ - \mu N$$

$$\Rightarrow F_{\text{net}} = F \cos 30^\circ - \mu (mg - F \sin 30^\circ)$$

If acceleration is now a_2 , then

$$\begin{aligned} a_2 &= \frac{F_{\text{net}}}{m} \\ &= \frac{F \cos 30^\circ - \mu (mg - F \sin 30^\circ)}{m} \end{aligned}$$

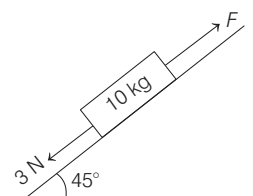
$$\begin{aligned} &= \frac{20 \times \frac{\sqrt{3}}{2} - 0.2 \left(5 \times 10 - 20 \times \frac{1}{2} \right)}{5} \\ &= \frac{10\sqrt{3} - 8}{5} \end{aligned}$$

$$\Rightarrow a_2 \approx 1.8 \text{ ms}^{-2}$$

$$\begin{aligned} \text{So, difference} &= a_2 - a_1 \\ &= 1.8 - 1 = 0.8 \text{ ms}^{-2} \end{aligned}$$

- 63** A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force F , such that the block does not move downward? (Take, $g = 10 \text{ ms}^{-2}$)

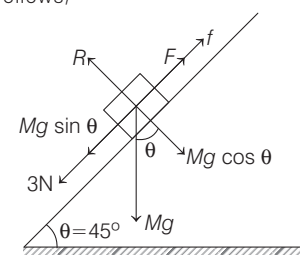
[2019, 9 Jan Shift-I]



- (a) 32 N (b) 25 N
(c) 23 N (d) 18 N

Ans. (a)

Free body diagram, for the given figure is as follows,



For the block to be in equilibrium i.e., so that it does not move downward, then $\Sigma f_x = 0$

$$\therefore 3 + Mg \sin \theta - F - f = 0$$

$$\text{or } 3 + Mg \sin \theta = F + f$$

As, frictional force, $f = \mu R$

$$\therefore 3 + Mg \sin \theta = F + \mu R \quad \dots(i)$$

Similarly, $\Sigma f_y = 0$

$$-Mg \cos \theta + R = 0$$

$$\text{or } Mg \cos \theta = R \quad \dots(ii)$$

Substituting the value of 'R' from Eq. (ii) to Eq. (i), we get

$$3 + Mg \sin \theta = F + \mu (Mg \cos \theta) \quad \dots(iii)$$

Here, $M = 10 \text{ kg}$, $\theta = 45^\circ$, $g = 10 \text{ m/s}^2$

and $\mu = 0.6$

Substituting these values in Eq. (iii), we get

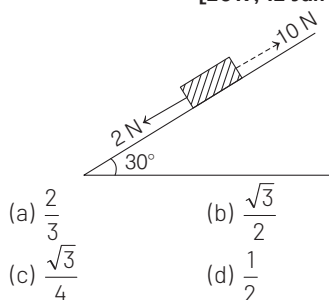
$$3 + (10 \times 10 \sin 45^\circ) - (0.6 \times 10 \times 10 \cos 45^\circ) = F$$

$$\Rightarrow F = 3 + \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}} = 3 + \frac{40}{\sqrt{2}}$$

$$= 3 + 20\sqrt{2} = 31.8 \text{ N or } F \approx 32 \text{ N}$$

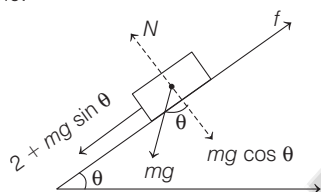
- 64** A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is (Take, $g = 10 \text{ m/s}^2$)

[2019, 12 Jan Shift-II]



Ans. (b)

Block does not move upto a maximum applied force of 2 N down the inclined plane.

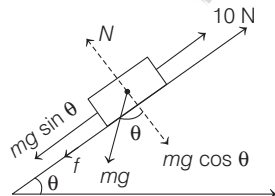


So, equating forces, we have

$$2 + mg \sin \theta = f$$

$$\text{or } 2 + mg \sin \theta = \mu mg \cos \theta \quad \dots(i)$$

Similarly, block also does not move upto a maximum applied force of 10 N up the plane.



Now, equating forces, we have

$$mg \sin \theta + f = 10 \text{ N}$$

$$\text{or } mg \sin \theta + \mu mg \cos \theta = 10 \quad \dots(ii)$$

Now, solving Eqs. (i) and (ii), we get

$$mg \sin \theta = 4 \quad \dots(iii)$$

$$\text{and } \mu mg \cos \theta = 6 \quad \dots(iv)$$

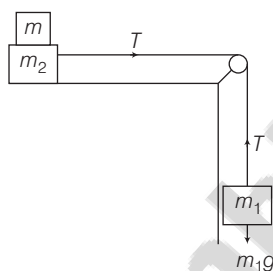
Dividing, Eqs. (iii) and (iv) we get

$$\mu \cot \theta = \frac{3}{2}$$

$$\Rightarrow \mu = \frac{3 \tan \theta}{2} = \frac{3 \tan 30^\circ}{2} \Rightarrow \mu = \frac{\sqrt{3}}{2}$$

- 65** Two masses $m_1 = 5 \text{ kg}$ and $m_2 = 10 \text{ kg}$ connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is

[JEE Main 2018]



- (a) 18.3 kg (b) 27.3 kg
(c) 43.3 kg (d) 10.3 kg

Ans. (b)

Motion stops when pull due to $m_1 \leq$ force of friction between m and m_2 and surface.

$$\Rightarrow m_1 g \leq \mu(m_2 + m)g$$

$$\Rightarrow 5 \times 10 \leq 0.15(10 + m) \times 10$$

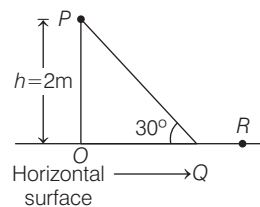
$$\Rightarrow m \geq 23.33 \text{ kg}$$

Here, nearest value is 27.3 kg

So, $m_{\min} = 27.3 \text{ kg}$

- 66** A point particle of mass m , moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ . The particle is released, from rest, from the point P and it comes to rest at a point R . The energies, lost by the ball, over the parts, PQ and QR , of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR . The values of the coefficient of friction μ and the distance $x(=QR)$, are respectively close to

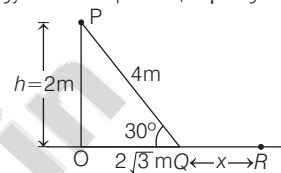
[JEE Main 2016]



- (a) 0.2 and 6.5 m (b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

Ans. (c)

Energy lost over path $PQ = \mu mg \cos \theta \times 4$



Energy lost over path $QR = \mu mg x$

$$\text{i.e. } \mu mg \cos 30^\circ \times 4 = \mu mg x \quad (\because \theta = 30^\circ)$$

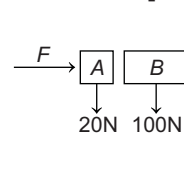
$$x = 2\sqrt{3} = 3.45 \text{ m}$$

From Q to R energy loss is half of the total energy loss.

$$\text{i.e. } \mu mg x = \frac{1}{2} \times mgh \Rightarrow \mu = 0.29$$

The values of the coefficient of friction μ and the distance $x(=QR)$ are 0.29 and 3.5.

- 67** Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall in block B is [JEE Main 2015]

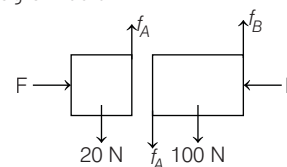


- (a) 100 N (b) 80 N (c) 120 N (d) 150 N

Ans. (c)

Key Idea In vertical direction, weights are balanced by frictional forces.

Consider FBD of block A and B as shown in diagram below.



As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

For the system of blocks (A + B).

$$F = N$$

For block A, $f_A = 20$ N and for block B,

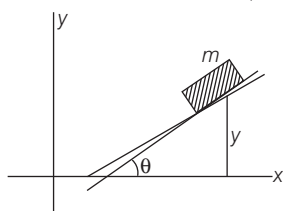
$$f_B = f_A + 100 = 120 \text{ N}$$

- 68** A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is [JEE Main 2014]

- (a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m (c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

Ans. (a)

A block of mass m is placed on a surface with a vertical cross-section, then



$$\tan = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan \theta, 0.5 = x^2/2$$

$$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Now, putting the value of x in $y = x^3/6$, we get

When $x = 1$	When $x = -1$
$\therefore y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = -\frac{1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is $\frac{1}{6}$ m.

- 69** The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2 . If the inclined plane makes an angle θ from the horizontal such that $\tan \theta = 2\mu$, then the ratio F_1/F_2 is [AIEEE 2011]
- (a) 4 (b) 1 (c) 2 (d) 3

Ans. (d)

$F_1 = mg(\sin \theta + \mu \cos \theta)$ [as body just in position to move up, friction force downward]

$$F_2 = mg(\sin \theta - \mu \cos \theta)$$

[as body just in position

to slide down, friction upward]

$$\therefore \frac{F_1}{F_2} = \frac{\sin \theta + \mu \cos \theta}{\sin \theta - \mu \cos \theta} = \frac{\tan \theta + \mu}{\tan \theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = 3$$

- 70** Consider a car moving on a straight road with a speed of 100 ms^{-1} . The distance at which car can be stopped, is [$\mu_k = 0.5$] [AIEEE 2005]

- (a) 800 m (b) 1000 m
(c) 100 m (d) 400 m

Ans. (b)

From Newton's equations, we have

$$v^2 = u^2 - 2as$$

Given, $v = 0$ [car is stopped]

As friction provide the retardation

$$a = \mu g, v = 100 \text{ ms}^{-1}$$

$$\therefore (100)^2 = 2\mu g s$$

$$\Rightarrow s = \frac{100 \times 100}{2 \times 0.5 \times 10} = \frac{100 \times 100}{5 \times 2} = 1000 \text{ m}$$

- 71** A smooth block is released at rest on a 45° incline and then slides a distance d . The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is [AIEEE 2005]

- (a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$
(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

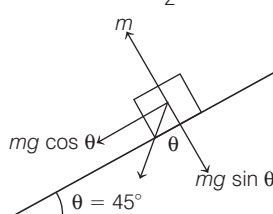
Ans. (a)

When friction is absent

$$ma_1 = mg \sin \theta$$

$$a = g \sin \theta$$

$$\therefore s_1 = \frac{1}{2} a_1 t_1^2 \quad \dots (i)$$



When friction is present, friction is in opposite to the direction of motion

$$a_2 = g \sin \theta - \mu_k g \cos \theta$$

$$\therefore s_2 = \frac{1}{2} a_2 t_2^2 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

$$\Rightarrow a_1 t_1^2 = a_2 (n t_1)^2 \quad [\because t_2 = n t_1]$$

$$\text{or } a_1 = n^2 a_2$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{g \sin \theta - \mu_k g \cos \theta}{g \sin \theta} = \frac{1}{n^2}$$

$$\text{or } \frac{g \sin 45^\circ - \mu_k g \cos 45^\circ}{g \sin 45^\circ} = \frac{1}{n^2}$$

$$\text{or } 1 - \mu_k = \frac{1}{n^2}$$

$$\text{or } \mu_k = 1 - \frac{1}{n^2}$$

- 72** The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by [AIEEE 2005]

- (a) $2 \sin \phi$ (b) $2 \cos \phi$
(c) $2 \tan \phi$ (d) $\tan \phi$

Ans. (c)

According to work-energy theorem,

Work done = Change in kinetic energy

$$W = \Delta K = 0$$

$$\Rightarrow \text{Work done by friction} + \text{Work done by gravity} = 0$$

$$\Rightarrow -(\mu mg \cos \phi) \frac{l}{2} + mgl \sin \phi = 0$$

$$\text{or } \frac{\mu}{2} \cos \phi = \sin \phi$$

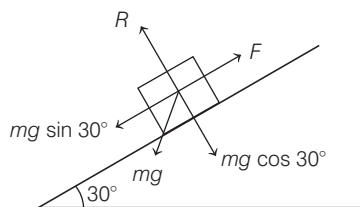
$$\text{or } \mu = 2 \tan \phi$$

- 73** A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is ($g = 10 \text{ m/s}^2$) [AIEEE 2004]

- (a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5

Ans. (a)

Let mass of the block be m .



Frictional force in rest position

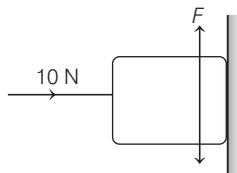
$$F = mg \sin 30^\circ$$

[this is static frictional force and may be less than the limiting frictional force]

$$\therefore 10 = m \times 10 \times \frac{1}{2}$$

$$\text{or } m = \frac{2 \times 10}{10} = 2 \text{ kg}$$

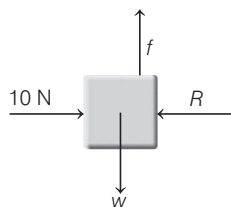
- 74** A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is [AIEEE 2003]



- (a) 20 N (b) 50 N (c) 100 N (d) 2 N

Ans. (d)

Let R be the normal contact force by wall on the block.



$$R = 10 \text{ N}$$

$$f_L = w \text{ and } f_L = \mu R$$

$$\therefore \mu R = w \text{ or } w = 0.2 \times 10 = 2 \text{ N}$$

- 75** A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. Then, the coefficient of friction is [AIEEE 2003]

- (a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01

Ans. (c)

Let coefficient of friction be μ , then retardation will be μg . From equation of motion, $v = u + at$

$$\Rightarrow 0 = 6 - \mu g \times 10$$

$$\text{or } \mu = \frac{6}{100} = 0.06$$

TOPIC 4

Dynamics of Circular Motion

- 76** A particle of mass m is suspended from a ceiling through a string of length L . The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle

will be

(a) $\sqrt{rg}$

(c) $2\sqrt{rg}$

[2021, 26 Aug Shift-II]

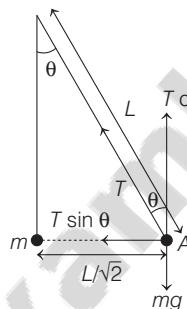
(b) $\sqrt{2rg}$

(d) $\sqrt{\frac{rg}{2}}$

Ans. (a)

Given, radius of horizontal circle, $r = L / \sqrt{2}$

Figure illustrating the particle of mass m moving in a horizontal circle, while suspended from a ceiling is shown



In equilibrium condition at point A,

$$T \cos \theta = mg \quad \dots(i)$$

$$T \sin \theta = \frac{mv^2}{r} \quad \dots(ii)$$

Divide Eq. (ii) by Eq. (i),

$$\tan \theta = v^2 / rg$$

$$\Rightarrow v = \sqrt{rg \tan \theta} \quad (iii)$$

Now, from figure, we can write

$$\sin \theta = \frac{L/\sqrt{2}}{L} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

Substituting the value of θ in Eq. (iii), we get

$$v = \sqrt{rg \tan 45^\circ} = \sqrt{rg}$$

Thus, the value of speed of particle is $v = \sqrt{rg}$.

- 77** Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B , respectively. If T_A and T_B are the time period of star A and star B respectively, then

[2021, 20 July Shift-II]

$$(a) \frac{T_A}{T_B} = \left(\frac{r_A}{r_B} \right)^{\frac{3}{2}}$$

$$(b) T_A = T_B$$

$$(c) T_A > T_B \text{ (if } m_A > m_B \text{)}$$

$$(d) T_A > T_B \text{ (if } r_A > r_B \text{)}$$

Ans. (b)

As per question, a binary star system of two stars A and B with masses m_A and m_B are revolving in a circular orbit of radii r_A and r_B , respectively.

It means both stars A and B will have same angular velocity because to remain perfectly align w.r.t. each other they need to cover equal angular displacement in equal time intervals.

Also, we know that

$$\text{Time period, } T = \frac{2\pi}{\omega}$$

$\therefore T_A$ and T_B are the time periods of stars A and B respectively, therefore, we can write

$$T_A = \frac{2\pi}{\omega} \quad \dots(i)$$

$$\text{and } T_B = \frac{2\pi}{\omega} \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we can say

$$T_A = T_B$$

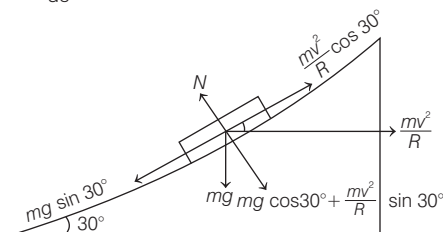
- 78** The normal reaction N for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is $\dots \times 10^3 \text{ kg-m/s}^2$. [Take, $\mu_s = 0.2$]

[2021, 20 July Shift-I]

- (a) 10.2 (b) 7.2 (c) 12.4 (d) 6.96

Ans. (a)

The given situation can be represented as



Equating forces perpendicular to the inclined plane,

$$N = mg \cos 30^\circ + \frac{mv^2}{R} \sin 30^\circ$$

$$\Rightarrow N - mg \cos 30^\circ = \frac{mv^2}{R} \sin 30^\circ \quad \dots(i)$$

Equating forces along the inclined plane,

$$mg \sin 30^\circ + \mu_s N = \frac{mv^2}{R} \cos 30^\circ \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{N - mg \cos 30^\circ}{mg \sin 30^\circ + \mu_s N} = \tan 30^\circ$$

$$[\because \cos 30^\circ = \frac{\sqrt{3}}{2} \text{ and } \sin 30^\circ = \frac{1}{2}]$$

$$\Rightarrow \frac{N - mg(\sqrt{3}/2)}{mg(1/2) + (0.2)N} = \frac{1}{\sqrt{3}}$$

$$N\sqrt{3} - mg \frac{\sqrt{3}}{2} \cdot \sqrt{3} = \frac{mg}{2} + 0.2N$$

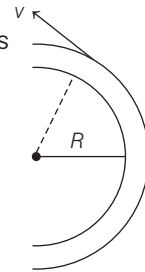
$$\Rightarrow (\sqrt{3} - 0.2)N = mg \frac{(1 + \sqrt{3})}{2} = 2mg$$

$$\Rightarrow N = \frac{2mg}{\sqrt{3} - 0.2} = \frac{2 \times 800 \times 10}{1.532}$$

$$= 10.44 \times 10^3 \text{ N}$$

Therefore, $N = 10.2 \times 10^3 \text{ kg-m/s}^2$

- 79** A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v . If the coefficient of static friction between the tyres and the track is μ_s , then the magnitude of negative lift f_L acting downwards on the car is



(Assume forces on the four tyres are identical and g = acceleration due to gravity) [2021, 17 March Shift-I]

(a) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (b) $m\left(\frac{v^2}{\mu_s R} - g\right)$

(c) $m\left(g - \frac{v^2}{\mu_s R}\right)$ (d) $-m\left(g + \frac{v^2}{\mu_s R}\right)$

Ans. (b)

We know that, static friction force, $f_s = \mu_s N$

where, μ_s is the coefficient of static friction and N is the normal force acting on the body. As the car is travelling on a circular track, so centripetal force is also acting on it.

$$\Rightarrow f_c = \frac{mv^2}{R}$$

In limiting condition,

$$f_s = f_c \Rightarrow \mu_s N = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{\mu_s R} \quad \dots(i)$$

The magnitude of negative lift f_L acting downwards on the car is given by

$$f_L = mg - N = mg - \frac{mv^2}{\mu_s R}$$

$$\Rightarrow f_L = m\left(g - \frac{v^2}{\mu_s R}\right) \Rightarrow f_L = -m\left(\frac{v^2}{\mu_s R} - g\right)$$

$$\Rightarrow |f_L| = m\left(\frac{v^2}{\mu_s R} - g\right)$$

- 80 Statement I** A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2 m without reducing the speed. The static friction coefficient is 0.2 . The cyclist will not slip and pass the curve ($g = 9.8 \text{ m/s}^2$)

Statement II If the road is banked at an angle of 45° , cyclist can cross the curve of 2 m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

[2021, 16 March Shift-II]

- (a) Statement I is false and statement II is true.
- (b) Statement I is true and statement II is false.
- (c) Both statement I and statement II are false.
- (d) Both statement I and statement II are true.

Ans. (d)

The maximum speed of cyclist on turn of unbanked road without slipping is given as

$$v_{\max} = \sqrt{\mu_s g R} = \sqrt{0.2 \times 10 \times 2} = 2 \text{ ms}^{-1}$$

$$[\because \mu = 0.2 (\text{given})]$$

Given, speed = 7 km/h

$$= \frac{7000}{3600} \text{ ms}^{-1} = \frac{70}{36} = 1.94 \text{ ms}^{-1}$$

As given speed is lesser than $v_{\max}$, so the cyclist will not slip. Therefore, Statement I is true.

As per Statement II, angle of banking, $\theta = 45^\circ$

We know that, for banked road,

$$v_{\max} = \sqrt{\frac{gR(\mu + \tan \theta)}{(1 - \mu \tan \theta)}}$$

$$\text{and } v_{\min} = \sqrt{\frac{gR(\tan \theta - \mu)}{1 + \mu \tan \theta}}$$

$$\Rightarrow v_{\max} = \sqrt{\frac{10 \times 2(0.2 + \tan 45^\circ)}{1 - 0.2 \tan 45^\circ}}$$

$$\text{and } v_{\min} = \sqrt{\frac{10 \times 2(1 - 0.2)}{1 + 0.2}}$$

$$\Rightarrow v_{\max} = 5.47 \text{ ms}^{-1}$$

$$\text{and } v_{\min} = 3.65 \text{ ms}^{-1}$$

$$\therefore v = 18.5 \text{ km/h} = \frac{18.5 \times 1000}{3600} = 5.13 \text{ ms}^{-1}$$

[: given speed = 18.5 kmh^{-1}]

As, $v_{\min} < v < v_{\max}$, so the cyclist will not slip.

$\therefore$ Statement II is also true.

Hence, option (d) is the correct.

- 81** A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm . If the block takes 40 s to complete one round, the normal force by the side walls of the groove is [2021, 16 March Shift-I]

- (a) 0.0314 N (b) $9.859 \times 10^{-2} \text{ N}$
- (c) $6.28 \times 10^{-3} \text{ N}$ (d) $9.859 \times 10^{-4} \text{ N}$

Ans. (d)

The normal force by the side walls of the groove will be equal to the centripetal force acting on it.

$$\text{i.e. } N = \frac{mv^2}{r} \quad \dots(i)$$

where, $r = 20 \text{ cm} = 0.2 \text{ m}$

$$m = 200 \text{ g} = 200 \times 10^{-3} \text{ kg}$$

$$\text{and } v = r\omega = \frac{2\pi r}{T} = \frac{2\pi \times 0.2}{40} \text{ m/s}$$

Substituting the given values in Eq. (i), we get

$$N = \frac{(200 \times 10^{-3}) \times \left(\frac{2\pi \times 0.2}{40}\right)^2}{0.2}$$

$$\approx 9.859 \times 10^{-4} \text{ N}$$

- 82** A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3 . Its time period of revolution will be given by [26 Feb 2021 Shift-I]

- (a) $T \propto R^2$ (b) $T \propto R^{3/2}$
- (c) $T \propto R^{5/2}$ (d) $T \propto R^{4/3}$

Ans. (a)

Given, radius of circle = R

Central fictitious force is, $F \propto \frac{1}{R^3}$.

Let T be the time period of revolution, m, ω be the mass and angular velocity of Earth.

$$\therefore F = m\omega^2 R \propto \frac{1}{R^3}$$

$$\Rightarrow \omega^2 \propto \frac{1}{R^4} \Rightarrow \omega = \frac{2\pi}{T} \propto \frac{1}{R^2}$$

$$\Rightarrow T \propto R^2$$

- 83** A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB , as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then, the value of ω^2 is equal to

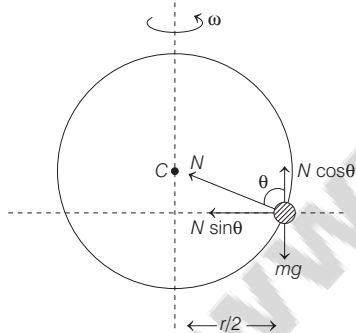
[2019, 12 April Shift-II]

- (a) $\frac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$
(c) $(g\sqrt{3})/r$ (d) $2g/r$

Ans. (b)

Key Idea For revolution in a circular path, there should be a force which balances the necessary centripetal force.

Let N = normal reaction of wire loop acting towards centre.



Then, component $N \cos \theta$ balances weight of bead,

$$\Rightarrow N \cos \theta = mg \quad \dots(i)$$

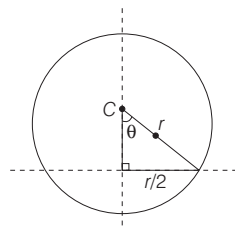
and component $N \sin \theta$ provides necessary centripetal pull on the bead,

$$\Rightarrow N \sin \theta = m \left(\frac{r}{2} \right) \omega^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\tan \theta = \frac{r\omega^2}{2g} \quad \dots(iii)$$

Now, from geometry of figure,



$$\tan \theta = \frac{\frac{r}{2}}{\sqrt{r^2 - \left(\frac{r}{2}\right)^2}} = \frac{r}{2\left(\frac{\sqrt{3}}{2}\right)r} = \frac{1}{\sqrt{3}} \quad \dots(iv)$$

Put this value in Eq. (iii), we get

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

- 84** A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n th power of R . If the period of rotation of the particle is T , then : [JEE Main 2018]

- (a) $T \propto R^{3/2}$ for any n
(b) $T \propto R^{\frac{n+1}{2}}$
(c) $T \propto R^{(n+1)/2}$
(d) $T \propto R^{n/2}$

Ans. (c)

$$\therefore \text{Force} = \text{Mass} \times \text{Acceleration} = m\omega^2 R$$

$$\text{and given, } F \propto \frac{1}{R^n} \Rightarrow F = \frac{k}{R^n}$$

$$\text{So, we have } \frac{k}{R^n} = m \left(\frac{2\pi}{T} \right)^2 R$$

$$\Rightarrow T^2 = \frac{4\pi^2 m}{k} \cdot R^{n+1} \Rightarrow T \propto R^{\frac{n+1}{2}}$$

- 85** An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed (d) The ratio of the forces experienced by the two

particles situated on the inner and outer parts of the ring, F_1 / F_2 is

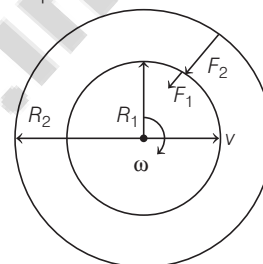
[AIEEE 2005]

- (a) $\frac{R_2}{R_1}$ (b) $\left(\frac{R_1}{R_2} \right)^2$
(c) 1 (d) $\frac{R_1}{R_2}$

Ans. (d)

Since, ω is constant, so no net force or torque is acting on ring.

The force experienced by any particle is only along radial direction or we can say the centripetal force.



The force experienced by inner part, $F_1 = m\omega^2 R_1$

and the force experienced by outer part,

$$F_2 = m\omega^2 R_2 \Rightarrow \frac{F_1}{F_2} = \frac{R_1}{R_2}$$

- 86** The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is [AIEEE 2002]

- (a) 60 (b) 30
(c) 15 (d) 25

Ans. (b)

As centrifugal force is balanced by the centripetal force i.e., frictional force.

Using the relation

$$\frac{mv^2}{r} = \mu R, \quad R = mg$$

$$\Rightarrow \frac{mv^2}{r} = \mu mg \quad \text{or} \quad v^2 = \mu rg$$

$$\Rightarrow v^2 = 0.6 \times 150 \times 10$$

$$\text{or} \quad v = 30 \text{ ms}^{-1}$$

05

Work, Energy and Power

TOPIC 1

Work and Energy

- 01** A block moving horizontally on a smooth surface with a speed of 40 m/s splits into two parts with masses in the ratio of 1:2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is

[2021, 31 Aug Shift-II]

- (a) $1/3$ (b) $2/3$ (c) $1/8$ (d) $1/4$

Ans. (c)

Let a block of mass M splits into two masses m_1 and m_2 in ratio 1:2.

Then, mass of smaller part,

$$m_1 = \frac{M}{1+2} = \frac{M}{3}$$

Mass of bigger part, $m_2 = 2M/3$

Given, speed of mass M , $v = 40 \text{ m/s}$

Speed of mass m_1 , $v_1 = 60 \text{ m/s}$

Let speed of mass m_2 , $v_2 = v$

Using conservation of linear momentum,

$$\begin{aligned} p_i &= p_f \\ Mv &= m_1v_1 + m_2v_2 \\ M \times 40 &= \frac{M}{3} \times 60 + \frac{2M}{3} \times v \Rightarrow v = 30 \text{ m/s} \end{aligned}$$

$\therefore$ The fractional change in kinetic energy,

$$\begin{aligned} \frac{\Delta K}{K} &= \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 \\ &= \frac{\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2}{\frac{1}{2}Mv^2} - 1 \\ &= \frac{\frac{1}{2} \left[\frac{M}{3} \times (60)^2 + \frac{2M}{3} \times (30)^2 \right]}{\frac{1}{2}M \times (40)^2} - 1 \\ &= \frac{9}{8} - 1 = \frac{1}{8} \end{aligned}$$

- 02** A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be $x : 4$, where x is

[2021, 31 Aug Shift-I]

Ans. (5)

Given, initial speed of block, $u = 40 \text{ ms}^{-1}$

Let total mass of block = m

$\therefore$ Broken masses, $m_1 = m_2 = \frac{m}{2}$

Final speed of m_1 , $v_1 = 60 \text{ ms}^{-1}$

As we know that, kinetic energy,

$$KE = \frac{1}{2}mv^2$$

$\therefore$ Initial kinetic energy $KE_i = \frac{1}{2}mu^2$

$$= \frac{1}{2}m(40)^2 = 800m \quad \dots (i)$$

By using law of conservation of momentum,

$$\begin{aligned} mu &= m_1v_1 + m_2v_2 \\ \therefore v_2 &= \frac{m \times 40 - m/2 \times 60}{m/2} \\ &= \frac{10m}{m/2} = 20 \text{ ms}^{-1} \end{aligned}$$

So, final kinetic energy

$$\begin{aligned} KE_f &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ &= \frac{1}{2} \frac{m}{2} (v_1^2 + v_2^2) = \frac{m}{4} (60^2 + 20^2) \\ &= m/4 (3600 + 400) \\ &= 1000m \quad \dots (ii) \end{aligned}$$

$\therefore$ Divide Eq (ii) by Eq. (i), we get

$$\frac{KE_f}{KE_i} = \frac{1000m}{800} = \frac{5}{4}m$$

So, $x = 5$

- 03** Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angle 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is

[2021, 27 Aug Shift-I]

Ans. (2)

Given, work done by both person is same,

$$W_A = W_B$$

Direction of first force with displacement, $\theta_1 = 45^\circ$

Direction of second force with displacement, $\theta_2 = 60^\circ$

Ratio of force applied by person A to force applied by person B is

$$F_A : F_B = 1 : \sqrt{x}$$

$F_A : F_B = 1 : \sqrt{x}$.

Work done by both person is same,

$$W_A = W_B \Rightarrow F_A d \cos \theta_1 = F_B d \cos \theta_2$$

($\because$ distance is same for both = d)

$$\frac{F_A}{F_B} = \frac{\cos \theta_1}{\cos \theta_2} = \frac{\cos 60^\circ}{\cos 45^\circ}$$

$$\Rightarrow \frac{F_A}{F_B} = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{F_A}{F_B} = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}} \Rightarrow x = 2$$

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- 04** A uniform chain of length 3 m and mass 3 kg overhangs a smooth table with 2 m laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off

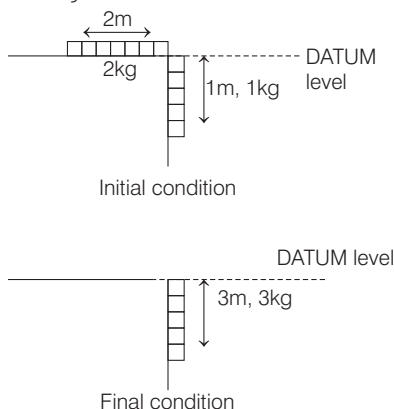
the table, then the value of k is

.....
(Take, $g = 10 \text{ m/s}^2$)

[2021, 26 Aug Shift-I]

Ans. (40)

The given situation is shown below as



Applying law of conservation of energy, we get

$$K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}$$

$$K_{\text{initial}} = 0$$

$$\Rightarrow U_{\text{initial}} = -1 \times 10 \times \frac{1}{2} \text{ (1 m lie below DATUM level, so negative)}$$

$$K_{\text{final}} = ?$$

$$U_{\text{final}} = -3 \times 10 \times \frac{3}{2}$$

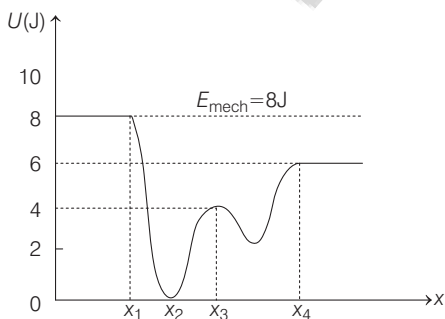
(3 m lie below DATUM level, so negative)

$$0 - 1 \times 10 \times \frac{1}{2} = K_{\text{final}} - 3 \times 10 \times \frac{3}{2}$$

$$\Rightarrow -5 = K_{\text{final}} - 45$$

$$\Rightarrow K_{\text{final}} = 45 - 5 = 40 \text{ J}$$

- 05** Given below is the plot of a potential energy function $U(x)$ for a system, in which a particle is in one-dimensional motion, while a conservative force $F(x)$ acts on it. Suppose that $E_{\text{mech}} = 8 \text{ J}$, the incorrect statement for this system is
- [2021, 27 July Shift-II]



[where, KE = kinetic energy]

- (a) at $x > x_4$, KE is constant throughout the region.
 (b) at $x < x_1$, KE is smallest and the particle is moving at the slowest speed.
 (c) at $x = x_2$, KE is greatest and the particle is moving at the fastest speed.
 (d) at $x = x_3$, KE = 4 J.

Ans. (b)

From the diagram given in question, Case I When $x > x_4$;

$$U = \text{constant} = 6 \text{ J}$$

$$K = E_{\text{mech}} - U = 2 \text{ J} = \text{constant}$$

Case II When $x < x_1$;

$$U = \text{constant} = 8 \text{ J}$$

$$K = E_{\text{mech}} - U = 8 - 8 = 0$$

It means that particle is at rest.

Case III When $x = x_2$; $U = 0$

$$\Rightarrow K = E_{\text{mech}} = 8 \text{ J}$$

It means that KE is greatest and particle is moving at the fastest speed.

Case IV When $x = x_3$; $U = 4 \text{ J}$

$$\Rightarrow U + K = 8 \text{ J}$$

$$\Rightarrow K = 4 \text{ J}$$

- 06** A force of $F = (5y + 20) \hat{j} \text{ N}$ acts on a particle. The work done by this force when the particle is moved from $y = 0 \text{ m}$ to $y = 10 \text{ m}$ is J.

[2021, 25 July Shift-II]

Ans. (450)

Given, force, $F = (5y + 20) \hat{j} \text{ N}$

Since, work done $(W) = \int_0^{10} F \cdot dy$

$$\therefore W = \int_0^{10} (5y + 20) dy$$

$$= \frac{5y^2}{2} \Big|_0^{10} + 20y \Big|_0^{10}$$

$$= \frac{5}{2} (10^2 - 0^2) + 20(10 - 0)$$

$$= \frac{5}{2} (100) + 200$$

$$= 250 + 200 = 450 \text{ J}$$

- 07** A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. [Take, $g = 9.8 \text{ ms}^{-2}$]

[2021, 22 July Shift-II]

- (a) -62720.0 J (b) -627.2 J
 (c) +627.2 J (d) 784.0 J

Ans. (b)

Given, mass of suitcase, $m = 80 \text{ kg}$

Let height lowered by porter, $h = 80 \text{ cm}$
 $= 80 \times 10^{-2} \text{ m}$

As we know that,

Work $(W) = -mgh$

$$\therefore W = -80 \times 9.8 \times 80 \times 10^{-2}$$

$$= -627.2 \text{ J}$$

- 08** If the kinetic energy of a moving body becomes four times of its initial kinetic energy, then the percentage change in its momentum will be

[2021, 20 July Shift-II]

- (a) 100% (b) 200%
 (c) 300% (d) 400%

Ans. (a)

We know that,

$$\text{Kinetic energy, } K = \frac{p^2}{2m} \Rightarrow$$

$$p \propto \sqrt{K} \quad \dots(i)$$

where, p = linear momentum of the body and m = mass of the body.

Considering Eq. (i), we can write

$$\frac{p_2}{p_1} = \frac{\sqrt{K_2}}{\sqrt{K_1}} \quad \dots(ii)$$

According to question,

$$\Rightarrow K_2 = 4K_1 \quad \dots(iii)$$

Putting the value of K_2 in Eq. (ii), we get

$$\frac{p_2}{p_1} = \frac{\sqrt{4K_1}}{\sqrt{K_1}} \Rightarrow \frac{p_2}{p_1} = 2$$

$$\Rightarrow p_2 = 2p_1$$

$\therefore$ Percentage change in momentum

$$= \frac{p_2 - p_1}{p_1} \times 100$$

$$= \frac{2p_1 - p_1}{p_1} \times 100$$

$$= \frac{p_1}{p_1} \times 100 = 100\%$$

- 09** In a spring gun having spring constant 100 N/m a small ball B of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance d on the ground, so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is m.

(Take, $g = 10 \text{ m/s}^2$)

[2021, 20 July Shift-I]

Ans. (0.003)

Given, $k = 100 \text{ N/m}$

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

$$x = 0.05 \text{ m and } H = 2 \text{ m}$$

By energy conservation,

$$\frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

$$\Rightarrow v = x \sqrt{\frac{k}{m}}$$

$$= 0.05 \times \sqrt{\frac{100}{0.1}} = 0.5\sqrt{10} \text{ ms}^{-1} \dots(i)$$

$$\text{Time of flight of ball, } t = \sqrt{\frac{2H}{g}}$$

$$\Rightarrow t = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}} \text{ s} \dots(ii)$$

$$\therefore \text{Range of ball, } d = vt$$

$$= 0.5\sqrt{10} \times \left(\frac{2}{\sqrt{10}}\right)$$

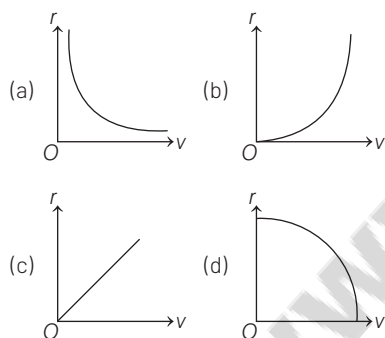
[From Eqs. (i) and (ii)]

$$= 0.5 \times 2 = 1 \text{ m} = 100 \text{ cm}$$

- 10** A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$, where C is a positive constant.

The correct radius-velocity graph of the particle's motion is

[2021, 18 March Shift-II]



Ans. (a)

The central potential field when particle moves in circular orbit,

$$U(r) = \frac{-C}{r}$$

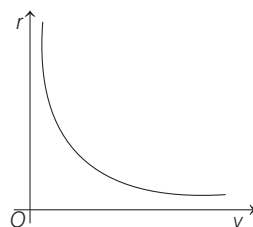
We know that,

$$F = -\frac{dU}{dr}$$

$$\Rightarrow F = -\frac{d}{dr} \left(\frac{-C}{r} \right)$$

$$\Rightarrow |F| = \frac{C}{r^2}$$

$$\Rightarrow \frac{mv^2}{r} = \frac{C}{r^2} \Rightarrow v^2 \propto \frac{1}{r}$$

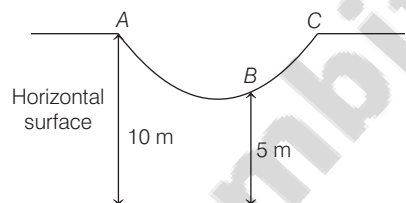


The graph between velocity and radius is hyperbolic.

- 11** As shown in the figure, a particle of mass 10 kg is placed at a point a . When the particle is slightly displaced to its right, it starts moving and reaches the point b . The speed of the particle at B is $x \text{ m/s}$. (Take, $g = 10 \text{ m/s}^2$)

The value of x to the nearest integer is

[2021, 18 March Shift-I]



Ans. (10)

Given,

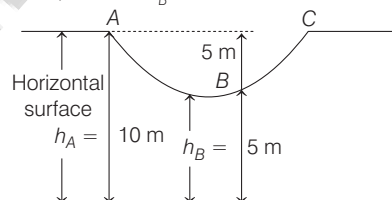
The mass of the particle, $m = 10 \text{ kg}$

The speed of the particle at point A,

$$v_A = 0 \text{ m/s}$$

The elevation of the point A from the point B, $h_A = 5 + h_B$

Let's consider the speed of the particle at point B $= v_B$



Using the law of conservation of energy.

Energy at point A = Energy at point B

$$\frac{1}{2} mv_A^2 + mgh_A = \frac{1}{2} mv_B^2 + mgh_B$$

Substituting the values in the above equations, we get

$$\frac{1}{2} (10)(0)^2 + 10 \times 10 \times (5 + h_B)$$

$$= \frac{1}{2} (10)v_B^2 + 10 \times 10 \times h_B$$

$$v_B = 10 \text{ m/s} = x \text{ m/s}$$

$$\therefore x = 10$$

- 12** Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is $n : 2$. The value of n will be

[2021, 25 Feb Shift-II]

Ans. (1)

Given, mass of particle A, $m_A = 4 \text{ g}$

Mass of particle B, $m_B = 16 \text{ g}$

Kinetic energy of A and B is same

$$\text{i.e. } KE_A = KE_B$$

As, kinetic energy $(KE) = \frac{p^2}{2m}$

where, p is momentum and m is mass.

$$\therefore \frac{(p_A)^2}{m_A} = \frac{(p_B)^2}{m_B} \Rightarrow \frac{p_A^2}{4} = \frac{p_B^2}{16} \Rightarrow \frac{p_A}{p_B} = \frac{1}{2}$$

$\therefore$ linear momentum is $n : 2$.

$$\therefore n = 1$$

- 13** Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(KE)_A : (KE)_B$ will be $\frac{A}{1}$, so the value of A will be

[2021, 24 Feb Shift-II]

Ans. (2)

Given, $m_A = 1 \text{ kg}$, $m_B = 2 \text{ kg}$,

$$(KE)_A : (KE)_B = A : 1$$

Linear momentum of A and B are equal.

$$\Rightarrow p_A = p_B$$

$$\therefore \text{Kinetic energy } (KE) = \frac{p^2}{2m}$$

$$\therefore \frac{KE_A}{KE_B} = \frac{m_B}{m_A} = \frac{2}{1} = \frac{A}{1} \Rightarrow A = 2$$

- 14** A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine, so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m , while launching the ball, the value of F (in N) is (Take, $g = 10 \text{ ms}^{-2}$)

[2020, 3 Sep Shift-I]

Ans. (150)

Work done by bowling machine = Initial kinetic energy of ball = Final potential energy of ball.

$$\Rightarrow \text{Force} \times \text{displacement} = mgh$$

$$\Rightarrow F(0.2) = (0.15)(10)(20)$$

$$F = 150 \text{ N}$$

- 15** A particle moving in the xy-plane experiences a velocity dependent force $\mathbf{F} = k(v_y \hat{i} + v_x \hat{j})$, where v_x and v_y are the x and y components of its velocity $\mathbf{v}$.

If $\mathbf{a}$ is the acceleration of the particle, then which of the following statements is true for the particle?

[2020, 6 Sep Shift-II]

- (a) Quantity $\mathbf{v} \times \mathbf{a}$ is constant in time.
 (b) $\mathbf{F}$ arises due to a magnetic field.
 (c) Kinetic energy of particle is constant in time.
 (d) Quantity $\mathbf{v} \cdot \mathbf{a}$ is constant in time.

Ans. (a)

Given that, force, $\mathbf{F} = k(v_y \hat{i} + v_x \hat{j})$

Acceleration, $\mathbf{a} = \frac{k}{m}(v_y \hat{i} + v_x \hat{j})$

or $a_x \hat{i} + a_y \hat{j} = \frac{k}{m}(v_y \hat{i} + v_x \hat{j})$

$$\frac{dv_x}{dv_y} = \frac{k}{m} v_y \text{ and } \frac{dv_y}{dt} = \frac{k}{m} v_x$$

On dividing, $\frac{dv_x}{dv_y} = \frac{v_y}{v_x}$ or $v_x dv_x = v_y dv_y$

Integrating both sides, we get

$$v_x^2 = v_y^2 + c \quad \dots(i)$$

Option (a)

$$\mathbf{v} \times \mathbf{a} = (v_x \hat{i} + v_y \hat{j}) \times \frac{k}{m}(v_y \hat{i} + v_x \hat{j})$$

$$= \frac{k}{m}(v_x^2 - v_y^2) \hat{k} = \left(\frac{k}{m} \times c\right) \hat{k}$$

[From Eq. (i)]

which is constant.

Option (b)

Magnetic force can never change the speed of a charged particle. So, force ($\mathbf{F}$) does not arise due to magnetic field.

Option (c)

The given force is not central force. So, work has to be done by this force and it will bring the change in kinetic energy.

Option (d)

$$\mathbf{v} \cdot \mathbf{a} = (v_x \hat{i} + v_y \hat{j}) \cdot \frac{k}{m}(v_y \hat{i} + v_x \hat{j})$$

$$= \frac{2k}{m} v_x v_y$$

which is not constant as force is not zero.

Hence, correct option is (a).

- 16** A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a

distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

[2020, 4 Sep Shift-II]

- (a) 5250 J (b) 2780 J
 (c) 3280 J (d) 5690 J

Ans. (a)

For $0 \leq x \leq 15$ m, $F = 200$ N ...(i)

For $15 \text{ m} < x \leq 30$ m,

Force F is linearly decreasing from 200 N to 100 N.

So, using two-point form of straight line, we have

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1} \right) (x - x_1)$$

$$(F - F_1) = \left(\frac{F_2 - F_1}{x_2 - x_1} \right) (x - x_1)$$

Here, $x_1 = 15$ m, $F_1 = 200$ N

$x_2 = 30$ m, $F_2 = 100$ N

$$\text{So, } (F - 200) = \left(\frac{100 - 200}{30 - 15} \right) (x - 15)$$

$$F - 200 = \frac{-100}{15} (x - 15)$$

$$F = 200 - \frac{20}{3} (x - 15)$$

$$F = 200 - \frac{20}{3} x + 100$$

$$F = \left(300 - \frac{20}{3} x \right) \text{ N} \quad \dots(ii)$$

Therefore,

$$F = \begin{cases} 200 \text{ N; } & 0 \leq x \leq 15 \text{ m} \\ \left(300 - \frac{20}{3} x \right) \text{ N; } & 15 \text{ m} < x \leq 30 \text{ m} \end{cases}$$

Now, work done during the complete movement of the box,

$$W = \int_0^{30} F dx$$

$$= \int_0^{15} 200 dx + \int_{15}^{30} \left(300 - \frac{20x}{3} \right) dx$$

$$= 200[x]_0^{15} + \left[300x - \frac{20}{3} \cdot \frac{x^2}{2} \right]_{15}^{30}$$

$$= 200[15 - 0] + \left[300x - \frac{10}{3} x^2 \right]_{15}^{30}$$

$$= 200 \times 15 + \left[\left\{ 300(30) - \frac{10}{3} (30)^2 \right\} - \left\{ 300(15) - \frac{10}{3} (15)^2 \right\} \right]$$

$$= 3000 + [(9000 - 3000) - (4500 - 750)]$$

$$= 3000 + [6000 - 3750]$$

$$= 3000 + 2250 = 5250 \text{ J}$$

Hence, option (a) is correct.

- 17** If the potential energy between two molecules is given by

$$U = -\frac{A}{r^6} + \frac{B}{r^{12}}, \text{ then at equilibrium,}$$

separation between molecules and the potential energy are

[2020, 6 Sep Shift-I]

$$(a) \left(\frac{B}{2A} \right)^{1/6}, -\frac{A^2}{2B} \quad (b) \left(\frac{B}{A} \right)^{1/6}, 0$$

$$(c) \left(\frac{2B}{A} \right)^{1/6}, -\frac{A^2}{4B} \quad (d) \left(\frac{2B}{A} \right)^{1/6}, -\frac{A^2}{2B}$$

Ans. (c)

Potential energy between two molecules is given by

$$U = -\frac{A}{r^6} + \frac{B}{r^{12}}$$

From the relation between force and potential energy,

Force acting between them, $F = -\frac{dU}{dr}$

$$\therefore F = -\frac{d}{dr} \left(-\frac{A}{r^6} + \frac{B}{r^{12}} \right)$$

$$= -\left(\frac{6A}{r^7} - \frac{12B}{r^{13}} \right)$$

$$= \frac{6}{r^7} \left(-A + \frac{2B}{r^6} \right)$$

At equilibrium, $F = 0$

$$\Rightarrow \frac{6}{r^7} \left(-A + \frac{2B}{r^6} \right) = 0$$

$$\Rightarrow -A + \frac{2B}{r^6} = 0 \quad (\because r \neq 0)$$

$$\Rightarrow r^6 = \frac{2B}{A}$$

$$\text{or } r = \left(\frac{2B}{A} \right)^{1/6}$$

The above calculated value of r is the separation between molecules at equilibrium.

Now, putting this value in the expression of potential energy, we get

$$U = -\frac{A}{\left\{ \left(\frac{2B}{A} \right)^{1/6} \right\}^6} + \frac{B}{\left\{ \left(\frac{2B}{A} \right)^{1/6} \right\}^{12}}$$

$$= -\frac{A^2}{2B} + \frac{A^2}{4B}$$

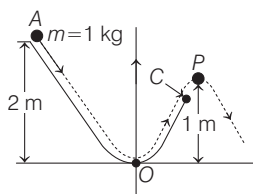
$$\therefore U = -\frac{A^2}{4B}$$

- 18** A particle ($m=1\text{ kg}$) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C , the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the kinetic energy of the particle (in J) is : (Figure drawn is schematic and not to scale; take $g=10\text{ ms}^{-2}$)

..... [2020, 7 Jan Shift-I]

Ans. (10)

Following is the situation given :
Since, the given path AOC is frictionless.
So, it starting from point A , the particle during the path AOC will attain maximum height at P .

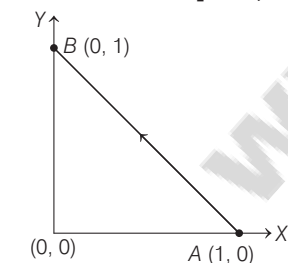


Energy conservation at A and P gives
PE at $A = (\text{PE} + \text{KE})$ at P

$$\begin{aligned} \Rightarrow U_A &= U_P + K_P \\ mgh_A &= mgh_P + K_P \\ 1 \times 10 \times 2 &= 1 \times 10 \times 1 + K_P \\ \therefore K_P &= 10\text{ J} \end{aligned}$$

- 19** Consider a force $\mathbf{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point $A(1,0)$ to $B(0,1)$ along the line segment is (all quantities are in SI units)

[2020, 9 Jan Shift-I]



- (a) $\frac{3}{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

Ans. (c)

Work done by a variable force on the particle,

$$W = \int \mathbf{F} \cdot d\mathbf{r} = \int \mathbf{F} \cdot (dx\hat{i} + dy\hat{j})$$

$\therefore$ In two dimension, $d\mathbf{r} = dx\hat{i} + dy\hat{j}$
and it is given $\mathbf{F} = -x\hat{i} + y\hat{j}$.

$$\begin{aligned} \therefore W &= \int (-x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) \\ &= \int -x dx + y dy = \int -x dx + \int y dy \end{aligned}$$

As particle is displaced from $A(1,0)$ to $B(0,1)$, so x varies from 1 to 0 and y varies from 0 to 1.

So, with limits, work will be

$$\begin{aligned} W &= \int_1^0 -x dx + \int_0^1 y dy \\ &= \left[-\frac{x^2}{2} \right]_1^0 + \left[\frac{y^2}{2} \right]_0^1 \\ &= \frac{1}{2} (0 - (-1)^2 + (1)^2 - 0) = 1\text{ J} \end{aligned}$$

- 20** A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

[2019, 9 Jan Shift-II]

- (a) 850 J (b) 900 J (c) 950 J (d) 875 J

Ans. (b)

Here, the displacement of an object is given by

$$x = (3t^2 + 5)\text{ m}$$

$$\text{Therefore, velocity } (v) = \frac{dx}{dt} = \frac{d(3t^2 + 5)}{dt}$$

$$\text{or } v = 6t\text{ m/s} \quad \dots(i)$$

The work done in moving the object from $t=0$ to $t=5\text{ s}$

$$W = \int_{x_0}^{x_5} \mathbf{F} \cdot d\mathbf{x} \quad \dots(ii)$$

The force acting on this object is given by

$$\begin{aligned} F &= ma = m \times \frac{dv}{dt} \\ &= m \times \frac{d(6t)}{dt} \quad [\text{using Eq. (i)}] \end{aligned}$$

$$F = m \times 6 = 6m = 12\text{ N}$$

$$\text{Also, } x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5\text{ m}$$

and at $t=5\text{ s}$,

$$x_5 = 3 \times (5)^2 + 5 = 80\text{ m}$$

Put the values in Eq. (ii), we get

$$W = 12 \times \int_{x_0}^{x_5} dx = 12 [80 - 5]$$

$$W = 12 \times 75 = 900\text{ J}$$

Alternate Solution

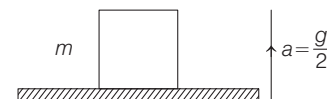
To using work - kinetic energy theorem is,

$$W = \Delta K \cdot E = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$= \frac{1}{2} m \times (30^2 - 0^2)$$

$$= \frac{1}{2} \times 2 \times 900 = 900\text{ J}$$

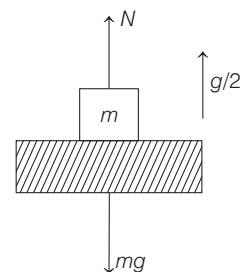
- 21** A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards as shown in figure. Work done by normal reaction on block in time t is [2019, 10 Jan Shift-I]



- (a) $\frac{mg^2 t^2}{8}$ (b) $\frac{3mg^2 t^2}{8}$
(c) 0 (d) $-\frac{mg^2 t^2}{8}$

Ans. (b)

Normal reaction force on the block is



$$N = ma_{\text{net}}$$

where, a_{net} = net acceleration of block.

$$= g + a$$

$$= g + \frac{g}{2} = \frac{3g}{2}$$

$$\Rightarrow N = m \left(g + \frac{g}{2} \right) = \frac{3mg}{2}$$

Now, in time t block moves by a displacement s given by

$$s = 0 + \frac{1}{2} at^2$$

$$= \frac{1}{2} \left(\frac{g}{2} \right) t^2 \quad (\because u=0)$$

$$\text{Here, } a = \frac{g}{2} \text{ (given)}$$

$\therefore$ Work done = Force $\times$ Displacement

$$\begin{aligned} \Rightarrow W &= \frac{3mg}{2} \times \frac{gt^2}{4} \\ &= \frac{3mg^2 t^2}{8} \end{aligned}$$

- 22** A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is [JEE Main 2018]

- (a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$
 (c) zero (d) $-\frac{3}{2} \frac{k}{a^2}$

Ans. (c)

$$\therefore \text{Force} = -\frac{dU}{dr}$$

$$\Rightarrow F = -\frac{d}{dr} \left(\frac{-k}{2r^2} \right) = -\frac{k}{r^3}$$

As particle is on circular path, this force must be centripetal force.

$$\Rightarrow |F| = \frac{mv^2}{r}$$

$$\text{So, } \frac{k}{r^3} = \frac{mv^2}{r} \Rightarrow \frac{1}{2} mv^2 = \frac{k}{2r^2}$$

$$\therefore \text{Total energy of particle} = \text{KE} + \text{PE} \\ = \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

Total energy = 0

- 23** A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8} mv_0^2$, the value of k will be

[JEE Main 2017]

- (a) 10^{-3} kgs^{-1} (b) 10^{-4} kgm^{-1}
 (c) $10^{-1} \text{ kgm}^{-1}\text{s}^{-1}$ (d) 10^{-3} kgm^{-1}

Ans. (b)

Given, force, $F = -kv^2$

$$\therefore \text{Acceleration, } a = \frac{-k}{m} v^2$$

$$\text{or } \frac{dv}{dt} = \frac{-k}{m} v^2 \Rightarrow \frac{dv}{v^2} = -\frac{k}{m} dt$$

Now, with limits, we have

$$\int_{10}^v \frac{dv}{v^2} = -\frac{k}{m} \int_0^t dt \\ \Rightarrow \left(-\frac{1}{v} \right)_{10}^v = -\frac{k}{m} t \Rightarrow \frac{1}{v} = 0.1 + \frac{kt}{m}$$

$$\Rightarrow v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\Rightarrow \frac{1}{2} \times m \times v^2 = \frac{1}{8} mv_0^2 \Rightarrow v = \frac{v_0}{2} = 5$$

$$\Rightarrow \frac{1}{0.1 + 1000k} = 5$$

$$\Rightarrow 1 = 0.5 + 5000k$$

$$\Rightarrow k = \frac{0.5}{5000}$$

$$\Rightarrow k = 10^{-4} \text{ kg/m}$$

- 24** A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. (d) How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted into mechanical energy with a 20% of efficiency rate. (Take, $g = 9.8 \text{ ms}^{-2}$)

[JEE Main 2016 (Offline)]

- (a) $2.45 \times 10^{-3} \text{ kg}$
 (b) $6.45 \times 10^{-3} \text{ kg}$
 (c) $9.89 \times 10^{-3} \text{ kg}$
 (d) $12.89 \times 10^{-3} \text{ kg}$

Ans. (d)

Given, potential energy burnt by lifting weight

$$= mgh = 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4$$

If mass lost by a person be m , then energy dissipated

$$= m \times \frac{2}{10} \times 3.8 \times 10^7 \text{ J}$$

$$\Rightarrow 9.8 \times 10^4 = m \times \frac{1}{5} \times 3.8 \times 10^7$$

$$\Rightarrow m = \frac{5}{3.8} \times 10^{-3} \times 9.8 \\ = 12.89 \times 10^{-3} \text{ kg}$$

- 25** When a rubber band is stretched by a distance x , it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is

[JEE Main 2014]

- (a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$
 (c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2} \left(\frac{aL^2}{2} + \frac{bL^3}{3} \right)$

Ans. (c)

Key Idea As, we know that change in potential energy of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f F \cdot dr$$

Given, $F = ax + bx^2$

According to work-energy theorem, we know that work done in stretching the rubber band by t is

$$|dW| = F dx$$

$$|W| = \int_0^L (ax + bx^2) dx \\ = \left[\frac{ax^2}{2} \right]_0^L + \left[\frac{bx^3}{3} \right]_0^L \\ = \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2} \right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3} \right] \\ \therefore |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

- 26** This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2 respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2 .

Statement I If stretched by the same amount, work done on S_1 will be more than that on S_2 .

Statement II $k_1 < k_2$ [AIEEE 2012]

- (a) Statement I is false, Statement II is true
 (b) Statement I is true, Statement II is false
 (c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
 (d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. (a)

As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about Statement I. But as in Statement II, $k_1 < k_2$.

Then, for same force,

$$W = F \cdot x = F \cdot \frac{F}{k} = \frac{F^2}{k} \Rightarrow W \propto \frac{1}{k}$$

$$\text{i.e., } W_1 > W_2$$

But for same displacement,

$$W = F \cdot x = \frac{1}{2} kx \cdot x = \frac{1}{2} kx^2 \Rightarrow W \propto k$$

$$\text{i.e., } W_1 < W_2$$

Thus, Statement II is true and Statement I is false.

- 27** An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

[AIEEE 2008]

- (a) 200 J – 500 J
 (b) 2×10^5 J – 3×10^5 J
 (c) 20000 J – 50000 J
 (d) 2000 J – 5000 J

Ans. (d)

The given question is somewhat based on approximations. Let mass of athlete be 65 kg.

Approx velocity from the given data is 10 m/s.

$$\text{So, } KE = \frac{65 \times 100}{2} = 3250 \text{ J}$$

So, option (d) is the most probable answer.

- 28** A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is [AIEEE 2008]

- (a) 0.16 J (b) 1.00 J
 (c) 0.67 J (d) 0.34 J

Ans. (c)

From law of conservation of momentum, we have

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

$$\text{Given, } m_1 = 0.50 \text{ kg, } v_1 = 2 \text{ ms}^{-1}, \\ m_2 = 1 \text{ kg, } v_2 = 0 \quad [\text{at rest}] \\ 0.5 \times 2 + 1 \times 0 = 1.5 v$$

[assumed that 2nd body is at rest]

$$\Rightarrow v = \frac{2}{3}$$

$$\therefore \Delta K = K_f - K_i$$

$$= \frac{1.5 \times \left(\frac{2}{3}\right)^2}{2} - (0.5) \times \frac{2^2}{2} = -\frac{2}{3} \text{ J} \\ = -0.67 \text{ J}$$

So, energy lost is 0.67 J.

- 29** A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by [AIEEE 2007]

- (a) 5.5 cm (b) 2.5 cm
 (c) 11.0 cm (d) 8.5 cm

Ans. (a)

According to work-energy theorem

Loss in kinetic energy = Work done against friction + Potential energy of spring

$$\frac{1}{2} m v^2 = f x + \frac{1}{2} k x^2 \\ \Rightarrow \frac{1}{2} \times 2 (4)^2 = 15 x + \frac{1}{2} \times 10000 x^2 \\ \Rightarrow 5000 x^2 + 15 x - 16 = 0 \\ \therefore x = 0.055 \text{ m} = 5.5 \text{ cm}$$

- 30** A particle is projected at 60° to the horizontal with a kinetic energy K . The kinetic energy at the highest point is [AIEEE 2007]

- (a) $\frac{K}{4}$ (b) zero
 (c) $\frac{K}{2}$ (d) $\frac{K}{4}$

Ans. (c)

Kinetic energy at highest point,

$$(KE)_H = \frac{1}{2} m v^2 \cos^2 \theta \\ = K \cos^2 \theta = K (\cos 60^\circ)^2 = K/4$$

- 31** The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2} \right) \text{ J}$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in ms^{-1}) is [AIEEE 2006]

- (a) $\frac{3}{2}$ (b) 2
 (c) $\frac{1}{2}$ (d) 2

Ans. (a)

$$\text{Given, } V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2} \right)$$

For minimum value of V , $\frac{dV}{dx} = 0$

$$\Rightarrow \frac{4x^3}{4} - \frac{2x}{2} = 0$$

$$\Rightarrow x = 0, x = \pm 1$$

$$\text{So, } V_{\min} (x = \pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \text{ J}$$

Now, $K_{\max} + V_{\min} = \text{Total mechanical energy}$

$$\Rightarrow K_{\max} = \left(\frac{1}{4} \right) + 2 \text{ or } K_{\max} = \frac{9}{4}$$

$$\text{or } \frac{mv^2}{2} = \frac{9}{4} \text{ or } v = \frac{3}{2} \text{ ms}^{-1}$$

- 32** A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a

smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is [AIEEE 2005]

- (a) 40 m/s (b) 20 m/s
 (c) 10 m/s (d) 10 30 m/s

Ans. (a)

According to conservation of energy, potential energy at height H is sum of kinetic energy and potential energy at h_2 .

$$H = 100 \text{ m} \quad h_1 = 30 \text{ m} \quad h_2 = 20 \text{ m}$$

$$\therefore mgH = \frac{1}{2} m v^2 + mgh_2$$

$$\Rightarrow mg(H - h_2) = \frac{1}{2} m v^2$$

$$\text{or } v = 2g(100 - 20)$$

$$\text{or } v = 2 \times 10 \times 80 = 40 \text{ m/s}$$

- 33** The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L . The maximum momentum of the block after collision is [AIEEE 2005]

M

- (a) MkL (b) $\frac{kL^2}{2M}$
 (c) zero (d) $\frac{ML^2}{k}$

Ans. (a)

Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.

According to conservation of energy,

$$\frac{1}{2} k L^2 = \frac{1}{2} M v^2$$

$$\Rightarrow k L^2 = \frac{(M v)^2}{M}$$

$$\text{or } M k L^2 = p^2 \quad [\because p = Mv]$$

$$\Rightarrow p = L \sqrt{Mk}$$

- 34** A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic

energy for any displacement x is proportional to [AIEEE 2004]

- (a) x^2 (b) e^x (c) x (d) $\log_e x$

Ans. (a)

From given information $a = -kx$, where a is acceleration, x is displacement and k is a proportionality constant,

$$\frac{v dv}{dx} = -kx \Rightarrow v dv = -kx dx$$

Let for any displacement from 0 to x , the velocity changes from v_0 to v .

$$\Rightarrow \int_{v_0}^v v dv = - \int_0^x kx dx$$

$$\Rightarrow \left[\frac{v^2}{2} \right]_{v_0}^v = -k \left[\frac{x^2}{2} \right]_0^x$$

$$\Rightarrow \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$$

$$\Rightarrow m \left(\frac{v^2 - v_0^2}{2} \right) = -\frac{mkx^2}{2}$$

$$\frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 = -\frac{mkx^2}{2}$$

$$\Rightarrow \Delta K \propto x^2 \quad [\Delta K \text{ is loss in KE}]$$

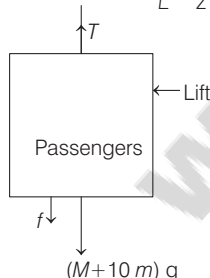
- 35** A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

[AIEEE 2004]

- (a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

Ans. (b)

Mass per unit length = $\frac{M}{L} = \frac{4}{2} = 2 \text{ kg/m}$



The mass of 0.6 m of chain = $0.6 \times 2 = 1.2 \text{ kg}$

The height of centre of mass of hanging part

$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table = Work done against gravity force

$$\begin{aligned} \text{i.e., } W &= mgh \\ &= 1.2 \times 10 \times 0.3 = 3.6 \text{ J} \end{aligned}$$

- 36** A force $\mathbf{F} = (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \text{ N}$ is applied over a particle which displaces it from its origin to the point $\mathbf{r} = (2\mathbf{i} - \mathbf{j}) \text{ m}$. The work done on the particle in joule is [AIEEE 2004]

- (a) -7 (b) +7 (c) +10 (d) +13

Ans. (b)

Work done in displacing the particle

$$\begin{aligned} W &= \mathbf{F} \cdot \mathbf{r} \\ &= (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j}) \\ &= 5 \times 2 + 3 \times (-1) + 2 \times 0 = 10 - 3 = 7 \text{ J} \end{aligned}$$

- 37** A spring of spring constant $5 \times 10^3 \text{ N/m}$ is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is [AIEEE 2003]

- (a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

Ans. (b)

Work done to stretch the spring by 5 cm from mean position = $W_1 = \frac{1}{2} kx_1^2$

$$\begin{aligned} &= \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2 \\ &= 6.25 \text{ J} \end{aligned}$$

Work done to stretch the spring by 10 cm from mean position, $W_2 = \frac{1}{2} k(x_1 + x_2)^2$

$$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2 = 25 \text{ J}$$

Net work done to stretch the spring from 5 cm to 10 cm = $W_2 - W_1$

$$\begin{aligned} &= 25 - 6.25 \\ &= 18.75 \text{ J} = 18.75 \text{ N-m} \end{aligned}$$

- 38** Consider the following two statements.

- I. Linear momentum of a system of particles is zero.
II. Kinetic energy of a system of particles is zero. Then,

[AIEEE 2003]

- (a) I does not imply II and II does not imply I
(b) I implies II but II does not imply I
(c) I does not imply II but II implies I
(d) I implies II and II implies I

Ans. (c)

Here, II is implying I but I is not implying II as kinetic energy of a system of particles is zero means speed of each and every particle is zero which says that momentum of every particle is zero.

But statement I means linear momentum of a system of particles is

zero, which may be true even, if particles have equal and opposite momentums and hence having non-zero kinetic energy.

- 39** A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is [AIEEE 2002]

- (a) 16 J (b) 8 J (c) 32 J (d) 24 J

Ans. (b)

The work done on the spring is stored as the PE of the body and is given by

$$U = \int_{x_1}^{x_2} F_{\text{ext}} dx$$

$$\text{or } U = \int_{x_1}^{x_2} kx dx = \frac{1}{2} k(x_2^2 - x_1^2)$$

$$= \frac{800}{2} [(0.15)^2 - (0.05)^2]$$

$$= 400(0.2 \times 0.1) = 8 \text{ J}$$

- 40** A ball whose kinetic energy is E , is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be [AIEEE 2002]

- (a) E (b) $\frac{E}{\sqrt{2}}$ (c) $\frac{E}{2}$ (d) zero

Ans. (c)

At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u \cos \theta$$

Given, $\theta = 45^\circ$

$$\therefore u_x = u \cos 45^\circ = \frac{u}{\sqrt{2}}$$

Hence, at the highest point, kinetic energy is

$$E' = \frac{1}{2} m u_x^2 = \frac{1}{2} m \left(\frac{u}{\sqrt{2}} \right)^2 = \frac{1}{2} m \left(\frac{u^2}{2} \right)$$

$$= \frac{E}{2} \quad \left[\because \frac{1}{2} m u^2 = E \right]$$

TOPIC 2

Work energy theorem, Power and Vertical Circle

- 41** A body of mass m dropped from a height h reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is [2021, 1 Sep Shift-II]

- (a) $-0.68 mgh$ (b) mgh
(c) $1.64 mgh$ (d) $0.64 mgh$

Ans. (a)

Given, the mass of the body = m

The height from which the body dropped = h

The speed of the body when reached the ground, $v_f = 0.8\sqrt{gh}$

Initial velocity of the body, $v = 0 \text{ m/s}$

Using the work-energy theorem,

Work done by gravity + Work done by air-friction = Final kinetic energy – Initial kinetic energy

$$W_{mg} + W_{\text{air-friction}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Here, work done by gravity = mgh

$\Rightarrow$

$$mgh + W_{\text{air-friction}} = \frac{1}{2}m(0.8\sqrt{gh})^2 - \frac{1}{2}m(0)^2$$

$$\Rightarrow W_{\text{air-friction}} = \frac{0.64mgh}{2} - mgh$$

$$= 0.32mgh - mgh$$

$$= -0.68mgh$$

The value of the work done by the air friction is $-0.68mgh$.

- 42** An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40000 kg is moving with a speed of 72 km h^{-1} , when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is $\times 10^5 \text{ N/m}$.

[2021, 1 Sep Shift-II]

Ans. (16)

Given, the length of the shock absorber, $l = 1.5 \text{ m}$

The total mass of the system, $M = 40000 \text{ kg}$

The speed of the wagon, $v = 72 \text{ km/h}$

When brakes are applied, the final velocity, $v_f = 0$

The compressed spring of the shock absorber, $x = 1 \text{ m}$

Applying the work-energy theorem,

Work done by the system = Change in kinetic energy

$$W = \Delta KE$$

$$W_{\text{friction}} + W_{\text{spring}} = \frac{1}{2}mv_f^2 + \frac{1}{2}mv_i^2$$

$$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{\text{spring}} = 0 - \frac{1}{2}mv_i^2$$

($\because$ 90% energy lost due to friction)

$$W_{\text{spring}} = -\frac{10}{100} \times \frac{1}{2}mv^2$$

$$-\frac{1}{2}kx^2 = \frac{1}{20}mv^2$$

$$k = \frac{mv^2}{10 \times x^2}$$

Substituting the values in the above equation, we get

$$k = \frac{40000 \times \left(72 \times \frac{5}{18}\right)^2}{10(1)^2}$$

$$= 16 \times 10^5 \text{ N/m}$$

Comparing the spring constant,

$$k = x \times 10^5$$

The value of the $x = 16$.

- 43** An automobile of mass m accelerates starting from origin and initially at rest, while the engine supplies constant power P . The position is given as a function of time by [2021, 27 July Shift-II]

$$(a) \left(\frac{9P}{8m}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$$

$$(b) \left(\frac{8P}{9m}\right)^{\frac{1}{2}} \cdot t^{\frac{2}{3}}$$

$$(c) \left(\frac{9m}{8P}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$$

$$(d) \left(\frac{8P}{9m}\right)^{\frac{1}{2}} \cdot t^{\frac{3}{2}}$$

Ans. (d)

According to given situation energy supplied in delivering the constant power P is equal to the kinetic energy of the automobile.

$\therefore$ Energy supplied = Kinetic energy

$$\Rightarrow Pt = \frac{1}{2}mv^2 \Rightarrow v = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$\text{or } \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \quad \left[\because v = \frac{ds}{dt}\right]$$

Integrating both sides of above equation, we get

$$\int_s^0 ds = \left(\frac{2P}{m}\right)^{1/2} \int_0^t t^{1/2} \cdot dt$$

$$\Rightarrow s = \left(\frac{2P}{m}\right)^{1/2} \cdot \frac{2}{3}t^{3/2}$$

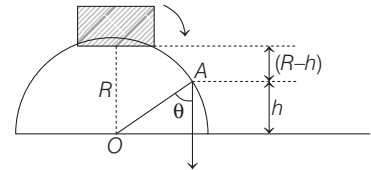
$$\Rightarrow s = \frac{2}{3} \left(\frac{2P}{m}\right)^{1/2} \cdot t^{3/2}$$

$$\Rightarrow s = \left(\frac{8P}{9m}\right)^{1/2} \cdot t^{3/2}$$

- 44** A small block slides down from the top of hemisphere of radius $R = 3 \text{ m}$ as shown in the figure. The height h

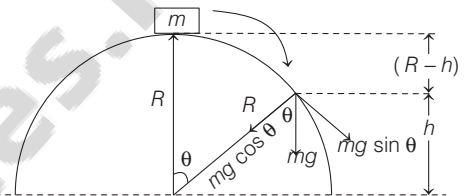
at which the block will lose contact with the surface of the sphere is m.

(Assume there is no friction between the block and the hemisphere) [2021, 27 July Shift-II]



Ans. (2)

The given figure can also be represented as



$\therefore$ From work - energy theorem, we have

Work done = Change in kinetic energy

$$\Rightarrow W = \Delta KE$$

$$\Rightarrow Mg(R - R \cos \theta) = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{2gR(1 - \cos \theta)} \quad \dots(i)$$

To loose contact,

$$\frac{mv^2}{R} = mg \cos \theta \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{m[2gR(1 - \cos \theta)]}{R} = mg \cos \theta$$

$$\Rightarrow 2(1 - \cos \theta) = \cos \theta$$

$$\Rightarrow 2 - 2 \cos \theta = \cos \theta$$

$$\Rightarrow 2 = 3 \cos \theta$$

$$\Rightarrow \cos \theta = \frac{2}{3}$$

$$\therefore \cos \theta = \frac{h}{R} \quad (\text{using the figure})$$

$$\Rightarrow \cos \theta = \frac{2}{3} = \frac{h}{R} = \frac{h}{3}$$

$$\Rightarrow h = 2 \text{ m}$$

- 45** A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob when the length makes an angle of 60° to the vertical will be m/s. (Take, $g = 10 \text{ m/s}^2$) [2021, 25 July Shift-I]

Ans. (2)

Given, speed of bob at lowest point,

$$v_1 = 3 \text{ ms}^{-1}$$

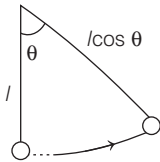
Length of string, $l = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$
Angle, $\theta = 60^\circ$

Let v_2 be the speed of bob at 60° from mean position.

By using law of conservation of energy

$$KE_i + PE_i = KE_f + PE_f$$

$$\Rightarrow \frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_2^2 + mgl(1 - \cos\theta)$$



$$\Rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mgl(1 - \cos\theta)$$

$$\Rightarrow v_2^2 = v_1^2 - 2gl(1 - \cos\theta)$$

$$\Rightarrow v_2 = \sqrt{3^2 - 2 \times 10 \times 50 \times 10^{-2} (1 - \cos 60^\circ)}$$

$$= \sqrt{9 - 10 \left(1 - \frac{1}{2}\right)} = \sqrt{9 - 5}$$

$$= \sqrt{4} = 2 \text{ ms}^{-1}$$

- 46** A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

[2021, 20 July Shift-II]

(a) $t^{\frac{3}{2}}$ (b) $t^{\frac{1}{2}}$ (c) $t^{\frac{1}{4}}$ (d) $t^{\frac{3}{4}}$

Ans. (a)

According to the question, we can say that, the energy of the machine is delivering a constant power which is further used up in moving a body (from rest) along a horizontal straight line.

$\therefore$ Energy supply $= Pt$

where, P = power

and t = time taken.

$$\Rightarrow \frac{1}{2}mv^2 = Pt$$

$$\Rightarrow v^2 \propto t \Rightarrow v \propto \sqrt{t}$$

$$\text{or } \frac{ds}{dt} = c\sqrt{t} \quad \dots(i)$$

where, c is any constant.

Now, integrating Eq. (i), we get

$$\int_0^s ds = \int_0^t c\sqrt{t} dt \Rightarrow s = c \int_0^t t^{\frac{1}{2}} \cdot dt$$

$$\Rightarrow s = \frac{2c}{3} t^{\frac{3}{2}} \Rightarrow s \propto t^{\frac{3}{2}}$$

So, the distance moved by the body in time t is proportional to $t^{\frac{3}{2}}$.

- 47** A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time t is proportional to

[2021, 18 March Shift-I]

(a) $t^{2/3}$ (b) $t^{3/2}$ (c) t (d) $t^{1/2}$

Ans. (b)

We know that,

Power = force $\times$ velocity [$\because$ Given, power = constant]

$\therefore$ Force $\times$ velocity = constant

$$\Rightarrow ma \times v = \text{constant}$$

$$\Rightarrow a \times v = \text{constant}$$

$$\Rightarrow \left(v \frac{dv}{dx}\right) \left(\frac{dx}{dt}\right) = C \Rightarrow v dv = C dt$$

$$\Rightarrow \frac{v^2}{2} = Ct \Rightarrow v = \sqrt{2Ct}$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{2Ct} \quad \left[\because v = \frac{dx}{dt}\right]$$

$$\Rightarrow \int dx = \int \sqrt{2Ct} dt$$

$$\Rightarrow x = \sqrt{2C} \left(\frac{t^{3/2}}{3/2}\right) \Rightarrow x \propto t^{3/2}$$

- 48** A small bob tied at one end of a thin string of length 1 m is describing a vertical circle, so that the maximum and minimum tension in the string are in the ratio $5 : 1$. The velocity of the bob at the highest position is m/s .

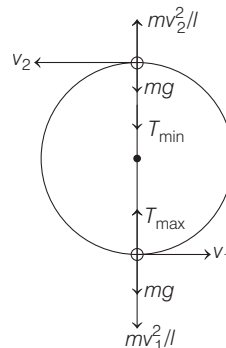
(Take, $g = 10 \text{ m/s}^2$)

[2021, 25 Feb Shift-I]

Ans. (5)

Given, length of string, $l = 1 \text{ m}$

$T_{\max}$ and $T_{\min}$ be the tension in string and v_1 and v_2 be the velocities of bob at bottom and top in vertical circle.



$$T_{\max} = mg + mv_1^2/l$$

$$\text{and } T_{\min} = mv_2^2/l - mg$$

$$\therefore \frac{T_{\max}}{T_{\min}} = \frac{mg + mv_1^2/l}{mv_2^2/l - mg} = \frac{5}{1} \quad (\text{given})$$

$$\Rightarrow mg + mv_1^2/l = 5mv_2^2/l - 5mg$$

$$\text{Here, } v_1 = \sqrt{v_2^2 + 4gl}$$

$$\Rightarrow mg + \frac{m}{l}(v_2^2 + 4gl) = \frac{5mv_2^2}{l} - 5mg$$

$$\Rightarrow g + \frac{v_2^2 + 4gl}{l} = \frac{5v_2^2}{l} - 5g$$

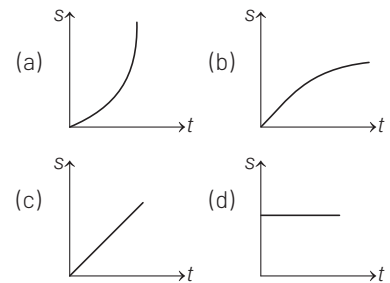
$$\Rightarrow 6gl = 5v_2^2 - v_2^2 - 4gl$$

$$\Rightarrow 10gl = 4v_2^2$$

$$\Rightarrow v_2 = \sqrt{\frac{10gl}{4}} = \sqrt{\frac{10 \times 10 \times 1}{4}} = \sqrt{25} = 5 \text{ m/s}$$

- 49** A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s)-time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale)

[2020, 3 Sep Shift-II]



Ans. (a)

As, power $= \frac{d}{dt}(\text{KE}) = \text{constant}$

$$\Rightarrow \frac{d}{dt} \left(\frac{1}{2}mv^2 \right) = P \Rightarrow \frac{1}{2}m \cdot 2v \frac{dv}{dt} = P$$

$$\Rightarrow v dv = \frac{P}{m} dt$$

Integrating both sides, we get

$$\int v dv = \frac{P}{m} \int dt \Rightarrow \frac{v^2}{2} = \frac{P}{m} t$$

$$\Rightarrow v = \sqrt{\frac{2P}{m}} \cdot t^{1/2} \Rightarrow \frac{ds}{dt} = \sqrt{\frac{2P}{m}} \cdot t^{1/2}$$

$$\Rightarrow ds = \sqrt{\frac{2P}{m}} \cdot t^{1/2} dt$$

Again, integrating both sides, we get

$$s = \int ds = \int \sqrt{\frac{2P}{m}} \cdot t^{1/2} dt \Rightarrow s = C \cdot t^{3/2}$$

where, C is a constant $= \frac{2}{3} \sqrt{\frac{2P}{m}}$.

Hence, displacement (s)-time (t) graph is correctly represented in option (a).

- 50** A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 s, the body has moved a distance (in m)

[2020, 5 Sep Shift-II]

Ans. (18)

Let s be the required distance.

$$P = \text{constant (1 J/s)}$$



From Work-Energy theorem,

Work = Change in kinetic energy

$$\Rightarrow \text{Power} \times \text{Time} = \Delta K$$

$$\text{i.e., } Pt = \Delta K \Rightarrow Pt = \frac{1}{2}mv^2 \quad \dots(i)$$

Given, $P = 1 \text{ Js}^{-1}$, $t = 9 \text{ s}$, $m = 2 \text{ kg}$

Substituting all the given values in eq. (i), we get

$$1 \times 9 = \frac{1}{2}(2)v^2$$

$$v^2 = 9 \Rightarrow v = 3 \text{ m/s (at } t = 9 \text{ s)}$$

$$\text{As, } Fv = P \Rightarrow (ma)v = P \quad [\because F = ma]$$

$$\Rightarrow m \left[\frac{dv}{dt} \right] v = P \Rightarrow m \left[\frac{ds}{dt} \frac{dv}{ds} \right] v = P$$

$$\Rightarrow m \left[v \frac{dv}{ds} \right] v = P$$

$$\Rightarrow 2v^2 dv = ds \quad [\because P = 1 \text{ J/s and } m = 2 \text{ kg}]$$

Integrating both sides,

$$\int_0^s 2v^2 dv = \int_0^s ds \Rightarrow \frac{2}{3} [v^3]_0^s = s$$

$$\frac{2}{3} [27 - 0] = s \Rightarrow s = 18 \text{ m}$$

Hence, after 9 s, the body has moved a distance of 18 m.

- 51** A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Take, 1 HP = 746 W, $g = 10 \text{ ms}^{-2}$)

[2020, 7 Jan Shift-I]

- (a) 2.0 ms^{-1} (b) 1.5 ms^{-1}
(c) 1.9 ms^{-1} (d) 1.7 ms^{-1}

Ans. (c)

At maximum load, force provided by motor to pull the lift,

$$F = \text{weight carried} + \text{friction} = mg + f$$

$$= (2000 \times 10) + 4000 = 24000 \text{ N}$$

Power delivered by motor at speed v of load, $P = F \times v$

$$\Rightarrow v = \frac{P}{F} = \frac{60 \times 746}{24000} = 1.865 = 1.9 \text{ ms}^{-1}$$

- 52** An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N.

If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10 \text{ m/s}^2$) must be at least

[2020, 7 Jan Shift-II]

- (a) 62360 W (b) 48000 W
(c) 56300 W (d) 66000 W

Ans. (d)

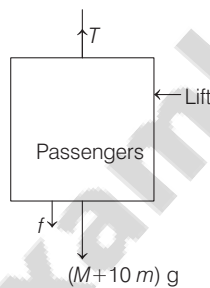
Mass of elevator, $M = 920 \text{ kg}$

Mass of all '10' passengers carried by elevator = $10 \times m = 10 \times 68 = 680 \text{ kg}$

Total weight of elevator and passengers

$$= (M + 10m)g = (920 + 680) \times 10$$

$$= 16000 \text{ N}$$



Force of friction = 6000 N

Total force (T) applied by the motor of elevator

$$= 16000 + 6000 = 22000 \text{ N}$$

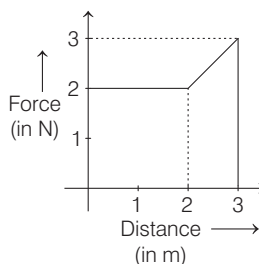
Power delivered by elevator's motor,

$$P = F \cdot v = 22000 \times 3 \quad [\because v = 3 \text{ ms}^{-1}]$$

$$= 66000 \text{ W}$$

- 53** A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is

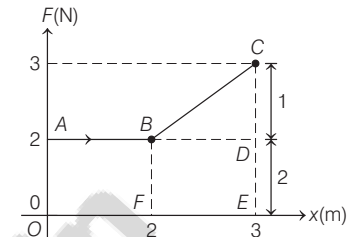
[2019, 8 April Shift-I]



- (a) 4 J (b) 2.5 J
(c) 6.5 J (d) 5 J

Ans. (c)

Key Idea Area under force-displacement graph gives the value of work done.



$\therefore$ Work done on the particle

= Area under the curve ABC

$W = \text{Area of square ABFO} + \text{Area of } \triangle BCD$

$$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1$$

$$= 6.5 \text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$$\Rightarrow K_f = \Delta W = 6.5 \text{ J} \quad [\because K_i = 0]$$

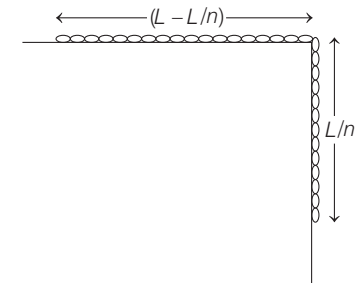
- 54** A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$ th part is

hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

[2019, 9 April Shift-I]

- (a) $\frac{2MgL}{n^2}$ (b) $nMgL$
(c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

Ans. (d)



Given, mass of the cable is M .

So, mass of $\frac{1}{n}$ th part of the cable, i.e.

hanged part of the cable is $= M/n \quad \dots(i)$

Now, centre of mass of the hanged part will be its middle point.

So, its distance from the top of the table will be $L/2n$.

∴ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow U_i = -\frac{MgL}{2n^2} \quad \dots(ii)$$

When whole cable is on the table, its potential energy will be zero.

$$\therefore U_f = 0 \quad \dots(iii)$$

Now, using work-energy theorem,

$$W_{\text{net}} = \Delta U = U_f - U_i$$

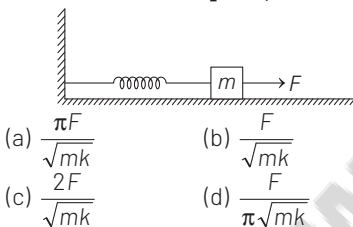
$$\Rightarrow W_{\text{net}} = 0 - \left(-\frac{MgL}{2n^2}\right)$$

[using Eqs. (ii) and (iii)]

$$\Rightarrow W_{\text{net}} = \frac{MgL}{2n^2}$$

- 55** A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k . The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F , the maximum speed of the block is

[2019, 9 Jan Shift-I]



Ans. (b)

In a spring-block system, when a block is pulled with a constant force F , then its speed is maximum at the mean position. Also, its acceleration will be zero.

In that case, force on the system is given as,

$$F = kx \quad \dots(i)$$

where, x is the extension produced in the spring.

$$\text{or } x = \frac{F}{k}$$

Now we know that, for a system vibrating at its mean position, its maximum velocity is given as,

$$v_{\text{max}} = A\omega$$

where, A is the amplitude and ω is the angular velocity.

Since, the block is at its mean position.

$$\begin{aligned} \text{So, } A = x &= \frac{F}{k} \\ v_{\text{max}} &= \frac{F}{k} \sqrt{\frac{k}{m}} \quad \left[\because \omega = \sqrt{\frac{k}{m}} \right] \\ &= \frac{F}{\sqrt{km}} \end{aligned}$$

Alternate Solution

According to the work-energy theorem, net work done = change in the kinetic energy

Here, net work done = work done due to external force (W_{ext}) + work done due to the spring (W_{spr}).

$$\text{As, } W_{\text{ext}} = F \cdot x$$

$$\text{and } W_{\text{spr}} = -\frac{1}{2} kx^2$$

$$\Rightarrow \Delta KE = F \cdot x + \left(-\frac{1}{2} kx^2\right)$$

$$(\Delta KE)_f - (\Delta KE)_i = F \cdot x - \frac{1}{2} kx^2$$

$$\Rightarrow \frac{1}{2} mv_{\text{max}}^2 - \frac{1}{2} m(0)^2 = F \cdot \left(\frac{F}{k}\right) - \frac{1}{2} k \left(\frac{F}{k}\right)^2$$

[using Eq. (i)]

$$\Rightarrow \frac{1}{2} mv_{\text{max}}^2 = \frac{F^2}{k} - \frac{F^2}{2k} = \frac{F^2}{2k}$$

$$\text{or } v_{\text{max}}^2 = \frac{F^2}{km}$$

$$\Rightarrow v_{\text{max}} = F / \sqrt{km}$$

- 56** A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\mathbf{d} = 4\hat{i}$.

If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? [2019, 10 Jan Shift-II]

- (a) 9 J (b) 15 J
(c) 12 J (d) 10 J

Ans. (b)

We know that, work done in displacing a particle at displacement $\mathbf{d}$ under force $\mathbf{F}$ is given by $\Delta W = \mathbf{F} \cdot \mathbf{d}$

By substituting given values, we get

$$\Rightarrow \Delta W = (3\hat{i} - 12\hat{j}) \cdot (4\hat{i})$$

$$\Rightarrow \Delta W = 12 \text{ J} \quad \dots(i)$$

Now, using work-energy theorem, we get

work done (ΔW) = change in kinetic energy (ΔK)

$$\text{or } \Delta W = K_2 - K_1 \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12 \text{ J}$$

$$\text{or } K_2 = K_1 + 12 \text{ J}$$

Given, initial kinetic energy, $K_1 = 3 \text{ J}$

∴ Final kinetic energy,

$$K_2 = 3 \text{ J} + 12 \text{ J} = 15 \text{ J}$$

- 57** A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be [JEE Main 2017]

- (a) 22 J (b) 9 J
(c) 18 J (d) 4.5 J

Ans. (d)

From Newton's second law,

$$\frac{\Delta p}{\Delta t} = F \Rightarrow \Delta p = F \Delta t$$

$$\therefore p = \int dp = \int_0^1 F dt$$

$$\Rightarrow p = \int_0^1 6t dt = 3 \text{ kg} \left(\frac{\text{m}}{\text{s}}\right)$$

Also, change in kinetic energy

$$\Delta K = \frac{\Delta p^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

From work-energy theorem,

work done = change in kinetic energy.

So, work done = $\Delta K = 4.5 \text{ J}$

- 58** At time $t = 0$, particle starts moving along the x -axis. If its kinetic energy increases uniformly with time t , the net force acting on it must be proportional to [AIEEE 2011]

- (a) $\sqrt{t}$ (b) constant
(c) t (d) $1/\sqrt{t}$

Ans. (d)

$$\text{Given, } k \propto t \Rightarrow \frac{dk}{dt} = \text{constant}$$

$$\Rightarrow K \propto t$$

$$\frac{1}{2} mv^2 \propto t \Rightarrow v \propto \sqrt{t}$$

$$\text{Also, } P = Fv = \frac{dK}{dt} = \text{constant}$$

$$\Rightarrow F \propto \frac{1}{v}$$

$$\Rightarrow F \propto \frac{1}{\sqrt{t}}$$

Alternate Solution

$$K \propto t \Rightarrow v \propto \sqrt{t},$$

$$F = ma = m \frac{dv}{dt}$$

$$\text{As, } a = \frac{dv}{dt} = k \frac{1}{2} t^{-1/2}$$

$$a \propto \frac{1}{\sqrt{t}}, \text{ so } F \propto \frac{1}{\sqrt{t}}$$

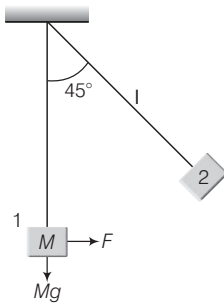
- 59** A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is [AIEEE 2006]

- (a) $Mg(\sqrt{2} + 1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2} - 1)$

Ans. (d)

Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem.

Let us assume that body be taken slowly, so that its speed does not change, then



$$\Delta K = 0 = W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

$$W_F = F \times l \sin 45^\circ = \frac{Fl}{\sqrt{2}}$$

$$W_{Mg} = Mg(l - l \cos 45^\circ), W_{\text{tension}} = 0$$

$$\therefore F = Mg(\sqrt{2} - 1)$$

- 60** A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion? [AIEEE 2005]

- (a) 3.0 cm (b) 2.0 cm
(c) 1.5 cm (d) 1.0 cm

Ans. (d)

According to work-energy theorem,
Total work done = Change in kinetic energy

$$W = \Delta K$$

Case I $-F \times 3 = \frac{1}{2} m \left(\frac{v_0}{2} \right)^2 - \frac{1}{2} m v_0^2$

where, F is resistive force and v_0 is initial speed

Case II Let the further distance travelled by the bullet before coming to rest be s .

$$\therefore -F(3 + s) = K_f - K_i = -\frac{1}{2} m v_0^2$$

$$\Rightarrow -\frac{1}{8} m v_0^2 (3 + s) = -\frac{1}{2} m v_0^2$$

$$\text{or } \frac{1}{4} (3 + s) = 1 \text{ or } \frac{3}{4} + \frac{s}{4} = 1$$

$$\text{or } s = 1 \text{ cm}$$

- 61** A body of mass m is accelerated uniformly from rest to a speed v in a time T . The instantaneous power delivered to the body as a function of time, is given by [AIEEE 2005, 04]

- (a) $\frac{mv^2}{T^2} t$ (b) $\frac{mv^2}{T^2} t^2$
(c) $\frac{1}{2} \frac{mv^2}{T^2} t$ (d) $\frac{1}{2} \frac{mv^2}{T^2} t^2$

Ans. (a)

A body of mass m with uniform acceleration, then force

$$F = ma = \frac{mv}{T} \left[\because a = \frac{v - 0}{T} \right]$$

Instantaneous power = $Fv = mav$

$$= \frac{mv}{T} \cdot at$$

$$= \frac{mv}{T} \cdot \frac{v}{T} \cdot t = \frac{mv^2}{T^2} t$$

- 62** A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that [AIEEE 2004]

- (a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line

Ans. (c)

When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

- 63** A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to [AIEEE 2003]

- (a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

Ans. (b)

Delivering power of a machine

$$P = \text{constant}$$

$$P = F \cdot v$$

$$\left[\because \text{Power} = \frac{W}{t} = F \cdot \frac{s}{t} = F \cdot v \right]$$

$$= mav = m \frac{dv}{dt} v$$

$$P = mv \frac{dv}{dt}$$

$$v dv = \frac{P}{m} dt$$

Integrating on both sides, we get

$$\int_0^v v dv = \int_0^t \frac{P}{m} dt$$

$$\frac{v^2}{2} = \frac{Pt}{m}, v = \left(\frac{2Pt}{m} \right)^{1/2}$$

$$v = \frac{ds}{dt} = \left(\frac{2Pt}{m} \right)^{1/2}$$

$$ds = \left(\frac{2Pt}{m} \right)^{1/2} dt$$

$$\int_0^s ds = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt$$

$$s = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2}$$

$$\therefore s \propto t^{3/2}$$

TOPIC 3 Collision

- 64** A body of mass M moving at speed v_0 collides elastically with a mass m at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M . The largest possible value of the ratio M/m , for which the angles θ_1 and θ_2 will be equal, is [2021, 31 Aug Shift-I]

- (a) 4 (b) 1 (c) 3 (d) 2

Ans. (c)

Given, mass of body 1 = M

Mass of body 2 = m

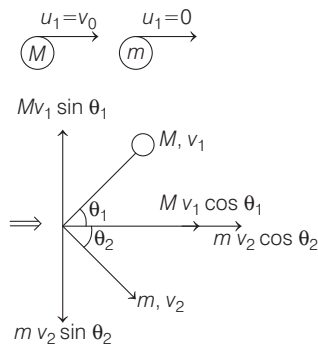
Initial speed of body 1, $u_1 = v_0$

Initial speed of body 2, $u_2 = 0$

Final speed of body 1 and 2 = v_1 and v_2

Angle made by body 1 and 2 after collision with respect to initial direction

$$= \theta_1, \theta_2$$



By using law of conservation of momentum,
Along X-axis

$$Mv_0 + m \cdot 0 = Mv_1 \cos \theta_1 + mv_2 \cos \theta_2$$

If $\theta_1 = \theta_2 = \theta$

$$\therefore Mv_0 = Mv_1 \cos \theta + mv_2 \cos \theta \quad \dots (i)$$

Along Y-axis,

$$Mv_1 \sin \theta_1 = mv_2 \sin \theta_2$$

[since, $\theta_1 = \theta_2 = \theta$]

$$v_2 = \frac{Mv_1}{m} \quad \dots (ii)$$

Substituting the value of v_2 in Eq. (i), we get

$$\begin{aligned} Mv_0 &= Mv_1 \cos \theta + m \left(\frac{Mv_1}{m} \right) \cos \theta \\ &= 2Mv_1 \cos \theta \\ v_1 &= \frac{v_0}{2 \cos \theta} \quad \dots (iii) \end{aligned}$$

By using law of conservation of energy
along X-axis

$$\begin{aligned} \frac{1}{2} Mv_0^2 &= \frac{1}{2} Mv_1^2 + \frac{1}{2} mv_2^2 \\ \Rightarrow Mv_0^2 &= Mv_1^2 + m \left(\frac{M}{m} v_1 \right)^2 \end{aligned}$$

[from Eq. (ii)]

$$\Rightarrow Mv_0^2 = Mv_1^2 + \frac{M^2 v_1^2}{m} = \frac{Mv_1^2}{m} (m + M)$$

$$\Rightarrow v_0^2 = \left(\frac{v_0}{2 \cos \theta} \right)^2 \left(\frac{m + M}{m} \right)$$

[From Eq. (iii)]

$$\Rightarrow v_0^2 = \frac{v_0^2}{4 \cos^2 \theta} \left(1 + \frac{M}{m} \right)$$

$$\Rightarrow 4 \cos^2 \theta = 1 + \frac{M}{m}$$

$$\Rightarrow \frac{M}{m} = 4 \cos^2 \theta - 1$$

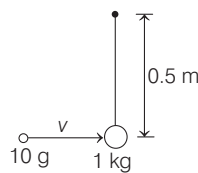
$$\left[\text{For largest possible value of } \frac{M}{m}, \theta = 0 \right]$$

$$= 4 \cos^2 0^\circ - 1 = 4 - 1 = 3$$

- 65** A bullet of 10 g, moving with velocity v , collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m

and mass of the bob is 1 kg. The minimum value of v m/s, so that the pendulum describes a circle. (Assume, the string to be inextensible and $g = 10 \text{ m/s}^2$)

[2021, 27 Aug Shift-II]



Ans. (400)

Given, mass of bullet (m_b) = 10 g
= $10 \times 10^{-3} \text{ kg}$

Initial speed of bullet is v .

Length of pendulum, $l = 0.5 \text{ m}$

Mass of bob, $m = 1 \text{ kg}$

Initial speed of bob, $u = 0$

Final speed of bullet, $v_b = 100 \text{ ms}^{-1}$

Final speed of bob for making complete circle at bottom, $v' = \sqrt{5gl}$

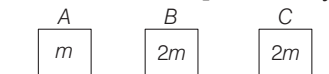
Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

By using law of conservation of momentum

$$\begin{aligned} m_b u_b + m u &= -m_b v_b + m v' \\ \Rightarrow \frac{10}{1000} v + 0 &= \frac{-10}{1000} \times 100 + 1 \sqrt{5gl} \\ \Rightarrow \frac{v}{100} &= -1 + \sqrt{5 \times 10 \times \frac{5}{10}} \\ \Rightarrow \frac{v}{100} &= -1 + 5 \\ \Rightarrow v &= 4 \times 100 = 400 \text{ ms}^{-1} \end{aligned}$$

- 66** Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are $m, 2m$ and $2m$, respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. There after B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is

[2021, 27 July Shift-I]



(a) 6 m/s (b) 9 m/s (c) 4 m/s (d) 3 m/s

Ans. (d)

If v'_A and v'_B be the velocity of body A and B after first collision, then by conservation of linear momentum

$$\begin{aligned} mv_A &= mv'_A + 2mv'_B \\ v_A &= v'_A + 2v'_B \quad \dots (i) \end{aligned}$$

$$\begin{aligned} \text{Again, } e &= \frac{v'_A - v'_B}{0 - v_A} \\ 1 &= \frac{v'_A - v'_B}{-v_A} \end{aligned}$$

[For elastic collision, $e = 1$]

$$\Rightarrow -v_A = v'_A - v'_B \quad \dots (ii)$$

Adding Eqs. (i) and (ii), we have

$$\begin{aligned} 0 &= 2v'_A + v'_B \\ \Rightarrow v'_B &= -2v'_A \quad \dots (iii) \end{aligned}$$

From Eqs. (ii) and (iii), we get

$$\begin{aligned} -v_A &= v'_A - (-2v'_A) \\ \Rightarrow -v_A &= v'_A + 2v'_A = 3v'_A \\ \Rightarrow v'_A &= -\frac{v_A}{3} = -\frac{9}{3} = -3 \text{ m/s} \end{aligned}$$

$\therefore$ From Eq. (iii), $v'_B = -2(-3) = 6 \text{ m/s}$

Again, after 2nd collision,

$$\begin{aligned} 2mv'_B &= (2m + 2m)v_c = 4mv_c \\ \Rightarrow v_c &= \frac{v'_B}{2} = \frac{6}{2} = 3 \text{ m/s} \end{aligned}$$

- 67** A ball of mass 4 kg, moving with a velocity of 10 ms^{-1} , collides with a spring of length 8 m and force constant 100 Nm^{-1} . The length of the compressed spring is x m. The value of x to the nearest integer, is
- [2021, 18 March Shift-II]

Ans. (6)

Given, mass of the ball, $m = 4 \text{ kg}$

Velocity of the ball, $v = 10 \text{ m/s}$

Force constant, $k = 100 \text{ N/m}$

The length of the spring, $x = 8 \text{ m}$

Let x is the compressed length of the spring.

Using the work-energy theorem,

"It states that kinetic energy of the ball is converted into the stored energy of spring".

$$\frac{mv^2}{2} = \frac{kx^2}{2} \Rightarrow \frac{4(10)^2}{2} = \frac{(100)x^2}{2}$$

$$\Rightarrow x = 2 \text{ m}$$

The final length (compressed) of the spring

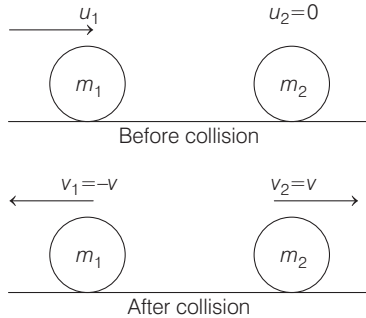
$$= 8 - 2 = 6 \text{ m}$$

Hence, the value of x to the nearest integer is 6.

- 68** An object of mass m_1 collides with another object of mass m_2 , which is at rest. After the collision, the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is
- [2021, 18 March Shift-II]
- (a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 1

Ans. (a)

The mass m_1 is moving with speed u_1 initially and mass m_2 is at rest. After the collision, the mass m_1 and m_2 move with speed v in opposite directions.



Using the law of conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 u_1 + m_2 (0) = m_1 (-v) + m_2 v$$

$$\Rightarrow m_1 u_1 = (-m_1 + m_2) v \quad \dots(i)$$

Since, the collision is elastic because they move with same speed after the collision. Hence, coefficient of restitution, $e = 1$

$$\therefore e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow 1 = \frac{v - (-v)}{u_1 - 0}$$

$$u_1 = 2v \quad \dots(ii)$$

Putting the above value in Eq. (i), we get

$$m_1 (2v) = (-m_1 + m_2) v$$

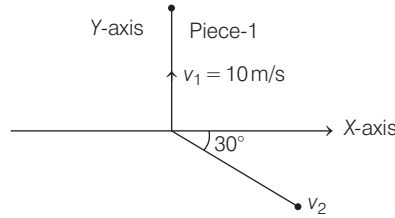
$$\Rightarrow 3m_1 = m_2 \Rightarrow \frac{m_2}{m_1} = \frac{3}{1}$$

$$\Rightarrow m_2 : m_1 = 3 : 1$$

- 69** A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the X-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along Y-axis with a speed of 10 m/s.

The second piece starts moving at an angle of 30° with respect to the X-axis. The velocity of the ball moving at 30° with X-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is

[2021, 18 March Shift-I]



Ans. (20)

Given,

The mass of the first ball, $m_1 = 10$ kg

The mass of the second ball, $m_2 = 20$ kg

The initial velocity of the first ball,

$$u_1 = 10\sqrt{3} \text{ m/s}$$

The initial velocity of the second ball,

$$u_2 = 0 \text{ m/s}$$

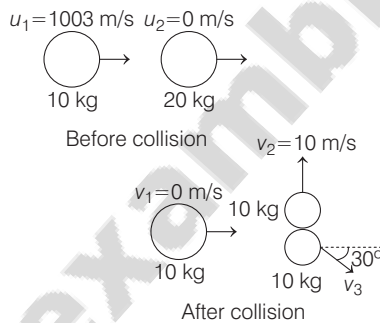
The final velocity of the first ball,

$$v_1 = 0 \text{ m/s}$$

The final velocity of the first piece of the second ball, $v_2 = 10$ m/s

Let's consider the final velocity of the second piece of the second ball $= v_3$

As shown in the figure,



The net external force on the system is to be zero. Hence, we can use the law of conservation of linear momentum in both directions.

In x-direction the linear momentum conserved,

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x}$$

$$\Rightarrow 10(10\sqrt{3}) + 0 = 10(0) + 0 + 10v_3 \cos 30^\circ$$

$$\Rightarrow v_3 = 20 \text{ m/s}$$

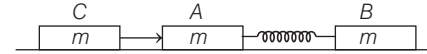
Hence, the velocity of the ball moving at 30° with respect to the X-axis is 20 m/s.

So, the value of x to the nearest integer is 20.

- 70** Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural length L and spring constant k . A third block C of mass m moving with a speed v along the line joining A and B

B collides with A. The maximum compression in the spring is

[2021, 17 March Shift-II]



(a) $v\sqrt{\frac{m}{2k}}$ (b) $\sqrt{\frac{mv}{2k}}$ (c) $\sqrt{\frac{mv}{k}}$ (d) $\sqrt{\frac{m}{2k}}$

Ans. (a)

Let v is the speed of the third block C.

The velocity of centre of mass of A and B is

$$v_{CM} = \frac{v}{2}$$

The spring is compressed maximum by x distance.

Using the law of conservation of energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{4}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{mv^2}{2k}}$$

$$\Rightarrow x = v\sqrt{\frac{m}{2k}}$$

- 71** A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through

which it falls. Find the average speed of the ball.

(Take, $g = 10 \text{ ms}^{-2}$)

[2021, 17 March Shift-II]

(a) 3.0 ms^{-1} (b) 3.5 ms^{-1}
(c) 2.0 ms^{-1} (d) 2.5 ms^{-1}

Ans. (d)

Velocity of rubber ball when it strikes to the ground, $v_0 = \sqrt{2gh_0}$

Using the formula of coefficient of restitution,

$$e = \frac{\text{velocity after collision}}{\text{velocity before collision}}$$

$$\Rightarrow e = \frac{v_1 - 0}{v_0 - 0} \Rightarrow e = \frac{v_1}{\sqrt{2gh_0}}$$

$$\Rightarrow v_1 = e\sqrt{2gh_0} \quad \dots(i)$$

As, initial height of the ball $= h_0$.

$$\therefore \text{The first height of the rebound, } h_1 = \frac{v_1^2}{2g}$$

$$\Rightarrow h_1 = e^2 h_0 \quad [\text{Using Eq. (i)}]$$

The n th height of the ball to the rebound,

$$h_n = \frac{v_n^2}{2g} \Rightarrow h_n = e^{2n} h_0$$

The velocity of the ball after n th rebound, $v_n = e^n v_0$

Now, the total distance travelled by the ball after n th rebound is

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 \dots$$

$$H = h_0 + 2e^2 h_0 + 2e^4 h_0 + 2e^6 h_0 \dots$$

$$H = h_0 [1 + 2e^2 (1 + e^2 + e^4 + e^6 \dots)]$$

Using the formula,

$$1 + e^2 + e^4 + \dots = \frac{1}{1 - e^2}$$

$$\Rightarrow H = h_0 \left[1 + 2e^2 \left(\frac{1}{1 - e^2} \right) \right]$$

$$\Rightarrow H = h_0 \left(\frac{1 + e^2}{1 - e^2} \right)$$

$$\Rightarrow H = 5 \left(\frac{1 + (0.81)}{1 - (0.81)} \right) \left(\because e^2 = \frac{h_1}{h_0} = \frac{81}{100} \right)$$

$$H = 47.6 \text{ m}$$

Now, the total time taken by the ball to come to rest

$$T = t_0 + 2t_1 + 2t_2 + \dots$$

$$T = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \dots$$

$$T = \sqrt{\frac{2h_0}{g}} [1 + 2e + 2e^2 + \dots]$$

$$T = \sqrt{\frac{2h_0}{g}} [1 + 2e(1 + e + e^2 + \dots)]$$

$$T = \sqrt{\frac{2h_0}{g}} \left(\frac{1 + e}{1 - e} \right)$$

$$\Rightarrow T = \sqrt{\frac{2(5)}{10}} \left(\frac{1 + \sqrt{0.81}}{1 - \sqrt{0.81}} \right)$$

$$T = 19 \text{ s}$$

The average velocity,

$$v_{\text{avg}} = \frac{H}{T} = \frac{47.6}{19}$$

$$= 2.5 \text{ m/s}$$

Hence, the average velocity is 2.5 m/s.

- 72** A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1} . The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now?

[2021, 17 March Shift-I]

- (a) 19.0 ms^{-1} (b) 4.47 ms^{-1}
(c) 14.41 ms^{-1} (d) 1.00 ms^{-1}

Ans. (b)

Given, mass of rolling ball, $m = 0.5 \text{ kg}$

Speed of ball, $u = 20 \text{ ms}^{-1}$

Before deflection,

Initial kinetic energy, $KE_i = \frac{1}{2} mu^2$

$$= \frac{1}{2} \times 0.5 \times (20)^2$$

$$= \frac{1}{2} \times 0.5 \times 400 = \frac{1}{2} \times \frac{1}{2} \times 400 = 100 \text{ J}$$

It is given in the question that after deflection the ball moves with 5% of its initial kinetic energy

$$KE_f = 5\% \text{ of } KE_i \Rightarrow KE_f = \frac{5}{100} \times 100 = 5 \text{ J}$$

If the final speed of the ball is $v \text{ ms}^{-1}$, then

$$KE_f = \frac{1}{2} mv^2$$

$$\Rightarrow 5 = \frac{1}{2} \times 0.5 \times v^2 \Rightarrow 10 = 0.5 \times v^2$$

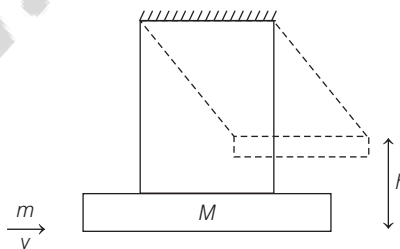
$$\Rightarrow v^2 = \frac{10}{0.5} = \frac{100}{5} \Rightarrow v^2 = 20$$

$$\Rightarrow v = \sqrt{20} = 4.47 \text{ ms}^{-1}$$

$$\Rightarrow v = 4.47 \text{ ms}^{-1}$$

- 73** A large block of wood of mass $M = 5.99 \text{ kg}$ is hanging from two long massless cords. A bullet of mass $m = 10 \text{ g}$ is fired into the block and gets embedded in it. The system (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8 \text{ cm}$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is (Take $g = 9.8 \text{ ms}^{-2}$)

[2021, 16 March Shift-II]



- (a) 841.4 m/s (b) 811.4 m/s
(c) 831.4 m/s (d) 821.4 m/s

Ans. (c)

Given,

Mass of large block of wood, $M = 5.99 \text{ kg}$

Mass of bullet, $m = 10 \text{ g}$

Height at which their centre of mass rise, $h = 9.8 \text{ cm}$

From the law of conservation of energy, Energy of the system when bullet gets

embedded = Energy of the system till it momentarily comes to rest.

$$\Rightarrow \frac{1}{2} (M + m) v_1^2 = (M + m) gh$$

where, v_1 = velocity of bullet + block system

$$\Rightarrow v_1 = \sqrt{2gh} \quad \dots (i)$$

According to law of conservation of momentum,

Momentum before collision = Momentum after collision.

$$\Rightarrow mv = (M + m) v_1$$

[where, v = velocity of bullet before collision]

$$\Rightarrow mv = (M + m) \sqrt{2gh} \quad [\text{using Eq. (i)}]$$

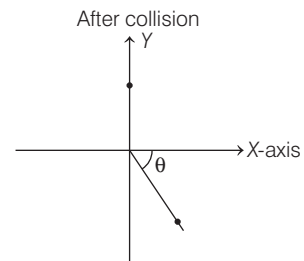
$$\Rightarrow v = \left(\frac{M + m}{m} \right) \sqrt{2gh}$$

$$\Rightarrow v = \frac{(5.99 + 0.01)}{10 \times 10^{-3}} \times \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}}$$

$$\Rightarrow v = 831.55 \text{ ms}^{-1}$$

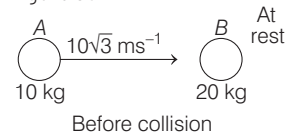
- 74** A ball of mass 10 kg moving with a velocity $10\sqrt{3} \text{ ms}^{-1}$ along X-axis, hits another ball of mass 20 kg , which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s . The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis. The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is

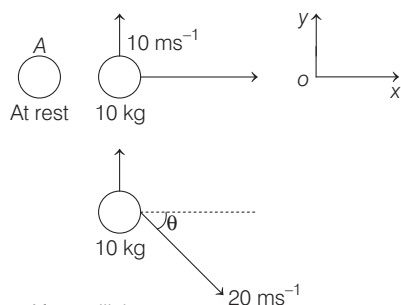
[2021, 16 March Shift-I]



Ans. (30)

We can represent the given situation in figure as





After collision
It means linear momentum is conserved along X-axis.

According to the law of conservation of linear momentum,

$$p_i = p_f$$

$$m_A u_A + m_B u_B = m_A v_A + (m_B v_B \cos \theta)$$

$$= m_A v_A + m_{B_1} v_{B_1} \cos 90^\circ + m_{B_2} v_{B_2} \cos \theta$$

$$\Rightarrow 10 \times 10\sqrt{3} + 20 \times 0 = 10 \times 0 + 10 \times 10 \times 0 + 10 \times 20 \cos \theta$$

$$\Rightarrow 10 \times 10\sqrt{3} = 200 \cos \theta$$

where, $\cos \theta$ being the horizontal component i.e., along X-axis

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 30^\circ \quad [\because \cos 30^\circ = \sqrt{3}/2]$$

- 75** Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m \ll M$, body Q will have a maximum speed equal to 2u after collision.

Reason (R) During elastic collision, the momentum and kinetic energy are both conserved. In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 26 Feb Shift-I]

- (a) A is not correct but R is correct.
(b) Both A and R are correct but R is not the correct explanation of A.
(c) Both A and R are correct and R is the correct explanation of A.
(d) A is correct but R is not correct.

Ans. (c)

Let v_1 and v_2 are the speed of P and Q after collision.

By using law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow Mu + m \cdot 0 = Mv_1 + mv_2$$

$$\Rightarrow \frac{M(u - v_1)}{m} = v_2 \quad \dots(i)$$

and by using law of conservation of energy,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow Mu^2 + 0 = Mv_1^2 + mv_2^2$$

$$\Rightarrow M(u^2 - v_1^2) = mv_2^2$$

$$\Rightarrow M(u - v_1)(u + v_1) = m v_2^2 \quad \dots(ii)$$

Substituting the value of $M \frac{(u - v_1)}{m}$ from

Eq. (i) in Eq. (ii), we get

$$v_2(u + v_1) = v_2^2$$

$$\Rightarrow u + v_1 = v_2$$

$$\therefore M \gg m$$

$$\therefore v_1 = u$$

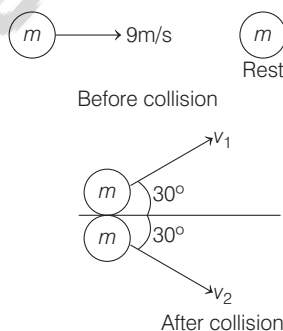
$$\text{and } v_2 = 2u$$

Hence, option (c) is the correct.

- 76** A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x:y, where x is
[2021, 24 Feb Shift-I]

Ans. (1)

The situation is shown below



Using conservation of linear momentum in y-direction,

$$p_i = p_f$$

As, $p_i = 0$

$$\text{and } p_f = m v_1 \sin 30^\circ - m v_2 \sin 30^\circ$$

$$\Rightarrow 0 = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$$\Rightarrow v_1 = v_2 \text{ or } v_1 : v_2 = 1 : 1$$

Since, $v_1 : v_2 = x : y$ (given)

$$\therefore x = 1$$

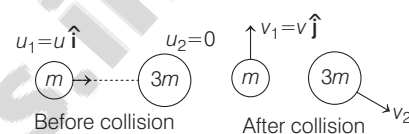
- 77** A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3m at rest. It moves with a velocity $v\hat{j}$ after collision, then v is given by

[2020, 2 Sep Shift-I]

- (a) $v = \sqrt{\frac{2}{3}}u$ (b) $v = \frac{1}{\sqrt{6}}u$
(c) $v = \frac{u}{\sqrt{2}}$ (d) $v = \frac{u}{\sqrt{3}}$

Ans. (c)

As collision is elastic as shown below, both momentum and KE are conserve(d)



Momentum conservation gives,

$$mu\hat{i} = mv\hat{j} + 3mv_2$$

$$\Rightarrow v_2 = \frac{1}{3}(u\hat{i} - v\hat{j})$$

$$\Rightarrow |v_2| = \sqrt{\frac{u^2 + v^2}{9}}$$

$$\text{or } v_2^2 = (u^2 + v^2)/9 \quad \dots(i)$$

Kinetic energy conservation gives,

$$\Rightarrow \frac{1}{2} mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} 3mv_2^2$$

$$\Rightarrow u^2 = v^2 + 3v_2^2 \quad \dots(ii)$$

Substituting value of v_2 from Eq (i) into Eq (ii), we get

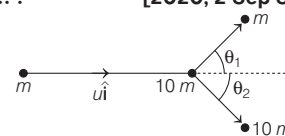
$$u^2 = v^2 + 3 \left(\frac{u^2 + v^2}{9} \right)$$

$$\Rightarrow \frac{2}{3} u^2 = \frac{4}{3} v^2$$

$$\Rightarrow v = \frac{u}{\sqrt{2}}$$

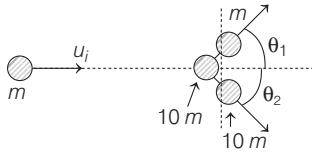
Hence, correct option is (c).

- 78** A particle of mass m is moving along the X-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10 m at rest and then moves with half its initial kinetic energy (see figure). If $\sin \theta_1 = \sqrt{n} \sin \theta_2$, then value of n is
[2020, 2 Sep Shift-II]



Ans. (10)

Given, impact is shown below,



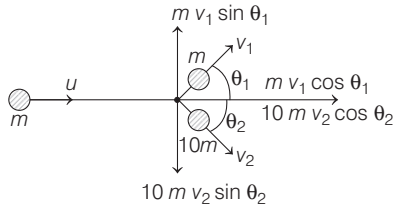
For particle 1, final KE is equals to half of its initial value,

$$\Rightarrow K_f = \frac{1}{2} K_i$$

$$\Rightarrow \frac{1}{2} m_1 v_1^2 = \left(\frac{1}{2} m_1 u^2 \right) \times \frac{1}{2}$$

$$\Rightarrow \text{Final velocity of } m_1 \text{ will be, } v_1 = \frac{u}{\sqrt{2}}$$

Momentum in y-direction is conserved,



$$\text{i.e. } 10m v_2 \sin \theta_2 = m v_1 \sin \theta_1$$

$$\text{Here, } v_1 = \frac{u}{\sqrt{2}} \text{ and } \sin \theta_1 = \sqrt{n} \sin \theta_2$$

So, we have

$$10m v_2 \cdot \sin \theta_2 = \frac{mu}{\sqrt{2}} \cdot \sqrt{n} \sin \theta_2$$

$$\Rightarrow v_2 = \frac{u\sqrt{n}}{10\sqrt{2}} \quad \dots(i)$$

Also collision is elastic, so KE is conserve(d)

$$\Rightarrow \frac{1}{2} m u^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} (10m) v_2^2$$

Substituting values of v_1 and v_2 , we have

$$u^2 = \left(\frac{u}{\sqrt{2}} \right)^2 + 10 \left(\frac{u^2 n}{100 \times 2} \right)$$

$$\Rightarrow \frac{u^2}{2} = \frac{u^2 n}{10 \times 2} \Rightarrow n = 10$$

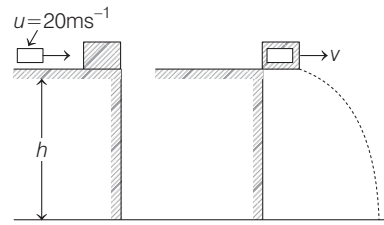
- 79** A block of mass 1.9 kg is at rest at the edge of a table of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the collision, then the kinetic energy just before the combined system strikes the floor, is (Take, $g = 10 \text{ m/s}^2$ and assume there is no rotational motion and

loss of energy after the collision is negligible) [2020, 3 Sep Shift-II]

- (a) 20 J
(b) 19 J
(c) 21 J
(d) 23 J

Ans. (c)

When the bullet undergoes an inelastic collision with block, a part of KE of bullet is lost.

When bullet + block system falls from height h , then its total energy (kinetic + potential) becomes kinetic energy, so kinetic energy of bullet + block system at bottom just before collision is equal to total energy just after collision.

Now, by law of conservation of momentum, we have

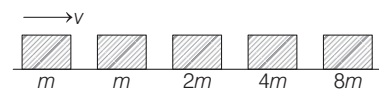
$$\begin{aligned} mu &= (m + M)v \\ \Rightarrow v &= \frac{mu}{m + M} = \frac{0.1 \times 20}{(0.1 + 1.9)} = 1 \text{ ms}^{-1} \end{aligned}$$

Total energy of bullet and block just after collision

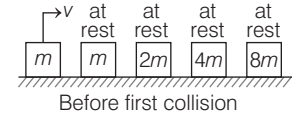
$$\begin{aligned} &= KE + PE = \frac{1}{2} (m + M) v^2 + (m + M) gh \\ &= \frac{1}{2} \times 2 \times 1^2 + 2 \times 10 \times 1 \\ &= 1 + 20 = 21 \text{ J} \end{aligned}$$

Hence, correct option is (c).

- 80** Blocks of masses $m, 2m, 4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m , moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. (c) By the time, the last block of mass $8m$ starts moving, the total energy loss is $p\%$ of the original energy. Value of p is close to [2020, 4 Sep Shift-I]

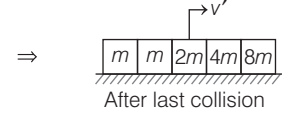


- (a) 77 (b) 87
(c) 94 (d) 37

Ans. (c)

Before first collision

(Assume)



After last collision

Since, all the collisions are perfectly inelastic, so after the final collision, all blocks will be moving together. Let their final velocity be v' .

By law of conservation of linear momentum,

$$\begin{aligned} (p_{\text{sys}})_i &= (p_{\text{sys}})_f \\ \Rightarrow mv + m(0) + 2m(0) + 4m(0) + 8m(0) &= (m + m + 2m + 4m + 8m)v' \\ \Rightarrow mv &= 16mv' \\ \Rightarrow v' &= \frac{v}{16} \quad \dots(i) \end{aligned}$$

Now, initial kinetic energy of system,

$$(K_{\text{sys}})_i = \frac{1}{2} mv^2 + 0 + 0 + 0 + 0 = \frac{1}{2} mv^2$$

And final kinetic energy of system,

$$\begin{aligned} (K_{\text{sys}})_f &= \frac{1}{2} (m + m + 2m + 4m + 8m) (v')^2 \\ &= \frac{1}{2} \times 16m \times \left(\frac{v}{16} \right)^2 \\ &= \frac{1}{2} \times 16m \times \frac{v^2}{256} \\ &= \frac{1}{32} mv^2 \end{aligned}$$

Loss in kinetic energy,

$$\begin{aligned} (\Delta K_{\text{sys}})_{\text{loss}} &= (K_{\text{sys}})_i - (K_{\text{sys}})_f \\ &= \frac{1}{2} mv^2 - \frac{1}{32} mv^2 = \frac{15}{32} mv^2 \end{aligned}$$

% loss in kinetic energy,

$$\begin{aligned} \%(\Delta K_{\text{sys}})_{\text{loss}} &= \frac{(\Delta K_{\text{sys}})_{\text{loss}}}{(K_{\text{sys}})_i} \times 100\% \\ &= \frac{\frac{15}{32} mv^2}{\frac{1}{2} mv^2} \times 100\% = \frac{15}{16} \times 100\% \\ &= 93.75\% \end{aligned}$$

Given that,

$$\%(\Delta K_{\text{sys}})_{\text{loss}} = p\%$$

so, $p = 93.75 \approx 94$

Hence, correct option is (c).

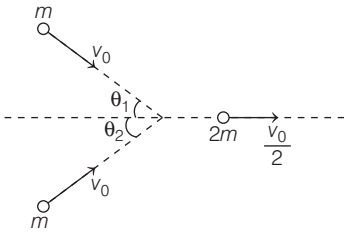
- 81** Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic

collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is

..... [2020, 6 Sep Shift-I]

Ans. (120)

Let initial velocities of two bodies are making angle θ_1 and θ_2 with horizontal direction as shown in figure.



$$\begin{aligned} \text{Initial momentum, } \mathbf{p}_i &= \mathbf{p}_1 + \mathbf{p}_2 \\ &= \{mv_0 \cos \theta_1 \hat{i} + mv_0 \sin \theta_1 \hat{j}\} \\ &\quad + \{mv_0 \cos \theta_2 \hat{i} + mv_0 \sin \theta_2 \hat{j}\} \\ &= mv_0 (\cos \theta_1 + \cos \theta_2) \hat{i} \\ &\quad + mv_0 (\sin \theta_1 + \sin \theta_2) \hat{j} \end{aligned}$$

$$\text{Final momentum, } \mathbf{p}_f = (2m) \left(\frac{v_0}{2} \right) \hat{i}$$

$$\Rightarrow \mathbf{p}_f = mv_0 \hat{i}$$

In collision momentum remains conserved, so, applying momentum conservation,

$$\begin{aligned} \mathbf{p}_f &= \mathbf{p}_i \\ \text{or } mv_0 \hat{i} &= mv_0 (\cos \theta_1 + \cos \theta_2) \hat{i} \\ &\quad + mv_0 (\sin \theta_1 + \sin \theta_2) \hat{j} \end{aligned}$$

$$\Rightarrow \sin \theta_1 + \sin \theta_2 = 0$$

$$\Rightarrow \theta_1 = \theta_2$$

$$\text{and } mv_0 = 2mv_0 \cos \theta$$

$$\text{or } \cos \theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

But angle between initial velocities is $\theta_1 + \theta_2$ which is equal to $60^\circ + 60^\circ = 120^\circ$.

- 82** Particle A of mass m_1 moving with velocity $(\sqrt{3}\hat{i} + \hat{j})\text{ms}^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of particles A and B after collision, respectively. If $m_1 = 2m_2$ and after collision $\mathbf{v}_1 = (\hat{i} + \sqrt{3}\hat{j})\text{ms}^{-1}$, then the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is

[2020, 6 Sep Shift-II]

- (a) 15° (b) 60°
(c) -45° (d) 105°

Ans. (d)

Given that, $\mathbf{u}_1 = (\sqrt{3}\hat{i} + \hat{j})\text{m/s}$, $\mathbf{u}_2 = 0$

$\mathbf{v}_1 = (\hat{i} + \sqrt{3}\hat{j})\text{m/s}$ and $m_1 = 2m_2$

Using conservation of linear momentum,

$$\begin{aligned} \mathbf{p}_i &= \mathbf{p}_f \\ m_1 \mathbf{u}_1 + m_2 \mathbf{u}_2 &= m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 \\ 2m_2(\sqrt{3}\hat{i} + \hat{j}) &+ m_2(0) \end{aligned}$$

$$= 2m_2(\hat{i} + \sqrt{3}\hat{j}) + m_2 \mathbf{v}_2$$

$$\mathbf{v}_2 = 2(\sqrt{3}\hat{i} + \hat{j}) - 2(\hat{i} + \sqrt{3}\hat{j})$$

$$= 2(\sqrt{3} - 1)\hat{i} - 2(\sqrt{3} - 1)\hat{j} \text{ m/s}$$

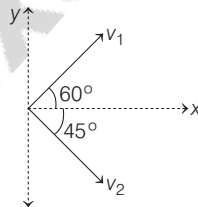
Let the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ be θ , then

$$\begin{aligned} \cos \theta &= \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{v_1 v_2} = \frac{(\hat{i} + \sqrt{3}\hat{j}) \cdot 2(\sqrt{3} - 1)(\hat{i} - \hat{j})}{2 \cdot 2\sqrt{2}(\sqrt{3} - 1)} \\ [\because v_1 = 2\text{ m/s}, v_2 = 2\sqrt{2}(\sqrt{3} - 1)\text{ m/s}] \\ &= \frac{2(\sqrt{3} - 1) - 2\sqrt{2}(\sqrt{3} - 1)}{4\sqrt{2}(\sqrt{3} - 1)} \\ &= \frac{2(\sqrt{3} - 1)(1 - \sqrt{2})}{4\sqrt{2}(\sqrt{3} - 1)} \\ \cos \theta &= \frac{1 - \sqrt{2}}{2\sqrt{2}} = -0.259 \Rightarrow \theta = 105^\circ \end{aligned}$$

Alternate solution

Directly observing the direction of $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\theta_1 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 60^\circ$$



$$\Rightarrow \theta_2 = \tan^{-1}(-1) = 45^\circ$$

$$\therefore \theta = \theta_1 + \theta_2 = 60^\circ + 45^\circ = 105^\circ$$

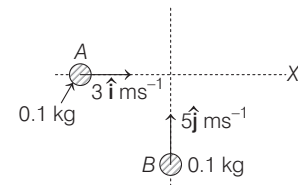
Hence, correct option is (d).

- 83** A body A, of mass $m = 0.1\text{kg}$ has an initial velocity of $3\hat{i}\text{ms}^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\text{ms}^{-1}$. After collision, A moves with a velocity $\mathbf{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}\text{J}$. The value of x is

..... [2020, 8 Jan Shift-I]

Ans. (1)

Given situation is as shown in the figure.



Total initial momentum,

$$\begin{aligned} \mathbf{p}_i &= m_A \mathbf{v}_A + m_B \mathbf{v}_B \\ &= 0.1 \times 3\hat{i} + 0.1 \times 5\hat{j} \\ &= 0.3\hat{i} + 0.5\hat{j} \text{ (kg-m/s)} \end{aligned}$$

Final velocity of A, $\mathbf{v}_A = 4(\hat{i} + \hat{j})$

Let final velocity of B is $\mathbf{v}_B$.

Then, final momentum after collision,

$$\begin{aligned} \mathbf{p}_f &= m_A \mathbf{v}_A + m_B \mathbf{v}_B \\ &= 0.1 \times 4(\hat{i} + \hat{j}) + 0.1 \times \mathbf{v}_B \end{aligned}$$

Now, by conservation of momentum, we have

$$\begin{aligned} \mathbf{p}_i &= \mathbf{p}_f \\ \Rightarrow 0.3\hat{i} + 0.5\hat{j} &= 0.4\hat{i} + 0.4\hat{j} + 0.1\mathbf{v}_B \\ \Rightarrow \mathbf{v}_B &= -\hat{i} + \hat{j} \end{aligned}$$

Kinetic energy of B after collision will be

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times |(-\hat{i} + \hat{j})|^2 \\ &= \frac{1}{2} \times 0.1 \times 2 = 0.1\text{J} \end{aligned}$$

It is given, energy of B after collision is $\frac{x}{10}$.

$$\text{So, } \frac{x}{10} = 0.1 \text{ or } x = 1$$

- 84** A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, then the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is

[2020, 8 Jan Shift-II]

- (a) $\sqrt{\frac{1}{2}}$ (b) $\frac{1}{2}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{3}{4}}$

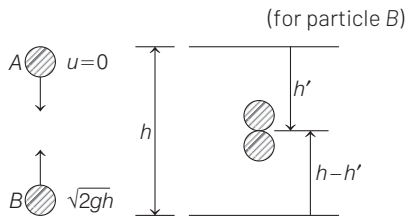
Ans. (c)

Let particles collide at some distance h' from top at time t_0 . Then,

$$h' = \frac{1}{2}gt_0^2 \quad \dots (i)$$

(for particle A)

$$\text{and } h-h' = \sqrt{2gh} \cdot t_0 - \frac{1}{2}gt_0^2 \quad \dots (ii)$$



From these equations, particles meet after time t_0 given by

$$t_0 = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

Velocities of particles A and B at instant of collision are $v_A = gt_0$

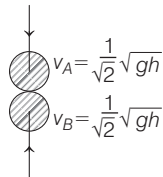
$$\text{and } v_B = \sqrt{2gh} - gt_0.$$

Hence,

$$v_A = g \times \sqrt{\frac{h}{2g}} = \sqrt{\frac{1}{2}gh} = \frac{1}{\sqrt{2}}\sqrt{gh}$$

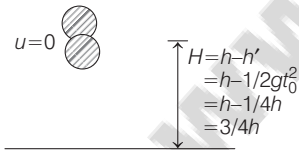
$$\text{and } v_B = \sqrt{2gh} - g\sqrt{\frac{h}{2g}} = \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\sqrt{gh} = \frac{1}{\sqrt{2}}\sqrt{gh}$$

So, particles collide as shown in the figure.



From momentum conservation, we can see that particles stick, $p_{\text{initial}} = p_{\text{final}}$. This means the combined system of particles comes to rest ($v_{\text{combined mass}} = 0$) instantaneously.

Now, we have to calculate time of fall of combined mass.



Combined mass starts with $u=0$ and its height above earth's surface is $H = \frac{3}{4}h$.

So, time taken by combined mass to reach ground is given by

$$H = ut + \frac{1}{2}at^2$$

$$\Rightarrow \frac{3}{4}h = \frac{1}{2}g \times t^2$$

$$\Rightarrow \sqrt{\frac{3}{2}}\sqrt{\frac{h}{g}} = t$$

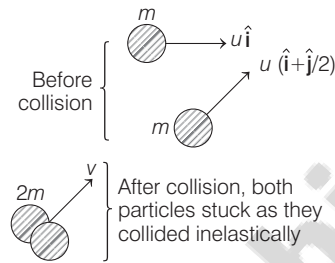
85 Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is

[2020, 9 Jan Shift-I]

- (a) $\frac{3}{4}mu^2$ (b) $\frac{\sqrt{2}}{3}mu^2$
(c) $\frac{1}{3}mu^2$ (d) $\frac{1}{8}mu^2$

Ans. (d)

Collision between particles are as shown in the figure.



From momentum conservation, we have

$$m(u\hat{i}) + m\left(u\frac{\hat{i}+\hat{j}}{2}\right) = 2m\mathbf{v}$$

$$\Rightarrow \mathbf{v} = \frac{u}{2}\hat{i} + u\left(\frac{\hat{i}+\hat{j}}{4}\right) = \frac{3}{4}u\hat{i} + \frac{1}{4}u\hat{j}$$

Initial kinetic energy of particles,

$$K_1 = \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}mu^2 + \frac{1}{4}mu^2 = \frac{3}{4}mu^2$$

Final kinetic energy of combined particles,

$$K_2 = \frac{1}{2}(2m)v^2 = \frac{1}{2} \times 2m \times \left(\sqrt{\left(\frac{3}{4}u\right)^2 + \left(\frac{1}{4}u\right)^2}\right)^2 = \frac{5}{8}mu^2$$

Change in kinetic energy or energy lost

$$= K_1 - K_2 = \frac{3}{4}mu^2 - \frac{5}{8}mu^2 = \frac{1}{8}mu^2$$

86 A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (X-axis). When it has reached its

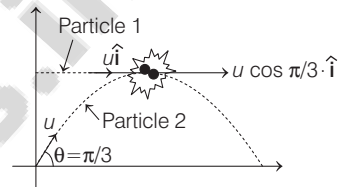
maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is

[2020, 9 Jan Shift-II]

- (a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$
(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

Ans. (a)

Collision is as shown in the figure.



Velocity of the particle projected from origin at its topmost point,

$$u_2 = u \cos \frac{\pi}{3} \cdot \hat{i} = \frac{u}{2} \hat{i}$$

By conservation of momentum (velocity of combined mass after collision (v), we have

$$mu\hat{i} + m\frac{u}{2}\hat{i} = 2m\mathbf{v}$$

$$\Rightarrow \mathbf{v} = \frac{3}{4}u\hat{i}$$

Time of fall of combined mass from h_{max}

$$t = \frac{u \sin \theta}{g} = \frac{u \sin \frac{\pi}{3}}{g} = \frac{\sqrt{3}}{2} \frac{u}{g}$$

During this time, combined particle keeps on moving with a horizontal speed of $|\mathbf{v}| = \frac{3}{4}u$.

So, horizontal distance covered by combined mass before reaching the ground,

$$R = \text{speed} \times \text{time} = \frac{3}{4}u \times \frac{\sqrt{3}}{2} \frac{u}{g} = \frac{3\sqrt{3}}{8} \frac{u^2}{g}$$

87 A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively.

If $m_2 = 0.5m_1$ and $v_3 = 0.5v_1$, then v_4 is

[2019, 8 April Shift-II]

- (a) $v_4 + v_2$ (b) $v_4 - \frac{v_2}{4}$
 (c) $v_4 - \frac{v_2}{2}$ (d) $v_4 - v_2$

Ans. (d)

Key Idea Total linear momentum is conserved in all collisions, i.e. the initial momentum of the system is equal to final momentum of the system.

Given,

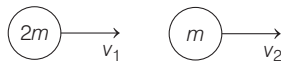
$$m_2 = 0.5m_1 \Rightarrow m_1 = 2m_2$$

Let $m_2 = m$, then, $m_1 = 2m$

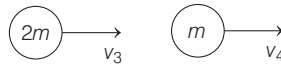
Also, $v_3 = 0.5v_1$

Given situation of collision is as shown below

Before collision,



After collision,



$\therefore$ According to the conservation of linear momentum, Initial momentum = Final momentum

$$m_1 v_1 \hat{i} + m_2 v_2 \hat{i} = m_1 v_3 \hat{i} + m_2 v_4 \hat{i}$$

$$\Rightarrow 2mv_1 \hat{i} + mv_2 \hat{i} = 2m(0.5v_1) \hat{i} + mv_4 \hat{i}$$

$$\Rightarrow v_4 = v_1 + v_2 \Rightarrow v_1 = v_4 - v_2$$

- 88** A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body? **[2019, 9 April Shift-I]**

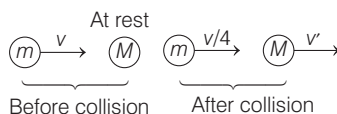
- (a) 1.5 kg (b) 1.2 kg
 (c) 1.8 kg (d) 1.0 kg

Ans. (b)

Key Idea For an elastic collision, coefficient of restitution (e), i.e. the ratio of relative velocity of separation after collision to the relative velocity of approach before collision is 1.

Given, mass of small body, $m = 2$ kg

Given situation is as shown



Using momentum conservation law for the given system,

$$(\text{Total momentum})_{\text{before collision}} =$$

$$(\text{Total momentum})_{\text{after collision}}$$

$$\Rightarrow m(v) + M(0) = m\left(\frac{v}{4}\right) + M(v') \quad \dots(i)$$

$\therefore e = 1$ and we know that,

$$e = -\frac{v_2 - v_1}{u_2 - u_1}$$

$$\Rightarrow 1 = -\frac{v' - v/4}{0 - v} \Rightarrow v = v' - v/4$$

$$\text{or } v' = 5v/4 \quad \dots(ii)$$

Using value from Eq. (ii) into Eq. (i), we get

$$mv = \frac{mv}{4} + M\left(\frac{5v}{4}\right)$$

$$\Rightarrow m\left(v - \frac{v}{4}\right) = M\left(\frac{5v}{4}\right)$$

$$\Rightarrow \frac{3}{4}mv = \frac{5}{4}Mv$$

$$M = \frac{3}{5}m = \frac{3}{5} \times 2 = 1.2 \text{ kg}$$

- 89** A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m , which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be **[2019, 9 April Shift-II]**

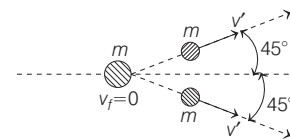
- (a) $\sqrt{2}v$ (b) $\frac{v}{\sqrt{2}}$
 (c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}v$

Ans. (d)

According to the questions, Initial condition,



Final condition,



As we know that, in collision, linear momentum is conserved in both x and y directions separately.

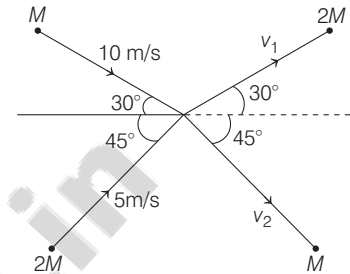
So, $(p_x)_{\text{initial}} = (p_x)_{\text{final}}$

$$m(2v) + 2m(v) = 0 + mv' \cos 45^\circ + mv' \cos 45^\circ$$

$$\Rightarrow 4mv = \frac{2m}{\sqrt{2}}v'$$

$$\Rightarrow v' = 2\sqrt{2}v$$

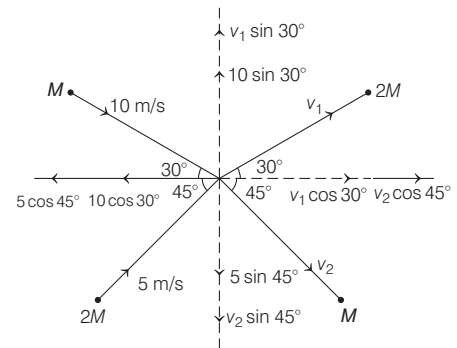
- 90** Two particles of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **[2019, 10 April Shift-I]**



- (a) 6.5 m/s and 3.2 m/s
 (b) 3.2 m/s and 6.3 m/s
 (c) 3.2 m/s and 12.6 m/s
 (d) 6.5 m/s and 6.3 m/s

Ans. (d)

The given condition can be drawn as shown below



Applying linear momentum conservation law in x-direction, we get

Initial momentum = Final momentum

$$(M \times 10 \cos 30^\circ) + (2M \times 5 \cos 45^\circ)$$

$$= (M \times v_2 \cos 45^\circ) + (2M \times v_1 \cos 30^\circ)$$

$$\Rightarrow \left(M \times 10 \times \frac{\sqrt{3}}{2}\right) + \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$

$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) + \left(2M \times v_1 \times \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow 5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \quad \dots(i)$$

Similarly, applying linear momentum conservation law in y-direction, we get

$$(M \times 10 \sin 30^\circ) - (2M \times 5 \sin 45^\circ)$$

$$= (M \times v_2 \sin 45^\circ) - (2M \times v_1 \sin 30^\circ)$$

$$\Rightarrow \left(M \times 10 \times \frac{1}{2}\right) - \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$

$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}} \right) - \left(2M \times v_1 \times \frac{1}{2} \right)$$

$$\Rightarrow 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1 \quad \dots (ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(5\sqrt{3} + 5\sqrt{2}) - (5 - 5\sqrt{2})$$

$$= \left(\frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \right) - \left(\frac{v_2}{\sqrt{2}} - v_1 \right)$$

$$\Rightarrow 5\sqrt{3} + 10\sqrt{2} - 5 = v_1\sqrt{3} + v_1$$

$$\Rightarrow v_1 = \left(\frac{5\sqrt{3} + 10\sqrt{2} - 5}{1 + \sqrt{3}} \right)$$

$$= \frac{8.66 + 14.142 - 5}{1 + 1.732}$$

$$= \frac{17.802}{2.732}$$

$$\Rightarrow v_1 = 6.516 \text{ m/s}$$

$$\approx 6.5 \text{ m/s} \quad \dots (iii)$$

Substituting the value from Eq. (iii) in Eq. (i), we get

$$5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + 6.51 \times \sqrt{3}$$

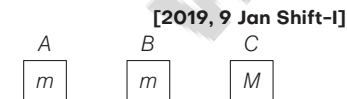
$$\Rightarrow v_2 = (5\sqrt{3} + 5\sqrt{2} - 6.51 \times \sqrt{3}) \sqrt{2}$$

$$v_2 = (8.66 + 7.071 - 11.215) 1.414$$

$$\Rightarrow v_2 = 4.456 \times 1.414$$

$$\Rightarrow v_2 \approx 6.3 \text{ m/s}$$

- 91** Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M . Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$?

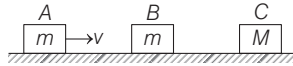


- (a) 4 (b) 2
(c) 3 (d) 5

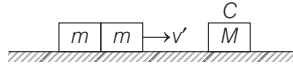
Ans. (a)

Key Idea For a perfectly inelastic collision, the momentum of the system remains conserved but there is some loss of kinetic energy. Also, after collision the objects of the system are stuck to each other and move as a combined system.

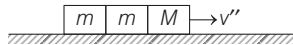
Initially, block A is moving with velocity v as shown in the figure below,



Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity ' v ' as shown below,



Then, if this combined system is collided inelastically again with the block C. So, now the velocity of system be ' v'' ' as shown below.



Thus, according to the principle of conservation of momentum,

$$\text{initial momentum of the system} = \text{final momentum of the system}$$

$$\Rightarrow mv = (2m + M)v''$$

$$\text{or } v'' = \left(\frac{mv}{2m + M} \right) \quad \dots (i)$$

$$\text{Initial kinetic energy of the system, } (KE)_i = \frac{1}{2}mv^2 \quad \dots (ii)$$

$$\text{Final kinetic energy of the system, } (KE)_f = \frac{1}{2}(2m + M)v''^2 = \frac{1}{2}(2m + M) \left(\frac{mv}{2m + M} \right)^2$$

$$= \frac{1}{2} \cdot \frac{m^2 v^2}{(2m + M)} \quad [\because \text{using Eq. (i)}]$$

$$= \frac{1}{2} \cdot \frac{v^2 m^2}{(2m + M)} \quad \dots (iii)$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{(KE)_f}{(KE)_i} = \frac{\frac{1}{2}m^2 v^2}{\frac{1}{2}mv^2} = \frac{m}{2m + M} \quad \dots (iv)$$

It is given that $\frac{5}{6}$ th of (KE) is lost in this process.

$$\Rightarrow (KE)_f = \frac{1}{6}(KE)_i$$

$$\Rightarrow \frac{(KE)_f}{(KE)_i} = \frac{1}{6} \quad \dots (v)$$

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

$$4m = M \Rightarrow \frac{M}{m} = 4$$

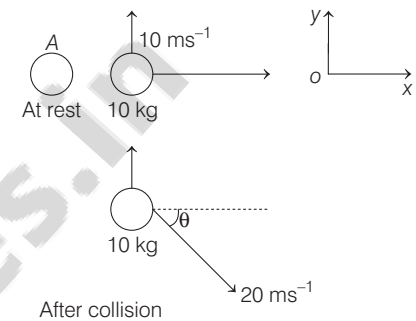
- 92** A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity

100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10 \text{ ms}^{-2}$)

[2019, 10 Jan Shift-I]

- (a) 20 m (b) 30 m
(c) 10 m (d) 40 m

Ans. (d)



Key Idea As bullet gets embedded in the block of wood so, it represents a collision which is perfectly inelastic and hence only momentum of the system is conserved.

Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{\text{rel}}} = \frac{100}{100} = 1 \text{ s}$$

Speed of block just before collision is;

$$v_1 = gt = 10 \times 1 = 10 \text{ ms}^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt$$

$$= 100 - 10 \times 1 = 90 \text{ ms}^{-1}$$

Let v = velocity of bullet + block system, then by conservation of linear momentum, we get

$$-(0.03 \times 10) + (0.02 \times 90) = (0.05)v$$

$$\Rightarrow v = 30 \text{ ms}^{-1}$$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g} \Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$$\Rightarrow h = 45 \text{ m}$$

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 s.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95 \text{ m}$$

Distance of point of collision from the top of the building

$$= 100 - 95 = 5 \text{ m}$$

∴ Maximum height to which the combined system reaches above the top of the building before falling below
 $= 45 - 5 = 40 \text{ m}$

- 93** A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6 \text{ N/m}$. The body sticks to the platform and the spring's maximum compression is found to be x . Given that $g = 10 \text{ ms}^{-2}$, the value of x will be close to

[2019, 11 Jan Shift-I]

- (a) 8 cm (b) 4 cm
 (c) 40 cm (d) 80 cm

Ans. (*)

Initial compression of the spring,

$$mg = k \left(\frac{x_0}{100} \right) \quad (x_0 \text{ in cm})$$

$$\Rightarrow x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

Which is very small and can be neglected.

Applying conservation of momentum before and after the collision i.e.,
 momentum before collision =
 momentum after collision.

$$m \times \sqrt{2gh} = (m + M)v \quad (\because \text{velocity of the block just before the collision is } v^2 - 0^2 = 2gh)$$

$$\text{or } v = \sqrt{2gh}$$

After substituting the given values, we get

$$1 \times \sqrt{2 \times 10 \times 100} = 4v$$

$$\text{or } 4v = 20\sqrt{5}$$

$$\text{So, } v = 5\sqrt{5} \text{ m/s}$$

Let this be the maximum velocity, then for the given system, using

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 1.25 \times 10^6 \times \left(\frac{x}{100} \right)^2$$

$$\Rightarrow 4 = 10^4 \times \frac{x^2}{10^4} \text{ or } x = 2 \text{ cm}$$

∴ No option given is correct.

- 94** A simple pendulum is made of a string of length l and a bob of mass m , is released from a small angle θ_0 . It strikes a block of mass M , kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and

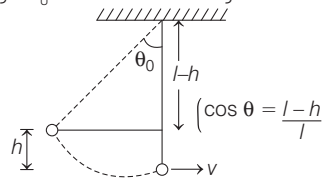
goes up to an angle θ_1 . Then, M is given by [2019, 12 Jan Shift-I]

- (a) $m \left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1} \right)$ (b) $\frac{m}{2} \left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \right)$
 (c) $m \left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \right)$ (d) $\frac{m}{2} \left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1} \right)$

Ans. (a)

Pendulum's velocity at lowest point just before striking mass m is found by equating its initial potential energy (PE) with final kinetic energy (KE).

Initially, when pendulum is released from angle θ_0 as shown in the figure below,



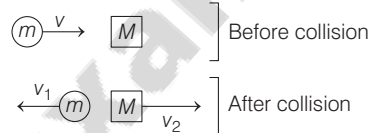
We have,

$$mgh = \frac{1}{2}mv^2$$

$$\text{Here, } h = l - l \cos \theta_0$$

$$\text{So, } v = \sqrt{2gl(1 - \cos \theta_0)} \quad \dots(i)$$

With velocity v , bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown



Then if pendulum is deflected back upto angle θ_1 , then

$$v_1 = \sqrt{2gl(1 - \cos \theta_1)} \quad \dots(ii)$$

Using definition of coefficient of restitution to get

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \sqrt{2gl(1 - \cos \theta_0)} = v_2 + \sqrt{2gl(1 - \cos \theta_1)}$$

$$\Rightarrow v_2 = \sqrt{2gl} (\sqrt{1 - \cos \theta_0} - \sqrt{1 - \cos \theta_1}) \quad \dots(iv)$$

According to the momentum conservation, initial momentum of the system = final momentum of the system

$$\Rightarrow mv = Mv_2 - mv_1$$

$$\Rightarrow Mv_2 = m(v + v_1)$$

$$Mv_2 = m\sqrt{2gl} (\sqrt{1 - \cos \theta_0} + \sqrt{1 - \cos \theta_1})$$

Dividing Eq. (v) and Eq. (iv), we get

$$\begin{aligned} \Rightarrow \frac{M}{m} &= \frac{\sqrt{1 - \cos \theta_0} + \sqrt{1 - \cos \theta_1}}{\sqrt{1 - \cos \theta_0} - \sqrt{1 - \cos \theta_1}} \\ &= \frac{\sqrt{\sin^2 \left(\frac{\theta_0}{2} \right)} + \sqrt{\sin^2 \left(\frac{\theta_1}{2} \right)}}{\sqrt{\sin^2 \left(\frac{\theta_0}{2} \right)} - \sqrt{\sin^2 \left(\frac{\theta_1}{2} \right)}} \\ \frac{M}{m} &= \frac{\sin \left(\frac{\theta_0}{2} \right) + \sin \left(\frac{\theta_1}{2} \right)}{\sin \left(\frac{\theta_0}{2} \right) - \sin \left(\frac{\theta_1}{2} \right)} \end{aligned}$$

For small θ_0 , we have

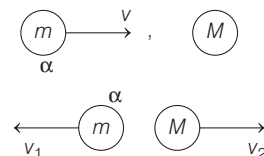
$$\frac{M}{m} = \frac{\frac{\theta_0}{2} + \frac{\theta_1}{2}}{\frac{\theta_0}{2} - \frac{\theta_1}{2}} \text{ or } M = m \left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1} \right)$$

- 95** An α -particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is [2019, 12 Jan Shift-II]

- (a) 1.5 m (b) 4 m
 (c) 3.5 m (d) 2 m

Ans. (b)

We have following collision, where mass of α particle = m and mass of nucleus = M



Let α particle rebounds with velocity v_1 , then

Given; final energy of $\alpha = 36\%$ of initial energy

$$\Rightarrow \frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv^2$$

$$\Rightarrow v_1 = 0.6v \quad \dots(i)$$

As unknown nucleus gained 64% of energy of α , we have

$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}mv^2$$

$$\Rightarrow v_2 = \sqrt{\frac{m}{M}} \times 0.8v \quad \dots(ii)$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

$$mv = M\sqrt{\frac{m}{M}} \times 0.8v - m \times 0.6v$$

$$\Rightarrow 1.6mv = \sqrt{mM} \times 0.8v \Rightarrow 2m = \sqrt{mM}$$

$$\Rightarrow 4m^2 = mM \Rightarrow M = 4m$$

- 96** In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is [JEE Main 2018]

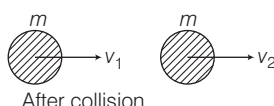
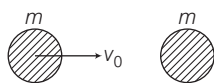
- (a) $\frac{v_0}{4}$ (b) $\sqrt{2} v_0$
(c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

Ans. (c)

Key Idea Momentum is conserved in all type of collisions,

Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100} \times \frac{1}{2}mv_0^2 \dots(i)$$



Conservation of momentum gives,

$$mv_0 = mv_1 + mv_2 \\ v_0 = v_2 + v_1 \dots(ii)$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2 \Rightarrow 2v_1v_2 = \frac{-v_0^2}{2}$$

$$\therefore (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2 \\ \text{or } v_{\text{rel}} = \sqrt{2}v_0$$

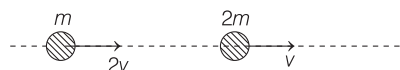
- 97** A particle of mass m moving in the x -direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y -direction with speed v . If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to [JEE Main 2015]

- (a) 44% (b) 50%
(c) 56% (d) 62%

Ans. (c)

Key Idea Conservation of linear momentum can be applied but energy is not conserved.

Consider the movement of two particles as shown below.



Conserving linear momentum in x -direction

$$(p_i)_x = (p_f)_x \\ \text{or } 2mv = (2m + m)v_x \\ \text{or } v_x = \frac{2}{3}v$$

Conserving linear momentum in y -direction

$$(p_i)_y = (p_f)_y \text{ or } 2mv = (2m + m)v_y \\ \text{or } v_y = \frac{2}{3}v$$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(v)^2 \\ = \frac{1}{2} \times 4mv^2 + \frac{1}{2} \times 2mv^2 \\ = 2mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_f = \frac{1}{2}(3m)(v_x^2 + v_y^2) \\ = \frac{1}{2}(3m)\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right] \\ = \frac{3m}{2}\left[\frac{8v^2}{9}\right] = \frac{4mv^2}{3}$$

$$\text{Loss in the energy } \Delta E = E_i - E_f \\ = mv^2 \left[3 - \frac{4}{3}\right] = \frac{5}{3}mv^2$$

Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i} \times 100 = \frac{(5/3)mv^2}{3mv^2} \times 100 = \frac{5}{9} \times 100 \\ = 56\%$$

- 98** This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M . If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then

$$f = \left(\frac{m}{M+m}\right)$$

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision. [JEE Main 2013]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

Ans. (d)

Energy $E = \frac{p^2}{2m}$, where p is momentum, m

is the mass moving of the particle. Maximum energy loss occurs when the particles get stuck together as a result of the collision.

$$\text{Maximum energy loss } (\Delta E) \\ = \frac{p^2}{2m} - \frac{p^2}{2(m+M)}$$

where, $(m+M)$ is the resultant mass when the particles get stuck.

$$\Delta E = \frac{p^2}{2m} \left[1 - \frac{m}{m+M}\right] = \frac{p^2}{2m} \left[\frac{M}{m+M}\right]$$

Also, $p = mv$

$$\therefore \Delta E = \frac{m^2v^2}{2m} \left[\frac{M}{m+M}\right] = \frac{mv^2}{2} \left[\frac{M}{m+M}\right]$$

Comparing the expression with

$$\Delta E = f\left(\frac{1}{2}mv^2\right), f = \frac{M}{m+M}$$

- 99 Statement I** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement II Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE 2010]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is true
(d) Statement I is true, Statement II is false

Ans. (a)

If it is a completely inelastic collision, then

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2},$$

$$KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As p_1 and p_2 both simultaneously cannot be zero. Therefore, total KE cannot be lost.

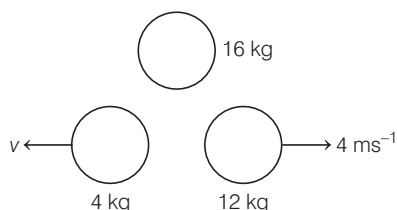
- 100** A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1} . The kinetic energy of the other mass is

[AIEEE 2006]

- (a) 144 J (b) 288 J
(c) 192 J (d) 96 J

Ans. (b)

Here, momentum of the system is remaining conserved as no external force is acting on the bomb (system).



Initial momentum (before explosion)
= Final momentum (after explosion)

Let velocity of 4 kg mass be $v\text{ ms}^{-1}$. From momentum conservation, we can say that its direction is opposite to velocity of 12 kg mass.

$$\begin{aligned} \text{From } p_i &= p_f \\ \Rightarrow 0 &= 12 \times 4 - 4 \times v \\ \text{or } v &= 12\text{ m/s} \\ \therefore \text{KE of 4 kg mass} &= \frac{4 \times (12)^2}{2} \\ &= 288\text{ J} \end{aligned}$$

- 101** A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction

perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

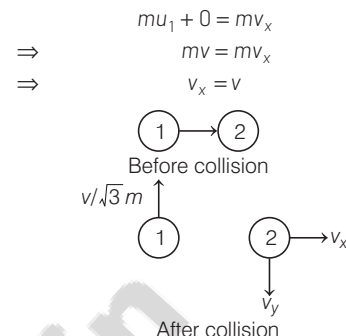
[AIEEE 2005]

- (a) v (b) $\sqrt{3}v$
(c) $\frac{2}{\sqrt{3}}v$ (d) $\frac{v}{\sqrt{3}}$

Ans. (c)

In x-direction,

Apply conservation of momentum, we get



In y-direction, apply conservation of momentum, we get

$$0 + 0 = m \left(\frac{v}{\sqrt{3}} \right) - mv_y \Rightarrow v_y = \frac{v}{\sqrt{3}}$$

Velocity of second mass after collision

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}} \right)^2 + v^2} = \sqrt{\frac{4}{3} v^2}$$

$$\text{or } v' = \frac{2}{\sqrt{3}} v$$

06

System of Particles and Rotational Motion

TOPIC 1

Centre of Mass, Torque and Angular Momentum

- 01** Angular momentum of a single particle moving with constant speed along circular path

[2021, 31 Aug Shift-1]

- (a) changes in magnitude but remains same in the direction
- (b) remains same in magnitude and direction
- (c) remains same in magnitude but changes in the direction
- (d) is zero

Ans. (b)

As we know that,

Angular momentum, $L = r \times p = mvr \sin \theta$
 where, m = mass, v = velocity, r = radius,
 θ = angle between v and $r = 90^\circ$ (for circular motion) Since, m , v , r are constant.

$\therefore L$ will always remain same in magnitude and direction.

- 02** A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is $\pi \times 10^{-1}$ N-m. [2021, 25 July Shift-1]

Ans. (4)

Given, radius of disc, $R = 20$ cm
 $= 20 \times 10^{-2}$ m

Mass of disc, $m = 10$ kg

Angular velocity, $\omega = 600$ rpm

$$\omega = 600 \times \frac{2\pi}{60} = 20\pi \text{ rads}^{-1}$$

Time taken, $t = 10$ s

Let torque be τ ,

As we know that,

$$\tau = I\alpha \quad \dots(i)$$

where, I is moment of inertia of disc about centre of mass $= \frac{1}{2}MR^2$

and α is angular acceleration.

$$\therefore \alpha = \frac{\omega}{t} = \frac{20\pi}{10} = 2\pi \text{ rads}^{-2}$$

$\therefore$ Put value of I in Eq. (i), we get

$$\begin{aligned} \tau &= \frac{1}{2}MR^2\alpha \\ &= \frac{1}{2} \times 10 \times \left(\frac{20}{100}\right)^2 \times 2\pi \\ &= \frac{1}{2} \times 10 \times \frac{4}{100} \times 2\pi \\ &= 0.4\pi \\ &= 4\pi \times 10^{-1} \text{ N-m} \end{aligned}$$

- 03** A body of mass 2 kg moving with a speed of 4 m/s makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10}$ m/s. Then, the value of x is

[2021, 25 July Shift-1]

Ans. (25)

Given, mass of 1st body, $m_1 = 2$ kg

Initial speed of 1st body, $u_1 = 4 \text{ ms}^{-1}$

Initial speed of 2nd body, $u_2 = 0 \text{ ms}^{-1}$

Let v_1 be the speed of 1st body after collision. So, as per question,

$$v_1 = \frac{u_1}{4} = \frac{4}{4} = 1 \text{ ms}^{-1}$$

and v_2 be the speed of 2nd body after collision.

By using law of conservation of momentum,

Initial momentum of system (p_i)

= Final momentum of system (p_f)

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow 2 \times 4 + m_2 \times 0 = 2 \times 1 + m_2v_2$$

$$\Rightarrow m_2v_2 = 8 - 2 = 6 \text{ kg} \cdot \text{ms}^{-1} \quad \dots(ii)$$

Since it is an elastic collision.

$$\text{Coefficient of restitution } (e) = \frac{v_2 - v_1}{u_1 - u_2} = 1$$

$$\Rightarrow v_2 - 1 = 4 - 0 \Rightarrow v_2 = 5 \text{ ms}^{-1}$$

Substituting in Eq. (i), we get

$$m_2 \times 5 = 6 \text{ kg} \cdot \text{ms}^{-1} \Rightarrow m_2 = 6/5 \text{ kg}$$

As we know that,

Speed of centre of mass,

$$\begin{aligned} v_{CM} &= \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \\ &= \frac{2 \times 1 + \frac{6}{5} \times 5}{2 + \frac{6}{5}} = \frac{8 \times 5}{16} \\ &= \frac{5}{2} = 2.5 \text{ ms}^{-1} = \frac{25}{10} \text{ ms}^{-1} \end{aligned}$$

$$\therefore x = 25$$

- 04** A particle of mass m is moving in time t on a trajectory given by

$$\mathbf{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$$

where, α and β are dimensional constants. The angular

momentum of the particle becomes the same as it was for $t=0$ at time t is s.

[2021, 25 July Shift-1]

Ans. (10)

Given,

$$\mathbf{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5) \hat{j}$$

Since,

$$\text{Velocity } (\mathbf{v}) = \frac{d\mathbf{r}}{dt}$$

$$\Rightarrow \mathbf{v} = \frac{d}{dt} [10\alpha t^2 \hat{i} + 5\beta(t-5) \hat{j}]$$

$$= 20\alpha t \hat{i} + 5\beta \hat{j}$$

$$\text{Now } \mathbf{v}(t=0) = 5\beta \hat{j}$$

and angular momentum (L) at $t=0$ will be

$$L = mvr$$

$$\text{and } \mathbf{r}(t=0) = 5\beta(0-5) \hat{j} = -25\beta \hat{j}$$

$$L(t=0) = m(\mathbf{r} \times \mathbf{v}) = 0$$

[$\because \sin\theta$ (angle between $\mathbf{v}$ and $\mathbf{r}$) is 0].

$$\text{Now, } \mathbf{r}(t) \times \mathbf{v}(t) = \mathbf{v}(t) \times \mathbf{r}(t) = 0$$

$$\Rightarrow \mathbf{v} \times \mathbf{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 20\alpha t & 5\beta & 0 \\ 10\alpha t^2 & 5\beta(t-5) & 0 \end{vmatrix}$$

$$\Rightarrow \hat{i}(0) - \hat{j}(0) + \hat{k}$$

$$[20\alpha t \times 5\beta(t-5) - 5\beta \times 10\alpha t^2] = 0$$

$$\Rightarrow \hat{k}[100\alpha\beta t(t-5) - 50\alpha\beta t^2] = 0$$

$$\Rightarrow 100\alpha\beta t(t-5) = 50\alpha\beta t^2$$

$$\Rightarrow 2(t-5) = t$$

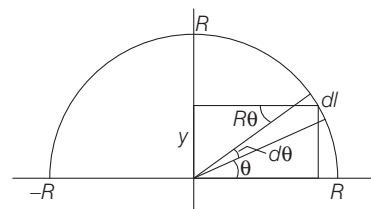
$$\Rightarrow 2t - 10 = t \Rightarrow t = 10 \text{ s}$$

- 05** The position of the centre of mass of a uniform semi-circular wire of radius R placed in XY -plane with its centre at the origin and the line joining its ends as X -axis is given by $(0, \frac{xR}{\pi})$. Then, the value of $|x|$ is

[2021, 22 July Shift-II]

Ans. (2)

Let R be the radius of semicircle, dl be the elemental arc length for $d\theta$ angle



$$\therefore dl = R d\theta$$

For πR length, mass be m

$$\therefore \text{Mass per unit length} = \frac{m}{\pi R}$$

$$\therefore dm = \frac{m}{\pi R} dl$$

$$\Rightarrow dm = \frac{m}{\pi R} R d\theta = \frac{m}{\pi} d\theta$$

$$\sin\theta = \frac{y}{R}$$

$$y = R \sin\theta$$

and as we know that

$$X_{CM} = 0 \text{ (by symmetry)}$$

$$\text{and } Y_{CM} = \frac{1}{m} \int y dm = \frac{1}{m} \int_0^\pi R \sin\theta \frac{m}{\pi} d\theta$$

$$= -\frac{R}{\pi} [\cos\theta]_0^\pi = \frac{2R}{\pi}$$

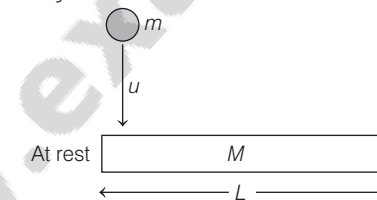
Hence, $x = 2$

- 06** A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits at one end of the rod with a velocity u in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $(\frac{m}{M})$ is $\frac{1}{x}$. The value of x will be

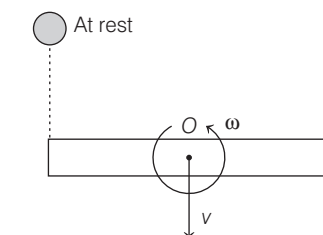
[2021, 20 July Shift-I]

Ans. (4)

The given situation can be shown as



Before collision,



After collision

As the collision is perfectly elastic, therefore momentum is conserved, i.e.

$$p_{\text{initially}} = p_{\text{finally}}$$

$$\Rightarrow mu = Mv \quad \dots(i)$$

Angular momentum will also be conserved about point O .

$$\Rightarrow mv \cdot \frac{L}{2} = \frac{ML^2}{12} \omega$$

$$\Rightarrow \omega = \frac{6mv}{ML} \quad \dots(ii)$$

$\therefore$ Coefficient of restitution,

$$e = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$$

$$\Rightarrow 1 = \frac{v + \frac{\omega L}{2}}{u}$$

$$\Rightarrow v + \frac{\omega L}{2} = u \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$v + \frac{3mu}{M} = u$$

$$\Rightarrow \frac{mu}{M} + \frac{3mu}{M} = u \quad [\text{using Eq.(i)}]$$

$$\Rightarrow \frac{4mu}{M} = u \Rightarrow \frac{m}{M} = \frac{1}{4} \quad \dots(iv)$$

According to question,

$$\text{ratio of masses } \left(\frac{m}{M}\right) = \frac{1}{x}$$

Comparing it with Eq. (iv), we get

$$x = 4$$

- 07** A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω . Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become

[2021, 18 March Shift-I]

$$(a) \omega \frac{M}{M+m}$$

$$(b) \omega \frac{M+2m}{M}$$

$$(c) \omega \frac{M}{M+2m}$$

$$(d) \omega \frac{M-2m}{M+2m}$$

Ans. (c)

Angular momentum, $L = I\omega$

The moment of inertia of the circular ring, $I = Mr^2$

When masses are attached at diametrically opposite ends, the moment of inertia, $I' = Mr^2 + 2mr^2$

The new angular speed $= \omega'$

So, the new angular momentum, $L' = I'\omega'$

Using the law of conservation of angular momentum

$$\Rightarrow I\omega = I'\omega'$$

$$\Rightarrow Mr^2\omega = (Mr^2 + 2mr^2)\omega'$$

$$\Rightarrow \omega' = \frac{M}{M+2m} \omega$$

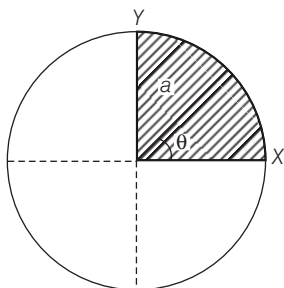
- 08** The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area)

is at the position $\frac{x}{3\pi}a, \frac{x}{3\pi}a$, where x

is

(Round off to the nearest integer)
(a is an area as shown in the figure)

[2021, 17 March Shift-II]



Ans. (4)

As we know the centre of mass of the quarter disc,

$$= \left(\frac{4R}{3\pi}, \frac{4R}{3\pi} \right)$$

Since, $R = a$

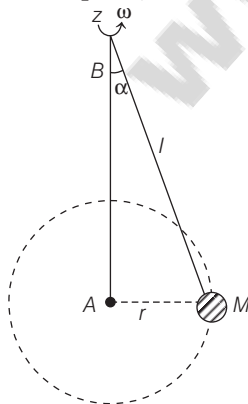
$\therefore$ The centre of mass of the quarter disc

is $\left(\frac{4a}{3\pi}, \frac{4a}{3\pi} \right)$.

Hence, the value of x is 4.

- 09** A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω . The angular momentum of M about point A is L_A which lies in the positive z -direction and the angular momentum of M about B is L_B . The correct statement for this system is

[2021, 17 March Shift-I]



- (a) L_A and L_B are both constant in magnitude and direction

- (b) L_B is constant in direction with varying magnitude
(c) L_B is constant, both in magnitude and direction
(d) L_A is constant, both in magnitude and direction

Ans. (d)

It is given that the system is assumed to be in steady circular motion with constant angular velocity ω .

We know that $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

where, $\mathbf{L}$ = angular momentum,
 $\mathbf{v}$ = velocity of the particle,

m = mass of the particle

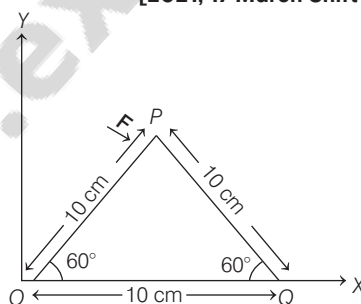
and r = radius of the circular path traced by the particle.

Since, L_A is the angular momentum of M about point A which lies in positive z -direction. Therefore, with respect to point A , we will get the direction of $\mathbf{L}$ along positive Z -axis and of constant magnitude of mvr .

Since, L_B is the angular momentum of M about point B , so with respect to point B , we will get the constant magnitude of $\mathbf{L}$ but its direction will be continuously changing. Hence, option (d) is correct.

- 10** A triangular plate is shown below. A force $\mathbf{F} = 4\hat{i} - 3\hat{j}$ is applied at point P . The torque at point P with respect to point O and Q are

[2021, 17 March Shift-I]



- (a) $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$
(b) $15 + 20\sqrt{3}, 15 - 20\sqrt{3}$
(c) $15 - 20\sqrt{3}, 15 + 20\sqrt{3}$
(d) $-15 + 20\sqrt{3}, 15 + 20\sqrt{3}$

Ans. (b)

Given, $\mathbf{F} = 4\hat{i} - 3\hat{j}$

Resolving the components of position vector in horizontal and vertical direction.

$$\begin{aligned} \therefore \text{Horizontal component, } r_x &= r \cos \theta \\ &= 10 \cos 60^\circ \quad \left[\because \cos 60^\circ = \frac{1}{2} \right] \\ &= 10 \times \frac{1}{2} = 5 \text{ units} \end{aligned}$$

$$\begin{aligned} \text{Vertical component, } r_y &= r \sin \theta \\ &= 10 \sin 60^\circ \quad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right] \\ &= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ units} \end{aligned}$$

$\therefore$ Position vector w.r.t. O , $\mathbf{r}_O = 5\hat{i} + 5\sqrt{3}\hat{j}$

and position vector w.r.t. Q ,

$$\mathbf{r}_Q = -5\hat{i} + 5\sqrt{3}\hat{j}$$

Torque at point P w.r.t. O ,

$$\begin{aligned} \boldsymbol{\tau}_O &= \mathbf{r}_O \times \mathbf{F} = (5\hat{i} + 5\sqrt{3}\hat{j}) \times (4\hat{i} - 3\hat{j}) \\ &= (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})(-\hat{k}) \end{aligned}$$

Torque at point P w.r.t. Q ,

$$\begin{aligned} \boldsymbol{\tau}_Q &= \mathbf{r}_Q \times \mathbf{F} = (4\hat{i} - 3\hat{j}) \times (-5\hat{i} + 5\sqrt{3}\hat{j}) \\ &= (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})\hat{k} \end{aligned}$$

- 11** A force $\mathbf{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of $x=2$ plane and X -axis. The magnitude of torque of this force about a point $(2, 3, 4)$ is

(Round off to the nearest integer)

[2021, 16 March Shift-II]

Ans. (20)

Given,

Force, $\mathbf{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

We know that

Torque, $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

where, $\mathbf{r}$ is the perpendicular distance.

$$\mathbf{r} = (2\hat{i}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -3\hat{j} - 4\hat{k}$$

$$\Rightarrow \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$\Rightarrow \boldsymbol{\tau} = \hat{i}(-12 + 12) - \hat{j}(0 + 16) + \hat{k}(0 + 12)$$

$$\Rightarrow \boldsymbol{\tau} = -16\hat{j} + 12\hat{k}$$

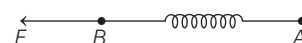
$\therefore$ Magnitude of torque

$$\begin{aligned} |\boldsymbol{\tau}| &= \sqrt{(16)^2 + (12)^2} \\ &= \sqrt{256 + 144} = \sqrt{400} \end{aligned}$$

$$\Rightarrow |\boldsymbol{\tau}| = 20$$

- 12** Two masses A and B , each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration a , then the acceleration of mass B will be

[2021, 26 Feb Shift-II]



$$(a) \frac{Ma-F}{M} \quad (b) \frac{MF}{F+Ma}$$

$$(c) \frac{F+Ma}{M} \quad (d) \frac{F-Ma}{M}$$

Ans. (d)

Let a_{CM} be the acceleration of centre of mass of system and F be the applied force on spring.

$$\therefore a_{CM} = \frac{F}{m_A + m_B} = \frac{m_A a_A + m_B a_B}{m_A + m_B}$$

$$\Rightarrow a_{CM} = \frac{F}{2M} = \frac{Ma_A + Ma_B}{2M}$$

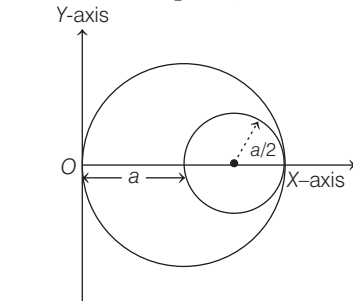
$$(\because m_A = m_B = M)$$

$$\Rightarrow a_B = \frac{F - Ma_A}{M} = \frac{F - Ma}{M}$$

13 A circular hole of radius $\left(\frac{a}{2}\right)$ is cut

out of a circular disc of radius a as shown in figure. The centroid of the remaining circular portion with respect to point O will be

[2021, 24 Feb Shift-II]



$$(a) \frac{1}{6}a \quad (b) \frac{10}{11}a \quad (c) \frac{5}{6}a \quad (d) \frac{2}{3}a$$

Ans. (c)

Given, radius of hole, $r = a/2$
and radius of disc, $R = a$

Let x_{CM} be the centre of mass of system,

m_1, x_1 be the mass and centre of mass of disc

m_2, x_2 be the mass and centre of mass of circular hole.

$$\therefore m_2 = \frac{m_1}{\pi R^2} \pi r^2$$

$$\Rightarrow m_2 = \frac{m_1 r^2}{R^2} = \frac{m_1 (a/2)^2}{a^2} = \frac{m_1}{4}$$

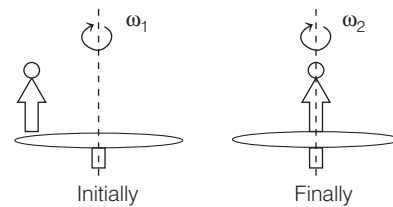
and $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$\therefore x_{CM} = \frac{m_1 a - (m_1/4)(3a/2)}{m_1 - (m_1/4)}$$

$$\Rightarrow x_{CM} = \frac{m_1 a(1 - 3/8)}{3m_1/4} \Rightarrow x_{CM} = \frac{5a}{6}$$

14 A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 rpm. The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre? [2020, 03 Sep Shift-I]

Ans. (9)



According to conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$

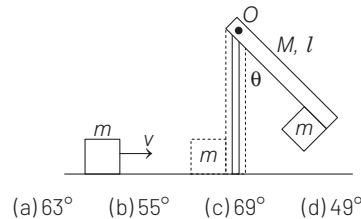
$$\Rightarrow \left(80R^2 + \frac{200R^2}{2}\right) \omega_1 = \left(\frac{200R^2}{2}\right) \omega_2$$

$$\Rightarrow \frac{360R^2}{2} \times 5 = \frac{200R^2}{2} \times \omega_2$$

$$\omega_2 = 9 \text{ rpm}$$

15 A block of mass $m = 1 \text{ kg}$ slides with velocity $v = 6 \text{ m/s}$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision, making angle θ before momentarily coming to rest. If the rod has mass $M = 2 \text{ kg}$ and length $l = 1 \text{ m}$, then the value of θ is approximately (Take, $g = 10 \text{ m/s}^2$)

[2020, 3 Sep Shift-I]



Ans. (a)

As there is no external torque so, angular momentum about pivot O is conserved.

$$\Rightarrow L_i = L_f \Rightarrow mvl = (I_{\text{system}}) \omega$$

$$\Rightarrow mvl = (I_{\text{rod}} + I_{\text{block}}) \omega$$

$$\Rightarrow mvl = \left(\frac{Ml^2}{3} + ml^2\right) \omega$$

(where, ω = initial angular speed of rod and block)

$$\omega = \frac{mvl}{\left(\frac{Ml^2}{3} + ml^2\right)} \Rightarrow \omega = \frac{mv}{\left(\frac{Ml}{3} + ml\right)}$$

$$\Rightarrow \omega = \frac{1 \times 6}{\left(\frac{2 \times 1}{3} + 1 \times 1\right)} = \frac{6 \times 3}{5} = \frac{18}{5} \text{ rad/s}$$

Now, by law of conservation of energy, Initial rotational kinetic energy of rod and block = Final potential energy at angle θ of rod and block

$$\Rightarrow \frac{1}{2} (I_{\text{system}}) \omega^2 = Mgh_1 + mgh_2$$

where,

h_1 = height raised by centre of mass of rod
 $= \frac{l}{2} (1 - \cos \theta)$ and h_2 = height raised by

block $= l(1 - \cos \theta)$. Hence,

$$\frac{1}{2} (I_{\text{rod}} + I_{\text{block}}) \omega^2 = Mg \frac{l}{2} (1 - \cos \theta) + mgl(1 - \cos \theta)$$

$$\Rightarrow \frac{1}{2} \left(\frac{Ml^2}{3} + ml^2\right) \omega^2 = gl \left(\frac{M}{2} + m\right)$$

$\times (1 - \cos \theta)$

$$\Rightarrow \frac{1}{2} \left(\frac{M}{3} + m\right) \omega^2 = \left(\frac{M}{2} + m\right) g$$

$$\Rightarrow \frac{1}{2} \left(\frac{2}{3} + 1\right) \times 1 \times \left(\frac{18}{5}\right)^2 = \left(\frac{2}{2} + 1\right) \times 10$$

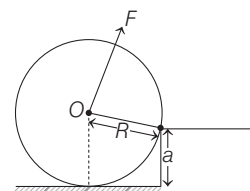
$$\Rightarrow \frac{27}{50} = 1 - \cos \theta \Rightarrow \cos \theta = \frac{23}{50} = 0.46$$

$$\Rightarrow \theta = \cos^{-1}(0.46) = 62.61^\circ \approx 63^\circ$$

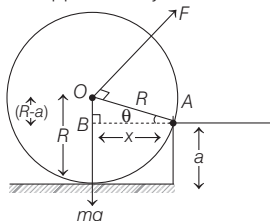
The nearest answer is 63° .

Hence, option (a) is correct.

16 A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre O perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is [2020, 02 Sep Shift-I]



(a) $Mg \frac{a}{R}$ (b) $Mg \sqrt{1 - \frac{a^2}{R^2}}$
 (c) $Mg \sqrt{\left(\frac{R}{R-a}\right)^2 - 1}$ (d) $Mg \sqrt{1 - \left(\frac{R-a}{R}\right)^2}$

Ans. (d)Force F is applied to cylinder as shown,

Force F must be large enough to create a torque about point A, so that it is able to counteract the torque due to weight about point A.

$\therefore$ Torque due to force about point A = Torque due to weight about point A

$$\Rightarrow F \cdot R = Mg x$$

$$\text{From } \triangle OAB, x = \sqrt{R^2 - (R-a)^2}$$

$$\Rightarrow F \cdot R = Mg \sqrt{R^2 - (R-a)^2}$$

$$\Rightarrow F = \frac{Mg}{R} \sqrt{R^2 - (R-a)^2}$$

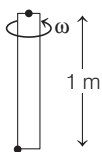
$$\Rightarrow F = Mg \sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

Hence, correct option is (d).

- 17** A thin rod of mass 0.9 kg and length 1 m is suspended at rest from one end, so that it can freely oscillate in the vertical plane. A particle of mass 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom-most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be [2020, 05 Sep Shift-II]

**Ans. (20)**

The given situation is shown in the following fig.,



Applying law of conservation of angular momentum about pivotal point,

$$L_i = L_f \Rightarrow mvl = I\omega$$

$$\Rightarrow mvl = \left(\frac{1}{3} ML^2 + mL^2\right) \omega \quad \dots(i)$$

Given, $m = 0.1 \text{ kg}$, $v = 80 \text{ ms}^{-1}$, $M = 0.9 \text{ kg}$, $l = 1 \text{ m}$

Substituting these values in eq. (i), we get

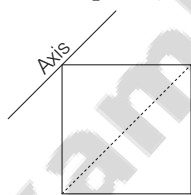
$$0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right) \omega$$

$$8 = \left(\frac{3}{10} + \frac{1}{10}\right) \omega \Rightarrow 8 = \frac{4}{10} \omega$$

$$\Rightarrow \omega = 20 \text{ rad/s}$$

- 18** Four point masses, each of mass m are fixed at the corners of a square of side l . The square is rotating with angular frequency ω , about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is

[2020, 06 Sep Shift-I]



(a) $ml^2\omega$ (b) $4ml^2\omega$ (c) $3ml^2\omega$ (d) $2ml^2\omega$

Ans. (c)

The angular momentum of the square about the given axis will be the sum of angular momentum due to each point masses.

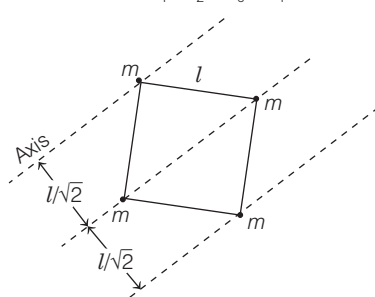
$$\text{i.e. } L = L_1 + L_2 + L_3 + L_4$$

$$= l_1\omega_1 + l_2\omega_2 + l_3\omega_3 + l_4\omega_4$$

Here, angular frequency will be same.

$$\text{i.e., } \omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega \quad (\text{say})$$

$$\therefore L = (l_1 + l_2 + l_3 + l_4)\omega$$



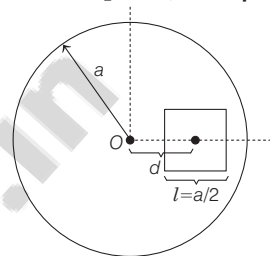
$$L = \left\{ m(0)^2 + 2 \times m \left(\frac{l}{\sqrt{2}}\right)^2 + m(\sqrt{2}l)^2 \right\} \omega$$

$$= 3ml^2\omega$$

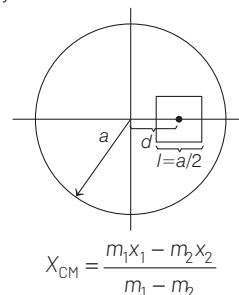
- 19** A square shaped hole of side $l = \frac{a}{2}$ is curved out at a distance $d = \frac{a}{2}$

from the centre O of a uniform circular disc of radius a . If the distance of the centre of mass of the remaining portion from O is $-a/X$, value of X (to the nearest integer) is

[2020, 02 Sep Shift-II]

**Ans. (23)**

Centre of mass of remaining portion is given by



Here note that, removed mass (m_2) is treated as a negative mass.

Let x = mass per unit area of uniform disc.

Then, m_1 = mass of complete disc
 $= x \cdot \pi a^2$

m_2 = mass of square portion $= x \cdot \frac{a^2}{4}$

Also,

x_1 = x-coordinate of centre of mass of $m_1 = 0$

x_2 = x-coordinate of centre of mass of $m_2 = d$

$$\Rightarrow X_{CM} = \frac{x\pi a^2 \times 0 - x \frac{a^2}{4} \times d}{x\pi a^2 - x \frac{a^2}{4}}$$

$$= \frac{-a}{2(4\pi - 1)} \quad \dots(i)$$

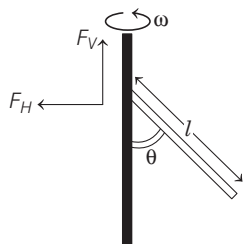
$$\left[\text{Since, } d = \frac{a}{2} \right]$$

$$\text{Given, } X_{CM} = -\frac{a}{X} \quad \dots(ii)$$

Comparing the Eqs. (i) and (ii), we get

$$\therefore X = 2(4\pi - 1) = 23.12 \text{ or } X \approx 23$$

20



A uniform rod of length l is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω , the rod makes an angle θ with it (see figure). To find θ , equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta \cos\theta$ about

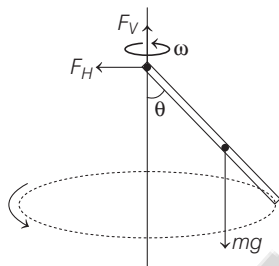
the centre of mass to the torque provided by the horizontal and vertical forces F_H and F_V about the centre of mass. The value of θ is then such that

[2020, 03 Sep Shift-II]

- (a) $\cos\theta = \frac{g}{l\omega^2}$ (b) $\cos\theta = \frac{g}{2l\omega^2}$
 (c) $\cos\theta = \frac{2g}{3l\omega^2}$ (d) $\cos\theta = \frac{3g}{2l\omega^2}$

Ans. (d)

Rod rotates around vertical axis as



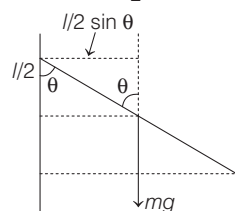
Given that,

Rate of change of angular momentum of rod = Torque provided by horizontal and vertical forces about centre of mass

$$\Rightarrow \frac{ml^2}{12}\omega^2 \sin\theta \cos\theta$$

= Net torque of F_H and F_V ... (i)

Here, $F_V = mg$ and its perpendicular distance from axis is $\frac{l}{2} \sin\theta$.



$$\therefore \text{Torque of } F_V = mg \cdot \frac{l}{2} \sin\theta$$

$$\text{and } F_H = \text{centripetal force} = m\omega^2 \frac{l}{2} \sin\theta$$

Also, distance of F_H from axis is also $\frac{l}{2} \cos\theta$.

$$\therefore \text{Torque of } F_H = m\omega^2 \frac{l}{2} \sin\theta \cdot \frac{l}{2} \cos\theta$$

So, from Eq. (i), we have

$$mg \frac{l}{2} \sin\theta - m\omega^2 \frac{l}{2} \sin\theta \cdot \frac{l}{2} \cos\theta = \frac{ml^2}{12}\omega^2 \sin\theta \cos\theta$$

$$\Rightarrow mg \frac{l}{2} \sin\theta = ml^2 \omega^2 \left(\frac{1}{12} + \frac{1}{4} \right) \sin\theta \cos\theta$$

$$\Rightarrow \cos\theta = \frac{3}{2} \left(\frac{g}{\omega^2 l} \right)$$

Hence, correct option is (d).

21 A force $\mathbf{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then, the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m.

The value of x is

[2020, 05 Sep Shift-I]

Ans. (195)

Given, $\mathbf{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

$$\begin{aligned} \mathbf{r} &= (4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k}) \\ &= (4-1)\hat{i} + (3-2)\hat{j} + (-1-1)\hat{k} \\ &= (3\hat{i} + \hat{j} - 2\hat{k}) \text{ m} \end{aligned}$$

Torque, $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = (3+4)\hat{i} - (9+2)\hat{j} + (6-1)\hat{k} = (7\hat{i} - 11\hat{j} + 5\hat{k}) \text{ N-m}$$

Magnitude of torque, $|\boldsymbol{\tau}|$

$$\begin{aligned} &= \sqrt{(7)^2 + (-11)^2 + (5)^2} \\ &= \sqrt{49 + 121 + 25} \\ &= \sqrt{195} \text{ N-m} \end{aligned}$$

So, the value of $x = 195$.

22 The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then, value of x is

[2020, 06 Sep Shift-II]

Ans. (3)

The distance of centre of mass of a solid hemisphere from the centre of the flat surface is given by

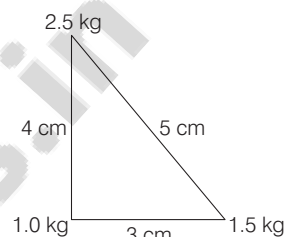
$$d = \frac{3R}{8}$$

Here, $R = 8$ cm

$$\therefore d = \frac{3 \times 8}{8} = 3 \text{ cm}$$

Given, $d = x$ cm $\therefore x = 3$

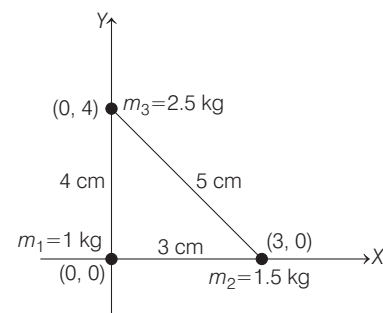
23 Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at a point [2020, 7 Jan Shift-I]



- (a) 2.0 cm right and 0.9 cm above 1 kg mass
 (b) 0.6 cm right and 2.0 cm above 1 kg mass
 (c) 1.5 cm right and 1.2 cm above 1 kg mass
 (d) 0.9 cm right and 2.0 cm above 1 kg mass

Ans. (d)

We choose origin as shown in the figure.



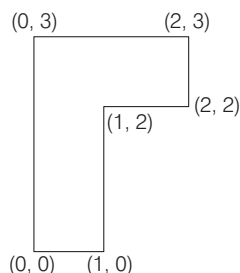
$$\begin{aligned} \text{Using } x_{\text{CM}} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}, \text{ we have} \\ x_{\text{CM}} &= \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = 0.9 \text{ cm} \end{aligned}$$

Similarly,

$$y_{\text{CM}} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = 2.0 \text{ cm}$$

So, centre of mass (CM) is 0.9 cm right and 2.0 cm above 1 kg mass.

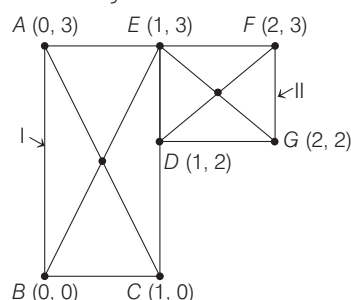
24 The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are [2020, 8 Jan Shift-I]



- (a) (1.25 m, 1.50 m) (b) (1 m, 1.75 m)
(c) (0.75 m, 0.75 m) (d) (0.75 m, 1.75 m)

Ans. (d)

Given lamina consists of 2 parts I and II as shown in the figure.



As mass of uniform lamina is 4 kg, mass of part I is $m_1 = 3$ kg and mass of part II is $m_2 = 1$ kg.

These masses can be assumed to be concentrated at geometrical centres of sections I and II.

So, $m_1 = 3$ kg has coordinates $x_1 = 0.5$ m, $y_1 = 1.5$ m

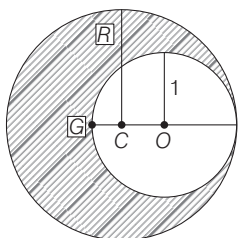
and $m_2 = 1$ kg has coordinates $x_2 = 1.5$ m, $y_2 = 2.5$ m

Now, we use formula of centre of mass (CM) to find X_{CM} and Y_{CM} .

$$\text{So, } X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(3 \times 0.5) + (1 \times 1.5)}{4} = 0.75 \text{ m}$$

$$\text{and } Y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{(3 \times 1.5) + (1 \times 2.5)}{4} = 1.75 \text{ m}$$

25



As shown in figure, when a spherical cavity (centred at O) of radius 1 is

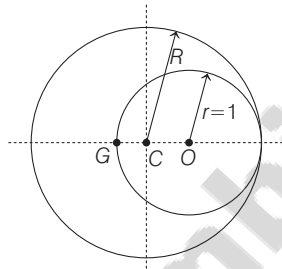
cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e. on the surface of the cavity. R can be determined by the equation

[2020, 8 Jan Shift-II]

- (a) $(R^2 + R + 1)(2 - R) = 1$
(b) $(R^2 + R - 1)(2 - R) = 1$
(c) $(R^2 - R - 1)(2 - R) = 1$
(d) $(R^2 - R + 1)(2 - R) = 1$

Ans. (a)

If we combine the remaining and removed spherical parts at their initial relative positions, we get complete sphere as shown in the figure.



$$\text{So, for complete sphere, } m_{\text{remaining}} \times x_{\text{remaining}} + m_{\text{removed}} \times x_{\text{removed}} = 0$$

$$X_{CM} = \frac{m_{\text{remaining}} \times x_{\text{remaining}} + m_{\text{removed}} \times x_{\text{removed}}}{m_{\text{remaining}} + m_{\text{removed}}} = 0$$

As, X_{CM} is at C which is taken at origin of our chosen reference axis, so

$$X_{CM} = 0 \Rightarrow m_{\text{remaining}} \times x_{\text{remaining}} + m_{\text{removed}} \times x_{\text{removed}} = 0$$

$$m_{\text{removed}} \times x_{\text{removed}} = 0 \dots (i)$$

$$\text{Here, } m_{\text{remaining}} = \left(\frac{4}{3} \pi R^3 - \frac{4}{3} \pi 1^3 \right) \rho$$

where, ρ = density of sphere,

$$x_{\text{remaining}} = 2 - R, \quad m_{\text{removed}} = - \left(\frac{4}{3} \pi 1^3 \right) \rho$$

Here, mass removed is negative and $x_{\text{removed}} = R - 1$.

So, from Eq. (i), we get

$$\left(\frac{4}{3} \pi R^3 - \frac{4}{3} \pi \right) \rho \times (2 - R) = \left(\frac{4}{3} \pi \right) \rho \times (R - 1)$$

$$\Rightarrow (R^3 - 1)(2 - R) = R - 1$$

$$\Rightarrow (R^2 + R + 1)(2 - R) = 1$$

- 26** Consider a uniform rod of mass $M = 4m$ and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the

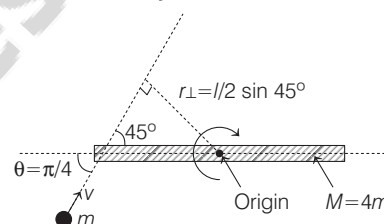
rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is

[2020, 8 Jan Shift-I]

- (a) $\frac{3\sqrt{2}}{7} \frac{v}{l}$ (b) $\frac{4}{7} \frac{v}{l}$
(c) $\frac{3}{7} \frac{v}{l}$ (d) $\frac{3}{7\sqrt{2}} \frac{v}{l}$

Ans. (a)

Collision of particle and rod occurs as shown in the figure.



Angular momentum of particle about origin (centre of rod) before collision,

$$L_i = m \cdot v \cdot r_{\perp}$$

where, $r_{\perp}$ = perpendicular distance of direction of velocity from centre of rod.

From geometry of figure,

$$r_{\perp} = \frac{l}{2} \sin 45^\circ = \frac{l}{2\sqrt{2}}$$

So, initial angular momentum of system,

$$L_i = m \cdot v \cdot \frac{l}{2\sqrt{2}}$$

Now, when mass m sticks to one of the ends of rod, let system rotates about origin with initial angular speed ω , then final angular momentum of system just after collision,

$$\begin{aligned} L_f &= I_{\text{system}} \cdot \omega \\ &= (I_{\text{rod}} + I_{\text{particle}}) \omega \\ &= \left(\frac{4m \cdot l^2}{12} + m \left(\frac{l}{2} \right)^2 \right) \omega \\ &= \frac{7}{12} m l^2 \omega \end{aligned}$$

By angular momentum conservation,

$$\begin{aligned} L_i &= L_f \\ \Rightarrow \frac{m v l}{2\sqrt{2}} &= \frac{7}{12} m l^2 \omega \\ \Rightarrow \omega &= \frac{3\sqrt{2}}{7} \left(\frac{v}{l} \right) \end{aligned}$$

- 27** A rod of length L has non-uniform linear mass density given by

$$\rho(x) = a + b\left(\frac{x}{L}\right)^2,$$

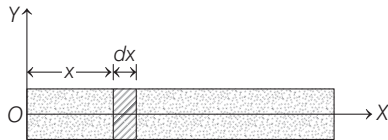
where a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is at

[2020, 9 Jan Shift-II]

- (a) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$
 (b) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$
 (c) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$
 (d) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

Ans. (a)

For a continuous mass distribution,



$$x_{CM} = \frac{\int x dm}{\int dm}$$

Here, mass of an elemental length dx of rod,

$$dm = \rho dx = \left(a + b\left(\frac{x}{L}\right)^2\right) dx$$

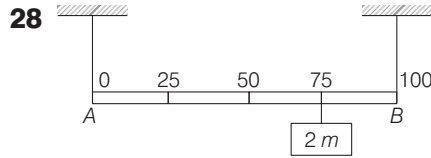
$$\text{So, } x_{CM} = \frac{\int_0^L x \left(a + \frac{bx^2}{L^2}\right) dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right) dx}$$

$$= \frac{\int_0^L \left(ax + \frac{bx^3}{L^2}\right) dx}{\int_0^L \left(a + \frac{bx^2}{L^2}\right) dx}$$

$$= \frac{\left[\frac{ax^2}{2} + \frac{b}{L^2} \cdot \frac{x^4}{4}\right]_0^L}{\left[\frac{ax}{1} + \frac{bx^3}{3L^2}\right]_0^L}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{4}}{aL + \frac{bL}{3}} = \frac{\frac{a}{2} + \frac{b}{4}}{a + \frac{b}{3}} L$$

$$\text{So, } x_{CM} = \frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$$



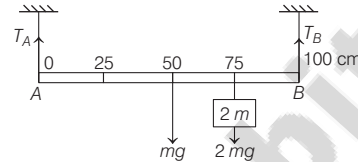
Shown in the above figure, a rigid and uniform 1 m long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass $2m$ hung at a distance of 75 cm from A. The tension in the string at A is

[2020, 2 Sep Shift-I]

- (a) 1 mg (b) 2 mg
 (c) 0.75 mg (d) 0.5 mg

Ans. (a)

Given, arrangement is shown below,



As rod is in equilibrium, net torque about point B is zero.

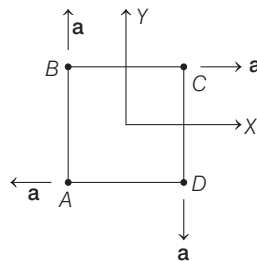
$$\Rightarrow T_A \times 100 - mg \times 50 - 2mg \times 25 = 0$$

$$\Rightarrow T_A = 1mg$$

- 29** Four particles A, B, C and D with

masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles (in ms^{-2}) is

[2019, 8 April Shift-I]

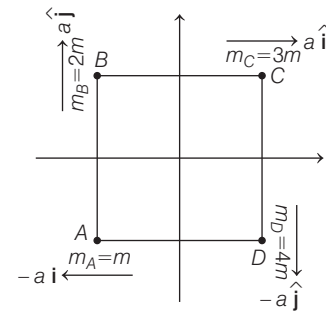


- (a) $\frac{a}{5}(\hat{i} - \hat{j})$ (b) $a(\hat{i} + \hat{j})$
 (c) zero (d) $\frac{a}{5}(\hat{i} + \hat{j})$

Ans. (a)

For a system of discrete masses, acceleration of centre of mass (CM) is given by

$$\mathbf{a}_{CM} = \frac{m_A \mathbf{a}_A + m_B \mathbf{a}_B + m_C \mathbf{a}_C + m_D \mathbf{a}_D}{m_A + m_B + m_C + m_D}$$



where, $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$, $|\mathbf{a}_A| = |\mathbf{a}_B| = |\mathbf{a}_C| = |\mathbf{a}_D| = a$ (according to the question)

$$\mathbf{a}_{CM} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{m + 2m + 3m + 4m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j}) \text{ ms}^{-2}$$

- 30** The time dependence of the position of a particle of mass $m = 2$ is given by $\mathbf{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its

angular momentum, with respect to the origin, at time $t = 2$ is

[2019, 10 April Shift-II]

- (a) $36\hat{k}$ (b) $-34(\hat{k} - \hat{i})$
 (c) $-48\hat{k}$ (d) $48(\hat{i} + \hat{j})$

Ans. (c)

Position of particle is, $\mathbf{r} = 2t\hat{i} - 3t^2\hat{j}$

where, t is instantaneous time.

Velocity of particle is $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2\hat{i} - 6t\hat{j}$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$= m[(2t\hat{i} - 3t^2\hat{j}) \times (2\hat{i} - 6t\hat{j})]$$

$$= m(-12t^2(\hat{i} \times \hat{j}) - 6t^2(\hat{j} \times \hat{i}))$$

As, $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{j} \times \hat{i} = -\hat{k}$

$$\Rightarrow \mathbf{L} = m(-12t^2\hat{k} + 6t^2\hat{k})$$

As, $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{j} \times \hat{i} = -\hat{k}$

$$\Rightarrow \mathbf{L} = -m(6t^2)\hat{k}$$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{k} = -48\hat{k}$$

- 31** A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size

are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be

[2019, 9 April Shift-II]

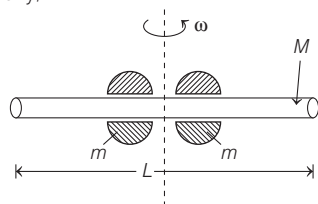
- (a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+m}$
(c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

Ans. (d)

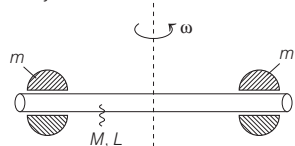
As there is no external torque on system.
∴ Angular momentum of system is conserved.

$$\Rightarrow I_i \omega_i = I_f \omega_f$$

Initially,



Finally,

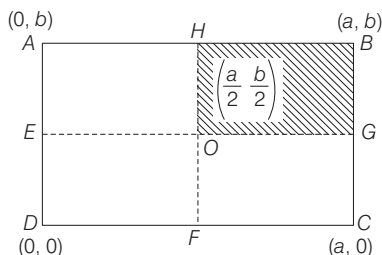


$$\Rightarrow \frac{ML^2}{12} \cdot \omega_0 + 0 = \left(\frac{ML^2}{12} + 2m \left(\frac{L}{2} \right)^2 \right) \omega$$

So, final angular speed of system is

$$\Rightarrow \omega = \frac{\frac{ML^2}{12} \cdot \omega_0}{\left(\frac{ML^2}{12} + 6mL^2 \right)} = \frac{M\omega_0}{M+6m}$$

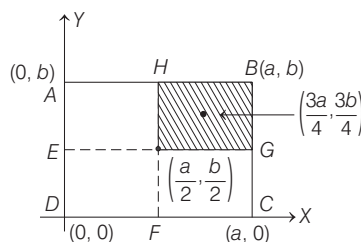
- 32** A uniform rectangular thin sheet ABCD of mass M has length a and breadth b , as shown in the figure. If the shaded portion HBGD is cut-off, the coordinates of the centre of mass of the remaining portion will be [2019, 8 April Shift-II]



- (a) $\left(\frac{2a}{3}, \frac{2b}{3} \right)$ (b) $\left(\frac{5a}{12}, \frac{5b}{12} \right)$
(c) $\left(\frac{3a}{4}, \frac{3b}{4} \right)$ (d) $\left(\frac{5a}{3}, \frac{5b}{3} \right)$

Ans. (b)

The given rectangular thin sheet ABCD can be drawn as shown in the figure below,



Here,

Area of complete lamina, $A_1 = ab$

Area of shaded part of lamina

$$= \frac{a}{2} \times \frac{b}{2} = \frac{ab}{4}$$

(x_1, y_1) = coordinates of centre of mass of complete lamina = $\left(\frac{a}{2}, \frac{b}{2} \right)$

(x_2, y_2) = coordinates of centre of mass of shaded part of lamina = $\left(\frac{3a}{4}, \frac{3b}{4} \right)$

∴ Using formula for centre of mass, we have

$$X_{CM} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{ab \left(\frac{a}{2} \right) - \frac{ab}{4} \left(\frac{3a}{4} \right)}{ab - \frac{ab}{4}} = \frac{\frac{8a^2b}{8} - \frac{3a^2b}{16}}{\frac{3ab}{4}} = \frac{5a}{12}$$

$$\text{Similarly, } Y_{CM} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{ab \left(\frac{b}{2} \right) - \frac{ab}{4} \left(\frac{3b}{4} \right)}{ab - \frac{ab}{4}} = \frac{5b}{12}$$

Alternate Solution

Let m be the mass of entire rectangular lamina.

So, the mass of the shaded portion of lamina = $\frac{m}{4}$

Using the relation,

$$X_{CM} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}, \text{ we get}$$

$$X_{CM} = \frac{m \left(\frac{a}{2} \right) - \frac{m}{4} \left(\frac{3a}{4} \right)}{m - \frac{m}{4}} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

Similarly, $Y_{CM} = \frac{m_1 y_1 - m_2 y_2}{m_1 - m_2}$, we get

$$Y_{CM} = \frac{m \left(\frac{b}{2} \right) - \frac{m}{4} \left(\frac{3b}{4} \right)}{m - \frac{m}{4}} = \frac{\frac{b}{2} - \frac{3b}{16}}{\frac{3}{4}} = \frac{5b}{12}$$

∴ The coordinates of the centre of mass of remaining portion will be $\left(\frac{5a}{12}, \frac{5b}{12} \right)$.

- 33** A particle of mass m is moving along a trajectory given by $x = x_0 + a \cos \omega_1 t$ and $y = y_0 + b \sin \omega_2 t$. The torque acting on the particle about the origin at $t = 0$ is [2019, 10 April Shift-I]

- (a) zero
(b) $m(-x_0 b + y_0 a) \omega_1^2 \hat{k}$
(c) $-m(x_0 b \omega_2^2 - y_0 a \omega_1^2) \hat{k}$
(d) $+m y_0 a \omega_1^2 \hat{k}$

Ans. (d)

Given equations of trajectory are

$$x = x_0 + a \cos \omega_1 t$$

$$y = y_0 + b \sin \omega_2 t$$

Now, $v_x = dx/dt = -a \omega_1 \sin \omega_1 t$

$$\text{and } v_y = \frac{dy}{dt} = b \omega_2 \cos \omega_2 t$$

$$\text{Similarly, } a_x = \frac{dv_x}{dt} = -a \omega_1^2 \cos(\omega_1 t)$$

$$\text{Similarly, } a_y = \frac{dv_y}{dt} = -b \omega_2^2 \sin(\omega_2 t)$$

Now, at $t = 0$; $x = x_0 + a$; $y = y_0$

$$\text{and } a_x = -a \omega_1^2; a_y = 0 \quad \dots (i)$$

Torque acting on the particle is given as

$$\tau = \mathbf{r} \times \mathbf{F} = m(\mathbf{r} \times \mathbf{a})$$

Here, $\mathbf{r} = (x_0 + a)\hat{i} + y_0\hat{j} + 0\hat{k}$

and $\mathbf{a} = -a \omega_1^2 \hat{i} + 0\hat{j} + 0\hat{k}$

So, torque at $t = 0$,

$$\tau = m(-a \omega_1^2) \times y_0 (-\hat{k})$$

[using Eq. (i)]

$$\Rightarrow \tau = +m y_0 a \omega_1^2 \hat{k}$$

Alternate Solution

We can calculate total force from Eq. (i),

$$\mathbf{F} = -m(a \omega_1^2 \cos \omega_1 t \hat{i} + b \omega_2^2 \sin \omega_2 t \hat{j})$$

$$\text{and } \mathbf{r} = x\hat{i} + y\hat{j} = (x_0 + a \cos \omega_1 t)\hat{i} + (y_0 + b \sin \omega_2 t)\hat{j}$$

Now, torque, $\tau = \mathbf{r} \times \mathbf{F}$

$$= [(x_0 + a \cos \omega_1 t)\hat{i} + (y_0 + b \sin \omega_2 t)\hat{j}] \times [-m(a \omega_1^2 \cos \omega_1 t \hat{i} + b \omega_2^2 \sin \omega_2 t \hat{j})]$$

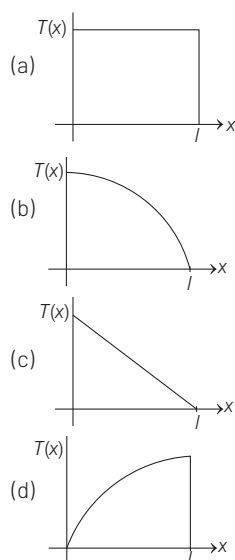
$$\begin{aligned}
 &= -m \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_0 + a \cos \omega_1 t & y_0 + b \sin \omega_2 t & 0 \\ a \omega_1^2 \cos \omega_1 t & b \omega_2^2 \sin \omega_2 t & 0 \end{bmatrix} \\
 &= -m [x_0 b \omega_2^2 \sin \omega_2 t + a b \omega_2^2 \sin \omega_2 t \cos \omega_1 t \\
 &\quad - y_0 a \omega_1^2 \cos \omega_1 t \\
 &\quad - a b \omega_1^2 \sin \omega_2 t \cos \omega_1 t] \hat{k}
 \end{aligned}$$

On solving it at $t=0$, we get

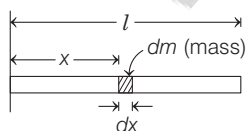
$$\tau = m a \omega_1^2 y_0 \hat{k}$$

- 34** A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely?

[2019, 12 April Shift-I]



Key Idea In a uniform rod, mass per unit length remains constant. If it is denoted by λ , then $\lambda = \frac{m}{l}$ = constant for all segments of rod.



To find tension at x distance from fixed end, let us assume an element of dx length and dm mass. Tension on this part due to rotation is

$$dT = Kx \quad \dots(i)$$

$$\text{As, } K = m\omega^2$$

$$\text{For this element, } K = (dm)\omega^2 \quad \dots(ii)$$

$$\therefore dT = (dm)\omega^2 x \quad \dots(iii)$$

To find complete tension in the rod, we need to integrate Eq. (iii),

$$\int_0^T dT = \int_0^m (dm)\omega^2 x \quad \dots(iv)$$

Using linear mass density,

$$\lambda = \frac{m}{l} = \frac{dm}{dx}$$

$$\Rightarrow dm = \frac{m}{l} \cdot dx \quad \dots(v)$$

Putting the value of Eq. (v) in Eq. (iv), we get

$$T = \int_x^l \frac{m}{l} \cdot \omega^2 x \cdot dx = \frac{m}{l} \cdot \omega^2 \left[\frac{x^2}{2} \right]_x^l$$

$$\Rightarrow T = \frac{m\omega^2}{2l} [l^2 - x^2] \text{ or } T \propto -x^2$$

- 35** A person of mass M is sitting on a swing to length L and swinging with an angular amplitude θ_0 . If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l ($l \ll L$), is close to

[2019, 12 April Shift-I]

- (a) $Mgl(1 - \theta_0^2)$ (b) $Mgl(1 + \theta_0^2)$
(c) Mgl (d) $Mgl \left(1 + \frac{\theta_0^2}{2} \right)$

Ans. (b)

Initially, centre of mass is at distance L from the top end of the swing. It shifts to $(L-l)$ distance when the person stands up on the swing.

$\therefore$ Using angular momentum conservation law, if v_0 and v_1 are the velocities before standing and after standing of the person, then

$$Mv_0L = Mv_1(L-l)$$

$$\Rightarrow v_1 = \left(\frac{L}{L-l} \right) v_0 \quad \dots(i)$$

Now, total work done by (person + gravitation) system will be equal to the change in kinetic energy of the person, i.e.

$$W_g + W_p = KE_1 - KE_0$$

$$\Rightarrow -Mgl + W_p = \frac{1}{2} Mv_1^2 - \frac{1}{2} Mv_0^2$$

$$\Rightarrow W_p = Mgl + \frac{1}{2} M(v_1^2 - v_0^2)$$

$$= Mgl + \frac{1}{2} M \left[\left(\frac{L}{L-l} \right)^2 v_0^2 - v_0^2 \right]$$

[from Eq. (i)]

$$\begin{aligned}
 &= Mgl + \frac{1}{2} Mv_0^2 \left[\left(\frac{L-l}{L} \right)^2 - 1 \right] \\
 &= Mgl + \frac{1}{2} Mv_0^2 \left[\left(1 - \frac{l}{L} \right)^2 - 1 \right] \\
 &= Mgl + \frac{1}{2} Mv_0^2 \left[\left(1 + \frac{2l}{L} \right) - 1 \right]
 \end{aligned}$$

[using $(1+x)^n = 1 + nx$ as higher terms can be neglected, if $n \ll 1$]

$$\Rightarrow W_p = Mgl + \frac{1}{2} Mv_0^2 \times \frac{2l}{L}$$

$$\text{or } W_p = Mgl + Mv_0^2 \frac{l}{L} \quad \dots(ii)$$

$$\text{Here, } v_0 = \omega A = \left(\sqrt{\frac{g}{L}} \right) (\theta_0 L)$$

$$\Rightarrow v_0 = \theta_0 \sqrt{gL}$$

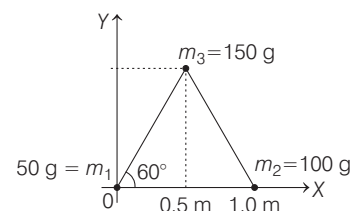
$\therefore$ Using this value of v_0 in Eq. (ii), we get

$$W_p = Mgl + M\theta_0^2 gL \cdot \frac{l}{L}$$

$$\Rightarrow W_p = Mgl [1 + \theta_0^2]$$

- 36** Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be

[2019, 12 April Shift-II]



- (a) $\left(\frac{\sqrt{3}}{4} \text{ m}, \frac{5}{12} \text{ m} \right)$ (b) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{8} \text{ m} \right)$
(c) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{4} \text{ m} \right)$ (d) $\left(\frac{\sqrt{3}}{8} \text{ m}, \frac{7}{12} \text{ m} \right)$

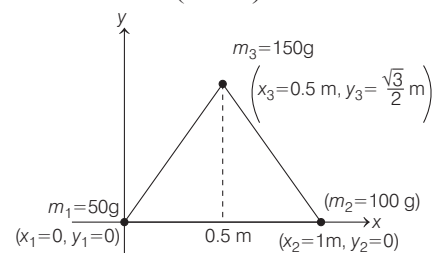
Ans. (c)

The height of equilateral Δ is

$$h = y_3 = \sqrt{(1)^2 - (0.5)^2} = \sqrt{3}/2 \text{ m}$$

Thus, coordinates of three masses are

$$(0, 0), (1, 0) \text{ and } \left(0.5, \frac{\sqrt{3}}{2} \right)$$



Using, $X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$,

we have

$$X_{CM} = \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

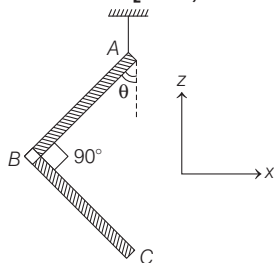
$$= \frac{175}{300} = \frac{7}{12} \text{ m}$$

Similarly, $Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150}$$

$$= \frac{\sqrt{3}}{4} \text{ m}$$

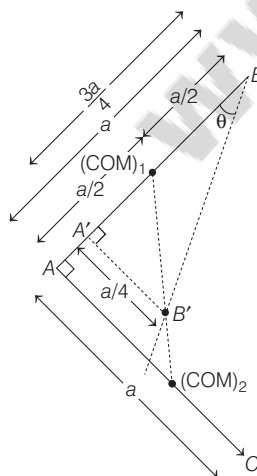
- 37** An L-shaped object made of thin rods of uniform mass density is suspended with a string as shown in figure. If $AB=BC$ and the angle is made by AB with downward vertical is θ , then [2019, 9 Jan Shift-I]



- (a) $\tan\theta = \frac{2}{\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2\sqrt{3}}$
 (c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = \frac{1}{3}$

Ans. (d)

Key Idea The centre of mass of a thin rod of uniform density lies at its centre. The given system of rods can be drawn using geometry as,

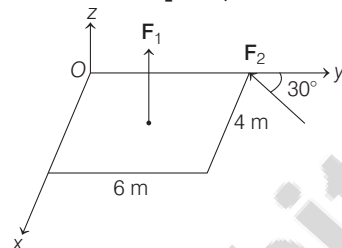


where, $(COM)_1$ and $(COM)_2$ are the centre of mass of both rods AB and AC , respectively.

So, in $\Delta A'B'B'$,

$$\tan\theta = \frac{A'B'}{A'B} = \frac{\frac{a}{4}}{\frac{3a}{4}} = \frac{1}{3} \text{ or } \tan\theta = \frac{1}{3}$$

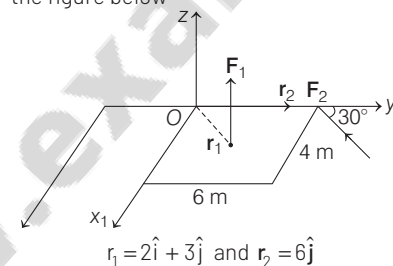
- 38** A slab is subjected to two forces F_1 and F_2 of same magnitude F as shown in the figure. Force F_2 is in xy -plane while force F_1 acts along Z -axis at the point $(2\hat{i} + 3\hat{j})$. The moment of these forces about point O will be [2019, 11 Jan Shift-I]



- (a) $(3\hat{i} + 2\hat{j} - 3\hat{k})F$ (b) $(3\hat{i} - 2\hat{j} + 3\hat{k})F$
 (c) $(3\hat{i} - 2\hat{j} - 3\hat{k})F$ (d) $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

Ans. (b)

According to the question as shown in the figure below



$$r_1 = 2\hat{i} + 3\hat{j} \text{ and } r_2 = 6\hat{j}$$

$$F_1 = F\hat{k}$$

$$\text{and } F_2 = (-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j})F$$

Moment of force is given as, $\tau = r \times F$

where, r is the perpendicular distance and F is the force.

$\therefore$ Moment due to F_1

$$\tau_1 = (2\hat{i} + 3\hat{j}) \times (F\hat{k})$$

$$= -2F\hat{j} + 3F\hat{i} \quad \dots(i)$$

Moment due to F_2

$$\tau_2 = (6\hat{j}) \times (-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j})F$$

$$= 6\sin 30^\circ F\hat{k} = 3F\hat{k} \quad \dots(ii)$$

$\therefore$ Resultant torque,

$$\tau = \tau_1 + \tau_2$$

$$= 3F\hat{i} - 2F\hat{j} + 3F\hat{k}$$

$$= (3\hat{i} - 2\hat{j} + 3\hat{k})F.$$

- 49** The magnitude of torque on a particle of mass 1 kg is 2.5 N-m about the origin. If the force acting on it is 1 N and the distance of the particle from the origin is 5 m , then the angle between the force and the position vector is (in radian) [2019, 11 Jan Shift-II]

- (a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Ans. (d)

Given, $m = 1 \text{ kg}$

$$|\tau| = 2.5 \text{ N-m}, F = 1 \text{ N} \text{ and } r = 5 \text{ m}$$

We know that,

$$\text{torque } |\tau| = rF \sin\theta \Rightarrow 2.5 = 5 \times 1 \times \sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{2} \text{ or } \theta = \frac{\pi}{6} \text{ rad}$$

- 40** A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m . If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) [2019, 11 Jan Shift-II]

- (a) 10 rad/s^2 (b) 16 rad/s^2
 (c) 20 rad/s^2 (d) 12 rad/s^2

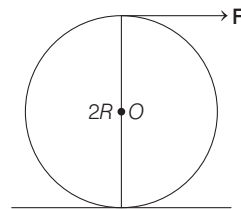
Ans. (b)

Given, $m = 5 \text{ kg}$, $R = 0.5 \text{ m}$.

Horizontal force, $F = 40 \text{ N}$

As, Cylinder is rolling without slipping.

Hence, torque is producing rotation about centre O .



So, $\tau = r \times F$ (Here, $r = R$)

$$\theta = 90^\circ$$

So, $\tau = r \times F = RF$

or $\tau = 0.5 \times 40 = 20 \text{ N-m} \quad \dots(i)$

If α is acceleration of centre of mass ' O '

then torque is, $\tau = I\alpha$

where, $I = MR^2$

$$\therefore \tau = MR^2\alpha \quad \dots(ii)$$

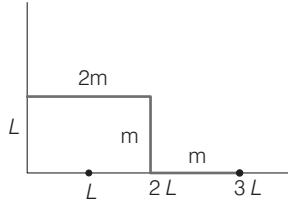
Comparing Eq. (i) with Eq. (ii),

$$MR^2\alpha = 20$$

$$\Rightarrow \alpha = \frac{20}{5 \times (0.5)^2}$$

$$\text{or } \alpha = 16 \text{ rad/s}^2$$

- 41** The position vector of the centre of mass r_{cm} of an asymmetric uniform bar of negligible area of cross-section as shown in figure is
[2019, 12 Jan Shift-I]



- (a) $r = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$
 (b) $r = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$
 (c) $r = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$
 (d) $r = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

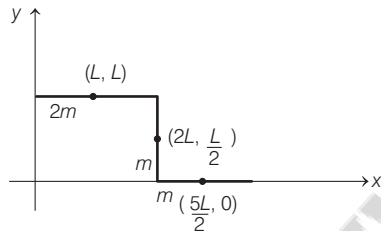
Ans. (a)

Coordinates of centre of mass (COM) are given by

$$X_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

and $Y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

For given system of rods, masses and coordinates of centre of rods are as shown.



$$\text{So, } X_{COM} = \frac{2mL + m \times \frac{L}{2} + m \times 0}{4m} = \frac{13}{8}L$$

$$\text{and } Y_{COM} = \frac{2m \times 0 + m \times \frac{L}{2} + m \times L}{4m} = \frac{5}{8}L$$

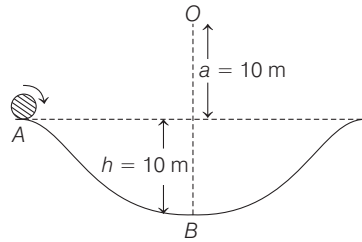
So, position vector of COM is

$$r_{COM} = X_{COM}\hat{x} + Y_{COM}\hat{y} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$$

- 42** A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless

surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10 \text{ m/s}^2$)

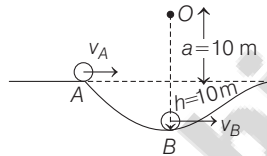
[2019, 12 Jan Shift-II]



- (a) $8 \text{ kg-m}^2/\text{s}$
 (b) $3 \text{ kg-m}^2/\text{s}$
 (c) $2 \text{ kg-m}^2/\text{s}$
 (d) $6 \text{ kg-m}^2/\text{s}$

Ans. (d)

The given figure is shown below as



As friction is absent, energy at A = energy at B

$$\Rightarrow \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$$

$$\Rightarrow v_B^2 = v_A^2 + 2gh = 5^2 + 2 \times 10 \times 10 = 225$$

$$\Rightarrow v_B = 15 \text{ ms}^{-1}$$

Angular momentum about point 'O',

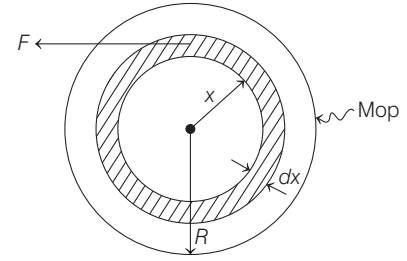
$$\begin{aligned} &= mv_B r_B \\ &= 20 \times 10^{-3} \times 15 \times 20 \\ &= 6 \text{ kg-m}^2 \text{ s}^{-1} \end{aligned}$$

- 43** To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ , the torque applied by the machine on the mop in (N-m) is [2019, 10 Jan Shift-I]

- (a) $\frac{2}{3}\mu FR$ (b) $\frac{\mu FR}{6}$
 (c) $\frac{\mu FR}{3}$ (d) $\frac{\mu FR}{2}$

Ans. (a)

Let a small strip of mop has width dx and radius x , as shown below,



Torque applied to move this strip is

$$d\tau = \text{Force on strip}$$

$$\times \text{Perpendicular distance from the axis}$$

$$\Rightarrow d\tau = \text{Force per unit area} \times \text{Area of strip} \times \text{Perpendicular distance from the axis.}$$

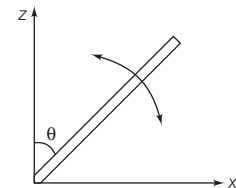
$$= \frac{\mu F}{\pi R^2} \cdot 2\pi x dx \cdot x$$

$$\Rightarrow d\tau = \frac{2\mu F x^2}{R^2} \cdot dx$$

So, total torque to be applied on the mop is

$$\begin{aligned} \tau &= \int_{x=0}^{x=R} d\tau = \int_0^R \frac{2\mu F x^2}{R^2} \cdot dx \\ &= \frac{2\mu F}{R^2} \times \frac{R^3}{3} \\ &= \frac{2}{3}\mu FR \text{ (N-m)} \end{aligned}$$

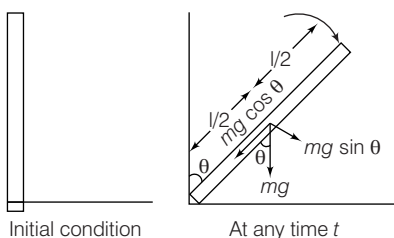
- 44** A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is [JEE Main 2017]



- (a) $\frac{2g}{3l} \sin \theta$ (b) $\frac{3g}{2l} \cos \theta$
 (c) $\frac{2g}{3l} \cos \theta$ (d) $\frac{3g}{2l} \sin \theta$

Ans. (d)

As the rod rotates in vertical plane so a torque is acting on it, which is due to the vertical component of weight of rod.



Initial condition At any time t

Now, torque $\tau = \text{force} \times \text{perpendicular distance of line of action of force from axis of rotation}$

$$= mg \sin \theta \times \frac{l}{2}$$

Again, torque $\tau = I\alpha$

where, $I = \text{moment of inertia} = \frac{ml^2}{3}$

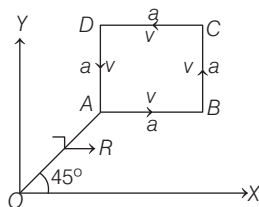
[Force and torque frequency along axis of rotation passing through in end]

$\alpha = \text{angular acceleration}$

$$\therefore mg \sin \theta \times \frac{l}{2} = \frac{ml^2}{3} \alpha$$

$$\therefore \alpha = \frac{3g \sin \theta}{2l}$$

- 45** A particle of mass m is moving along the side of a square of side a , with a uniform speed v in the X-Y plane as shown in the figure.



Which of the following statements is false for the angular momentum L about the origin? [JEE Main 2016]

- (a) $L = -\frac{mv}{\sqrt{2}} R \hat{k}$, when the particle is moving from A to B.
 (b) $L = mv \left(\frac{R}{\sqrt{2}} + a \right) \hat{k}$, when the particle is moving from B to C.
 (c) $L = mv \left(\frac{R}{\sqrt{2}} - a \right) \hat{k}$, when the particle is moving from C to D.
 (d) $L = \frac{mv}{\sqrt{2}} R \hat{k}$, when the particle is moving from D to A.

Ans. (b, d)

For a particle of mass m is moving along the side of a square of side a . Such that Angular momentum L about the origin

$$= L = r \times p = rps \sin \theta \hat{n} \text{ or } L = r(p) \hat{n}$$

When a particle is moving from D to A,

$$L = \frac{R}{\sqrt{2}} mv(-\hat{k})$$

A particle is moving from A to B,

$$L = \frac{R}{\sqrt{2}} mv(-\hat{k})$$

and it moves from C to D,

$$L = \left(\frac{R}{\sqrt{2}} + a \right) mv(\hat{k})$$

For B to C, we have

$$L = \left(\frac{R}{\sqrt{2}} + a \right) mv(\hat{k})$$

Hence, options (b) and (d) are incorrect.

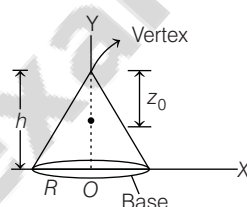
- 46** Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h , then z_0 is equal to

[JEE Main 2015]

- (a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$
 (c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

Ans. (b)

We know that centre of mass of a uniform solid cone of height h is at height $h/4$ from base, therefore



$$h - z_0 = \frac{h}{4}$$

$$\text{or } z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

- 47** A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical support. About the point of suspension,

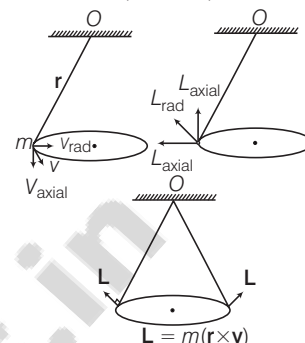
[JEE Main 2014]

- (a) angular momentum is conserved
 (b) angular momentum changes in magnitude but not in direction
 (c) angular momentum changes in direction but not in magnitude

(d) angular momentum changes both in direction and magnitude

Ans. (c)

Angular momentum of the pendulum about the suspension point O is



Then, v can be resolved into two components, radial component v_{rad} and axial component v_{axial} . Due to v_{rad} , L will be axial and due to v_{axial} , L will be radially outwards as shown.

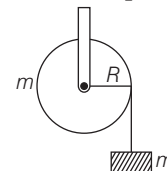
So, net angular momentum will be as shown in figure whose value will be constant ($|L| = mv$). But its direction will change as shown in the figure.

$$L = m(r \times v)$$

where, $r = \text{position vector of particle point } O$.

- 48** A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R . If the string does not slip on the cylinder, with what acceleration will the mass fall on release?

[JEE Main 2014]



- (a) $\frac{2g}{3}$ (b) $\frac{g}{2}$
 (c) $\frac{5g}{6}$ (d) g

Ans. (b)

For the mass m , $mg - T = ma$

As we know, $a = R\alpha$

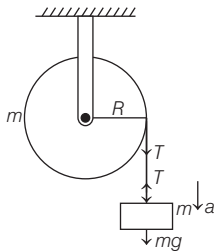
So, $mg - T = mR\alpha$... (i)

As torque $= T \times R = mR^2\alpha$... (ii)

From Eqs. (i) and (ii), we get

$$a = g/2$$

Hence, the acceleration with the mass of a body fall is $g/2$.



- 49** A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

[JEE Main 2013]

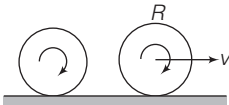
- (a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$
(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

Ans. (c)

From the principle of conservation of angular momentum, we have

$$\Sigma l\omega = \text{constant}$$

where, l is moment of inertia ($l = mr^2$).



By conservation law of angular momentum,

$$mr^2\omega_0 = mvr + mr^2 \times \frac{v}{r}$$

$$\Rightarrow v = \frac{\omega_0 r}{2}$$

- 50** A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

[AIEEE 2011]

- (a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

Ans. (c)

$$\text{Moment of inertia, } I = \frac{1}{2}MR^2 + mx^2$$

where, m = mass of insect,
and x = distance of insect from centre.

Clearly, as the insect moves along the diameter of the disc, Moment of inertia first decreases, then increases.

By conservation of angular momentum, angular speed first increases, then decreases.

- 51** A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R . Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m , if the string does not slip on the pulley, is

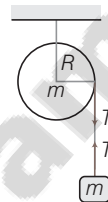
[AIEEE 2011]

- (a) g (b) $\frac{2}{3}g$ (c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

Ans. (b)

For the motion of the block,

$$mg - T = ma \quad \dots(i)$$



For the rotation of the pulley,

$$\tau = TR = I\alpha \Rightarrow T = \frac{1}{2}mR^2 \alpha \quad \dots(ii)$$

As string does not slip on the pulley

$$a = R\alpha \quad \dots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

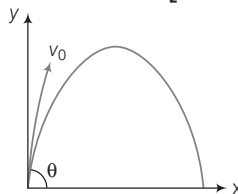
$$a = 2g/3$$

- 52** A small particle of mass m is projected at an angle θ with the x -axis with an initial velocity v_0 in the xy plane as shown in the figure.

At a time $t < \frac{v_0 \sin \theta}{g}$, the angular

momentum of the particle is

[AIEEE 2010]



- (a) $-mgv_0 t^2 \cos \theta \hat{j}$
(b) $mgv_0 t \cos \theta \hat{k}$
(c) $-\frac{1}{2}mgv_0 t^2 \cos \theta \hat{k}$
(d) $\frac{1}{2}mgv_0 t^2 \cos \theta \hat{i}$

Ans. (c)

Angular momentum of the particle is,

$$L = m(r \times v)$$

$$\Rightarrow L = m \left[v_0 \cos \theta \hat{i} + (v_0 \sin \theta - gt) \hat{j} \right]$$

$$\times [v_0 \cos \theta \hat{i} + (v_0 \sin \theta - gt) \hat{j}]$$

$$= mv_0 \cos \theta \left(-\frac{1}{2}gt \right) \hat{k}$$

$$= -\frac{1}{2}mgv_0 t^2 \cos \theta \hat{k}$$

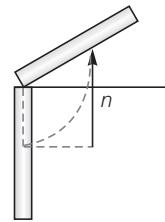
- 53** A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of

[AIEEE 2009]

- (a) $\frac{1}{3} \frac{l^2 \omega^2}{g}$ (b) $\frac{1}{6} \frac{l\omega}{g}$
(c) $\frac{1}{2} \frac{l^2 \omega^2}{g}$ (d) $\frac{1}{6} \frac{l^2 \omega^2}{g}$

Ans. (d)

By conservation of energy,



Initial total energy = Final total energy

$$TE_i = TE_f$$

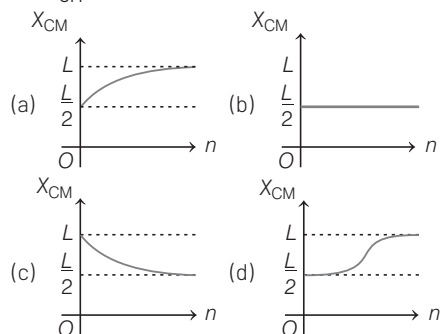
$$\frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2} \times \frac{1}{3}ml^2\omega^2 = mgh$$

$$\Rightarrow h = \frac{1}{6} \frac{l^2 \omega^2}{g}$$

- 54** A thin rod of length L is lying along the x -axis with its ends at $x=0$ and $x=L$. Its linear density (mass/length) varies with x as

$k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n , which of the following graphs best approximates the dependence of x_{CM} on n ? [AIEEE 2008]



Ans. (a)

$$x_{CM} = \frac{\int x dm}{\int dm}$$

If $n=0$, then $x_{CM} = \frac{L}{2}$

As n increases, the centre of mass shift away from $x = \frac{L}{2}$ towards $x = L$ which only option (a) is satisfying.

Alternately, you can use basic concept.

$$x_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n \times x dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = L \left[\frac{n+1}{n+2} \right]$$

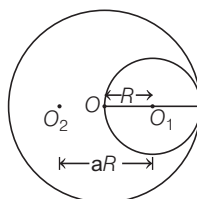
55 A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is [AIEEE 2007]

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{6}$ (d) $\frac{1}{4}$

Ans. (a)

In this question, distance of centre of mass of new disc from the centre of mass of remaining disc is αR .

Mass of original disc = M



$$\text{Mass of disc removed} = \frac{M}{\pi(2R)^2} \times \pi R^2 = \frac{M}{4}$$

$$\text{Mass of remaining disc} = M - \frac{M}{4} = \frac{3M}{4}$$

$$\therefore -\frac{3M}{4}\alpha R + \frac{M}{4}R = 0 \Rightarrow \alpha = \frac{1}{3}$$

Note In this question, the given distance must be αR for real approach to the solution.

56 Angular momentum of the particle rotating with a central force is constant due to [AIEEE 2007]

- (a) constant force
(b) constant linear momentum
(c) zero torque
(d) constant torque

Ans. (c)

According to the principle of conservation of angular momentum, in the absence of external torque, the total angular momentum of the system is constant.

i.e., zero torque $\left(\tau = \frac{dL}{dt}\right)$

57 Consider a two particle system with particles having masses m_1 and m_2 . If the first particle is pushed towards the centre of mass through a distance d , by what distance should the second particle be moved, so as to keep the centre of mass at the same position? [AIEEE 2006]

- (a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

Ans. (c)

To keep the centre of mass at the same position, velocity of centre of mass is zero, so

$$\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

where, v_1 and v_2 are the velocities of particles 1 and 2, respectively.

$$\Rightarrow m_1 \frac{dr_1}{dt} + m_2 \frac{dr_2}{dt} = 0$$

$$\left[\because v_1 = \frac{dr_1}{dt} \text{ and } v_2 = \frac{dr_2}{dt} \right]$$

$$\Rightarrow m_1 dr_1 + m_2 dr_2 = 0$$

dr_1 and dr_2 represent the change in displacement of particles.

Let 2nd particle has been displaced by distance x .

$$\Rightarrow m_1(d) + m_2(x) = 0$$

or $x = -\frac{m_1 d}{m_2}$

Alternate Solution

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \quad \dots(i)$$

After moving m_1 through a distance d towards CM and to keep the position of CM unchanged, let d' be the shift of m_2 .

$$x_{CM} = \frac{m_1(x_1 - d) + m_2(x_2 + d')}{m_1 + m_2} \quad \dots(ii)$$

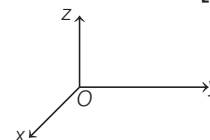
From Eqs. (i) and (ii), we get

$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1(x_1 - d) + m_2(x_2 + d')}{m_1 + m_2}$$

$$\Rightarrow -m_1 d + m_2 d' = 0$$

$$\therefore d' = \frac{m_1}{m_2} d$$

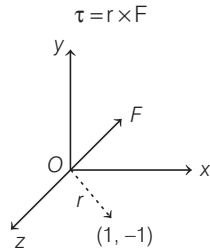
58 A force of $-F\hat{k}$ acts on O , the origin of the coordinate system. The torque about the point $(1, -1)$ is [AIEEE 2006]



- (a) $F(\hat{i} - \hat{j})$ (b) $-F(\hat{i} + \hat{j})$
(c) $F(\hat{i} + \hat{j})$ (d) $-F(\hat{i} - \hat{j})$

Ans. (c)

The torque about the given position,



Here, $r = \hat{i} - \hat{j}$ and $F = -F\hat{k}$

$$\begin{aligned}\therefore \tau &= (\hat{i} - \hat{j}) \times (-F\hat{k}) \\ &= F [(-\hat{i} \times \hat{k}) + (\hat{j} \times \hat{k})] \\ &= F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j})\end{aligned}$$

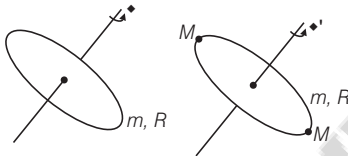
- 59** A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω . Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

[AIEEE 2006]

- (a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$
(c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

Ans. (d)

As no external torque is acting on the system, angular momentum of system remains conserved.



So, by conservation of angular momentum,

$$\begin{aligned}i.e., I_1 \omega &= I_2 \omega' \\ \Rightarrow mR^2 \omega &= (mR^2 + 2MR^2) \omega' \\ \Rightarrow \omega' &= \left(\frac{m}{m+2M} \right) \omega\end{aligned}$$

- 60** A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}M$.

The centre of mass of bodies B and C taken together shifts compared to that of body A towards [AIEEE 2005]

- (a) depends on height of breaking

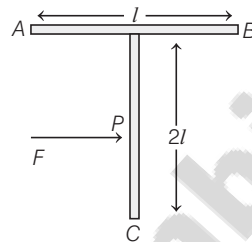
- (b) does not shift
(c) body C
(d) body B

Ans. (b)

Since, the acceleration of centre of mass in both the cases is same equal to g , so the centre of mass of the bodies B and C taken together does not shift compared to that of body A.

- 61** A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force F is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.

[AIEEE 2005]



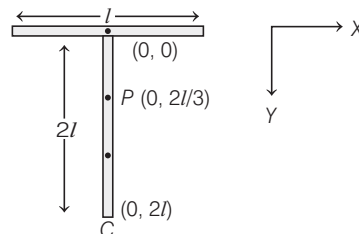
- (a) $\frac{2}{3}l$ (b) $\frac{3}{2}l$
(c) $\frac{4}{3}l$ (d) l

Ans. (c)

For pure translatory motion, net torque about centre of mass should be zero. Thus, F is applied at centre of mass of system,

$$OP = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2}$$

where, m_1 and m_2 are masses of horizontal and vertical section of the object. Assuming object is uniform,



$$\begin{aligned}m_2 &= 2m_1 \Rightarrow OP = \frac{2l}{3} \\ PC &= \left(l - \frac{2l}{3} + l \right) \\ &= \left(2l - \frac{2l}{3} \right) = \frac{4l}{3}\end{aligned}$$

- 62** A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected? [AIEEE 2004]

- (a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

Ans. (b)

In free space, neither acceleration due to gravity nor external torque act on the rotating solid sphere. Therefore, taking the same mass of sphere, if radius is increased, then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

- 63** A particle performing uniform circular motion has angular momentum L . If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is [AIEEE 2003]

- (a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $L/2$

Ans. (a)

Angular momentum,

$$L = I\omega \quad \dots(i)$$

$$\text{Kinetic energy, } K = \frac{1}{2} I \omega^2 = \frac{1}{2} L \omega$$

[from Eq. (i)]

$$\therefore L = \frac{2K}{\omega}$$

$$\text{Now, } L' = \frac{2\left(\frac{K}{2}\right)}{2\omega}$$

$$\Rightarrow L' = \frac{L}{4}$$

- 64** Let F be the force acting on a particle having position vector r and τ be the torque of this force about the origin. Then, [AIEEE 2003]

- (a) $r \cdot \tau = 0$ and $F \cdot \tau \neq 0$
(b) $r \cdot \tau \neq 0$ and $F \cdot \tau = 0$
(c) $r \cdot \tau \neq 0$ and $F \cdot \tau \neq 0$
(d) $r \cdot \tau = 0$ and $F \cdot \tau = 0$

Ans. (d)

As $\tau = r \times F$

τ is perpendicular to both r and F , so $r \cdot \tau$ as well as $F \cdot \tau$ has to be zero.

- 65** Initial angular velocity of a circular disc of mass M is ω_1 . Then, two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

[AIEEE 2002]

- (a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
 (c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

Ans. (c)

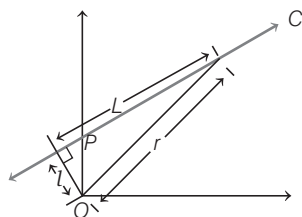
Conservation of angular momentum gives

$$\frac{1}{2}MR^2\omega_1 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega_2$$

$$\Rightarrow \frac{1}{2}MR^2\omega_1 = \frac{1}{2}R^2(M+4m)\omega_2$$

or $\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$

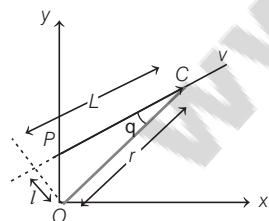
- 66** A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about O ? [AIEEE 2002]



- (a) mvL (b) $mv l$ (c) mvr (d) Zero

Ans. (b)

Angular momentum of particle about O ,



$$\mathbf{V} = m(\mathbf{r} \times \mathbf{v})$$

$$|\mathbf{V}| = mrv \sin\theta = mv(r \sin\theta) = mv l$$

- 67** Two identical particles move towards each other with velocity $2v$ and v , respectively. The velocity of centre of mass is [AIEEE 2002]

- (a) v (b) $\frac{v}{3}$ (c) $\frac{v}{2}$ (d) zero

Ans. (c)

Let mass of each body be m . Their motion is represented as shown in figure.



$$\text{From } v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_{CM} = \frac{m \times 2v - mv}{m + m} = \frac{v}{2}$$

[the direction of motion of first particle is taken as positive]

So, velocity of centre of mass of the system is $\frac{v}{2}$ in the direction of motion of particle having larger speed.

TOPIC 2 Moment of Inertia

- 68** A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity. Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is ms^{-1} . (Take, $g = 10 \text{ ms}^{-2}$)

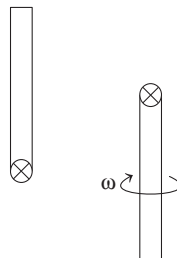
[2021, 31 Aug Shift-I]

Ans. (6)

Given, the mass of the steel rod, $m = 2 \text{ kg}$

The length of the steel rod, $l = 0.6 \text{ m}$

According to question, the diagram can be as shown below



By using the energy conservation,

$$mgl = \frac{1}{2}I\omega^2$$

$$mgl = \frac{1}{2} \times \frac{ml^2}{3} \times \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{6g}{l}}$$

As we know the relation between the linear speed and angular speed,

$$v = \omega r = \omega l = \sqrt{6gl}$$

Substituting the values in the above equation, we get

$$v = \sqrt{6 \times 10 \times 0.6} = 6 \text{ m/s}$$

Hence, the speed of the free end of the rod when it passes through its lowest position is 6 m/s.

- 69** A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the end of light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its mid-point? [2021, 31 Aug Shift-II]

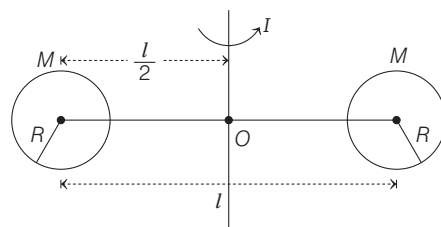
- (a) 18.75 kgm^2 (b) $1.905 \times 10^5 \text{ kgm}^2$
 (c) 19.05 kgm^2 (d) $1.875 \times 10^5 \text{ kgm}^2$

Ans. (c)

Given, mass of each sphere, $M = 1.5 \text{ kg}$

Radius of each sphere, $R = 50 \text{ cm}$
 $= 50 \times 10^{-2} \text{ m}$

Distance between centre of spheres,
 $l = 5 \text{ m}$



By using parallel axis theorem, moment of inertia of system is

$$I = 2 \left[\frac{2}{5}MR^2 + \frac{Ml^2}{4} \right]$$

$$I = 2M \left[\frac{2}{5}R^2 + \frac{l^2}{4} \right]$$

Substituting the values, we get

$$I = 2 \times 1.5 \left[\frac{2}{5} \times (50 \times 10^{-2})^2 + \frac{(5)^2}{4} \right]$$

$$= 19.05 \text{ kg-m}^2$$

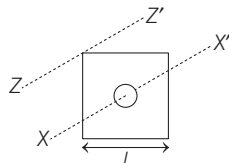
- 70** Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by

[2021, 27 Aug Shift-I]

- (a) $\frac{Ml^2}{6}$ (b) Ml^2 (c) $\frac{Ml^2}{12}$ (d) $\frac{2}{3}Ml^2$

Ans. (d)

Let the length of side of square plate is l .
The diagram of square plate having an axis passing from centre as $X-X'$ and an axis passing through one of the corner and perpendicular to plane of square plate $Z-Z'$ is shown below.



The distance between the centre of square plate and the corner of square plate is

$$d = \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$$

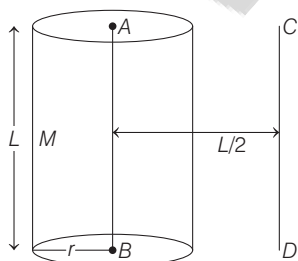
We know that, moment of inertia of square plate about centre,
 $I_{XX'} = \frac{Ml^2}{6}$

By parallel axis theorem, moment of inertia of square plate about axis $Z-Z'$ will be

$$\begin{aligned} I_{ZZ'} &= I_{XX'} + M\left(\frac{l}{\sqrt{2}}\right)^2 \\ I_{ZZ'} &= \frac{Ml^2}{6} + \frac{Ml^2}{2} \\ &= Ml^2 \left[\frac{1}{6} + \frac{1}{2} \right] \\ I_{ZZ'} &= \frac{2Ml^2}{3} \end{aligned}$$

Thus, the moment of inertia about an axis passing through corner of square plate and perpendicular to plane of plate is $2Ml^2/3$.

- 71** The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used, if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2 . [2021, 26 Aug Shift-II]



- (a) 14.9 kg/m^3 (b) $7.5 \times 10^1 \text{ kg/m}^3$
(c) $7.5 \times 10^2 \text{ kg/m}^3$ (d) $1.49 \times 10^2 \text{ kg/m}^3$

Ans. (d)

Given, the length of cylinder,
 $L = 80 \text{ cm} = 0.8 \text{ m}$

The radius of cylinder, $r = 20 \text{ cm} = 0.2 \text{ m}$

The moment of inertia about CD ,
 $I_{CD} = 2.7 \text{ kg-m}^2$

Now, the moment of inertia of cylinder about its axis AB is

$$I_{AB} = \frac{1}{2} Mr^2$$

By parallel axis theorem, the moment of inertia of cylinder about CD will be calculated as

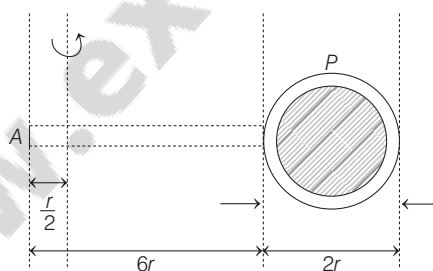
$$\begin{aligned} I_{CD} &= I_{AB} + M\left(\frac{L}{2}\right)^2 \\ \Rightarrow 2.7 &= \frac{Mr^2}{2} + \frac{ML^2}{4} \\ \Rightarrow 2.7 &= M \left[\frac{(0.2)^2}{2} + \frac{(0.8)^2}{4} \right] \\ \Rightarrow M &= 15 \text{ kg} \end{aligned}$$

Density of cylinder is given as

$$\begin{aligned} \rho &= \frac{M}{V} = \frac{M}{\pi r^2 L} = \frac{15}{\pi (0.2)^2 (0.8)} \\ &= 149.2 \text{ kg m}^{-3} \\ &= 1.49 \times 10^2 \text{ kg m}^{-3} \end{aligned}$$

Thus, the density of cylinder is $1.49 \times 10^2 \text{ kg m}^{-3}$.

- 72** Consider a badminton racket with length scales as shown in the figure.

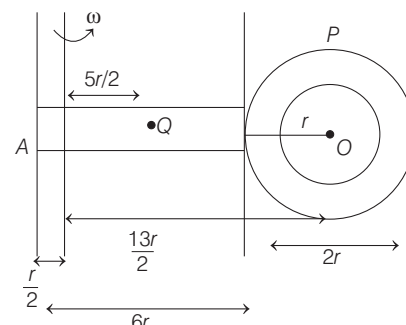


If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be Mr^2 .

[2021, 26 Aug Shift-I]

Ans. (52)

The given situation is shown in the following figure



The moment of inertia of the racket about an axis perpendicular to the handle and in the plane of ring at, $\frac{r}{2}$ distance from end A of the handle is given by

$$I = I_{\text{due to handle}} + I_{\text{due to disc}} \quad \dots(i)$$

$$\begin{aligned} I_{\text{due to handle}} &= \frac{1}{12} M(6r)^2 + M\left(\frac{5r}{2}\right)^2 \\ &= \frac{1}{12} M36r^2 + \frac{25Mr^2}{4} = 3Mr^2 + \frac{25}{4} Mr^2 \\ &= Mr^2 \left(\frac{37}{4} \right) = \frac{37}{4} Mr^2 \end{aligned}$$

$$\begin{aligned} I_{\text{due to disc}} &= \frac{Mr^2}{2} + M\left(\frac{13r}{2}\right)^2 \\ &= \frac{Mr^2}{2} + \frac{169Mr^2}{4} = \frac{171Mr^2}{4} \end{aligned}$$

$\therefore$ From Eq. (i), we get

$$\begin{aligned} I &= I_{\text{due to handle}} + I_{\text{due to disc}} \\ &= \frac{37}{4} Mr^2 + \frac{171Mr^2}{4} = \frac{208}{4} Mr^2 = 52Mr^2 \end{aligned}$$

73

	List I	List II
A.	Moment of inertia of the rod (length L , mass M , about an axis perpendicular to the rod passing through the mid-point)	I. $\frac{8ML^2}{3}$
B.	Moment of inertia of the rod (length L , mass $2M$, about an axis perpendicular to the rod passing through one of its end)	II. $\frac{ML^2}{3}$
C.	Moment of inertia of the rod (length $2L$, mass M , about an axis perpendicular to the rod passing through its midpoint)	III. $\frac{ML^2}{12}$
D.	Moment of inertia of the rod (length $2L$, mass $2M$, about an axis perpendicular to the rod passing through one of its end)	IV. $\frac{2ML^2}{3}$

Choose the correct answer from the options given below.

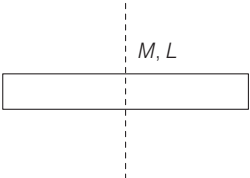
[2021, 27 July Shift-I]

- | | | | | |
|-----|-------|-------|-------|------|
| | A | B | C | D |
| (a) | (II) | (III) | (I) | (IV) |
| (b) | (II) | (I) | (III) | (IV) |
| (c) | (III) | (IV) | (II) | (I) |
| (d) | (III) | (IV) | (I) | (II) |

Ans. (c)

Case I

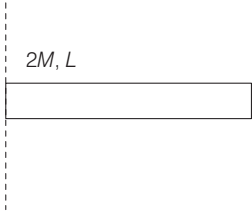
Moment of inertia of the rod (length L , mass M , about an axis perpendicular to the rod passing through the mid point)



$$= \frac{ML^2}{12}$$

Case II

Moment of inertia of rod (length L , mass $2M$ about an axis perpendicular to the rod passing through one of its end)



$$= \frac{(2M)(L^2)}{3} = \frac{2ML^2}{3}$$

Case III

Moment of inertia of rod (length $2L$, mass M , about an axis perpendicular to the rod passing through its mid point)



$$= \frac{M(2L)^2}{12} = \frac{ML^2}{3}$$

Case IV

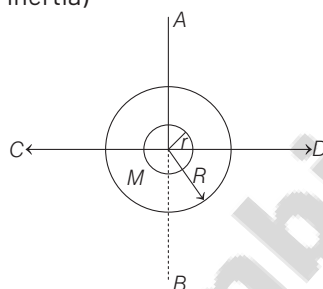
Moment of inertia of rod (length $2L$, mass $2M$ about an axis perpendicular to the rod passing through one of its end)



$$= \frac{2M(2L)^2}{3} = \frac{8}{3} ML^2$$

Hence, option (c) is the correct.

- 74** The figure shows two solid discs with radius R and r , respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (which is perpendicular to the plane of the disc and passing through its centre) of MI of smaller disc around one of its diameters lying on its plane? [2021, 27 July Shift-I]
- Given, M is the mass of the larger disc. (MI stands for moment of inertia)



- (a) $R^2 : r^2$ (b) $2r^4 : R^4$
 (c) $2R^2 : r^2$ (d) $2R^4 : r^4$

Ans. (d)

As per question, mass per unit area of both disc is same.

Let mass per unit area be σ .

$\therefore$ Mass of smaller disc, $M' = \pi r^2 \sigma$

Mass of larger disc, $M = \pi R^2 \sigma$

$\therefore$ Moment of inertia of smaller disc around one of its diameter lying on its plane, $I' = \frac{1}{4} M' r^2$

$$\Rightarrow I' = \frac{\pi \sigma r^4}{4}$$

and moment of inertia of bigger disc around axis AB (perpendicular to the plane) of disc and passing through its centre,

$$I = \frac{1}{2} MR^2$$

$$= \frac{\pi \sigma R^4}{2}$$

$$\therefore \frac{I}{I'} = \frac{\frac{\pi \sigma R^4}{2}}{\frac{\pi \sigma r^4}{4}} = \frac{2R^4}{r^4}$$

$$\Rightarrow \frac{I}{I'} = \frac{2R^4}{r^4}$$

- 75 Assertion A** Moment of inertia of a circular disc of mass M and radius R about X, Y -axes (passing through its plane) and Z -axis which is perpendicular to its plane were found to be I_x, I_y and I_z , respectively. The respective radii of gyration about all the three axes will be the same.

Reason R A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 25 July Shift-1]

- (a) Both A and R are correct and R is the correct explanation of A.
 (b) Both A and R are correct but R is not the correct explanation of A.
 (c) A is correct but R is not correct.
 (d) A is not correct but R is correct.

Ans. (d)

Given, I_x, I_y and I_z be the moment of inertia of disc about X, Y and Z -axes.

By using perpendicular axis theorem,

$$I_z = I_x + I_y$$

where, I_z is the moment of inertia about its centre of mass $= \frac{MR^2}{2}$

and in symmetrical body, $I_x = I_y$ (Disc in given case)

$$\therefore I_z = 2I_x$$

$$\Rightarrow I_x = I_y = \frac{I_z}{2} = \frac{MR^2}{4}$$

Now, $I_x = I_y \neq I_z$

As, radius of gyration, $k = \sqrt{I/M}$

$$\Rightarrow k \propto I$$

So, radii of gyration about all axis will not be the same.

$\therefore$ Assertion is not correct but Reason is correct because in a rigid body making rotational motion has fixed mass M and radius R ($I_x = I_y$).

- 76** Consider a uniform wire of mass M and length L . It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is

[2021, 18 March Shift-II]

- (a) $\frac{1}{4} \frac{ML^2}{\pi^2}$ (b) $\frac{2}{5} \frac{ML^2}{\pi^2}$ (c) $\frac{ML^2}{\pi^2}$ (d) $\frac{1}{2} \frac{ML^2}{\pi^2}$

Ans. (c)

The uniform wire whose mass is M and length is L and bent into a semicircle of radius r .

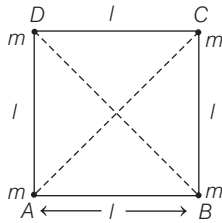
The length of the semicircle wire, $L = \pi r$
 $\Rightarrow r = \frac{L}{\pi}$

The moment of inertia,
 $I = Mr^2$

$\Rightarrow I = M \left(\frac{L}{\pi} \right)^2$

$\Rightarrow I = \frac{ML^2}{\pi^2}$

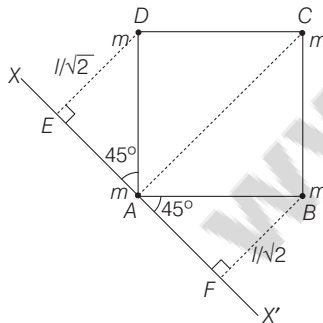
- 77** Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be **[2021, 16 March Shift-I]**



- (a) $1ml^2$ (b) $2ml^2$
 (c) $3ml^2$ (d) $\sqrt{3}ml^2$

Ans. (c)

The given situation can be represented as follows



In the above figure, XX' be the axis which passes through A and is parallel to DB .

$\therefore$ Moment of inertia of the system

= Mass $\times$ (Perpendicular distance from axis) ^{2}

$$I = m(AC)^2 + m(ED)^2 + m(FB)^2$$

$$= m(0)^2 + m(l/\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 3ml^2$$

- 78** Four identical solid spheres each of mass m and radius a are placed with their centres on the four corners of a square of side b . The moment of inertia of the system about one side of square, where the axis of rotation is parallel to the plane of the square is **[2021, 26 Feb Shift-I]**

- (a) $\frac{4}{5}ma^2 + 2mb^2$
 (b) $\frac{8}{5}ma^2 + mb^2$
 (c) $\frac{8}{5}ma^2 + 2mb^2$
 (d) $\frac{4}{5}ma^2$

Ans. (c)

Given, mass of solid sphere = m

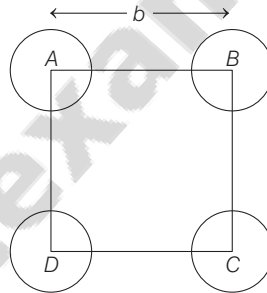
Radius of solid sphere = a

Side of square = b

Let I_{net} be the net moment of inertia of system.

Moment of inertia of sphere, $I_s = \frac{2}{5}ma^2$

Axis of rotation is BC .



$\therefore$ Moment of inertia of any body at distance,

$$d = ma^2 + mb^2 \quad (\because d = b)$$

$$\therefore I_{\text{net}} = 4 \left(\frac{2}{5}ma^2 \right) + 2(mb^2)$$

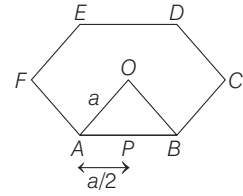
$$\Rightarrow I_{\text{net}} = \frac{8}{5}ma^2 + 2mb^2$$

- 79** A uniform thin bar of mass 6 kg and length 2.4 m is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is $\times 10^{-1} \text{ kg} \cdot \text{m}^2$. **[2021, 24 Feb Shift-II]**

Ans. (8)

Given, mass of uniform bar, $m = 6 \text{ kg}$

Length of bar, $l = 2.4 \text{ m}$



Side of hexagon, $a = \frac{2.4}{6} = 0.4 \text{ m}$

Mass of each side, $m' = 6/6 = 1 \text{ kg}$

and $OP = \sqrt{a^2 - (a/2)^2} = a\sqrt{3}/2$

$$= 0.2\sqrt{3} \text{ m}$$

Now, by using parallel axes theorem,

$$I_{OP} = \frac{m'a^2}{12} + m' \left(\frac{\sqrt{3}a}{2} \right)^2$$

$$= \frac{a^2}{12} + \frac{3a^2}{4} = \frac{10a^2}{12} = \frac{5a^2}{6}$$

$$\Rightarrow I_{OP} = \frac{5}{6} \times 0.4 \times 0.4$$

and I_{net} (net moment of inertia at O)

$$= 6 \times \frac{5}{6} \times 0.4 \times 0.4$$

$$= 8 \times 10^{-1} \text{ kg} \cdot \text{m}^2$$

- 80** Moment of inertia (MI) of four bodies, having same mass and radius, are reported as
 I_1 = MI of thin circular ring about its diameter,
 I_2 = MI of circular disk about an axis perpendicular to the disk and going through the centre,
 I_3 = MI of solid cylinder about its axis
 and I_4 = MI of solid sphere about its diameter. Then,

[2021, 24 Feb Shift-I]

(a) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$

(b) $I_1 + I_3 < I_2 + I_4$

(c) $I_1 = I_2 = I_3 < I_4$

(d) $I_1 = I_2 = I_3 > I_4$

Ans. (d)

Let M and R be the mass and radius of four bodies. Then, as per question, their moment of inertia are

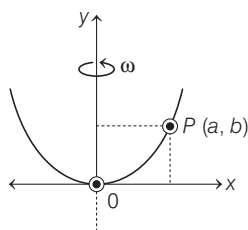
$$I_1 = \frac{MR^2}{2}, I_2 = \frac{MR^2}{2}, I_3 = \frac{MR^2}{2},$$

$$I_4 = \frac{2}{5}MR^2$$

Clearly, $I_1 = I_2 = I_3 > I_4$

- 81** A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction)

[2020, 02 Sep Shift-I]



(a) $2\sqrt{2gC}$

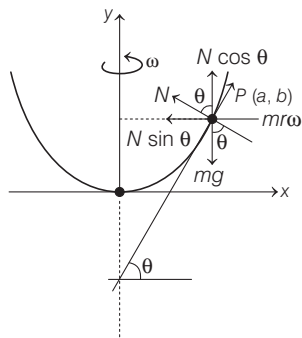
(b) $\sqrt{\frac{2g}{C}}$

(c) $2\sqrt{gC}$

(d) $\sqrt{\frac{2gC}{ab}}$

Ans. (a)

Various forces acting on bead at point P can be resolved as shown in figure.



Normal force of rotating wire are

$$N \cos \theta = mg \text{ and } N \sin \theta = m\omega^2 x$$

$$\Rightarrow \tan \theta = \frac{dy}{dx} = \frac{x\omega^2}{g} \quad \dots(i)$$

Given, $y = 4Cx^2$

$$\Rightarrow \frac{dy}{dx} = 8Cx$$

$$\left. \frac{dy}{dx} \right|_{\text{at } x=a, y=b} = 8Ca \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{x\omega^2}{g} = 8Ca$$

At $x=a$, we have

$$\frac{a\omega^2}{g} = 8Ca$$

$$\Rightarrow \omega = \sqrt{8Cg} = 2\sqrt{2gC}$$

- 82** Moment of inertia of a cylinder of mass M , length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is

$$I = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$$

If such a cylinder is to be made for a given mass of a material. To have minimum possible moment of inertia, the ratio L/R for cylinder is

[2020, 03 Sep Shift-I]

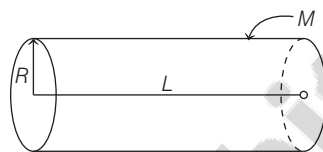
(a) $\frac{2}{3}$

(b) $\frac{3}{2}$

(c) $\sqrt{\frac{3}{2}}$

(d) $\sqrt{\frac{2}{3}}$

Ans. (c)



$$\text{Mass of cylinder, } M = \rho \pi R^2 L \quad \dots(i)$$

(where, ρ = density of material)

$$\Rightarrow L = \frac{M}{\rho \pi R^2} \quad \dots(ii)$$

Moment of inertia of a cylinder,

$$I = M \left(\frac{R^2}{4} + \frac{L^2}{12} \right)$$

$$\Rightarrow I = M \left(\frac{R^2}{4} + \frac{M^2}{12\rho^2\pi^2R^4} \right)$$

[From Eq. (ii)]

For moment of inertia to be minimum,

$$\frac{dI}{dR} = 0$$

$$\Rightarrow \frac{d}{dR} \left[M \left(\frac{R^2}{4} + \frac{M^2}{12\rho^2\pi^2R^4} \right) \right] = 0$$

$$\Rightarrow \frac{2R}{4} + \frac{M^2}{12\rho^2\pi^2} \cdot (-4R^{-5}) = 0$$

[$\because M = \text{constant}$]

$$\Rightarrow \frac{2R}{4} = \frac{4M^2}{12\rho^2\pi^2R^5}$$

$$\Rightarrow R^6 = \frac{2}{3} \frac{M^2}{\rho^2\pi^2}$$

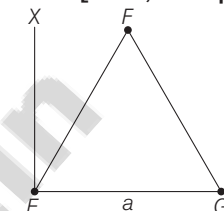
$$\Rightarrow R^6 = \frac{2}{3} \frac{\rho^2\pi^2R^4L^2}{\rho^2\pi^2} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \left(\frac{R}{L} \right)^2 = \frac{2}{3} \Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

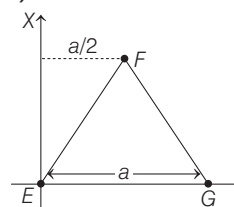
Hence, option (c) is correct.

- 83** A massless equilateral triangle EFG of side a (as shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20} ma^2$, where N is an integer. The value of N is

[2020, 03 Sep Shift-II]



Ans. (25)



Moment of inertia of system

$$= \sum_{i=1}^3 m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

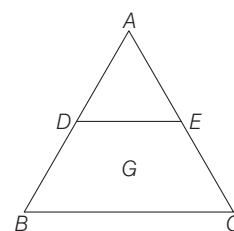
$$= m \times 0 + m \times a^2 + m \times \left(\frac{a}{2} \right)^2$$

$$= \frac{5}{4} ma^2 = \frac{25}{20} ma^2 = \frac{N}{20} ma^2$$

$$\therefore N = 25$$

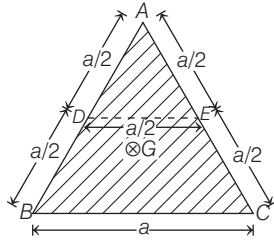
- 84** ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid-points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0 . If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$, where N is an integer. Value of N is

[2020, 04 Sep Shift-I]



Ans. (11)

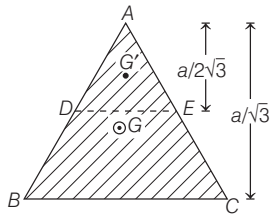
Let the mass of plane lamina having a shape of equilateral triangle be m and its side be a .



Then, the moment of inertia of this lamina ABC about the axis passing through its centroid G and perpendicular to its plane is

$$I_0 = \frac{ma^2}{12} \quad \dots(i)$$

Now, part ADE is removed.



The side of the new triangular lamina ADE is $\frac{a}{2}$.

It is also equilateral in shape.

If its centroid is G' , so $AG' = \frac{a/2}{\sqrt{3}} = \frac{a}{2\sqrt{3}}$

We know that for triangular lamina ABC , the centroid is G .

$$\text{So, } AG = \frac{a}{\sqrt{3}}$$

$$\begin{aligned} \text{Now, side } G'G &= AG - AG' \\ &= \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}} \end{aligned}$$

Now, the moment of inertia of lamina ADE about the axis passing through point G and perpendicular to its plane (using parallel axis theorem)

$$I_1 = \frac{m_1 \left(\frac{a}{2}\right)^2}{12} + m_1 (G'G)^2 \quad \dots(ii)$$

Let m_1 be the mass of lamina ADE .

$$\therefore m_1 = \frac{\text{mass of lamina } ABC}{\text{area of lamina } ABC}$$

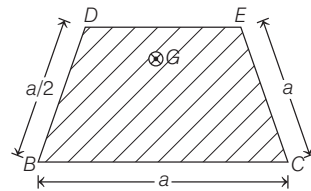
$$ADE = \frac{m}{\sqrt{3}(a)^2} \times \frac{\sqrt{3}}{4} \left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

So, putting all the value in eq. (ii), we get

$$\begin{aligned} I_1 &= \frac{\left(\frac{m}{4}\right) \left(\frac{a}{2}\right)^2}{12} + \left(\frac{m}{4}\right) \left(\frac{a}{2\sqrt{3}}\right)^2 \\ &= \frac{ma^2}{192} + \frac{ma^2}{48} = \frac{5ma^2}{192} \quad \dots(iii) \end{aligned}$$

So, the moment of inertia of remaining part about the axis passing through point G and perpendicular to its plane is given by

$$\begin{aligned} I_{\text{net}} &= I_0 - I_1 \\ &= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{192} \\ &= \frac{11}{16} \times \frac{ma^2}{12} = \frac{11}{16} I_0 \end{aligned}$$

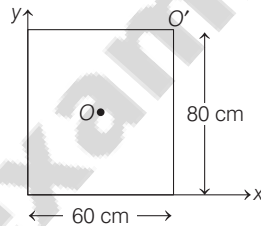


It is given that, $I_{\text{net}} = \frac{N}{16} I_0$

On comparing, we get

$$N = 11$$

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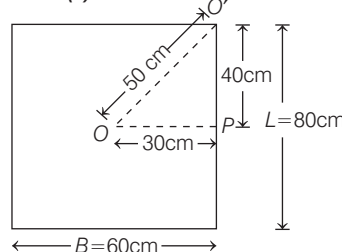


For a uniform rectangular sheet, shown in the above figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is

[2020, 04 Sep Shift-II]

- (a) 1/8 (b) 2/3 (c) 1/4 (d) 1/2

Ans. (c)



Let the mass of this rectangular sheet be M .

From figure,

$$O'P = \frac{L}{2} = \frac{80}{2} \text{ cm} = 40 \text{ cm}$$

$$OP = \frac{B}{2} = \frac{60}{2} \text{ cm} = 30 \text{ cm}$$

$$OO' = \sqrt{(OP)^2 + (O'P)^2}$$

$$\begin{aligned} &\text{(by Pythagoras' theorem)} \\ &= \sqrt{(30)^2 + (40)^2} \\ &= \sqrt{900 + 1600} \\ &= \sqrt{2500} = 50 \text{ cm} \end{aligned}$$

Now, moment of inertia about the axis perpendicular to the plane of sheet and passing through point O is

$$\begin{aligned} I_0 &= \frac{M}{12} [L^2 + B^2] = \frac{M}{12} [(80)^2 + (60)^2] \\ &= \frac{M}{12} [6400 + 3600] = \frac{M}{12} [10000] \quad \dots(i) \end{aligned}$$

And moment of inertia about the axis perpendicular to the plane of sheet and passing through point O' is

$$I_{O'} = I_0 + Md^2$$

(using theorem of parallel axis)

Here, $d = OO' = 50 \text{ cm}$

$$\begin{aligned} \text{So, } I_{O'} &= \frac{M}{12} [10000] + M[50]^2 \\ &= \frac{M}{12} [10000] + M[2500] \\ &= \frac{M[10000] + M[30000]}{12} \\ &= \frac{M[40000]}{12} \quad \dots(ii) \end{aligned}$$

Dividing eq. (i) by eq. (ii), we get

$$\frac{I_0}{I_{O'}} = \frac{\frac{M}{12} [10000]}{\frac{M}{12} [40000]} = \frac{1}{4}$$

Hence, option (c) is correct.

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Consider two uniform discs of same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2 respectively, about their axes is $I_1 : I_2 = 1 : 16$, then the value of α is

[2020, 04 Sep Shift-II]

- (a) 2 (b) $2\sqrt{2}$
(c) 4 (d) $\sqrt{2}$

Ans. (a)

Both the discs are made up of same material.

Let the density of material used for both discs be ρ and their thickness be t .

Now, moment of inertia of first disc,

$$I_1 = \frac{M_1 R_1^2}{2} = \frac{\rho A_1 t R_1^2}{2}$$

$$= \frac{\rho(\pi R_1^2)tR_1^2}{2} = \frac{\pi\rho R_1^4 t}{2}$$

$$\therefore R_1 = R \quad (\text{given})$$

$$\therefore I_1 = \frac{\pi\rho R^4 t}{2} \quad \dots(i)$$

And, moment of inertia of second disc,

$$I_2 = \frac{M_2 R_2^2}{2} = \frac{\rho A_2 t R_2^2}{2}$$

$$= \frac{\rho(\pi R_2^2)tR_2^2}{2} = \frac{\pi\rho R_2^4 t}{2}$$

$$\therefore R_2 = \alpha R \quad (\text{given})$$

$$\therefore I_2 = \frac{\pi\rho(\alpha R)^4 t}{2} = \frac{\alpha^4 \pi\rho R^4 t}{2} \quad \dots(ii)$$

Dividing eq. (i) by eq. (ii), we get

$$\frac{I_1}{I_2} = \frac{\left(\frac{\pi\rho R^4 t}{2}\right)}{\left(\frac{\alpha^4 \pi\rho R^4 t}{2}\right)} \Rightarrow \frac{I_1}{I_2} = \frac{1}{\alpha^4} \quad \dots(iii)$$

It is given that,

$$\frac{I_1}{I_2} = \frac{1}{16} \quad \dots(iv)$$

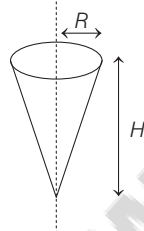
On comparing the eqs. (iii) and (iv), we get

$$\frac{1}{\alpha^4} = \frac{1}{16}$$

$$\Rightarrow \alpha = (16)^{1/4} = (2^4)^{1/4} = 2$$

Hence, option (a) is correct.

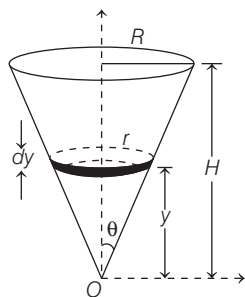
- 87** Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M , radius of its top R and height H , then its moment of inertia about its axis is
[2020, 06 Sep Shift-I]



- (a) $\frac{MR^2}{2}$ (b) $\frac{M(R^2 + H^2)}{4}$
(c) $\frac{MH^2}{3}$ (d) $\frac{MR^2}{3}$

Ans. (a)

The hollow icecream cone can be assumed to be formed of several parts of rings having different radii. In general we shall consider a ring (element) which is at distance y away from vertex O , of thickness dy , having radius r as shown in figure.



$$\text{Area of cone, } A = \pi Rl = \pi R\sqrt{R^2 + H^2}$$

$$\text{Area of element, } dA = 2\pi r dl = 2\pi r \frac{dy}{\cos\theta}$$

Mass of element, $dm = \text{Surface density} \times \text{Area}$

$$= \frac{M}{\pi R\sqrt{R^2 + H^2}} \times \frac{2\pi r dy}{\cos\theta}$$

From above diagram,

$$\tan\theta = \frac{r}{y} \text{ or } r = y \tan\theta$$

$$\therefore dm = \frac{2M \tan\theta y dy}{R \cos\theta \sqrt{R^2 + H^2}}$$

Moment of inertia of element about its axis

$$dl = (dm)r^2 = \left(\frac{2M \tan\theta y dy}{R \cos\theta \sqrt{R^2 + H^2}} \right) (y \tan\theta)^2$$

$$= \frac{2M \tan^3\theta y^3 dy}{R \cos\theta \sqrt{R^2 + H^2}}$$

$\therefore$ Total moment of inertia,

$$I = \int dl = \frac{2M \tan^3\theta}{R \cos\theta \sqrt{R^2 + H^2}} \int_0^H y^3 dy$$

$$= \frac{2M \tan^3\theta}{R \cos\theta \sqrt{R^2 + H^2}} \cdot \frac{H^4}{4}$$

Again, from above diagram,

$$\tan\theta = \frac{R}{H}$$

$$\cos\theta = \frac{H}{\sqrt{R^2 + H^2}}$$

Putting these values in the expression of I , we get

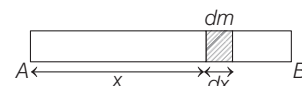
$$I = \frac{MR^2}{2}$$

- 88** The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0 \left(1 + \frac{x}{L}\right)$, where x is the distance from A . If M is mass of the rod, then its moment of inertia about an axis passing through A and perpendicular to the rod is
[2020, 06 Sep Shift-II]

- (a) $\frac{5}{12} ML^2$ (b) $\frac{7}{18} ML^2$
(c) $\frac{2}{5} ML^2$ (d) $\frac{3}{7} ML^2$

Ans. (b)

According to question,



$$\text{Linear mass density, } \lambda(x) = \lambda_0 \left(1 + \frac{x}{L}\right)$$

Moment of inertia of small element about given axis,

$$dl = (dm)x^2 = (\lambda dx)x^2$$

$$\Rightarrow dl = \lambda_0 \left(1 + \frac{x}{L}\right) x^2 dx$$

Total moment of inertia about same axis due to the complete rod,

$$I = \int dl = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) x^2 dx$$

$$= \lambda_0 \left[\frac{x^3}{3} + \frac{x^4}{4L} \right]_0^L$$

$$= \frac{7L^3 \lambda_0}{12} \quad \dots(i)$$

Now, mass of complete rod,

$$M = \int dm = \int_0^L \lambda dx = \int_0^L \lambda_0 \left(1 + \frac{x}{L}\right) dx$$

$$= \lambda_0 \left[x + \frac{x^2}{2L} \right]_0^L$$

$$= \lambda_0 \left[L + \frac{L^2}{2L} - 0 \right] = \frac{3\lambda_0 L}{2} \Rightarrow \lambda_0 = \frac{2M}{3L}$$

Putting the value of λ_0 in Eq. (i), we get

$$I = \frac{7}{18} ML^2$$

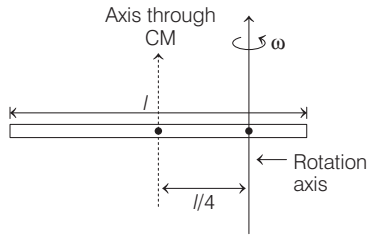
Hence, correct option is (b).

- 89** The radius of gyration of a uniform rod of length L , about an axis passing through a point $\frac{L}{4}$ away from the centre of the rod and perpendicular to it, is
[2020, 7 Jan Shift-I]

- (a) $\frac{1}{8} L$ (b) $\sqrt{\frac{3}{8}} L$
(c) $\sqrt{\frac{7}{48}} L$ (d) $\frac{1}{4} L$

Ans. (c)

To calculate radius of gyration, first we calculate moment of inertia of rod about given axis.



Moment of inertia of rod about an axis through

centre of mass, $I_c = \frac{ml^2}{12}$

where, m = mass and l = length of rod.
Moment of inertia of rod about given axis of rotation by parallel axes theorem will be

$$I = I_c + mh^2 = m \frac{l^2}{12} + m \left(\frac{l}{4} \right)^2 = \frac{7}{48} ml^2$$

If k = radius of gyration of rod, then moment of inertia of rod, $I = mk^2$

$$\Rightarrow mk^2 = \frac{7}{48} ml^2$$

$$\therefore \text{Radius of gyration of rod, } k = \sqrt{\frac{7}{48} l}$$

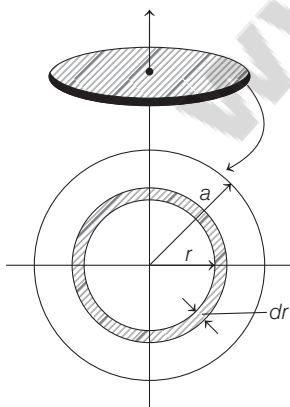
- 90** Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is

[2020, 7 Jan Shift-II]

- (a) $2\pi a^4 \left(\frac{A}{4} + \frac{B}{5} \right)$ (b) $2\pi a^4 \left(\frac{aA}{4} + \frac{B}{5} \right)$
(c) $\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$ (d) $2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$

Ans. (d)

Consider an elemental ring of thickness dr at a distance from the centre.



Area of this elemental ring, $dA = 2\pi r dr$
Given, mass per unit area, $\sigma(r) = A + Br$
Mass of elemental ring,

$$dm = \sigma(r) \cdot dA = (A + Br) 2\pi r dr$$

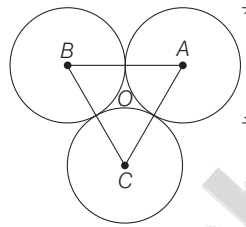
Moment of inertia of this elemental ring about axis, $dI = dm \cdot r^2$

$$= 2\pi r^3 (A + Br) \cdot dr$$

Moment of inertia of complete disc of radius a ,

$$\begin{aligned} I &= \int_0^a dI = \int_0^a 2\pi r^3 (A + Br) dr \\ &= \left[2\pi A \frac{r^4}{4} + 2\pi B \frac{r^5}{5} \right]_0^a \\ &= \frac{2\pi A a^4}{4} + \frac{2\pi B a^5}{5} = 2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right) \end{aligned}$$

91



Three solid sphere each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d . The ratio I_0 / I_A of moment of inertia I_0 of the system about an axis passing through the centroid and about centre of any of the spheres I_A and perpendicular to the plane of the triangle is

[2020, 9 Jan Shift-I]

- (a) $\frac{15}{13}$ (b) $\frac{13}{15}$ (c) $\frac{13}{23}$ (d) $\frac{23}{13}$

Ans. (c)

In given arrangement, mass of each sphere = m

$$\text{Radius of each sphere} = \frac{d}{2}$$

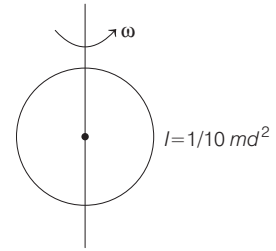
Distance of centre of each sphere from axis of rotation or centroidal axis = $\frac{2}{3} \times$

altitude of equilateral triangle of side d

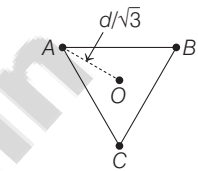
$$= \frac{2}{3} \times \frac{\sqrt{3}}{2} \times d = \frac{d}{\sqrt{3}}$$

Now, moment of inertia of each sphere along an axis through its centre,

$$I_1 = \frac{2}{5} mr^2 = \frac{2}{5} m \left(\frac{d}{2} \right)^2 = \frac{1}{10} md^2$$



Now, by using parallel axes theorem, moment of inertia of each sphere around axis through O ,



$$\begin{aligned} I' &= I_1 + m(h^2) \\ &= I_1 + m \left(\frac{d}{\sqrt{3}} \right)^2 \\ &= \frac{1}{10} md^2 + \frac{md^2}{3} = \frac{13}{30} md^2 \end{aligned}$$

So, moment of inertia of system of spheres about O ,

$$I_0 = 3I' = 3 \times \frac{13}{30} md^2 = \frac{13}{10} md^2 \quad \dots(i)$$

Again by using parallel axes theorem, moment of inertia of system about point A ,

$$\begin{aligned} I_A &= I_0 + m_{\text{system}} \times h^2 \\ &= \frac{13}{10} md^2 + 3m \times \left(\frac{d}{\sqrt{3}} \right)^2 \\ &= \frac{23}{10} md^2 \quad \dots(ii) \end{aligned}$$

Hence, required ratio from Eqs. (i) and (ii), we get

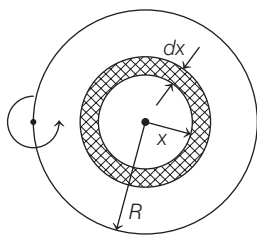
$$\frac{I_0}{I_A} = \frac{13}{10} md^2 \times \frac{10}{23 md^2} \Rightarrow \frac{I_0}{I_A} = \frac{13}{23}$$

- 92** A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is [2019, 8 April Shift-I]

- (a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{3}{2}$

Ans. (*)

Consider an elementary ring of thickness dx and radius x .



Moment of inertia of this ring about a perpendicular axes through centre is

$$dl_c = dm \cdot x^2 = \rho_0 (2\pi x) dx \cdot x^2 = 2\pi\rho_0 x^4 dx$$

Moment of inertia of this elementary ring about a perpendicular axes at a point through edge, (by parallel axes theorem)

$$dl = dm x^2 + dm R^2 \\ = 2\pi\rho_0 x^4 dx + 2\pi\rho_0 R^2 x^2 dx$$

Moment of inertia of complete disc is

$$I = \int_0^R dl = \int_0^R 2\pi\rho_0 x^4 dx + \int_0^R 2\pi\rho_0 R^2 x^2 dx \\ = \frac{2\pi\rho_0 R^5}{5} + \frac{2\pi\rho_0 R^5}{3} = \frac{16\pi\rho_0 R^5}{15}$$

$$\therefore a = \frac{16}{15}$$

- 93** A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is

[2019, 10 April Shift-I]

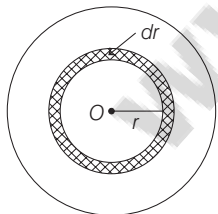
(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{6}$ (c) $\frac{MR^2}{3}$ (d) $\frac{2MR^2}{3}$

Ans. (d)

Given, surface mass density,

$$\sigma = kr^2$$

So, mass of the disc can be calculated by considering small element of area $2\pi r dr$ on it and then integrating it for complete disc, i.e.



$$dm = \sigma dA = \sigma \times 2\pi r dr$$

$$\int dm = M = \int_0^R (kr^2) 2\pi r dr$$

$$\Rightarrow M = 2\pi k \frac{R^4}{4} = \frac{1}{2} \pi k R^4 \quad \dots (i)$$

Moment of inertia about the axis of the disc,

$$I = \int dl = \int dm r^2 = \int \sigma dA r^2$$

$$= \int_0^R kr^2 (2\pi r dr) r^2 \\ \Rightarrow I = 2\pi k \int_0^R r^5 dr = \frac{2\pi k R^6}{6} = \frac{\pi k R^6}{3} \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$I = \frac{2}{3} MR^2$$

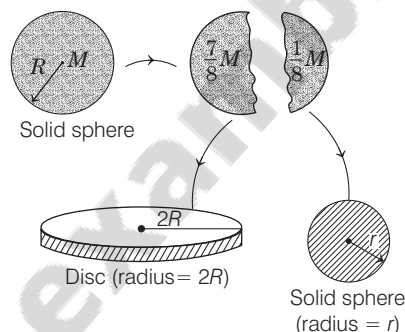
- 94** A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $7M/8$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1 / I_2 is given by

[2019, 10 April Shift-II]

- (a) 285 (b) 185 (c) 65 (d) 140

Ans. (d)

The given situation is shown in the figure given below



Density of given sphere of radius R is

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

Let radius of sphere formed from second part is r , then

mass of second part = volume $\times$ density

$$\frac{1}{8}M = \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore r^3 = \frac{R^3}{8} \Rightarrow r = \frac{R}{2}$$

Now, I_1 = moment of inertia of disc (radius $2R$ and mass $\frac{7}{8}M$) about its axis

$$= \frac{\text{Mass} \times (\text{Radius})^2}{2}$$

$$= \frac{\frac{7}{8}M \times (2R)^2}{2} = \frac{7}{4}MR^2$$

and I_2 = moment of inertia of sphere

(radius $\frac{R}{2}$ and mass $\frac{1}{8}M$) about its axis

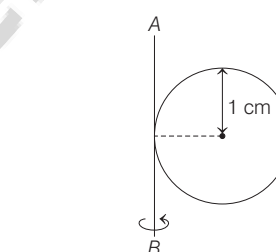
$$= \frac{2}{5} \times \text{Mass} \times (\text{Radius})^2$$

$$= \frac{2}{5} \times \frac{1}{8}M \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}$$

$$\therefore \text{Ratio } \frac{I_1}{I_2} = \frac{\frac{7}{4}MR^2}{\frac{1}{80}MR^2} = 140$$

- 95** A metal coin of mass 5g and radius 1cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s , is close to

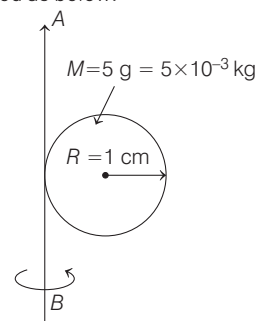
[2019, 10 April Shift-II]



- (a) $4.0 \times 10^{-6} \text{ N}\cdot\text{m}$ (b) $2.0 \times 10^{-5} \text{ N}\cdot\text{m}$
(c) $1.6 \times 10^{-5} \text{ N}\cdot\text{m}$ (d) $7.9 \times 10^{-6} \text{ N}\cdot\text{m}$

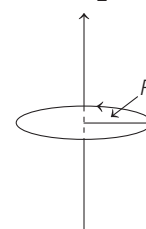
Ans. (b)

Moment of inertia (MI) of a disc about a tangential axis in the plane of disc can be obtained as below.



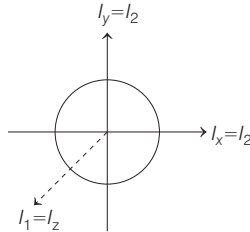
Moment of inertia of disc about its axis,

$$I_1 = \frac{MR^2}{2}$$



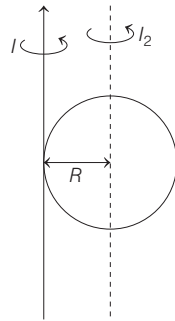
From perpendicular axes theorem, moment of inertia of disc about an axis along its diameter is

$$\begin{aligned} I_x + I_y &= I_z \\ \Rightarrow 2I_2 &= I_1 \\ \Rightarrow I_2 &= \frac{I_1}{2} = \frac{MR^2}{4} \end{aligned}$$



So, moment of inertia about a tangential axis from parallel axes theorem is

$$\begin{aligned} I &= I_2 + MR^2 = \frac{MR^2}{4} + MR^2 \\ &= \frac{5}{4}MR^2 \end{aligned}$$



Now, using torque, $\tau = I\alpha$, we have

$$\tau = I\alpha = \frac{5}{4}MR^2 \left(\frac{\omega_f - \omega_i}{\Delta t} \right)$$

Here,

$$M = 5 \times 10^{-3} \text{ kg}, R = 1 \times 10^{-2} \text{ m}$$

$$\omega_i = 25 \text{ rps} = 25 \times 2\pi \frac{\text{rad}}{\text{s}}$$

$$= 50\pi \frac{\text{rad}}{\text{s}}$$

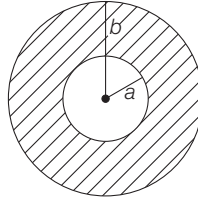
$$\omega_f = 0, \Delta t = 5 \text{ s}$$

$$\text{So, } \tau = \frac{\frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 50\pi}{5}$$

$$\approx 2 \times 10^{-5} \text{ N-m}$$

- 96** A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is

[2019, 12 April Shift-I]



$$\begin{aligned} \text{(a)} \quad & \sqrt{\frac{a^2 + b^2 + ab}{2}} \quad \text{(b)} \quad \frac{a+b}{2} \\ \text{(c)} \quad & \sqrt{\frac{a^2 + b^2 + ab}{3}} \quad \text{(d)} \quad \frac{a+b}{3} \end{aligned}$$

Ans. (c)

Key Idea Radius of gyration K of any structure is given by

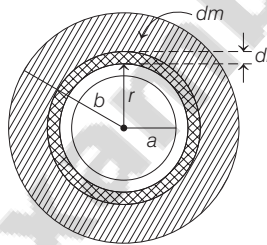
$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

To find K , we need to find both moment of inertia I and mass M of the given structure.

Given, variation in mass per unit area (surface mass density),

$$\sigma = \frac{\sigma_0}{r} \quad \dots(i)$$

Calculation of Mass of Disc



Let us divide whole disc in small area elements, one of them shown at r distance from the centre of the disc with its width as dr .

Mass of this element is

$$dm = \sigma \cdot dA$$

$$\Rightarrow dm = \frac{\sigma_0}{r} \times 2\pi r dr \quad [\text{from Eq. (i)}]$$

...(ii)

Mass of the disc can be calculated by integrating it over the given limits of r ,

$$\int_0^M dm = \int_a^b \sigma_0 \times 2\pi \times dr$$

$$\Rightarrow M = \sigma_0 2\pi (b-a) \quad \dots(iii)$$

Calculation of Moment of Inertia

$$I = \int_0^M r^2 dm = \int_a^b r^2 \cdot \frac{\sigma_0}{r} \times 2\pi r dr \quad [\text{from Eq. (i)}]$$

$$= \sigma_0 2\pi \int_a^b r^2 dr = \sigma_0 2\pi \left[\frac{r^3}{3} \right]_a^b$$

$$\Rightarrow I = \frac{1}{3} \sigma_0 2\pi [b^3 - a^3] \quad \dots(iv)$$

Now, radius of gyration,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{1}{3} \sigma_0 2\pi (b^3 - a^3)}{\sigma_0 2\pi (b-a)}}$$

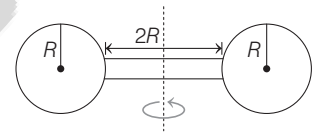
$$\Rightarrow K = \sqrt{\frac{1}{3} \frac{(b^3 - a^3)}{b-a}}$$

As we know, $b^3 - a^3 = (b-a)(b^2 + a^2 + ab)$

$$\therefore K = \sqrt{\frac{1}{3} (b^2 + a^2 + ab)}$$

$$\text{or } K = \sqrt{\frac{a^2 + b^2 + ab}{3}}$$

- 97** Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure).



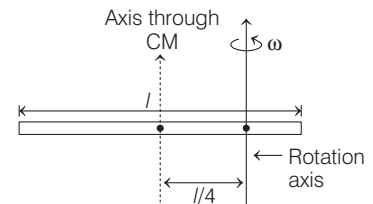
The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is

[2019, 10 Jan Shift-II]

$$\begin{aligned} \text{(a)} \quad & \frac{137}{15} MR^2 \quad \text{(b)} \quad \frac{209}{15} MR^2 \\ \text{(c)} \quad & \frac{17}{15} MR^2 \quad \text{(d)} \quad \frac{152}{15} MR^2 \end{aligned}$$

Ans. (a)

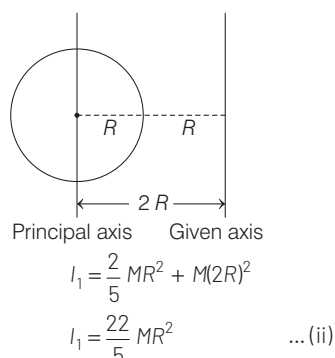
Key Idea This problem will be solved by applying parallel axis theorem, which states that moment of inertia of a rigid body about any axis is equals to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the axis.



We know that moment of inertia (MI) about the principle axis of the sphere is given by

$$I_{\text{sphere}} = \frac{2}{5} MR^2 \quad \dots(i)$$

Using parallel axis theorem, moment of inertia about the given axis in the figure below will be



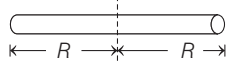
Considering both spheres at equal distance from the axis, moment of inertia due to both spheres about this axis will be

$$2I_1 = 2 \times \frac{22}{5} MR^2$$

Now, moment of inertia of rod about its perpendicular bisector axis is given by

$$I_2 = \frac{1}{12} ML^2$$

Here, given that $L = 2R$



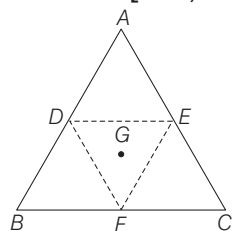
$$\therefore I_2 = \frac{1}{12} M(2R)^2 = \frac{1}{3} MR^2 \quad \dots (iii)$$

So, total moment of inertia of the system is

$$I = 2I_1 + I_2 = 2 \times \frac{22}{5} MR^2 + \frac{1}{3} MR^2$$

$$\Rightarrow I = \left(\frac{44}{5} + \frac{1}{3} \right) MR^2 = \frac{137}{15} MR^2$$

- 98** An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0 . If the smaller triangle DEF is removed from ABC , the moment of inertia of the remaining figure about the same axis is I . Then **[2019, 11 Jan Shift-I]**

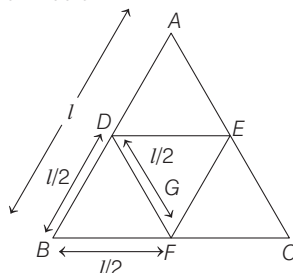


- (a) $I' = \frac{3}{4} I_0$ (b) $I' = \frac{15}{16} I_0$
(c) $I' = \frac{I_0}{4}$ (d) $I' = \frac{9}{16} I_0$

Ans. (b)

Suppose the mass of the $\triangle ABC$ be ' M ' and length of the side be ' l '.

When the $\triangle DEF$ is being removed from it, then the mass of the removed $\triangle$ will be ' $M/4$ ' and length of its side will be ' $l/2$ ' as shown below



Since we know, moment of inertia of the triangle about the axis, passing through its centre of gravity is, $I = kml^2$, where k is a constant.

Then for $\triangle DEF$, moment of inertia of the triangle about the axis,

$$I = k \left(\frac{M}{4} \right) \left(\frac{l}{2} \right)^2 = \frac{kMl^2}{16} \quad \dots (i)$$

Moment of inertia of $\triangle ABC$ is

$$I_0 = kMl^2 \quad \dots (ii)$$

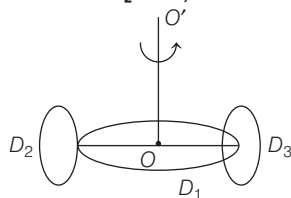
The moment of inertia of the remaining part will be

$$I' = I_0 - I = kMl^2 - \frac{kMl^2}{16}$$

$$[\because \text{using Eqs. (i) and (ii)}]$$

$$= \frac{15kMl^2}{16} \quad \text{or} \quad I' = \frac{15}{16} I_0$$

- 99** A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1 , as shown in the figure will be **[2019, 11 Jan Shift-II]**

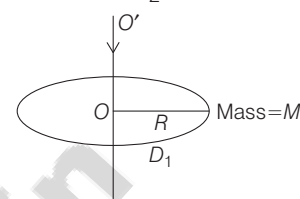


- (a) $\frac{2}{3} MR^2$ (b) $\frac{4}{5} MR^2$
(c) $3MR^2$ (d) MR^2

Ans. (c)

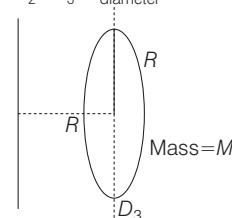
For disc D_1 , moment of inertia across axis OO' will be

$$I_1 = \frac{1}{2} MR^2 \quad \dots (i)$$



For discs D_2 and D_3 , OO' is an axis parallel to the diameter of disc. Using parallel axis theorem,

$$I_2 = I_3 = I_{\text{diameter}} + Md^2 \quad \dots (ii)$$



Here, $I_{\text{diameter}} = \frac{1}{4} MR^2$ and $d = R$

$$\therefore I_2 = I_3 = \frac{1}{4} MR^2 + MR^2 = \frac{5}{4} MR^2$$

Now, total MI of the system

$$I = I_1 + I_2 + I_3 = \frac{1}{2} MR^2 + 2 \times \frac{5}{4} MR^2 = 3MR^2$$

- 100** Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I . The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I , is

[2019, 12 Jan Shift-I]

- (a) 16 cm (b) 14 cm
(c) 12 cm (d) 18 cm

Ans. (a)

Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2} (R_1^2 + R_2^2)$$

where, R_1 = inner radius and R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is.

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M). So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

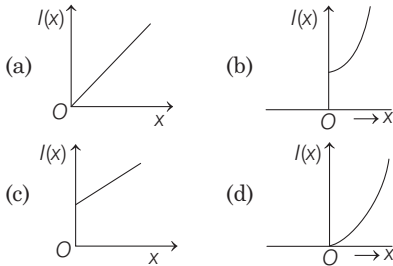
$$(10^2 + 20^2)/2 = R^2$$

$$\Rightarrow R^2 = 250 = 15.8$$

$$R \approx 16 \text{ cm}$$

- 101** The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is $I(x)$. Which one of the graphs represents the variation of $I(x)$ with x correctly?

[2019, 12 Jan Shift-II]

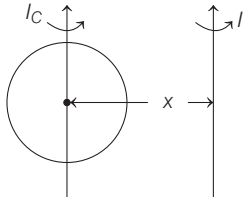


Ans. (b)

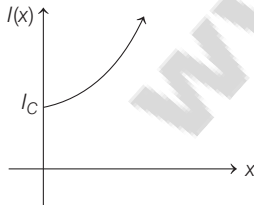
Moment of inertia of a solid sphere about an axis through its centre of mass is

$$I_c = \frac{2}{5}MR^2$$

Moment of inertia about a parallel axis at a distance x from axis through its COM is

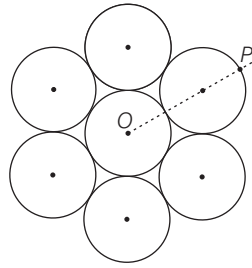


$I = I_c + Mx^2$ (by parallel axis theorem)
So graph of I versus x is parabolic as shown



- 102** Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is

[JEE Main 2018]



- (a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$
(c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

Ans. (d)

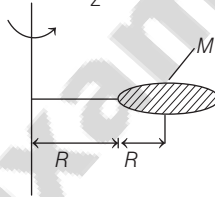
Key Idea First we found moment of inertia (MI) of system using parallel axis theorem about centre of mass, then we use it to find moment of inertia about given axis.

Moment of inertia of an outer disc about the axis through centre is

$$= \frac{MR^2}{2} + M(2R)^2$$

$$= MR^2 \left(4 + \frac{1}{2} \right)$$

$$= \frac{9}{2}MR^2$$



For 6 such discs,

$$\text{Moment of inertia} = 6 \times \frac{9}{2}MR^2 = 27MR^2$$

So, moment of inertia of system

$$= \frac{MR^2}{2} + 27MR^2 = \frac{55}{2}MR^2$$

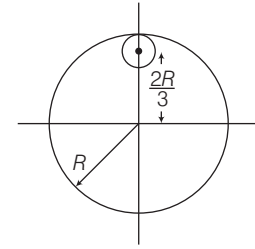
$$\text{Hence, } I_P = \frac{55}{2}MR^2 + (7M \times 9R^2)$$

$$\Rightarrow I_P = \frac{181}{2}MR^2$$

$$I_{\text{system}} = \frac{181}{2}MR^2$$

- 103** From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

[JEE Main 2018]



- (a) $4MR^2$ (b) $\frac{40}{9}MR^2$
(c) $10MR^2$ (d) $\frac{37}{9}MR^2$

Ans. (a)

Moment of inertia of remaining solid

= Moment of inertia of complete solid

$$- \text{Moment of inertia of removed portion}$$

$$\therefore I = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M \left(\frac{2R}{3} \right)^2 \right]$$

$$\Rightarrow I = 4MR^2$$

- 104** The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I . What is the ratio I/R such that the moment of inertia is minimum?

[JEE Main 2017]

- (a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (d)

MI of a solid cylinder about its perpendicular bisector of length is

$$I = m \left(\frac{l^2}{12} + \frac{R^2}{4} \right)$$

$$\Rightarrow I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12}$$

$$[\because \rho\pi R^2 l = m]$$

For I to be minimum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho} \left(\frac{1}{l^2} \right) + \frac{ml}{6} = 0 \Rightarrow$$

$$\frac{m^2}{4\pi\rho} = \frac{ml^3}{6}$$

Now, putting $m = \rho\pi R^2 l$

$$\therefore I^3 = \frac{3}{2\pi\rho} \cdot \rho\pi R^2 l \Rightarrow \frac{l^2}{R^2} = \frac{3}{2}$$

$$\therefore \frac{l}{R} = \sqrt{\frac{3}{2}}$$

- 105** From a solid sphere of mass M and radius R , a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

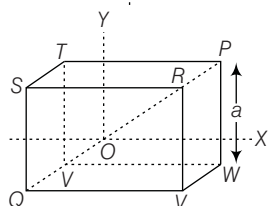
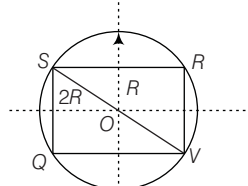
[JEE Main 2015]

- (a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$
 (c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

Ans. (c)

Key Idea Use geometry of the figure to calculate mass and side length of the cube in terms of M and R respectively.

Consider the cross-sectional view of a diametric plane as shown in the adjacent diagram.



Using geometry of the cube

$$PQ = 2R = (\sqrt{3})a \text{ or } a = \frac{2R}{\sqrt{3}}$$

Volume density of the solid sphere

$$\rho = \frac{M}{(4/3)\pi R^3} = \frac{3}{4\pi} \left(\frac{M}{R^3} \right)$$

Mass of cube (m) = (ρ) (a)³

$$= \left(\frac{3}{4\pi} \times \frac{M}{R^3} \right) \left[\frac{2R}{\sqrt{3}} \right]^3$$

$$= \frac{3M}{4\pi R^3} \times \frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about given axis is

$$I_y = \frac{ma^2}{12} (a^2 + a^2) = \frac{ma^2}{6}$$

$$\Rightarrow I_y = \frac{ma^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{1}{6} \times \frac{4R^2}{3} = \frac{4MR^2}{9\sqrt{3}\pi}$$

- 106** Consider a uniform square plate of side a and mass m . The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

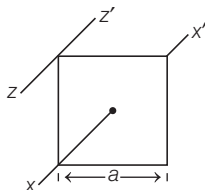
[AIEEE 2008]

- (a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$
 (c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

Ans. (d)

Moment of inertia of square plate about xx' is $\frac{ma^2}{6}$, therefore moment of inertia

about zz' can be computed using parallel axes theorem,

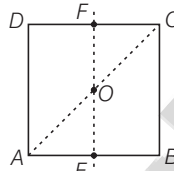


$$I_{zz'} = I_{xx'} + m \left(\frac{a}{\sqrt{2}} \right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2}$$

$$= \frac{2ma^2}{3}$$

- 107** For the given uniform square lamina $ABCD$, whose centre is O

[AIEEE 2007]



- (a) $\sqrt{2}I_{AC} = I_{EF}$
 (b) $I_{AD} = 3I_{EF}$
 (c) $I_{AC} = 4I_{EF}$
 (d) $I_{AC} = \sqrt{2}I_{EF}$

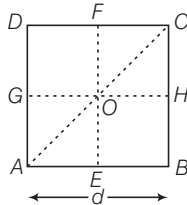
Ans. (c)

Let the each side of square lamina be d .

So, $I_{EF} = I_{GH}$ [due to symmetry]

and $I_{AC} = I_{BD}$ [due to symmetry]

Now, according to theorem of perpendicular axes,



$$I_{AC} + I_{BD} = I_O$$

$$\Rightarrow 2I_{AC} = I_O \quad \dots(i)$$

$$\text{and } I_{EF} + I_{GH} = I_O$$

$$\Rightarrow 2I_{EF} = I_O \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$I_{AC} = I_{EF}$$

$$\therefore I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left[\text{as } I_{EF} = \frac{md^2}{12} \right]$$

$$\text{So, } I_{AD} = \frac{md^2}{3} = 4I_{EF}$$

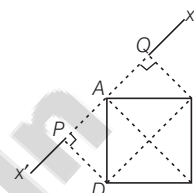
- 108** Four point masses, each of value m , are placed at the corners of a square $ABCD$ of side l . The moment of inertia of this system about an axis passing through A and parallel to BD is

[AIEEE 2006]

- (a) $2ml^2$ (b) $\sqrt{3}ml^2$ (c) $3ml^2$ (d) ml^2

Ans. (c)

The situation is shown in figure.



$$I_{xx'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$

$$= m \times 2 \times \left(\frac{\sqrt{2}l}{2} \right)^2 + m \times (\sqrt{2}l)^2 = 3ml^2$$

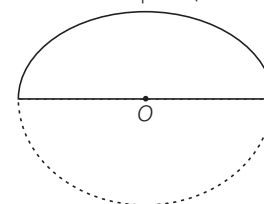
- 109** The moment of inertia of uniform semi-circular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is

[AIEEE 2005]

- (a) $\frac{1}{4}Mr^2$ (b) $\frac{2}{5}Mr^2$
 (c) Mr^2 (d) $\frac{1}{2}Mr^2$

Ans. (d)

The mass of complete (circular) disc is



The moment of inertia of disc about the given axis is

$$I = \frac{2Mr^2}{2} = Mr^2$$

Let the moment of inertia of semi-circular disc be I_1 .

The disc may be assumed as combination of two semi-circular parts.

$$\text{Thus, } I_1 = I - I_1 \text{ or } I_1 = \frac{I}{2} = \frac{Mr^2}{2}$$

- 110** One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that

[AIEEE 2004]

- (a) $I_A = I_B$ (b) $I_A > I_B$
 (c) $I_A < I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where, d_A and d_B are their densities.

Ans. (c)

Let same mass and same outer radii of solid sphere and hollow sphere be M and R , respectively.

The moment of inertia of solid sphere A about its diameter,

$$I_A = \frac{2}{5} MR^2 \quad \dots(i)$$

Similarly, the moment of inertia of hollow sphere (spherical shell) B about its diameter,

$$I_B = \frac{2}{3} MR^2 \quad \dots(ii)$$

It is clear from Eqs. (i) and (ii), $I_A < I_B$.

TOPIC 3

Kinematics and Dynamics of Rotational Motion

- 111** Two discs have moments of inertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation co-axial. The loss in kinetic energy of the system in the process is given by

[2021, 27 Aug Shift-II]

- (a) $\frac{I_1 I_2}{(I_1 + I_2)} (\omega_1 - \omega_2)^2$ (b) $\frac{(I_1 - I_2)^2 \omega_1 \omega_2}{2(I_1 + I_2)}$
 (c) $\frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2$ (d) $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Ans. (c)

Given, moment of inertia of two discs are I_1, I_2 .

Their initial angular velocities is ω_1 and ω_2 .

Let final angular velocity is ω .

By using law of conservation of angular momentum,

$$\Rightarrow I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega$$

$$\Rightarrow \omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$$

$\therefore$ Change in kinetic energy ΔKE

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} (I_1 + I_2) \omega^2$$

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} (I_1 + I_2) \left(\frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2} \right)^2$$

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{I_1 + I_2}$$

$$= \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} \frac{(I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + 2 I_1 I_2 \omega_1 \omega_2)}{I_1 + I_2}$$

$$= \frac{1}{2} \frac{I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + 2 I_1 I_2 \omega_1 \omega_2}{I_1 + I_2} - \frac{1}{2} \frac{I_1^2 \omega_1^2 + I_2^2 \omega_2^2 + 2 I_1 I_2 \omega_1 \omega_2}{I_1 + I_2}$$

$$= \frac{1}{2} \frac{I_1 I_2 (\omega_1^2 + \omega_2^2 - 2 \omega_1 \omega_2)}{I_1 + I_2}$$

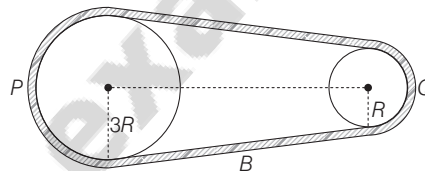
$$= \frac{1}{2} \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{I_1 + I_2}$$

- 112** In the given figure, two wheels P and Q are connected by a belt B . The radius of P is three times as that of Q . In case of same rotational kinetic energy, the ratio of

rotational inertias $\left(\frac{I_1}{I_2} \right)$ will be $x:1$.

The value of x will be

[2021, 27 July Shift-II]



Ans. (9)

From the given situation,

$$KE_P = KE_Q$$

$$\Rightarrow \frac{1}{2} I_1 (\omega_1)^2 = \frac{1}{2} I_2 (\omega_2)^2$$

$$\Rightarrow \frac{1}{2} I_1 \left(\frac{v}{3R} \right)^2 = \frac{1}{2} I_2 \left(\frac{v}{R} \right)^2$$

$[\because \text{Radius of } P \text{ is 3 times to that of } Q]$

$$\Rightarrow I_1 \times \frac{v^2}{9R^2} = I_2 \times \frac{v^2}{R^2}$$

$$\Rightarrow \frac{I_1}{9} = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{9}{1} \quad \dots(i)$$

$\therefore$ According to question, the ratio of rotational inertias $\frac{I_1}{I_2} = \frac{x}{1}$. So, on

comparing it with Eq. (i), we can write $x = 9$.

- 113** The centre of a wheel rolling on a plane surface moves with a speed v_0 . A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x} v_0$. Then, the value of x is

[2021, 22 July Shift-II]

Ans. (4)

Given, speed of centre of mass of wheel be v_0

Speed of particle at rim be $v_0 \sqrt{x}$.

$\therefore$ Velocity of particle placed at rim of wheel will be

$$v_0 \sqrt{x} = v_0 + R\omega = v_0 + R \cdot \frac{v_0}{R}$$

$$\Rightarrow v_0 \sqrt{x} = 2v_0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$$

- 114** Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The correct statement for this situation.

[2021, 22 July Shift-II]

- (a) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.
 (b) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.
 (c) All of them will have same velocity.
 (d) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Ans. (a)

Given, three shapes be ring, solid cylinder and solid sphere. As we know that,

Moment of inertia of ring, solid cylinder and solid sphere about its centre of mass be

$$I_{\text{ring}} = MR^2 \quad \dots(i)$$

$$I_{\text{solid cylinder}} = \frac{MR^2}{2} \quad \dots(ii)$$

$$I_{\text{solid sphere}} = \frac{2}{5} MR^2 \quad \dots(iii)$$

and speed of body to roll on inclined plane is

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}} \quad \dots(iv)$$

where, k is the of gyration.

Now from Eqs. (i), (ii) and (iii), we get

$$k_{\text{ring}} = R$$

$$k_{\text{solid cylinder}} = \frac{R}{\sqrt{2}}$$

and $k_{\text{solid sphere}} = R\sqrt{\frac{2}{5}}$

Substituting these values sequentially in Eq. (iv), we get

$$v_{\text{ring}} = \sqrt{\frac{2gh}{1 + \frac{R^2}{R^2}}} = \sqrt{gh}$$

$$v_{\text{solid cylinder}} = \sqrt{\frac{2gh}{1 + \frac{R^2/2}{R^2}}} = \sqrt{\frac{4gh}{3}}$$

$$= \sqrt{1.33gh}$$

$$v_{\text{solid sphere}} = \sqrt{\frac{2gh}{1 + \frac{2}{5} \frac{R^2}{R^2}}} = \sqrt{\frac{10}{7}gh} = \sqrt{1.43gh}$$

$$v_{\text{solid sphere}} > v_{\text{solid cylinder}} > v_{\text{ring}}$$

Hence, the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

115 A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is [2021, 20 July Shift-II]

- (a) solid sphere (b) solid cylinder
(c) hollow cylinder (d) ring

Ans. (b)

It is given that $K_{\text{rotational}} = 50\%$ of $K_{\text{translational}}$

$$\Rightarrow \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) \quad \dots(i)$$

where, I is moment of inertia, ω is angular speed, m is mass and v is linear speed.

$\therefore$ This case is of pure rolling, therefore $v = \omega R \quad \dots(iii)$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2 \Rightarrow I = \frac{1}{2}mR^2$$

As this the value of moment of inertia is for solid cylinder. Therefore, the body is solid cylinder.

116 A circular disc reaches from top to bottom of an inclined plane of length L . When it slips down the plane, it takes time t_1 . When it rolls

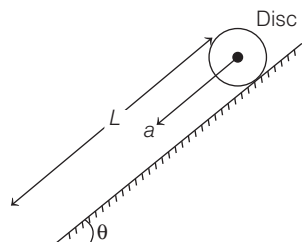
down the plane, it takes time t_2 .

The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of

x will be [2021, 20 July Shift-I]

Ans. (2)

According to question, a circular disc reaches from top to bottom of an inclined plane of length L . This can be shown as



When the disc slips down the inclined plane, it takes time t_1 .

Therefore, in this case its acceleration, $a_1 = g \sin \theta$

$$\therefore s = ut_1 + \frac{1}{2}a_1t_1^2$$

$$\Rightarrow s = \frac{1}{2}g \sin \theta t_1^2 \quad \dots(i)$$

And when the disc rolls down the inclined plane, it takes time t_2 .

Therefore in this case, its acceleration, $a_2 = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} = \frac{g \sin \theta}{1 + \frac{1}{2}} = \frac{2}{3}g \sin \theta$
[$\therefore$ for disc, $\frac{K^2}{R^2} = \frac{1}{2}$]

$$\therefore s = ut_2 + \frac{1}{2}a_2t_2^2 = \frac{1}{2} \cdot \frac{2}{3}g \sin \theta t_2^2 \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}} \quad \dots(iii)$$

According to question, value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$.

Comparing it with Eq. (iii), we get $x = 2$

117 Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is [2021, 20 July Shift-II]

Ans. (3)

Let us consider the case for ring first.

Rotational kinetic energy of ring,

$$K = \frac{1}{2}I\omega^2$$

$\therefore$ Moment of inertia for ring about the point of contact,

$$I_{\text{ring}} = 2mR^2$$

$$\therefore K = \frac{1}{2}(2mR^2)\omega^2$$

$$\therefore k = \frac{1}{2}(2mR^2) \frac{v_R^2}{R^2} \quad \left\{ \therefore \omega = \frac{v}{R} \right\}$$

As, this is a case of pure rolling and the ring is rolling down, then by conservation of energy its potential energy will be equal to its kinetic energy at the bottom

$$\therefore PE = mgh = \frac{1}{2}(2mR^2) \frac{v_R^2}{R^2}$$

$$\Rightarrow v_R = \sqrt{gh} \quad \dots(i)$$

Now, consider the case of solid cylinder,

$\therefore$ Moment of inertia in this case as well as about the point of contact is given as

$$I_{\text{cylinder}} = \frac{3}{2}mR^2$$

$$mgh = \frac{1}{2}I_{\text{cylinder}}\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}\left(\frac{3}{2}mR^2\right) \cdot \frac{v_C^2}{R^2} \quad \left\{ \therefore \omega = \frac{v}{R} \right\}$$

$$\Rightarrow v_C = \sqrt{\frac{4gh}{3}} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{v_R}{v_C} = \frac{\sqrt{3}}{2} \quad \dots(iii)$$

As per question, the ratio of velocity of centre of mass at the bottom of inclined plane of the ring to that of cylinder is $\frac{\sqrt{x}}{2}$. So, on comparing it with Eq. (iii), we

can write $x = 3$.

118 A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 s. The number of rotations made in the process is [2021, 20 July Shift-II]

Ans. (32)

Given, initial angular speed,

$$\omega_0 = 600 \text{ rpm} = 10 \text{ rps}$$

Final angular speed,

$$\omega_F = 1800 \text{ rpm} = 30 \text{ rps}$$

Time, $t = 10 \text{ s}$

∴ We know that

$$\begin{aligned}\omega_F &= \omega_0 + \alpha t \\ \Rightarrow 30 &= 10 + \alpha \times 10 \\ \Rightarrow \frac{30 - 10}{10} &= \alpha \\ \Rightarrow \alpha &= \frac{20}{10} = 2 \text{ rps}^2\end{aligned}$$

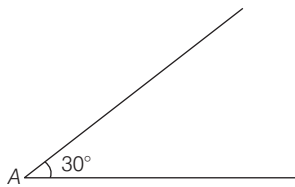
Angular displacement is calculated by the rotational equation of motion as

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

where, θ is the angular displacement.

$$\begin{aligned}\Rightarrow \theta &= 10 \times 10 + \frac{1}{2} \times 2 \times (10)^2 \\ \Rightarrow \theta &= 100 + 100 \\ \Rightarrow \theta &= 200 \text{ rad} \\ \text{Number of rotations} &= \frac{\theta}{2\pi} = \frac{200}{2\pi} = 31.8 \approx 32\end{aligned}$$

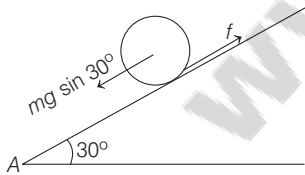
- 119** A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A? [2021, 17 March Shift-II]



(a) 0.60 s (b) 0.52 s (c) 0.57 s (d) 0.80 s

Ans. (c)

As we know the moment of inertia of the solid sphere, $I = \frac{2}{5} mR^2$



Acceleration of sphere on inclined plane, $a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}}$

Substituting the values in the above equation, we get

$$a = \frac{g \sin 30^\circ}{1 + \frac{\frac{2}{5} mR^2}{mR^2}} = \frac{10 + 1/2}{(1 + 2/5)} = \frac{5 \times 5}{7}$$

$$a = 3.5 \text{ m/s}^2$$

At the top of incline,

$$\begin{aligned}v &= u + at \Rightarrow 0 = 1 - 3.5t \\ t &= 1/3.5 \text{ s}\end{aligned}$$

∴ Total time = time of ascent + time of decent

As, time of ascent = time of decent

⇒ Total time = 2(time of ascent)

Total time = $2(1/3.5) \text{ s}$

Total time = 0.57 s

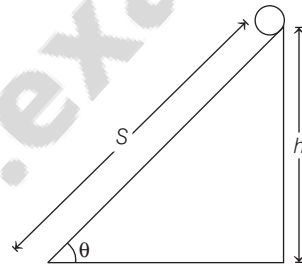
Hence, the total time taken to sphere to return the starting point A is 0.57 s.

- 120** The following bodies,

1. a ring
2. a disc
3. a solid cylinder
4. a solid sphere

of same mass m and radius R are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is

(Mark the body as per their respective numbering given in the question) [2021, 17 March Shift-I]



Ans. (4)

For rolling without slipping on an inclined plane, we can write

$$Rmg \sin \theta = (mK^2 + mR^2)\alpha$$

$$\Rightarrow Rmg \sin \theta = m(K^2 + R^2)\alpha$$

$$\Rightarrow \alpha = \frac{Rg \sin \theta}{K^2 + R^2} \Rightarrow \frac{\alpha}{R} = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}}$$

$$\Rightarrow a = \frac{g \sin \theta}{1 + \frac{K^2}{R^2}} \quad \left[\because a = \frac{\alpha}{R} \right] \dots (i)$$

$$\text{Time period, } t = \sqrt{\frac{2s}{a}} \dots (ii)$$

From Eqs. (i) and (ii), we get

$$t = \sqrt{\frac{2s}{g \sin \theta} \left(1 + \frac{K^2}{R^2} \right)}$$

For least time acceleration a should be maximum and K should be minimum and we know that K is least for solid sphere. So, time will be least for sphere.

It means the body which will reach first at the bottom of the inclined plane is 4, i.e. solid sphere.

- 121** The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 s. The number of revolutions by the truck engine during this time is

(Assuming the acceleration to be uniform). [2021, 17 March Shift-I]

Ans. (728)

Initial angular speed, $\omega_0 = 900 \text{ rpm}$
 $= 900 \times \frac{2\pi}{60} = 30\pi \text{ rad/s}$

Final angular speed, $\omega = 2460 \text{ rpm}$

$$= 2460 \times \frac{2\pi}{60} = 41 \times 2\pi = 82\pi \text{ rad/s}$$

We know that,

$$\begin{aligned}\omega &= \omega_0 + \alpha t \Rightarrow \omega - \omega_0 = \alpha t \\ \Rightarrow 82\pi - 30\pi &= \alpha \times 26 \Rightarrow 52\pi = 26\alpha \\ \Rightarrow \alpha &= 2\pi \text{ rad/s}^2\end{aligned}$$

Also we know that,

$$\begin{aligned}\theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ \Rightarrow 2\pi n &= 30\pi \times 26 + \frac{1}{2} \times 2\pi \times (26)^2 \\ \Rightarrow 2n &= 30 \times 26 + \frac{(26)^2}{2} \\ \Rightarrow n &= \frac{30 \times 26 + \frac{(26)^2}{2}}{2} \\ &= \frac{26(30 + 26)}{2} = 13 \times 56 = 728\end{aligned}$$

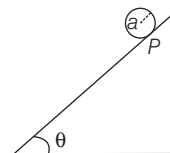
- 122** A solid disc of radius a and mass m rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b} g \sin \theta$, where b is

(Round off to the nearest integer)

(g = acceleration due to gravity)

(θ = angle as shown in figure)

[2021, 16 March Shift-II]



Ans. (3)

We know that, on an inclined plane,

$$\text{Acceleration, } a = \frac{g \sin \theta}{1 + \frac{I}{mR^2}}$$

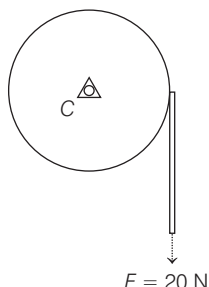
$$\Rightarrow a = \frac{g \sin \theta}{1 + \frac{1}{2}} \quad \left[\because \text{For disc, } I = \frac{mR^2}{2} \right]$$

$$\Rightarrow a = \frac{2}{3} g \sin \theta \quad \dots (i)$$

As per question, acceleration of the disc will be $\frac{2}{b} g \sin \theta$.

Comparing it with Eq. (i), we get
 $b = 3$

- 123** Consider a 20 kg uniform circular disc of radius 0.2 m. It is pin supported at its centre and is at rest initially. The disc is acted upon by a constant force $F = 20 \text{ N}$ through a massless string wrapped around its periphery as shown in the figure.



Suppose the disc makes n number of revolutions to attain an angular speed of 50 rad s^{-1} . The value of n to the nearest integer, is
(Given, in one complete revolution, the disc rotates by 6.28 rad .) **[2021, 16 March Shift-I]**

Ans. (20)

Given, $M = 20 \text{ kg}$, $R = 0.2 \text{ m}$,
 $F = 20 \text{ N}$, $\omega = 50 \text{ rad s}^{-1}$,
 $\theta = 6.28 \text{ rad}$ (for one revolution).

We know that,

Torque = Moment of inertia $\times$ Angular acceleration

$$\Rightarrow \tau = I \times \alpha$$

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{F \times R}{MR^2} = \frac{2F}{MR} \quad [$$

$$\because \tau = F \times R \text{ and } I_{\text{disc}} = \frac{1}{2} MR^2]$$

$$\Rightarrow \alpha = \frac{2 \times 20}{20 \times (0.2)} = 10 \text{ rad/s}^2$$

In angular terms, third equation of motion,

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\Rightarrow (50)^2 = (0)^2 + 2 \times 10 \times \Delta\theta$$

$$\Rightarrow \Delta\theta = \frac{2500}{20}$$

$$\Rightarrow \Delta\theta = 125 \text{ rad}$$

$$\therefore \text{Number of revolution} = \frac{\Delta\theta}{\theta} = \frac{125}{6.28} \approx 20$$

- 124** A cord is wound round the circumference of wheel of radius r . The axis of the wheel is horizontal and the moment of inertia about it is I . A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance h , the square of angular velocity of wheel will be

[2021, 26 Feb Shift-II]

(a) $\frac{2mgh}{I + 2mr^2}$

(b) $\frac{2mgh}{I + mr^2}$

(c) $2gh$

(d) $\frac{2gh}{I + mr^2}$

Ans. (b)

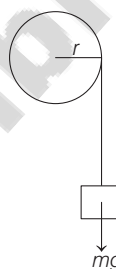
Moment of inertia of wheel,
 $I = mR^2$

Let v be the velocity and
 $v = r\omega \Rightarrow \omega = v/r$

By using law of conservation of energy,

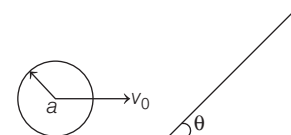
$$1/2(I + mr^2)\omega^2 = mgh$$

$$\Rightarrow \omega^2 = \frac{2mgh}{I + mr^2}$$



- 125** A sphere of radius a and mass m rolls along a horizontal plane with constant speed v_0 . It encounters an inclined plane at angle θ and climbs upwards. Assuming that it rolls without slipping, how far up the sphere will travel?

[2021, 25 Feb Shift-II]



(a) $\frac{v_0^2}{2g \sin \theta}$

(b) $\frac{v_0^2}{5g \sin \theta}$

(c) $\frac{10v_0^2}{7g \sin \theta}$

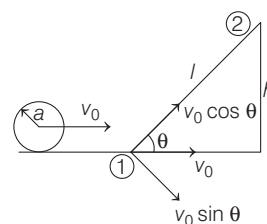
(d) $\frac{2}{5} \frac{v_0^2}{g \sin \theta}$

Ans. (d)

Given, radius of sphere is a , mass of sphere is m , horizontal speed of sphere is v_0 .

Moment of inertia of solid sphere,
 $I = \frac{2}{5} ma^2$

Let, h be the height of inclined plane and l be the length of inclined plane.



By using law of conservation of energy,
Total energy at 1 = Total energy at 2

$$\therefore PE_1 + KE_1 = PE_2 + KE_2$$

$$\Rightarrow 0 + 1/2 mv_0^2 + 1/2 I \omega^2 = mgh \quad \dots (i)$$

$$\text{As, } v_0 = a\omega \Rightarrow \omega = \frac{v_0}{a}$$

$$\therefore \frac{1}{2} mv_0^2 + \frac{1}{2} \left(\frac{2}{5} ma^2 \right) \frac{v_0^2}{a^2} = mgh$$

$$\Rightarrow \frac{1}{2} mv_0^2 (1 + 2/5) = mgh$$

$$\Rightarrow \frac{7v_0^2}{10} = gh \quad \dots (ii)$$

$$\text{As } \sin \theta = h/l$$

$$\Rightarrow h = l \sin \theta$$

Put this value in Eq. (ii), we get

$$\Rightarrow \frac{7}{10} v_0^2 = gl \sin \theta$$

$$\Rightarrow l = \frac{7v_0^2}{10 g \sin \theta}$$

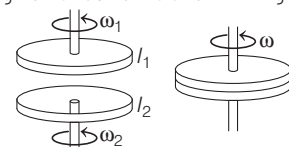
- 126** Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively, while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} , respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is

[2020, 02 Sep Shift-II]

(a) $\frac{10}{3} \text{ J}$ (b) $\frac{20}{3} \text{ J}$ (c) $\frac{5}{3} \text{ J}$ (d) $\frac{2}{3} \text{ J}$

Ans. (b)

The given situation is shown in figure.



As there is no external torque, angular momentum of system is conserved

$$L_1 + L_2 = L_{\text{system}}$$

$$\Rightarrow I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega$$

$$\Rightarrow \omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$$

$$\omega = \frac{0.1 \times 10 + 0.2 \times 5}{(0.1 + 0.2)} \text{ or } \omega = \frac{20}{3} \text{ rad/s}$$

Kinetic energy of combined system is

$$K = \frac{1}{2} I_{\text{system}} \omega^2$$

$$= \frac{1}{2} (0.1 + 0.2) \times \left(\frac{20}{3} \right)^2 = \frac{20}{3} \text{ J}$$

Hence, correct option is (b).

- 127** A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1 . If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually, both discs attain constant angular speed ω_2 . The energy lost in the process is $p\%$ of the initial energy. Value of p is

[2020, 04 Sep Shift-I]

Ans. (20)

In this process, no external torque is applied, so the angular momentum will remain conserved.

From law of conservation of angular momentum,

$$(L_{\text{sys}})_i = (L_{\text{sys}})_f$$

$$\Rightarrow I_1 \omega_1 + 0 = (I_1 + I_2) \omega_2$$

$$\Rightarrow \frac{MR^2}{2} \times \omega_1 = \left[\frac{MR^2}{2} + \frac{M(R/2)^2}{2} \right] \omega_2$$

$$\Rightarrow \frac{MR^2}{2} \times \omega_1 = \left[\frac{MR^2}{2} + \frac{MR^2}{8} \right] \omega_2$$

$$\Rightarrow \frac{MR^2}{2} \times \omega_1 = \frac{5MR^2}{8} \times \omega_2$$

$$\Rightarrow \omega_1 = \frac{5}{4} \omega_2 \Rightarrow \omega_2 = \frac{4}{5} \omega_1 \quad \dots(i)$$

Now, the loss in rotational kinetic energy,

$$(\Delta K_{\text{sys}})_{\text{loss}} = (K_{\text{sys}})_i - (K_{\text{sys}})_f$$

$$= \left[\frac{1}{2} I_1 \omega_1^2 + 0 \right] - \left[\frac{1}{2} (I_1 + I_2) \omega_2^2 \right]$$

$$= \left[\frac{1}{2} \times \frac{MR^2}{2} \times \omega_1^2 + 0 \right]$$

$$- \left[\frac{1}{2} \times \left(\frac{MR^2}{2} + \frac{MR^2}{8} \right) \times \left(\frac{4}{5} \omega_1 \right)^2 \right]$$

$$= \frac{MR^2 \omega_1^2}{4} - \frac{1}{2} \times \frac{5MR^2}{8} \times \frac{16\omega_1^2}{25}$$

$$= \frac{MR^2 \omega_1^2}{4} - \frac{MR^2 \omega_1^2}{5} = \frac{MR^2 \omega_1^2}{20}$$

Now, the % loss in rotational kinetic energy,

$$\%(\Delta K_{\text{sys}})_{\text{loss}} = \frac{(\Delta K_{\text{sys}})_{\text{loss}}}{(K_{\text{sys}})_i} \times 100\%$$

$$= \left(\frac{\frac{MR^2 \omega_1^2}{20}}{\frac{MR^2 \omega_1^2}{4}} \right) \times 100\% = \frac{4}{20} \times 100\% = 20\%$$

and given that $\%(\Delta K_{\text{sys}})_{\text{loss}} = p\%$

On comparing, we get $p = 20$

- 128** A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is

[2020, 05 Sep Shift-I]

(a) $\frac{3}{4}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{4}$

Ans. (a)

Let ω_c be the final angular speed.

According to law of conservation of angular momentum,

Initial angular momentum = Final angular momentum

$$I\omega + 0 = I\omega_c + 3I\omega_c$$

$$I\omega = 4I\omega_c \Rightarrow \omega_c = \frac{\omega}{4}$$

Loss of rotational kinetic energy,

$$(\Delta KE)_{\text{loss}} = \text{Initial KE} - \text{Final KE}$$

$$\Rightarrow (\Delta KE)_{\text{loss}} = \frac{1}{2} I \omega^2 - \frac{1}{2} (I + 3I) \omega_c^2$$

$$= \frac{1}{2} I \omega^2 - \frac{1}{2} (I + 3I) \left(\frac{\omega}{4} \right)^2$$

$$= \frac{1}{2} I \omega^2 - \frac{1}{2} (4I) \left(\frac{\omega}{4} \right)^2 = \frac{1}{2} I \omega^2 \left(1 - \frac{1}{4} \right) = \frac{3}{8} I \omega^2$$

The resultant fractional loss in KE of the system

$$= \frac{(\Delta KE)_{\text{loss}}}{(KE)_{\text{initial}}} = \frac{3/8 I \omega^2}{1/2 I \omega^2}$$

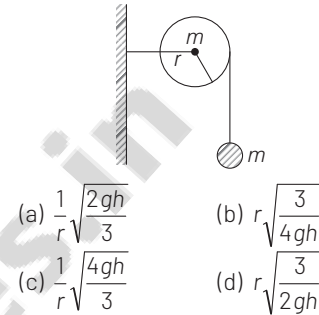
$$\left(\because (KE)_{\text{initial}} = \frac{1}{2} I \omega^2 \right)$$

$$= \frac{3}{4}$$

Hence, correct option is (a).

- 129** As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a flywheel (disc) of radius r and mass m . When released from rest the bob starts falling vertically. When it has covered a distance of h , the angular speed of the wheel will be

[2020, 7 Jan Shift-I]



Ans. (c)

When bob falls, its potential energy is converted into kinetic energy of bob and rotational kinetic energy of disc. So, decrease in potential energy of bob = kinetic energy of bob + kinetic energy of disc

$$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \quad \dots(i)$$

If there is no slip, then $v = r\omega$

Also, moment of inertia of disc,

$$I = \frac{mr^2}{2}$$

Substituting these in Eq. (i), we get

$$mgh = \frac{1}{2} m(r\omega)^2 + \frac{1}{2} \times \frac{mr^2}{2} \times \omega^2$$

$$\Rightarrow mgh = \frac{3}{4} mr^2 \omega^2$$

$$\Rightarrow \omega^2 = \frac{4gh}{3r^2} \text{ or } \omega = \frac{1}{r} \sqrt{\frac{4gh}{3}}$$

- 130** A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is

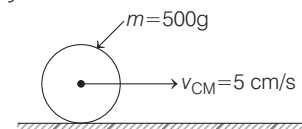
[2020, 8 Jan Shift-II]

(a) $6.25 \times 10^{-4} \text{ J}$ (b) $1.13 \times 10^{-3} \text{ J}$

(c) $8.75 \times 10^{-4} \text{ J}$ (d) $8.75 \times 10^{-3} \text{ J}$

Ans. (c)

The given situation is as shown in the figure below.



In pure rolling object possesses both translational kinetic energy and rotational kinetic energy.

$$\text{So, } KE_{\text{sphere}} = \frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2$$

$$\text{Here, } I = \frac{2}{5}mr^2 \text{ and } \omega = \frac{v}{r} \quad [\because v_{\text{CM}} = v]$$

$$\text{So, } KE_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2}$$

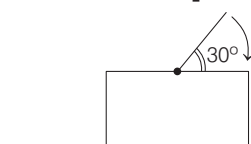
$$= \left(\frac{1}{2} + \frac{1}{5}\right)mv^2$$

$$= \frac{7}{10} \times 500 \times 10^{-3} \times 25 \times 10^{-4}$$

$$= 8.75 \times 10^{-4} \text{ J}$$

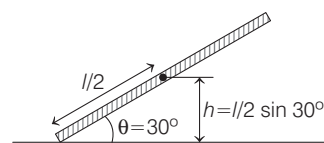
- 131** One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n} \text{ s}^{-1}$, where n is an integer. The value of n is

[2020, 9 Jan Shift-I]



Ans. ($\sqrt{15}$)

When bar is released, its potential energy appears in form of rotational kinetic energy.



Now, from geometry of figure, height of centre of mass above surface,

$$h = \frac{l}{2} \cdot \sin 30^\circ = \frac{l}{4}$$

If angular speed of bar as it hits the table is ω , then

Gravitational potential energy = Rotational kinetic energy

$$\Rightarrow mgh = \frac{1}{2}I\omega^2$$

$$\Rightarrow mg \frac{l}{4} = \frac{1}{2} \left(\frac{ml^2}{3} \right) \cdot \omega^2$$

$$\Rightarrow \omega^2 = \frac{3}{2} \frac{g}{l} = \frac{3}{2} \times \frac{10}{1} = 15$$

$$\Rightarrow \omega = \sqrt{15} \text{ s}^{-1}$$

Thus, $n = \sqrt{15}$

- 132** A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see figure). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descends by a distance h is

[2020, 9 Jan Shift-II]

$$(a) \left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + 1} \right]^{1/2}$$

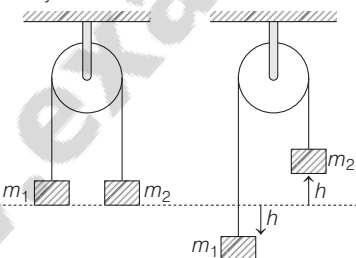
$$(b) \left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + 1} \right]^{1/2} gh$$

$$(c) \left[\frac{m_1 - m_2}{(m_1 + m_2)R^2 + 1} \right]^{1/2} gh$$

$$(d) \left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + 1} \right]^{1/2}$$

Ans. (d)

Loss of gravitational potential energy of system appears in form of kinetic energy of system.



Loss of potential energy of masses = Kinetic energy of masses + Kinetic energy of rotation of wheel

$$\Rightarrow m_1gh - m_2gh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

Assuming no slip, we have

$$v = \omega R$$

$$\therefore (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)\omega^2 R^2 + \frac{1}{2}I\omega^2$$

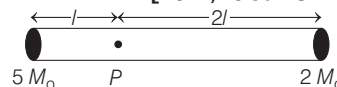
$$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + \frac{I}{R^2}}}$$

$$\text{or } \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$$

$$\text{or } \omega = \left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + 1} \right]^{1/2}$$

- 133** A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be

[2019, 10 Jan Shift-II]



$$(a) \frac{g}{13l}$$

$$(b) \frac{g}{2l}$$

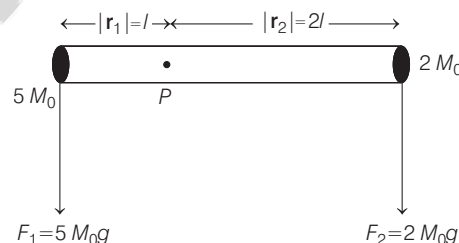
$$(c) \frac{7g}{3l}$$

$$(d) \frac{g}{3l}$$

Ans. (a)

Key Idea When a rod is pivoted at any point, its angular acceleration is given by $\tau_{\text{net}} = I\alpha$.

The given condition can be drawn in the figure below



Torque (τ) about

$$P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \quad \dots (i)$$

$$\Rightarrow \tau = l \times 5M_0g \text{ (outwards)} - 2l \times 2M_0g \text{ (inwards)}$$

$$\Rightarrow \tau = 5M_0gl - 4M_0gl \text{ (outwards)}$$

$$\Rightarrow \tau = M_0gl \text{ (outwards)}$$

$$\text{or } \tau = M_0gl \quad \dots (ii)$$

Now we know that, torque is also given by

$$\tau = I\alpha \quad \dots (iii)$$

Here, I = moment of inertia (w.r.t. point P) of rod and α = angular acceleration.

$$\text{For point } P, I = (5M_0) \times l^2 + (2M_0) \times (2l)^2 \quad [\because I = MR^2]$$

$$\Rightarrow I = 13M_0l^2 \quad \dots (iv)$$

Putting value of I from Eq. (iv) in Eq. (iii), we get

$$\tau = (13M_0l^2)\alpha \quad \dots (v)$$

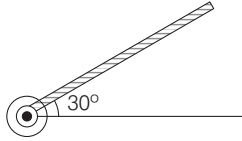
From Eqs. (ii) and (v), we get

$$M_0gl = 13M_0l^2\alpha \Rightarrow \alpha = \frac{g}{13l}$$

- 134** A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released

from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (Take, $g = 10\text{ms}^{-2}$)

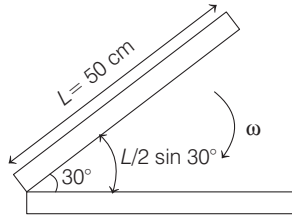
[2019, 9 Jan Shift-II]



- (a) $\frac{\sqrt{30}}{2}$ (b) $\sqrt{30}$
(c) $\frac{\sqrt{20}}{3}$ (d) $\sqrt{\frac{30}{2}}$

Ans. (b)

Lose of potential energy of rod = Gain of kinetic energy



$$\therefore \Delta PE = \frac{1}{2} I \omega^2$$

(where, I = moment of inertia of rod and ω = angular frequency of rod)

$$\Rightarrow Mg \times \frac{L}{2} \sin 30^\circ = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow Mg \frac{L}{2} \times \frac{1}{2} = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \frac{MgL \times 2}{4 \times I} = \omega^2$$

$$\Rightarrow \sqrt{\frac{MgL}{4 \times \frac{ML^2}{3}}} \times \frac{2}{1} = \omega \left[\because I = \frac{ML^2}{3} \text{ for rod} \right]$$

$$\omega = \sqrt{30} \text{ rad/s}$$

- 135** A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline.

The ratio $\frac{h_{\text{sph}}}{h_{\text{cyl}}}$ is given by

[2019, 8 April Shift-II]



- (a) $\frac{2}{\sqrt{5}}$ (b) $\frac{14}{15}$ (c) 1 (d) $\frac{4}{5}$

Ans. (b)

When a spherical/circular body of radius r rolls without slipping, its total kinetic energy is

$$K_{\text{total}} = K_{\text{translation}} + K_{\text{rotation}}$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} I \cdot \frac{v^2}{r^2} \quad \left[\because \omega = \frac{v}{r} \right]$$

Let v be the linear velocity and R be the radius for both solid sphere and solid cylinder.

$\therefore$ Kinetic energy of the given solid sphere will be

$$K_{\text{sph}} = \frac{1}{2} mv^2 + \frac{1}{2} I_{\text{sph}} \frac{v^2}{R^2}$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{2}{5} mR^2 \times \frac{v^2}{R^2} = \frac{7}{10} mv^2 \quad \dots(i)$$

Similarly, kinetic energy of the given solid cylinder will be

$$K_{\text{cyl}} = \frac{1}{2} mv^2 + \frac{1}{2} I_{\text{cyl}} \frac{v^2}{R^2}$$

$$= \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{mR^2}{2} \times \frac{v^2}{R^2}$$

$$= \frac{3}{4} mv^2 \quad \dots(ii)$$

Now, from the conservation of mechanical energy,

$$mgh = K_{\text{total}}$$

$\therefore$ For solid sphere,

$$mgh_{\text{sph}} = \frac{7}{10} mv^2 \quad \dots(iii) \text{ [using Eq. (i)]}$$

Similarly, for solid cylinder,

$$mgh_{\text{cyl}} = \frac{3}{4} mv^2 \quad \dots(iv) \text{ [using Eq. (ii)]}$$

Taking the ratio of Eqs. (iii) and (iv), we get

$$\frac{mgh_{\text{sph}}}{mgh_{\text{cyl}}} = \frac{\frac{7}{10} mv^2}{\frac{3}{4} mv^2} \Rightarrow \frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{7}{10} \times \frac{4}{3} = \frac{14}{15}$$

Alternate Solution

Total kinetic energy for a rolling body without slipping can also be given as

$$K_{\text{total}} = \frac{1}{2} mv^2 \left[1 + \frac{K^2}{R^2} \right]$$

where, K is the radius of gyration.

$\therefore$ From law of conservation,

$$mgh = \frac{1}{2} mv^2 \left[1 + \frac{K^2}{R^2} \right] \text{ or } h \propto \left(1 + \frac{K^2}{R^2} \right)$$

As we know that, for solid sphere,

$$K = \sqrt{\frac{2}{5}} R \Rightarrow \frac{K^2}{R^2} = \frac{2}{5}$$

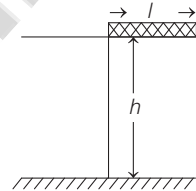
Similarly, for solid cylinder,

$$K = \frac{R}{\sqrt{2}} \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$$

$$\text{So, } \frac{h_{\text{sph}}}{h_{\text{cyl}}} = \frac{1 + \frac{2}{5}}{1 + \frac{1}{2}} = \frac{\frac{7}{5}}{\frac{3}{2}} = \frac{7}{5} \times \frac{2}{3} = \frac{14}{15}$$

- 136** A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to

[2019, 8 April Shift-II]



- (a) 0.02 (b) 0.3
(c) 0.5 (d) 0.28

Ans. (c)

Key Idea The rectangular box rotates due to torque of weight about its centre of mass.

Now, angular impulse of weight

= Change in angular momentum.

$$\therefore mg \frac{l}{2} \times \tau = \frac{ml^2}{3} \omega \Rightarrow \omega = \frac{3g \times \tau}{2 \times l}$$

Substituting the given values, we get

$$= \frac{3 \times 10 \times 0.01}{2 \times 0.3} = 0.5 \text{ rad s}^{-1}$$

Time of fall of box,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ s}$$

So, angle turned by box in reaching ground is $\theta = \omega t = 0.5 \times 1 = 0.5 \text{ rad}$

- 137** A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ , where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I , then the angular acceleration of the disc is

[2019, 9 April Shift-I]

- (a) $\frac{k}{2I}$ (b) $\frac{k}{I}$
(c) $\frac{k}{4I}$ (d) $\frac{2k}{I}$

Ans. (d)Given, kinetic energy = $k\theta^2$ We know that, kinetic energy of a rotating body about its axis = $\frac{1}{2}I\omega^2$ where, I is moment of inertia and ω is angular velocity.

$$\text{So, } \frac{1}{2}I\omega^2 = k\theta^2 \text{ or } \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{2k}{I}} \theta \quad \dots (i)$$

Differentiating the above equation w.r.t. time on both sides, we get

$$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \frac{d\theta}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega \quad \left[\because \omega = \frac{d\theta}{dt} \right]$$

 $\therefore$ Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}} \theta \quad [\text{using Eq. (i)}]$$

$$\text{or } \alpha = \frac{2k}{I} \theta$$

Alternate Solution

$$\text{As, } \omega^2 = \frac{2k\theta^2}{I}$$

$$\Rightarrow 2\omega \frac{d\omega}{dt} = \frac{2k}{I} \cdot 2\theta \frac{d\theta}{dt}$$

$$\text{or } \omega d\omega = \frac{2k}{I} \theta d\theta$$

$$\omega \frac{d\omega}{d\theta} (= \alpha) = \frac{2k}{I} \theta$$

$$\text{or } \alpha = \frac{2k}{I} \theta$$

- 138** The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R , (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is

[2019, 9 April Shift-I]

- (a) 10:15:7 (b) 4:3:2
(c) 14:15:20 (d) 2:3:4

Ans. (c)

From question,

let height attained by ring = h_1 Height attained by cylinder = h_2 Height attained by sphere = h_3

As we know that for a body which is rolling up an inclined plane (without slipping), follows the law of conservation of energy.

 $\therefore$ For ring, using energy conservation law at its height h_1 .

$$(KE)_{\text{linear}} + (KE)_{\text{rotational}} = (PE)$$

$$\Rightarrow \frac{1}{2}m_1v_0^2 + \frac{1}{2}I_1\omega^2 = m_1gh_1$$

$$\Rightarrow \frac{1}{2}m_1v_0^2 + \frac{1}{2}m_1R^2\omega^2 = m_1gh_1$$

$$(\because I = mR^2 \text{ for ring})$$

$$\Rightarrow gh_1 = \frac{v_0^2}{2} + \frac{v_0^2}{2} \quad (\because v_0 = \omega R)$$

$$\Rightarrow h_1 = v_0^2 / g \quad \dots (i)$$

Similarly, for solid cylinder, applying the law of conservation of energy,

$$\frac{1}{2}m_2v_0^2 + \frac{1}{2}I_2\omega^2 = m_2gh_2$$

$$\Rightarrow \frac{1}{2}m_2v_0^2 + \frac{1}{2}\left(\frac{1}{2} \times m_2 \times \left(\frac{R}{2}\right)^2\right)\omega^2 = m_2gh_2$$

$$\left[\because I = \frac{1}{2}mR^2 \text{ for cylinder} \right]$$

$$\text{and } R = \frac{R}{2}$$

$$\Rightarrow \frac{1}{2}m_2v_0^2 + \frac{1}{2} \times \frac{1}{8}m_2R^2 \times \frac{v_0^2}{(R/2)^2} = m_2gh_2$$

$$\Rightarrow \frac{1}{2}v_0^2 + \frac{1}{4}v_0^2 = gh_2 \Rightarrow gh_2 = \frac{3}{4}v_0^2$$

$$\Rightarrow h_2 = \frac{3}{4}\left(\frac{v_0^2}{g}\right) \quad \dots (ii)$$

Similarly, for solid sphere,

$$\frac{1}{2}m_3v_0^2 + \frac{1}{2}I_3\omega^2 = m_3gh_3$$

$$\Rightarrow \frac{1}{2}m_3v_0^2 + \frac{1}{2}\left[\frac{2}{5}m_3\left(\frac{R}{4}\right)^2\right]\omega^2 = m_3gh_3$$

$$\left[\because I = \frac{2}{5}mR^2 \text{ for solid sphere} \right]$$

$$\text{and } R = \frac{R}{4}$$

$$\Rightarrow \frac{1}{2}m_3v_0^2 + \frac{1}{2} \times \frac{2}{5} \times m_3 \frac{R^2}{16} \times \frac{v_0^2}{(R/4)^2} = m_3gh_3$$

$$\Rightarrow \frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = gh_3 \Rightarrow gh_3 = \frac{7}{10}v_0^2$$

$$\text{or } h_3 = \frac{7}{10}\left(\frac{v_0^2}{g}\right) \quad \dots (iii)$$

 $\therefore$ Taking the ratio of h_1 , h_2 and h_3 by using Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{v_0^2}{g} : \frac{3}{4} \frac{v_0^2}{g} : \frac{7}{10} \frac{v_0^2}{g} = 1 : \frac{3}{4} : \frac{7}{10}$$

$$\Rightarrow h_1 : h_2 : h_3 = 40 : 30 : 28 = 20 : 15 : 14$$

 $\therefore$ The most appropriate option is (c).

Although, it is still not in the correct sequence.

Alternate Solution

Total kinetic energy of a rolling body is also given as

$$E_{\text{total}} = \frac{1}{2}mv^2 \left[1 + \frac{K^2}{R^2} \right]$$

where, K is the radius of gyration.

Using conservation law of energy,

$$\frac{1}{2}mv^2 \left[1 + \frac{K^2}{R^2} \right] = mgh$$

$$\text{or } h = \frac{v^2}{2g} \left[1 + \frac{K^2}{R^2} \right]$$

$$\text{For ring, } \frac{K^2}{R^2} = 1$$

$$\Rightarrow h_1 = \frac{v^2}{2g} [1 + 1] = \frac{2v^2}{2g} = \frac{v^2}{g}$$

For solid cylinder,

$$\frac{K^2}{R^2} = \frac{(R/2\sqrt{2})^2}{(R/2)^2} = \frac{R^2 \times 4}{8 \times R^2} = \frac{1}{2}$$

$$\Rightarrow h_2 = \frac{v^2}{2g} \left[1 + \frac{1}{2} \right] = \frac{3v^2}{4g}$$

$$\text{For solid sphere, } \frac{K^2}{R^2} = \frac{2}{5}$$

$$\Rightarrow h_3 = \frac{v^2}{2g} \left[1 + \frac{2}{5} \right] = \frac{7v^2}{10g}$$

So, the ratio of h_1 , h_2 and h_3 is

$$h_1 : h_2 : h_3 = \frac{v^2}{g} : \frac{3v^2}{4g} : \frac{7v^2}{10g} = 1 : \frac{3}{4} : \frac{7}{10} = 20 : 15 : 14$$

- 139** A wedge of mass $M = 4 \text{ m}$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v . There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by

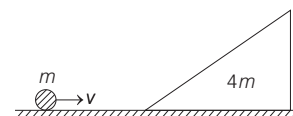
[2019, 9 April Shift-II]

$$(a) \frac{2v^2}{7g} \quad (b) \frac{v^2}{g} \quad (c) \frac{2v^2}{5g} \quad (d) \frac{v^2}{2g}$$

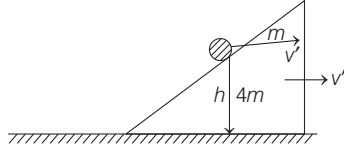
Ans. (c)

Key Idea Since, the ground is frictionless, so when the particle will collide and climb over the wedge, then the wedge will also move. Thus, by using conservation laws for momentum and energy, maximum height climbed by the particle can be calculated.

Initial condition can be shown in the figure below



As mass m collides with wedge, let both wedge and mass move with speed v' . Then,



By applying linear momentum conservation, we have Initial momentum of the system

= Final momentum of the system

$$mv + 0 = (m + 4m)v'$$

$$\Rightarrow v' = \frac{v}{5} \quad \dots (i)$$

Now, if m rises upto height h over wedge, then by applying conservation of mechanical energy, we have Initial energy of the system = Final energy of the system

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv'^2 + mgh + \frac{1}{2}(4m)v'^2$$

$$mv^2 = (m + 4m)v'^2 + 2mgh$$

$$\Rightarrow v^2 = 5v'^2 + 2gh$$

$$\Rightarrow v^2 = \frac{1}{5}v^2 + 2gh \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \frac{4}{5}v^2 = 2gh \Rightarrow h = \frac{2v^2}{5g}$$

- 140** Moment of inertia of a body about a given axis is 1.5 kg m^2 . Initially, the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of

[2019, 9 April Shift-II]

- (a) 5 s (b) 2 s (c) 3 s (d) 2.5 s

Ans. (b)

Rotation kinetic energy of a body is given by

$$KE_{\text{rotational}} = \frac{1}{2}I\omega^2$$

where $\omega = \omega_0 + \alpha t$

$$\text{So, } KE_{\text{rotational}} = \frac{1}{2}I(\omega_0 + \alpha t)^2 \quad \dots (i)$$

Here, $I = 1.5 \text{ kg m}^2$,

$$KE = 1200 \text{ J and}$$

$$\alpha = 20 \text{ rad/s}^2$$

and $\omega_0 = 0$

Substituting these values in Eq. (i), we get

$$1200 = \frac{1}{2}(1.5)(20 \times t)^2$$

$$\Rightarrow t^2 = \frac{2 \times 1200}{1.5 \times 400} = 4 \therefore t = 2 \text{ s}$$

- 141** Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis.

They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then

$(E_f - E_i)$ is [2019, 10 April Shift-I]

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $-\frac{I_1\omega_1^2}{12}$

(c) $\frac{3}{8}I_1\omega_1^2$ (d) $\frac{I_1\omega_1^2}{6}$

Ans. (a)

Initial kinetic energy of the given system,

$$KE_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{16}\right)I_1\omega_1^2 = \frac{9}{16}I_1\omega_1^2 \quad \dots (i)$$

Now, using angular momentum conservation law (assuming angular velocity after contact is ω)

Initial angular momentum = Final angular momentum

$$I_1\omega_1 + \left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right) = I_1\omega' + \frac{I_1}{2}\omega'$$

$$\Rightarrow \frac{5}{4}\omega_1 = \frac{3}{2}\omega' \text{ or } \omega' = \frac{5}{6}\omega_1 \quad \dots (ii)$$

Now, final kinetic energy (after contact) is

$$KE_f = \frac{1}{2}I_1\omega'^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\omega'^2$$

$$= \frac{1}{2}I_1\left(\frac{5}{6}\omega_1\right)^2 + \frac{1}{4}I_1\left(\frac{5}{6}\omega_1\right)^2$$

$$= \left(\frac{25}{72} + \frac{25}{144}\right)I_1\omega_1^2 = \frac{25}{48}I_1\omega_1^2 \quad \dots (iii)$$

Hence, change in KE,

$$\Delta KE = KE_f - KE_i = \frac{25}{48}I_1\omega_1^2 - \frac{9}{16}I_1\omega_1^2$$

$$[\text{using Eqs. (i)}]$$

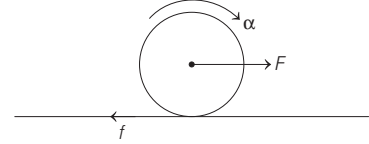
$$\Delta KE = -\frac{1}{24}I_1\omega_1^2$$

- 142** A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is [2019, 10 Jan Shift-I]

(a) $\frac{F}{2mR}$ (b) $\frac{2F}{3mR}$ (c) $\frac{3F}{2mR}$ (d) $\frac{F}{3mR}$

Ans. (b)

When force F is applied at the centre of roller of mass m as shown in the figure below



Its acceleration is given by

$$\frac{(F - f)}{m} = \alpha \quad \dots (i)$$

where, f = force of friction and mass of roller. Torque on roller is provided by friction f and it is

$$\tau = fR = I\alpha \quad \dots (ii)$$

where, I = moment of inertia of solid cylindrical roller.

$= mR^2/2$ and α = angular acceleration of cylinder $= a/R$.

$$\text{Hence, } \tau = \frac{mR^2}{2} \cdot \frac{a}{R} = \frac{maR}{2}$$

From eq. (ii), $(\tau = fR)$

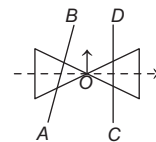
$$f = \frac{ma}{2} \quad \dots (iii)$$

From Eqs. (i) and (iii), we get

$$F = \frac{3}{2}ma \Rightarrow a = \frac{2F}{3m}$$

$$\text{So, } \alpha = \frac{2F}{3mR} \quad \left[\because \alpha = \frac{a}{R} \right]$$

- 143** A roller is made by joining together two corners at their vertices O . It is kept on two rails AB and CD which are placed a symmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light path, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to [JEE Main 2016]



- (a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately

Ans. (a)

As, the wheel rolls forward the radius of the wheel decreases along AB , hence for the same number of rotations it moves less distance along AB , hence it turns left.

- 144** A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ N (where, t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 , then the number of rotations made by the pulley before its direction of motion is reversed, is [AIEEE 2011]

- (a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9
(d) less than 3

Ans. (a)

Given force, $F = 20t - 5t^2$

$$\alpha = \frac{FR}{I}$$

$$= \frac{(20t - 5t^2)2}{10} = 4t - t^2$$

$$\Rightarrow \frac{d\omega}{dt} = 4t - t^2$$

$$\Rightarrow \int_0^\omega d\omega = \int_0^t (4t - t^2) dt$$

$$\Rightarrow \omega = 2t^2 - \frac{t^3}{3}$$

When direction is reversed, $\omega = 0$, i.e., $t = 0, 6s$

Now, $d\theta = \omega dt$

$$\Rightarrow \int_0^\theta d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3} \right) dt$$

$$\Rightarrow \theta = \left[\frac{2t^3}{3} - \frac{t^4}{12} \right]_0^6$$

$$\Rightarrow \theta = 144 - 108 = 36 \text{ rad}$$

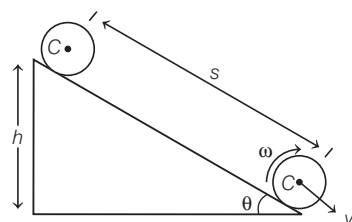
$$\therefore \text{Number of rotations, } n = \frac{\theta}{2\pi} = \frac{36}{2\pi} < 6$$

- 145** A round uniform body of radius R , mass M and moment of inertia I , rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then, its acceleration is [AIEEE 2007]

- (a) $\frac{g \sin \theta}{1 + I/MR^2}$ (b) $\frac{g \sin \theta}{1 + MR^2/I}$
(c) $\frac{g \sin \theta}{1 - I/MR^2}$ (d) $\frac{g \sin \theta}{1 - MR^2/I}$

Ans. (a)

Assuming that no energy is used up against friction, the total loss in potential energy is equal to the total gain in the kinetic energy.



$$Mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} Mv^2 \quad [\text{as } v = r\omega]$$

$$\text{i.e., } Mgh = \frac{1}{2} I \left(\frac{v^2}{R^2} \right) + \frac{1}{2} Mv^2$$

$$\text{or } \frac{1}{2} v^2 \left(M + \frac{I}{R^2} \right) = Mgh$$

$$\text{or } v^2 = \frac{2Mgh}{M + I/R^2} = \frac{2gh}{1 + I/MR^2}$$

If s is the distance covered along the plane, then

Alternate Solution

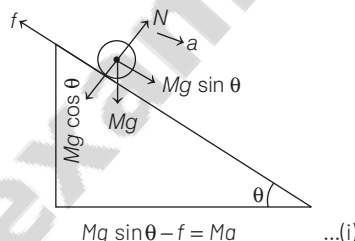
$$h = s \sin \theta$$

$$\therefore v^2 = \frac{2gs \sin \theta}{1 + I/MR^2}$$

$$\text{Now, } v^2 = 2as$$

$$\therefore 2as = \frac{2gs \sin \theta}{1 + I/MR^2}$$

$$\text{or } a = \frac{g \sin \theta}{1 + I/MR^2}$$



$$Mg \sin \theta - f = Ma \quad \dots(i)$$

$$\text{As torque} = I\alpha = I \frac{a}{R} \quad [\text{as } a = R\alpha]$$

$$\text{Torque } \tau = f \cdot R$$

$$\text{So, } f \cdot R = \frac{Ia}{R} \Rightarrow f = \frac{Ia}{R^2}$$

From Eq. (i),

$$Mg \sin \theta - \frac{Ia}{R^2} = Ma$$

$$\Rightarrow a \left[M + \frac{I}{R^2} \right] = Mg \sin \theta$$

$$a = \frac{Mg \sin \theta}{M + \frac{I}{R^2}}$$

$$\therefore a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

- 146** A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius $4R$ is made from an iron plate of

thickness $t/4$. Then, the relation between the moment of inertia

I_X and I_Y is

[AIEEE 2003]

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64I_X$

Ans. (d)

Mass of disc (X), $m_X = \pi R^2 t \rho$

where, ρ = density of material of disc

$$\therefore I_X = \frac{1}{2} m_X R^2 = \frac{1}{2} \pi R^2 t \rho R^2$$

$$\text{or } I_X = \frac{1}{2} \pi \rho t R^4 \quad \dots(i)$$

Mass of disc (Y),

$$m_Y = \pi (4R)^2 \frac{t}{4} \rho = 4\pi R^2 t \rho$$

$$\text{and } I_Y = \frac{1}{2} m_Y (4R)^2 = \frac{1}{2} 4\pi R^2 t \rho \cdot 16R^2$$

$$\text{or } I_Y = 32\pi \rho t R^4 \quad \dots(ii)$$

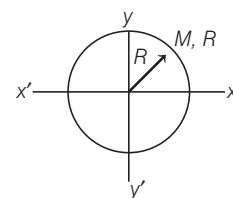
$$\therefore \frac{I_Y}{I_X} = \frac{32\pi \rho t R^4}{\frac{1}{2} \pi \rho t R^4} = 64 \quad \text{or } I_Y = 64I_X$$

- 147** Moment of inertia of a circular wire of mass M and radius R about its diameter is [AIEEE 2002]

- (a) $\frac{MR^2}{2}$ (b) MR^2 (c) $2MR^2$ (d) $\frac{MR^2}{4}$

Ans. (a)

Moment of inertia of circular wire about its axis is MR^2 . Consider two diameters xx' and yy' moment of inertia about any of these diameters is same let us say I .



From perpendicular axes theorem,

$$I + I = MR^2 \quad \text{or } I = \frac{MR^2}{2}$$

- 148** A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless), so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling) [AIEEE 2002]

- (a) solid sphere (b) hollow sphere
(c) ring (d) All same

Ans. (d)

Since, the inclined plane is frictionless, then there will be no rolling and the mass will only slide down.

Hence, acceleration is same for all the given bodies.

07

Gravitation

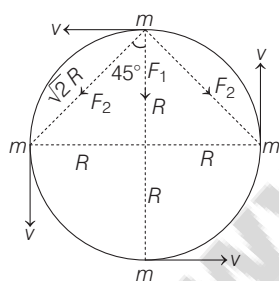
TOPIC 1

Kepler's Law and Universal Law of Gravitation

- 01** Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be [2021, 24 Feb Shift-I]

- (a) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$
 (b) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$
 (c) $\sqrt{G(1+2\sqrt{2})}$
 (d) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

Ans. (*)



Given, $m = 1 \text{ kg}$, $R = 1 \text{ m}$

We know that,

$$F = \frac{Gm_1m_2}{r^2}$$

$$\therefore F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

$$\text{and } F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

Net force on one particle,

$$F_{\text{net}} = F_1 + F_2 \cos 45^\circ + F_2 \cos 45^\circ \\ = F_1 + 2F_2 \cos 45^\circ$$

$$= \frac{Gm^2}{4R^2} + 2 \left(\frac{Gm^2}{2R^2} \right) \cdot \frac{1}{\sqrt{2}} \\ = \frac{Gm^2}{4R^2} + \frac{Gm^2}{\sqrt{2}R^2} \\ = \frac{Gm^2}{R^2} \left[\frac{1}{4} + \frac{1}{\sqrt{2}} \right]$$

As the gravitational force provides the necessary centripetal force, so

$$F_{\text{net}} = F_c = \frac{mv^2}{R}$$

Here, F_c = centripetal force.

$$\Rightarrow \frac{Gm^2}{R^2} \left[\frac{1}{4} + \frac{1}{\sqrt{2}} \right] = \frac{mv^2}{R}$$

$$\Rightarrow v = \frac{1}{2} \sqrt{\frac{Gm}{R}(1+2\sqrt{2})}$$

$$\Rightarrow v = \frac{1}{2} \sqrt{G(1+2\sqrt{2})}$$

- 02** A cylindrical wire of radius 0.5 mm and conductivity $5 \times 10^7 \text{ S/m}$ is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3 \pi \text{ mA}$. The value of x is [2021, 24 Feb Shift-II]

Ans. (5)

Given, radius of cylindrical wire, $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Conductivity, $\sigma = 5 \times 10^7 \text{ S/m}$

Electric field, $E = 10 \text{ mV/m} = 10 \times 10^{-3} \text{ V/m}$

We know that current density,

$$\therefore J = \sigma E = 5 \times 10^7 \times 10 \times 10^{-3} \\ = 5 \times 10^5 \text{ A/m}^2$$

Also, $J = I/A \Rightarrow I = JA$

$$\Rightarrow I = 5 \times 10^5 \times \pi \times (0.5 \times 10^{-3})^2$$

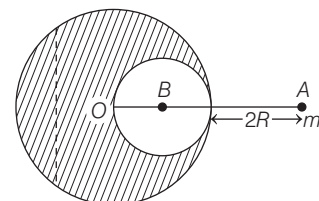
$$= 5 \times 10^5 \times \pi \times 25 \times 10^{-8} \\ = 125 \pi \times 10^{-3}$$

$$\Rightarrow x^3 \pi \text{ mA} = 125 \pi \text{ mA}$$

$$\Rightarrow x^3 = 5^3$$

$$\Rightarrow x = 5$$

- 03** A solid sphere of radius R gravitationally attracts a particle placed at $3R$ from its centre with a force F_1 . Now, a spherical cavity of radius $(R/2)$ is made in the sphere (as shown in figure) and the force becomes F_2 . The value of $F_1 : F_2$ is [2021, 25 Feb Shift-I]



- (a) 25 : 36 (b) 36 : 25
 (c) 50 : 41 (d) 41 : 50

Ans. (c)

Given, radius of sphere = R

Distance of particle from centre of Earth = $3R$

Force between sphere and particle = F_1

Radius of cavity = $R/2$

Let mass of sphere = M

Mass of sphere with cavity = M'

Mass of particle = m

$$\text{Now, } M' = \rho \cdot \frac{4}{3} \pi \left(\frac{R}{2} \right)^3 = \frac{M}{8}$$

By using concept of gravitational force,

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2} \quad \dots (i)$$

$$\text{and } F_{\text{cavity}} = \frac{GM'm}{(AB)^2} = \frac{G(M/8)m}{(2R + R/2)^2} \\ = \frac{4GMm}{25 \times 8 R^2} \quad \dots (ii)$$

$$\therefore F_2 = F_1 - F_{\text{cavity}} = \frac{GMm}{9R^2} - \frac{4}{25 \times 8} \frac{GMm}{R^2} \\ = \frac{GMm}{R^2} \left(\frac{1}{9} - \frac{1}{50} \right) = \frac{41}{50 \times 9} \frac{GMm}{R^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{50}{41}$$

04 A planet revolving in elliptical orbit has

- I. a constant velocity of revolution
- II. has the least velocity when it is nearest to the Sun
- III. its areal velocity is directly proportional to its velocity
- IV. areal velocity is inversely proportional to its velocity.
- V. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below.

[2021, 26 Feb Shift-I]

- (a) Only I (b) Only IV
(c) Only III (d) Only V

Ans. (d)

According to Kepler's second law of planetary motion, areal velocity of every planet moving around the sun should remain constant in elliptical orbit.

05 The maximum and minimum distance of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m, respectively. If the speed of the comet at the nearest point is $6 \times 10^4 \text{ ms}^{-1}$, then the speed at the farthest point is

[2021, 16 March Shift-I]

- (a) $1.5 \times 10^3 \text{ m/s}$ (b) $6.0 \times 10^3 \text{ m/s}$
(c) $3.0 \times 10^3 \text{ m/s}$ (d) $4.5 \times 10^3 \text{ m/s}$

Ans. (c)

Given, maximum distance of comet from the Sun, $r_1 = 1.6 \times 10^{12}$ m

Minimum distance of comet from the Sun,

$$r_2 = 8.0 \times 10^{10} \text{ m}$$

Speed of the comet at nearest point, $v_2 = 6 \times 10^4 \text{ ms}^{-1}$

Applying law of conservation of angular momentum,

$$\begin{aligned} m v_1 r_1 &= m v_2 r_2 \\ (\because \text{mass of comet will remain same}) \\ \Rightarrow v_1 &= \frac{v_2 r_2}{r_1} \\ &= \frac{6 \times 10^4 \times 8 \times 10^{10}}{1.6 \times 10^{12}} = \frac{48 \times 10^{14}}{1.6 \times 10^{12}} \\ &= 3 \times 10^3 \text{ m/s} \end{aligned}$$

06 A geostationary satellite is orbiting around an arbitrary planet P at a height of $11R$ above the surface of P , R being the radius of P . The time period of another satellite in hours

at a height of $2R$ from the surface of P is P has the time period of 24 h.

[2021, 17 March Shift-II]

- (a) $6\sqrt{2}$ (b) $6/\sqrt{2}$ (c) 3 (d) 5

Ans. (c)

Given,

Height of satellite from planet's surface, $h = 11R$

So, the total distance from the centre of planet = $11R + R = 12R$

Time period of planet = 24 h

Similarly, the total distance of second satellite from the centre of planet = $2R + R = 3R$

Using the Kepler's law

$$T^2 \propto R^3$$

$$\Rightarrow \left(\frac{T_1}{T_2} \right) = \left(\frac{R_1}{R_2} \right)^{3/2}$$

Substituting the values in the above equation, we get

$$\left(\frac{24}{T} \right) = \left(\frac{12R}{3R} \right)^{3/2}$$

$$T = 3 \text{ h}$$

$\therefore$ The time period of another satellite is 3 h.

07 The time period of a satellite in a circular orbit of radius R is T . The period of another satellite in a circular orbit of radius $9R$ is

[2021, 18 March Shift-I]

- (a) $9T$ (b) $27T$ (c) $12T$ (d) $3T$

Ans. (b)

According to Kepler's third law,

$$T^2 \propto R^3$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1} \right)^{3/2}$$

$$\Rightarrow \frac{T_2}{T} = \left(\frac{9R}{R} \right)^{3/2} \quad \left[\because R_1 = R \right]$$

$$\Rightarrow T_2 = 27T$$

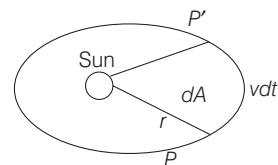
08 The angular momentum of a planet of mass M moving around the Sun in an elliptical orbit is L . The magnitude of the areal velocity of the planet is [2021, 18 March Shift-II]

- (a) $\frac{4L}{M}$ (b) $\frac{L}{M}$ (c) $\frac{2L}{M}$ (d) $\frac{L}{2M}$

Ans. (d)

According to the Kepler's second law,

$$\frac{dA}{dt} = \text{constant}$$



Area covered from P to P' is dA , then

$$dA = \frac{1}{2} r^2 d\theta$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \omega \quad \dots(i)$$

Angular momentum,

$$L = Mvr = M(\omega r)r = M\omega r^2$$

$$\Rightarrow \omega = \frac{L}{mr^2} \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{dA}{dt} = \frac{1}{2} r^2 \left(\frac{L}{Mr^2} \right)$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2M}$$

09 Two identical particles of mass 1 kg each go round a circle of radius R , under the action of their mutual gravitational attraction. The angular speed of each particle is

[2021, 22 July Shift-II]

- (a) $\sqrt{\frac{G}{2R^3}}$ (b) $\frac{1}{2} \sqrt{\frac{G}{R^3}}$
(c) $\frac{1}{2R} \sqrt{\frac{1}{G}}$ (d) $\sqrt{\frac{2G}{R^3}}$

Ans. (b)

$\therefore$ Gravitational force between both particles will be

$$F = \frac{Gm^2}{R^2} \quad \dots(i)$$

$\therefore$ Both are tracing a circular path, centripetal force will be acting on them.

$$F = \frac{Gm^2}{(2R)^2}$$

$$\Rightarrow F_c = \frac{m(\omega R)^2}{R} \quad [\because v = \omega R]$$

$$\Rightarrow F_c = m\omega^2 R \quad \dots(ii)$$

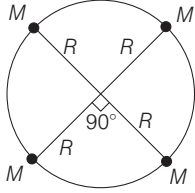
During this circular motion,

Gravitational force between the particles = Centripetal force acting on the particles

$$\Rightarrow \frac{Gm^2}{4R^2} = m\omega^2 R$$

$$\Rightarrow \omega = \frac{1}{2} \sqrt{\frac{Gm}{R^3}} = \frac{1}{2} \sqrt{\frac{G}{R^3}} m \quad [\because m = 1 \text{ kg}]$$

- 10** Four particles each of mass M , move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is



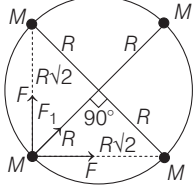
[2021, 1 Sep Shift-I]

- (a) $\frac{1}{2} \sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$ (b) $\frac{1}{2} \sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$
 (c) $\frac{1}{2} \sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (d) $\sqrt{\frac{GM}{R}}$

Ans. (b)

Let us consider the gravitational force acting on each mass M by adjacent particles be F .

and the gravitational force acting on each mass M diagonally be F_1 .



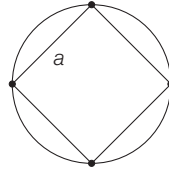
$$\text{The net force, } F_{\text{net}} = \frac{Mv^2}{R}$$

Along the centre of circle,

$$\begin{aligned} \sqrt{2}F + F_1 &= \frac{Mv^2}{R} \\ \sqrt{2} \left(\frac{GMM}{(\sqrt{2}R)^2} \right) + \left(\frac{GMM}{(2R)^2} \right) &= \frac{Mv^2}{R} \\ \Rightarrow \frac{GM}{R^2} \left(\frac{1}{\sqrt{2}} + \frac{1}{4} \right) &= \frac{v^2}{R} \\ \Rightarrow v &= \frac{1}{2} \sqrt{\frac{GM(2\sqrt{2}+1)}{R}} \end{aligned}$$

Hence, the speed of each particle is $\frac{1}{2} \sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$.

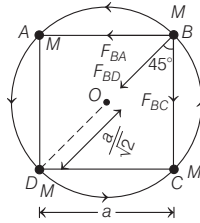
- 11** Four identical particles of mass M are located at the corners of a square of side a . What should be their speed, if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square? [2019, 8 April Shift-I]



- (a) $1.35 \sqrt{\frac{GM}{a}}$ (b) $1.16 \sqrt{\frac{GM}{a}}$
 (c) $1.21 \sqrt{\frac{GM}{a}}$ (d) $1.41 \sqrt{\frac{GM}{a}}$

Ans. (b)

Key Idea In given configuration of masses, net gravitational force provides the necessary centripetal force for rotation.



Net force on mass M at position B towards centre of circle is

$$\begin{aligned} F_{B0\text{net}} &= F_{BD} + F_{BA} \sin 45^\circ + F_{BC} \cos 45^\circ \\ &= \frac{GM^2}{(\sqrt{2}a)^2} + \frac{GM^2}{a^2} \left(\frac{1}{\sqrt{2}} \right) + \frac{GM^2}{a^2} \left(\frac{1}{\sqrt{2}} \right) \\ &\quad \left[\text{where, diagonal length } BD \text{ is } \sqrt{2}a \right] \\ &= \frac{GM^2}{2a^2} + \frac{GM^2}{a^2} \left(\frac{2}{\sqrt{2}} \right) = \frac{GM^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right) \end{aligned}$$

This force will act as centripetal force. Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

$$\text{Here, } F_{\text{centripetal}} = \frac{Mv^2}{r} = \frac{Mv^2}{\frac{a}{\sqrt{2}}} = \frac{\sqrt{2}Mv^2}{a} \quad \left(\because r = \frac{a}{\sqrt{2}} \right)$$

So, for rotation about the centre,

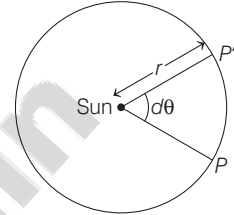
$$\begin{aligned} F_{\text{centripetal}} &= F_{B0\text{net}} \\ \Rightarrow \sqrt{2} \frac{Mv^2}{a} &= \frac{GM^2}{a^2} \left(\frac{1}{2} + \sqrt{2} \right) \\ \Rightarrow v^2 &= \frac{GM}{a} \left(1 + \frac{1}{2\sqrt{2}} \right) = \frac{GM}{a} (1.35) \\ \Rightarrow v &= 1.16 \sqrt{\frac{GM}{a}} \end{aligned}$$

- 12** If the angular momentum of a planet of mass m , moving around sun in a circular orbit is L about the centre of the sun, its areal velocity is [2019, 9 Jan Shift-I]

- (a) $\frac{4L}{m}$ (b) $\frac{2L}{m}$
 (c) $\frac{L}{2m}$ (d) $\frac{L}{m}$

Ans. (c)

According to Kepler's second law, "the line joining the planet to sun sweeps out equal areas in equal interval of time". This means the rate of change of area with time is constant.



The area covered from P to P' is dA , which is given by

$$\begin{aligned} dA &= \frac{d\theta}{2\pi} \times \pi r^2 \\ \Rightarrow dA &= \frac{1}{2} r^2 d\theta \\ \text{or } \frac{dA}{dt} &= \frac{1}{2} r^2 \frac{d\theta}{dt} \\ \text{where, } \frac{dA}{dt} &= \text{areal velocity.} \\ \frac{dA}{dt} &= \frac{1}{2} r^2 \omega \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \cdot \frac{L}{mr^2} \\ (\text{because angular momentum, } L &= mr^2 \omega) \\ \frac{dA}{dt} &= \frac{L}{2m} \end{aligned}$$

- 13** A straight rod of length L extends from $x=a$ to $x=L+a$. The gravitational force it exerts on a point mass m at $x=0$, if the mass per unit length of the rod is $A+Bx^2$, is given by [2019, 12 Jan Shift-I]

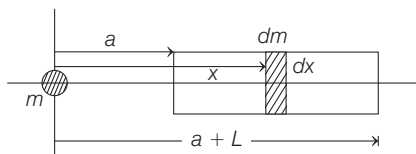
- (a) $Gm \left[A \left(\frac{1}{a+L} - \frac{1}{a} \right) - BL \right]$
 (b) $Gm \left[A \left(\frac{1}{a+L} - \frac{1}{a} \right) + BL \right]$
 (c) $Gm \left[A \left(\frac{1}{a} - \frac{1}{a+L} \right) + BL \right]$
 (d) $Gm \left[A \left(\frac{1}{a} - \frac{1}{a+L} \right) - BL \right]$

Ans. (c)

Given, situation is,

Force of attraction between mass ' m ' and an elemental mass ' dm ' of rod is

$$dF = \frac{Gm dm}{x^2} = \frac{Gm(A+Bx^2) dx}{x^2}$$



Total attraction force is sum of all such differential forces produced by elemental parts of rod from $x = a$ to $x = a + L$.

$$\begin{aligned} \therefore F &= \int dF = \int_{x=a}^{x=a+L} \frac{Gm(A + Bx^2)}{x^2} dx \\ &= Gm \int_{x=a}^{x=a+L} \left(\frac{A}{x^2} + B \right) dx \\ &= Gm \left[-\frac{A}{x} + Bx \right]_{x=a}^{x=a+L} \\ &= Gm \left[\frac{-A}{a+L} + B(a+L) + \frac{A}{a} - Ba \right] \\ &= Gm \left[A \left(\frac{1}{a} - \frac{1}{a+L} \right) + BL \right] \end{aligned}$$

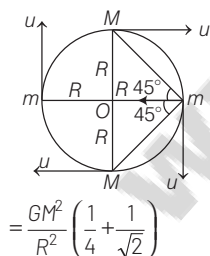
- 14** Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is [JEE Main 2014]

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2} \frac{GM}{R}}$
 (c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (d) $\frac{1}{2} \sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Ans. (d)

Net force acting on any one particle M

$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2} \cos 45^\circ + \frac{GM}{(R\sqrt{2})^2} \cos 45^\circ$$



This force will equal to centripetal force.

$$\text{So, } \frac{Mu^2}{R} = \frac{GM^2}{R^2} \left(\frac{1}{4} + \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow u = \sqrt{\frac{GM}{4R}(1+2\sqrt{2})} = \frac{1}{2} \sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

Hence, speed of each particle in a circular motion is

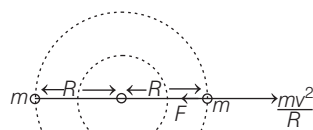
$$\frac{1}{2} \sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

- 15** Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is [AIEEE 2011]

(a) $\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{Gm}{4R}}$ (c) $\sqrt{\frac{Gm}{3R}}$ (d) $\sqrt{\frac{Gm}{2R}}$

Ans. (b)

As gravitational force provides necessary centripetal force.



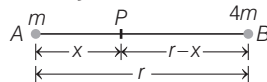
$$\begin{aligned} \text{i.e., } F &= \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R} \\ \Rightarrow v &= \sqrt{\frac{Gm}{4R}} \end{aligned}$$

- 16** Two bodies of masses m and $4m$ are placed at a distance r . The gravitational potential at a point on the line joining them, where the gravitational field is zero, is [AIEEE 2011]

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$
 (c) $-\frac{9Gm}{r}$ (d) zero

Ans. (c)

Let gravitational field be zero at P as shown in figure.



$$\begin{aligned} \therefore \frac{Gm}{x^2} &= \frac{G(4m)}{(r-x)^2} \\ \Rightarrow 4x^2 &= (r-x)^2 \\ \Rightarrow 2x &= r-x \\ \Rightarrow x &= \frac{r}{3} \\ \therefore V_p &= -\frac{Gm}{x} - \frac{G(4m)}{r-x} \\ &= -\frac{3Gm}{r} - \frac{6Gm}{r} = -\frac{9Gm}{r} \end{aligned}$$

- 17** The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where, g is the acceleration due to gravity on the surface of the

earth) in terms of R , the radius of the earth is [AIEEE 2009]

(a) $2R$ (b) $\frac{R}{\sqrt{3}}$
 (c) $\frac{R}{2}$ (d) $\sqrt{2}R$

Ans. (a)

As value of acceleration due to gravity above the earth surface is g .

$$g' = \frac{GM}{(R+h)^2}$$

acceleration due to gravity at height h

$$\Rightarrow \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2} = g \left(\frac{R}{R+h} \right)^2$$

$$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h} \right)^2$$

$$\Rightarrow \frac{R}{R+h} = \frac{1}{3} \Rightarrow 3R = R+h$$

$$\Rightarrow 2R = h$$

- 18** If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will [AIEEE 2002]

- (a) continue to move in its orbit with same velocity
 (b) move tangentially to the original orbit with the same velocity
 (c) become stationary in its orbit
 (d) move towards the earth

Ans. (b)

When gravitational force becomes zero, then centripetal force required cannot be provided.

So, the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, i.e., it moves tangentially to the original orbit.

TOPIC 2

Acceleration Due to Gravity and Gravitational PE

- 19** If the angular velocity of Earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately

(Take $2g = 10 \text{ ms}^{-2}$, the radius of Earth, $R = 6400 \times 10^3 \text{ m}$, take $\pi = 3.14$) [2021, 18 March Shift-II]

- (a) 60 min (b) does not change
(c) 1200 min (d) 84 min

Ans. (d)

Condition of weightlessness, $g_{\text{equator}} = 0$

$$g_{\text{equator}} = g - \omega^2 R$$

The radius of the Earth, $R = 6400 \text{ km}$
 $= 6.4 \times 10^6 \text{ m}$

$$\Rightarrow \omega = \sqrt{\frac{g}{R}}$$

$$\Rightarrow \omega = \sqrt{\frac{10}{6.4 \times 10^6 \text{ m}}} \Rightarrow \omega = \frac{1}{800} \text{ rad/s}$$

The duration of the day,

$$T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{\frac{1}{800}}$$

$$\Rightarrow T = 5024 \text{ s} \Rightarrow T = \frac{5024}{60} \text{ min}$$

$$\Rightarrow T = 84 \text{ min}$$

Hence, the duration of the day would be 84 min.

- 20** A body weights 49 N on a spring balance at the North pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

(Use, $g = \frac{GM}{R^2} = 9.8 \text{ ms}^{-2}$ and radius of earth, $R = 6400 \text{ km}$)

[2021, 24 Feb Shift-II]

- (a) 49 N (b) 48.83 N
(c) 49.83 N (d) 49.17 N

Ans. (b)

Given, weight of body at North pole,
 $w_p = mg = 49 \text{ N}$

Radius of Earth, $R = 6400 \text{ km}$

Let weight of body at equator be w_e .

At equator, $g_e = g - R\omega^2$

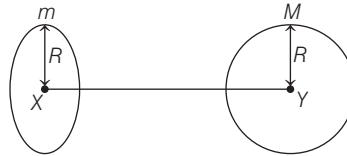
$$\therefore w_e = mg_e = m(g - R\omega^2)$$

Since, $w_p > w_e \Rightarrow w_e < 49 \text{ N}$

Hence, above condition is satisfied by only option (b).

- 21** Find the gravitational force of attraction between the ring and sphere as shown in the figure, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass m) and a sphere (of mass M), where both have equal radius R .

[2021, 26 Feb Shift-I]



- (a) $\frac{\sqrt{8}}{9} \cdot \frac{GMm}{R}$ (b) $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$
(c) $\frac{1}{3\sqrt{8}} \cdot \frac{GMm}{R^2}$ (d) $\frac{\sqrt{8}}{27} \cdot \frac{GMm}{R^2}$

Ans. (d)

Given, distance between centre of ring and sphere, $d = \sqrt{8}R$

Since, gravitational field at the axis of ring, $E = \frac{Gmd}{(d^2 + R^2)^{3/2}}$

Here, G is the gravitational constant.

$$\Rightarrow E = \frac{GmR\sqrt{8}}{(8R^2 + R^2)^{3/2}} = \frac{GmR\sqrt{8}}{(3R)^3}$$

$$\Rightarrow E = \frac{GmR\sqrt{8}}{27R^3} = \frac{Gm\sqrt{8}}{27R^2}$$

Force between ring and sphere,
 $F = ME$... (i)

Substituting the value of E in Eq. (i), we get

$$F = \frac{\sqrt{8}}{27} \frac{GMm}{R^2}$$

- 22** A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person, if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be N.

[$g = 10 \text{ m/s}^2$] [2021, 26 Feb Shift-I]

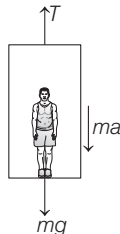
Ans. (492)

Given, mass of man (m) = 60 kg

Downward acceleration of lift, $a = 1.8 \text{ ms}^{-2}$

Let T be the tension in the rope connected with lift, g be the acceleration due to gravity (10 ms^{-2}).

As, lift is moving in downward direction



$$\therefore mg - T = ma$$

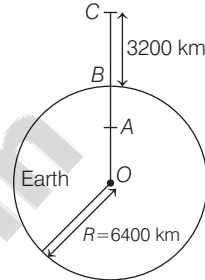
$$\Rightarrow T = mg - ma = m(g - a)$$

$$= 60(10 - 1.8) = 60 \times 8.2 = 492 \text{ N}$$

Hence, the weight of the man during downward acceleration is 492 N.

- 23** In the reported figure of Earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the Earth). The value of $OA : AB$ will be $x : y$. The value of x is

[2021, 26 Feb Shift-II]



Ans. (4)

Given, $g_A = g_C$

Radius of Earth, $R = 6400 \text{ km}$

Height, $h = 3200 \text{ km} = \frac{R}{2}$

$$\text{Since, } g = \frac{GM}{R^2}$$

where, G is gravitational constant.

$$\therefore g_A = \frac{GM(OA)}{R^3} \dots (i)$$

$$\text{and } g_C = \frac{GM}{(OC)^2} = \frac{GM}{R+h} = \frac{GM}{(R + R/2)^2} = \frac{GM}{(3R/2)^2} \dots (ii)$$

According to given information from Eqs. (i) and (ii), we get

$$\frac{GM(OA)}{R^3} = \frac{GM}{9R^2/4}$$

$$\Rightarrow OA = \frac{4}{9}R = \frac{4}{9} \times 6400$$

$$\text{and } AB = OB - OA$$

$$\Rightarrow AB = 6400 - \frac{4}{9} \times 6400$$

$$= 6400(1 - 4/9)$$

$$= 6400 \times 5/9$$

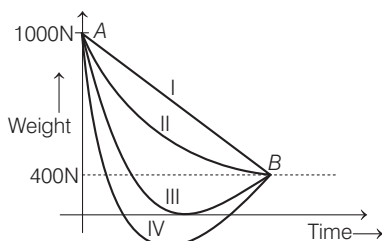
$$\text{Now, } OA : AB = \frac{6400 \times 4/9}{5/9 \times 6400} = \frac{4}{5}$$

Hence, $x = 4$

- 24** A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 , respectively.

Identify from the below figures, the curve that fits best for the

weight of the passenger as a function of time. [2021, 20 July Shift-I]



- (a) III (b) I
(c) IV (d) II

Ans. (a)

Given,

Acceleration due to gravity on Earth,

$$g_E = 10 \text{ m/s}^2$$

and acceleration due to gravity on Mars,

$$g_M = 4 \text{ m/s}^2$$

We know that,

$$g_E \text{ (at height } h) = g_{\text{Earth}} \left(1 - \frac{2h}{R}\right)$$

$\therefore$ Weight at Earth,

$$mg_E = 100 \times 10 = 1000 \text{ N}$$

As the spaceship moves far away from Earth, the value of g_E decreases to zero at a point where $g_E + g_M = 0$ and hence weight will also be zero.

This point is called neutral point and is shown by graph (III) in the given figure.

Then, g_E increases 4 ms^{-2} at Mars surface and weight becomes 400N which is also exhibited by graph (III).

- 25** A body is projected vertically upwards from the surface of Earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is s.

[2021, 22 July Shift-II]

- (a) $\sqrt{\frac{R_e}{2g}} \left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1 \right]$
 (b) $\sqrt{\frac{2R_e}{g}} \left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1 \right]$
 (c) $\frac{1}{3} \sqrt{\frac{R_e}{2g}} \left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1 \right]$
 (d) $\frac{1}{3} \sqrt{\frac{2R_e}{g}} \left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1 \right]$

Ans. (d)

Given, let v be the initial velocity, h be the height achieved in time t

and U , KE and TE be the potential energy kinetic energy and total energy, respectively.

Now, by using law of conservation of energy

$$TE = 0 = -\frac{GMm}{h} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{h}$$

$$\Rightarrow v = \sqrt{\frac{2GM}{h}}$$

$$\text{Since, } v = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \sqrt{\frac{2GM}{h}}$$

$$\Rightarrow \int_{R_e}^{R_e+h} h^{1/2} dh = \int_0^t \sqrt{2GM} dt$$

$$\Rightarrow \left[\frac{h^{3/2+1}}{3/2+1} \right]_{R_e}^{R_e+h} = \sqrt{2GM} \cdot t$$

$$\Rightarrow \frac{2}{3} [h^{3/2}]_{R_e}^{R_e+h} = \sqrt{2GM} t$$

$$\Rightarrow \frac{2}{3} [(R_e+h)^{3/2} - R_e^{3/2}] = \sqrt{2GM} \cdot t$$

$$\Rightarrow t = \frac{2}{3} \frac{[(R_e+h)^{3/2} - R_e^{3/2}]}{\sqrt{2GM}} \quad \dots(i)$$

$$\therefore \text{Acceleration due to gravity (g)} = \frac{GM}{R_e^2}$$

$$\therefore 2GM = 2gR_e^2$$

Substituting in Eq. (i), we get

$$t = \frac{2}{3} \left[\frac{(R_e+h)^{3/2} - R_e^{3/2}}{(2gR_e^2)^{1/2}} \right]$$

$$= \frac{1}{3} \left[\sqrt{\frac{4}{2gR_e^2}} \{ (R_e+h)^{3/2} - R_e^{3/2} \} \right]$$

$$= \frac{1}{3} \sqrt{\frac{R_e}{g}} \left[\left(1 + \frac{h}{R_e}\right)^{3/2} - 1 \right]$$

- 26** Consider a planet in some solar system which has a mass double the mass of Earth and density equal to the average density of Earth. If the weight of an object on Earth is w , the weight of the same object on that planet will be

[2021, 25 July Shift-II]

- (a) $2w$ (b) w (c) $2^{1/3}w$ (d) $\sqrt{2}w$

Ans. (c)

Let the mass of planet be m_p and that of earth be m .

$$\text{Given, } m_p = 2m$$

Density of planet (ρ_p) = Density of earth (ρ)

As we know that,

Weight of object on earth is w

$$w = mg \quad \dots(i)$$

where, g is acceleration due to gravity.

$$\text{and } g = \frac{GM}{R^2}$$

where, G = universal gravitational constant,

R = radius of earth

and m_e = mass of earth.

$$\Rightarrow g = \frac{G \rho \frac{4}{3} \pi R^3}{R^2}$$

$$\Rightarrow g = G \rho \frac{4}{3} \pi R$$

Putting this value in Eq. (i), we get

$$w = m G \rho \frac{4}{3} \pi R \quad \dots(ii)$$

As densities of planets earth is same.

i.e., $\rho_p = \rho$

$$\Rightarrow \frac{m_p}{\frac{4}{3} \pi R_p^3} = \frac{m}{\frac{4}{3} \pi R^3}$$

$$\Rightarrow R_p = \left(\frac{m_p R^3}{m} \right)^{1/3}$$

$$= \left(\frac{2m R^3}{m} \right)^{1/3}$$

$$\Rightarrow R_p = (2)^{1/3} R$$

Weight on planet,

$$w_p = \frac{4}{3} (2)^{1/3} \pi m G \rho R \quad \dots(iii)$$

Dividing Eq. (iii) by Eq. (ii), we get

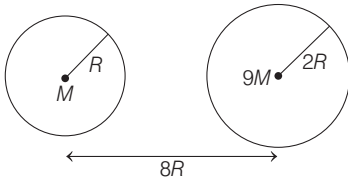
$$\frac{w_p}{w} = \frac{\frac{4}{3} (2)^{1/3} \pi m G \rho R}{m G \frac{4}{3} \pi R}$$

$$\Rightarrow w_p = 2^{1/3} w$$

- 27** Suppose two planets (spherical in shape) of radii R and $2R$, but mass M and $9M$ respectively have a centre to centre separation $8R$ as shown in the figure. A satellite of mass m is projected from the surface of the planet of mass M directly towards the centre of the second planet. The minimum speed v required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7} \frac{GM}{R}}$, then

the value of a is

[Take, the two planets are fixed in their position] [2021, 27 July Shift-I]

**Ans. (4)**

Assume that at a distance x from the planet of mass M , the net gravitational field becomes zero.

$$\begin{aligned} \therefore \frac{GM}{x^2} &= \frac{G \times 9M}{(8R-x)^2} \\ \Rightarrow \frac{1}{x^2} &= \frac{9}{(8R-x)^2} \\ \Rightarrow \left(\frac{1}{x}\right) &= \left(\frac{3}{8R-x}\right)^2 \Rightarrow \frac{1}{x} = \frac{3}{8R-x} \\ \Rightarrow 3x &= 8R - x \\ \Rightarrow 4x &= 8R \\ \Rightarrow x &= 2R \end{aligned} \quad \dots(i)$$

Now, a satellite should be projected in such a way that it covers a minimum distance of $2R$.

$$\begin{aligned} \therefore \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G(9M)m}{7R} \\ = -\frac{GMm}{2R} - \frac{G(9M)m}{6R} \end{aligned}$$

where, m is the mass of satellite.

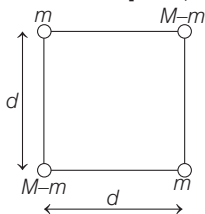
$$\Rightarrow \frac{1}{2}v^2 = \frac{2GM}{7R} \Rightarrow v = \sqrt{\frac{4}{7} \frac{GM}{R}} \quad \dots(ii)$$

According to question, the minimum speed v required for the satellite to reach the surface of the second planet is $\sqrt{\frac{aGM}{7R}}$. So, on comparing it with Eq. (ii), we can write

$$a = 4$$

- 28** A body of mass $2M$ splits into four masses $\{m, M-m, m, M-m\}$, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$

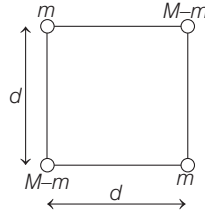
for which, the gravitational potential energy of the system becomes maximum is $x:1$. The value of x is.....

[2021, 27 Aug Shift-I]**Ans. (2)**

Given, total mass of body is $2M$.

Potential energy is maximum at $M/m = x/1$

The arrangement of masses to form a square is shown in diagram.



The gravitational potential energy of a body is given

$$U = -\frac{GMm}{r}$$

For the given system, the potential energy will be

$$\begin{aligned} U_T &= -\frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} \\ &\quad - \frac{Gm^2}{(\sqrt{2}d)^2} - \frac{G(M-m)^2}{(\sqrt{2}d)^2} \\ U_T &= -\frac{4Gm(M-m)}{d} - \frac{Gm^2}{\sqrt{2}d} - \frac{G(M-m)^2}{\sqrt{2}d} \end{aligned}$$

For maximum potential energy,

$$\begin{aligned} \frac{dU_T}{dm} &= 0 \\ -\frac{4G}{d} [M-2m] - \frac{G}{\sqrt{2}d} [2m] \\ -\frac{G}{\sqrt{2}d} [2(M-m)x-1] &= 0 \\ \Rightarrow 4M-8m+\sqrt{2}m &= \sqrt{2}(M-m) \\ (4-\sqrt{2})M &= (8-2\sqrt{2})m \\ \frac{M}{m} &= \frac{2(4-\sqrt{2})}{4-\sqrt{2}} = 2 \end{aligned}$$

Comparing it with the given value, we get $x=2$

Thus, potential energy will be maximum when $x=2$.

- 29** A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is

[2021, 27 Aug Shift-II]

- (a) $-60G$ (b) $+2G$
(c) $-20G$ (d) $-4G$

Ans. (d)

Given, mass of uniform spherical shell, $m_s = 100$ kg

Radius of sphere, $R = 50$ m

Mass of inner body, $m = 50$ kg

Distance from inner mass, $r = 25$ m

Since, gravitational potential, $V = -\frac{Gm}{r}$

where, G is gravitational constant.

$$\begin{aligned} \therefore \text{Net potential, } V_{\text{net}} &= -\frac{Gm}{r} - \frac{Gm_s}{R} \\ &= -G \left(\frac{50}{25} + \frac{100}{50} \right) = -4G \end{aligned}$$

- 30** If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth r below and a height r above the Earth surface is (Given, $r < R_E$)

[2021, 31 Aug Shift-II]

- (a) $1 - \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$
(b) $1 + \frac{r}{R_E} + \frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}$
(c) $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} + \frac{r^3}{R_E^3}$
(d) $1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$

Ans. (d)

Given that, radius of Earth $= R_E$

Height above Earth's surface, $h = r$

Depth below Earth surface, $d = r$

We know that, acceleration due to gravity at height (r) and depth (r) is (for $r < R_E$)

$$\begin{aligned} g_h &= g \left(\frac{R_E}{R_E + h} \right)^2 \\ &= g \left(\frac{R_E}{R_E + r} \right)^2 \end{aligned} \quad \dots(i)$$

$$\begin{aligned} \text{and } g_d &= g \left(\frac{R_E - d}{R_E} \right)^2 \\ &= g \left(\frac{R_E - r}{R_E} \right)^2 \end{aligned} \quad \dots(ii)$$

$$\therefore \frac{g_d}{g_h} = \frac{g \left(\frac{R_E - r}{R_E} \right)^2}{g \left(\frac{R_E}{R_E + r} \right)^2} \quad [\text{from Eqs. (i) and (ii)}]$$

$$\begin{aligned} \Rightarrow \frac{g_d}{g_h} &= \frac{(R_E - r)(R_E + r)^2}{R_E^3} \\ &= \frac{R_E^3 + R_E^2 r - R_E r^2 - r^3}{R_E^3} \\ &= 1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3} \end{aligned}$$

31 Inside a uniform spherical shell
[2021, 26 Aug Shift I]

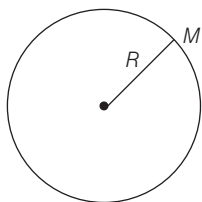
- the gravitational field is zero.
- the gravitational potential is zero.
- the gravitational field is same everywhere.
- the gravitation potential is same everywhere.
- All of the above

Choose the most appropriate answer from the options given below .

- (a) I, III and IV (b) Only V
(c) I, II and III (d) II, III and IV

Ans. (a)

The uniform spherical shell is shown in the figure below



Inside the shell, there is no mass, so by applying Gauss's law for gravitation

$$\oint \mathbf{g} \cdot d\mathbf{a} = -4\pi G M_{\text{enclosed}}$$

where, g = gravitational field intensity.

da = area enclosed

G = gravitational constant

and M_{enclosed} = mass enclosed.

As, $M_{\text{enclosed}} = 0$

So, gravitational field intensity is zero inside the shell, i.e. $g = 0$... (i)

We also know that,

$$g = -\frac{dV}{dr}$$

where, dV is change in potential due to gravity.

$\Rightarrow V = \text{constant}$ [using Eq. (i)]

So, gravitational field is zero everywhere inside the shell and gravitational potential (V) is constant everywhere.

32 The height h at which the weight of a body will be the same as that at the same depth h from the surface of the earth is
(Radius of the earth is R and effect of the rotation of the earth is neglected) [2020, 2 Sep Shift-II]

- (a) $\frac{\sqrt{5}}{2}(R-R)$ (b) $\frac{R}{2}$
(c) $\frac{\sqrt{5}R-R}{2}$ (d) $\frac{\sqrt{3}R-R}{2}$

Ans. (c)

Acceleration due to gravity at height h above earth's surface is

$$g_h = g \left(\frac{R^2}{(R+h)^2} \right)$$

Also, acceleration at depth $d = h$ below surface is

$$g_d = g \left(1 - \frac{d}{R} \right) = g \left(1 - \frac{h}{R} \right)$$

Given, weight of body at height h = weight of body at depth h ,

$$\text{So, } mg_h = mg_d \Rightarrow g_h = g_d$$

$$\Rightarrow g \left(\frac{R^2}{(R+h)^2} \right) = g \left(1 - \frac{h}{R} \right)$$

$$\Rightarrow \frac{R^2}{(R+h)^2} = 1 - \frac{h}{R}$$

$$\Rightarrow R^3 = (R-h)(R+h)^2$$

$$\Rightarrow R^3 = (R^3 - hR^2 + h^2R - h^3) + 2R^2h - 2Rh^2$$

$$\Rightarrow h^2 + Rh - R^2 = 0$$

$$\Rightarrow h = \frac{-R \pm \sqrt{R^2 - 4(1)(-R^2)}}{2(1)}$$

$$\Rightarrow h = \frac{-R \pm \sqrt{5}R}{2}$$

As h is a positive number,

$$h = \frac{-R + \sqrt{5}R}{2} \text{ or } \frac{\sqrt{5}R - R}{2}$$

Hence, correct option is (c).

33 The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2} \right)$.

Then, the gravitational field is maximum at [2020, 3 Sep Shift-II]

- (a) $r = \frac{\sqrt{3}}{4}R$ (b) $r = R$
(c) $r = \frac{\sqrt{5}}{9}R$ (d) $r = \frac{1}{\sqrt{3}}R$

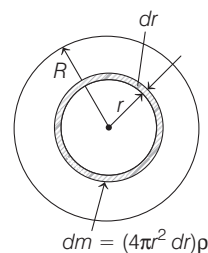
Ans. (c)

Gravitational field intensity at a distance x from centre of planet is $E = \frac{GM}{x^2}$,

where M = mass enclosed with in a spherical volume of radius x .

$$\text{So, } E = \frac{G}{x^2} \int_0^x (4\pi r^2 dr) [\rho(r)] \quad \dots (i)$$

$$\text{Here, } M = \int_0^x dm = \int_0^x 4\pi r^2 dr \cdot \rho(r)$$



So, from Eq. (i), we get

$$\begin{aligned} E &= \frac{G}{x^2} \int_0^x 4\pi r^2 \cdot \rho_0 \left(1 - \frac{r^2}{R^2} \right) dr \\ &= \frac{G}{x^2} \int_0^x 4\pi \rho_0 \left(r^2 - \frac{r^4}{R^2} \right) dr \\ &= \frac{4\pi G \rho_0}{x^2} \left[\frac{r^3}{3} - \frac{r^5}{5R^2} \right]_0^x \\ &= \frac{4\pi G \rho_0}{x^2} \left(\frac{x^3}{3} - \frac{x^5}{5R^2} \right) \\ &= 4\pi G \rho_0 \left(\frac{x}{3} - \frac{x^3}{5R^2} \right) \end{aligned}$$

For gravitational field to be maximum, $\frac{dE}{dx} = 0$

$$\Rightarrow \frac{d}{dx} \left[4\pi G \rho_0 \left(\frac{x}{3} - \frac{x^3}{5R^2} \right) \right] = 0$$

$$\Rightarrow 4\pi G \rho_0 \left(\frac{1}{3} - \frac{3x^2}{5R^2} \right) = 0$$

$$\Rightarrow x = \frac{\sqrt{5}}{9} R$$

So, E is maximum at a distance of $\frac{\sqrt{5}}{9} R$

from centre of planet. Hence, correct option is (c).

34 On the X-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2 + a^2)^{3/2}}$ in the

x -direction. The magnitude of gravitational potential on the X-axis at a distance x , taking its value to be zero at infinity, is [2020, 4 Sep Shift-I]

- (a) $\frac{A}{(x^2 + a^2)^{1/2}}$ (b) $A(x^2 + a^2)^{3/2}$
(c) $\frac{A}{(x^2 + a^2)^{3/2}}$ (d) $A(x^2 + a^2)^{1/2}$

Ans. (a)

Here, gravitational field,

$$E_g = \frac{Ax}{(x^2 + a^2)^{3/2}}$$

and also gravitational potential at infinity, $V_\infty = 0$

Now, $dV = -\mathbf{E}_G \cdot d\mathbf{x}$

$$\int_{V_\infty}^{V_x} dV = -\int_{\infty}^x \mathbf{E}_G \cdot d\mathbf{x}$$

$$[V]_{V_\infty}^{V_x} = -\int_{\infty}^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx \cos 0^\circ$$

$$V_x - V_\infty = -\int_{\infty}^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx \times 1$$

$$V_x - 0 = -\int_{\infty}^x \frac{Ax}{(x^2 + a^2)^{3/2}} dx$$

$$V_x = -A \int_{\infty}^x \frac{x}{(x^2 + a^2)^{3/2}} dx \quad \dots(i)$$

Putting $x^2 + a^2 = t$ then, $2x dx + 0 = dt$

$$x dx = \frac{dt}{2}$$

At $x = \infty, (\infty)^2 + a^2 = t$

$$t = \infty \quad (\text{lower limit})$$

Again, at $x = x$

$$x^2 + a^2 = t \Rightarrow t = x^2 + a^2 \quad (\text{upper limit})$$

From eq (i), we get

$$V_x = -A \int_{\infty}^{x^2 + a^2} \frac{dt/2}{(t)^{3/2}}$$

$$= -\frac{A}{2} \int_{\infty}^{x^2 + a^2} (t)^{-3/2} dt$$

$$= -\frac{A}{2} \left[\frac{(t)^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \right]_{\infty}^{x^2 + a^2}$$

$$= -\frac{A}{2} \left[\frac{(t)^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_{\infty}^{x^2 + a^2} = A \left[\frac{1}{(t)^{\frac{1}{2}}} \right]_{\infty}^{x^2 + a^2}$$

$$= A \left[\frac{1}{(x^2 + a^2)^{1/2}} - \frac{1}{(\infty)^{1/2}} \right]$$

$$= A \left[\frac{1}{(x^2 + a^2)^{1/2}} - 0 \right]$$

$$= \frac{A}{(x^2 + a^2)^{1/2}}$$

Hence, correct option is (a).

- 35** The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$

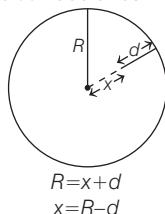
(where, R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals

[2020, 5 Sep Shift-I]

- (a) $\frac{7}{9}$ (b) $\frac{1}{3}$
(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Ans. (d)

Given that, acceleration due to gravity at height h from the surface of earth = acceleration due to gravity at depth d below the surface of earth = g_1



$$\Rightarrow g_1 = \frac{GM}{(R+h)^2} = \frac{GM}{R^3} (R-d)$$

$$\Rightarrow \frac{GM}{\left(R + \frac{R}{2}\right)^2} = \frac{GM}{R^3} (R-d) \quad \left[\because h = \frac{R}{2}\right]$$

$$\Rightarrow \frac{4GM}{9R^2} = \frac{GM}{R^2} \left(1 - \frac{d}{R}\right)$$

$$\Rightarrow 1 - \frac{d}{R} = \frac{4}{9} \Rightarrow \frac{d}{R} = 1 - \frac{4}{9} = \frac{5}{9}$$

Hence, correct option is (d).

- 36** The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω . An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is ($h \ll R$, where R is the radius of the earth)

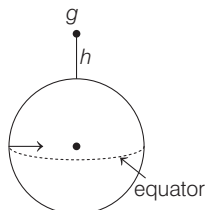
[2020, 5 Sep Shift-II]

- (a) $\frac{R^2 \omega^2}{2g}$ (b) $\frac{R^2 \omega^2}{g}$
(c) $\frac{R^2 \omega^2}{4g}$ (d) $\frac{R^2 \omega^2}{8g}$

Ans. (a)

Weight of the object near the equator = $m(g - R\omega^2)$ and at height h near the poles = $mg \frac{R^2}{(R+h)^2}$

$$mg \frac{R^2}{(R+h)^2}$$



According to question, both the weights are same.

$$\Rightarrow m(g - R\omega^2) = \frac{mgR^2}{(R+h)^2}$$

$$(g - R\omega^2) = gR^2 \left[\frac{1}{R^2 + h^2 + 2Rh} \right]$$

$$= \frac{gR^2}{R^2} \left[\frac{1}{1 + \frac{h^2}{R^2} + \frac{2Rh}{R^2}} \right] = g \left[\frac{1}{1 + \frac{2h}{R}} \right] \quad [\because h \ll R]$$

$$= g \left[1 - \frac{2h}{R} \right]$$

$$\Rightarrow R\omega^2 = \frac{2gh}{R}$$

$$\Rightarrow h = \frac{R^2 \omega^2}{2g}$$

Hence, correct option is (a).

- 37** Two planets have masses M and $16M$ and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is

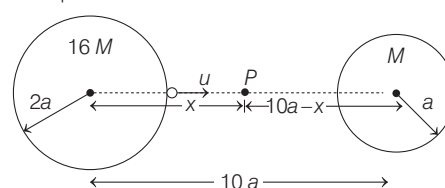
[2020, 6 Sep Shift-II]

- (a) $2\sqrt{\frac{GM}{a}}$ (b) $4\sqrt{\frac{GM}{a}}$
(c) $\sqrt{\frac{GM^2}{ma}}$ (d) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Ans. (d)

For the body to be able to reach at the surface of smaller planet, it just has to overcome the gravitational field of bigger planet and at that point (let say P in given figure) net force will be zero. After crossing this P point, body will move towards smaller planet itself and will reach with minimum speed (d)

Let the distance of point P from bigger planet be x .



At point P , $F_{\text{net}} = 0$

$$\Rightarrow F_{16M} = F_M$$

$$\Rightarrow \frac{G(16M)m}{x^2} = \frac{G(M)m}{(10a - x)^2}$$

$$\Rightarrow x = 8a \quad \dots(i)$$

Since, there is no net external force on system, so we can apply conservation of mechanical energy from the surface of bigger planet to point P.

$$\text{i.e., } K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mu^2 + \left(-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right) = 0 + \left(-\frac{G(16M)m}{8a} - \frac{GMm}{2a}\right)$$

$$\frac{1}{2}mu^2 - \frac{65GMm}{8a} = -\frac{5GMm}{2a}$$

$$\frac{1}{2}mu^2 = \frac{45GMm}{8a}$$

$$\Rightarrow u = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

Hence, correct option is (d).

- 38** A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance, if it is shifted to the equator is close to (Take, $g = 10 \text{ ms}^{-2}$ at the north pole and the radius of the earth $= 6400 \text{ km}$) [2020, 7 Jan Shift-II]

- (a) 195.66 N (b) 195.32 N
(c) 194.66 N (d) 194.32 N

Ans. (b)

Acceleration due to gravity at poles,

$$g_p = 10 \text{ ms}^{-2}$$

Weight of box at poles, $w_p = 196 \text{ N}$

So, mass of box, $m = \frac{w_p}{g_p} = 19.6 \text{ kg}$

Now, due to rotation of earth acceleration due to gravity at equator,

$$g_e = g_p - R\omega^2 \quad \dots(i)$$

Here $g_p = 10 \text{ ms}^{-2}$, $R = 6400 \times 10^3 \text{ m}$,

$$\omega = \frac{2\pi}{T} \text{ and } T = 24 \times 3600 \text{ s}$$

Now, substituting the given values in Eq. (i), we get

$$g_e = g_p - 0.034 = 10 - 0.034 = 9.966 \text{ ms}^{-2}$$

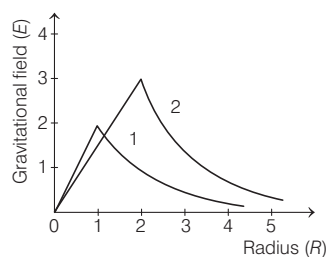
So, weight of box at equator,

$$w_e = g_e \times m = 9.966 \times 19.6 = 195.32 \text{ N}$$

- 39** Consider two solid spheres of radii $R_1 = 1 \text{ m}$, $R_2 = 2 \text{ m}$ and masses M_1 and M_2 , respectively. The gravitational field due to sphere 1 and 2 are shown. The value of $\frac{M_1}{M_2}$

is

[2020, 8 Jan Shift-I]



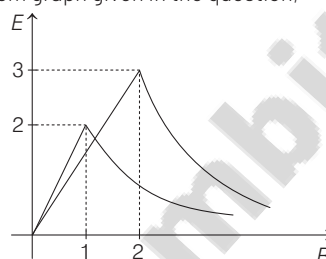
- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

Ans. (c)

Gravitational field of a solid sphere is maximum at its surface ($r = R$) and its value at surface,

$$E = \frac{GM}{R^2}$$

From graph given in the question,



We can observe that

$$E_1 = \frac{GM_1}{(1)^2} = 2 \text{ and } E_2 = \frac{GM_2}{(2)^2} = 3$$

$$\text{or } GM_1 = 2 \quad \dots(i)$$

$$\text{and } GM_2 = 12 \quad \dots(ii)$$

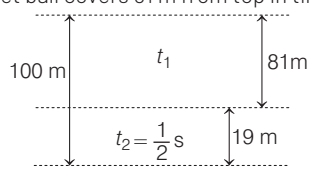
On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M_1}{M_2} = \frac{2}{12} = \frac{1}{6}$$

- 40** A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2} \text{ s}$ before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is [2020, 8 Jan Shift-II]

Ans. (8)

Let ball covers 81 m from top in time t ,



$$\text{Then by } s = ut + \frac{1}{2}at^2 \Rightarrow 81 = \frac{1}{2}at_1^2 \quad \dots(i)$$

As ball covers 100 m in time $t_1 + \frac{1}{2}$ seconds,

$$\text{we have } 100 = \frac{1}{2}a\left(t_1 + \frac{1}{2}\right)^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$t_1 = \frac{9\sqrt{2}}{\sqrt{a}} \text{ and } t_1 + \frac{1}{2} = \frac{10\sqrt{2}}{\sqrt{a}}$$

$$\text{So, } \frac{9\sqrt{2}}{\sqrt{a}} + \frac{1}{2} = \frac{10\sqrt{2}}{\sqrt{a}} \Rightarrow \frac{1}{2} = \frac{\sqrt{2}}{\sqrt{a}}$$

$$\Rightarrow a = 8 \text{ ms}^{-2}$$

- 41** A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and $2M$. The gravitational field at distance $3a$ from the centre will be

[2019, 9 April Shift-I]

- (a) $\frac{GM}{9a^2}$ (b) $\frac{2GM}{9a^2}$ (c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

Ans. (c)

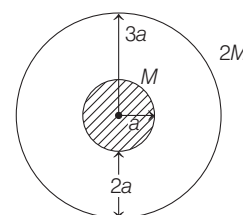
According to question, diagram will be as follows

Gravitational field due to solid sphere of radius a at a distance, $r = 3a$, i.e. ($r > R$) is

$$E_1 = \frac{GM}{r^2} = \frac{GM}{(3a)^2} = \frac{GM}{9a^2}$$

Similarly, gravitational field due to spherical shell at a distance, $r = 3a$,

$$\text{i.e. } E_2 = \frac{GM}{R^2} = \frac{G(2M)}{(3a)^2} = \frac{2GM}{9a^2}$$



Both fields are attractive in nature, so direction will be same.

$$\text{Net field, } E_{\text{net}} = \frac{GM}{9a^2} + \frac{2GM}{9a^2}$$

$$E_{\text{net}} = \frac{GM}{3a^2}$$

- 42** The value of acceleration due to gravity at earth's surface is 9.8 ms^{-2} . The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2} , is close to (Take, radius of earth $= 6.4 \times 10^6 \text{ m}$)

[2019, 10 April Shift-I]

- (a) $9.0 \times 10^6 \text{ m}$ (b) $2.6 \times 10^6 \text{ m}$
(c) $6.4 \times 10^6 \text{ m}$ (d) $1.6 \times 10^6 \text{ m}$

Ans. (b)

Key Idea The relation of gravitational acceleration at an altitude h above the earth's surface is given as

$$g_h = g \left(1 + \frac{h}{R_e} \right)^{-2}$$

where, g is the acceleration due to gravity at earth's surface and R_e is the radius of the earth.

Given that at some height h , acceleration due to gravity,

$$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2} \quad \dots (i)$$

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e} \right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \quad [\text{From Eq. (i)}]$$

$$\therefore \frac{h}{R_e} = \sqrt{2} - 1 \text{ or}$$

$$h = 0.414 \times R_e$$

$$h = 0.414 \times 6400 \text{ km}$$

$$(\because \text{given, radius of earth, } R_e = 6400 \text{ km} = 6.4 \times 10^6 \text{ m})$$

$$\text{or } h = 2649.6 \text{ km} = 2.6 \times 10^6 \text{ m}$$

Thus, at $2.6 \times 10^6 \text{ m}$ above the earth's surface, acceleration due to gravity decreases to 4.9 m/s^2 .

- 43** The ratio of the weights of a body on the earth's surface, so that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$ th of that of

the earth. If R is the radius of the earth, what is the radius of the planet? (Take, the planets to have the same mass density)

[2019, 12 April Shift-II]

- (a) $\frac{R}{3}$ (b) $\frac{R}{4}$
(c) $\frac{R}{9}$ (d) $\frac{R}{2}$

Ans. (d)

Let mass of given body is m . Then, it's weight on earth's surface = mg_e

where, g_e = acceleration due to gravity on earth's surface and weight on the surface of planet = mg_p

g_p = acceleration due to gravity on planet's surface.

$$\text{Given, } \frac{mg_e}{mg_p} = \frac{9}{4} \Rightarrow \frac{g_e}{g_p} = \frac{9}{4}$$

$$\text{But } g = \frac{GM}{R^2}, \text{ so we have } \frac{\left(\frac{GM}{R^2}\right)}{\left(\frac{GM_p}{R_p^2}\right)} = \frac{9}{4}$$

where, M = mass of earth, R = radius of earth,

$$M_p = \text{mass of planet} = \frac{M}{9} \quad (\text{given})$$

and R_p = radius of planet.

$$\Rightarrow \frac{M}{M_p} \cdot \frac{R_p^2}{R^2} = \frac{9}{4}$$

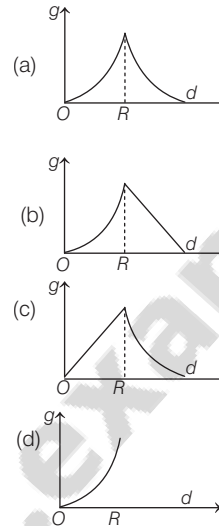
$$\Rightarrow 9 \cdot \left(\frac{R_p}{R}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \frac{R_p}{R} = \frac{1}{2}$$

$$\Rightarrow R_p = \frac{R}{2}$$

- 44** The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)

[JEE Main 2017]

**Ans. (d)**

Inside the earth's surface, $g = \frac{GM}{R^3} r$

i.e. $g \propto r$

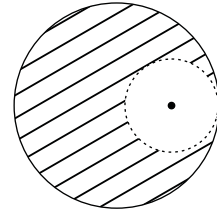
Out the earth's surface, $g = \frac{Gm}{r^2}$

i.e. $g \propto \frac{1}{r^2}$

So, till earth's surface g increases linearly with distance r , shown only in graph (c).

- 45** From a solid sphere of mass M and radius R , a spherical portion of radius $(R/2)$ is removed as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)

[JEE Main 2015]



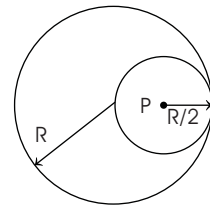
- (a) $-\frac{GM}{2R}$ (b) $-\frac{GM}{R}$
(c) $-\frac{2GM}{3R}$ (d) $-\frac{2GM}{R}$

Ans. (b)

Key Idea Consider cavity as negative mass and apply superposition of gravitational potential.

Consider the cavity formed in a solid sphere as shown in figure.

$$V(\infty) = 0$$



According to the question, we can write potential at an internal point P due to complete solid sphere,

$$\begin{aligned} V_s &= -\frac{GM}{2R^3} \left[3R^2 - \left(\frac{R}{2}\right)^2 \right] \\ &= -\frac{GM}{2R^3} \left[3R^2 - \frac{R^2}{4} \right] \\ &= -\frac{GM}{2R^3} \left[\frac{11R^2}{4} \right] = -\frac{11GM}{8R} \end{aligned}$$

Mass of removed part

$$= \frac{M}{(4/3) \times \pi R^3} \times \frac{4}{3} \pi \left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Potential at point P due to removed part

$$V_c = \frac{-3}{2} \times \frac{GM/8}{R/2} = -\frac{3GM}{8R}$$

Thus, potential due to remaining part at point P ,

$$\begin{aligned} V_P &= V_s - V_c = -\frac{11GM}{8R} - \left(-\frac{3GM}{8R}\right) \\ &= \frac{(-11+3)GM}{8R} = -\frac{GM}{R} \end{aligned}$$

- 46 Statement I** For a mass M kept at the centre of a cube of side a , the flux of gravitational field passing through its sides is $4\pi GM$.

Statement II If the direction of a field due to a point source is radial and its dependence on the

distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

[AIEEE 2008]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
 (b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
 (c) Statement I is true, Statement II is false
 (d) Statement I is false, Statement II is true

Ans. (a)

Correct option is (a), you can make an analogy with Gauss's law in electrostatics.

- 47** If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio electronic charge on the moon to electronic charge on the earth be

[AIEEE 2007]

- (a) 1 (b) zero (c) $\frac{g_E}{g_M}$ (d) $\frac{g_M}{g_E}$

Ans. (a)

According to Millikan's oil drop experiment, electronic charge is given by

$$q = \frac{6\pi\eta r(v_1 + v_2)}{E}$$

which is independent of g .

So, $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$

- 48** Average density of the earth

[AIEEE 2005]

- (a) does not depend on g
 (b) is a complex function of g
 (c) is directly proportional to g
 (d) is inversely proportional to g

Ans. (c)

Mass of earth = Volume $\times$ Density

$$\text{As } g = \frac{GM}{R^2}; M = \left(\frac{4}{3}\pi R^3\right)\rho$$

$$\therefore g = \frac{4G}{3} \frac{\pi R^3}{R^2} \rho$$

$$\text{or } g = \left(\frac{4G\pi R}{3}\right)\rho$$

[where ρ = average density]

$$\Rightarrow g \propto \rho \text{ or } \rho \propto g$$

- 49** The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct?

[AIEEE 2005]

- (a) $d = \frac{h}{2}$ (b) $d = \frac{3h}{2}$
 (c) $d = 2h$ (d) $d = h$

Ans. (c)

Acceleration due to gravity at height h .

$$g_h = g \left(1 - \frac{2h}{R}\right) \quad [\text{if } h < R] \dots (i)$$

Acceleration due to gravity at depth (d)

$$g_d = g \left(1 - \frac{d}{R}\right) \dots (ii)$$

As per statement of the problem,

$$g_h = g_d$$

$$\text{i.e., } g \left(1 - \frac{2h}{R}\right) = g \left(1 - \frac{d}{R}\right)$$

$$\Rightarrow 2h = d$$

- 50** A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere, (you may take $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$)

[AIEEE 2005]

- (a) $13.34 \times 10^{-10} \text{ J}$
 (b) $3.33 \times 10^{-10} \text{ J}$
 (c) $6.67 \times 10^{-9} \text{ J}$
 (d) $6.67 \times 10^{-10} \text{ J}$

Ans. (d)

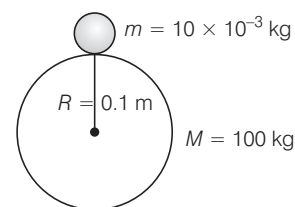
The initial potential energy of the particle = Work done.

$$U_i = -\frac{GMm}{r}$$

$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$

$$U_i = -\frac{6.67 \times 10^{-11}}{0.1}$$

$$= -6.67 \times 10^{-10} \text{ J}$$



We know that, work done = Difference in potential energy

$$\therefore W = \Delta U = U_f - U_i$$

$$\Rightarrow W = -U_i \quad [\because U_f = 0]$$

$$= 6.67 \times 10^{-10} \text{ J}$$

- 51** If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is

[AIEEE 2004]

- (a) $2mgR$ (b) $\frac{1}{2}mgR$
 (c) $\frac{1}{4}mgR$ (d) mgR

Ans. (b)

Gravitational potential energy of body on the earth's surface is

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{(R+h)}$$

$$= -\frac{GM_e m}{2R} \quad [\text{as } h = R]$$

where, M_e = mass of earth, m = mass of body

and R = radius of earth.

$\therefore$ Gain in potential energy

$$= U_h - U = -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$

$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$

$$= \frac{GM_e m}{2R} = \frac{gR^2 m}{2R} \quad \left[\text{as } g = \frac{GM_e}{R^2}\right]$$

$$= \frac{1}{2}mgR$$

- 52** Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

[AIEEE 2002]

- (a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$
 (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

Ans. (d)

Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

where, M = mass of earth,
 m = mass of the body
 and R = radius of earth.

At $r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $r = 3R$, $E_2 = -\frac{GMm}{(3R)}$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GMm}{R} \left[\frac{1}{2} - \frac{1}{3} \right] = \frac{GMm}{6R}$$

TOPIC 3

Escape Speed and Motion of Satellites

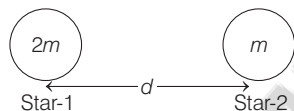
- 53** Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is

[2021, 24 Feb Shift-I]

- (a) $\frac{1}{2\pi} \sqrt{\frac{3Gm}{d^3}}$ (b) $2\pi \sqrt{\frac{d^3}{3Gm}}$
 (c) $2\pi \sqrt{\frac{3Gm}{d^3}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{d^3}{3Gm}}$

Ans. (b)

The given situation is shown below



The gravitational force between these two stars provide the required centripetal force for rotation in a circle about their common centre.

Assuming $2m$ at origin, the centre of mass of the system lies at

$$x = \frac{2m \times 0 + m \times d}{2m + m} = \frac{d}{3}$$

Hence, $F_g = F_c$

where, F_g is gravitational force between them

and F_c is centripetal force.

$$\Rightarrow \frac{Gm_1m_2}{r^2} = 2m\omega^2 x$$

$$\Rightarrow \frac{G(2m)(m)}{d^2} = 2m\omega^2 \times \frac{d}{3} \Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}} \quad \dots(i)$$

We know that, $\omega = \frac{2\pi}{T}$

$$\therefore T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}} = 2\pi \sqrt{\frac{d^3}{3Gm}} \quad [\text{using Eq. (i)}]$$

- 54** Consider two satellites S_1 and S_2 with periods of revolution 1 h and 8 h respectively, revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is

[2021, 24 Feb Shift-I]

- (a) 8 : 1 (b) 1 : 8 (c) 2 : 1 (d) 1 : 4

Ans. (a)

Given, period of revolution of first satellite,

$$T_1 = 1 \text{ h}$$

Period of revolution of second satellite,

$$T_2 = 8 \text{ h}$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{8}$$

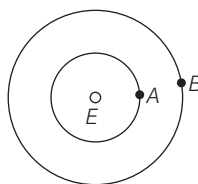
We know that, $\omega = \frac{2\pi}{T} \Rightarrow \omega \propto \frac{1}{T}$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$$

or $\omega_1 : \omega_2 = 8 : 1$

- 55** Two satellites A and B of masses 200 kg and 400 kg are revolving around the Earth at height of 600 km and 1600 km, respectively. If T_A and T_B are the time periods of A and B respectively, then the value of $T_B - T_A$ is

[2021, 25 Feb Shift-I]



(Given, radius of Earth = 6400 km, mass of Earth = 6×10^{24} kg)

- (a) 1.33×10^3 s (b) 3.33×10^2 s
 (c) 4.24×10^3 s (d) 4.24×10^2 s

Ans. (a)

Given,

$M_A = 200$ kg, $M_B = 400$ kg, $H_A = 600$ km,
 $H_B = 1600$ km

and

$$R_A = R_E + H_A = 6400 + 600 = 7000 \text{ km}$$

$$R_B = R_E + H_B = 6400 + 1600 = 8000 \text{ km}$$

Let $T_A, T_B, \omega_A, \omega_B, R_A$ and R_B be the time period, angular frequencies and radii of satellites A and B, respectively.

Force on satellite A,

$$F_A = m_A \omega_A^2 R_A = \frac{GMm_A}{R_A^2}$$

$$\Rightarrow \omega_A^2 = \frac{GM}{R_A^3}$$

$$\text{but, } \omega_A = \frac{2\pi}{T_A}$$

$$\therefore \left(\frac{2\pi}{T_A} \right)^2 = \frac{GM}{R_A^3} \Rightarrow T_A = \sqrt{\frac{4\pi^2 R_A^3}{GM}}$$

$$\text{Similarly, } T_B = \sqrt{\frac{4\pi^2 R_B^3}{GM}}$$

$$\therefore T_B - T_A = \sqrt{\frac{4\pi^2}{GM}} (\sqrt{R_B^3} - \sqrt{R_A^3})$$

$$= \frac{2\pi}{\sqrt{GM}}$$

$$[\sqrt{(8 \times 10^3)^3} - \sqrt{(7 \times 10^3)^3}]$$

$$= \frac{2\pi \times 10^9}{\sqrt{GM}} (8\sqrt{8} - 7\sqrt{7})$$

$$= \frac{2\pi \times 10^9}{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}} (4.107)$$

$$= \frac{2\pi}{\sqrt{4 \times 10^{14}}} \times 10^9 (4.107)$$

$$= \pi \times 10^2 \times 4.107 = 12.9 \times 10^2 \text{ s}$$

$$\approx 1.33 \times 10^3 \text{ s}$$

- 56** Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

[2021, 25 Feb Shift-I]

Assertion (A) The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason (R) The product of their mass and radius must be same, $M_1 R_1 = M_2 R_2$

In the light of the above statements, choose the most appropriate answer from the options given below.

- (a) Both A and R are correct but R is not the correct explanation of (a)
 (b) A is correct but R is not correct.
 (c) Both A and R are correct and R is the correct explanation of (a)
 (d) A is not correct but R is correct.

Ans. (b)

Let v_A, v_B, M_A, M_B and R_A, R_B be the escape velocities, masses and radii of planet A and B, respectively.

As we know that,

$$v = \sqrt{2gR} = \sqrt{\frac{2GM}{R^2} \cdot R} = \sqrt{\frac{2GM}{R}}$$

where, G is the gravitational constant.

$$\Rightarrow v \propto \sqrt{\frac{M}{R}}$$

Since, $v_A = v_B$

$$\therefore \frac{M_A}{R_A} = \frac{M_B}{R_B} \Rightarrow M_A R_B = M_B R_A$$

or $M_1 R_2 = M_2 R_1$

Hence, option (b) is the correct.

- 57** The initial velocity v_i required to project a body vertically upward from the surface of the Earth to reach a height of $10R$, where R is the radius of the Earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$.

The value of x will be

[2021, 25 Feb Shift-II]

Ans. (10)

By using law of conservation of energy,
Energy on the surface of earth
(E_{Surface}) = Energy at height ($h = 10R$)

$$\Rightarrow \frac{-GMm}{R} + \frac{1}{2}mv_i^2 = \frac{-GMm}{R+10R} + 0$$

$$= \frac{-GMm}{11R}$$

$$\Rightarrow \frac{1}{2}mv_i^2 = \frac{11}{11} \frac{GMm}{R} - \frac{GMm}{11R}$$

$$\Rightarrow \frac{1}{2}mv_i^2 = \frac{10}{11} \frac{GMm}{R} \Rightarrow v_i^2 = \frac{20GM}{11R}$$

$$\Rightarrow v_i = \sqrt{\frac{10}{11}} v_e$$

$$(\because v_e = \sqrt{\frac{2GM}{R}} = \text{escape velocity})$$

Hence, $x = 10$

- 58** If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x GM^2}{5 R}$, where x is
(Round off to the nearest integer)

(M is the mass of earth, R is the radius of earth and G is the gravitational constant.)

[16 March 2021 Shift-II]

Ans. (3)

We know that binding energy of earth,

$$BE = -\frac{3}{5} \frac{GM^2}{R}$$

$\therefore$ Energy required to break the earth into pieces

$$= -BE = \frac{3}{5} \frac{GM^2}{R} \quad \dots (i)$$

According to question, the amount of energy that needs to be supplied is

$$\frac{x}{5} \frac{GM^2}{R}$$

Comparing it with value in Eq. (i), we get,
 $x = 3$

- 59** The radius in kilometre to which the present radius of Earth ($R = 6400$ km) to be compressed so that the escape velocity is increased 10 times is

[2021, 17 March Shift-I]

Ans. (64)

$$\text{We know that, } v_e = \sqrt{\frac{2Gm}{R_1}}$$

where, v_e = escape velocity,
 G = gravitational constant,
 R = radius of earth

and m = mass of the body.

Now, the escape velocity is increased to 10 times,

$$\Rightarrow 10v_e = \sqrt{\frac{2Gm}{R_2}} \quad \dots (ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\Rightarrow 10 = \sqrt{\frac{R_1}{R_2}} \Rightarrow 100 = \frac{R_1}{R_2}$$

$$\Rightarrow R_2 = \frac{R_1}{100} = \frac{6400}{100} \Rightarrow R_2 = 64 \text{ km}$$

- 60** A satellite is launched into a circular orbit of radius R around Earth, while a second satellite is launched into a circular orbit of radius $1.02 R$. The percentage difference in the time periods of the two satellites is

[2021, 20 July Shift-II]

- (a) 1.5 (b) 2.0 (c) 0.7 (d) 3.0

Ans. (d)

According to question, a satellite is launched into a circular orbit of radius R

R around the Earth, while a second satellite is launched into a circular orbit of radius $1.02 R$.

As we know, according to Kepler's law of planetary motion,

$$T^2 \propto R^3$$

where, T = time period

and R = length of the semi-major axis of planet's orbit.

$$\Rightarrow T \propto R^{3/2}$$

Differentiating above equation, we get

$$\frac{dT}{T} = \frac{3}{2} \frac{dR}{R}$$

Here, $dR = 1.02R - R = 0.02R$

$$\Rightarrow \frac{dT}{T} = \frac{3}{2} \times \frac{0.02R}{R} = 0.03$$

$\therefore$ % difference in time periods of two satellites is

$$\frac{dT}{T} \times 100 = 0.03 \times 100 = 3\%$$

- 61** The planet Mars has two Moons, if one of them has a period 7 h, 30 min and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

[2021, 27 July Shift-II]

$$\left[\text{Take, } \frac{4\pi^2}{G} = 6 \times 10^{11} \text{ N}^{-1} \text{m}^{-2} \text{kg}^2 \right]$$

- (a) 5.96×10^{19} kg (b) 3.25×10^{21} kg
(c) 7.02×10^{25} kg (d) 6.00×10^{23} kg

Ans. (d)

Given, $r = 9 \times 10^3 \text{ km} = 9 \times 10^6 \text{ m}$

$$T = 7\text{h} + 30 \text{ min}$$

$$= 7 \times 60 \times 60 + 30 \times 60 = 27000 \text{ s}$$

$$\frac{4\pi^2}{G} = 6 \times 10^{11} \text{ N}^{-1} \text{m}^{-2} \text{kg}^2$$

$\therefore$ Both Mars and its one Moon will have gravitational force of attraction between them and as both of them are revolving in a circular orbit, then centripetal force will be acting on them.

Let m be the mass of Moon and M be the mass of Mars.

$\therefore$ Gravitational force acting between them = Centripetal force making both the them move in circular orbit

$$\Rightarrow \frac{GMm}{r^2} = m\omega^2 r \Rightarrow \omega^2 = \frac{GM}{r^3}$$

$$\Rightarrow \frac{4\pi^2}{T^2} = \frac{GM}{r^3} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\Rightarrow M = \frac{4\pi^2 r^3}{GT^2} \quad \dots (i)$$

$$= \frac{6 \times 10^{11} \times (9 \times 10^6)^3}{(27000)^2}$$

$$M = 6.0 \times 10^{23} \text{ kg}$$

- 62** The masses and radii of the Earth and Moon are (m_1, R_1) and (m_2, R_2) , respectively. Their centres are at a distance r apart. Find the minimum escape velocity for a particle of mass m to be projected from the middle of these two masses.

[2021, 31 August Shift-I]

$$(a) v = \frac{1}{2} \sqrt{\frac{4G(m_1 + m_2)}{r}}$$

$$(b) v = \sqrt{\frac{4G(m_1 + m_2)}{r}}$$

$$(c) v = \frac{1}{2} \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

$$(d) v = \frac{\sqrt{2G(m_1 + m_2)}}{r}$$

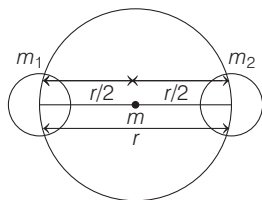
Ans. (b)

Given, masses and radii of Earth and Moon are (m_1, R_1) , (m_2, R_2) and separation between their centre of mass = r

If v be the minimum escape velocity of particle of mass m , then by using law of conservation of energy,

Initial energy = Final energy = 0

$$\frac{1}{2}mv^2 - \frac{Gm_1m}{r/2} - \frac{Gm_2m}{r/2} = 0$$



$$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{r}(m_1 + m_2)$$

$$\Rightarrow v = \sqrt{\frac{4G}{r}(m_1 + m_2)}$$

- 63** Two satellites revolve around a planet in coplanar circular orbits in anti-clockwise direction. Their period of revolutions are 1 h and 8 h, respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x} \text{ rad h}^{-1}$, where x is

[2021, 1 Sep. Shift-II]

Ans. (3)

Given, the time period of the first satellite, $T_1 = 1 \text{ h}$

The time period of the second satellite,

$$T_2 = 8 \text{ h}$$

The radius of the orbit of nearer satellite, $R_1 = 2000 \text{ km}$

Applying the Kepler's third law,

$$T^2 \propto R^3$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\Rightarrow \frac{2000}{R_2} = \left(\frac{1}{8}\right)^{2/3}$$

$$\Rightarrow R_2 = 8000 \text{ km}$$

As we know the relation between the angular speed and time period

$$\omega = \frac{\theta}{T}$$

So, the angular speed of the nearer satellite to the orbit,

$$\omega_1 = \frac{2\pi}{1} \text{ rad/h}$$

and the angular speed of the farther satellite to the orbit,

$$\omega_2 = \frac{2\pi}{8} = \frac{\pi}{4} \text{ rad/h}$$

The speed of the nearer satellite to orbit,

$$v_1 = \omega_1 R_1 = (2\pi) \times 2 \times 10^3 \text{ km/h}$$

The speed of the farther satellite to the orbit

$$v_2 = \omega_2 R_2 = \left(\frac{\pi}{4}\right) \times 8000 = \pi \times 2 \times 10^3 \text{ km/h}$$

Thus, the relative angular speed of the nearer satellite to the farther satellite,

$$\omega = \frac{v_1 - v_2}{R_1 - R_2} = \frac{2\pi \times 2 \times 10^3 - \pi \times 2 \times 10^3}{8000 - 2000} = \frac{2\pi \times 10^3}{6000} = \frac{\pi}{3} \text{ rad/h}$$

Comparing with, $\omega = \frac{\pi}{x}$

The value of $x = 3$.

- 64** The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large

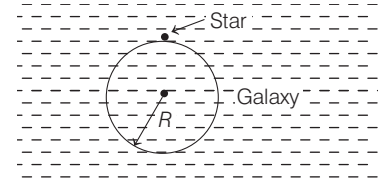
distance r from its centre. In that region, a small star is in a circular orbit of radius R . Then, the period of revolution T depends on R as

[2020, 2 Sep Shift-I]

- (a) $T \propto R$ (b) $T^2 \propto R^3$
(c) $T^2 \propto R$ (d) $T^2 \propto \frac{1}{R^3}$

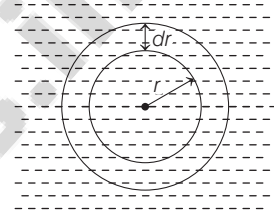
Ans. (c)

Force on star is provided by the mass enclosed inside region of radius R as shown below.



where, R = orbital radius of star.

Now, consider a spherical region of radius r and thickness dr as shown below.



Mass enclosed in this region is given by

$$dm = \text{density} \times \text{volume} = \frac{k}{r} \cdot 4\pi r^2 \cdot dr$$

So, total mass enclosed in a region of radius R is given by

$$M = \int_0^R dm = \int_0^R \frac{k}{r} \cdot 4\pi r^2 dr = 4\pi k \left[\frac{r^2}{2} \right]_0^R = 2k\pi R^2$$

Gravitational pull of this mass provides necessary centripetal force for rotation of star.

$$\text{So, } \frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow v^2 = \frac{GM}{R} = \frac{G2k\pi R^2}{R}$$

$$\Rightarrow v^2 = 2Gk\pi R \Rightarrow v = \sqrt{2Gk\pi R}$$

Time period of rotation,

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{2Gk\pi R}}$$

$$\Rightarrow T \propto \sqrt{R} \text{ or } T^2 \propto R$$

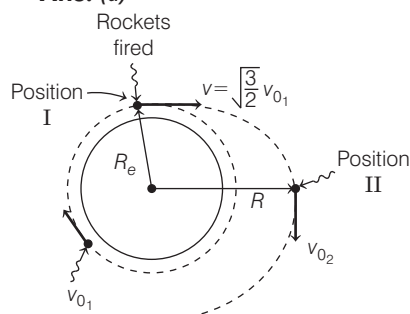
Hence, correct option is (c).

- 65** A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e . By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion, so that it becomes $\sqrt{\frac{3}{2}}$ times larger. Due to

this, the farthest distance from the centre of the earth that the satellite reaches is R . Value of R is

[2020, 3 Sep Shift-I]

- (a) $4R_e$ (b) $2.5R_e$ (c) $3R_e$ (d) $2R_e$

Ans. (d)

Using law of conservation of energy for position I and position II of satellite, we have

$$\begin{aligned} (PE + KE)_{\text{Position I}} &= (PE + KE)_{\text{Position II}} \\ \Rightarrow \frac{-GMm}{R_e} + \frac{1}{2}mv_1^2 &= \frac{-GMm}{R} + \frac{1}{2}mv_2^2 \\ \Rightarrow \frac{-GMm}{R_e} + \frac{1}{2}m\left(\frac{3}{2}v_{01}^2\right) &= \frac{-GMm}{R} + \frac{1}{2}mv_{02}^2 \end{aligned}$$

As, orbital speed is

$$v_0 = \sqrt{\frac{GM}{R}} \quad (\text{where, } R = \text{orbital radius})$$

Therefore,

$$\begin{aligned} \frac{-GMm}{R_e} + \frac{1}{2}m\left(\frac{3}{2}\frac{GM}{R_e}\right) &= \frac{-GMm}{R} + \frac{1}{2}m\left(\frac{GM}{R}\right) \\ \Rightarrow \frac{-1}{4}\left(\frac{GMm}{R_e}\right) &= \frac{-1}{2}\left(\frac{GMm}{R}\right) \\ \Rightarrow R &= 2R_e \end{aligned}$$

- 66** A body is moving in a low circular orbit about a planet of mass M and radius R . The radius of the orbit can be taken to be R itself. Then, the ratio of the speed of this body in the orbit to the escape velocity from the planet is

[2020, 4 Sep Shift-II]

- (a) 1
(b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$
(d) 2

Ans. (b)

Orbital velocity of a body revolving around a planet,

$$v_o = \sqrt{\frac{GM}{r}} \quad \dots(i)$$

where, M = mass of the planet,
 r = orbital radius of revolving body = R (given)
 and G = universal gravitational constant.
 Escape velocity of any body from the surface of a planet,

$$v_e = \sqrt{\frac{2GM}{R}} \quad \dots(ii)$$

where, M = mass of the planet,
 R = radius of the planet and G = universal gravitational constant. Ratio of body's orbital velocity to the escape velocity from the planet,

$$\frac{v_o}{v_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

Hence, option (b) is correct.

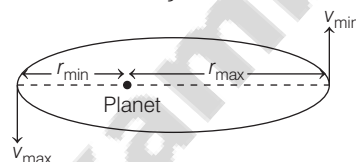
- 67** A satellite is in an elliptical orbit around a planet P . It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is

[2020, 6 Sep Shift-I]

- (a) 1 : 6
(b) 1 : 3
(c) 1 : 2
(d) 3 : 4

Ans. (a)

The motion of a satellite in an elliptical orbit is shown in figure



Applying, angular momentum conservation about planet at closest and farthest point,

$$\begin{aligned} L_{\text{closest}} &= L_{\text{farthest}} \\ mv_{\text{max}}r_{\text{min}} &= mv_{\text{min}}r_{\text{max}} \\ \text{or } \frac{r_{\text{min}}}{r_{\text{max}}} &= \frac{v_{\text{min}}}{v_{\text{max}}} \end{aligned}$$

$$\text{Given that, } v_{\text{max}} = 6v_{\text{min}} \text{ or } \frac{v_{\text{min}}}{v_{\text{max}}} = \frac{1}{6}$$

$$\therefore \frac{r_{\text{min}}}{r_{\text{max}}} = \frac{1}{6}$$

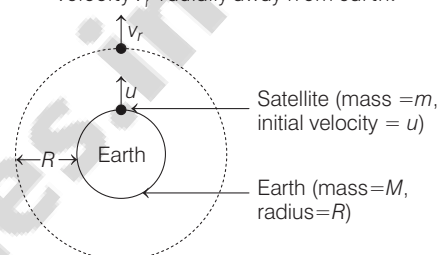
Hence, correct option is (a).

- 68** A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$, so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant M , is the mass of the earth) [2020, 7 Jan Shift-I]

- (a) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$
 (b) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$
 (c) $\frac{m}{20}\left(u^2 + \frac{113GM}{200R}\right)$
 (d) $5m\left(u^2 - \frac{119GM}{200R}\right)$

Ans. (d)

Let us assume that, when satellite reaches in orbit at height R , it has a velocity v_r radially away from earth.



According to conservation of energy, Total energy of satellite at surface of earth = Total energy at height R

$$\begin{aligned} \Rightarrow (KE + PE)_{\text{initial}} &= (KE + PE)_{\text{final}} \\ \Rightarrow \frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) &= \frac{1}{2}mv_r^2 + \left(\frac{-GMm}{2R}\right) \end{aligned}$$

So, radial velocity of satellite at height R ,

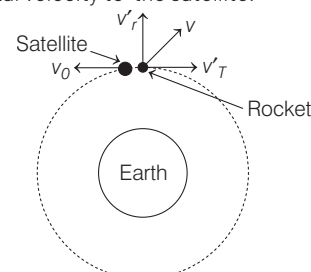
$$v_r = \left(u^2 - \frac{GM}{R}\right)^{1/2} \quad \dots(i)$$

To provide satellite an orbital velocity

$$\left(v_o = \sqrt{\frac{GM}{2R}}\right) \text{ a rocket (mass} = \frac{m}{10}\text{) is}$$

fired.

Rocket must be given a velocity v such that it destroyed or opposes radial momentum of satellite and also provides orbital velocity to the satellite.



Now, we apply conservation of momentum to calculate radial (v_r') and tangential (v_T') components of rocket's velocity.

When rocket is ejected from satellite, mass of satellite will be

$$m - \frac{m}{10} = \frac{9m}{10}$$

By applying conservation of tangential momentum of rocket and satellite system, we have

$$\frac{m}{10} \times v_r' + \frac{9m}{10}(-v_0) = 0 \quad \dots(ii)$$

Similarly, by applying conservation of radial momentum of rocket and satellite system, we have

$$\frac{m}{10} \times v_r' + \frac{9m}{10}(v_r) = 0 \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we have

$$v_r' = 9\sqrt{\frac{GM}{2R}}$$

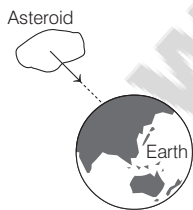
and $v_r' = 10\sqrt{u^2 - \frac{GM}{R}}$

So, kinetic energy of the rocket,

$$\begin{aligned} K &= \frac{1}{2} \left(\frac{m}{10} \right) v^2 \\ &= \frac{1}{2} \left(\frac{m}{10} \right) \left(\sqrt{v_r'^2 + v_t'^2} \right)^2 \\ &= \frac{1}{2} \left(\frac{m}{10} \right) (v_r'^2 + v_t'^2) \\ &= \frac{1}{2} \left(\frac{m}{10} \right) \left[81 \times \frac{GM}{2R} + 100 \times \left(u^2 - \frac{GM}{R} \right) \right] \\ &= 5m \left(u^2 - \frac{119 GM}{200 R} \right) \end{aligned}$$

- 69** An asteroid is moving directly towards the centre of the earth. When at a distance of $10R$ (R is the radius of the earth) from the earth's centre, it has a speed of 12 km/s . Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in km/s [2020, 8 Jan Shift-II]

Ans. (16)



Applying energy conservation, we have
Total energy of asteroid at 12 km/s = Total energy of asteroid at surface of earth

$$\begin{aligned} \Rightarrow U_1 + K_1 &= U_2 + K_2 \\ \Rightarrow \frac{-GM_e m}{10R_e} + \frac{1}{2} m v_0^2 &= \frac{-GM_e m}{R_e} + \frac{1}{2} m v^2 \\ \Rightarrow \frac{9 GM_e m}{10 R_e} + \frac{1}{2} m v_0^2 &= \frac{1}{2} m v^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{9}{10} \times \frac{2GM_e}{R_e} + v_0^2 &= v^2 \\ \Rightarrow \frac{9}{10} \times (\sqrt{v_e})^2 + v_0^2 &= v^2 \\ \Rightarrow \frac{9}{10} \times (11.2)^2 + (12)^2 &= v^2 \\ \Rightarrow v^2 &= \sqrt{\frac{9}{10} \times (11.2)^2 + (12)^2} \\ \Rightarrow v &\approx 16 \text{ km/s}^{-1} \end{aligned}$$

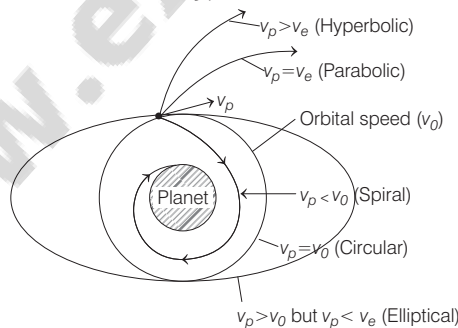
- 70** A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{v}{2}\right)$, the instantaneous velocity v of A. The collision is completely inelastic (c) Then, the combined body

[2020, 9 Jan Shift-I]

- escapes from the planet's gravitational field
- starts moving in an elliptical orbit around the planet
- falls vertically downward towards the planet
- continues to move in a circular orbit

Ans. (b)

For a particle or satellite with speed v_p around a planet whose escape speed is v_e and orbital speed of particle v_0 , we have following possibilities,



Now, by momentum conservation in given collision, we have, $p_i = p_f$

$$\Rightarrow m v_A + \frac{m}{2} \left(\frac{v_A}{2} \right) = \left(m + \frac{m}{2} \right) v_p \quad \dots(i)$$

Body A is orbiting in orbit of radius $R + h$, hence for a small value of h ,

$$v_A = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$$

So, final speed of combined mass system,

$$v_p = \frac{5}{6} v_0 = \frac{5}{6} \times \frac{1}{\sqrt{2}} v_e$$

As v_p is less than v_0 , so it must spiral towards planet. Most nearest option is (b), if we take spiral close to elliptical.

- 71** Planet A has mass M and radius R . Planet B has half the mass and half the radius of planet A. If the escape velocities from the planets A and B are v_A and v_B respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is

[2020, 9 Jan Shift-II]

- 1
- 2
- 3
- 4

Ans. (d)

Escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

where, m = mass, R = radius of planet.

So, $v_A = \sqrt{\frac{2GM}{R}}$

and $v_B = \sqrt{\frac{2G\left(\frac{M}{2}\right)}{\left(\frac{R}{2}\right)}} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \frac{v_A}{v_B} = 1$$

or $\frac{n}{4} = 1 \Rightarrow n = 4$

- 72** A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have, if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

[2019, 8 April Shift-II]

- $\frac{E}{64}$
- $\frac{E}{16}$
- $\frac{E}{32}$
- $\frac{E}{4}$

Ans. (b)

Given, volume of earth (V_e) is 64 times of volume of moon (V_m), i.e.

$$\frac{V_e}{V_m} = 64 = \frac{\frac{4}{3} \pi R_e^3}{\frac{4}{3} \pi R_m^3}$$

where, R_e and R_m are the radius of earth and moon, respectively.

$$\text{Then, } \frac{R_e}{R_m} = 4 \quad \dots(i)$$

Also, since the density of moon and earth are equal, i.e. $\rho_m = \rho_e$

$$\Rightarrow \frac{M_e}{V_e} = \frac{M_m}{V_m}, \text{ where } M_e \text{ and } M_m \text{ are the}$$

mass of the earth and moon, respectively.

$$\Rightarrow \frac{M_e}{M_m} = \frac{V_e}{V_m} = 64 \quad \dots(ii)$$

The minimum energy or escape energy delivered by the rocket launcher, so that the rocket never returns to earth is

$$E_e = \frac{GM_e m}{R_e} = E$$

where, m is the mass of the rocket.

Similarly, minimum energy that a launcher should have to escape or to never return, if rocket is launched from surface of the moon is

$$E_m = \frac{GM_m m}{R_m}$$

$\therefore$ Ratio of escape energies E_e and E_m is

$$\frac{E_e}{E_m} = \frac{\left(\frac{GM_e m}{R_e}\right)}{\left(\frac{GM_m m}{R_m}\right)} = \frac{M_e}{M_m} \cdot \frac{R_m}{R_e}$$

$$= 64 \times \frac{1}{4} = 16 \quad [\text{using Eqs. (i) and (ii)}]$$

$$\therefore E_m = \frac{E_e}{16} = \frac{E}{16}$$

- 73** A test particle is moving in a circular orbit in the gravitational field produced by mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T [2019, 9 April Shift-II]

(a) $\frac{T^2}{R^3}$ is a constant

(b) $\frac{T}{R^2}$ is a constant

(c) TR is a constant

(d) $\frac{T}{R}$ is a constant

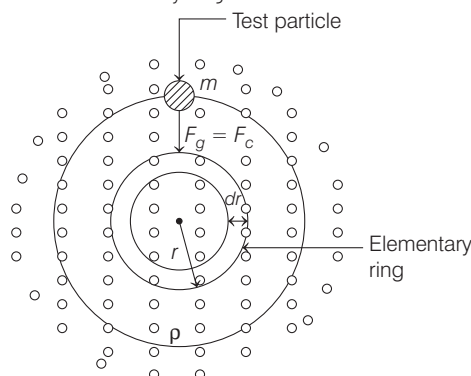
Ans. (d)

Let the mass of the test particle be m and its orbital linear speed be v .

Force of gravity of the mass-density would provide the necessary centripetal pull on test particle.

$\Rightarrow$ Gravitational force (F_g) = Centripetal force (F_c)

Let us now assume an elementary ring at a distance r from the centre of the mass density, such that mass of this elementary ring



$$\Rightarrow \frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\text{where, } M = \int_0^R \rho \cdot 4\pi r^2 \cdot dr$$

$$\Rightarrow \frac{G \left(\int_0^R \frac{K}{r^2} \cdot 4\pi r^2 \cdot dr \right)}{R^2} = \frac{v^2}{R} \quad \left[\because \text{given, } \rho(r) = \frac{K}{r^2} \right]$$

$$\Rightarrow G \cdot 4\pi K = v^2$$

$$\therefore \text{Orbital speed of mass, } m = v = \sqrt{G4\pi K} \quad \dots(i)$$

Time period of rotation of the test particle is

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G \cdot 4\pi K}} \quad [\because \text{using Eq. (i)}]$$

$$\text{Hence, } \frac{T}{R} = \sqrt{\frac{\pi}{GK}} = \text{a constant}$$

Alternate Solution

Since, $F_g = F_c$

This relation can also be written directly as,

$$\frac{G \int_0^R \left(\frac{K}{r^2} \cdot 4\pi r^2 \cdot dr \right)}{R^2} = m\omega^2 R = m \left(2\frac{\pi}{T} \right)^2 R$$

$$\Rightarrow \frac{G \cdot 4\pi KR}{R^2} = \frac{4\pi^2}{T^2} R$$

$$\text{or } \frac{T^2}{R^2} = \frac{\pi}{GK}$$

$$\Rightarrow \frac{T}{R} = \sqrt{\frac{\pi}{GK}} = \text{a constant.}$$

- 74** A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the

number of complete revolutions made by the spaceship in 24 hours around the planet?

[Take, mass of planet = 8×10^{22} kg, radius of planet = 2×10^6 m, gravitational constant $G = 6.67 \times 10^{-11}$ N - m² / kg²]

[2019, 10 April Shift-II]

(a) 11

(b) 17

(c) 13

(d) 9

Ans. (a)

A satellite or spaceship in a circular orbit at a distance $(R + h)$ from centre of a planet experiences a gravitational force given by

$$F_g = \frac{GMm}{(R + h)^2}$$

where, M = mass of planet,

m = mass of spaceship,

R = radius of planet

and h = height of spaceship above surface.

This gravitational pull provides necessary centripetal pull for orbital motion of spaceship.

$$\text{So, } F_g = F_{\text{centripetal}} \Rightarrow \frac{GMm}{(R + h)^2} = \frac{mv^2}{(R + h)}$$

where, v = orbital speed of spaceship.

Orbital speed of spaceship is

$$v = \sqrt{\frac{GM}{R + h}} \quad \dots(i)$$

Here, $G = 6.67 \times 10^{-11}$ N-m²/kg²,

$M = 8 \times 10^{22}$ kg

$$R + h = (2 \times 10^6 + 20 \times 10^3) \text{ m} = 2.02 \times 10^6 \text{ m}$$

So, substituting these values in Eq. (i), we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 8 \times 10^{22}}{2.02 \times 10^6}} = 1.6 \times 10^3 \text{ ms}^{-1}$$

Time period of rotation of spaceship will be

$$T = \frac{2\pi(R + h)}{v}$$

$$\Rightarrow T = \frac{2\pi \times 2.02 \times 10^6}{1.6 \times 10^3} \approx 8 \times 10^3 \text{ s} = \frac{8 \times 10^3}{60 \times 60} \text{ (h)}$$

$$= 2.2 \text{ h}$$

So, number of revolutions made by spaceship in 24 h,

$$n = \frac{24}{T} = \frac{24}{2.2} \approx 11 \text{ rev}$$

- 75** The energy required to take a satellite to a height h above earth surface (where, radius of earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2 . The value of h for which E_1 and E_2 are equal is

[2019, 9 Jan Shift-II]

- (a) 3.2×10^3 km
(b) 1.28×10^4 km
(c) 6.4×10^3 km
(d) 1.6×10^3 km

Ans. (a)

The energy required for taking a satellite upto a height h from earth's surface is the difference between the energy at height and energy at surface, then

$$\Rightarrow E_1 = U_f - U_i$$

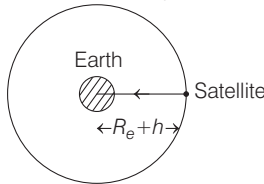
$$E_1 = -\frac{GM_e m}{R_e + h} + \frac{GM_e m}{R_e} \quad \dots(i)$$

(where, U = potential energy)

$\therefore$ Orbital velocity of satellite,

$$v_o = \sqrt{\frac{GM_e}{R_e + h}}$$

(where, M_e = mass of earth)



So energy required to perform circular motion

$$\Rightarrow E_2 = \frac{1}{2}mv_o^2 = \frac{GM_e m}{2(R_e + h)}$$

$$E_2 = \frac{GM_e m}{2(R_e + h)} \quad \dots(ii)$$

According to the question,

$$\therefore \frac{-GM_e m}{R_e + h} + \frac{GM_e m}{R_e} = \frac{GM_e m}{2(R_e + h)}$$

$$\Rightarrow 3R_e = 2R_e + 2h$$

$$h = \frac{R_e}{2}$$

As radius of earth, $R_e \approx 6.4 \times 10^3$ km

Hence,

$$h = \frac{6.4 \times 10^3}{2} \text{ km or } 3.2 \times 10^3 \text{ km}$$

- 76** A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass m' is ejected from the satellite such that it just escapes

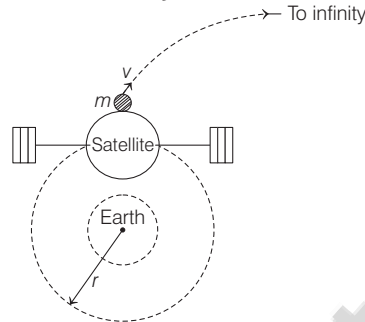
from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

[2019, 10 Jan Shift-I]

- (a) $\frac{3}{2}mv^2$ (b) $2mv^2$ (c) mv^2 (d) $\frac{1}{2}mv^2$

Ans. (c)

Kinetic energy must be given to an object so that it just escapes to infinity as shown in the figure below



Applying energy-conservation for mass m , we get

Initial energy of m at satellite = Final energy of m at infinity.

$$\Rightarrow U_i + K_i = U_f + K_f \quad \dots(i)$$

Here, U_i = initial PE of m

$$= \frac{-G \cdot Mm}{r} = -(v^2)m$$

As, $v = \sqrt{\frac{GM}{r}}$ = orbital speed of the satellite.

K_i = initial kinetic energy of m ,

U_f = final potential energy of m which is zero and

K_f = final kinetic energy of m which is also zero.

So, from Eq. (i), we have

$$-v^2 m + K_i = 0 + 0 \text{ or } K_i = mv^2.$$

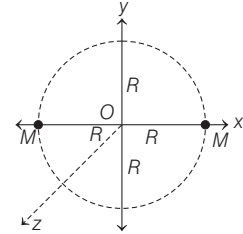
- 77** Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O . A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take, gravitational constant, $G = 6.67 \times 10^{-11} \text{ N-m}^2 \text{ kg}^{-2}$)

[2019, 10 Jan Shift-II]

- (a) 2.8×10^5 m/s (b) 3.8×10^4 m/s
(c) 2.4×10^4 m/s (d) 1.4×10^5 m/s

Ans. (a)

Let us assume that stars are moving in x y -plane with origin as their centre of mass as shown in the figure below



According to question, mass of each star,

$$M = 3 \times 10^{31} \text{ kg}$$

and diameter of circle, $2R = 2 \times 10^{11}$ m

$$\Rightarrow R = 10^{11} \text{ m}$$

Potential energy of meteorite at O , origin is,

$$U_{\text{total}} = -\frac{2GMm}{r}$$

If v is the velocity of meteorite at O then

Kinetic energy K of the meteorite is

$$K = \frac{1}{2}mv^2$$

To escape from this dual star system, total mechanical energy of the meteorite at infinite distance from stars must be at least zero.

By conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{2GMm}{R} = 0$$

$$\Rightarrow v^2 = \frac{4GM}{R}$$

$$= \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}$$

$$\Rightarrow v = 2.83 \times 10^5 \text{ m/s}$$

- 78** A satellite is revolving in a circular orbit at a height h from the earth surface such that $h \ll R$, where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

[2019, 11 Jan Shift-I]

- (a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$
(c) $\sqrt{2gR}$ (d) $\sqrt{gR}(\sqrt{2} - 1)$

Ans. (d)

Orbital velocity of the satellite is given as,

$$v_o = \sqrt{\frac{GM}{R+h}}$$

Since, $R \gg h$

$$\therefore v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left[\because g = \frac{GM}{R^2} \right]$$

Escape velocity of the satellite,

$$v_e = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Since, we know that in order to escape the earth's gravitational field a satellite must get escape velocity.

$\therefore$ Change in velocity,

$$\Delta v = v_e - v_0 = \sqrt{gR} (\sqrt{2} - 1)$$

- 79** A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be [2019, 12 Jan Shift-I]

- (a) in the same circular orbit of radius R
 (b) in an elliptical orbit
 (c) such that it escapes to infinity
 (d) in a circular orbit of a different radius

Ans. (b)

According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force. Hence, orbit must be elliptical.

- 80** Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is [2019, 12 Jan Shift-II]

- (a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

Ans. (d)

Orbital speed of a satellite in a circular orbit is

$$v_0 = \sqrt{\left(\frac{GM}{r_0}\right)}$$

where r_0 is the radius of the circular orbit. So, kinetic energies of satellites A and B are

$$T_A = \frac{1}{2} m_A v_{0A}^2 = \frac{GMm}{2R}$$

$$T_B = \frac{1}{2} m_B v_{0B}^2 = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}$$

So, ratio of their kinetic energies is

$$\frac{T_A}{T_B} = 1$$

- 81** A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth R , $h \ll R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) [JEE Main 2017]

- (a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
 (c) $\sqrt{gR/2}$ (d) $\sqrt{gR} (\sqrt{2} - 1)$

Ans. (d)

Given, a satellite is revolving in a circular orbit at a height h from the Earth's surface having radius of Earth R , i.e. $h \ll R$.

Orbital velocity of a satellite,

$$v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}} \quad (\text{as } h \ll R)$$

Velocity required to escape,

$$\frac{1}{2} m v'^2 = \frac{GMm}{R+h}$$

$$v' = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} \quad (h \ll R)$$

$\therefore$ Minimum increase in its orbital velocity required to escape from the Earth's gravitational field.

$$v' - v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = \sqrt{2gR} - \sqrt{gR}$$

$$= \sqrt{gR} (\sqrt{2} - 1) \quad \left(\because g = \frac{GM}{R^2} \right)$$

- 82** What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? [JEE Main 2013]

- (a) $\frac{5GMm}{6R}$ (b) $\frac{2GMm}{3R}$ (c) $\frac{GMm}{2R}$ (d) $\frac{GMm}{3R}$

Ans. (a)

From conservation of energy,

Total energy at the planet = Total energy at altitude

$$-\frac{GMm}{R} + \frac{1}{2} m v_A^2 = -\frac{GMm}{R} + \frac{1}{2} m v_A^2 + (KE)_{\text{surface}} \quad \dots(i)$$

In the orbital of planet, the necessary centripetal force is obtained by gravitational force.

$$\therefore \frac{mv_A^2}{R+2R} = \frac{GMm}{(R+2R)^2}$$

$$\Rightarrow \frac{v_A}{2} = \frac{GM}{3R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$(KE)_{\text{surface}} = \frac{5GMm}{6R}$$

- 83** The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km , respectively. The required energy for this work will be [AIEEE 2012]

- (a) $6.4 \times 10^{11} \text{ J}$ (b) $6.4 \times 10^8 \text{ J}$
 (c) $6.4 \times 10^9 \text{ J}$ (d) $6.4 \times 10^{10} \text{ J}$

Ans. (d)

Potential energy on earth surface is

$-mgR$ while in free space, it is zero. So,

to free the spaceship, minimum required energy is

$$K = mgR = 10^3 \times 10 \times 6400 \times 10^3 \text{ J}$$

$$= 6.4 \times 10^{10} \text{ J}$$

$$\therefore PE = \frac{-GmM}{R}$$

On earth surface, as $g = \frac{GM}{R^2}$

$$\text{So, } PE = -\frac{gR^2 m}{R} = -mgR$$

- 84** A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1} , the escape velocity from the surface of the planet would be [AIEEE 2008]

- (a) 1.1 kms^{-1} (b) 11 kms^{-1}
 (c) 110 kms^{-1} (d) 0.11 kms^{-1}

Ans. (c)

Mass of planet, $M_p = 10M_e$, where M_e is mass of earth. Radius of planet,

$$R_p = \frac{R_e}{10}, \text{ where } R_e \text{ is radius of earth.}$$

Escape velocity is given by, $v_e = \sqrt{\frac{2GM}{R}}$

$$\text{For planet, } v_p = \sqrt{\frac{2G \times M_p}{R_p}} = \sqrt{\frac{100 \times 2GM_e}{R_e}}$$

$$= 10 \times v_e = 10 \times 11 = 110 \text{ kms}^{-1}$$

- 85** A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is [AIEEE 2004]

(a) gx (b) $\frac{gR}{R-x}$
 (c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

Ans. (d)

The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where, M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$$

where, v_o is orbital speed of satellite.

$$\Rightarrow \frac{GM_e m}{(R+x)} = mv_o^2$$

$$\text{or } \frac{gR^2 m}{(R+x)} = mv_o^2 \quad \left[\because g = \frac{GM_e}{R^2} \right]$$

$$\text{or } v_o = \sqrt{\left[\frac{gR^2}{(R+x)} \right]} = \left[\frac{gR^2}{(R+x)} \right]^{1/2}$$

- 86** The time period of an earth satellite in circular orbit is independent of [AIEEE 2004]
- (a) the mass of the satellite
 (b) radius of its orbit
 (c) both the mass and radius of the orbit
 (d) neither the mass of the satellite nor the radius of its orbit

Ans. (a)

$$\text{Time period of satellite, } T = 2\pi \sqrt{\frac{(R+h)^3}{GM_e}}$$

where, $R+h$ = orbital radius of satellite

and M_e = mass of earth.

Thus, time period does not depend on mass of satellite.

- 87** Suppose the gravitational force varies inversely as the n th power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to [AIEEE 2004]

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$
 (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

Ans. (a)

The necessary centripetal force required for a planet to move round the sun

= Gravitational force exerted on it

$$\text{i.e., } \frac{mv^2}{R} = \frac{GM_e m}{R^n} \quad \text{or } v = \left(\frac{GM_e}{R^{n-1}} \right)^{1/2}$$

$$\text{Now, } T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e} \right)^{1/2}$$

$$= 2\pi \left(\frac{R^2 \times R^{n-1}}{GM_e} \right)^{1/2}$$

$$= 2\pi \left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}} \right)$$

$$\text{or } T \propto R^{(n+1)/2}$$

- 88** The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become [AIEEE 2003]

(a) 10 h (b) 80 h
 (c) 40 h (d) 20 h

Ans. (c)

According to Kepler's law,

$$T^2 \propto r^3 \Rightarrow 5^2 \propto r^3 \quad \dots(i)$$

$$\text{and } (T')^2 \propto (4r)^3 \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

or $T' = \sqrt{1600}$ or $T' = 40$ h

- 89** The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, then the escape velocity will be [AIEEE 2003]

(a) $11\sqrt{2}$ km/s (b) 22 km/s
 (c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ m/s

Ans. (c)

The escape velocity is independent of angle of projection, hence it will remain same i.e., 11 km/s.

- 90** The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is [AIEEE 2002]

(a) $\frac{mgR}{2}$ (b) $2mgR$
 (c) mgR (d) $\frac{mgR}{4}$

Ans. (c)

The minimum kinetic energy required to project a body of mass m from earth's surface to infinity is known as escape energy. Therefore,

$$KE = \frac{GM_e m}{R} = mgR \quad \left[\because gR = \frac{GM_e}{R} \right]$$

- 91** The escape velocity of a body depends upon mass as [AIEEE 2002]
- (a) m^0
 (b) m^1
 (c) m^2
 (d) m^3

Ans. (a)

$$\text{Escape velocity} = \sqrt{2gR_e}$$

So, escape velocity is independent of m .

So, it depends upon mass m as m^0 .

Since, $m^0 = 1$.

08

Mechanical Properties of Solids

TOPIC 1

Stress, Strain and Hooke's Laws

- 01** A uniform heavy rod of weight 10 kg ms^{-2} , cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young's modulus of the material of the rod is $2 \times 10^{11} \text{ Nm}^{-2}$. Neglecting the lateral contraction, find the elongation of rod due to its own weight.

[2021, 31 Aug Shift-I]

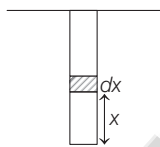
- (a) $2 \times 10^{-9} \text{ m}$ (b) $5 \times 10^{-8} \text{ m}$
(c) $4 \times 10^{-8} \text{ m}$ (d) $5 \times 10^{-10} \text{ m}$

Ans. (d)

Given, weight of rod, $w = 10 \text{ kg ms}^{-2}$

Area of cross-section, $A = 100 \text{ cm}^2$
 $= 100 \times 10^{-4} \text{ m}^2$

Length of rod, $l = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$



Young's modulus (Y) $= 2 \times 10^{11} \text{ Nm}^{-2}$

Let, elongation $= \Delta l$

Since, Young's modulus, $Y = \frac{dmg \cdot x}{A \Delta l}$

Mass of element dx at distance x ,

$$dm = \frac{m}{l} dx$$

$$\Rightarrow \Delta l = \left(\frac{mg}{AY} \right) \times dx$$

$$\Rightarrow \Delta l = \frac{mg}{AY} \cdot \frac{x^2}{2} \Big|_0^l$$

$$\Rightarrow \Delta l = \frac{mg}{AY} \cdot \frac{l^2}{2} = \frac{mgl}{2AY} = \frac{wl}{2AY}$$

$$= \frac{10 \times 20 \times 10^{-2}}{2 \times 100 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= 5 \times 10^{-10} \text{ m}$$

- 02** When a rubber ball is taken to a depth of m in deep sea, its volume decreases by 0.5% . (The bulk modulus of rubber $= 9.8 \times 10^8 \text{ Nm}^{-2}$. Density of sea water $= 10^3 \text{ kg m}^{-3}$, $g = 9.8 \text{ m/s}^2$)

[2021, 31 Aug Shift-I]

Ans. (500)

Given, decrease in volume

$$\left(\frac{\Delta V}{V} \right) = -\frac{0.5}{100}$$

Bulk modulus of rubber,

$$B = 9.8 \times 10^8 \text{ Nm}^{-2}$$

Density of sea water, $\rho = 10^3 \text{ kg m}^{-3}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Let h be the depth at which ball is dipped.

$$\text{Since, } B = \frac{\Delta p}{-\Delta V/V}$$

where, Δp is change in pressure $= \rho gh$

$$\therefore \rho gh = -B \frac{\Delta V}{V}$$

$$\Rightarrow h = -\frac{1}{\rho g} B \frac{\Delta V}{V}$$

$$\Rightarrow h = -\frac{1}{10^3 \times 9.8} \times 9.8 \times 10^8 \times \left(-\frac{0.5}{100} \right)$$

$$= 0.5 \times 10^{8-3-2} = 0.5 \times 10^3 = 500 \text{ m}$$

- 03** Four identical hollow cylindrical columns of mild steel support a big structure of mass $50 \times 10^3 \text{ kg}$. The inner and outer radii of each column are 50 cm and 100 cm ,

respectively. Assuming, uniform local distribution, calculate the compression strain of each column. [Use, $Y = 2.0 \times 10^{11} \text{ Pa}$, $g = 9.8 \text{ m/s}^2$].

[2021, 31 Aug Shift-II]

- (a) 3.60×10^{-8} (b) 2.60×10^{-7}
(c) 1.87×10^{-3} (d) 7.07×10^{-4}

Ans. (b)

Let inner and outer radii of hollow cylindrical column are r and R , respectively.

Given that, $r = 50 \text{ cm}$, $R = 100 \text{ cm}$

Mass supported on four columns,

$$M = 50 \times 10^3 \text{ kg}$$

Mass supported on each column, $m = \frac{M}{4}$

$$\Rightarrow m = \frac{50 \times 10^3}{4} = 12.5 \times 10^3 \text{ kg}$$

Now, weight, $w = mg = 12.5 \times 9.8 \times 10^3 \text{ N}$
 $= 1.225 \times 10^5 \text{ N}$

Area of cross-section of each column

$$A = \pi(R^2 - r^2)$$

$$= 3.14 \{ (100)^2 - (50)^2 \} \times 10^{-4}$$

$$\text{m}^2 = 2.35 \text{ m}^2$$

Young's modulus, $Y = 2.0 \times 10^{11} \text{ Pa}$

By using Hooke's law,

$$\text{Stress} = Y \times \text{Strain}$$

$$\therefore \text{Compressive strain} = \frac{\text{Stress}}{Y} = \frac{w}{AY}$$

Substituting the values, we get

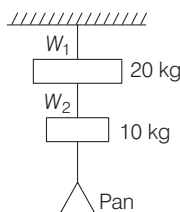
$$\text{Compressive strain} = \frac{1.225 \times 10^5}{2.35 \times 2.0 \times 10^{11}}$$

$$= 2.60 \times 10^{-7}$$

- 04** Wires W_1 and W_2 are made of same material having the breaking stress of $1.25 \times 10^9 \text{ N/m}^2$. W_1 and W_2 have cross-sectional area of $8 \times 10^{-7} \text{ m}^2$ and $4 \times 10^{-7} \text{ m}^2$, respectively. Masses of 20 kg and 10 kg hang

from them as shown in the figure.
The maximum mass that can be placed in the pan without breaking the wires is kg.
(Use, $g = 10 \text{ m/s}^2$)

[2021, 27 Aug Shift-II]

**Ans. (40)**

Given, breaking stress of wires 1 and 2 are $\sigma_1 = \sigma_2 = 1.25 \times 10^9 \text{ N/m}^2$

Cross-sectional area of wire 1,

$$A_1 = 8 \times 10^{-7} \text{ m}^2$$

Cross-sectional area of wire 2,

$$A_2 = 4 \times 10^{-7} \text{ m}^2$$

Mass hanging from first wire, $m_1 = 20 \text{ kg}$

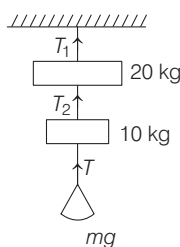
Mass hanging from second wire,

$$m_2 = 10 \text{ kg}$$

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

Let m be the maximum mass placed in pan without breaking the wire.

According to free body diagram of given figure,



$$\text{Since, stress } (\sigma) = \frac{\text{Tension } (T)}{\text{Area } (A)}$$

$$\begin{aligned} \therefore T_1 &= \sigma A_1 \\ &= 1.25 \times 10^9 \times 8 \times 10^{-7} \\ &= 10.00 \times 10^2 = 1000 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{and } T_2 &= \sigma A_2 \\ &= 1.25 \times 10^9 \times 4 \times 10^{-7} \\ &= 5.00 \times 10^2 = 500 \text{ N} \end{aligned}$$

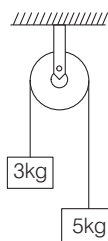
By using concept of tension in string

$$\begin{aligned} T_2 &= (10 + m)g \\ \Rightarrow 500 &= (10 + m)10 \\ \Rightarrow m &= 50 - 10 = 40 \text{ kg} \end{aligned}$$

- 05** Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is

$(24/\pi) \times 10^2 \text{ Nm}^{-2}$. What is the minimum radius of the wire?
(Take, $g = 10 \text{ ms}^{-2}$)

[2021, 26 Aug Shift-II]



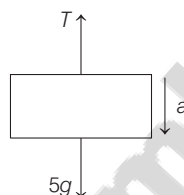
- (a) 125 cm (b) 1250 cm
(c) 12.5 cm (d) 1.25 cm

Ans. (c)

Given, breaking stress of wire,

$$\sigma = \frac{24}{\pi} \times 10^2 \text{ Nm}^{-2}$$

Free body diagram of 5 kg block is given as



where, a is common acceleration.

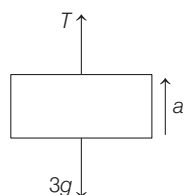
Value of acceleration due to gravity,

$$g = 10 \text{ ms}^{-2}$$

From free body diagram of block of mass 5 kg

$$\begin{aligned} 5g - T &= 5a \\ \Rightarrow 5 \times 10 - T &= 5a \\ \Rightarrow 50 - T &= 5a \quad \dots (i) \end{aligned}$$

Free body diagram of 3 kg block is given as



From free body diagram of block of mass 3 kg,

$$\begin{aligned} T - 3g &= 3a \\ \Rightarrow T - 3 \times 10 &= 3a \\ \Rightarrow T - 30 &= 3a \quad \dots (ii) \end{aligned}$$

Add Eqs. (i) and (ii), we get

$$\begin{aligned} 50 - T + T - 30 &= 5a + 3a \\ \Rightarrow 20 &= 8a \Rightarrow a = 2.5 \text{ ms}^{-2} \end{aligned}$$

Substituting the value of a in Eq. (i), we get

$$50 - T = 5 \times 2.5 \Rightarrow T = 37.5 \text{ N}$$

Let us assume the minimum radius of wire is r .

The breaking stress is expressed as

$$\begin{aligned} \sigma &= \frac{T}{\pi r^2} \\ \frac{24}{\pi} \times 10^2 &= \frac{37.5}{\pi r^2} \\ \Rightarrow r^2 &= \frac{37.5}{24 \times 10^2} \\ &= \frac{1}{64} \\ r &= \frac{1}{8} \text{ m} \\ &= \frac{100}{8} \text{ cm} \\ &= 12.5 \text{ cm} \end{aligned}$$

Thus, the minimum radius of wire should be 12.5 cm.

- 06** Two wires of same length and radius are joined end-to-end and loaded. The Young's moduli of the materials of the two wires are Y_1 and Y_2 . The combination behaves as a single wire, then its Young's modulus is [2021, 25 July Shift-I]

$$\begin{aligned} \text{(a)} Y &= \frac{2Y_1Y_2}{3(Y_1 + Y_2)} & \text{(b)} Y &= \frac{2Y_1Y_2}{Y_1 + Y_2} \\ \text{(c)} Y &= \frac{Y_1Y_2}{2(Y_1 + Y_2)} & \text{(d)} Y &= \frac{Y_1Y_2}{Y_1 + Y_2} \end{aligned}$$

Ans. (b)

Given, length of two wires, $l_1 = l_2 = l$

Radius of two wires, $r_1 = r_2 = r$

Since, Young's moduli are different.

$\therefore$ Change in length of wire 1 and 2 will be Δl_1 and Δl_2 .

As we know that,

$$\begin{aligned} Y &= \frac{Fl}{A\Delta l} \\ \Rightarrow \Delta l &= \frac{Fl}{YA} \end{aligned}$$

where, F = force

and A = area of cross-section.

As the wires are in series,

$\therefore$ Net change in length,

$$\begin{aligned} \Delta l &= \Delta l_1 + \Delta l_2 \\ \frac{F\Delta l}{Y\pi r^2} &= \frac{Fl}{Y_1\pi r^2} + \frac{Fl}{Y_2\pi r^2} \\ \Rightarrow \frac{2}{Y} &= \frac{1}{Y_1} + \frac{1}{Y_2} \Rightarrow \frac{2}{Y} = \frac{Y_2 + Y_1}{Y_1Y_2} \\ \Rightarrow Y &= \frac{2Y_1Y_2}{Y_1 + Y_2} \end{aligned}$$

This is the Young's modulus for the combination of curves.

- 07** The length of a metal wire is l_1 , when the tension in it is T_1 and is l_2 when the tension is T_2 . The natural length of the wire is

[2021, 20 July Shift-II]

(a) $\sqrt{l_1 l_2}$ (b) $\frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$
 (c) $\frac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$ (d) $\frac{l_1 + l_2}{2}$

Ans. (b)

According to question, length of metal wire is l_1 when the tension in it is T_1 and l_2 when the tension is T_2 .

Let initial length of metal wire be l_0 and area of cross-section A .

We know that,

$$\frac{T}{A} = \frac{Y \Delta l}{l} \quad \dots(i)$$

where, T = tension in the metal wire,

A = area of cross-section of wire,

Y = Young's modulus of the material of wire

and Δl = change in length of wire.

Considering Eq. (i), we can write

$$\frac{T_1}{A} = \frac{Y(l_1 - l_0)}{l_0} \quad \dots(ii)$$

and $\frac{T_2}{A} = \frac{Y(l_2 - l_0)}{l_0} \quad \dots(iii)$

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\frac{T_1}{A}}{\frac{T_2}{A}} = \frac{\frac{Y(l_1 - l_0)}{l_0}}{\frac{Y(l_2 - l_0)}{l_0}}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{l_1 - l_0}{l_2 - l_0}$$

$$\Rightarrow T_1 l_2 - T_1 l_0 = T_2 l_1 - T_2 l_0$$

$$\Rightarrow T_1 l_0 - T_2 l_0 = T_1 l_2 - T_2 l_1$$

$$\Rightarrow l_0 = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$$

or $l_0 = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$

- 08** The value of tension in a long thin metal wire has been changed from T_1 to T_2 . The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 , respectively. The actual length of the metal wire is [2021, 20 July Shift-I]

(a) $\frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$ (b) $\frac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$
 (c) $\frac{l_1 + l_2}{2}$ (d) $\sqrt{l_1 l_2}$

Ans. (a)

Suppose, l_0 be the actual length of metal wire and Y be its Young's modulus.

From Hooke's law,

$$Y = \frac{T l_0}{A \Delta l}$$

where, $\Delta l = l - l_0$

$$\Rightarrow Y = \frac{T l_0}{A(l - l_0)}$$

or $l - l_0 = \frac{T l_0}{A Y}$

$$\therefore \frac{l_1 - l_0}{l_2 - l_0} = \frac{T_1 l_0}{T_2 l_0} \times \frac{A Y}{T_2 l_0} = \frac{T_1}{T_2}$$

$$\Rightarrow l_1 T_2 - l_0 T_2 = l_2 T_1 - l_0 T_1$$

$$\Rightarrow l_0 = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1} = \frac{T_1 l_2 - T_2 l_1}{(T_1 - T_2)}$$

- 09** If Y , K and η are the values of

Young's modulus, bulk modulus and modulus of rigidity of any material, respectively. Choose the correct relation for these parameters.

[2021, 24 Feb Shift-I]

(a) $Y = \frac{9K\eta}{2\eta + 3K} \text{ N/m}^2$

(b) $Y = \frac{9K\eta}{3K - \eta} \text{ N/m}^2$

(c) $K = \frac{Y\eta}{9\eta - 3Y} \text{ N/m}^2$

(d) $\eta = \frac{3YK}{9K + Y} \text{ N/m}^2$

Ans. (c)

We know that,

$$Y = 3K(1 - 2\sigma)$$

$$\Rightarrow \sigma = \frac{1}{2} \left(1 - \frac{Y}{3K} \right) \quad \dots(i)$$

Also, $Y = 2\eta(1 + \sigma)$

$$\Rightarrow \sigma = \frac{Y}{2\eta} - 1 \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

$$\left(1 - \frac{Y}{3K} \right) \frac{1}{2} = \frac{Y}{2\eta} - 1$$

On solving, we get

$$K = \frac{\eta Y}{9\eta - 3Y} \text{ N/m}^2$$

- 10** A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F . The elongation, if its length and diameter is doubled and subjected to the same force will be cm. [2021, 24 Feb Shift-II]

Ans. (2)

Let initial length and diameter be l_1 and d_1 , whereas final length and diameter be l_2 and d_2 .

Given, $l_2 = 2l_1$, $d_2 = 2d_1$, $\Delta l_1 = 0.04 \text{ m}$

By using formula of Young's modulus of elasticity,

$$Y = \frac{F \cdot l}{A \Delta l}$$

$$\therefore Y_1 = Y_2$$

$$\Rightarrow \frac{F l_1}{A_1 \times \Delta l_1} = \frac{F l_2}{A_2 \times \Delta l_2}$$

$$\Rightarrow \frac{F l_1}{\pi (d_1/2)^2 \times 0.04} = \frac{F l_2}{\pi (2d_1/2)^2 \Delta l_2}$$

$$\Rightarrow \frac{1}{1/4 \times 0.04} = \frac{2}{\Delta l_2}$$

$$\Rightarrow \Delta l_2 = 0.02 \text{ m} = 2 \text{ cm}$$

- 11** The normal density of a material is p and its bulk modulus of elasticity is K . The magnitude of increase in density of material, when a pressure p is applied uniformly on all sides, will be [2021, 26 Feb Shift-I]

(a) $\frac{pK}{p}$ (b) $\frac{p\rho}{K}$ (c) $\frac{K}{p\rho}$ (d) $\frac{pK}{p}$

Ans. (b)

Given, density of material = p

Bulk modulus of elasticity = K

and applied pressure = p

Let change in volume and density be ΔV and Δp respectively and initial volume and density be V and p .

Since, $K = \frac{p}{-\frac{\Delta V}{V}} \quad \dots(i)$

and density (p) = $\frac{\text{mass (m)}}{\text{volume (V)}}$

$$\therefore \frac{\Delta p}{p} = -\frac{\Delta V}{V}$$

Substituting it in Eq. (i), we get

$$\frac{-\Delta V}{V} = \frac{p}{K} = \frac{\Delta p}{p}$$

$$\therefore \Delta p = \frac{p\rho}{K}$$

- 12** The length of metallic wire is l_1 when tension in it is T_1 . It is l_2 when the tension is T_2 . The original length of the wire will be [2021, 26 Feb Shift-II]

(a) $\frac{l_1 + l_2}{2}$ (b) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$
 (c) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$ (d) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$

Ans. (c)

Let l_0 be the original length, A be the area of cross-section, α be the coefficient of linear expansion,

Δl be the change in length and Y be the Young's modulus of elasticity.

$$\text{As, } l_1 = l_0(1 + \alpha \Delta T)$$

$$\Rightarrow l_1 - l_0 = l_0 \alpha \Delta T \Rightarrow \Delta l = l_0 \alpha \Delta T$$

$$\text{Initially, } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\Delta l/l_0}$$

$$\Rightarrow Y = \frac{T_1/A}{(l_1 - l_0)/l_0} \quad \dots (i)$$

$$\text{Finally, } Y = \frac{T_2/A}{(l_2 - l_0)/l_0} \quad \dots (ii)$$

Now, from Eqs. (i) and (ii), we get

$$\frac{T_1/A}{(l_1 - l_0)/l_0} = \frac{T_2/A}{(l_2 - l_0)/l_0}$$

$$\Rightarrow \frac{T_1}{l_1 - l_0} = \frac{T_2}{l_2 - l_0}$$

$$\Rightarrow T_1 l_2 - T_1 l_0 = T_2 l_1 - T_2 l_0$$

$$\Rightarrow T_1 l_2 - T_2 l_1 = (T_1 - T_2) l_0$$

$$\Rightarrow l_0 = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2} \text{ or } l_0 = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

- 13** An object is located at 2 km beneath the surface of the water. If the fractional compression $\Delta V/V$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be

(Take, density of water is 1000 kg m^{-3} and $g = 9.81 \text{ ms}^{-2}$)

[2021, 17 March Shift-II]

- (a) $1.96 \times 10^7 \text{ Nm}^{-2}$ (b) $1.44 \times 10^7 \text{ Nm}^{-2}$
(c) $2.26 \times 10^9 \text{ Nm}^{-2}$ (d) $1.44 \times 10^9 \text{ Nm}^{-2}$

Ans. (d)

Given, The volumetric strain is $\frac{\Delta V}{V} = 1.36\%$

The depth beneath the water surface, $h = 2 \text{ km}$

The pressure inside the water surface up to 2 km, $p = \rho gh$

Substituting the values in the above equation, we get

$$p = 1000 \times 9.81 \times 2000$$

$$p = 19.62 \times 10^6 \text{ Pa}$$

The bulk modulus of the object, $\beta = \frac{p}{\frac{\Delta V}{V}}$

Substituting the values in the above equation, we get

$$\beta = \frac{19.62 \times 10^6}{\frac{1.36}{100}} = 1.44 \times 10^9 \text{ N/m}^2$$

Hence, the ratio of the hydraulic stress to the corresponding hydraulic strain will be $1.44 \times 10^9 \text{ N/m}^2$.

- 14** Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of $a:b$. Then, a/b can be expressed as $1/x$, where x is

[2021, 18 March Shift-I]

Ans. (32)

Given,

The change in the length of the wire A, $\Delta L_A = 2 \text{ mm} = 0.002 \text{ m}$

The change in the length of the wire B, $\Delta L_B = 4 \text{ mm} = 0.004 \text{ m}$

The force subjected to the wire, $F = 2 \text{ N}$

The radius of the wire B is 4 times the radius of the wire A, i.e., $\frac{r_B}{r_A} = \frac{4}{1}$

Since, the wire is made of the same material, so the Young's modulus of the elasticity of the wire is same.

$$\Rightarrow Y_A = Y_B$$

Using Hooke's law,

Stress = Y (Strain)

$$\frac{F}{A} = Y \left(\frac{\Delta L}{L} \right)$$

$$\Rightarrow L = \frac{Y \Delta L A}{F}$$

$$\Rightarrow \frac{L_A}{L_B} = \frac{Y_A}{Y_B} \times \frac{\Delta L_A}{\Delta L_B} \times \frac{A_A}{A_B} \times \frac{F_B}{F_A}$$

$$\Rightarrow \frac{L_A}{L_B} = \frac{Y_A}{Y_B} \times \frac{0.002}{0.004} \times \frac{\pi r_A^2}{\pi r_B^2} \times \frac{2}{2}$$

$$\Rightarrow \frac{L_A}{L_B} = \frac{0.002}{0.004} \times \frac{r_A^2}{16r_A^2}$$

$$\Rightarrow \frac{L_A}{L_B} = \frac{a}{b} = \frac{1}{32}$$

Comparing this equation with $1/x$, we get the value of the x is 32.

- 15** A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to (Take bulk modulus of metal, $B = 8 \times 10^{10} \text{ Pa}$) [2020, 4 Sep Shift-II]
(a) 1.67 (b) 0.6 (c) 20 (d) 5

Ans. (a)

$$\text{Bulk modulus, } B = \frac{\Delta p}{\left(-\frac{\Delta V}{V} \right)}$$

$$\Rightarrow \frac{\Delta V}{V} = -\frac{\Delta p}{B}$$

$$\frac{\Delta V}{V} \times 100\% = -\frac{\Delta p}{B} \times 100\%$$

$$= \frac{-4 \times 10^9}{8 \times 10^{10}} \times 100\%$$

$$= -\frac{1}{2} \times 10\%$$

$$= -0.5 \times 10\% = -5\%$$

Now, $V = l^3$ (for cube)

$$\Rightarrow l = (V)^{1/3}$$

$$\Rightarrow \frac{\Delta l}{l} \times 100\% = \frac{1}{3} \left(\frac{\Delta V}{V} \times 100\% \right)$$

$$= \frac{1}{3} (-5\%) = -1.67\%$$

So, length of cube will be decreased by 1.67%.

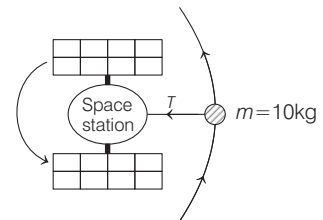
Hence, option (a) is correct.

- 16** A body of mass $m = 10 \text{ kg}$ is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (breaking stress of wire $= 4.8 \times 10^7 \text{ Nm}^{-2}$ and area of cross-section of the wire $= 10^{-2} \text{ cm}^2$) is

[2020, 9 Jan Shift-I]

Ans. (4)

Centripetal force is provided by the tension in wire.



$$\text{So, } T = m\omega^2 l$$

$$\text{Stress in wire, } \sigma = \frac{T}{A} = \frac{m\omega^2 l}{A} \quad \dots (i)$$

$$\text{Here, } \sigma_{\max} = 4.8 \times 10^7 \text{ Nm}^{-2},$$

$$A = 10^{-2} \text{ cm}^2 = 10^{-2} \times 10^{-4} \text{ m}^2 = 10^{-6} \text{ m}^2,$$

$$m = 10 \text{ kg and } l = 0.3 \text{ m}$$

If maximum angular speed of rotation is $\omega_{\max}$, then from Eq. (i), we have

$$\omega_{\max}^2 = \frac{\sigma_{\max} A}{ml} = \frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3} = 16$$

$$\text{or } \omega_{\max} = 4 \text{ rad s}^{-1}$$

- 17** A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi \text{ ms}^{-2}$, what will be the tensile stress that would be developed in the wire?

[2019, 8 April Shift-I]

- (a) $6.2 \times 10^6 \text{ Nm}^{-2}$ (b) $5.2 \times 10^6 \text{ Nm}^{-2}$
(c) $3.1 \times 10^6 \text{ Nm}^{-2}$ (d) $4.8 \times 10^6 \text{ Nm}^{-2}$

Ans. (c)

Given, radius of wire, $r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$
Weight of load, $m = 4 \text{ kg}$, $g = 3.1\pi \text{ ms}^{-2}$

$$\therefore \text{Tensile stress} = \frac{\text{Force (F)}}{\text{Area (A)}} = \frac{mg}{\pi r^2}$$

$$= \frac{4 \times 3.1 \times \pi}{\pi \times (2 \times 10^{-3})^2} = 3.1 \times 10^6 \text{ Nm}^{-2}$$

- 18** Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to

[2019, 8 April Shift-II]

- (a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

Ans. (d)

When a wire is stretched, then change in length of wire is

$$\Delta l = \frac{Fl}{\pi r^2 Y}, \text{ where } Y \text{ is its Young's modulus.}$$

Here, for wires A and B,

$$l_A = 2 \text{ m}, l_B = 1.5 \text{ m},$$

$$\frac{Y_A}{Y_B} = \frac{7}{4}, r_B = 2 \text{ mm} = 2 \times 10^{-3} \text{ m and } \frac{F_A}{F_B} = 1$$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow r_A^2 = \frac{F_A}{F_B} \cdot \frac{l_A}{l_B} \cdot \frac{Y_B}{Y_A} \cdot r_B^2$$

$$= 1 \times \frac{2}{1.5} \times \frac{4}{7} \times 4 \times 10^{-6} \text{ m}$$

$$= 3.04 \times 10^{-6} \text{ m}$$

$$\text{or } r_A = 1.7 \times 10^{-3} \text{ m or } r_A = 1.7 \text{ mm}$$

- 19** In an experiment, brass and steel wires of length 1 m each with areas of cross-section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation.

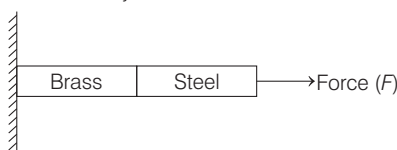
The stress requires to produce a net elongation of 0.2 mm is
[Take, the Young's modulus for steel and brass are respectively $120 \times 10^9 \text{ N/m}^2$ and $60 \times 10^9 \text{ N/m}^2$]

[2019, 10 April Shift-II]

- (a) $12 \times 10^6 \text{ N/m}^2$ (b) $0.2 \times 10^6 \text{ N/m}^2$
(c) $1.8 \times 10^6 \text{ N/m}^2$ (d) $4.0 \times 10^6 \text{ N/m}^2$

Ans. (*)

In given experiment, a composite wire is stretched by a force F.



Net elongation in the wire = elongation in brass wire + elongation in steel wire ... (i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied,

$$Y = \frac{Fl}{A\Delta l}$$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from relation (i), we have

$$\Delta l_{\text{net}} = \Delta l_{\text{brass}} + \Delta l_{\text{steel}}$$

$$\Rightarrow \Delta l_{\text{net}} = \left(\frac{Fl}{AY} \right)_{\text{brass}} + \left(\frac{Fl}{AY} \right)_{\text{steel}}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{\text{net}} = \frac{F}{A} \left(\frac{l}{Y_{\text{brass}}} + \frac{l}{Y_{\text{steel}}} \right)$$

$$\text{Here, } \Delta l_{\text{net}} = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m and } l = 1 \text{ m}$$

$$Y_{\text{brass}} = 60 \times 10^9 \text{ Nm}^{-2},$$

$$Y_{\text{steel}} = 120 \times 10^9 \text{ Nm}^{-2}$$

On putting the values, we have

$$0.2 \times 10^{-3} = \frac{F}{A} \left(\frac{1}{60 \times 10^9} + \frac{1}{120 \times 10^9} \right)$$

$$\Rightarrow \text{Stress} = \frac{F}{A} = 8 \times 10^6 \text{ Nm}^{-2}$$

- 20** A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston

to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r} \right)$ is

[JEE Main 2018]

- (a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$ (c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

Ans. (c)

$$\therefore \text{Bulk modulus, } K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$= \frac{\Delta p}{\frac{\Delta V}{V}}$$

$$\Rightarrow K = \frac{mg}{a \left(\frac{3\Delta r}{r} \right)} \left[\because V = \frac{4}{3} \pi r^3, \text{ so } \frac{\Delta V}{V} = \frac{3\Delta r}{r} \right]$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{mg}{3aK}$$

- 21** A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

[JEE Main 2017 (Offline)]

- (a) $\frac{1}{9}$ (b) 81 (c) $\frac{1}{81}$ (d) 9

Ans. (d)

$$\therefore \text{Stress} = \frac{\text{Weight}}{\text{Area}}$$

Volume will become (9^3) times.

So weight = volume $\times$ density $\times g$ will also become (9^3) times.

Area of cross-section will become $(9)^2$ times.

$$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9 \left(\frac{W_0}{A_0} \right)$$

Hence, the stress increases by a factor of 9.

- 22** A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by $t^\circ \text{C}$, the linear stress developed in it is

[AIEEE 2011]

- (a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$
(c) $Y\alpha t$ (d) $\frac{1}{Y\alpha t}$

Ans. (c)

As change in length,

$$\Delta L = \alpha L \Delta T = \frac{FL}{AY}$$

$$\Rightarrow \text{Stress} = \frac{F}{A} = Y\alpha\Delta T$$

- 23.** Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F , how much force is needed to stretch wire 2 by the same amount? [AIEEE 2009]

- (a) F (b) $4F$
(c) $6F$ (d) $9F$

Ans. (d)

As volume is same.

$$\therefore A_1 l_1 = A_2 l_2$$

$$\Rightarrow l_2 = \frac{A_1 l_1}{A_2} = \frac{A \times l_1}{3A} = \frac{l_1}{3}$$

$$\Rightarrow \frac{l_1}{l_2} = 3$$

$$\therefore \Delta x_1 = \frac{F_1}{AY} \times l_1 \quad \dots(i)$$

$$\text{and } \Delta x_2 = \frac{F_2}{3AY} l_2 \quad \dots(ii)$$

$$\text{Here, } \Delta x_1 = \Delta x_2$$

$$\Rightarrow \frac{F_2}{3AY} l_2 = \frac{F_1}{AY} l_1$$

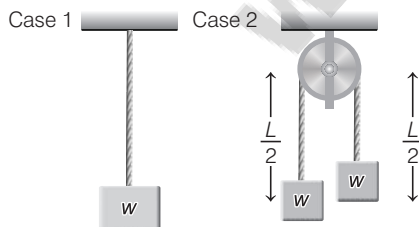
$$\Rightarrow F_2 = 3F_1 \times \frac{l_1}{l_2} = 3F_1 \times 3 = 9F$$

- 24** A wire elongates by l mm when a load w is hanged from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) [AIEEE 2006]

- (a) l (b) $2l$
(c) zero (d) $\frac{l}{2}$

Ans. (a)

Let us consider the length of wire as L and cross-sectional area A , the material of wire has Young's modulus as Y .



$$\text{Then, for 1st case, } Y = \frac{w/A}{l/L}$$

$$\text{For 2nd case, } Y = \frac{w/A}{2l'/L} \Rightarrow l' = \frac{l}{2}$$

$$\text{So, total elongation of both sides} = 2l' = l$$

TOPIC 2

Stress-Strain Curve, Thermal Stress and Elastic PE

- 25** A steel rod with $Y = 2.0 \times 10^{11} \text{ Nm}^{-2}$ and $\alpha = 10^{-5} \text{ }^\circ\text{C}^{-1}$ of length 4 m and area of cross-section 10 cm^2 is heated from 0°C to 400°C without being allowed to extend. The tension produced in the rod is $x \times 10^5 \text{ N}$, where the value of x is

[2021, 1 Sep Shift-II]

Ans. (8)

Given, the Young's modulus of the steel rod, $Y = 2 \times 10^{11} \text{ Pa}$

Thermal coefficient of the steel rod, $\alpha = 10^{-5} \text{ }^\circ\text{C}^{-1}$

The length of the steel rod, $l = 4 \text{ m}$

The area of the cross-section, $A = 10 \text{ cm}^2$

The temperature difference,

$$\Delta T = 400^\circ\text{C}$$

As we know that,

$$\text{Thermal strain} = \alpha \Delta T$$

Using the Hooke's law

Young's modulus

$$(Y) = \frac{\text{Thermal stress}}{\text{Thermal strain}} = \frac{F/A}{\alpha \Delta T}$$

$$\text{Thermal stress, } F = YA \alpha \Delta T$$

Substitute the values in the above equation, we get

$$F = 2 \times 10^{11} \times 10 \times 10^{-4} \times 10^{-5} \times (400) = 8 \times 10^5 \text{ N}$$

Comparing with, $F = x \times 10^5 \text{ N}$

The value of the $x = 8$.

- 26** A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross-section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is m/s.

(Take, Young's modulus of rubber $= 0.5 \times 10^9 \text{ N/m}^2$) [27 July 2021 Shift-I]

Ans. (20)

Given, mass of stone, $m = 20 \text{ g} = 0.020 \text{ kg}$

Length of catapult, $l = 0.1 \text{ m}$

Area of cross-section, $A = 10^{-6} \text{ m}^2$

Young's modulus of rubber,

$$Y = 0.5 \times 10^9 \text{ N/m}^2$$

During the projection of stone from catapult,

Work done in stretching a catapult = Kinetic energy of stone after releasing from catapult

$$\Rightarrow \frac{1}{2} \times Y \times (\text{Strain})^2 \times \text{Volume} = \frac{1}{2} mv^2$$

$$\Rightarrow \frac{1}{2} \times 0.5 \times 10^9 \times 16 \times 10^{-2} \times 10^{-7}$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times v^2$$

$$\Rightarrow v^2 = 400$$

$$\Rightarrow v = 20 \text{ ms}^{-1}$$

- 27** The area of cross-section of a railway track is 0.01 m^2 . The temperature variation is 10°C . Coefficient of linear expansion of material of track is $10^{-5} \text{ }^\circ\text{C}^{-1}$. The energy stored per metre in the track is J/m. (Take, Young's modulus of material of track is 10^{11} Nm^{-2})

[2021, 22 July Shift-II]

Ans. (5)

Given, area of cross-section, $A = 0.01 \text{ m}^2$

Change of temperature, $\Delta T = 10^\circ\text{C}$

Coefficient of linear expansion, $\alpha = 10^{-5} \text{ }^\circ\text{C}^{-1}$

Young's modulus, $Y = 10^{11} \text{ Nm}^{-2}$

Energy stored per unit length

$$U = \text{Energy stored per unit volume} \times \text{Area}$$

$$= \frac{1}{2} Y (\text{strain})^2 \times A$$

$$U = \frac{1}{2} Y (\alpha \Delta T)^2 \times A \left[\because \frac{\Delta l}{l} = \alpha \Delta T \right]$$

$$= \frac{1}{2} \times 10^{11} (10^{-5} \times 10)^2 \times 0.01$$

$$= \frac{1}{2} \times 10^{11} \times 10^{-8} \times 0.01 = 5 \text{ J/m}$$

- 28** Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) When a rod lying freely is heated, no thermal stress is developed in it.

Reason (R) On heating, the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below [2021, 25 Feb Shift-I]

- (a) Both A and R are true but R is not the correct explanation of A.
 (b) A is false but R is true.
 (c) A is true but R is false.
 (d) Both A and R are true and R is the correct explanation of A.

Ans. (a)

Thermal stress is defined as the stress, experienced by any rod on heating between two fixed rigid supports. On heating, the size of the rod increases but, if the two ends are free, rod will not experience any stress. i.e., there is no thermal stress will be produced in it. Hence, option (a) is the correct.

- 29** Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is

[2020, 9 Jan Shift-II]

- (a) $\sqrt{2} : 1$ (b) $1 : \sqrt{2}$
 (c) $2 : 1$ (d) $1 : 2$

Ans. (a)

Elastic potential energy stored in a loaded wire,

$$U = \frac{1}{2} (\text{Stress} \times \text{Strain} \times \text{Volume})$$

$\therefore$ Energy stored per unit volume,

$$u = \frac{U}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \left(\frac{F}{A} \right) \times \frac{1}{Y}$$

Here, both wires are of same material and under same load, so the ratio of stored energies per unit volume, for both the wires will be

$$\frac{u_A}{u_B} = \frac{\frac{1}{2Y} \cdot \frac{F^2}{A_A^2}}{\frac{1}{2Y} \cdot \frac{F^2}{A_B^2}} = \frac{A_B^2}{A_A^2}$$

$$\Rightarrow \frac{u_A}{u_B} = \frac{d_B^4}{d_A^4} \quad \left(\because A = \pi \frac{d^2}{4} \right)$$

Here, $\frac{u_A}{u_B} = \frac{1}{4}$

So, $\frac{d_B^4}{d_A^4} = \frac{1}{4}$ or $\frac{d_B}{d_A} = \frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{d_A}{d_B} = \sqrt{2} : 1$$

- 30** A rod of length L at room temperature and uniform area of cross-section A , is made of a metal

having coefficient of linear expansion $\alpha / ^\circ\text{C}$. It is observed that an external compressive force F , is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is

[2019, 9 Jan Shift-I]

- (a) $\frac{F}{2A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$
 (c) $\frac{2F}{A\alpha\Delta T}$ (d) $\frac{F}{A\alpha\Delta T}$

Ans. (d)

If a rod of length L and coefficient of linear expansion $\alpha / ^\circ\text{C}$, then with the rise in temperature by ΔT K, its change in length is given as,

$$\Delta L = L \alpha \Delta T$$

$$\Rightarrow \frac{\Delta L}{L} = \alpha \Delta T \quad \dots(i)$$

Also, when a rod is subjected to some compressive force (F), then its Young's

modulus is given as $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$$\frac{\Delta L}{L} = \frac{F}{YA} \quad \dots(ii)$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha \Delta T = \frac{F}{YA} \Rightarrow Y = \frac{F}{A\alpha\Delta T}$$

- 31** The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit?

[2019, 10 April Shift-II]

- (a) 0.90 mm (b) 1.00 mm
 (c) 1.16 mm (d) 1.36 mm

Ans. (c)

Let $d_{\min}$ = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{\min}^2} \quad \left[\because A = \frac{\pi d^2}{4} \right]$$

For stress not to exceed elastic limit, we have

$$\sigma \leq 379 \text{ MPa}$$

$$\Rightarrow \frac{4F}{\pi d_{\min}^2} \leq 379 \times 10^6$$

$$\text{Here, } F = 400 \text{ N}$$

$$\therefore d_{\min}^2 = \frac{1600}{\pi \times 379 \times 10^6}$$

$$\Rightarrow d_{\min} = 1.16 \times 10^{-3} \text{ m} = 1.16 \text{ mm}$$

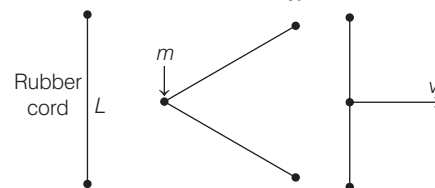
- 32** A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1} . Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to

[2019, 8 April Shift-I]

- (a) 10^6 Nm^{-2}
 (b) 10^4 Nm^{-2}
 (c) 10^8 Nm^{-2}
 (d) 10^3 Nm^{-2}

Ans. (a)

When rubber cord is stretched, then it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.



So, potential energy of stretched cord = kinetic energy of stone

$$\Rightarrow \frac{1}{2} Y \left(\frac{\Delta L}{L} \right)^2 A \cdot L = \frac{1}{2} mv^2$$

Here, $\Delta L = 20 \text{ cm} = 0.2 \text{ m}$,

$L = 42 \text{ cm} = 0.42 \text{ m}$,

$v = 20 \text{ ms}^{-1}$,

$m = 0.02 \text{ kg}$,

$d = 6 \text{ mm}$

$= 6 \times 10^{-3} \text{ m}$

$$\therefore A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2$$

$$= \pi \left(\frac{6 \times 10^{-3}}{2} \right)^2$$

$$= \pi (3 \times 10^{-3})^2$$

$$= 9\pi \times 10^{-6} \text{ m}^2$$

On substituting values, we get

$$Y = \frac{mv^2 L}{A(\Delta L)^2}$$

$$= \frac{0.02 \times (20)^2 \times 0.42}{9\pi \times 10^{-6} \times (0.2)^2}$$

$$\approx 3.0 \times 10^6 \text{ Nm}^{-2}$$

So, the closest value of Young's modulus is 10^6 Nm^{-2} .

- 33** The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is $2 \times 10^{11} \text{ Nm}^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5} \text{ K}^{-1}$) [JEE Main 2014]

- (a) $2.2 \times 10^8 \text{ Pa}$
 (b) $2.2 \times 10^9 \text{ Pa}$
 (c) $2.2 \times 10^7 \text{ Pa}$
 (d) $2.2 \times 10^6 \text{ Pa}$

Key Idea If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

Ans. (a)

By Hooke's law,

$$\text{Young's modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$\text{So, } Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire (CL) = 10 cm
 Temperature (t) = 100°C

As length is constant.

$$\therefore \text{Strain} = \frac{\Delta L}{L} \propto \Delta\theta \quad [\text{as } \Delta L = (L\Delta\theta)]$$

Now, pressure = stress = $Y \times \text{strain}$

$$\begin{aligned} [\text{given, } Y &= 2 \times 10^{11} \text{ N/m}^2 \text{ and} \\ \alpha &= 1.1 \times 10^{-5} \text{ K}^{-1}] \\ &= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100 \\ &= 2.2 \times 10^8 \text{ Pa} \end{aligned}$$

- 34** If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is [AIEEE 2005]

- (a) $2S^2Y$ (b) $\frac{S^2}{2Y}$
 (c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

Ans. (b)

Energy stored in wire

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$\text{and Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{S}{Y}$$

$$\therefore \frac{\text{Energy stored in wire}}{\text{Volume}}$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

- 35** A wire fixed at the upper end stretches by length l by applying a force F . The work done in stretching is [AIEEE 2004]

- (a) $\frac{F}{2l}$ (b) F/l
 (c) $2Fl$ (d) $\frac{Fl}{2}$

Ans. (d)

As work done in stretching the wire

= Potential energy stored

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL = \frac{1}{2} Fl$$

- 36** A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then, the elastic energy stored in the wire is [AIEEE 2003]

- (a) 0.2 J (b) 10 J
 (c) 20 J (d) 0.1 J

Ans. (d)

Elastic energy stored in the wire is

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{L} \times AL = \frac{1}{2} F \Delta l$$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1 \text{ J}$$

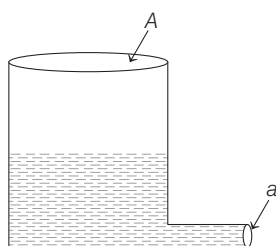
Mechanical Properties of Fluids

TOPIC 1

Pressure and Pascal Law

- 01** A light cylindrical vessel is kept on a horizontal surface. Area of base is A . A hole of cross-sectional area a is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is ($a \ll A$).

[2021, 27 July Shift-I]



- (a) $\frac{A}{2a}$ (b) None of these
(c) $\frac{2a}{A}$ (d) $\frac{a}{A}$

Ans. (c)

$\therefore$ Force exerted by leaving water on the container,

$$F = \left(\frac{dm}{dt} \right) v = (\rho a v) v$$

$$F = \rho a v^2$$

Also, by conservation of energy,

$$v = \sqrt{2gh} \Rightarrow F = \rho a (\sqrt{2gh})^2$$

$$\Rightarrow F = \rho a (2gh)$$

$\therefore$ For no sliding of vessel,

$$\mu mg \geq F \Rightarrow \mu mg \geq \rho a (2gh)$$

$$\Rightarrow \mu p(Ah)g \geq \rho a 2gh$$

$$\Rightarrow \mu_{\min} = \frac{2a}{A}$$

- 02** The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be (Assume that atmospheric pressure is 1×10^5 Pa, density of water is 10^3 kg m^{-3} , $g = 10 \text{ ms}^{-2}$) [2021, 16 March Shift-I]

- (a) $\frac{200}{3} \%$ (b) $\frac{200}{5} \%$
(c) $\frac{5}{200} \%$ (d) $\frac{3}{200} \%$

Ans. (a)

Given, pressure on submarine at a certain depth,

$$p_i = 3 \times 10^5 \text{ Pa}$$

Since, we know that at a certain depth or below the surface of a liquid of density ρ , pressure is given as

$$p_i = p_0 + \rho gh$$

$$\Rightarrow 3 \times 10^5 = p_0 + \rho gh$$

$$\Rightarrow 3 \times 10^5 = 1 \times 10^5 + \rho gh \quad (\because p_0 = 1 \times 10^5 \text{ Pa})$$

$$\Rightarrow 3 \times 10^5 - 1 \times 10^5 = \rho gh$$

$$\Rightarrow 2 \times 10^5 = \rho gh \quad \dots (i)$$

When the depth is doubled, then final pressure will be

$$\begin{aligned} p_f &= p_0 + \rho g 2h = 1 \times 10^5 + 2(\rho gh) \\ &= 1 \times 10^5 + 2(2 \times 10^5) \quad [\text{using Eq. (i)}] \\ &= 1 \times 10^5 + 4 \times 10^5 \end{aligned}$$

$$\Rightarrow p_f = 5 \times 10^5 \text{ Pa}$$

$\therefore$ Percentage increase in pressure

$$\begin{aligned} &= \frac{p_f - p_i}{p_i} \times 100 = \frac{(5 - 3) \times 10^5}{3 \times 10^5} \times 100 \\ &= \frac{200}{3} \% \end{aligned}$$

- 03** A hydraulic press can lift 100 kg when a mass m is placed on the smaller piston. It can lift kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass m on the smaller piston. [2021, 24 Feb Shift-I]

Ans. (25600)

According to Pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\text{Initially, } \frac{100g}{A_1} = \frac{mg}{A_2} \quad \dots (i)$$

$$\text{Finally, } \frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)} \quad \dots (ii)$$

On dividing Eqs. (i) by (ii), we get

$$\frac{100 \times 16}{M} = \frac{1}{16}$$

$$\therefore M = 25600 \text{ kg}$$

- 04** An air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2} . The density of water is 1 g/cm^3 and water offers negligible drag force on the bubble. The mass of the bubble is (Take, $g = 980 \text{ cm/s}^2$)

[2020, 4 Sep Shift-I]

- (a) 1.52 g (b) 4.51 g (c) 4.15 g (d) 3.15 g

Ans. (c)

From free body diagram,

$$B - mg = ma \quad (\text{here, } B = \text{buoyant force})$$

$$\rho_w Vg - mg = ma \quad (\text{here, } V = \text{volume})$$

$$\rho_w Vg = mg + ma$$

$$\rho_w Vg = m(g + a)$$

$$\Rightarrow m = \frac{\rho_w Vg}{g + a} \quad \dots (i)$$

Given, $\rho_w = 1 \text{ g/cm}^3$, $g = 980 \text{ cm/s}^2$,
 $a = 9.8 \text{ cm/s}^2$ and $r = 1 \text{ cm}$
 Substituting all values in eq. (i), we get

$$m = \frac{(1) \times \frac{4}{3} \pi r^3 \times 980}{980 + 9.8} = \frac{\frac{4}{3} \times \frac{22}{7} \times (1)^3 \times 980}{989.8}$$

$$= \frac{4 \times 22 \times 980}{3 \times 7 \times 989.8} = 4.148 \text{ g} \approx 4.15 \text{ g}$$

Hence, correct option is (c).

- 05** A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r . If the specific gravity of the shell material is $\frac{27}{8}$

with respect to water, value of r is

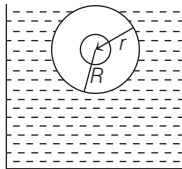
[2020, 5 Sep Shift-I]

- (a) $\frac{8}{9}R$ (b) $\frac{1}{3}R$ (c) $\frac{2}{3}R$ (d) $\frac{4}{9}R$

Ans. (a)

Relative density or specific gravity,

$$\rho_r = \frac{\text{Density of object}}{\text{Density of water}} = \frac{\rho_o}{\rho_w} = \frac{27}{8}$$



In equilibrium,

$$\frac{4}{3} \pi R^3 \rho_w g = \frac{4}{3} \pi (R^3 - r^3) \rho_o \cdot g$$

$$\Rightarrow R^3 = (R^3 - r^3) \left(\frac{27}{8} \right)$$

$$\Rightarrow 8R^3 = 27R^3 - 27r^3$$

$$\Rightarrow 27r^3 = 27R^3 - 8R^3 \Rightarrow 27r^3 = 19R^3$$

$$\Rightarrow \frac{r^3}{R^3} = \frac{19}{27} \Rightarrow r = \left(\frac{19}{27} \right)^{1/3} R$$

$$= \frac{2.66}{3} R = 0.88R \Rightarrow r \approx \frac{8}{9} R$$

Hence, correct option is (a).

- 06** Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2} \right)$

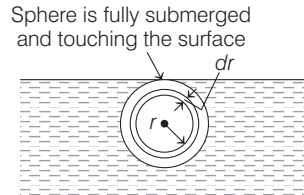
$0 < r \leq R$. The min. density of a liquid in which it will float is

[2020, 8 Jan Shift-I]

- (a) $\frac{\rho_0}{3}$ (b) $\frac{2\rho_0}{5}$ (c) $\frac{2\rho_0}{3}$ (d) $\frac{\rho_0}{5}$

Ans. (b)

If density of liquid is minimum, then its buoyant force is also minimum. In this case, floating pattern of sphere is as shown below.



So, weight of sphere = upthrust or weight of fluid displaced.

$$\Rightarrow M_{\text{sphere}} \times g = V_{\text{sphere}} \times \rho_l \times g$$

$$\Rightarrow \int_0^R \rho(r) \cdot 4\pi r^2 dr = \frac{4}{3} \pi R^3 \cdot \rho_l$$

$$\Rightarrow \int_0^R \rho_0 \left(1 - \frac{r^2}{R^2} \right) \cdot 4\pi r^2 dr = \frac{4}{3} \pi R^3 \cdot \rho_l$$

$$\Rightarrow \rho_0 \times 4\pi \int_0^R \left(r^2 - \frac{r^4}{R^2} \right) dr = \frac{4}{3} \pi R^3 \cdot \rho_l$$

$$\Rightarrow \rho_0 \times 4\pi \left(\frac{R^3}{3} - \frac{R^5}{5} \right) = \frac{4}{3} \pi R^3 \cdot \rho_l$$

$$\Rightarrow \rho_l = \frac{2}{5} \rho_0$$

- 07** A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C , the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4^\circ\text{C}$, relative to the density at $T = 0^\circ\text{C}$ is close to

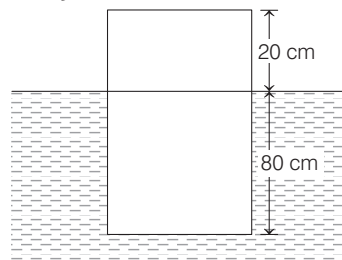
[2020, 8 Jan Shift-I]

- (a) 1.26 (b) 1.03 (c) 1.01 (d) 1.04

Ans. (c)

For a floating body, weight of body = upthrust or buoyant force = weight of water displaced.

Initially at 0°C , cylinder floats as shown in the figure.



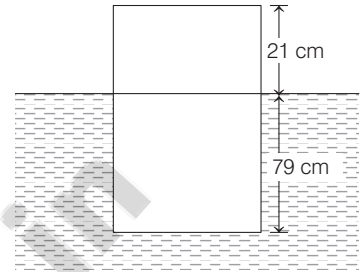
Here, weight of cylinder = weight of water displaced by cylinder of 80 cm length.

$$\Rightarrow mg = \rho_0 \cdot g \cdot 80 A \quad \dots(i)$$

where, ρ_0 = density of water at 0°C

and A = area of cross-section of cylinder.

When temperature is raised to 4°C , cylinder floats as shown in the figure.



In above case, weight of cylinder = weight of water displaced by cylinder of 79 cm length.

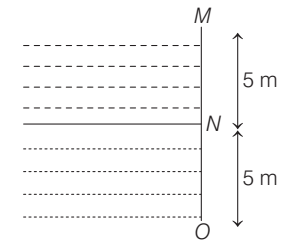
$$\Rightarrow mg = \rho_4 \cdot g \cdot 79 A \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$80 \rho_0 = \rho_4 \cdot 79$$

$$\Rightarrow \frac{\rho_4}{\rho_0} = \frac{80}{79} = 1.01$$

08



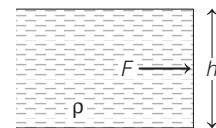
Two liquids of densities ρ_1 and ρ_2 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (assume that the liquids are not mixing)

[2020, 8 Jan Shift-II]

- (a) 1/2 (b) 2/3
(c) 1/4 (d) 1/3

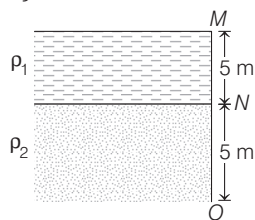
Ans. (c)

Force on a vertical surface of area A and dipped upto height h in a fluid of density ρ is



$$\text{Force, } F = \frac{1}{2} (\rho g h \times A)$$

So, in given case,



$$\text{Force on portion } MN = \frac{\rho_1 gh}{2} \times A$$

Force on portion NO = Force due to pressure of liquid in MN portion + Force due to pressure of liquid in NO portion

$$\begin{aligned} &= \rho_1 ghA + \frac{1}{2}(\rho_2 ghA) \\ &= \rho_1 ghA + \frac{1}{2} \times 2\rho_1 \times ghA \quad (\because \rho_2 = 2\rho_1) \\ &= 2\rho_1 ghA \end{aligned}$$

$$\text{Required ratio, } \frac{F_{MN}}{F_{NO}} = \frac{\frac{1}{2}(\rho_1 ghA)}{2\rho_1 ghA} = \frac{1}{4}$$

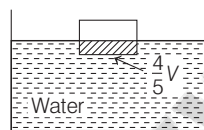
- 09** A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume

submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is [2019, 9 April Shift-II]

- (a) 0.6 (b) 0.8 (c) 0.7 (d) 0.5

Ans. (a)

For a floating body, upthrust = weight of the part of object, i.e. submerged in the fluid. In first situation,

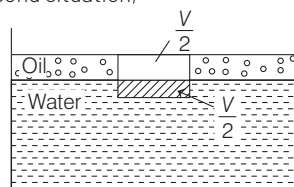


So, weight of block of volume V = weight of water of volume $\frac{4}{5}V \Rightarrow V\rho_b g = \frac{4}{5}V\rho_w g$

where, ρ_b = density of block and ρ_w = density of water.

$$\Rightarrow \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \dots (i)$$

In second situation,



So, weight of block of volume V = weight of oil of volume $\frac{V}{2}$ + weight of water of volume $\frac{V}{2}$.

$$\Rightarrow V\rho_b g = \frac{V}{2}\rho_o g + \frac{V}{2}\rho_w g$$

where, ρ_o = density of oil.

$$\Rightarrow 2\rho_b = \rho_o + \rho_w \Rightarrow \frac{2\rho_b}{\rho_w} = \frac{\rho_o}{\rho_w} + 1$$

$$\Rightarrow 2 \times \frac{4}{5} = \frac{\rho_o}{\rho_w} + 1 \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \rho_o / \rho_w = 8/5 - 1 = 3/5 = 0.6$$

- 10** A submarine experience a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2 , it experiences a pressure of 8.08×10^6 Pa, then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m³ and acceleration due to gravity = 10 ms^{-2})

[2019, 10 April Shift-II]

- (a) 500 m (b) 400 m
(c) 600 m (d) 300 m

Ans. (d)

Pressure inside a fluid volume open to atmosphere is

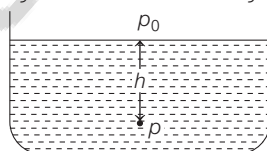
$$p = p_0 + h\rho g$$

where, p = pressure at depth h ,

h = depth,

ρ = density of fluid

and g = acceleration due to gravity.



In problem given,

when $h = d_1$, pressure $p_1 = 5.05 \times 10^6$ Pa

and when $h = d_2$,

pressure $p_2 = 8.08 \times 10^6$ Pa

So, we have

$$p_1 = p_0 + d_1 \rho g = 5.05 \times 10^6$$

$$\text{and } p_2 = p_0 + d_2 \rho g = 8.08 \times 10^6$$

$$\Rightarrow p_2 - p_1 = (d_2 - d_1) \rho g = 3.03 \times 10^6$$

$$\Rightarrow d_2 - d_1 = \frac{3.03 \times 10^6}{\rho g}$$

$$\text{Given, } \rho = 10^3 \text{ kg m}^{-3}$$

$$\text{and } g = 10 \text{ ms}^{-2}$$

$$\therefore d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10}$$

$$= 303 \text{ m}$$

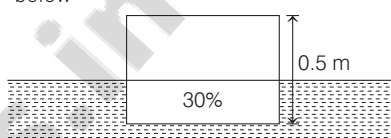
$$\approx 300 \text{ m}$$

- 11** A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg/m³] [2019, 10 April Shift-II]

- (a) 30.1 kg (b) 46.3 kg
(c) 87.5 kg (d) 65.4 kg

Ans. (c)

When only block is floating on water, 30% of its volume is in water as shown below



By Archimedes' principle, weight of block = weight of displaced water

$$\Rightarrow V\rho_b g = (30\%V)\rho_w g$$

where, ρ_b = density of block,

ρ_w = density of water

and V = volume of block.

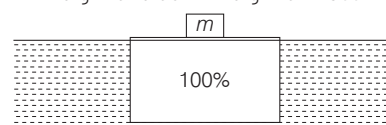
$$\text{So, } \rho_b = 0.3\rho_w$$

Now, let a mass m is placed over block to just keep the cube fully immersed in water.

Now, by Archimedes' principle,

weight of water displaced

= weight of block + weight of mass m



$$\Rightarrow V\rho_w g = V\rho_b g + mg$$

$$\Rightarrow V(\rho_w - \rho_b) = m$$

$$\Rightarrow V \times 0.7\rho_w = m \quad [\because \rho_b = 0.3\rho_w]$$

$$\text{Here, } V = (\text{side})^3 = (0.5)^3 \text{ m}^3$$

$$\rho_w = 10^3 \text{ kg m}^{-3}$$

Substituting these values in the above relation, we get $m = (0.5)^3 \times 0.7 \times 10^3 = 87.5 \text{ kg}$

- 12** A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected 25% losses all of its momentum and, 25% comes back with the same speed. The resultant pressure on the mesh will be [2019, 11 Jan Shift-I]

- (a) ρv^2 (b) $\frac{1}{2}\rho v^2$ (c) $\frac{1}{4}\rho v^2$ (d) $\frac{3}{4}\rho v^2$

Ans. (d)

Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho Av$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

$\therefore$ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4} \left(\frac{dm}{dt} \right) v = \frac{1}{4} \rho Av^2 \quad \dots(i)$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4} \left(\frac{dm}{dt} \right) \times 2v = \frac{1}{2} \rho Av^2 \quad \dots(ii)$$

[$\therefore$ Net change in velocity is $= 2v$]

$\therefore$ Net pressure on the mesh is

$$p = \frac{F_{\text{net}}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \left[\therefore F = \frac{dp}{dt} \right]$$

$\therefore$ From Eqs. (i) and (ii), we get

$$p = \frac{3}{4} \rho v^2 A / A \\ = \frac{3}{4} \rho v^2$$

- 13** A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now, the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is

[2019, 12 Jan Shift-II]

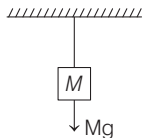
- (a) zero (b) 5.0 mm
(c) 4.0 mm (d) 3.0 mm

Ans. (d)

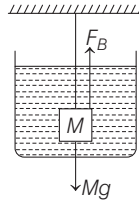
When load M is attached to wire, extension in length of wire is

$$\Delta l_1 = \frac{Mg}{A.Y} \quad \dots(i)$$

where, Y is the Young's modulus of the wire.



when load is immersed in liquid of relative density 2, increase in length of wire as shown in the figure is



$$\Delta l_2 = \frac{(Mg - F_B)l}{A.Y}$$

where, F_B = Buoyant force

$$\therefore \Delta l_2 = \frac{\left(Mg - Mg \cdot \frac{\rho_l}{\rho_b} \right) l}{A.Y} \\ \left[\therefore F_B = V\rho_l g = \frac{Mg}{\rho_b} \rho_l g \right]$$

$$\text{Here given that, } \frac{\rho_l}{\rho_b} = \frac{2}{8} = \frac{1}{4}$$

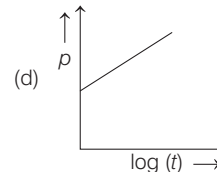
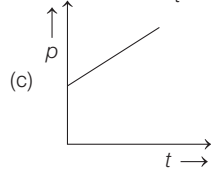
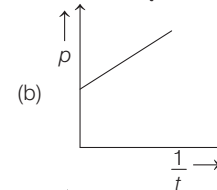
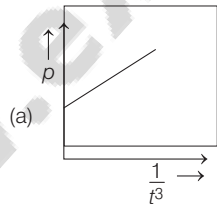
$$\text{So, } \Delta l_2 = \frac{\left(\frac{3}{4} Mg \right) l}{A.Y} \quad \dots(ii)$$

Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta l_2}{\Delta l_1} = \frac{3}{4} \Rightarrow \Delta l_2 = \frac{3}{4} \times \Delta l_1 = \frac{3}{4} \times 4 \text{ mm} = 3 \text{ mm}$$

- 14** A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by

[2019, 12 Jan Shift-II]



Ans. (b)

When soap bubble is being inflated and its temperature remains constant, then it follows Boyle's law, so

$$pV = \text{constant} (k)$$

$$\Rightarrow p = \frac{k}{V}$$

Differentiating above equation with time, we get

$$\frac{dp}{dt} = k \cdot \frac{d}{dt} \left(\frac{1}{V} \right)$$

$$\Rightarrow \frac{dp}{dt} = k \left(\frac{-1}{V^2} \right) \cdot \frac{dV}{dt}$$

It is given that, $\frac{dV}{dt} = c$ (a constant)

$$\text{So, } \frac{dp}{dt} = \frac{-kc}{V^2} \quad \dots(i)$$

Now, from $\frac{dV}{dt} = c$, we get

$$dV = c dt$$

$$\text{or } \int dV = \int c dt$$

$$\text{or } V = ct \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{dp}{dt} = \frac{-kc}{c^2 t^2}$$

$$\text{or } \frac{dp}{dt} = - \left(\frac{k}{c} \right) t^{-2}$$

$$\Rightarrow dp = - \frac{k}{c} \cdot t^{-2} dt$$

Integrating both sides, we get

$$\int dp = - \frac{k}{c} \int t^{-2} dt$$

$$p = - \frac{k}{c} \cdot \left(\frac{t^{-2+1}}{-2+1} \right)$$

$$= - \frac{k}{c} \cdot \frac{-1}{t} = \frac{k}{ct}$$

$$\text{or } p \propto \frac{1}{t}$$

Hence, p versus $\frac{1}{t}$ graph is a straight

line, which is correctly represented in option (b).

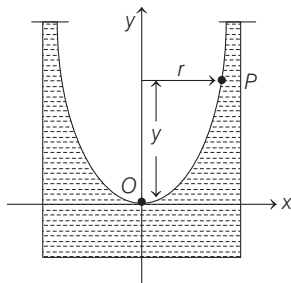
- 15** A long cylindrical vessel is half-filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides (in cm) will be

[2019, 12 Jan Shift-II]

- (a) 0.1 (b) 1.2
(c) 0.4 (d) 2.0

Ans. (d)

When liquid filled vessel is rotated the liquid profile becomes a paraboloid due to centripetal force, as shown in the figure below



Pressure at any point P due to rotation is

$$p_R = \frac{1}{2} \rho r^2 \omega^2$$

Gauge pressure at depth y is $p_G = -\rho gy$

If p_0 is atmospheric pressure, then total pressure at point P is

$$p = p_0 + \frac{1}{2} \rho r^2 \omega^2 - \rho gy$$

For any point on surface of rotating fluid,

$$p = p_0$$

Hence, for any surface point;

$$p_0 = p_0 + \frac{1}{2} \rho r^2 \omega^2 - \rho gy$$

$$\text{or } \frac{1}{2} \rho r^2 \omega^2 = \rho gy \quad \text{or } y = \frac{r^2 \omega^2}{2g} \quad \dots (i)$$

In the given case,

Angular speed,

$$\omega = 2\pi \text{ rps} = 2 \times 2\pi = 4\pi \text{ rad s}^{-1}$$

Radius of vessel, $r = 5 \text{ cm} = 0.05 \text{ m}$

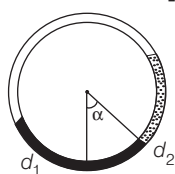
and $g = 10 \text{ ms}^{-2}$

Hence, substituting these values in Eq. (i), we get

$$y = \frac{\omega^2 r^2}{2g} = \frac{(4\pi)^2 (0.05)^2}{2 \times 10} = 0.02 \text{ m} = 2 \text{ cm}$$

- 16** There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is

[JEE Main 2014]

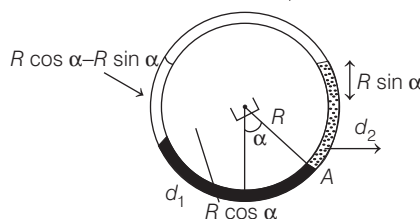


- (a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$
(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Ans. (c)

Equating pressure at A, we get

$$R \sin\alpha d_2 + R \cos\alpha d_2 + R(1-\cos\alpha)d_1 = R(1-\sin\alpha)d_1$$



$$(\sin\alpha + \cos\alpha)d_2 = d_1(\cos\alpha - \sin\alpha)$$

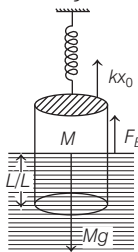
$$\Rightarrow \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

- 17** A uniform cylinder of length L and mass M having cross-sectional area A is suspended with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium, is [JEE Main 2013]

- (a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{M}\right)$
(c) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k} \left(1 + \frac{LA\sigma}{M}\right)$

Ans. (c)

The various forces acting on the cylinder are shown in the figure.



Let x_0 be the extension in spring.

At equilibrium, Upward forces = Downward forces

$$kx_0 + F_B = mg$$

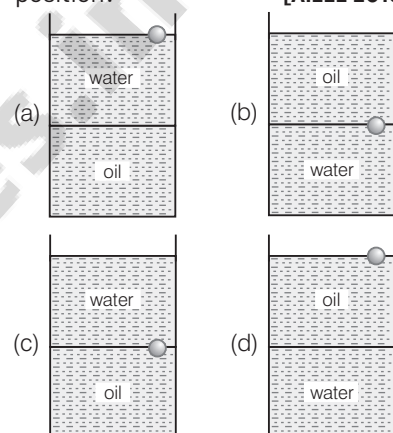
where, k is spring constant, F_B is force due to buoyancy which is equal to weight of liquid displaced.

$$F_B = \sigma \cdot \frac{L}{2} \cdot A \cdot g$$

$$\therefore kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$\Rightarrow x_0 = \frac{Mg - \frac{\sigma LA g}{2}}{k} = \frac{Mg}{k} \left(1 - \frac{\sigma LA}{2M}\right)$$

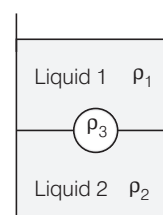
- 18** A ball is made of a material of density ρ , where $\rho_{\text{oil}} < \rho < \rho_{\text{water}}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position? [AIEEE 2010]

**Ans. (b)**

$$\rho_{\text{oil}} < \rho < \rho_{\text{water}}$$

Oil is the least dense of them, so it should settle at the top with water at the base. Now, the ball is denser than oil but less denser than water. So, it will sink through oil but will not sink in water. So, it will stay at the oil-water interface.

- 19** A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 , respectively. A solid ball, made of a material of density ρ_3 , is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1 , ρ_2 and ρ_3 ? [AIEEE 2008]



- (a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 > \rho_2 > \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

Ans. (d)

$\rho_1 < \rho_2$ as denser liquid acquires lowest position of vessel. $\rho_3 > \rho_1$ as ball sinks in liquid 1 and $\rho_3 < \rho_2$ as ball does not sink in liquid 2, so

$$\rho_1 < \rho_3 < \rho_2$$

TOPIC 2**Bernoulli's Principle and Viscosity**

- 20** In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density $1.2 \times 10^3 \text{ kg m}^{-3}$? Take viscosity of liquid $= 1.8 \times 10^{-5} \text{ N s m}^{-2}$. (Neglect buoyancy due to air).

[2021, 27 Aug Shift-I]

- (a) $3.8 \times 10^{-11} \text{ N}$ (b) $3.9 \times 10^{-10} \text{ N}$
(c) $1.8 \times 10^{-10} \text{ N}$ (d) $5.8 \times 10^{-10} \text{ N}$

Ans. (b)

Given, radius of oil drop, $r = 2.0 \times 10^{-5} \text{ m}$

Density of oil drop, $\rho = 1.2 \times 10^3 \text{ kg m}^{-3}$

Viscosity of liquid, $\eta = 1.8 \times 10^{-5} \text{ N s m}^{-2}$

To neglect buoyancy, consider the density of air, $\rho_a = 0$

Viscous force acting on drop can be given as

$$F = 6\pi\eta r v \quad \dots(i)$$

Here, v is terminal velocity.

The terminal velocity of drop can be given as

$$v = \frac{2r^2(\rho - \rho_a)g}{9\eta}$$

$$\Rightarrow v = \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}}$$

$$= 5.807 \times 10^{-2} \text{ ms}^{-1}$$

Substituting all values in Eq. (i), we get

$$F = 6\pi \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.807 \times 10^{-2}$$

$$= 3.94 \times 10^{-10} \text{ N}$$

$$\approx 3.9 \times 10^{-10} \text{ N}$$

- 21** A raindrop with radius $R = 0.2 \text{ mm}$ falls from a cloud at a height $h = 2000 \text{ m}$ above the ground. Assume that, the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is

(Take, density of water, $\rho_w = 1000 \text{ kg m}^{-3}$ and density of air, $\rho_a = 1.2 \text{ kg m}^{-3}$, $g = 10 \text{ m/s}^2$, coefficient of viscosity of air, $\eta = 1.8 \times 10^{-5} \text{ N-s m}^{-2}$)

[2021, 27 July Shift-II]

- (a) 250.6 ms^{-1} (b) 43.56 ms^{-1}
(c) 4.94 ms^{-1} (d) 14.4 ms^{-1}

Ans. (c)

Given,

Radius of rain drop, $R = 0.2 \text{ mm}$
 $= 2 \times 10^{-4} \text{ m}$

Height, $h = 2000 \text{ m}$

Density of water, $\rho_w = 1000 \text{ kg m}^{-3}$

Density of air, $\rho_a = 1.2 \text{ kg m}^{-3}$

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

Coefficient of viscosity of air,

$$\eta = 1.8 \times 10^{-5} \text{ N-s m}^{-2}$$

According to given situation, when spherical rain drop falls with terminal velocity v_T , then

$$\text{Viscous force} = \text{Weight of rain drop}$$

$$6\pi\eta R v_T = mg$$

$$\Rightarrow 6\pi\eta R v_T = \left(\frac{4}{3}\pi R^3\right)\rho_w \times g$$

$$\Rightarrow v_T = \frac{2R^2\rho_w g}{9 \times \eta}$$

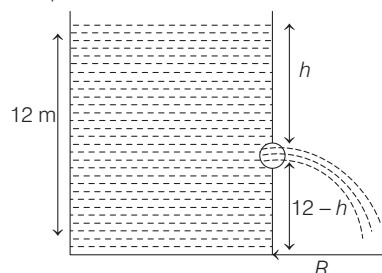
$$= \frac{2 \times (2 \times 10^{-4})^2 \times 1000 \times 10}{9 \times 1.8 \times 10^{-5}}$$

$$= 4.94 \text{ ms}^{-1}$$

- 22** The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth h below the water level. The value of h for which the emerging stream of water strikes the ground at the maximum range is m. **[2021, 27 July Shift-II]**

Ans. (6)

As per question, the water is filled up to a height of 12 m in a tank having vertical side walls. A hole is made in one of the walls at a depth h below the water level. The given description can be represented as follows



$\therefore$ We know that range will be given as

$$R = \sqrt{2gh} \times \sqrt{\frac{(12-h) \times 2}{g}}$$

$$\Rightarrow R = \sqrt{4h(12-h)}$$

For range to be maximum,

$$\frac{dR}{dh} = 0$$

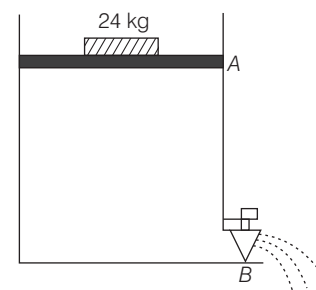
$$\Rightarrow \frac{d}{dh}(\sqrt{4h(12-h)}) = 0$$

On solving, we will get

$$h = 6 \text{ m}$$

- 23** Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2 . The tank has an opening B near the bottom whose cross-section area is 1 cm^2 . A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v \text{ ms}^{-1}$. The value of v , to the nearest integer, is

(Take value of g to be 10 ms^{-2})

[2021, 18 March Shift-II]**Ans. (3)**

Given, cross-sectional area of the tank, $A = 0.4 \text{ m}^2$

Cross-sectional area of the opening at B ,
 $A = 4 \text{ cm}^2$

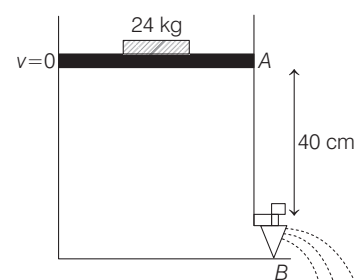
$$= 1 \times 10^{-4} \text{ m}^2$$

Load applied on the top of the tank,

$$m = 24 \text{ kg}$$

Height of the water level, $h = 40 \text{ cm}$

From the figure,



Using the Bernoulli's principle equation,

$$p_0 + \frac{mg}{A} + \rho gh + \frac{\rho v_A^2}{2} = p_0 + \frac{\rho v_B^2}{2}$$

$$\Rightarrow 0 + \frac{(24)10}{0.4} + 1000 \times 10 \times 0.4 + \frac{\rho(0)^2}{2}$$

$$= 0 + \frac{1000 \times v_B^2}{2}$$

$$v_B = 3 \text{ m/s}$$

Hence, the value of v to the nearest integer is 3.

- 24** What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa-s, respectively.

(Density of water = 10^3 kg/m^3)

[2021, 16 March Shift-II]

- (a) Unsteady to steady flow
(b) Remains steady flow
(c) Remains turbulent flow
(d) Steady flow to unsteady flow

Ans. (d)

As we know that the nature of flow of water is determined by Reynold's number,

$$R_e = \frac{\rho v D}{\eta}$$

Here, $\rho = 10^3 \text{ kgm}^{-3}$, $r = 0.5 \text{ cm}$

$$= 0.5 \times 10^{-2} \text{ m}$$

$$D = 2r = 1 \times 10^{-2} \text{ m}$$

$$\eta = 10^{-3} \text{ Pa-s}$$

$$Q_1 = 0.18 \text{ L/min} = \frac{0.18 \times 10^{-3}}{60} \text{ m}^3/\text{s}$$

$$Q_2 = 0.48 \text{ L/min} = \frac{0.48 \times 10^{-3}}{60} \text{ m}^3/\text{s}$$

$$\text{Also, } v = \frac{Q}{\pi r^2} \therefore v_1 = \frac{Q_1}{\pi r^2}$$

$$\therefore R_{e_{\text{initial}}} = \frac{\rho v_1 D}{\eta} = \frac{\rho Q_1 D}{\pi r^2 \eta}$$

$$= \frac{10^3 \times 0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$$= 381.97 \approx 382$$

$$\text{and } R_{e_{\text{final}}} = \frac{\rho v_2 D}{\eta} = \frac{\rho Q_2 D}{\pi r^2 \eta}$$

$$= \frac{10^3 \times 0.48 \times 10^{-3} \times 1 \times 10^{-2}}{\pi \times (0.5 \times 10^{-2})^2 \times 60 \times 10^{-3}}$$

$$= 1019.09$$

We know that,

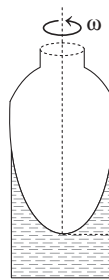
When $R_e < 1000$, the flow of water is considered to be steady.

When $1000 < R_e < 2000$, the flow of water becomes unsteady.

When $R_e > 2000$, the flow of water becomes turbulent.

As the $R_{e_{\text{final}}}$ value lies in between 1000 and 2000 and $R_{e_{\text{initial}}}$ value is less than 1000, so the nature of flow of water will be steady flow to the unsteady one.

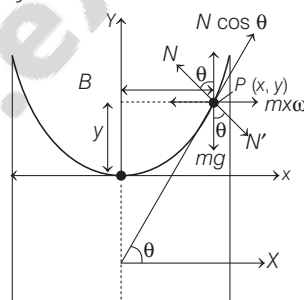
- 25** A cylindrical vessel containing a liquid is rotated about its axis, so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is $\omega \text{ rad s}^{-1}$. The difference in the height h (in cm) of liquid at the centre of vessel and at the side will be [2020, 2 Sep Shift-I]



- (a) $\frac{25\omega^2}{2g}$ (b) $\frac{5\omega^2}{2g}$ (c) $\frac{2\omega^2}{25g}$ (d) $\frac{2\omega^2}{5g}$

Ans. (a)

Let tangent at any point $P(x, y)$ makes angle θ with X-axis as shown below,



In rotating frame of vessel, let N be normal reaction. Then,

$$N \cos \theta = mg \text{ and } N \sin \theta = mx\omega^2$$

$$\Rightarrow \tan \theta = \frac{\omega^2 x}{g}$$

But $\tan \theta = \frac{dy}{dx}$ = slope of tangent

$$\therefore \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\Rightarrow y = \int dy = \int \frac{\omega^2 x}{g} \cdot dx \Rightarrow y = \frac{\omega^2}{g} \cdot \frac{x^2}{2}$$

At topmost point of profile of rotating fluid body $y = h$ and $x = R$

$$\Rightarrow h = \frac{\omega^2 R^2}{2g}$$

Given, $R = 5 \text{ cm}$

$$\Rightarrow h = \frac{\omega^2 (5)^2}{2g} = \frac{25\omega^2}{2g} \text{ cm}$$

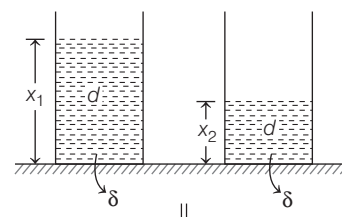
Hence, correct option is (a).

- 26** Two identical cylindrical vessels are kept on the ground and each contains the same liquid of density d . The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2 . When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is [2020, 4 Sep Shift-II]

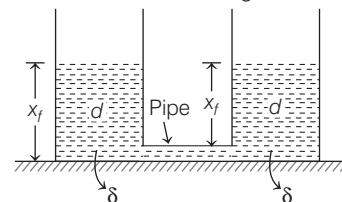
- (a) $gdS(x_2^2 + x_1^2)$
(b) $\frac{3}{4}gdS(x_2 - x_1)^2$
(c) $gdS(x_2 + x_1)^2$
(d) $\frac{1}{4}gdS(x_2 - x_1)^2$

Ans. (d)

Before connecting



After connecting



By conservation of volume,

$$(V_{\text{system}})_{\text{initial}} = (V_{\text{system}})_{\text{final}}$$

$$Sx_1 + Sx_2 = Sx_f + Sx_f$$

$$x_1 + x_2 = x_f + x_f$$

$$x_1 + x_2 = 2x_f$$

$$x_f = \frac{x_1 + x_2}{2}$$

...(i)

Now, initial energy of system,

$$(U_{\text{system}})_{\text{initial}} = M_1gh_1 + M_2gh_2$$

$$= dV_1gh_1 + dV_2gh_2$$

$$\begin{aligned}
 &= dSx_1g\left(\frac{x_1}{2}\right) + dSx_2g\left(\frac{x_2}{2}\right) \\
 &= \frac{1}{2}dSgx_1^2 + \frac{1}{2}dSgx_2^2 \\
 &= \frac{1}{2}dSg(x_1^2 + x_2^2) \quad \dots(ii)
 \end{aligned}$$

and final energy of system,

$$\begin{aligned}
 (U_{\text{system}})_{\text{final}} &= M'gh' + M'gh' \\
 &= dV'gh' + dV'gh' \\
 &= dSx_fg\left(\frac{x_f}{2}\right) + dSx_fg\left(\frac{x_f}{2}\right) \\
 &= \frac{1}{2}dSgx_f^2 + \frac{1}{2}dSgx_f^2 \\
 &= dSgx_f^2 \\
 &= dSg\left(\frac{x_1 + x_2}{2}\right)^2 \quad \dots(iii) \text{ [Using eq. (i)]}
 \end{aligned}$$

Therefore, the change in energy of system,

$$\begin{aligned}
 \Delta U_{\text{system}} &= (U_{\text{system}})_{\text{final}} - (U_{\text{system}})_{\text{initial}} \\
 &= dSg\left(\frac{x_1 + x_2}{2}\right)^2 - \frac{1}{2}dSg(x_1^2 + x_2^2) \\
 &= dSg\left(\frac{x_1^2 + x_2^2 + 2x_1x_2}{4}\right) - dSg\left(\frac{x_1^2 + x_2^2}{2}\right) \\
 &= dSg\left[\left(\frac{x_1^2 + x_2^2 + 2x_1x_2}{4}\right) - \left(\frac{x_1^2 + x_2^2}{2}\right)\right] \\
 &= dSg\left[\frac{x_1^2 + x_2^2 + 2x_1x_2 - 2(x_1^2 + x_2^2)}{4}\right] \\
 &= \frac{dSg}{4}(x_1^2 + x_2^2 + 2x_1x_2 - 2x_1^2 - 2x_2^2) \\
 &= \frac{dSg}{4}(-x_1^2 - x_2^2 + 2x_1x_2) \\
 &= -\frac{dSg}{4}(x_1^2 + x_2^2 - 2x_1x_2) \\
 &= -\frac{1}{4}gdS(x_2 - x_1)^2
 \end{aligned}$$

So, energy of the system will be decreased by $\frac{1}{4}gdS(x_2 - x_1)^2$. Hence, option (d) is correct.

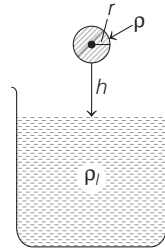
- 27** In an experiment to verify Stoke's law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to (Ignore viscosity of air)

[2020, 5 Sep Shift-II]

- (a) r^4 (b) r
(c) r^3 (d) r^2

Ans. (a)

Let ρ_l be the density of water.



Velocity of the ball after free fall,
 $v = \sqrt{2gh}$... (i)

After falling through distance h , the velocity of the ball will be equal to terminal velocity.

$$\text{Terminal velocity, } v = \frac{2r^2g(\rho_l - \rho)}{9\eta} \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$\begin{aligned}
 \sqrt{2gh} &= \frac{2r^2g}{9\eta}(\rho_l - \rho) \\
 \Rightarrow 2gh &= \frac{4r^4g^2}{81\eta^2}(\rho_l - \rho)^2 \\
 \Rightarrow h &= \frac{2r^4g^2}{81\eta^2}(\rho_l - \rho)^2 \Rightarrow h \propto r^4
 \end{aligned}$$

Hence, correct option is (a).

- 28** A fluid is flowing through a horizontal pipe of varying cross-section with speed $v \text{ ms}^{-1}$ at a point where the pressure is $p \text{ Pa}$. At another point, where pressure is $\frac{p}{2} \text{ Pa}$ its speed is $V \text{ ms}^{-1}$. If the density of the fluid is $\rho \text{ kg m}^{-3}$ and the flow is streamline, then V is equal to [2020, 6 Sep Shift-II]

- (a) $\sqrt{\frac{p}{\rho} + v}$ (b) $\sqrt{\frac{2p}{\rho} + v^2}$
(c) $\sqrt{\frac{p}{2\rho} + v^2}$ (d) $\sqrt{\frac{p}{\rho} + v^2}$

Ans. (d)

Since, pipe is horizontal. Applying Bernoulli's equation at two different cross-section,

$$\rho_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = \rho_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

$$\text{For same level, } p + \frac{1}{2}\rho v_1^2 = \frac{p}{2} + \frac{1}{2}\rho v_2^2$$

Given, $v_1 = v \text{ m/s}$ and $v_2 = V \text{ m/s}$

$$\frac{p}{2} = \frac{1}{2}\rho(V^2 - v^2) \Rightarrow V^2 - v^2 = \frac{p}{\rho}$$

$$\Rightarrow V = \sqrt{\frac{p}{\rho} + v^2}$$

Hence, correct option is (d).

- 29** An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is

[2020, 7 Jan Shift-II]

- (a) $\frac{9}{16}$ (b) $\frac{81}{256}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{4}$

Ans. (a)

Using equation of continuity, we have

$$A_{\min} v_{\max} = A_{\max} v_{\min}$$



We have,

$$\frac{\pi}{4}d_{\min}^2 \cdot v_{\max} = \frac{\pi}{4}d_{\max}^2 \cdot v_{\min}$$

$$\Rightarrow \frac{v_{\min}}{v_{\max}} = \left(\frac{d_{\min}}{d_{\max}}\right)^2$$

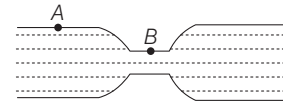
Here, $d_{\max} = 6.4 \text{ cm}$

and $d_{\min} = 4.8 \text{ cm}$

$$\text{So, } \frac{v_{\min}}{v_{\max}} = \left(\frac{4.8}{6.4}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

- 30** Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B, where the area of cross-section are 40 cm^2 and 20 cm^2 , respectively. Find the rate of flow of water through the tube. (Take, density of water = 1000 kg m^{-3})

[2020, 9 Jan Shift-I]

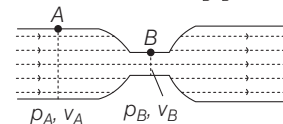


- (a) $3020 \text{ cm}^3/\text{s}$ (b) $2420 \text{ cm}^3/\text{s}$
(c) $2720 \text{ cm}^3/\text{s}$ (d) $1810 \text{ cm}^3/\text{s}$

Ans. (c)

By equation of continuity for sections A and B, we have

$$A_A v_A = A_B v_B$$



$$\begin{aligned}
 \Rightarrow 40 v_A &= 20 v_B \\
 \Rightarrow 2 v_A &= v_B \quad \dots(i)
 \end{aligned}$$

Now, using Bernoulli's equation (for horizontal tube), we have

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2$$

$$\Rightarrow p_A - p_B = \frac{1}{2}\rho(v_B^2 - v_A^2)$$

Here, $p_A - p_B = 700 \text{ Nm}^{-2}$
and $\rho = 1000 \text{ kg m}^{-3}$

$$\Rightarrow 700 = \frac{1}{2} \times 1000 (v_B^2 - v_A^2)$$

$$\Rightarrow v_B^2 - v_A^2 = 1.4 \quad \dots(\text{ii})$$

From Eqs. (i) and (ii), we have

$$3v_A^2 = 1.4$$

$$\Rightarrow v_A = \sqrt{0.467} = 0.68 \text{ ms}^{-1} = 68 \text{ cm s}^{-1}$$

So, volume flow rate of water = $A_A \cdot v_A$
 $= 40 \times 68 = 2720 \text{ cm}^3/\text{s}$

- 31** Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = 1000 kg m^{-3} , coefficient of viscosity of water = 1 mPa s)

[2019, 8 April Shift-I]

- (a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

Ans. (b)

Reynolds' number for flow of a liquid is given by

$$R_e = \frac{\rho v D}{\eta}$$

where, velocity of flow,

$$v = \frac{\text{volume flow rate}}{\text{area of flow}} = \frac{V/t}{A}$$

$$\text{So, } R_e = \frac{\rho V D}{\eta A t} = \frac{\rho V 2r}{\eta \times \pi r^2 \times t} = \frac{2\rho V}{\eta \pi r t}$$

Here, ρ = density of water = 1000 kg m^{-3}

$$\frac{V}{t} = \frac{100 \times 10^{-3}}{60} \text{ m}^3 \text{ s}^{-1}$$

where, η = viscosity of water
 $= 1 \times 10^{-3} \text{ Pa-s}$

and r = radius of pipe = $5 \times 10^{-2} \text{ m}$

$$R_e = \frac{2 \times 1000 \times 100 \times 10^{-3}}{1 \times 10^{-3} \times 60 \times 3.14 \times 5 \times 10^{-2}}$$

$$= 2123 \times 10^2$$

$$\approx 2.0 \times 10^4$$

So, order of Reynolds' number is of 10^4 .

- 32** Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1} . The cross-sectional area of the tap is 10^{-4} m^2 . Assume that the pressure is constant throughout the stream of water and that the flow is streamlined.

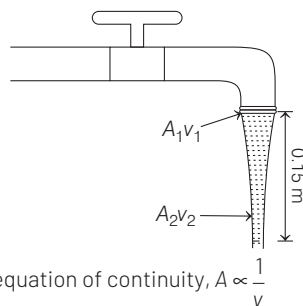
The cross-sectional area of the stream, 0.15 m below the tap would be [Take, $g = 10 \text{ ms}^{-2}$]

[2019, 10 April Shift-II]

- (a) $2 \times 10^{-5} \text{ m}^2$ (b) $1 \times 10^{-5} \text{ m}^2$
 (c) $5 \times 10^{-4} \text{ m}^2$ (d) $5 \times 10^{-5} \text{ m}^2$

Ans. (d)

Given situation is as shown in the figure below

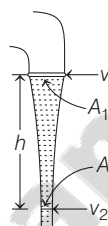


From equation of continuity, $A \propto \frac{1}{v}$

where, A = area of flow

and v = velocity of flow.

$\therefore$ Increase in speed of flow causes a decrease in area of flow.



Here given that height of fall, $h = 0.15 \text{ m}$

Area, $A = 10^{-4} \text{ m}^2$

Initial speed $v = 1 \text{ ms}^{-1}$

Velocity of water stream below height is

$$v_2 = \sqrt{v_1^2 + 2gh} \quad [\because v^2 - u^2 = 2gh]$$

Substituting the given values, we get

$$= \sqrt{1^2 + 2 \times 10 \times 0.15} = \sqrt{4} = 2 \text{ ms}^{-1}$$

Now, from equation of continuity, we have

$$A_1 v_1 = A_2 v_2 \text{ or } A_2 = \frac{A_1 v_1}{v_2}$$

$$\therefore A_2 = \frac{10^{-4} \times 1}{2} = 0.5 \times 10^{-4} = 5 \times 10^{-5} \text{ m}^2$$

- 33** A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η . The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2 when falling through the same fluid, the ratio (v_1 / v_2) equals [2019, 12 April Shift-II]
- (a) 9 (b) $1/27$ (c) $1/9$ (d) 27

Ans. (a)

Key Idea Terminal speed of a sphere falling in a viscous fluid is

$$v_T = \frac{2}{9} \frac{r^2}{\eta} (\rho_0 - \rho_f) g$$

where, η = coefficient of viscosity of fluid,

ρ_0 = density of falling sphere

and ρ_f = density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

$$\text{i.e. } v_T \propto r^2 \quad \dots(\text{i})$$

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r , then

Volume of sphere of radius $R = 27 \times$

Volume of sphere of radius r

$$\Rightarrow \frac{4}{3} \pi R^3 = 27 \times \frac{4}{3} \pi r^3$$

$$\Rightarrow R = 3r \Rightarrow r = \frac{R}{3}$$

So, from Eq. (i), we have

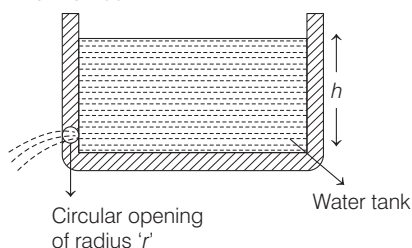
$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

- 34** The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius is its wall. The depth of the centre of the opening from the level of water in the tank is close to [2019, 9 Jan Shift-II]

- (a) 4.8 m (b) 6.0 m (c) 2.9 m (d) 9.6 m

Ans. (a)

For the given condition, a water tank is open to air and its water level maintained.



Suppose the depth of the centre of the opening from level of water in tank is h and the radius of opening is r .

According to question, the water per minute through a circular opening
 flow rate (Q) = $0.74 \text{ m}^3/\text{min} = \frac{0.74}{60} \text{ m}^3/\text{s}$

r = radius of circular opening = 2 cm

Here, the area of circular opening = $\pi(r^2)$

$$a = \pi \times (2 \times 10^{-2})^2 \\ = 4\pi \times 10^{-4} \text{ m}^2$$

Now flow rate through an area is given by

$$Q = \text{Area of circular opening} \\ \times \text{Velocity of water}$$

$$Q = a \times v = \pi(r^2) \times v \\ \Rightarrow \frac{0.74}{60} = (4\pi \times 10^{-4}) \times v \quad \dots(i)$$

According to Torricelli's law (velocity of efflux)

$$v = \sqrt{2gh} \quad \dots(ii)$$

Equation value of 'v' from Eqs. (i) and (ii), we get

$$\sqrt{2gh} = \frac{0.74 \times 10^4}{60 \times 4\pi} \\ \Rightarrow h = \left(\frac{0.74 \times 10^4}{60 \times 4\pi} \right)^2 \times \frac{1}{2g} \\ h \approx 4.8 \text{ m}$$

- 35** Water flows into a large tank with flat bottom at the rate of $10^{-4} \text{ m}^3 \text{ s}^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is **[2019, 10 Jan Shift-I]**

- (a) 4 cm (b) 2.9 cm
(c) 5.1 cm (d) 1.7 cm

Ans. (c)

As, level of water in tank remains constant with time, so (water inflow rate) = (outflow rate of water)

$$\Rightarrow 10^{-4} \text{ m}^3 \text{ s}^{-1} = \text{Area of orifice} \\ \times \text{Velocity of outflow} \\ \Rightarrow 10^{-4} \text{ m}^3 \text{ s}^{-1} = 10^{-4} \times \sqrt{2gh}$$

where, h = Height of water above the orifice or hole.

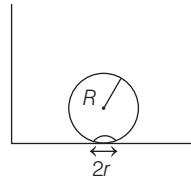
$$\Rightarrow \sqrt{2gh} = 1$$

$$\text{or } 2 \times 9.8 \times h = 1$$

$$\Rightarrow h = \frac{1}{19.6} \text{ m} = \frac{100}{19.6} \text{ cm}$$

$$\text{or } h = 5.1 \text{ cm}$$

- 36** On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r \ll R$ and the surface tension of water is T , value of r just before bubbles detach is (density of water is ρ) **[JEE Main 2014]**

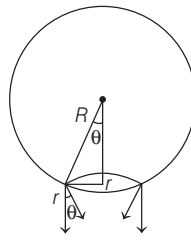


- (a) $R^2 \sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2 \sqrt{\frac{\rho_w g}{6T}}$
(c) $R^2 \sqrt{\frac{\rho_w g}{T}}$ (d) $R^2 \sqrt{\frac{3\rho_w g}{T}}$

Ans. (a)

The bubble will detach, if

Buoyant force $\geq$ Surface tension force



$$T \times dl = T (2\pi r) \sin \theta$$

$$\text{i.e., } \frac{4}{3} \pi R^3 \rho_w g \geq \int T \times dl \sin \theta$$

$$(\rho_w) \left(\frac{4}{3} \pi R^3 \right) g \geq (T) (2\pi r) \sin \theta \\ \sin \theta = \frac{r}{R}$$

$$\text{Solving, } r = \sqrt{\frac{2\rho_w R^4 g}{3T}} = R^2 \sqrt{\frac{2\rho_w g}{3T}}$$

No option matches with the correct answer.

- 37** If a ball of steel (density, $\rho = 7.8 \text{ g cm}^{-3}$) attains a terminal velocity of 10 cms^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{\text{water}} = 8.5 \times 10^{-4} \text{ Pa-s}$), then its terminal velocity in glycerine ($\rho = 12 \text{ g cm}^{-3}$, $\eta = 13.2 \text{ Pa-s}$) would be nearly **[AIEEE 2011]**

- (a) $1.6 \times 10^{-5} \text{ cms}^{-1}$
(b) $6.25 \times 10^{-4} \text{ cms}^{-1}$
(c) $6.45 \times 10^{-4} \text{ cms}^{-1}$
(d) $1.5 \times 10^{-5} \text{ cms}^{-1}$

Ans. (b)

As terminal velocity,

$$u = \frac{2}{9} r^2 \frac{(\rho - \rho_0)}{\eta} \Rightarrow v \propto \frac{\rho - \rho_0}{\eta}$$

$$\therefore \frac{v_2}{v_1} = \frac{\rho - \rho_{02}}{\rho - \rho_{01}} \times \frac{\eta_1}{\eta_2} \\ = \frac{7.8 - 1.2}{7.8 - 1} \times \frac{8.5 \times 10^{-4} \times 10}{13.2} \\ = 6.25 \times 10^{-4} \text{ cms}^{-1}$$

- 38** Water is flowing continuously from a tap having an internal diameter $8 \times 10^{-3} \text{ m}$. The water velocity as it leaves the tap is 0.4 ms^{-1} . The diameter of the water stream at a distance $2 \times 10^{-1} \text{ m}$ below the tap is close to **[AIEEE 2011]**

- (a) $7.5 \times 10^{-3} \text{ m}$ (b) $9.6 \times 10^{-3} \text{ m}$
(c) $3.6 \times 10^{-3} \text{ m}$ (d) $5.0 \times 10^{-3} \text{ m}$

Ans. (c)

From Bernoulli's theorem,

$$\rho gh = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Rightarrow gh = \frac{1}{2} v_1^2 \left[\left(\frac{v_2}{v_1} \right)^2 - 1 \right]$$

$$\Rightarrow gh = \frac{1}{2} v_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] \quad [\because A_1 v_1 = A_2 v_2]$$

$$\Rightarrow \left(\frac{A_1}{A_2} \right)^2 = 1 + \frac{2gh}{v_1^2} \Rightarrow \left(\frac{D_1}{D_2} \right)^4 = 1 + \frac{2gh}{v_1^2}$$

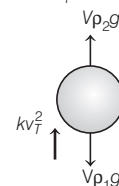
$$\Rightarrow D_2 = \frac{D_1}{\left(1 + \frac{2gh}{v_1^2} \right)^{1/4}} = \frac{8 \times 10^{-3}}{\left(1 + \frac{2 \times 10 \times 0.2}{(0.4)^2} \right)^{1/4}} \\ \approx 3.6 \times 10^{-3} \text{ m}$$

- 39** A spherical solid ball of volume V is made of a material of density ρ_1 . It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v , i.e., $F_{\text{viscous}} = -kv^2$ ($k > 0$)]. The terminal speed of the ball is **[AIEEE 2008]**

- (a) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\frac{Vg\rho_1}{k}$
(c) $\sqrt{\frac{Vg\rho_1}{k}}$ (d) $\frac{Vg(\rho_1 - \rho_2)}{k}$

Ans. (a)

The forces acting on the ball are gravity force, buoyancy force and viscous force. When ball acquires terminal speed, it is in dynamic equilibrium, let terminal speed of ball be v_T .



$$\text{So, } V\rho_2 g + kv_T^2 = V\rho_1 g$$

$$\text{or } v_T = \sqrt{\frac{V(\rho_1 - \rho_2)g}{k}}$$

- 40** If the terminal speed of a sphere of gold (density = 19.5 kg m^{-3}) is 0.2 ms^{-1} in a viscous liquid (density = 1.5 kg m^{-3}), find the terminal speed of a sphere of silver (density = 10.5 kg m^{-3}) of the same size in the same liquid. [AIEEE 2006]

- (a) 0.4 ms^{-1}
 (b) 0.133 ms^{-1}
 (c) 0.1 ms^{-1}
 (d) 0.2 ms^{-1}

Ans. (c)

Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where, ρ = density of substance of a body

σ = density of liquid.

From given data,

$$\begin{aligned} \frac{v_T(\text{Ag})}{v_T(\text{Gold})} &= \frac{\rho_{\text{Ag}} - \sigma_l}{\rho_{\text{Gold}} - \sigma_l} \\ \Rightarrow v_T(\text{Ag}) &= \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2 \\ &= \frac{9}{18} \times 0.2 \\ &= 0.1 \text{ ms}^{-1} \end{aligned}$$

- 41** Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v .

The retarding viscous force acting on the spherical ball is [AIEEE 2004]

- (a) directly proportional to R but inversely proportional to v
 (b) directly proportional to both radius R and velocity v
 (c) inversely proportional to both radius R and velocity v
 (d) inversely proportional to R but directly proportional to velocity v

Ans. (b)

Retarding force acting on a ball falling into a viscous fluid

$$F = 6\pi\eta Rv$$

where, R = radius of ball,

v = velocity of ball

and η = coefficient of viscosity.

$$\therefore F \propto R \text{ and } F \propto v$$

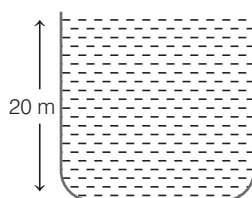
Or in other words, retarding force is directly proportional to both R and v .

- 42** A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is [AIEEE 2004]

- (a) 10 (b) 20
 (c) 25.5 (d) 5

Ans. (d)

Applying the Bernoulli's theorem just inside and outside the hole. Take reference line for gravitational potential energy at the bottom of the vessel.



Let p_0 be the atmospheric pressure, ρ be the density of liquid and v be the velocity at which water is coming out.

$$p_{\text{inside}} + \rho gh + 0 = p_{\text{outside}} + \frac{\rho v^2}{2}$$

$$\Rightarrow p_0 + \rho gh = p_0 + \frac{\rho v^2}{2}$$

$$\begin{aligned} \text{or } v &= \sqrt{2gh} \\ &= \sqrt{2 \times 10 \times 20} \\ &= 20 \text{ ms}^{-1} \end{aligned}$$

TOPIC 3

Surface Tension, Excess Pressure and Capillarity

- 43** Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube.

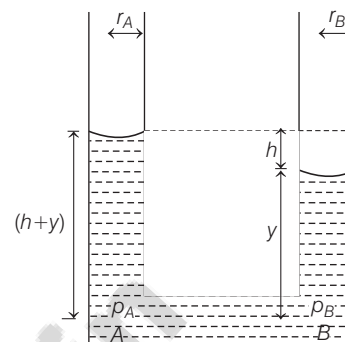
[Take surface tension of water $T = 7.3 \times 10^{-2} \text{ Nm}^{-1}$, angle of contact = 0 , $g = 10 \text{ ms}^{-2}$ and density of water = $1.0 \times 10^3 \text{ kg m}^{-3}$]

[2021, 26 Aug Shift-I]

- (a) 3.62 mm (b) 2.19 mm
 (c) 5.34 mm (d) 4.97 mm

Ans. (b)

The given situation can be shown as below.



Points A and B are at same horizontal level.

So, the pressure at A and B will be equal

$$\text{i.e. } p_A = p_B \quad \dots(i)$$

If p_0 is the atmospheric pressure, then from Bernoulli's theorem,

$$p_A = p_0 - \frac{2T}{r_A} + (h + y)\rho g$$

$$p_B = p_0 - \frac{2T}{r_B} + \rho gy$$

Substituting the value of p_A and p_B in Eq. (i), we get

$$p_0 - \frac{2T}{r_A} + (h + y)\rho g = p_0 - \frac{2T}{r_B} + \rho gy$$

$$\Rightarrow 2T \left(\frac{1}{r_B} - \frac{1}{r_A} \right) = \rho gy - (h + y)\rho g$$

$$\Rightarrow 2T \left(\frac{1}{r_B} - \frac{1}{r_A} \right) = -\rho gh$$

$$\text{or } 2T \left(\frac{1}{r_A} - \frac{1}{r_B} \right) = \rho gh \quad \dots(ii)$$

Here, $r_A = 2.5 \text{ mm}$

$$= 2.5 \times 10^{-3} \text{ m},$$

and $T = 7.3 \times 10^{-2} \text{ N m}^{-1}$,

$$r_B = 4 \text{ mm} = 4 \times 10^{-3} \text{ m},$$

$$g = 10 \text{ m s}^{-2}$$

and $\rho = 1 \times 10^3 \text{ kg m}^{-3}$

Substituting the given value in Eq. (ii), we get

$$h = \frac{2 \times 7.3 \times 10^{-2}}{1 \times 10^3 \times 10} \left(\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}} \right)$$

$$h = \frac{2 \times 7.3 \times 10^{-2}}{10^{-3} \times 10^4} \left(\frac{1}{2.5} - \frac{1}{4} \right)$$

$$h = 2.19 \times 10^{-3} \text{ m}$$

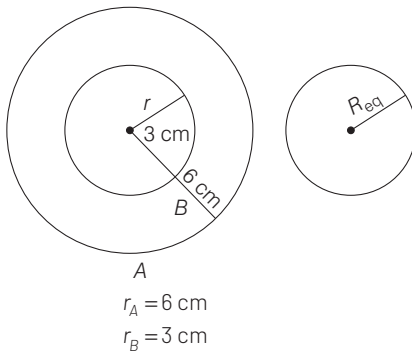
$$= 2.19 \text{ mm}$$

- 44** A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is cm.

[2021, 26 Aug Shift-I]

Ans. (2)

The given situation is shown below



Excess pressure inside the soap bubble is given by

$$(\Delta p)_{\text{excess}} = \frac{4S}{r_A} + \frac{4S}{r_B}$$

According to the given question,

$$\begin{aligned} \frac{4S}{r_A} + \frac{4S}{r_B} &= \frac{4S}{R_{\text{eq}}} \\ \Rightarrow \frac{1}{r_A} + \frac{1}{r_B} &= \frac{1}{R_{\text{eq}}} \\ \Rightarrow \frac{1}{6} + \frac{1}{3} &= \frac{1}{R_{\text{eq}}} \\ \Rightarrow \frac{1+2}{6} &= \frac{1}{R_{\text{eq}}} \\ \Rightarrow \frac{1}{R_{\text{eq}}} &= \frac{1}{2} \\ \Rightarrow R_{\text{eq}} &= 2 \text{ cm} \end{aligned}$$

- 45** Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to

[2021, 25 July Shift-II]

(a) $\frac{r_1 r_2}{r_1 + r_2}$ (b) $\sqrt{r_1 r_2}$
 (c) $\sqrt{r_1^2 + r_2^2}$ (d) $\frac{r_1 + r_2}{2}$

Ans. (c)Given, radius of bubbles is r_1 and r_2 , respectively.Let, the radius of combined new bubble be r .

Excess pressure inside a soap bubble is

$$p = \frac{4T}{r}$$

where, T is surface tension.

and volume of a soap bubble is

$$V = \frac{4}{3} \pi r^3$$

By using law of conservation of energy

$$\begin{aligned} pV &= p_1 V_1 + p_2 V_2 \\ \frac{4T}{r} \times \frac{4}{3} \pi r^3 &= \frac{4T}{r_1} \times \frac{4}{3} \pi r_1^3 + \frac{4T}{r_2} \times \frac{4}{3} \pi r_2^3 \\ \Rightarrow r^2 &= r_1^2 + r_2^2 \\ \Rightarrow r &= \sqrt{r_1^2 + r_2^2} \end{aligned}$$

- 46** Two small drops of mercury each of radius R coalesce to form a single large drop. The ratio of total surface energy before and after the change is

[2021, 20 July Shift-II]

(a) $2^{\frac{1}{3}} : 1$ (b) $1 : 2^{\frac{1}{3}}$ (c) $2 : 1$ (d) $1 : 2$

Ans. (a)According to question, two small drops of mercury each of radius R coalesce to form a single large drop of radius R' .

According to question,

Volume of 1st small drop + Volume of 2nd small drop = Volume of a big drop

$$\begin{aligned} \frac{4}{3} \pi R^3 + \frac{4}{3} \pi R^3 &= \frac{4}{3} \pi R'^3 \\ \Rightarrow R^3 + R^3 &= R'^3 \Rightarrow 2R^3 = R'^3 \\ \text{or } R'^3 &= 2R^3 \Rightarrow R' = (2)^{\frac{1}{3}} (R^3)^{\frac{1}{3}} \\ \Rightarrow R' &= 2^{\frac{1}{3}} (R) \quad \dots(i) \end{aligned}$$

 $\therefore$ So, considering the given scenario, we can write

$$\text{Area}_{\text{initially}} = 4\pi R^2 + 4\pi R^2 = 2(4\pi R^2)$$

$$\text{and Area}_{\text{finally}} = 4\pi R'^2$$

$$\therefore \text{Surface energy initially } U_i = \frac{A_i}{A_f}$$

$$\text{Surface energy finally } U_f = \frac{A_f}{A_i}$$

$$\Rightarrow \frac{U_i}{U_f} = \frac{2(4\pi R^2)}{4\pi R'^2}$$

$$\Rightarrow \frac{U_i}{U_f} = \frac{2R^2}{R'^2} \quad \dots(ii)$$

 $\therefore$ From Eqs. (i) and (ii), we get

$$\frac{U_i}{U_f} = \frac{2R^2}{(2^{\frac{1}{3}})^2 \cdot (R)^2}$$

$$\Rightarrow \frac{U_i}{U_f} = \frac{2R^2}{2^{\frac{2}{3}} \cdot R^2}$$

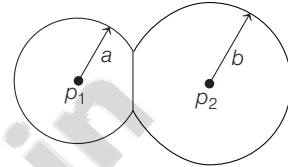
$$\Rightarrow \frac{U_i}{U_f} = 2^{1-\frac{2}{3}} \Rightarrow \frac{U_i}{U_f} = 2^{\frac{1}{3}}$$

$$\Rightarrow U_i : U_f = 2^{\frac{1}{3}} : 1$$

- 47** When two soap bubbles of radii a and b ($b > a$) coalesce, the radius of curvature of common surface is

[2021, 17 March Shift-I]

(a) $\frac{ab}{b-a}$ (b) $\frac{a+b}{ab}$ (c) $\frac{b-a}{ab}$ (d) $\frac{ab}{a+b}$

Ans. (a)When two soap bubbles of radii a and b ($b > a$) coalesce, it can be shown as follows

$$\text{Pressure inside the bubble 1, } p_1 = p_0 + \frac{4T}{a}$$

where, p_0 is the initial pressure, T is the surface tensionand a is the radius of the bubble 1.

Similarly, pressure inside the bubble 2,

$$p_2 = p_0 + \frac{4T}{b}$$

where, b is the radius of the bubble 2.

Excess pressure at common surface is given by

$$p_{\text{ex}} = p_1 - p_2$$

Let, r be the radius of common surface, then

$$\begin{aligned} \Rightarrow \frac{4T}{r} &= \left(p_0 + \frac{4T}{a} \right) - \left(p_0 + \frac{4T}{b} \right) \\ \Rightarrow \frac{4T}{r} &= p_0 + \frac{4T}{a} - p_0 - \frac{4T}{b} \\ \Rightarrow \frac{4T}{r} &= \frac{4T}{a} - \frac{4T}{b} \\ \Rightarrow \frac{1}{r} &= \frac{1}{a} - \frac{1}{b} \\ \Rightarrow \frac{1}{r} &= \frac{b-a}{ab} \\ \Rightarrow r &= \frac{ab}{b-a} \end{aligned}$$

Which is the required expression for radius of curvature at common surface.

- 48** A large number of water drops, each of radius r , combine to have a drop of radius R . If the surface tension is T and mechanical equivalent of heat is J , the rise in heat energy per unit volume will be

[2021, 26 Feb Shift-I]

(a) $\frac{2T}{J} \left(\frac{1}{r} - \frac{1}{R} \right)$ (b) $\frac{2T}{rJ}$
 (c) $\frac{3T}{rJ}$ (d) $\frac{3T}{J} \left(\frac{1}{r} - \frac{1}{R} \right)$

Ans. (d)Given, radius of small drop = r Radius of big drop = R Surface tension = T and mechanical equivalent of heat = J

As, small drops combine to form big drop.

 $\therefore$ Volume of big drop (V_B) = $n \times$ Volume of small drop (V_S)

$$\Rightarrow \frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$$

$$\Rightarrow nr^3 = R^3$$

$$\Rightarrow r = \frac{R}{n^{1/3}} \quad \dots(i)$$

Surface energy of small drop

(E_S) = Surface tension (T) $\times$ Area (A)

$$\Rightarrow E_S = n \times 4\pi r^2 T$$

and $E_B = 4\pi R^2 T$

Now, change in energy will be

$$\Delta E = E_B - E_S = 4\pi T(nr^2 - R^2)$$

 $\therefore$ Heat energy per unit volume

$$\begin{aligned} &= \frac{\Delta E}{V} = \frac{4\pi T(nr^2 - R^2)}{J \times \frac{4}{3}\pi R^3} \\ &= \frac{3T}{J} \left(\frac{nr^2}{R^3} - \frac{1}{R} \right) \\ &= \frac{3T}{J} \left(n \frac{R^2}{n^{2/3}R^3} - \frac{1}{R} \right) \\ &= \frac{3T}{J} \left[\frac{n^{1/3}}{R} - \frac{1}{R} \right] \text{ [from Eq. (i)]} \\ &= \frac{3T}{J} \left[\frac{1}{r} - \frac{1}{R} \right] \end{aligned}$$

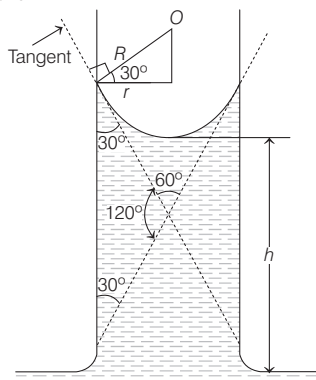
- 49** A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide, which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opposite sides of the capillary) make an angle of 60° with one another. Then, h is close to (Given, surface tension = 0.05 Nm^{-1} , density = 667 kg m^{-3} and $g = 10 \text{ ms}^{-2}$) **[2020, 2 Sep Shift-II]**

- (a) 0.049 m
(b) 0.087 m
(c) 0.137 m
(d) 0.172 m

Ans. (b)

It is given that tangents drawn from point of contact of meniscus makes 60° angle with each other.

We have following situation, as shown in figure



A normal drawn on the tangent, will pass through the centre O of meniscus. Let r be the radius of capillary tube and R be the radius of meniscus.

From geometry of figure,

$$\frac{\text{radius of capillary}}{\text{radius of meniscus}} = \frac{r}{R} = \cos 30^\circ$$

$$\Rightarrow \frac{r}{R} = \frac{\sqrt{3}}{2} \text{ or } R = \frac{2r}{\sqrt{3}} = \frac{2 \times 0.15 \times 10^{-3}}{\sqrt{3}}$$

$$\Rightarrow R = \sqrt{3} \times 10^{-4} \text{ m}$$

Now, by ascent formula,

$$\begin{aligned} h &= \frac{2T}{\rho g R} = \frac{2 \times 0.05}{667 \times 10 \times \sqrt{3} \times 10^{-4}} \\ &= 0.087 \text{ m} \end{aligned}$$

Alternative solution

Angle of contact with walls of capillary tube,

$$\theta = 30^\circ$$

$$\therefore \text{Height of vertical column, } h = \frac{2T \cos \theta}{\rho g}$$

$$= \frac{2 \times 0.05 \times \cos 30^\circ}{0.15 \times 10^{-3} \times 667 \times 10} = 0.087 \text{ m}$$

Hence, correct option is (b).

- 50** When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass is close to 0° , the surface tension of the liquid, (in millinewton m^{-1}) to the nearest integer is (Take, $\rho_{(\text{liquid})} = 900 \text{ kg m}^{-3}$, $g = 10 \text{ ms}^{-2}$) **[2020, 3 Sep Shift-I]**

Ans. (101)

Height of liquid in capillary tube,

$$h = \frac{2S \cos \theta}{\rho g r} \Rightarrow S = \frac{\rho g r h}{2 \cos \theta}$$

Here, $\rho = 900 \text{ kg m}^{-3}$, $g = 10 \text{ ms}^{-2}$,

$$r = 0.015 \text{ cm} = 15 \times 10^{-5} \text{ m},$$

$$h = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$$

$$\text{and } \theta \approx 0^\circ \Rightarrow \cos \theta = 1$$

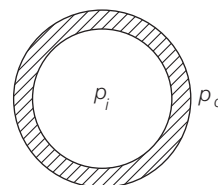
$$\begin{aligned} \text{Hence, } S &= \frac{900 \times 10 \times 15 \times 10^{-5} \times 15 \times 10^{-2}}{2 \times 1} \\ &= 101.25 \times 10^{-3} \text{ N/m} \approx 101 \text{ mN/m} \end{aligned}$$

- 51** Pressure inside two soap bubbles are 1.01 atm and 1.02 atm, respectively. The ratio of their volume is **[2020, 3 Sep Shift-I]**
- (a) 4 : 1 (b) 0.8 : 1
(c) 8 : 1 (d) 2 : 1

Ans. (c)

Excess of pressure inside a soap bubble is given by

$$\Delta p = p_i - p_o = \frac{4T}{R}$$

Given that, $(p_i)_I = 1.01 \text{ atm}$ and $(p_i)_{II} = 1.02 \text{ atm}$ As, $(p_o)_I = (p_o)_{II} = 1 \text{ atm}$ We have, $(\Delta p)_I = 0.01 \text{ atm}$ and $(\Delta p)_{II} = 0.02 \text{ atm}$ 

$$\text{Hence, } \frac{(\Delta p)_I}{(\Delta p)_{II}} = \frac{4T/R_I}{4T/R_{II}} = \frac{R_{II}}{R_I}$$

$$\Rightarrow \frac{0.01}{0.02} = \frac{R_{II}}{R_I} \Rightarrow \frac{R_I}{R_{II}} = \frac{2}{1}$$

Now, ratio of volumes of soap bubbles is

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \left(\frac{R_1}{R_2} \right)^3 = \left(\frac{2}{1} \right)^3 = 8 : 1$$

Hence, option (c) is correct.

- 52** A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T . The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) **[2020, 9 Jan Shift-II]**

$$(a) r = \sqrt{\frac{3T}{(2d - \rho)g}}$$

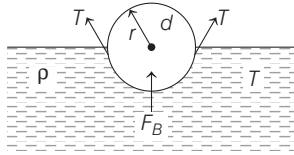
$$(b) r = \sqrt{\frac{T}{(d - \rho)g}}$$

$$(c) r = \sqrt{\frac{T}{(d + \rho)g}}$$

$$(d) r = \sqrt{\frac{2T}{3(d + \rho)g}}$$

Ans. (a)

Weight of the drop is balanced by upthrust or buoyant force (F_B) and surface tension (T).



$$\begin{aligned} \text{So, weight, } w &= F_B + T \times 2\pi r \\ \Rightarrow d \cdot V \cdot g &= \rho \cdot \frac{V}{2} g + T \times 2\pi r \\ [\because w &= dVg \text{ and } F_B = \rho \frac{V}{2} g] \\ \Rightarrow d \cdot \frac{4}{3} \pi r^3 \cdot g &= \rho \cdot \frac{2}{3} \pi r^3 \cdot g + T \times 2\pi r \\ [\because V &= \frac{4}{3} \pi r^3] \\ \Rightarrow r &= \sqrt{\frac{3T}{(2d - \rho)g}} \end{aligned}$$

- 53** If M is the mass of water that rises in a capillary tube of radius r , then mass of water which will rise in a capillary tube of radius $2r$ is [2019, 9 April Shift-I]

- (a) $2M$ (b) $4M$ (c) $\frac{M}{2}$ (d) M

Ans. (a)

Height of liquid rise in capillary tube,

$$\begin{aligned} h &= \frac{2T \cos \theta_c}{\rho r g} \\ \Rightarrow h &\propto \frac{1}{r} \end{aligned}$$

So, when radius is doubled, height becomes half.

$$\therefore h' = h/2$$

$$\text{Now, density } (\rho) = \frac{\text{mass}(M)}{\text{volume}(V)}$$

$$\Rightarrow M = \rho \times \pi r^2 h$$

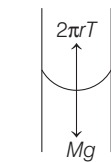
$$\therefore M' = \rho \pi r'^2 h'$$

$$\therefore \frac{M'}{M} = \frac{r'^2 h'}{r^2 h} = \frac{(2r)^2 (h/2)}{r^2 h} = 2$$

$$\Rightarrow M' = 2M$$

Alternate Solution

According to the given figure, force inside the capillary tube is



$$2\pi r T = Mg$$

$$\Rightarrow M \propto r$$

$$\begin{aligned} \text{When } r' &= 2r, \text{ then} \\ M' &= 2M \end{aligned}$$

- 54** The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles with glass are close to 135° and 0° , respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1 , while water rises by the same amount h in a capillary tube of radius r_2 . The ratio (r_1/r_2) , is then close to [2019, 10 April Shift-I]

- (a) $3/5$ (b) $2/3$
(c) $2/5$ (d) $4/5$

Ans. (c)

$$\text{Given, } \frac{T_{Hg}}{T_w} = 7.5, \frac{\rho_{Hg}}{\rho_w} = 13.6$$

$$\text{and } \frac{\cos \theta_{Hg}}{\cos \theta_w} = \frac{\cos 135^\circ}{\cos 0^\circ} = \frac{1}{\sqrt{2}}$$

Height of the fluid inside capillary tube is given by

$$h = \frac{2T \cos \theta}{\rho g r}$$

According to given situation, $h_w = h_{Hg}$

$$\therefore \frac{2T_w \cos \theta_w}{\rho_w g r_w} = \frac{2T_{Hg} \cos \theta_{Hg}}{\rho_{Hg} g r_{Hg}}$$

$$\therefore \frac{r_{Hg}}{r_w} = \left(\frac{T_{Hg}}{T_w} \right) \left(\frac{\cos \theta_{Hg}}{\cos \theta_w} \right) \left(\frac{\rho_w}{\rho_{Hg}} \right)$$

Given, $r_{Hg} = r_1$ and $r_w = r_2$, then

Substituting the given values, we get

$$\frac{r_{Hg}}{r_w} = \frac{r_1}{r_2} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6} = 0.4 = 2/5$$

- 55** Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T , density of liquid is ρ and L is its latent heat of vaporisation.

[JEE Main 2013]

- (a) $\rho L / T$ (b) $\sqrt{T / \rho L}$
(c) $T / \rho L$ (d) $2T / \rho L$

Ans. (d)

If a layer of thickness dr is evaporated, then

Change in surface energy

$$\begin{aligned} &= \text{Change in surface area} \times T \\ &= [4\pi(r + dr)^2 - 4\pi r^2] T \\ &= [4\pi r^2 + 4\pi dr^2 + 8\pi r dr - 4\pi r^2] \end{aligned}$$

As dr is very small, so dr^2 neglected.

So, change in surface energy $= 8\pi r dr T$
Energy required to evaporate layer of thickness

$$dr = (4\pi r^2 dr) \rho L$$

The process of evaporation just starts, if change in surface energy is just sufficient to evaporate the water layer.

$$\begin{aligned} \therefore (4\pi r^2 dr) \rho L &= (8\pi r dr) T \\ \Rightarrow r &= \frac{2T}{\rho L} \end{aligned}$$

- 56** A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is [AIEEE 2012]

- (a) 0.0125 Nm^{-1} (b) 0.1 Nm^{-1}
(c) 0.05 Nm^{-1} (d) 0.025 Nm^{-1}

Ans. (d)

At equilibrium, weight of the given block is balanced by force due to surface tension, i.e.,

$$2L \cdot S = W$$

$$\begin{aligned} \text{or } S &= \frac{W}{2L} = \frac{1.5 \times 10^{-2} \text{ N}}{2 \times 0.3 \text{ m}} \\ &= 0.025 \text{ Nm}^{-1} \end{aligned}$$

- 57** Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is [AIEEE 2011]

- (a) $2^{5/3} \pi r^2 T$ (b) $4\pi r^2 T$
(c) $2\pi r^2 T$ (d) $2^{8/3} \pi r^2 T$

Ans. (d)

Let R be the radius of the bigger drop, then

Volume of bigger drop $= 2 \times$ Volume of small drop

$$\frac{4}{3} \pi R^3 = 2 \times \frac{4}{3} \pi r^3 \Rightarrow R = 2^{1/3} r$$

Surface energy of bigger drop,

$$\begin{aligned} E &= 4\pi R^2 T = 4 \times 2^{2/3} \pi r^2 T \\ &= 2^{8/3} \pi r^2 T \end{aligned}$$

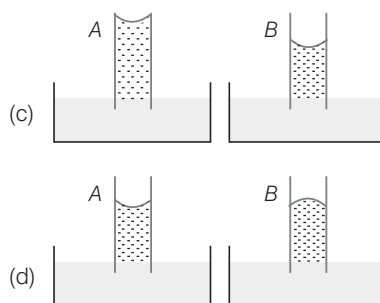
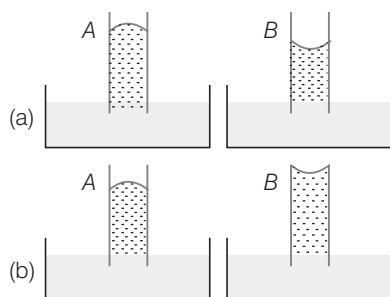
- 58** Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (surface tension of soap solution $= 0.03 \text{ Nm}^{-1}$) [AIEEE 2011]

- (a) $0.2 \pi \text{ mJ}$ (b) $2 \pi \text{ mJ}$
(c) $0.4 \pi \text{ mJ}$ (d) $4 \pi \text{ mJ}$

Ans. (c)

According to work-energy theorem,
 Work done = Change in surface energy
 $\Rightarrow W = 2T \times 4\pi(R_2^2 - R_1^2)$
 $= 2 \times 0.03 \times 4\pi[(5)^2 - (3)^2] \times 10^{-4} \text{ J}$
 $= 0.4\pi \text{ mJ}$

- 59** A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

[AIEEE 2008]**Ans. (c)**

Soap solution has lower surface tension, T as compared to pure water and capillary rise $h = \frac{2T \cos\theta}{\rho r g}$, so h is less for soap solution.

- 60** A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be

[AIEEE 2005]

- (a) 8 cm (b) 10 cm
(c) 4 cm (d) 20 cm

Ans. (d)

Water fills the tube entirely in gravity less condition.
 Hence, length of water column in the capillary tube is 20 cm.

- 61** If two soap bubbles of different radii are connected by a tube **[AIEEE 2004]**

- (a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
 (b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
 (c) air flows from the smaller bubble to the bigger
 (d) there is no flow of air

Ans. (c)

The excess pressure inside the soap bubble is inversely proportional to radius of soap bubble i.e., $p \propto 1/r$, r being the radius of bubble. It follows that pressure inside a smaller bubble is greater than that inside a bigger bubble. Thus, if these two bubbles are connected by a tube, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

10

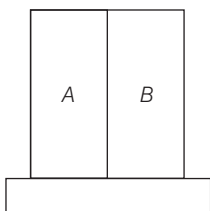
Thermal Properties of Matter

TOPIC 1

Thermometry and Thermal Expansion

- 01** A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will

[2021, 16 March Shift-II]



- (a) bend towards the right
(b) not bend but shrink
(c) Neither bend nor shrink
(d) bend towards the left

Ans. (d)

As per the question, coefficient of expansion of metal A is higher than that of B.

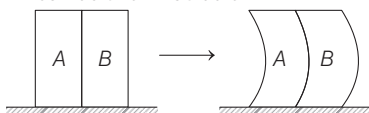
It means, $\alpha_A > \alpha_B$

If the bimetallic strip is placed in a cold bath, then the length of both strips will decrease.

i.e., $l_A > l_B$

$\therefore$ The bimetallic strip will bend towards left.

It can be shown as below.



- 02** Each side of a box made of metal sheet in cubic shape is a at room temperature T , the coefficient of linear expansion of the metal sheet is α . The metal sheet is heated uniformly, by a small temperature ΔT , so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

[2021, 24 Feb Shift-I]

- (a) $4\pi a^3 \alpha \Delta T$ (b) $4a^3 \alpha \Delta T$
(c) $\frac{4}{3} \pi a^3 \alpha \Delta T$ (d) $3a^3 \alpha \Delta T$

Ans. (d)

We know that, $\gamma = 3\alpha$... (i)
where, α is the coefficient of linear expansion
and γ is the coefficient of volume expansion.

We know that,

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\Rightarrow \frac{\Delta V}{V} = 3\alpha \Delta T \quad [\text{from Eq. (i)}]$$

$$\Delta V = 3\alpha^3 \alpha \Delta T$$

[$\because$ volume of cube $= a^3$]

- 03** A bakelite beaker has volume capacity of 500 cc at 30°C . When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})} = 6 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ and $\gamma_{(\text{mercury})} = 1.5 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to

[2020, 3 Sep Shift-I]

Ans. (20)

Volume of empty part

= Volume of beaker – Volume of mercury

Also it is given that, there is no change in unfilled volume of the beaker with the varying temperature.

So, change in volume of beaker

= change in volume of mercury

$$\Rightarrow V_b \gamma_b \Delta T = V_m \gamma_m \Delta T \Rightarrow V_b \gamma_b = V_m \gamma_m$$

$$\Rightarrow V_m = \frac{V_b \gamma_b}{\gamma_m} = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}}$$

$$= 20 \text{ cm}^3 = 20 \text{ c}$$

- 04** When the temperature of a metal wire is increased from 0°C to 10°C , its length increases by 0.02%. The percentage change in its mass density will be closest to

[2020, 2 Sep Shift-II]

- (a) 0.06 (b) 2.3 (c) 0.008 (d) 0.8

Ans. (a)

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Area} \times \text{Length}}$$

$$\Rightarrow \rho = \frac{M}{AL} \Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

But $\Delta M = 0$, as mass remains constant.

$$\therefore \frac{\Delta \rho}{\rho} = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta \Delta T + \alpha \Delta T$$

$$= 2\alpha \Delta T + \alpha \Delta T \quad [\because \beta = 2\alpha]$$

$$= 3\alpha \Delta T \quad \dots (i)$$

Now given that,

$$\frac{\Delta L}{L} \times 100 = \alpha \Delta T = 0.02\%$$

So, from Eq (i), we get

$$\frac{\Delta \rho}{\rho} \times 100 = 3 \times 0.02\% = 0.06\%$$

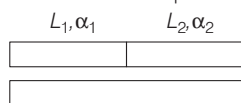
Hence, correct option is (a).

- 05** Two different wires having lengths L_1 and L_2 and respective temperature coefficients of linear expansion α_1 and α_2 , are joined end-to-end. Then the effective temperature coefficient of linear expansion is [2020, 5 Sep Shift-II]

- (a) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$
 (b) $2\sqrt{\alpha_1 \alpha_2}$
 (c) $\frac{\alpha_1 + \alpha_2}{2}$
 (d) $4 \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \frac{L_2 L_1}{(L_2 + L_1)^2}$

Ans. (a)

Let α be the effective temperature coefficient of linear expansion.



$$\alpha L = L_1 + L_2$$

Change in length of equivalent wire =
Sum of change in length of each wire

$$\begin{aligned} \Delta L &= \Delta L_1 + \Delta L_2 \\ \Rightarrow L \alpha \Delta T &= L_1 \alpha_1 \Delta T + L_2 \alpha_2 \Delta T \\ \therefore L &= L_1 + L_2 \\ \therefore (L_1 + L_2) \alpha \Delta T &= \Delta T (L_1 \alpha_1 + L_2 \alpha_2) \\ \Rightarrow \alpha &= \frac{L_1 \alpha_1 + L_2 \alpha_2}{L_1 + L_2} \end{aligned}$$

Hence, correct option is (a).

- 06** A non-isotropic solid metal cube has coefficients of linear expansion as : $5 \times 10^{-5}/^\circ\text{C}$ along the X-axis and $5 \times 10^{-6}/^\circ\text{C}$ along the Y and the Z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}/^\circ\text{C}$, then the value of C is [2020, 7 Jan Shift-I]

Ans. (60)

Let metal cube has sides of length x , y and z .

$\therefore$ Volume of cube,

$$V = x \cdot y \cdot z$$

$$\text{So, we have, } \frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

Dividing both sides with rise of temperature (ΔT), we have

$$\begin{aligned} \Rightarrow \frac{\Delta V}{V} \times \frac{1}{\Delta T} &= \frac{\Delta x}{x} \times \frac{1}{\Delta T} + \frac{\Delta y}{y} \times \frac{1}{\Delta T} + \frac{\Delta z}{z} \times \frac{1}{\Delta T} \\ \Rightarrow \gamma &= \alpha_x + \alpha_y + \alpha_z \end{aligned}$$

$$\begin{aligned} \Rightarrow C \times 10^{-6} &= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6} \\ &= 10^{-6} (50 + 5 + 5) \\ \Rightarrow C &= 60 \end{aligned}$$

- 07** Two rods A and B of identical dimensions are at temperature 30°C . If A is heated upto 180°C and B upto $T^\circ\text{C}$, then new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is [2019, 11 Jan Shift-II]

- (a) 230°C (b) 270°C
 (c) 200°C (d) 250°C

Ans. (a)

Let initial length of identical rods is l_0
Thermal expansion in length of rod due to heating is given by the relation

$$\Delta l = l_0 \alpha (\Delta T) = l_0 \alpha (T_2 - T_1)$$

Here, α is coefficient of linear expansion.

So, change in length of rods are

$$\Delta l_1 = l_0 \alpha_1 (180 - 30)$$

$$\Delta l_2 = l_0 \alpha_2 (T - 30)$$

Because new lengths are same, so

change in lengths of both rods are equal.

$$\begin{aligned} \text{i.e. } \Delta l_1 &= \Delta l_2 \\ \Rightarrow l_0 \alpha_1 (180 - 30) &= l_0 \alpha_2 (T - 30) \end{aligned}$$

$$\text{or } \frac{\alpha_1}{\alpha_2} = \frac{(T - 30)}{150}$$

Given, $\alpha_1 : \alpha_2 = 4 : 3$

$$\therefore \frac{T - 30}{150} = \frac{4}{3}$$

$$\Rightarrow T - 30 = \frac{4}{3} \times 150 = 200$$

$$\text{or } T = 200 + 30 = 230^\circ\text{C}$$

- 08** A thermometer graduated according to a linear scale reads a value x_0 , when in contact with boiling water and $x_0/3$, when in contact with ice. What is the temperature of an object in $^\circ\text{C}$, if this thermometer in the contact with the object reads $x_0/2$? [2019, 11 Jan Shift-II]

- (a) 35 (b) 60
 (c) 40 (d) 25

Ans. (d)

By principle of thermometry for any linear temperature scale,

$$\frac{T - T_{\text{LFP}}}{T_{\text{UFP}} - T_{\text{LFP}}} = a \quad (\text{constant})$$

where,

T = temperature measured

T_{LFP} = temperature of melting ice or lower fixed point.

T_{UFP} = temperature of boiling water or upper fixed point.

If, T = temperature of given object, then we have,

$$\frac{T - 0^\circ\text{C}}{100^\circ\text{C} - 0^\circ\text{C}} = \frac{\frac{x_0}{2} - \frac{x_0}{3}}{x_0 - \frac{x_0}{3}} \quad \text{or } \frac{T}{100} = \frac{1}{4}$$

$$\text{or } T = 25^\circ\text{C}$$

- 09** An external pressure p is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by [JEE Main 2017]

- (a) $\frac{p}{\alpha K}$ (b) $\frac{3\alpha}{pK}$
 (c) $3pK\alpha$ (d) $\frac{p}{3\alpha K}$

Ans. (d)

$$K = \frac{p}{(-\Delta V/V)} \Rightarrow \frac{\Delta V}{V} = \frac{p}{K}$$

$$\Rightarrow -\Delta V = \frac{pV}{K}$$

Change in volume, $\Delta V = \gamma V \Delta T$

where, γ = coefficient of volume expansion

Again, $\gamma = 3\alpha$

α is coefficient of linear expansion

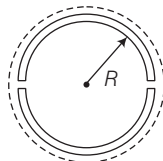
$$\therefore \Delta V = V(3\alpha) \Delta T$$

$$\therefore \frac{pV}{K} = V(3\alpha) \Delta T$$

$$\therefore \Delta T = \frac{p}{3\alpha K}$$

- 10** A wooden wheel of radius R is made of two semi-circular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L . L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over

the wheel. As it cools down to surrounding temperature, it presses the semi-circular parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y , the force that one part of the wheel applies on the other part is [AIEEE 2012]



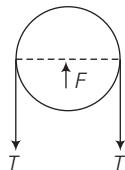
- (a) $2\pi SY\alpha\Delta T$
 (b) $SY\alpha\Delta T$
 (c) $\pi SY\alpha\Delta T$
 (d) $2SY\alpha\Delta T$

Ans. (d)

Elongation due to change in temperature,

$$\Delta l = L\alpha\Delta T$$

which is compensated by elastic strain, when temperature becomes normal, i.e.,



$$\Delta l = \frac{TL}{YS}$$

$$\text{Thus, } \frac{TL}{YS} = L\alpha\Delta T$$

$$\Rightarrow T = VS\alpha\Delta T$$

At equilibrium, force exerted by one half on other,

$$F = 2T = 2VS\alpha\Delta T$$

TOPIC 2

Specific Heat Capacity, Calorimetry and Change of State

- 11** Due to cold weather a 1 m water pipe of cross-sectional area 1 cm^2 is filled with ice at -10°C . Resistive heating is used to melt the ice. Current of 0.5 A is passed through $4 \text{ k}\Omega$ resistance. Assuming that, all the heat produced is used for melting, what is the minimum time required?

[Given, latent heat of fusion for water/ice $= 3.33 \times 10^5 \text{ J kg}^{-1}$, specific heat of ice $= 2 \times 10^3 \text{ J kg}^{-1}$ and density of ice $= 10^3 \text{ kg/m}^3$]

[2021, 1 Sep Shift-II]

- (a) 0.353 s (b) 35.3 s
 (c) 3.53 s (d) 70.6 s

Ans. (b)

Given, the length of the water pipe,
 $L = 1 \text{ m}$

The cross-sectional area of the water pipe, $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$

The temperature of the ice $= -10^\circ\text{C}$

Current passing in the conductor,
 $I = 0.5 \text{ A}$

Resistance of the conductor, $R = 4 \text{ k}\Omega$

The latent heat of fusion for ice,

$$L_f = 3.33 \times 10^5 \text{ J/kg}$$

The density of the ice, $d = 1000 \text{ kg/m}^3$

The specific heat of the ice,

$$c_{p, \text{ice}} = 2 \times 10^3 \text{ J/kg}$$

Heat required to melt the ice at 10°C to 0°C

$$Q = mc_p\Delta T + mL_f$$

$$Q = dVc_p\Delta T + dVL_f$$

$$= 1000 \times 10^{-4} \times 2 \times 10^3 \times (10) + 1000 \times 10^{-4} \times 3.33 \times 10^5$$

$$(\because V = A \times L)$$

$$= 35300 \text{ J}$$

According to the Joule's law of heating,

$$H = I^2 R t$$

$$\Rightarrow 35300 = (0.5)^2 (4000) (t)$$

$$t = 35.3 \text{ s}$$

Thus, the minimum time required to melt the ice is 35.3 s.

- 12** The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall?

[Given, $1 \text{ cal} = 4.2 \text{ J}$ and specific heat of water $= 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$]

[2021, 27 Aug Shift-II]

- (a) 0.147°C (b) 14.76°C
 (c) 1.476°C (d) 0.014°C

Ans. (a)

Given, height of fall, $H = 63 \text{ m}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Specific heat of water, $s = 1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$

$$= 4.2 \times 10^3 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$$

Since, energy, $E = mgH = ms\Delta T$

where, $m = \text{mass}$

and $\Delta T = \text{change in temperature.}$

$$\therefore \Delta T = \frac{gH}{s} = \frac{9.8 \times 63}{4.2 \times 10^3} = 0.147^\circ\text{C}$$

- 13** The temperature of equal masses of three different liquids x, y and z are 10°C , 20°C and 30°C , respectively. The temperature of mixture when x is mixed with y is 16°C and that when y is mixed with z is 26°C . The temperature of mixture when x and z are mixed will be [2021, 26 Aug Shift-II]

- (a) 28.32°C (b) 25.62°C
 (c) 23.84°C (d) 20.28°C

Ans. (c)

Given masses of three liquids are equal, i.e. $m_x = m_y = m_z = m$

Temperature of mixture of liquids x and y, $T_1 = 16^\circ\text{C}$

Temperature of mixture of liquids y and z, $T_2 = 26^\circ\text{C}$

While mixing the liquids x and y, the heat energy lost by liquid y will be equal to the heat energy absorbed by liquid x.

Consider the specific heat capacity of liquid x is s_x , of liquid y is s_y and of liquid z is s_z .

Now, for liquid x and y, using principle of calorimetry,

Heat gained by liquid x = Heat lost by liquid y

$$ms_x(T_1 - 10) = ms_y(20 - T_1)$$

$$\Rightarrow s_x(16 - 10) = s_y(20 - 16)$$

$$\Rightarrow \frac{s_x}{s_y} = \frac{2}{3} \quad \dots(i)$$

Similarly, for liquid y and z, using principle of calorimetry,

Heat gained by liquid y = Heat lost by liquid z

$$ms_y(T_2 - 20) = ms_z(30 - T_2)$$

$$\Rightarrow s_y(26 - 20) = s_z(30 - 26)$$

$$\Rightarrow \frac{s_y}{s_z} = \frac{2}{3} \quad \dots(ii)$$

Multiply Eq. (i) by Eq. (ii), we get

$$\frac{s_x}{s_z} = \frac{4}{9}$$

Consider the mixing of liquids x and z, let the temperature of mixture be T_3 .

Using principle of calorimetry for liquids x and z.

Heat gained by liquid x = Heat lost by liquid z

$$ms_x(T_3 - 10) = ms_z(30 - T_3)$$

$$\Rightarrow s_x(T_3 - 10) = s_z(30 - T_3)$$

$$\Rightarrow \frac{s_x}{s_z} = \frac{30 - T_3}{T_3 - 10}$$

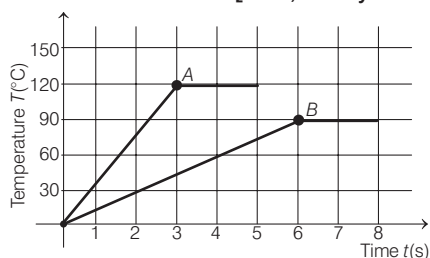
$$\Rightarrow \frac{4}{9} = \frac{30 - T_3}{T_3 - 10}$$

$$\Rightarrow 4T_3 - 40 = 270 - 9T_3$$

$$\Rightarrow T_3 = 23.84^\circ\text{C}$$

- 14** Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is

[2021, 25 July Shift-I]



- (a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Ans. (b)

Let masses of two bodies A and B be m_A and m_B and $m_A = m_B = m$.

As we know that

$$\Delta Q / \Delta t = ms \Delta T / \Delta t$$

where,

$\Delta Q / \Delta t$ = rate of heat change,

m = mass,

s = specific heat

and ΔT = change in temperature.

$$\text{Since, } \left(\frac{\Delta Q}{\Delta t} \right)_A = \left(\frac{\Delta Q}{\Delta t} \right)_B$$

$$\Rightarrow ms_A \left(\frac{\Delta T}{t} \right)_A = ms_B \left(\frac{\Delta T}{t} \right)_B$$

$$\Rightarrow \frac{s_A}{s_B} = \left(\frac{\Delta T}{t} \right)_B \frac{90}{120} = \frac{3}{4} = \frac{3}{4}$$

- 15** Three containers C_1 , C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each container and mixed (assume no loss of heat during the process)

[2020, 8 Jan Shift-II]

C_1	C_2	C_3	T
1l	2l	–	60°C
–	1l	2l	30°C
2l	–	1l	60°C
1l	1l	1l	θ

The value of θ (in $^\circ\text{C}$ to the nearest integer) is

Ans. (50)

We are given with following processes

Container	Volume taken (in litre)			Final temperature
	C_1	C_2	C_3	T
Process I	1	2	–	60°C
Process II	–	1	2	30°C
Process III	2	–	1	60°C
Process IV	1	1	1	θ

Let's assume that temperature of C_1 , C_2 and C_3 are T_1 , T_2 and T_3 .

In process I Heat lost by water from

C_1 = Heat gained by water from C_2

Let masses taken from C_1 and C_2 are m_1 and m_2 , then

$$m_1 s (T_1 - 60^\circ) = m_2 s (60^\circ - T_2)$$

$$V_1 \cdot \rho \cdot s (T_1 - 60^\circ) = V_2 \cdot \rho \cdot s (60^\circ - T_2)$$

Assuming density does not changes very largely

$$\Rightarrow 1(T_1 - 60^\circ) = 2(60^\circ - T_2)$$

$$\Rightarrow T_1 + 2T_2 = 180^\circ \quad \dots(i)$$

Similarly, for processes II and III, we have

$$T_2 + 2T_3 = 90^\circ \quad \dots(ii)$$

$$2T_1 + T_3 = 180^\circ \quad \dots(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$$T_1 + T_2 + T_3 = 150^\circ \quad \dots(iv)$$

Now for process IV,

$$T_1 + T_2 + T_3 = 3\theta \quad \dots(v)$$

From Eqs. (iv) and (v), we have

$$150^\circ = 3\theta \text{ or } \theta = 50^\circ\text{C}$$

- 16** A bullet of mass 5g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One-half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet, if the specific heat of this material is $0.030 \text{ cal g}^{-1}^\circ\text{C}^{-1}$, is close to

[2020, 5 Sep Shift-I]

- (a) 87.5°C
(b) 83.3°C
(c) 119.2°C
(d) 38.4°C

Ans. (a)

We know that,

Change in heat = mass $\times$ specific heat $\times$ change in temperature

$$\text{So, } \Delta Q = (ms\Delta T)_{\text{bullet}} \quad \dots(i)$$

and according to question,

$$\Delta Q = \frac{1}{2} \left(\frac{1}{2} mv^2 \right) \quad \dots(ii)$$

From eqs. (i) and (ii), we get

$$(ms\Delta T)_{\text{bullet}} = \frac{1}{2} \left(\frac{1}{2} mv^2 \right)$$

$$\Rightarrow \Delta T = \frac{v^2}{4s}$$

Given, specific heat, $s = 0.030 \text{ cal/g}^\circ\text{C}$

and $v = 210 \text{ m/s}$

Substituting these values in above equation, we get

$$\Delta T = \frac{(210)^2}{4 \times 4.2 \times 0.03 \times 1000} = 87.5^\circ\text{C}$$

Hence, correct option is (a).

- 17** M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporisation of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is ...

[2020, 7 Jan Shift-II]

Ans. (40)

M grams of steam at 100°C is mixed with 200 g of ice at 0°C and mixture obtained is at 40°C .

So, heat lost by steam in condensation at 100°C + Heat lost by water formed to reach at 40°C = Heat required by ice for melting + Heat gained by water formed at 0°C to reach upto a temperature of 40°C .

$$\Rightarrow M_{\text{steam}} \times L_v + M_{\text{steam}} \times s_w \times \Delta T$$

$$= M_{\text{ice}} \times L_f + M_{\text{ice}} \times s_w \times \Delta T$$

$$\Rightarrow M \times 540 + M \times 1 \times (100 - 40)$$

$$= 200 \times 80 + 200 \times 1 \times (40 - 0)$$

$$\Rightarrow M \times 600 = 24000 \Rightarrow M = 40 \text{ g}$$

- 18** A calorimeter of water equivalent 20 g contains 180 g of water at 25°C . ' m ' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C . The value of m is close to
(Take, latent heat of water = 540 cal g^{-1} , specific heat of water = $1 \text{ cal g}^{-1}^\circ\text{C}^{-1}$) [2020, 3 Sep Shift-II]
- (a) 2 g (b) 3.2 g (c) 2.6 g (d) 4 g

Ans. (a)

Heat lost by m grams of steam is gained by calorimeter and water in it.

(Steam at 100°C) $\xrightarrow{\text{Condensation}}$ (Water at 100°C) $\xrightarrow{\text{Cooling}}$ (Water at 31°C)
 (Calorimeter and water at 25°C) $\xrightarrow{\text{Heating}}$ (Calorimeter and water at 31°C)

Heat lost by steam

$$= mL_1 + m \times S_w \times (100 - 31) \\ = m[L_1 + S_w(100 - 31)] \\ = m[540 + 1 \times (100 - 31)] = m \times 609$$

Heat gained by calorimeter and water in calorimeter

$$= (M + W) \times S \times (31 - 25) \\ = (180 + 20) \times 1 \times 6 = 200 \times 6$$

As, heat lost = heat gained

$$\Rightarrow m \times 609 = 200 \times 6$$

$$\Rightarrow m = \frac{1200}{609} = 1.97 \approx 2 \text{ g}$$

Hence, correct option is (a).

- 19** The specific heat of water = $4200 \text{ J kg}^{-1}\text{K}^{-1}$ and the latent heat of ice = $3.4 \times 10^5 \text{ J kg}^{-1}$. 100 g of ice at 0°C is placed in 200 g of water at 25°C . The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams)

[2020, 4 Sep Shift-I]

- (a) 63.8 (b) 69.3 (c) 64.6 (d) 61.7

Ans. (d)

Let the mass of ice which gets melted be m_i . Now,

Heat lost by water = Heat gained by ice

$$\Rightarrow m_w c_w \Delta T_w = m_i L_i$$

$$\Rightarrow \frac{200}{1000} \times 4200 \times (25 - 0) = m_i \times 3.4 \times 10^5$$

$$\Rightarrow 21000 = m_i \times 3.4 \times 10^5$$

$$\Rightarrow m_i = \frac{21000}{3.4 \times 10^5} = 0.0617 \text{ kg} = 61.7 \text{ g}$$

So, the mass of ice melted = 61.7 g and remaining ice will remain unmelted.

Hence, correct option is (d).

- 20** When M_1 gram of ice at -10°C (specific heat = $0.5 \text{ cal g}^{-1}\text{C}^{-1}$) is added to M_2 gram of water at 50°C , finally no ice is left and the water is at 0°C . The value of latent heat of ice, in cal g^{-1} is [2019, 12 April Shift-I]

- (a) $\frac{50M_2}{M_1} - 5$ (b) $\frac{50M_1}{M_2} - 50$
 (c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

Ans. (a)

Key Idea In such kind of heat transfer problems,

Heat given by water = Heat gained by ice

and (heat)_{liquid} = mass $\times$ specific heat $\times$ temperature = $\{m_l s_l \Delta T_l\}$

(Heat)_{solid} = (mass $\times$ latent heat) + (mass $\times$ specific heat $\times$ temperature) = $(m_s \times L) + m_s \times s_s \times \Delta T_s$

Heat given by water is (specific heat of water is $1 \text{ cal g}^{-1}\text{C}^{-1}$)

$$(\Delta H)_{\text{water}} = M_2 \times 1 \times (50 - 0) = 50M_2 \dots (i)$$

Heat taken by ice is

$$(\Delta H)_{\text{ice}} = M_1 \times 0.5 \times [0 - (-10)] + M_1 \times L_{\text{ice}}$$

$$\Rightarrow (\Delta H)_{\text{ice}} = 5M_1 + M_1 L_{\text{ice}} \dots (ii)$$

Comparing Eqs. (i) and (ii), we get

$$(\Delta H)_{\text{water}} = (\Delta H)_{\text{ice}}$$

$$\therefore 50M_2 = 5M_1 + M_1 L_{\text{ice}}$$

$$\Rightarrow L_{\text{ice}} = \frac{50M_2 - 5M_1}{M_1}$$

$$\Rightarrow L_{\text{ice}} = 50 \frac{M_2}{M_1} - 5$$

- 21** When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C , the temperature of the mixture becomes 90°C . The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C will be

[2019, 11 Jan Shift-II]

- (a) 60°C (b) 80°C (c) 70°C (d) 85°C

Ans. (b)

In first case according to principle of calorimetry,

heat lost by liquid A = heat gained by liquid B

$$\text{or } m_A S_A \Delta T_A = m_B S_B \Delta T_B$$

where, S_A is specific heat capacity of A and S_B is specific heat capacity of B

$$\Rightarrow 100 \times S_A (100 - 90) = 50 \times S_B (90 - 75)$$

$$\Rightarrow 1000 S_A = 50 \times 15 S_B$$

$$\text{or } 4S_A = 3S_B \dots (i)$$

Similarly, in second case,

$$100 \times S_A (100 - T) = 50 \times S_B (T - 50)$$

where, T = Final temperature of the mixture.

$$\Rightarrow 4S_A (100 - T) = 2S_B (T - 50)$$

Using Eq. (i),

$$3S_B (100 - T) = 2S_B (T - 50)$$

$$\text{or } 300 - 3T = 2T - 100$$

$$\text{or } 5T = 400 \text{ or } T = 80^\circ\text{C}$$

- 22** An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C . Calculate the specific heat of the unknown metal, if water temperature stabilises at 21.5°C . (Take, specific heat of brass is $394 \text{ J kg}^{-1}\text{K}^{-1}$)

[2019, 10 Jan Shift-II]

- (a) $916 \text{ J kg}^{-1}\text{K}^{-1}$ (b) $654 \text{ J kg}^{-1}\text{K}^{-1}$
 (c) $1232 \text{ J kg}^{-1}\text{K}^{-1}$ (d) $458 \text{ J kg}^{-1}\text{K}^{-1}$

Ans. (a)

Key Idea The principle of calorimetry states that total heat lost by the hotter body equals to the total heat gained by colder body, provided that there is no exchange of heat with the surroundings.

Let specific heat of unknown metal is s and heat lost by this metal is ΔQ .

Heat lost and specific heat of a certain material/substance are related as

$$\Delta Q = ms\Delta T \dots (i)$$

For unknown metal, $m = 192 \text{ g}$ and

$$\Delta T = (100 - 21.5)^\circ\text{C}$$

$$\therefore \Delta Q = 192(100 - 21.5) \times s \dots (ii)$$

Now, this heat is gained by the calorimeter and water inside it.

As, heat gained by calorimeter can be calculated by Eq. (i).

So, for brass specific heat,

$$s = 394 \text{ J kg}^{-1}\text{K}^{-1} \text{ (given)}$$

$$= 0.394 \text{ J g}^{-1}\text{K}^{-1}$$

Mass of calorimeter, $m = 128 \text{ g}$

Change in temperature,

$$\Delta T = (21.5 - 8.4)^\circ\text{C}$$

So, using Eq. (i) for calorimeter, heat gained by brass

$$\Delta Q_1 = 128 \times 0.394 \times (21.5 - 8.4) \dots (iii)$$

Heat gained by water can be calculated as follows

mass of water, $m = 240 \text{ g}$,

specific heat of water,

$$s = 4.18 \text{ J g}^{-1}\text{K}^{-1},$$

change in temperature,

$$\Delta T = (21.5 - 8.4)^\circ\text{C}$$

Using Eq. (i) for water also, we get

heat gained by water,

$$\Delta Q_2 = 240 \times 4.18 \times (21.5 - 8.4) \dots (iv)$$

Now, according to the principle of calorimeter, the total heat gained by the

calorimeter and water must be equal to heat lost by unknown metal

$$\Delta Q' = \Delta Q_1 + \Delta Q_2$$

Using Eqs. (ii), (iii) and (iv), we get

$$\begin{aligned} \Rightarrow &= 192(100 - 21.5) \times s \\ &= 128 \times 0.394 \times (21.5 - 8.4) + 240 \\ &\quad \times 4.18 \times (21.5 - 8.4) \\ \Rightarrow &15072 s = 660.65 + 13142 \\ \Rightarrow &s = 0.916 \text{ J g}^{-1} \text{ K}^{-1} \\ \text{or} &s = 916 \text{ J kg}^{-1} \text{ K}^{-1}. \end{aligned}$$

- 23** Ice at -20°C is added to 50 g of water at 40°C . When the temperature of the mixture reaches 0°C , it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Take, specific heat of water $= 4.2 \text{ J/g}^\circ\text{C}$ specific heat of ice $= 2.1 \text{ J/g}^\circ\text{C}$ and heat of fusion of water at $0^\circ\text{C} = 334 \text{ J/g}$)

[2019, 11 Jan Shift-I]

- (a) 40 g (b) 50 g (c) 60 g (d) 100 g

Ans. (a)

Let amount of ice be 'x' gm.

According to the principle of calorimeter,

heat lost by water = heat gained by ice

Here, heat lost by water,

$$\Delta Q = m s_{\text{water}} \Delta T$$

Substituting the given values, we get

$$\Delta Q = 50 \times 4.2 \times 40$$

Heat gained by ice,

$$\begin{aligned} \Delta Q &= x s_{\text{ice}} \Delta T + (x - 20) L \\ &= x \times 2.1 \times 20 + (x - 20) \times 334 \\ &= 20 \times 2.1 + 334x - 6680 \end{aligned}$$

$$\therefore 20 \times 2.1 + 334x - 6680 = 50 \times 4.2 \times 40$$

$$42x + 334x - 6680 = 8400$$

$$\Rightarrow 376x = 15080 \text{ or } x = 40.10 \text{ g} \\ x \approx 40 \text{ g}$$

- 24** A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity 800 JK^{-1} and containing 0.5 kg water. The initial temperature of water and vessel is 30°C . What is the approximate percentage increment in the temperature of the water?

[Take, specific heat capacities of

water and metal are respectively $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ and $400 \text{ J kg}^{-1} \text{ K}^{-1}$]

[2019, 11 Jan Shift-II]

- (a) 25% (b) 15% (c) 30% (d) 20%

Ans. (d)

Using heat lost or gained without change in state is $\Delta Q = ms\Delta T$, where s is specific heat capacity and T = change in temperature

Let final temperature of ball be T .

Then heat lost by ball is,

$$\Delta Q = 0.1 \times 400(500 - T) \quad \dots(i)$$

This lost heat by ball is gained by water and vessel and given as

Heat gained by water,

$$\Delta Q_1 = 0.5 \times 4200(T - 30) \quad \dots(ii)$$

and heat gained by vessel is

$$\begin{aligned} \Delta Q_2 &= \text{heat capacity} \times \Delta T \\ &= 800 \times (T - 30) \quad \dots(iii) \end{aligned}$$

According to principle of calorimetry, total heat lost = total heat gained

$$\begin{aligned} \Rightarrow 0.1 \times 400(500 - T) \\ &= 0.5 \times 4200(T - 30) + 800(T - 30) \\ \Rightarrow (500 - T) &= \frac{(2100 + 800)(T - 30)}{40} \end{aligned}$$

$$\Rightarrow 500 - T = 72.5(T - 30)$$

$$\Rightarrow 500 + 217.5 = 72.5T \text{ or } T = 36.39 \text{ K}$$

So, percentage increment in temperature of water

$$= \frac{36.39 - 30}{30} \times 100 \approx 20\%$$

- 25** A copper ball of mass 100 g is at a temperature T . It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C . T is (Given, room temperature $= 30^\circ\text{C}$, specific heat of copper $= 0.1 \text{ cal/g}^\circ\text{C}$)

[JEE Main 2017]

- (a) 885°C (b) 1250°C
(c) 825°C (d) 800°C

Ans. (a)

Heat gained (water + calorimeter) = Heat lost by copper ball

$$\begin{aligned} \Rightarrow m_w s_w \Delta T + m_c s_c \Delta T &= m_B s_B \Delta T \\ \Rightarrow 170 \times 1 \times (75 - 30) + 100 \times 0.1 \times (75 - 30) \\ &= 100 \times 0.1 \times (T - 75) \end{aligned}$$

$$\therefore T = 885^\circ\text{C}$$

TOPIC 3

Heat Transfer

- 26** Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature θ_2 ($\theta_1 < \theta_2$). The rate at which heat flows radially through the material is

[2021, 31 Aug Shift-II]

$$(a) \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$

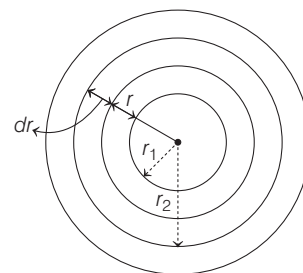
$$(b) \frac{\pi r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$

$$(c) \frac{K(\theta_2 - \theta_1)}{r_2 - r_1}$$

$$(d) \frac{K(\theta_2 - \theta_1)(r_2 - r_1)}{4\pi r_1 r_2}$$

Ans. (a)

Given that, r_1 and r_2 be the radii of inner and outer shells.



A material of thermal conductivity (K) is filled between region.

Consider an elementary sphere of radius r and thickness dr .

Now, thermal resistance of elementary

$$\text{sphere } dR = \frac{dr}{K 4\pi r^2}$$

[$\therefore$ inner surface area, $A = 4\pi r^2$]

So, total thermal resistance,

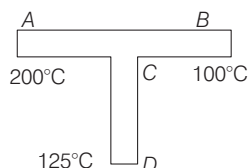
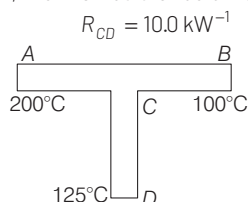
$$\begin{aligned} R &= \int dR = \int_{r_1}^{r_2} \frac{dr}{4\pi K r^2} = -\frac{1}{4\pi K} \left[\frac{1}{r} \right]_{r_1}^{r_2} \\ R &= \frac{r_2 - r_1}{4\pi K r_1 r_2} \end{aligned}$$

Hence, heat current (or the rate of flow of heat)

$$\frac{dQ}{dt} = \frac{\Delta \theta}{R} = \frac{(\theta_2 - \theta_1)}{\left(\frac{r_2 - r_1}{4\pi K r_1 r_2} \right)} = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}$$

- 27** A rod CD of thermal resistance 10.0 kW^{-1} is joined at the middle of an identical rod AB as shown in figure. The end A, B and D are maintained at 200°C , 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is

[2021, 27 Aug Shift-I]

**Ans. (2)**Given, thermal resistance of rod CD ,Temperature of end $A, T_A = 200^\circ\text{C}$ Temperature of end $B, T_B = 100^\circ\text{C}$ Temperature of end $D, T_D = 125^\circ\text{C}$

As rods are given as identical, then

$$R_{CD} = R_{CB} = 10 \text{ kW}^{-1}$$

As, C is mid-point of AB , and as we know

$$R \propto \text{length}$$

$$R_{AC} = R_{BC} = 5 \text{ kW}^{-1}$$

Now, we know that heat current is given as

$$\frac{Q}{t} = \frac{\Delta T}{R}$$

Here, ΔT is temperature difference and R is thermal resistance.Now, for point C , consider temperature of point C as T .Consider at point C heat is coming in from A and going out at point B and D .

Therefore, we can write,

$$\begin{aligned} \frac{T_A - T}{R_{AC}} &= \frac{T - T_D}{R_{CD}} = \frac{T - T_B}{R_{BC}} \\ \Rightarrow \frac{200 - T}{5} &= \frac{T - 125}{10} + \frac{T - 100}{5} \\ \Rightarrow \frac{200 - T}{5} &= \frac{T - 125}{10} + \frac{T - 100}{5} \\ \Rightarrow 400 - 2T &= T - 125 + 2T - 200 \\ 5T &= 725^\circ\text{C} \\ T &= 145^\circ\text{C} \end{aligned}$$

The heat current in CD is

$$I_{CD} = \frac{T - T_D}{R_{CD}} = \frac{145 - 125}{10} = \frac{20}{10} = 2 \text{ W}$$

Thus, the value of P is 2.

- 28** A body takes 4 min to cool from 61°C to 59°C . If the temperature of the surroundings is 30°C , then the time taken by the body to cool from 51°C to 49°C is

[2021, 27 July Shift-I]

- (a) 4 min (b) 3 min
(c) 8 min (d) 6 min

Ans. (d)As per question, a body takes 4 min to cool from 61°C to 59°C . Temperature of surroundings is 30°C . $\therefore$ We know that,

$$\frac{\Delta T}{\Delta t} = K(T_t - T_s)$$

where, ΔT = change in temperature, T_t = average temperature,and T_s = surrounding temperature Δt = change in time.

$$\Rightarrow \frac{61^\circ - 59^\circ}{4} = K \left(\frac{61^\circ + 59^\circ}{2} - 30^\circ \right) \dots (i)$$

$$\text{and } \frac{51^\circ - 49^\circ}{t} = K \left(\frac{51^\circ + 49^\circ}{2} - 30^\circ \right) \dots (ii)$$

On dividing Eqs. (i) and (ii), we get

$$\frac{\frac{61^\circ - 59^\circ}{4}}{\frac{51^\circ - 49^\circ}{t}} = \frac{K \left(\frac{61^\circ + 59^\circ}{2} - 30^\circ \right)}{K \left(\frac{51^\circ + 49^\circ}{2} - 30^\circ \right)}$$

On solving, we will get $t = 6 \text{ min}$

- 29** In 5 min, a body cools from 75°C to 65°C at room temperature of 25°C . The temperature of body at the end of next 5 min is

[2021, 22 July Shift-II]

Ans. (57)When, $t = 5 \text{ min}$, $T_1 = 75^\circ\text{C}$, $T_2 = 65^\circ\text{C}$, then

By using Newton's law of cooling,

$$\frac{T_1 - T_2}{t} = k \left[\left(\frac{T_1 + T_2}{2} \right) - T_s \right] \dots (i)$$

$$\Rightarrow \frac{75 - 65}{5} = k \left[\frac{140}{2} - 25 \right]$$

$$\Rightarrow \frac{10}{5} = k(70 - 25) \Rightarrow k = \frac{2}{45}$$

If T_3 be the temperature of body in next 5 min, then

By using Eq. (i) again

$$\frac{65 - T_3}{5} = \frac{2}{45} \left[\frac{65 + T_3}{2} - 25 \right]$$

$$\Rightarrow 9(65 - T_3) = 2 \left[\frac{65 + T_3}{2} - 25 \right]$$

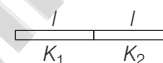
$$\Rightarrow 585 - 9T_3 = 15 + T_3 \Rightarrow 570 = 10T_3$$

$$\Rightarrow T_3 = 57^\circ\text{C}$$

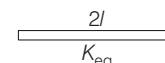
- 30** Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is

[2021, 17 March Shift-I]

- (a) $\frac{2K_1K_2}{K_1 + K_2}$ (b) $\frac{K_1 + K_2}{2K_1K_2}$
(c) $\frac{K_1 + K_2}{K_1K_2}$ (d) $\frac{K_1K_2}{K_1 + K_2}$

Ans. (a)Two identical metal wires of thermal conductivities K_1 and K_2 respectively connected in series are represented as follows

The above figure can also be represented as



Therefore, the effective thermal conductivity of the combination will be given by

$$\begin{aligned} R_{\text{eff}} &= \frac{l}{K_1A} + \frac{l}{K_2A} = \frac{2l}{K_{\text{eq}}A} \\ \Rightarrow \frac{2l}{K_{\text{eq}}A} &= \frac{l}{A} \left(\frac{1}{K_1} + \frac{1}{K_2} \right) \\ \Rightarrow \frac{2l}{K_{\text{eq}}A} &= \frac{l}{A} \left(\frac{K_1 + K_2}{K_1K_2} \right) \\ \Rightarrow \frac{2}{K_{\text{eq}}} &= \frac{K_1 + K_2}{K_1K_2} \\ \Rightarrow K_{\text{eq}} &= \frac{2K_1K_2}{K_1 + K_2} \end{aligned}$$

- 31** The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by

[2021, 26 Feb Shift-I]



- (a) $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$ (b) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$
(c) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$ (d) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$

Ans. (c)Let, Q = heat current, k = thermal conductivity, A = area, l = length of capacitorand $\Delta\theta$ = change in temperature.

$$\therefore Q = \frac{kA\Delta\theta}{l} = \frac{\Delta\theta}{R} \quad \left(\because R = \frac{l}{kA} \right)$$

$$\Rightarrow \frac{\theta_2 - \theta}{R_2} = \frac{\theta - \theta_1}{R_1}$$

$$\Rightarrow R_1\theta_2 - R_1\theta = R_2\theta - R_2\theta_1$$

$$\Rightarrow \theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$$

- 32** A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C , then the sphere's temperature after the next 5 min will be close to

[2020, 3 Sep Shift-II]

- (a) 31°C (b) 35°C
(c) 28°C (d) 33°C

Ans. (d)

By Newton's law of cooling, the rate of fall of temperature of an object is proportional to the difference of temperature of object and its surroundings.

$$\Rightarrow \frac{\theta_i - \theta_f}{\Delta t} = k(\theta_m - \theta_s)$$

where, θ_i = initial temperature of body = 50°C ,

 θ_f = final temperature of body = 40°C , θ_m = mean temperature of body

$$= \frac{(50 + 40)}{2} = \frac{90}{2} = 45^\circ\text{C},$$

θ_s = temperature of surroundings = 20°C ,

 Δt = time interval = 300 sand k = proportionality constant.

$$\text{So, } \frac{50 - 40}{300} = k(45 - 20) \quad \dots(i)$$

Now, after next 5 min (= 300 s), if object's temperature is T , then

$$\frac{40 - T}{300} = k\left(\frac{40 + T}{2} - 20\right) \quad \dots(ii)$$

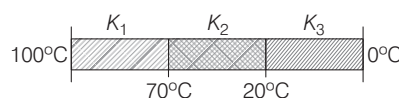
Dividing Eq. (i) by Eq. (ii), we have

$$\frac{50 - 40}{40 - T} = \frac{45 - 20}{\left(\frac{40 + T}{2} - 20\right)}$$

$$\Rightarrow 10\left(\frac{T}{2}\right) = 25(40 - T) \Rightarrow T = \frac{100}{3} = 33^\circ\text{C}$$

Hence, correct option is (d).

- 33** Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3 , respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is

[2020, 6 Sep Shift-II]

(a) $K_1 : K_3 = 2 : 3; K_2 : K_3 = 2 : 5$

(b) $K_1 < K_2 < K_3$

(c) $K_1 : K_2 = 5 : 2; K_1 : K_3 = 3 : 5$

(d) $K_1 > K_2 > K_3$

Ans. (a)

$$\text{Rate of heat transfer, } H = \frac{KA(\theta_H - \theta_L)}{L}$$

At steady state, rate of heat transfer through each rod is same.

$$\text{i.e., } H_1 = H_2 = H_3$$

For same cross-section and lengths,

$$K \times (\theta_H - \theta_L) = \text{constant}$$

$$K_1(100 - 70) = K_2(70 - 20) = K_3(20 - 0)$$

$$K_1 \times 30 = K_2 \times 50 = K_3 \times 20$$

$$3K_1 = 2K_3$$

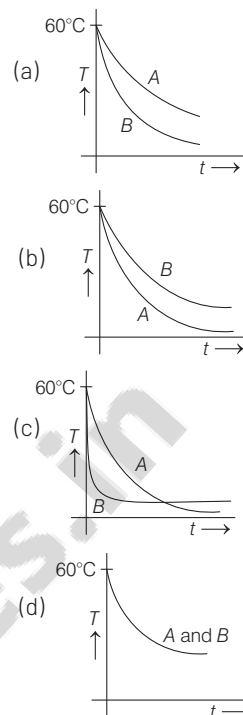
$$\text{or } \frac{K_1}{K_3} = \frac{2}{3} \quad \text{or } K_1 : K_3 = 2 : 3$$

$$\text{or } 5K_2 = 2K_3 \quad \text{or } \frac{K_2}{K_3} = \frac{2}{5}$$

$$\text{or } K_2 : K_3 = 2 : 5$$

Hence, correct option is (a).

- 34** Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of $8 \times 10^2 \text{ kg/m}^3$ and specific heat of $2000 \text{ J kg}^{-1}\text{K}^{-1}$ while liquid in B has density of 10^3 kg/m^3 and specific heat of $4000 \text{ J kg}^{-1}\text{K}^{-1}$. Which of the following best describes their temperature versus time graph schematically? (Assume the emissivity of both the beakers to be the same)

[2019, 8 April Shift-I]**Ans. (b)**

Key Idea From Newton's law of cooling, we have rate of cooling,

$$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0)$$

where, h = heat transfer coefficient, T = temperature of body, T_0 = temperature of surrounding, m = mass and s = specific heat.We know, $m = V \cdot \rho$ where, V = volume and ρ = density.

So, we have

$$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0) = \frac{h(T - T_0)}{V \cdot \rho s}$$

Since, $h, (T - T_0)$ and V are constant for both beaker.

$$\therefore \frac{dQ}{dt} \propto \frac{1}{\rho s}$$

We have given that $\rho_A = 8 \times 10^2 \text{ kgm}^{-3}$, $\rho_B = 10^3 \text{ kgm}^{-3}$, $s_A = 2000 \text{ J kg}^{-1}\text{K}^{-1}$ and $s_B = 4000 \text{ J kg}^{-1}\text{K}^{-1}$,

$$\rho_A s_A = 16 \times 10^5 \text{ and } \rho_B s_B = 4 \times 10^6$$

So, $\rho_A < \rho_B$, $s_A < s_B$ and $\rho_A s_A < \rho_B s_B$

$$\Rightarrow \frac{1}{\rho_A s_A} > \frac{1}{\rho_B s_B}$$

$$\Rightarrow \frac{dQ_A}{dt} > \frac{dQ_B}{dt}$$

So, for container B, rate of cooling is smaller than the container A. Hence, graph of B lies above the graph of A and it is not a straight line (slope of A is greater than B).

- 35** Two materials having coefficients of thermal conductivity $3K'$ and K' and thickness ' d ' and $3d$ ' respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2' and θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is

[2019, 9 April Shift-II]

- (a) $\frac{\theta_2 + \theta_1}{2}$ (b) $\frac{\theta_1 + 2\theta_2}{3}$
 (c) $\frac{\theta_1 + 5\theta_2}{6}$ (d) $\frac{\theta_1 + 9\theta_2}{10}$

Ans. (d)

Let interface temperature in steady state conduction is θ , then assuming no heat loss through sides;

$$\left(\begin{array}{c} \text{Rate of heat} \\ \text{flow through} \\ \text{first slab} \end{array} \right) = \left(\begin{array}{c} \text{Rate of heat} \\ \text{flow through} \\ \text{second slab} \end{array} \right)$$

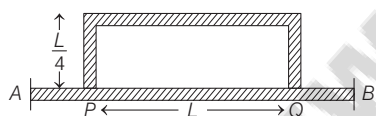
$$\Rightarrow \frac{(3K) A(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$$

$$\Rightarrow 9(\theta_2 - \theta) = \theta - \theta_1 \Rightarrow 9\theta_2 + \theta_1 = 10\theta$$

$$\Rightarrow \theta = \frac{9}{10}\theta_2 + \frac{1}{10}\theta_1$$

- 36** Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ , of same cross-section as AB and length $\frac{3L}{2}$ is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to

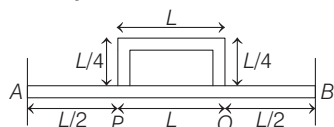
[2019, 9 Jan Shift-I]



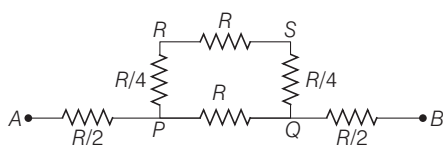
- (a) 45° (b) 35° (c) 75° (d) 60°

Ans. (a)

According to the given question, the given figure with its length for each section is given as below



The above figure considering that every section has the same thermal conductivity, then in terms of thermal resistance is shown in the figure below,



Net resistance of the section $PQRS$ is

$$= \frac{R \times \frac{3R}{2}}{\frac{5R}{2}} = \frac{3R}{5} \quad \dots(i)$$

Total resistance of the net network, R_{net}

$$= \frac{R}{2} + \frac{R}{2} + \frac{3R}{5} = \frac{8R}{5}$$

$\therefore$ Thermal current,

$$I = \frac{\Delta T_{AB}}{R_{\text{net}}} = \frac{120 - 0}{\left(\frac{8R}{5}\right)} = \frac{120 \times 5}{8R}$$

Thus, the net temperature difference between point P and Q is

$$T_P - T_Q = I \times \frac{3R}{5} \quad [\text{using Eq. (i)}]$$

$$= \frac{120 \times 5}{8R} \times \frac{3R}{5} = 45^\circ\text{C}$$

- 37** A heat source at $T = 10^3$ K is

connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is $0.1 \text{ WK}^{-1}\text{m}^{-1}$, the energy flux through it in the steady state is

[2019, 10 Jan Shift-I]

- (a) 90 Wm^{-2} (b) 65 Wm^{-2}
 (c) 120 Wm^{-2} (d) 200 Wm^{-2}

Ans. (a)

Energy flux is the rate of heat flow per unit area through the rod.

Also, rate of flow of heat per unit time through a material of area of cross-section A and thermal conductivity k between the temperatures T_1 and T_2 ($T_1 > T_2$) is given as,

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{l} \quad \dots(i)$$

Energy flux using Eq. (i), we get

$$= \frac{1}{A} \cdot \frac{\Delta Q}{\Delta t} = \frac{k(T_1 - T_2)}{l}$$

Here, $k = 0.1 \text{ K}^{-1}\text{m}^{-1}$, $l = 1 \text{ m}$

$$T_1 = 1000 \text{ K and } T_2 = 100 \text{ K}$$

$$\therefore \text{Energy flux} = \frac{0.1(1000 - 100)}{1} = 90 \text{ Wm}^{-2}$$

- 38** A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2 . Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is

[2019, 12 Jan Shift-I]

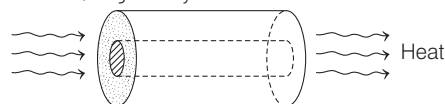
- (a) $\frac{K_1 + K_2}{2}$ (b) $\frac{K_1 + 3K_2}{4}$
 (c) $\frac{2K_1 + 3K_2}{5}$ (d) $K_1 + K_2$

Ans. (b)

Both the given cylinders are in parallel as heat flow is given along length. In parallel, equivalent thermal conductivity of system is

$$K_{\text{eq}} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

So, in given system



$$K_{\text{eq}} = \frac{K_1(\pi R^2) + K_2[\pi(2R)^2 - \pi R^2]}{(\pi R^2) + (4\pi R^2 - \pi R^2)}$$

$$\text{or } K_{\text{eq}} = \frac{K_1 + 3K_2}{4}$$

- 39** Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2 . End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C . Lengths of the copper, brass and steel rods are 46, 13 and 12 cm, respectively. The rods are thermally insulated from surrounding except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units, respectively. Rate of heat flow through copper rod is

[JEE Main 2014]

- (a) 1.2 cal/s (b) 2.4 cal/s
 (c) 4.8 cal/s (d) 6.0 cal/s

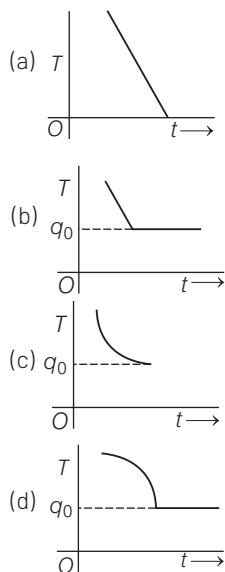
Ans. (c)

$$\text{Then, } \frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$$

According to thermal conductivity, we get

$$\begin{aligned}
 & \begin{array}{c} 100^\circ\text{C} \\ \downarrow l_1 \\ K_1 \frac{dQ_1}{dt} \\ \swarrow \searrow \\ K_2 \frac{dQ_2}{dt} \quad K_2 \frac{dQ_3}{dt} \\ \downarrow \downarrow \\ 0^\circ\text{C} \quad 0^\circ\text{C} \\ l_2 \quad l_3 \end{array} \\
 & \frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt} \\
 & \frac{0.92(100-T)}{46} = \frac{0.26(T-0)}{13} + \frac{0.12(T-0)}{12} \\
 & T = 40^\circ\text{C} \\
 \therefore \frac{dQ_1}{dt} &= \frac{0.92 \times 4(100-40)}{46} \\
 &= 4.8 \text{ cal/s}
 \end{aligned}$$

- 40** If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 , the graph between the temperature T of the metal and time t will be closed to [JEE Main 2013]



Ans. (c)

According to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and surroundings.

$$\frac{dQ}{dt} = k(T_2 - T_1) \quad \dots(i)$$

$$\begin{aligned}
 & \text{and } dQ = msdT_2 \\
 \therefore \frac{dQ}{dt} &= \frac{msdT_2}{dt} \quad \dots(ii)
 \end{aligned}$$

From Eqs. (i) and (ii), we get

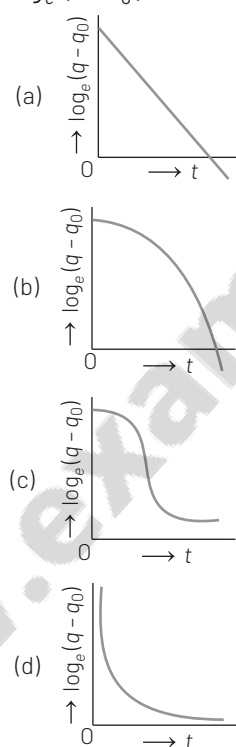
$$\begin{aligned}
 -\frac{msdT_2}{dt} &= k(T_2 - T_1) \\
 \Rightarrow \frac{dT_2}{T_2 - T_1} &= -\frac{k}{ms} dt = -K dt \left[\because K = \frac{k}{ms} \right]
 \end{aligned}$$

On integrating, we get

$$\begin{aligned}
 \log_e(T_2 - T_1) &= -Kt + C \\
 \text{or } T_2 - T_1 &= C'e^{-Kt} \quad [\text{where, } C' = e^C]
 \end{aligned}$$

This variation is correctly represented in option (c).

- 41** A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling, the correct graph between $\log_e(\theta - \theta_0)$ and t is [AIEEE 2012]



Ans. (a)

According to Newton's law of cooling, rate of fall in temperature is proportional to the difference in temperature of the body with surrounding, i.e.,

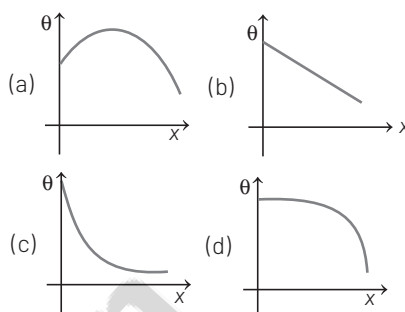
$$-\frac{d\theta}{dt} = k(\theta - \theta_0) \Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k dt$$

$$\Rightarrow \ln(\theta - \theta_0) = kt + C$$

which is a straight line with negative slope.

- 42** A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along

the length x of the bar from its hot end is best described by which of the following figure. [AIEEE 2009]



Ans. (b)

We know that,

Rate of flow of heat,

$$\frac{dQ}{dt} = kA \frac{d\theta}{dx}$$

In steady state, flow of heat,

$$d\theta = \frac{dQ}{dt} \cdot \frac{1}{kA} dx$$

$$\Rightarrow \theta_H - \theta = k'x$$

$$\Rightarrow \theta = \theta_H - k'x$$

Equation $\theta = \theta_H - k'x$ represents a straight line.

- 43** One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2 . The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is [AIEEE 2007]

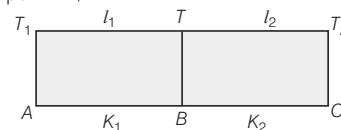


$$\begin{aligned}
 (a) & \frac{(K_2 l_2 T_1 + K_1 l_1 T_2)}{(K_1 l_1 + K_2 l_2)} & (b) & \frac{(K_2 l_1 T_1 + K_1 l_2 T_2)}{(K_2 l_1 + K_1 l_2)} \\
 (c) & \frac{(K_1 l_2 T_1 + K_2 l_1 T_2)}{(K_1 l_2 + K_2 l_1)} & (d) & \frac{(K_1 l_1 T_1 + K_2 l_2 T_2)}{(K_1 l_1 + K_2 l_2)}
 \end{aligned}$$

Ans. (c)

Let temperature at the interface be T .

For part AB,



As rate of flow of heat,

$$\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$$

For part BC, $\frac{Q_2}{t} \propto \frac{(T - T_2)K_2}{l_2}$

At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$

$$\Rightarrow \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$

or $T = \frac{T_1 K_1 l_2 + T_2 K_2 l_1}{K_1 l_2 + K_2 l_1}$

- 44** Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun.

[AIEEE 2006]

(a) $\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (b) $\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$
 (c) $\frac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$ (d) $\frac{R^2 \sigma T^4}{r^2}$

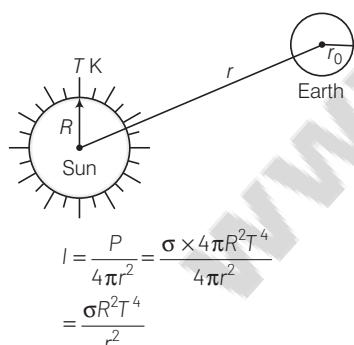
where, r_0 is the radius of the earth and σ is Stefan's constant.

Ans. (b)

From Stefan's law, the rate at which energy is radiated by sun at its surface is $P = \sigma \times 4\pi R^2 \times T^4$

[sun is a perfectly black body as it emits radiations of all wavelengths and so for it, $e = 1$]

The intensity of this power at earth's surface (under the assumption $r \gg r_0$) is

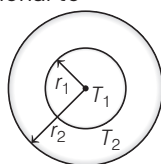


The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be πr_0^2 .

$\therefore$ The total radiant power as received by earth

$$= \pi r_0^2 \times I = \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2} = \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

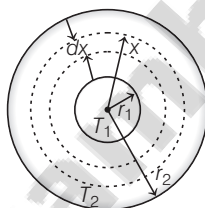
- 45** The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 , respectively. The radial rate of flow of heat in a substance between the two concentric spheres, is proportional to [AIEEE 2005]



(a) $\frac{(r_2 - r_1)}{(r_1 r_2)}$ (b) $\ln\left(\frac{r_2}{r_1}\right)$
 (c) $\frac{r_1 r_2}{(r_2 - r_1)}$ (d) $(r_2 - r_1)$

Ans. (c)

To measure the radial rate of heat flow, we have to go for integration technique as here, the area of the surface through which heat will flow is not constant.



Let us consider an element (spherical shell) of thickness dx and radius x as shown in figure. Let us first find the equivalent thermal resistance between inner and outer spheres.

Thermal resistance of shell

$$= dR = \frac{dx}{K \times 4\pi x^2}$$

[from $R = \frac{l}{KA}$
 where, K is thermal conductivity]

$$\Rightarrow \int dR = R = \int_{r_1}^{r_2} \frac{dx}{4\pi K x^2} = \frac{1}{4\pi K} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{r_2 - r_1}{4\pi K (r_1 r_2)}$$

Rate of heat flow $= H = \frac{T_1 - T_2}{R} = \frac{T_1 - T_2}{\frac{r_2 - r_1}{4\pi K (r_1 r_2)}} \propto \frac{r_1 r_2}{r_2 - r_1}$

- 46** If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be [AIEEE 2004]

(a) 4 (b) 16 (c) 32 (d) 64

Ans. (d)

From Stefan's law, the energy radiated by sun is given by $P = \sigma e A T^4$.

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi (2R)^2 \times (2T)^4 = \sigma e \times 4\pi R^2 \times T^4 \times 64$

$= 64P_1$

The rate at which energy received at earth is

$$E = \frac{P}{4\pi R_{SE}^2} \times A_E$$

where, A_E = area of earth

and R_{SE} = distance between sun and earth

So, in 1st case,

$$E_1 = \frac{P_1}{4\pi R_{SE}^2} \times A_E$$

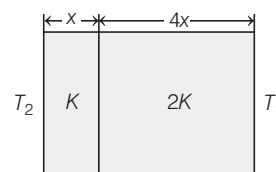
$$E_2 = \frac{P_2}{4\pi R_{SE}^2} \times A_E = 64E_1$$

Hence, ratio is 64.

- 47** The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$ respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, then

f is equal to

[AIEEE 2004]



(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

Ans. (d)

Let the temperature of common interface be $T^\circ\text{C}$. Rate of heat flow

$$H = \frac{Q}{t} = \frac{KA \Delta T}{l}$$

$$\therefore H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2KA(T - T_1)}{4x}$$

and $H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA(T_2 - T)}{x}$

In steady state, the rate of heat flow should be same in whole system i.e.,

$$\Rightarrow \frac{2KA(T - T_1)}{4x} = \frac{KA(T_2 - T)}{x}$$

or $\frac{T - T_1}{2} = T_2 - T$

or $T - T_1 = 2T_2 - 2T$

or $T = \frac{2T_2 + T_1}{3} \quad \dots(i)$

Hence, heat flow from composite slab is

$$\begin{aligned} H &= \frac{KA(T_2 - T)}{x} \\ &= \frac{KA}{x} \left(T_2 - \frac{2T_2 + T_1}{3} \right) \\ &= \frac{KA}{3x} (T_2 - T_1) \quad \dots(ii) \end{aligned}$$

Accordingly, $H = \left[\frac{A(T_2 - T_1)K}{x} \right] f \quad \dots(iii)$

By comparing Eqs. (ii) and (iii), we get

$$f = \frac{1}{3}$$

- 48** According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, then n is equal to

[AIEEE 2003]

- (a) 2 (b) 3
(c) 4 (d) 1

Ans. (d)

According to Newton's law of cooling,

Rate of loss of heat, $\frac{dQ}{dt} \propto \Delta\theta$

But $\frac{dQ}{dt} \propto (\Delta\theta)^n$ [given]

$\therefore n = 1$

- 49** The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by

- (a) Rayleigh Jeans law [AIEEE 2003]
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law

Ans. (a)

The spectrum is correctly given by Rayleigh Jeans law

- 50** Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K, respectively. The ratio of the energy radiated per second by the first sphere to that by the second is

[AIEEE 2002]

- (a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9

Ans. (a)

Energy radiated per second by a body which has surface area A at temperature T is given by Stefan's law,

$$E = \sigma AT^4 \quad [\text{as } A = \pi r^2]$$

Therefore, $\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4$

$$= \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4$$

[since, bodies are of same material, so $e_1 = e_2$]

$$\Rightarrow \frac{E_1}{E_2} = \frac{16}{16} = \frac{1}{1} = 1:1$$

- 51** If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should [AIEEE 2002]

- (a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease

Ans. (a)

According to the mass-energy equivalence, mass and energy remain conserved. So, when water is cooled to form ice, water loses its energy, so change in energy increases the mass of water.

- 52** Which of the following is more close to a black body? [AIEEE 2002]

- (a) Black board paint
(b) Green leaves
(c) Black holes
(d) Red roses

Ans. (c)

Black body is one which absorbs all radiations and emits none, out of the four examples given black holes are more close to a black body.

11

Thermodynamics

TOPIC 1

Zeroth and First Law of Thermodynamics

- 01** An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by 2.5×10^3 J? [2021, 26 Aug Shift-I]
- (a) 2.5×10^2 s (b) 4.1×10^1 s
(c) 2.4×10^3 s (d) 2.5×10^1 s

Ans. (a)

Given, heat supplied to the system,
 $\frac{\Delta Q}{\Delta t} = 6000 \text{ J/min} = \frac{6000}{60} \text{ J/s} = 100 \text{ J/s}$

Power

delivered, $P = \frac{\Delta W}{\Delta t} = 90 \text{ W}$

Increase in internal energy,
 $\Delta U = 2.5 \times 10^3 \text{ J}$

From first law of thermodynamics, we have

$$\Delta Q = \Delta U + \Delta W$$

$$\text{or } \frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t} \quad \dots(i)$$

Substituting the given values in Eq. (i), we get

$$100 = \frac{2.5 \times 10^3}{\Delta t} + 90$$

$$\Rightarrow 10 = \frac{2.5 \times 10^3}{\Delta t}$$

$$\Rightarrow \Delta t = \frac{2.5 \times 10^3}{10} \Rightarrow \Delta t = 2.5 \times 10^2 \text{ s}$$

- 02** The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is
 (R is the universal gas constant)
 [2021, 20 July Shift-I]

- (a) 250 R (b) 750 R
(c) 175 R (d) 500 R

Ans. (d)

According to first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \quad \dots(i)$$

where, ΔQ = quantity of heat energy supplied to the system,

ΔU = change in the internal energy of a closed system

and ΔW = work done by the system on its surroundings.

As per question, no work is done

$$\therefore \Delta W = 0 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\Delta Q = 0 + \Delta U \Rightarrow \Delta Q = \Delta U$$

or

$$\Delta Q = \Delta U = nC_V\Delta T$$

where,

C_V = specific heat capacity at constant volume for diatomic gas = $\frac{5R}{2}$

ΔT = change in temperature
 $= (50 - 0) = 50^\circ\text{C}$

n = number of moles = 4

$$\Rightarrow \Delta Q = nC_V\Delta T$$

$$= 4 \times \frac{5R}{2} \times (50) = 500R = 500R$$

- 03** In thermodynamics, heat and work are [2021, 16 March Shift-I]

- (a) path functions
(b) intensive thermodynamic state variables
(c) extensive thermodynamic state variables
(d) point functions

Ans. (a)

According to first law of thermodynamics,

$$dQ = dU + dW \quad \dots(i)$$

As we know that, work done by the gas depends on the type of process, i.e.

path and dU depends on the initial and final states. So, considering Eq. (i), dQ will also be dependent on path. It means that, in thermodynamics, heat and work are path functions.

- 04** 1 mole of rigid diatomic gas performs a work of $Q/5$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$, The value of x

is

[R = universal gas constant]

[2021, 26 Feb Shift-II]

Ans. (25)

Given, number of mole, $n = 1$

Heat supplied = Q

Work done = $Q/5$

By using first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow Q = \Delta U + Q/5$$

$$\Rightarrow \Delta U = 4Q/5 = nC_V\Delta T \quad \dots(i)$$

where, C_V is heat coefficient at constant volume and ΔT is change in temperature.

Degree of freedom of diatomic gas, $f = 5$

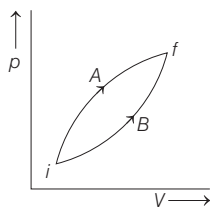
$$\therefore Q = \frac{5f}{4} R\Delta T \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{Q}{\Delta T} = C = \frac{5}{4} \times \frac{5}{2} R = \frac{25}{8} R$$

Hence, $x = 25$

- 05** Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies respectively, then

[2019, 9 April Shift-I]



- (a) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$
 (b) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$
 (c) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$
 (d) $\Delta Q_A = \Delta Q_B$, $\Delta U_A = \Delta U_B$

Ans. (c)

According to the first law of thermodynamics,

Heat supplied (ΔQ) = Work done (W) +
 Change in internal energy of the system (ΔU)

$$\Delta Q_A = \Delta U_A + W_A$$

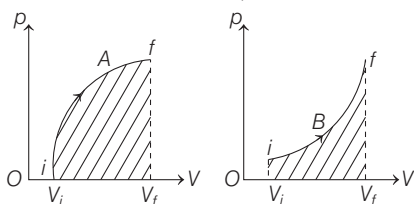
Similarly, for process B,

$$\Delta Q_B = \Delta U_B + W_B$$

Now, we know that,

work done for a process

= area under its p - V curve



Thus, it is clear from the above graphs,

$$W_A > W_B \quad \dots(i)$$

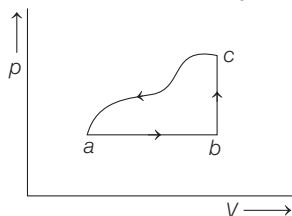
Also, since the initial and final state are same in both process, so

$$\Delta U_A = \Delta U_B \quad \dots(ii)$$

So, from Eqs. (i) and (ii), we can conclude that

$$\Delta Q_A > \Delta Q_B$$

- 06** A sample of an ideal gas is taken through the cyclic process $abca$ as shown in the figure. The change in the internal energy of the gas along the path ca is -180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc . The work done by the gas along the path abc is [2019, 12 April Shift-I]



- (a) 120 J (b) 130 J (c) 100 J (d) 140 J

Ans. (b)

Key Idea In p - V curve, work done dW , change in internal energy ΔU and heat absorbed ΔQ are connected with first law of thermodynamics, i.e.

$$\Delta Q = \Delta U + dW \quad \dots(i)$$

and total change in internal energy in complete cycle is always zero. Using this equation in different part of the curve, we can solve the given problem.

In Process $a \rightarrow b$

Given, $\Delta Q_{ab} = 250$ J

$$\therefore 250 \text{ J} = \Delta U_{ab} + dW_{ab} \quad \dots(ii)$$

In Process $b \rightarrow c$

Given, $\Delta Q_{bc} = 60$ J

Also, V is constant, so $dV = 0$

$$\Rightarrow dW_{bc} = p(dV)_{bc} = 0 \quad \dots(iii)$$

$$\therefore 60 \text{ J} = \Delta U_{bc} + 0$$

$$\Rightarrow \Delta U_{bc} = 60 \text{ J}$$

In Process $c \rightarrow a$

Given, $\Delta U_{ca} = -180$ J

$$\text{Now, for complete cycle, } \Delta U_{abca} = \Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0 \quad \dots(v)$$

From Eqs. (iii), (iv) and (v), we get

$$\Delta U_{ab} = -\Delta U_{bc} - \Delta U_{ca}$$

$$\Delta U_{ab} = -60 + 180 = 120 \text{ J} \quad \dots(vi)$$

From Eq. (ii), we get

$$250 \text{ J} = 120 \text{ J} + dW_{ab}$$

$$\Rightarrow dW_{ab} = 130 \text{ J} \quad \dots(vii)$$

From Eqs. (i) and (vii), we get

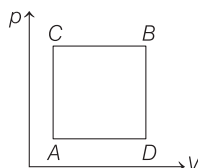
Work done by the gas along the path abc ,

$$dW_{abc} = dW_{ab} + dW_{bc}$$

$$= 130 \text{ J} + 0 \text{ J}$$

$$\Rightarrow dW_{abc} = 130 \text{ J}$$

- 07** A gas can be taken from A to B via two different processes ACB and ADB.



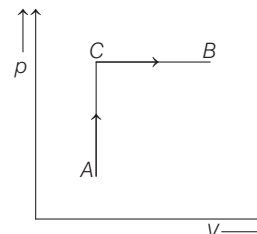
When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J the heat flow into the system in path ADB is

[2019, 9 Jan Shift-I]

- (a) 80 J (b) 40 J
 (c) 100 J (d) 20 J

Ans. (b)

For the ABC as shown in the figure below,



According to the first law of thermodynamics, heat supplied,

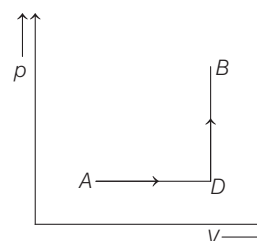
$$\Delta Q = \text{work done } (\Delta W) + \text{internal energy } (\Delta U)$$

$$\Rightarrow \Delta Q_{CB} = \Delta W_{ACB} + (U_B - U_A) \quad [\text{where, } \Delta U = U_B - U_A]$$

Substituting the given values,

$$U_B - U_A = 60 - 30 = 30 \text{ J} \quad \dots(i)$$

Similarly for the ADB as shown in the figure below,



$$\Delta Q_{ADB} = \Delta W_{ADB} + (U_B - U_A)$$

$$\Rightarrow \Delta Q_{ADB} = 10 + 30 \quad [\text{using Eq. (i)}]$$

$$= 40 \text{ J}$$

- 08** 100g of water is heated from 30°C to 50°C . Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) [AIEEE 2011]

- (a) 8.4 kJ (b) 84 kJ
 (c) 2.1 kJ (d) 4.2 kJ

Ans. (a)

As work done = 0

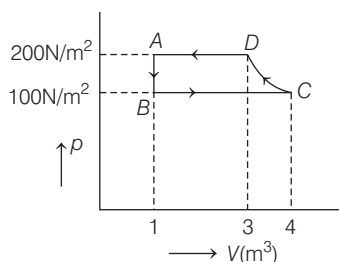
[as $W = p\Delta V$ and $\Delta V = 0$, so $W = 0$]

$$\Delta U = mC\Delta T$$

$$= 100 \times 10^{-3} \times 4184 \times (50 - 30)$$

$$\approx 8.4 \text{ kJ}$$

- 09** When a system is taken from state i to state f along the path iaf , it is found that $Q = 50 \text{ cal}$ and $W = 20 \text{ cal}$. Along the path ibf , $Q = 36 \text{ cal}$. W along the path ibf is [AIEEE 2007]



- (a) 6 cal (b) 16 cal (c) 66 cal (d) 14 cal

Ans. (a)

From first law of thermodynamics,

$$\Delta Q = \Delta U + W$$

For path iaf,

$$50 = \Delta U + 20$$

$$\therefore \Delta U = U_f - U_i = 30 \text{ cal}$$

For path ibf,

$$\Delta Q = \Delta U + W$$

$$\text{or } W = Q - \Delta U = 36 - 30 = 6 \text{ cal}$$

- 10** A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then [AIEEE 2005]

- (a) $\Delta U_1 = \Delta U_2$
 (b) relation between ΔU_1 and ΔU_2 cannot be determined
 (c) $\Delta U_2 > \Delta U_1$
 (d) $\Delta U_2 < \Delta U_1$

Ans. (a)

The change in internal energy does not depend upon path followed by the process. It only depends on initial and final states.

$$\text{Hence, } \Delta U_1 = \Delta U_2$$

- 11** Which of the following is incorrect regarding the first law of thermodynamics? [AIEEE 2005]

- (a) It is not applicable to any cyclic process
 (b) It is a restatement of the principle of conservation of energy
 (c) It introduces the concept of the internal energy
 (d) Both (a) and (c)

Ans. (d)

Statements (a) and (d) are wrong. Concept of entropy is associated with second law of thermodynamics.

- 12** Which of the following statements is correct for any thermodynamic system? [AIEEE 2004]

- (a) The internal energy changes in all processes
 (b) Internal energy and entropy are state functions
 (c) The change in entropy can never be zero
 (d) The work done in an adiabatic process is always zero

Ans. (b)

Internal energy does not change in isothermal process. ΔS can be zero for adiabatic process. Work done in adiabatic process may be non-zero.

- 13** Which of the following parameters does not characterise the thermodynamic state of matter? [AIEEE 2003]

- (a) Temperature (b) Pressure
 (c) Work (d) Volume

Ans. (c)

Work does not characterise the thermodynamic state of matter, it is a path function giving only relationship between two quantities.

TOPIC 2

Thermodynamics Process

- 14** A sample of gas with $\gamma = 1.5$ is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3 . If the initial pressure is 200 kPa . The absolute value of the work done by the gas in the process is J. [2021, 31 Aug Shift-II]

Ans. (480)

Given, sample of an ideal gas with ($\gamma = 1.5$) is taken through adiabatic process, where

$$\text{Initial volume, } V_1 = 1200 \text{ cm}^3 = 12 \times 10^{-4} \text{ m}^3$$

$$\text{Final volume, } V_2 = 300 \text{ cm}^3 = 3 \times 10^{-4} \text{ m}^3$$

$$\text{Initial pressure, } p_1 = 200 \text{ kPa} = 2 \times 10^5 \text{ Pa}$$

$$\text{Let final pressure} = p_2$$

Using equation of adiabatic process,

$$p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = 2 \times 10^5 \left(\frac{12 \times 10^{-4}}{3 \times 10^{-4}} \right)^{1.5}$$

$$= 16 \times 10^5 \text{ Pa}$$

Work done in adiabatic process,

$$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

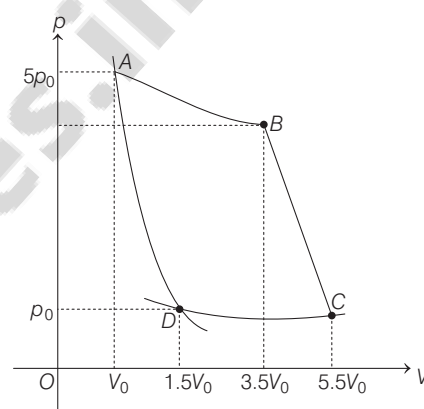
Substituting the values, we get

$$W = \frac{2 \times 10^5 \times 12 \times 10^{-4} - 16 \times 10^5 \times 3 \times 10^{-4}}{1.5 - 1}$$

$$= -480 \text{ J}$$

Hence, work done in adiabatic compression is 480 J .

- 15** In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process $A \rightarrow B$ and $C \rightarrow D$ are T_1 and T_2 ($T_1 > T_2$), respectively.



Choose the correct option out of the following for work done, if processes BC and DA are adiabatic. [2021, July 27 Shift-I]

- (a) $W_{AB} = W_{DC}$
 (b) $W_{AD} = W_{BC}$
 (c) $W_{BC} + W_{DA} > 0$
 (d) $W_{AB} < W_{CD}$

Ans. (b)

As per question, the process BC and DA are adiabatic.

$\therefore$ We know that work done in adiabatic process is given as

$$W = \frac{-nR}{\gamma - 1} (T_f - T_i)$$

where, n = number of moles,

γ = specific heat ratio,

R = gas constant

T_f = final temperature

and T_i = initial temperature.

$$\therefore W_{AD} = \frac{-nR}{\gamma - 1} (T_2 - T_1)$$

$$\text{and } W_{BC} = \frac{-nR}{\gamma - 1} (T_2 - T_1)$$

$$\Rightarrow W_{AD} = W_{BC}$$

- 16** A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$

will be [2021, 25 July Shift-I]

- (a) $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$ (b) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$
 (c) $\frac{l_2}{l_1}$ (d) $\frac{l_1}{l_2}$

Ans. (b)

As we know that,

In adiabatic condition,

$$TV^{\gamma-1} = \text{constant}$$

$$T(AI)^{\gamma-1} = \text{constant}$$

where, A is constant.

$$\Rightarrow Tl^{\gamma-1} = \text{constant} \quad \dots(i)$$

where, γ = adiabatic constant.

For monoatomic gas, $\gamma = 5/3$

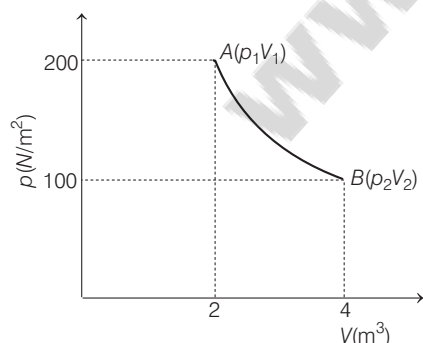
$$\Rightarrow T_1 l_1^{\frac{5}{3}-1} = T_2 l_2^{\frac{5}{3}-1} \Rightarrow \frac{T_1}{T_2} = \left(\frac{l_2}{l_1}\right)^{2/3}$$

- 17** One mole of an ideal gas at 27°C is taken from A to B as shown in the given p - V indicator diagram. The work done by the system will be $\times 10^{-1}$ J.

[Take, $R = 8.3 \text{ J/mol-K}$, $\ln 2 = 0.6931$]

(Round off to the nearest integer)

[2021, 20 July Shift-II]



Ans. (17258)

According to given p - V diagram in question, work done in isothermal process,

$$W = nRT \ln\left(\frac{V_F}{V_i}\right)$$

where, R = gas constant,

n = number of moles,

T = temperature

V_F = final volume

and V_i = initial volume.

$$\Rightarrow W = 8.3 \times 300 \times \ln(2)$$

$$[\text{Since, } T = 273 + 27 = 300 \text{ K}]$$

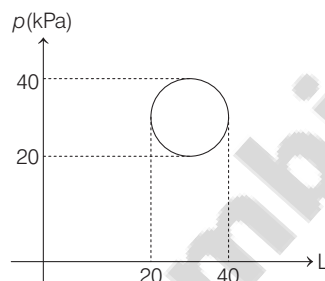
$$W = 1725.819 \text{ J}$$

$$W = 17258 \times 10^{-1} \text{ J}$$

This is the required value of the work done by the system.

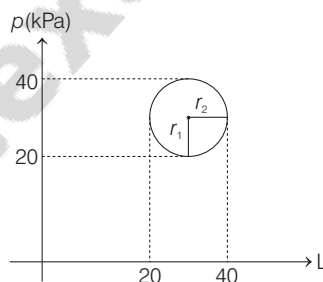
- 18** In the reported figure, heat energy absorbed by a system in going through a cyclic process is $\pi \text{ J}$.

[2021, 20 July Shift-I]



Ans. (100)

Consider the given diagram,



Here, $r_1 = 10 \times 10^{-3} \text{ m}$ and $r_2 = 10 \times 10^{-3} \text{ m}$

We know that for a complete cyclic process, change in internal energy,

$$(\Delta U) = 0 \quad \dots(i)$$

According to 1st law of thermodynamics,

$$\Delta Q = \Delta U + W \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \Delta Q = 0 + W$$

$$\Rightarrow \Delta Q = W \quad \dots(iii)$$

$$\therefore W = \text{Area}$$

$$= \pi r_1 r_2$$

$$= \pi \times (10 \times 10^{-3}) \times (10 \times 10^{-3})$$

$$W = 100 \pi \text{ J} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\Delta Q = 100 \pi \text{ J}$$

- 19** For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where, γ is the ratio of specific heats)

[2021, 18 March Shift-II]

- (a) $-\gamma \frac{dV}{V}$ (b) $-\gamma \frac{V}{dV}$
 (c) $-\frac{1}{\gamma} \frac{dV}{V}$ (d) $\frac{dV}{V}$

Ans. (a)

The ideal gas equation for adiabatic expansion process,

$$pV^\gamma = C$$

$$\ln(p) + \gamma \ln(V) = C$$

where, C is constant.

On differentiating both sides, we get

$$\frac{dp}{p} + \gamma \left(\frac{dV}{V}\right) = 0$$

$$\Rightarrow \frac{dp}{p} = -\gamma \left(\frac{dV}{V}\right)$$

- 20** An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2 . Given that $S_1 > S_2$. If the piston is removed, then the total entropy of the system will be

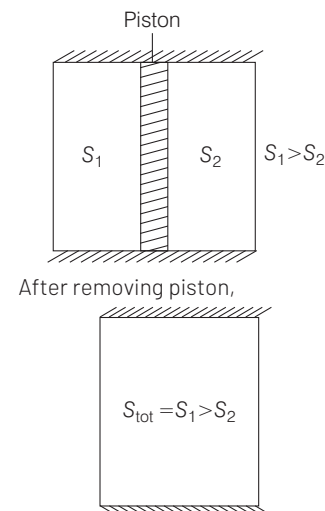
[2021, 18 March Shift-II]

- (a) $S_1 \times S_2$ (b) $S_1 - S_2$
 (c) $\frac{S_1}{S_2}$ (d) $S_1 + S_2$

Ans. (d)

Entropy present in one part of the system is S_1 .

Entropy present in other part of the system is S_2 .



We know that,

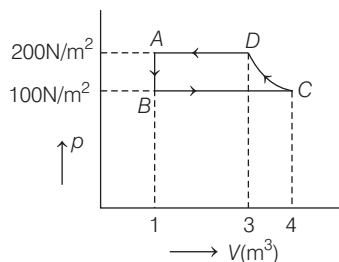
$$\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}}$$

Thus, after removing the piston, the total entropy present in system,

$$S_{\text{tot}} = S_1 + S_2$$

- 21** The p - V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use, $\gamma = 1.4$)

[2021, 18 March Shift-I]



- (a) -500 J (b) -400 J
(c) 400 J (d) 200 J

Ans. (a)

Given,

The final pressure of the adiabatic process, $p_f = 200 \text{ N/m}^2$

The initial pressure of the adiabatic process, $p_i = 100 \text{ N/m}^2$

The initial volume of the adiabatic process, $V_i = 4 \text{ m}^3$

The final volume of the adiabatic process, $V_f = 3 \text{ m}^3$

We know that, for an adiabatic process

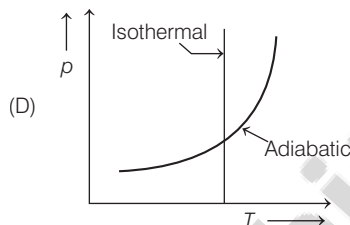
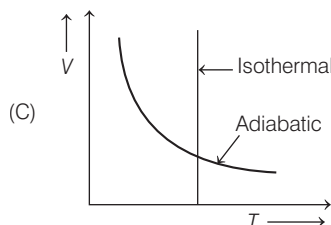
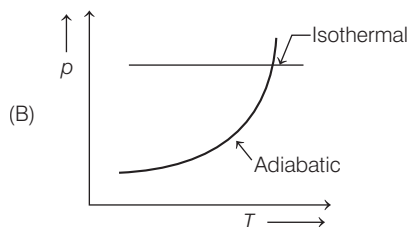
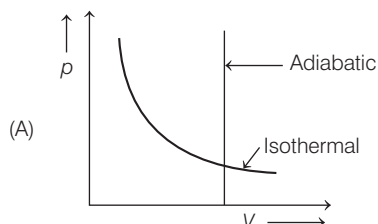
$$W = \frac{p_f V_f - p_i V_i}{1 - \gamma}$$

$$\Rightarrow W = \frac{200(3) - (100)(4)}{1 - 1.4}$$

$$\Rightarrow W = -500 \text{ J}$$

So, the work done by an adiabatic process is 500 J and negative sign represents the work done on the system.

- 22** Which one is the correct option for the two different thermodynamic processes? [2021, 17 March Shift-II]



- (a) C and A (b) C and D
(c) Only A (d) B and C

Ans. (b)

In graph (A) vertical line is isochoric process, so the graph (A) is incorrect.

In graph (B) horizontal line is isobaric process, so the graph (B) is incorrect.

In graph (C) and (D) vertical line is isothermal process and the curve line is adiabatic process. So, the correct representation of thermodynamic process graph is (C) and (D).

So, the correct option is (b)

- 23** The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = kT^{2/3}$. The work done when temperature changes by 90 K will be xR . The value of x is.....

[R = universal gas constant]

[2021, 26 Feb Shift-II]

Ans. (60)

$$\text{Given, } V = kT^{2/3} \quad \dots (i)$$

where, T is temperature.

Change in temperature, $\Delta T = 90 \text{ K}$

Let p be the pressure, dV be the change in volume and work done be W .

As we know that,

$$W = \int p dV \quad \dots (ii)$$

As,

$$pV = nRT$$

$$\therefore p = \frac{nRT}{V}$$

Substituting this value in Eq. (ii), we get

$$W = \int nRT \frac{dV}{V}$$

$$\Rightarrow W = \int nRT \frac{dV}{kT^{2/3}} \quad [\text{using Eq. (i)}] \quad \dots (iii)$$

On differentiating Eq. (i) w.r.t.

temperature on both sides, we get

$$\begin{aligned} \frac{dV}{dT} &= k \cdot \frac{2}{3} T^{-1/3} \\ &= \frac{2}{3} k T^{-1/3} = \frac{2}{3} \frac{k}{T^{1/3}} \\ dV &= \frac{2}{3} k T^{-1/3} dT \end{aligned}$$

Substituting this in Eq. (iii), we get

$$\begin{aligned} W &= \int nRT \frac{2/3 k T^{-1/3} dT}{k T^{2/3}} \\ \Rightarrow W &= \frac{2}{3} nR \int T dT = \frac{2}{3} nR [T]_{T_1}^{T_2} \\ &= \frac{2}{3} nR [T_2 - T_1] = \frac{2}{3} nR \Delta T \\ &= \frac{2}{3} nR 90 = 60 nR \end{aligned}$$

Here, $n = 1$

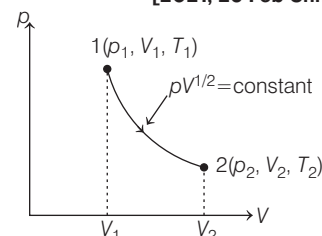
So, work done will be $60R$.

Hence, $x = 60$.

- 24** Thermodynamic process is shown below on a p - V diagram for one mole of an ideal gas.

If $V_2 = 2V_1$, then the ratio of temperature $\frac{T_2}{T_1}$ is

[2021, 25 Feb Shift-II]



- (a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) $1/2$ (d) 2

Ans. (d)

Here, p_1 and p_2 , T_1 and T_2 , V_1 and V_2 are initial and final pressures, temperatures and volumes, respectively.

Given, $V_2 = 2V_1$

$$pV^{1/2} = \text{constant}$$

From graph, γ = adiabatic constant = $1/2$

$$\Rightarrow p_1 V_1^{1/2} = p_2 V_2^{1/2} \Rightarrow \frac{p_1}{p_2} = \left(\frac{2V_1}{V_1} \right)^{1/2} = 2^{1/2}$$

Also, $p^{1-\gamma} T^\gamma = \text{constant}$

(for adiabatic process)

$$\Rightarrow p_1^{1-\gamma} T_1^\gamma = p_2^{1-\gamma} T_2^\gamma$$

$$\Rightarrow \left(\frac{p_1}{p_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{1-\gamma}{\gamma}} = \frac{T_2}{T_1}$$

$$\therefore T_2/T_1 = (2^{1/2})^{\frac{1-0.5}{0.5}} = \sqrt{2}$$

- 25** In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3 . The work done when the temperature changes from 100°C to 300°C will be nR , where n denotes number of moles of a gas.

[2021, 25 Feb Shift-I]

Ans. (50)Given, pressure $(p) \propto kV^3$

$$T_1 = 100^\circ\text{C}, T_2 = 300^\circ\text{C}$$

$$\Rightarrow \Delta T = T_2 - T_1 = 300 - 100 = 200^\circ\text{C}$$

By using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow kV^3 \cdot V = nRT \Rightarrow kV^4 = nRT$$

On differentiating both sides w.r.t temperature, we get

$$4kV^3 \frac{dV}{dT} = nR$$

$$\Rightarrow 4kV^3 dV = nRdT$$

$$\Rightarrow kV^3 dV = nRdT/4$$

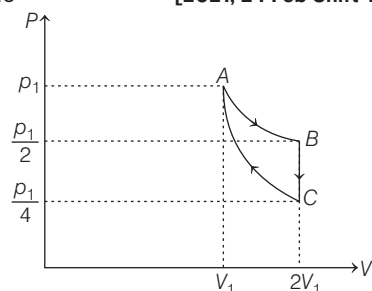
$$\Rightarrow p dV = nRdT/4$$

As, work done $(W) = p dV = nRdT/4$

$$= \frac{nR}{4} \Delta T = \frac{nR}{4} \times 200 = 50 nR$$

- 26** If one mole of an ideal gas at (p_1, V_1) is allowed to expand reversibly and isothermally (A to B), its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B to C). Then, it is restored to its initial state by a reversible adiabatic compression (C to A). The net work done by the gas is equal to

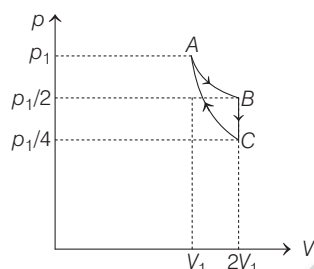
[2021, 24 Feb Shift-II]



$$(a) RT \left(\ln 2 - \frac{1}{2(\gamma-1)} \right) \quad (b) -\frac{RT}{2(\gamma-1)}$$

$$(c) 0$$

$$(d) RT \ln 2$$

Ans. (a)Let p_i, p_f, V_i and V_f be the initial and final pressure and volume.Given, AB is isothermal ($\Delta T = 0$),BC is isochoric ($\Delta V = 0$)and CA is adiabatic ($\Delta Q = 0$)Since, isothermal work $(W_{AB}) = p_1 V_1 \ln \frac{V_f}{V_i}$ where, V_i and V_f are volume at A and B, respectively.

$$\therefore W_{AB} = p_1 V_1 \ln \frac{2V_1}{V_1} = p_1 V_1 \ln 2$$

Since, at constant volume, work done is zero.

$$\therefore W_{BC} = 0$$

Since, W_{CA} is an adiabatic work done, i.e.

$$W_{CA} = \frac{1}{1-\gamma} (p_f V_f - p_i V_i)$$

$$\Rightarrow W_{CA} = \frac{1}{1-\gamma} (p_1 V_1 - \frac{p_1}{4} \times 2V_1)$$

$$= \frac{1}{1-\gamma} (p_1 V_1 - p_1 V_1 / 2) = \frac{1}{1-\gamma} \frac{p_1 V_1}{2}$$

 $\therefore$ Net work done,

$$W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$$

$$= p_1 V_1 \ln 2 + 0 + \frac{1}{1-\gamma} \frac{p_1 V_1}{2}$$

$$= p_1 V_1 \left[\ln 2 + \frac{1}{2(1-\gamma)} \right]$$

From ideal gas law, $pV = nRT$

$$\therefore W_{\text{net}} = RT \left[\ln 2 - \frac{1}{2(\gamma-1)} \right] \quad (\because n=1)$$

- 27** n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes.

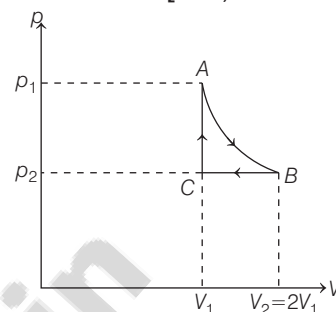
A $\rightarrow$ B : Isothermal expansion at temperature T , so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from p_1 to p_2 .

B $\rightarrow$ C : Isobaric compression at pressure p_2 to initial volume V_1 .

C $\rightarrow$ A : Isochoric change leading to change of pressure from p_2 to p_1 .

Total work done in the complete cycle ABCA is

[2021, 24 Feb Shift-I]



$$(a) 0$$

$$(b) nRT \ln 2$$

$$(c) nRT \left(\ln 2 + \frac{1}{2} \right) \quad (d) nRT \left(\ln 2 - \frac{1}{2} \right)$$

Ans. (d)

As, AB is isothermal process, so work done by isothermal process is given by

$$W_{AB} = nRT \ln \left(\frac{V_2}{V_1} \right)$$

$$W_{AB} = nRT \ln \left(\frac{2V_1}{V_1} \right) = nRT \ln 2$$

As, BC is isobaric process, so work done by isobaric process is given by

$$W_{BC} = p \Delta V = p_2 (V_1 - V_2)$$

$$= p_2 \left(\frac{V_2}{2} - V_2 \right) = -\frac{p_2 V_2}{2}$$

$$= \frac{-nRT}{2} \quad \left[\begin{array}{l} \because pV = nRT \\ \therefore p_2 V_2 = nRT \end{array} \right]$$

As, CA is isochoric process, so

work done by isochoric process is given by, $W_{CA} = 0$ [$\because \Delta V = 0$]

Total work done in the cycle ABCA,

$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$

$$= nRT \ln 2 + \left(\frac{-nRT}{2} \right) + 0$$

$$= nRT \left(\ln 2 - \frac{1}{2} \right)$$

- 28** Match List-I with List-II.

List-I	List-II
A. Isothermal	1. Pressure constant
B. Isochoric	2. Temperature constant
C. Adiabatic	3. Volume constant
D. Isobaric	4. Heat content is constant

Choose the correct answer from the options given below.

[2021, 24 Feb Shift-I]

	A	B	C	D
(a)	1	3	2	4
(b)	3	2	1	4
(c)	2	4	3	1
(d)	2	3	4	1

Ans. (d)

We know that, in isothermal process, $\Delta T = 0$

In isochoric process, $\Delta V = 0$

In adiabatic process, $\Delta Q = 0$

In isobaric process, $\Delta p = 0$

So, the correct match is,

A $\rightarrow$ 2, B $\rightarrow$ 3, C $\rightarrow$ 4, D $\rightarrow$ 1.

- 29.** A balloon filled with helium (32°C and 1.7 atm) bursts. Immediately afterwards; the expansion of helium can be considered as

[2020, 3 Sep Shift-I]

- (a) irreversible isothermal
(b) irreversible adiabatic
(c) reversible adiabatic
(d) reversible isothermal

Ans. (b)

Expansion after bursting of balloon is a fast process in which helium presses the atmosphere. This expansion occurs at the expense of internal energy of helium molecules.

So, this process is irreversible and adiabatic.

Hence, option (b) is correct.

- 30.** Match the thermodynamic processes taking place in a system with the correct conditions. In the table, ΔQ is heat supplied, ΔW is work done and ΔU is change in internal energy of the system.

[2020, 4 Sep Shift-II]

Process	Condition
(I) Adiabatic	(A) $\Delta W = 0$
(II) Isothermal	(B) $\Delta Q = 0$
(III) Isochoric	(C) $\Delta U \neq 0, \Delta W \neq 0$ and $\Delta Q \neq 0$
(IV) Isobaric	(D) $\Delta U = 0$

	I	II	III	IV
(a)	B	A	D	C
(b)	A	A	B	C
(c)	B	D	A	C
(d)	A	B	D	D

Ans. (c)

I. Adiabatic process

No heat is transferred between system and surroundings. Therefore,

$$\Delta Q = 0$$

II. Isothermal process

Temperature of the system remains constant,

$$\text{i.e., } T = \text{constant} \Rightarrow \Delta T = 0$$

So, change in internal energy,

$$\Delta U = nC_V \Delta T = nC_V(0) = 0$$

III. Isochoric process

Volume of system remains constant,

$$\text{i.e., } V = \text{constant} \Rightarrow \Delta V = 0$$

So, work done,

$$\Delta W = \int p dV = \int p(0) = 0$$

(IV) Isobaric process

Pressure of system remains constant,

$$\text{i.e., } p = \text{constant (C)}$$

$$\Rightarrow \Delta p = 0$$

So, change in internal energy,

$$\Delta U = nC_V \Delta T = n \left(\frac{fR}{2} \right) \Delta T$$

$$= \frac{f}{2} (nR \Delta T) = \frac{f}{2} (p \Delta V)$$

$$= \frac{f}{2} (C \Delta V) \neq 0$$

and work done,

$$\Delta W = \int p dV = \int C dV = C \Delta V \neq 0$$

and heat transferred,

$$\Delta Q = nC_p \Delta T = n \left[\frac{(f+2)R}{2} \right] \Delta T$$

$$= \left(\frac{f+2}{2} \right) (nR \Delta T) = \left(\frac{f+2}{2} \right) (p \Delta V)$$

$$= \left(\frac{f+2}{2} \right) (C \Delta V) \neq 0$$

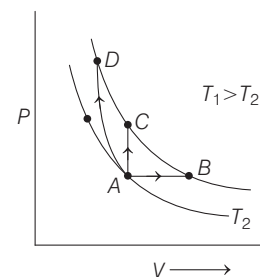
So, (I) $\rightarrow$ (B), (II) $\rightarrow$ (D), (III) $\rightarrow$ (A) and (IV) $\rightarrow$ (C).

Hence, option (c) is correct.

- 31** Three different processes that can occur in an ideal monoatomic gas are shown in the p versus V diagram. The paths are labelled as $A \rightarrow B$, $A \rightarrow C$ and $A \rightarrow D$. The change in internal energies during these process are taken as E_{AB} , E_{AC} and E_{AD} and the work done as W_{AB} , W_{AC} and W_{AD} .

The correct relation between these parameters are

[2020, 5 Sep Shift-I]



- (a) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(c) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(d) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

Ans. (*)

As initial and final temperatures for each process are same.

$\therefore$ Change in internal energy for each process will be equal, i.e.,

$$\Delta U = nC_V \Delta T = \text{same}$$

Thus, $E_{AB} = E_{AC} = E_{AD}$

Now, work done, $W = p dV$

For process AB, volume is increasing

$$\Rightarrow W_{AB} > 0$$

For process AD, volume is decreasing

$$\Rightarrow W_{AD} < 0$$

For process AC, volume is constant

$$\Rightarrow W_{AC} = 0$$

Hence, no option matches the answer.

- 32** In an adiabatic process, the density of a diatomic gas becomes 32 times of its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is

[2020, 5 Sep Shift-II]

- (a) 32 (b) 326
(c) 128 (d) $\frac{1}{32}$

Ans. (c)

Given that, density of the gas becomes 32 times and we know that density is inversely proportional to volume V .

So, volume becomes $\frac{1}{32}$ times of initial volume.

In adiabatic process, $pV^\gamma = \text{constant}$

$$\Rightarrow p_i V_i^\gamma = p_f V_f^\gamma$$

Now, $\gamma = \frac{7}{5}$ for diatomic gas

$$\Rightarrow \frac{p_f}{p_i} = \left(\frac{V_i}{V_f} \right)^\gamma = (32)^{\frac{7}{5}} = 128$$

Hence, correct option is (c).

- 33** A litre of dry air at STP expands adiabatically to a volume of 3 L. If $\gamma = 1.40$, the work done by air is ($3^{1.4} = 4.6555$)

[Take, air to be an ideal gas]

[2020, 7 Jan Shift-I]

- (a) 100.8 J (b) 90.5 J
(c) 48 J (d) 60.7 J

Ans. (b)

The given expansion process is adiabatic in nature and we have the following data :

Initial volume, $V_1 = 1 \text{ litre} = 10^{-3} \text{ m}^3$

Final volume, $V_2 = 3 \text{ litre} = 3 \times 10^{-3} \text{ m}^3$

Initial pressure, $p_1 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

Final pressure, $p_2 = ?$

Using $p_1 V_1^\gamma = p_2 V_2^\gamma$, we have

$$1 \times 1^\gamma = p_2 \times (3)^\gamma$$

Here, $\gamma = 1.40$

and $(3)^{1.4} = 4.6555$

$$\text{So, } p_2 = \frac{1}{4.6555} \approx 0.22 \text{ atm}$$

$$= 0.22 \times 1.01 \times 10^5 \text{ Pa}$$

Work done in an adiabatic expansion is given by

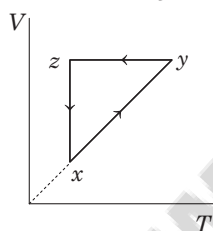
$$W = \frac{(p_1 V_1 - p_2 V_2)}{\gamma - 1}$$

$$= \frac{(1.01 \times 10^5 \times 10^{-3} - 0.22 \times 1.01 \times 10^5 \times 3 \times 10^{-3})}{(1.40 - 1)}$$

$$= 0.85 \times 1.01 \times 10^5 \times 10^{-3} = 85.85 \text{ J}$$

Closest value of work done, $W \approx 90.5 \text{ J}$

- 34** A thermodynamic cycle $xyzx$ is shown on a $V-T$ diagram.



The $p-V$ diagram that best describes this cycle is (diagrams are schematic and not to scale)

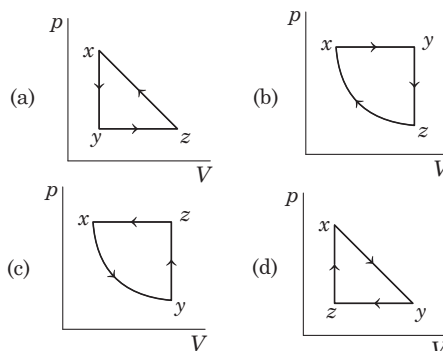
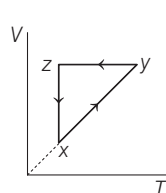
[2020, 8 Jan Shift-I]

Ans. (b)

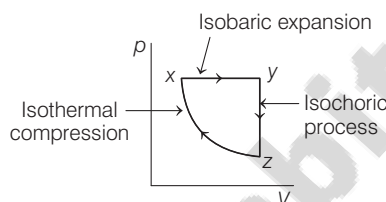
For the given $V-T$ graph of thermodynamic cycle $xyzx$,

In process xy , $V \propto T$

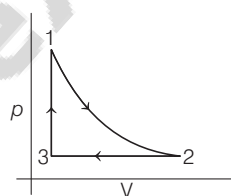
$\Rightarrow$ pressure is constant.



Process zx is an isothermal compression process, so pressure increases in this process. Process yz is an isochoric process in which temperature decreases, so pressure must decrease in this process. Hence, the $p-V$ graph is as shown in the figure.

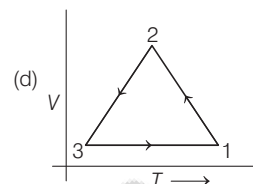
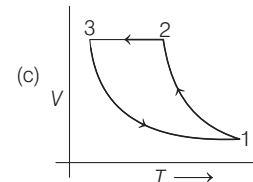
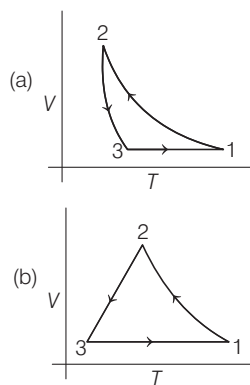


- 35** Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure? where, $1 \rightarrow 2$ is adiabatic.



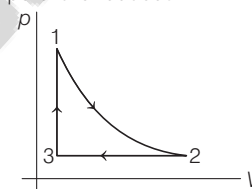
(Graphs are schematic and are not to scale)

[2020, 9 Jan Shift-I]



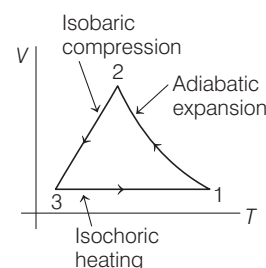
Ans. (b)

In given $p-V$ diagram, $1 \rightarrow 2$ is adiabatic expansion in which temperature reduces.



$2 \rightarrow 3$ is isobaric compression, so volume decreases and temperature increases in this process.

$3 \rightarrow 1$ is isochoric process in which pressure increases. So, it must be isochoric heating in which temperature increases. So, resultant $V-T$ graph will be



Thus, option (b) is correct.

- 36** Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand

isobarically to volume $2V_2$. If all the processes are the quasi-static, then the final temperature of the gas (in $^\circ\text{K}$) is (to the nearest integer)

..... [2020, 9 Jan Shift-II]

Ans. (1819)

Initially gas is compressed adiabatically.

Initial temperature, $T_1 = 300$ KNumber of moles = 1, $\gamma = 1.4$ Initial volume = V_1 Final volume = $\frac{V_1}{16}$

Process equation is,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow 300 V_1^{1.4-1} = T_2 \left(\frac{V_1}{16}\right)^{1.4-1}$$

$$\Rightarrow T_2 = 300 \times 2^{\frac{8}{5}} \quad \dots(i)$$

In next process, gas is expanded isobarically. Initial volume, $V_2 = \frac{V_1}{16}$ Final volume, $V_3 = 2\left(\frac{V_1}{16}\right) = \frac{V_1}{8}$

Now, for isobaric expansion,

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_2}{300 \times 2^{\frac{8}{5}}} = \frac{2V_2}{T_3}$$

$$\Rightarrow T_3 = 2 \times 300 \times 2^{\frac{8}{5}}$$

$$= 300 \times 2^{\frac{13}{5}} = 1818.85 \text{ or } T_3 = 1819 \text{ K}$$

- 37** A thermally insulated vessel contains 150 g of water at 0°C . Then, the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to (Latent heat of vaporisation of water $= 2.10 \times 10^6 \text{ J kg}^{-1}$ and latent heat of fusion of water $= 3.36 \times 10^5 \text{ J kg}^{-1}$)

[2019, 8 April Shift-I]

- (a) 150 g (b) 20 g (c) 130 g (d) 35 g

Ans. (b)Let x grams of water is evaporated.

According to the principle of calorimetry,

Heat lost by freezing water (that turns into ice) = Heat gained by evaporated water

Given, mass of water = 150 g

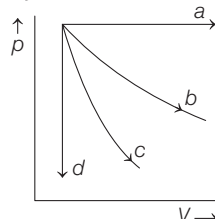
$$\Rightarrow (150 - x) \times 10^{-3} \times 3.36 \times 10^5$$

$$= x \times 10^{-3} \times 2.10 \times 10^6$$

$$\Rightarrow (150 - x) \times 3.36 = 21x$$

$$\Rightarrow x = \frac{150}{7.25} = 20.6 \therefore x \approx 20 \text{ g}$$

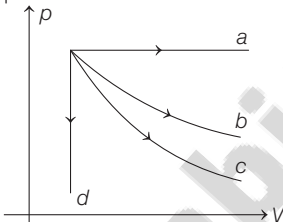
- 38** The given diagram shows four processes, i.e. isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by

[2019, 8 April Shift-II]

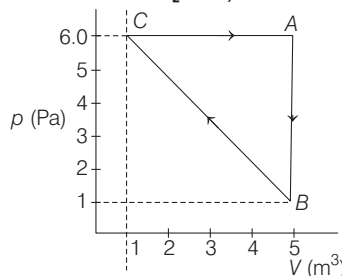
- (a) d a b c (b) a d b c
(c) d a c b (d) a d c b

Ans. (a)

Given processes are

For process a , pressure is constant. $\therefore a$ is isobaric.For process d , volume is constant. $\therefore d$ is isochoric.Also, as we know that, slope of adiabatic curve in p - V diagram is more than that of isothermal curve. $\therefore b$ is isothermal and c is adiabatic.

- 39** For the given cyclic process CAB shown for a gas, the work done is

[2019, 12 Jan Shift-I]

- (a) 5 J (b) 10 J
(c) 1 J (d) 30 J

Ans. (b)Key Idea In a cyclic thermodynamic process work done = area under p - V diagram.

Also in clockwise cycle, work done is positive.

In the given cyclic process,

work done = $\oint p dV$ = area enclosed by the cycle

$$= \frac{1}{2} \times \text{base} \times \text{height of triangle (CAB) made}$$

$$\text{by cycle} = \frac{1}{2} (V_2 - V_1) (p_2 - p_1)$$

From graph, given

$$V_2 = 5 \text{ m}^3, V_1 = 1 \text{ m}^3,$$

$$p_2 = 6 \text{ Pa}, p_1 = 1 \text{ Pa}$$

$$= \frac{1}{2} (5 - 1) (6 - 1) = \frac{1}{2} \times 4 \times 5 = 10 \text{ J}$$

- 40** n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is

[2019, 10 April Shift-I]

- (a) $\frac{4nR}{C_V + nR}$ (b) $\frac{4nR}{C_V - nR}$
(c) $\frac{nR}{C_V - nR}$ (d) $\frac{nR}{C_V + nR}$

Ans. (d)

For isobaric process, work done is given as

$$W = nR\Delta T$$

Heat supplied, $\Delta Q = nC_p\Delta T$ and

$$C_p - \frac{C_V}{n} = R \Rightarrow C_p = R + C_V/n$$

$$\therefore \frac{W}{\Delta Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{C_p} = \frac{R}{R + \frac{C_V}{n}}$$

$$\Rightarrow \frac{W}{\Delta Q} = \frac{nR}{nR + C_V}$$

- 41** Two moles of an ideal monoatomic gas occupies a volume V at 27°C . The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy.

[JEE Main 2018]

- (a) (i) 189 K (ii) 2.7 kJ
(b) (i) 195 K (ii) -2.7 kJ
(c) (i) 189 K (ii) -2.7 kJ
(d) (i) 195 K (ii) 2.7 kJ

Ans. (c)

For adiabatic process relation of temperature and volume is,

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\Rightarrow T_2 (2V)^{2/3} = 300 (V)^{2/3}$$

$$[\gamma = \frac{5}{3} \text{ for monoatomic gases}]$$

$$\Rightarrow T_2 = \frac{300}{2^{2/3}} \approx 189 \text{ K}$$

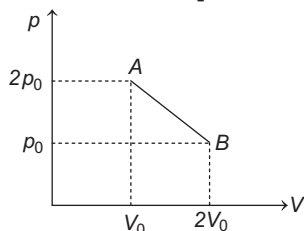
Also, in adiabatic process,

$$\Delta Q = 0, \Delta U = -\Delta W$$

$$\text{or } \Delta U = \frac{-nR(\Delta T)}{\gamma - 1}$$

$$= -2 \times \frac{3}{2} \times \frac{25}{3} (300 - 189) \\ \approx -2.7 \text{ kJ}$$

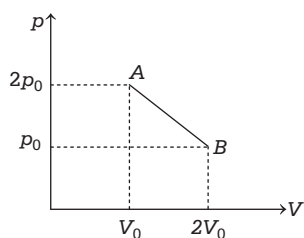
- 42** n moles of an ideal gas undergoes a process A and B as shown in the figure. The maximum temperature of the gas during the process will be [JEE Main 2016]



- (a) $\frac{9}{4} \frac{p_0 V_0}{nR}$ (b) $\frac{3}{2} \frac{p_0 V_0}{nR}$ (c) $\frac{9}{2} \frac{p_0 V_0}{nR}$ (d) $\frac{9p_0 V_0}{nR}$

Ans. (a)

As, T will be maximum temperature where product of pV is maximum



Equation of line AB, we have

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\Rightarrow p - p_0 = \frac{2p_0 - p_0}{V_0 - 2V_0} (V - 2V_0)$$

$$\Rightarrow p - p_0 = \frac{-p_0}{V_0} (V - 2V_0)$$

$$\Rightarrow p = \frac{-p_0}{V_0} V + 3p_0$$

$$pV = \frac{-p_0}{V_0} V^2 + 3p_0 V$$

$$nRT = \frac{-p_0}{V_0} V^2 + 3p_0 V$$

$$T = \frac{1}{nR} \left(\frac{-p_0}{V_0} V^2 + 3p_0 V \right)$$

For maximum temperature,

$$\frac{\partial T}{\partial V} = 0$$

$$\frac{-p_0}{V_0} (2V) + 3p_0 = 0$$

$$\frac{-p_0}{V_0} (2V) = -3p_0 \Rightarrow V = \frac{3}{2} V_0$$

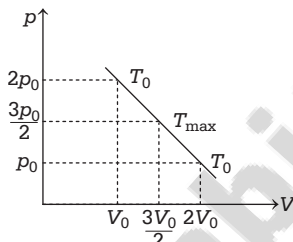
(condition for maximum temperature)

Thus, the maximum temperature of the gas during the process will be

$$T_{\max} = \frac{1}{nR} \left(\frac{-p_0}{V_0} \times \frac{9}{4} V_0^2 + 3p_0 \times \frac{3}{2} V_0 \right) \\ = \frac{1}{nR} \left(-\frac{9}{4} p_0 V_0 + \frac{9}{2} p_0 V_0 \right) \\ = \frac{9}{4} \frac{p_0 V_0}{nR}$$

Alternate Solution

Since, initial and final temperature are equal, hence maximum temperature is at the middle of line.



i.e.

$$pV = nRT \\ \Rightarrow \left(\frac{3}{2} p_0 \right) \left(\frac{3V_0}{2} \right) = T_{\max} \Rightarrow \frac{9}{4} \frac{p_0 V_0}{nR} = T_{\max}$$

- 43** An ideal gas undergoes a quasistatic, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure p and volume V is given by $pV^n = \text{constant}$, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively) [JEE Main 2016]

$$(a) n = \frac{C_p}{C_v}$$

$$(b) n = \frac{C - C_p}{C - C_v}$$

$$(c) n = \frac{C_p - C}{C - C_v}$$

$$(d) n = \frac{C - C_v}{C - C_p}$$

Ans. (b)

For polytropic process, specific heat for an ideal gas,

$$C = \frac{R}{1-n} + C_v$$

$$\therefore \frac{R}{1-n} + C_v = C \Rightarrow \frac{R}{1-n} = C - C_v$$

$$\Rightarrow \frac{R}{C - C_v} = 1 - n$$

(where, $R = C_p - C_v$)

$$\Rightarrow \frac{C_p - C_v}{C - C_v} = 1 - n$$

$$\Rightarrow n = 1 - \frac{C_p - C_v}{C - C_v} \Rightarrow n = \frac{C - C_p}{C - C_v}$$

Thus, number of moles n is given by

$$n = \frac{C - C_p}{C - C_v}$$

$$T_2 \approx 189 \text{ K}, \Delta U \approx 2.7 \text{ kJ}$$

- 44** Consider a spherical shell of radius R at temperature T . The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion, the relation between T and R is [JEE Main 2015]

$$(a) T \propto e^{-R}$$

$$(b) T \propto e^{-3R}$$

$$(c) T \propto \frac{1}{R}$$

$$(d) T \propto \frac{1}{R^3}$$

Ans. (c)

According to question,

$$p = \frac{1}{3} \left(\frac{U}{V} \right)$$

$$\Rightarrow \frac{nRT}{V} = \frac{1}{3} \left(\frac{U}{V} \right) \quad [\because pV = nRT]$$

$$\text{or } \frac{nRT}{V} \propto \frac{1}{3} T^4 \quad \text{or } VT^3 = \text{constant}$$

$$\text{or } \frac{4}{3} \pi R^3 T^3 = \text{constant} \quad \text{or } TR = \text{constant}$$

$$\Rightarrow T \propto \frac{1}{R}$$

- 45** One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K, respectively. Choose the correct statement.

[JEE Main 2014]

- (a) The change in internal energy in whole cyclic process is 250 R
(b) The change in internal energy in the process CA is 700 R
(c) The change in internal energy in the process AB is -350 R
(d) The change in internal energy in the process BC is -500 R

Ans. (d)

According to first law of thermodynamics, we get

(i) Change in internal energy from A to B i.e., ΔU_{AB}

$$\Delta U_{AB} = nC_V(T_B - T_A) = 1 \times \frac{5R}{2}(800 - 400) = 1000 R$$

(ii) Change in internal energy from B to C

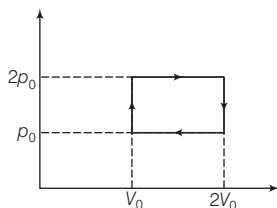
$$\Delta U_{BC} = nC_V(T_C - T_B) = 1 \times \frac{5R}{2}(600 - 800) = -500 R$$

(iii) $\Delta U_{\text{total}} = 0$

(iv) Change in internal energy from C to A i.e., ΔU_{CA}

$$\Delta U_{CA} = nC_V(T_A - T_C) = 1 \times \frac{5R}{2}(400 - 600) = -500 R$$

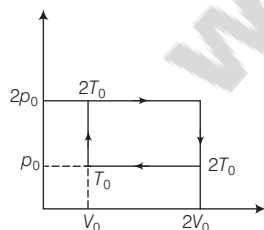
- 46** The given p - V diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is **[JEE Main 2013]**



- (a) $p_0 V_0$ (b) $\left(\frac{13}{2}\right) p_0 V_0$
(c) $\left(\frac{11}{2}\right) p_0 V_0$ (d) $4 p_0 V_0$

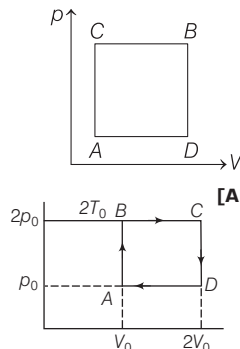
Ans. (b)

Heat supplied
 $= nC_V(2T_0 - T_0) + nC_p(2T_0 - T_0)$



$$\begin{aligned} &= \frac{n3RT_0}{2} + \frac{n5R}{2} 2T_0 \\ &= \frac{13}{2} nRT_0 \\ &= \frac{13}{2} p_0 V_0 \end{aligned}$$

- 47** Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas)

**[AIEEE 2012]**

- (a) 15.4% (b) 9.1% (c) 10.5% (d) 12.5%

Ans. (a)

Efficiency of a process is defined as the ratio of work done to energy supplied. Here,

$$\begin{aligned} \eta &= \frac{\Delta W}{\Delta Q} = \frac{\text{Area under } p\text{-}V \text{ diagram}}{\Delta Q_{AB} + \Delta Q_{BC}} \\ \therefore \eta &= \frac{p_0 V_0}{nC_V \Delta T_1 + nC_p \Delta T_2} \\ &= \frac{p_0 V_0}{\frac{3}{2} nR(T_B - T_A) + \frac{5}{2} nR(T_C - T_D)} \\ &= \frac{p_0 V_0}{\frac{3}{2} (2p_0 V_0 - p_0 V_0) + \frac{5}{2} (4p_0 V_0 - 2p_0 V_0)} \\ &= \frac{p_0 V_0}{\frac{3}{2} p_0 V_0 + \frac{5}{2} \cdot 2p_0 V_0} = \frac{1}{6.5} = 15.4\% \end{aligned}$$

- 48** A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ . It is moving with speed v and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by **[AIEEE 2011]**

- (a) $\frac{(\gamma-1)}{2\gamma R} M v^2 K$ (b) $\frac{\gamma M v^2}{2R} K$
(c) $\frac{(\gamma-1)}{2R} M v^2 K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R} M v^2 K$

Ans. (c)

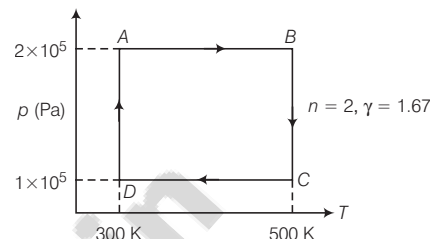
As no heat is lost,

Loss of kinetic energy = Gain of internal energy of gas

$$\frac{1}{2} m v^2 = nC_V \Delta T$$

$$\begin{aligned} \Rightarrow \frac{1}{2} m v^2 &= \frac{m}{M} \cdot \frac{R}{\gamma-1} \Delta T \\ \Rightarrow \Delta T &= \frac{M v^2 (\gamma-1)}{2R} K \end{aligned}$$

Direction (Q.Nos. 49 to 51) are based on the following figure.



- 49** Assume the gas to be ideal, the work done on the gas in taking it from A to B is **[AIEEE 2009]**

- (a) $200 R$ (b) $300 R$ (c) $400 R$ (d) $500 R$

Ans. (c)

$$\begin{aligned} W_{AB} &= \Delta Q - \Delta U = nC_p dT - nC_V \Delta T \\ &= n(C_p - C_V) dT \\ &= nR dT = 2 \times R \times (500 - 300) = 400 R \end{aligned}$$

- 50** The work done on the gas in taking it from D to A is (see above figure) **[AIEEE 2009]**

- (a) $-414 R$ (b) $+414 R$
(c) $-690 R$ (d) $+690 R$

Ans. (b)

At constant temperature (isothermal process),

$$\begin{aligned} W_{DA} &= nRT \ln \left(\frac{p_1}{p_2} \right) \\ &= 2.303 nRT \log_{10} \left(\frac{p_1}{p_2} \right) \\ &= 2.303 \times 2R \times 300 \log \left(\frac{10^5}{2 \times 10^5} \right) \\ &= 2.303 \times 600 R \log \left(\frac{1}{2} \right) \\ &= 0.693 \times 600 R = -414 R \end{aligned}$$

So, work done on the gas is $+414 R$.

- 51** The net work on the gas in the cycle ABCDA is (see above figure) **[AIEEE 2009]**

- (a) zero (b) $276 R$ (c) $1076 R$ (d) $1904 R$

Ans. (c)

$$W_{AB} = nR \Delta T = 2R \times 200 = 400 R$$

$$W_{BC} = nRT \log \left(\frac{p_1}{p_2} \right)$$

$$\begin{aligned}
 &= 2.303 \times R \times 500 \log \left(\frac{2 \times 10^5}{10^5} \right) \\
 &= 2 \times 346 R = 692 R \\
 W_{CD} &= nR\Delta T = 2R \times 200 = 400 R \\
 W_{DA} &= 2.303 \times R \times 300 \log \left(\frac{10^5}{2 \times 10^5} \right) \\
 &= -415.963 R \\
 \therefore \text{Net work done in a cycle} \\
 &= W_{AB} + W_{BC} + W_{CD} + W_{DA} \\
 &= 400 R + 692 R + 400 R - 415.963 R \\
 &= 1076 R
 \end{aligned}$$

- 52** During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v for the gas is [AIEEE 2003]

(a) $4/3$ (b) 2 (c) $5/3$ (d) $3/2$

Ans. (d)

Given, $p \propto T^3$... (i)

In an adiabatic process,

$$T^\gamma p^{1-\gamma} = \text{constant} \quad \left[\text{as } \gamma = \frac{C_p}{C_v} \right]$$

$$T \propto \frac{1}{p^{(1-\gamma)/\gamma}}$$

$$T^{(\gamma/\gamma-1)} \propto p \quad \dots (ii)$$

Comparing Eqs. (i) and (ii), we get

$$\frac{\gamma}{\gamma-1} = 3$$

or $3\gamma - 3 = \gamma$

or $2\gamma = 3$ or $\frac{C_p}{C_v} = \gamma = \frac{3}{2}$

TOPIC 3

Heat Engine, Second Law of Thermodynamics and Carnot Engine

- 53** A reversible engine has an efficiency of $1/4$. If the temperature of the sink is reduced by 58°C , its efficiency becomes double.

Calculate the temperature of the sink. [2021, 31 Aug Shift-I]

(a) 174°C (b) 280°C
(c) 180.4°C (d) 382°C

Ans. (a)

Given, initial efficiency of engine, $\eta_1 = 1/4$

Let, initial temperature of sink $= T_2$

Final temperature of sink $= T_2 - 58^\circ\text{C}$

Final efficiency of engine, $\eta_2 = 2\eta_1 = 2/4 = 1/2$

Temperature of source $= T_1$

Since, $\eta = 1 - \frac{T_2}{T_1}$

$\therefore \eta_1 = \frac{1}{4} = 1 - \frac{T_2}{T_1}$

$\Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{4} \Rightarrow T_2/T_1 = 3/4$

$\Rightarrow T_1 = \frac{4}{3}T_2$... (i)

and $\eta_2 = \frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$

$\Rightarrow \frac{T_2 - 58}{T_1} = 1 - \frac{1}{2}$

$\Rightarrow \frac{T_2 - 58}{4/3 T_2} = \frac{1}{2}$ [From Eq. (i)]

$\Rightarrow 2T_2 - 116 = \frac{4}{3}T_2 \Rightarrow \frac{2T_2}{3} = 116$

$\Rightarrow T_2 = \frac{116 \times 3}{2} = 58 \times 3 = 174^\circ\text{C}$

- 54** A heat engine operates between a cold reservoir at temperature $T_2 = 400\text{ K}$ and a hot reservoir at temperature T_1 . It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be K.

[2021, 27 Aug Shift-II]

Ans. (500)

Given, temperature of hot reservoir $= T_1$

Temperature of cold reservoir, $T_2 = 400\text{ K}$

Heat of hot reservoir, $Q_1 = 300\text{ J}$

Heat of cold reservoir, $Q_2 = 240\text{ J}$

Since, efficiency $(\eta) = 1 - \frac{T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$

$$1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1}$$

$\Rightarrow 1 - \frac{400}{T_1} = 1 - \frac{240}{300}$

$\Rightarrow \frac{400}{T_1} = \frac{4}{5} \Rightarrow T_1 = 500\text{ K}$

- 55** Two Carnot engines A and B operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T . engine B absorbs half of the heat rejected by engine A and rejects heat to the sink at T_3 . When work done in both the cases is equal, then the value of T is

[2021, 27 July Shift-II]

(a) $\frac{2}{3}T_1 + \frac{3}{2}T_3$

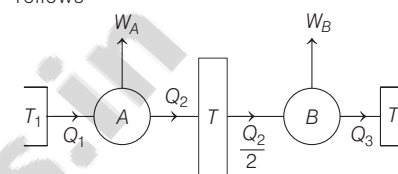
(b) $\frac{1}{3}T_1 + \frac{2}{3}T_3$

(c) $\frac{3}{2}T_1 + \frac{1}{3}T_3$

(d) $\frac{2}{3}T_1 + \frac{1}{3}T_3$

Ans. (d)

As per question, there are two Carnot engines A and B, which are connected in series such that the engine A absorbs heat at T_1 and rejects heat into sink at T . Engine B absorbs half of the heat rejected by engine A and rejects heat to sink at T_3 . It can be represented as follows



Efficiency of engine A,

$$\eta_A = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T}{T_1} = \frac{Q_2}{Q_1} = \frac{T}{T_1} \quad \dots (i)$$

Efficiency of engine B,

$$\eta_B = 1 - \frac{Q_3}{\left(\frac{Q_2}{2}\right)} = 1 - \frac{T_3}{T}$$

$\Rightarrow \eta_B = \frac{2Q_3}{Q_2} = \frac{T_3}{T} \quad \dots (ii)$

According to question,

Work done in both the cases is equal. It means

$$\Rightarrow W_A = W_B$$

$$\Rightarrow Q_1 - Q_2 = \frac{Q_2}{2} - Q_3$$

$\Rightarrow \frac{2Q_1}{Q_2} + \frac{2Q_3}{Q_2} = 3 \quad \dots (iii)$

$\therefore$ From Eqs. (i), (ii) and (iii), we get

$$\frac{2T_1}{T} + \frac{T_3}{T} = 3$$

$$\Rightarrow \frac{2T_1}{3} + \frac{T_3}{3} = T$$

- 56** A refrigerator consumes an average 35 W power to operate between temperature -10°C to 25°C . If there is no loss of energy, then how much average heat per second does it transfer?

[2021, 26 July Shift-II]

(a) 263 J/s (b) 298 J/s
(c) 350 J/s (d) 35 J/s

Ans. (a)

Given, power consumed by refrigerator $P = 35\text{ W} = 35\text{ Js}^{-1}$

$\therefore$ Energy consumed by refrigerator per second, $W = 35\text{ J}$

Lower temperature limit,

$$T_L = -10^\circ\text{C} = 263\text{K}$$

Higher temperature limit,

$$T_H = 25^\circ\text{C} = 298\text{K}$$

The coefficient of performance of refrigerator is given as

$$\text{COP} = \frac{T_L}{T_H - T_L} = \frac{Q}{W}$$

Here, Q is the heat transferred by the refrigerator.

Substituting the values, we get

$$\frac{263}{298 - 263} = \frac{Q}{35}$$

$$\Rightarrow Q = \frac{263}{35} \times 35 = 263\text{J/s}$$

Thus, the average heat transfer by refrigerator per second is 263J s^{-1} .

- 57** A heat engine has an efficiency of $1/6$. When the temperature of sink is reduced by 62°C , its efficiency get doubled. The temperature of the source is [2021, 25 July Shift-II]

- (a) 124°C (b) 37°C
(c) 62°C (d) 99°C

Ans. (d)

Given, efficiency of heat engine, $\eta_1 = \frac{1}{6}$

Let T_1, T_2 be the initial temperature of source and sink, respectively

and T'_1, T'_2 be the final temperature of source and sink, respectively.

Given, $T'_2 = T_2 - 62$ and $T'_1 = T_1$

Final efficiency, $\eta_2 = 2\eta_1 = \frac{2}{6} = \frac{1}{3}$

As we know that, $\eta = 1 - \frac{T_2}{T_1}$

For case 1

$$\therefore \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6} \quad \dots(1)$$

For case 2

$$\frac{1}{3} = 1 - \frac{T'_2}{T'_1} \Rightarrow \frac{T'_2}{T'_1} = 1 - \frac{1}{3}$$

$$\Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3} \Rightarrow \frac{T_2}{T_1} - \frac{62}{T_1} = \frac{2}{3}$$

$$\Rightarrow \frac{5}{6} - \frac{62}{T_1} = \frac{2}{3} \quad [\text{From Eq. (i)}]$$

$$\Rightarrow \frac{62}{T_1} = \frac{5}{6} - \frac{2}{3}$$

$$\Rightarrow \frac{62}{T_1} = \frac{1}{6} \Rightarrow T_1 = 372\text{K}$$

$$\therefore T_1 = 372 - 273^\circ\text{C} = 99^\circ\text{C}$$

- 58** A Carnot's engine working between 400K and 800K has a work output of 1200J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

[2021, 17 March Shift-I]

- (a) 3200J (b) 1800J
(c) 1600J (d) 2400J

Ans. (d)

Efficiency of Carnot heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1} = \frac{W}{Q_1} \quad \dots(i)$$

where, W = net work done by the gas

Q_1 = heat absorbed by the gas,

Q_2 = heat released by the gas,

T_1 = temperature of hot reservoir,

and T_2 = temperature of cold reservoir.

Using Eq. (i), we can write

$$1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow \frac{T_2}{T_1} = 1 - \frac{W}{Q_1}$$

$$\Rightarrow \frac{400}{800} = 1 - \frac{W}{Q_1} \Rightarrow \frac{W}{Q_1} = 1 - \frac{400}{800}$$

$$\Rightarrow \frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{W}{Q_1} = \frac{1}{2}$$

$$\Rightarrow Q_1 = 2 \times W = 2 \times 1200 \quad [\because W_1 = 1200\text{J}]$$

$$= 2400\text{J}$$

- 59** For an ideal heat engine, the temperature of the source is 127°C . In order to have 60% efficiency the temperature of the sink should be $^\circ\text{C}$.

(Round off to the nearest integer)

[2021, 16 March Shift-II]

Ans. (113)

Given, temperature of source, $T_H = 127^\circ\text{C}$
 $= 273 + 127 = 400\text{K}$

Efficiency, $\eta = 60\% = 0.6$

The efficiency of Carnot (ideal) heat engine is given by

$$\eta = \left(1 - \frac{T_L}{T_H}\right)$$

where, T_L = temperature of sink,

$$0.6 = \left(1 - \frac{T_L}{T_H}\right)$$

$$\Rightarrow \frac{T_L}{T_H} = 1 - 0.6$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times T_H$$

$$= 0.4 \times 400 = 160\text{K}$$

$$= 160 - 273 = -113^\circ\text{C}$$

- 60** A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52K , its efficiency is doubled. The temperature in kelvin of the source will be

[2021, 25 Feb Shift-II]

Ans. (104)

Given, initial efficiency (η_1) = $1/4$

$$\Rightarrow \eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{4}$$

$$\Rightarrow \frac{T_2}{T_1} = 1 - \frac{1}{4} = \frac{3}{4}$$

When temperature of sink is reduced by 50K ,

$$\eta_2 = 2\eta_1 = 1 - \frac{T_2 - 52}{T_1}$$

$$\Rightarrow 2 \times \frac{1}{4} = 1 - \frac{T_2}{T_1} + \frac{52}{T_1}$$

$$\Rightarrow \frac{1}{2} = 1 - \frac{3}{4} + \frac{52}{T_1}$$

$$\Rightarrow \frac{1}{2} - \frac{1}{4} = \frac{52}{T_1}$$

$$\Rightarrow \frac{2}{4} = \frac{52}{T_1}$$

$$\Rightarrow T_1 = 104\text{K}$$

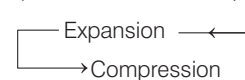
- 61** A heat engine is involved with exchange of heat of 1915J , -40J , $+125\text{J}$ and $-Q\text{J}$, during one cycle achieving an efficiency of 50.0% . The value of Q is [2020, 2 Sep Shift-II]

- (a) 640J (b) 40J (c) 980J (d) 400J

Ans. (c)

As processes involved in one cycle of a heat engine must be

Expansion $\rightarrow$ Compression



So, Q must be negative.

Now,

Work done in process = Heat supplied - Heat rejected

$$= (1915 + 125) - (40 + Q)\text{J}$$

And heat supplied to cycle = $1915 + 125\text{J}$

Given, efficiency = 50%

$$\Rightarrow \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$\Rightarrow \frac{50}{100} = \frac{(1915 + 125) - (40 + Q)}{1915 + 125}$$

$$\Rightarrow Q = 2000 - 1020 = 980\text{J}$$

Hence, correct option is (c).

- 62** If minimum possible work is done by a refrigerator in converting 100 g of water at 0°C to ice, how much heat (in cal) is released to the surroundings at temperature 27°C to the nearest integer

(Take, latent heat of ice = 80 cal/g)

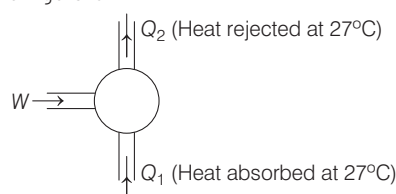
[2020, 3 Sep Shift-II]

Ans. (8791)

To convert 100 g water at 0°C to ice at 0°C , heat absorbed by refrigerator,

$$Q_1 = mL = 100 \times 80 = 8000\text{ cal}$$

W = work done or energy supplied to refrigerator



From figure, we have

$$W + Q_1 = Q_2$$

$$\Rightarrow W = Q_2 - Q_1$$

Now, coefficient of performance of refrigerator is

$$\beta = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_1}{T_2 - T_1}$$

Here, $T_1 = 0^\circ\text{C} = 273\text{ K}$

$T_2 = 27^\circ\text{C} = 300\text{ K}$

$$\text{So, } \frac{Q_1}{Q_2 - Q_1} = \frac{273}{300 - 273}$$

$$\Rightarrow 27Q_1 = (Q_2 - Q_1)273$$

$$\Rightarrow 300Q_1 = Q_2 \times 273$$

$$\Rightarrow Q_2 = 300 \times \frac{8000}{273}$$

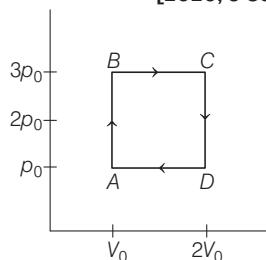
$$= \frac{2400000}{273}$$

$$= 8791.2\text{ cal}$$

$$\approx 8791\text{ cal}$$

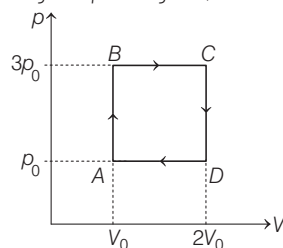
- 63** An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to

[2020, 6 Sep Shift-II]



Ans. (19)

From given p - V diagram,



Work done by gas in close loop, W = Area of loop

$$= (3p_0 - p_0) \times (2V_0 - V_0)$$

$$W = 2p_0V_0$$

During the cycle ABCDA, process AB and BC absorbs the heat.

Heat absorption during process AB,

$$Q_{AB} = \mu C_V \Delta T \text{ (as AB} \rightarrow \text{isochoric process)}$$

$$Q_{AB} = \mu \left(\frac{3}{2} R \right) (T_B - T_A)$$

$$(\because C_V \text{ for monatomic gas is } \frac{3}{2} R)$$

$$= \frac{3}{2} (\mu R T_B - \mu R T_A) = \frac{3}{2} (p_B V_B - p_A V_A)$$

$$(\because pV = \mu RT)$$

$$= \frac{3}{2} (3p_0 V_0 - p_0 V_0) = 3p_0 V_0$$

Heat absorption during process BC,

$$Q_{BC} = \mu C_p \Delta T$$

(as BC $\rightarrow$ isobaric process)

$$= \mu \left(\frac{5}{2} R \right) (T_C - T_B) \quad (\because C_p = C_V + R)$$

$$= \frac{5}{2} (\mu R T_C - \mu R T_B) = \frac{5}{2} (p_C V_C - p_B V_B)$$

$$= \frac{5}{2} (6p_0 V_0 - 3p_0 V_0) = \frac{15}{2} p_0 V_0$$

Total heat absorption, $Q = Q_{AB} + Q_{BC}$

$$= 3p_0 V_0 + \frac{15}{2} p_0 V_0 = \frac{21}{2} p_0 V_0$$

$\therefore$ Efficiency of engine,

$$\eta\% = \frac{W}{Q} \times 100$$

$$= \frac{2p_0 V_0}{\frac{21}{2} p_0 V_0} \times 100$$

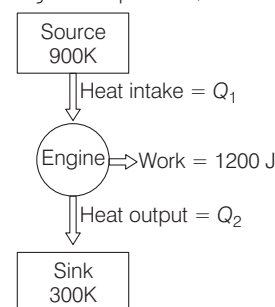
$$= 19\%$$

- 64** A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K . The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir in a cycle, is

[2020, 7 Jan Shift-I]

Ans. (600)

According to the question,



For the given Carnot engine, we have work output, $W = Q_1 - Q_2$ and ratio of heat taken and rejected is $Q_1/Q_2 = T_1/T_2$. So, we have

$$1200 = Q_1 - Q_2 \quad \text{and} \quad \frac{Q_1}{Q_2} = \frac{900}{300} = 3$$

On solving these, we get

$$Q_2 = 600\text{ J}$$

- 65** Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures T_1 and T_2 . The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2 . T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2 , if both the engines perform equal amount of work?

[2020, 7 Jan Shift-II]

$$(a) T = 0$$

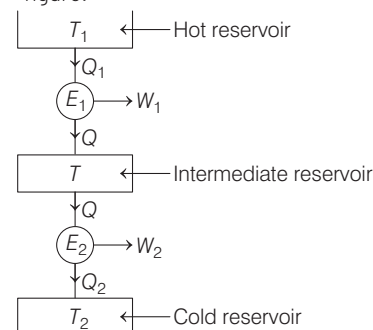
$$(b) T = \frac{2T_1 T_2}{T_1 + T_2}$$

$$(c) T = \sqrt{T_1 T_2}$$

$$(d) T = \frac{T_1 + T_2}{2}$$

Ans. (d)

Carnot engines given are operating in cascade configuration as shown in the figure.



Work done by engine 1,

$$W_1 = Q_1 - Q = k(T_1 - T)$$

Work done by engine 2,

$$W_2 = Q - Q_2 = k(T - T_2)$$

$$\text{So, } \frac{W_1}{W_2} = \frac{T_1 - T}{T - T_2}$$

$$\text{As, } \frac{W_1}{W_2} = 1 \quad (\text{given})$$

$$\Rightarrow T - T_2 = T_1 - T \Rightarrow T = \frac{T_1 + T_2}{2}$$

- 66** A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a

refrigerator. If the work done on the refrigerator is 10 J, then the amount of heat absorbed from the reservoir at lower temperature is:

[2020, 8 Jan Shift-II]

- (a) 99 J (b) 100 J (c) 90 J (d) 1 J

Ans. (c)

Relation of coefficient of performance β of refrigerator and efficiency η of engine,

$$\beta = \frac{1 - \eta}{\eta}$$

$$\text{In given case, } \eta = \frac{1}{10}$$

$$\text{So, } \beta = \frac{1 - \frac{1}{10}}{\frac{1}{10}} = \frac{9}{1} = 9$$

Now, for a Carnot refrigerator, we have

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done}}$$

$$\Rightarrow 9 = \frac{Q}{10} \Rightarrow Q = 90 \text{ J}$$

- 67** A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62°C , its efficiency is doubled. The temperatures of the source and the sink are respectively,

[2019, 12 April Shift-II]

- (a) 62°C , 124°C (b) 99°C , 37°C
(c) 124°C , 62°C (d) 37°C , 99°C

Ans. (b)

Efficiency of a Carnot engine working between source of temperature T_1 and sink of temperature T_2 is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Here, T_2 and T_1 are absolute temperatures.

$$\text{Initially, } \eta = \frac{1}{6}$$

$$\therefore \frac{1}{6} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$$

Finally, efficiency is doubled on reducing sink temperature by 62°C .

$$\therefore \eta = \frac{2}{6}, T_{\text{sink}} = T_2' = T_2 - 62$$

$$\text{So, } \eta = 1 - \frac{T_2'}{T_1}$$

$$\Rightarrow \frac{2}{6} = 1 - \frac{T_2 - 62}{T_1} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{4}{6}$$

$$\Rightarrow \frac{T_2 - 62}{T_1} = \frac{4}{6} \Rightarrow \frac{5}{6} - \frac{62}{T_1} = \frac{4}{6} \Rightarrow \left[\therefore \frac{T_2}{T_1} = \frac{5}{6} \right]$$

$$\Rightarrow T_1 = 6 \times 62 = 372 \text{ K} = 372 - 273 = 99^\circ\text{C}$$

$$\Rightarrow T_2 = \frac{5}{6} \times T_1 = 310 \text{ K} = 310 - 273 = 37^\circ\text{C}$$

- 68** Two Carnot engines A and B are operated in series. The first one, A receives heat at $T_1 (= 600 \text{ K})$ and rejects to a reservoir at temperature T_2 . The second engine B receives heat rejected by the first engine and in turn rejects to a heat reservoir at $T_3 (= 400 \text{ K})$.

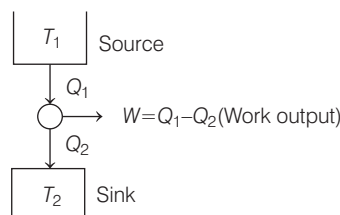
Calculate the temperature T_2 if the work outputs of the two engines are equal.

[2019, 9 Jan Shift-II]

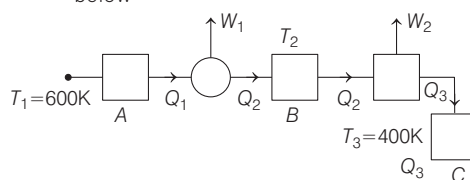
- (a) 600 K (b) 500 K (c) 400 K (d) 300 K

Ans. (b)

Key idea In a Carnot engine the heat flow from higher temperature source (at T_1) to lower temperature sink (at T_2) and give the work done equal to the $W = Q_1 - Q_2$.



For the given condition, Carnot engine A and B are operated in series as shown below



where,

Q_1 = heat rejected by engine A at T_1 K,

Q_2 = heat received by engine B at T_2 K and

Q_3 = heat rejected by engine B to source C at T_3 K.

According to Carnot engine principle,

$W_1 = Q_1 - Q_2$ (work-output from source A and B)

$W_2 = Q_2 - Q_3$ (work output from source B and C)

As per the given condition, if the work outputs of the two engines are equal, then

$$\begin{aligned} Q_1 - Q_2 &= Q_2 - Q_3 \\ \Rightarrow Q_1 + Q_3 &= 2Q_2 \\ \frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} &= 2 \quad \dots(i) \end{aligned}$$

$$\text{Therefore } \frac{T_1}{T_2} + \frac{T_3}{T_2} = 2$$

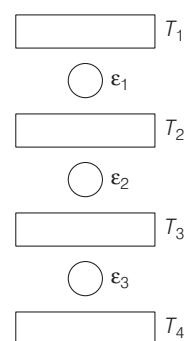
$$\text{So, } \frac{T_1 + T_3}{T_2} = 2$$

$$\Rightarrow T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2}$$

$$T_2 = 500 \text{ K}$$

- 69** Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperatures T_2 and T_3 , as shown with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if

[2019, 10 Jan Shift-I]



$$(a) T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$$

$$(b) T_2 = (T_1^2 T_4)^{1/3}; T_3 = (T_1 T_4^2)^{1/3}$$

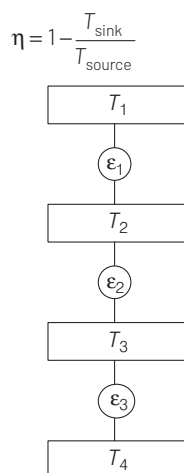
$$(c) T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$$

$$(d) T_2 = (T_1 T_4^2)^{1/3}; T_3 = (T_1^2 T_4)^{1/3}$$

Ans. (b)

Given, Carnot engines operates as,

As, efficiency of a Carnot's engine is given by



We have,

η_1 = efficiency of engine ϵ_1

$$= 1 - \frac{T_2}{T_1}$$

η_2 = efficiency of engine ϵ_2

$$= 1 - \frac{T_3}{T_2}$$

η_3 = efficiency of engine $\epsilon_3 = 1 - \frac{T_4}{T_3}$

For equal efficiencies,

$$\Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow T_2^2 = T_1 T_3 \text{ and } T_3^2 = T_2 T_4$$

$$\Rightarrow T_2^4 = T_1^2 T_3^2$$

$$T_2^3 = T_4 T_1^2 \text{ or } T_2 = (T_1^2 T_4)^{\frac{1}{3}}$$

$$\text{Also, } T_3^4 = T_2^2 T_4^2 = T_1 T_3^2 T_4^2$$

$$T_3^3 = T_1 T_4^2 \text{ or } T_3 = (T_1 T_4^2)^{\frac{1}{3}}$$

- 70** A solid body of constant heat capacity $1 \text{ J/}^\circ\text{C}$ is being heated by keeping it in contact with reservoirs in two ways

[JEE Main 2015]

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to

final temperature 200°C . Entropy change of the body in the two cases respectively, is

- (a) $\ln 2, 4 \ln 2$ (b) $\ln 2, \ln 2$
(c) $\ln 2, 2 \ln 2$ (d) $2 \ln 2, 8 \ln 2$

Ans. (b)

Since, entropy is a state function, therefore change in entropy in both the processes must be same. Therefore, correct option should be (b).

- 71** A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be

[AIEEE 2012]

- (a) efficiency of Carnot engine cannot be made larger than 50%
(b) 1200 K
(c) 750 K
(d) 600 K

Ans. (c)

$$\text{Efficiency, } \eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

$$\text{Now, } 0.4 = 1 - \frac{T_{\text{sink}}}{500 \text{ K}}$$

$$\Rightarrow T_{\text{sink}} = 0.6 \times 500 \text{ K} = 300 \text{ K}$$

$$\text{Thus, } \eta = 1 - \frac{300 \text{ K}}{T_{\text{source}}}$$

$$\Rightarrow T_{\text{source}} = \frac{300 \text{ K}}{0.4} = 750 \text{ K}$$

- 72** The specific heat capacity of a metal at low temperature (T) is given as

$$C_p (\text{kJ K}^{-1} \text{ kg}^{-1}) = 32 \left(\frac{T}{400} \right)^3$$

A 100 g vessel of this metal is to be cooled from 20 K to 4 K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is

[AIEEE 2011]

- (a) equal to 0.002 kJ
(b) greater than 0.148 kJ
(c) between 0.148 kJ and 0.028 kJ
(d) less than 0.028 kJ

Ans. (c)

Heat required to change the temperature of vessel by a small amount dT ,

$$-dQ = mC_p dT$$

Total heat required

$$-Q = m \int_{20}^4 32 \left(\frac{T}{400} \right)^3 dT$$

$$= \frac{100 \times 10^{-3} \times 32}{(400)^3} \left[\frac{T^4}{4} \right]_{20}^4$$

$$\Rightarrow Q = 0.001996 \text{ kJ}$$

Work done required to maintain the temperature of sink to T_2 ,

$$W = Q_1 - Q_2 = \frac{Q_1 - Q_2}{Q_2} Q_2 = \left(\frac{T_1}{T_2} - 1 \right) Q_2$$

$$\Rightarrow W = \left(\frac{T_1 - T_2}{T_2} \right) Q_2$$

For $T_2 = 20 \text{ K}$,

$$W_1 = \frac{300 - 20}{20} \times 0.001996 = 0.028 \text{ kJ}$$

For $T_2 = 4 \text{ K}$,

$$W_2 = \frac{300 - 4}{4} \times 0.001996 = 0.148 \text{ kJ}$$

As temperature is changing from 20 K to 4 K, work done required will be more than W_1 but less than W_2 .

- 73** A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then, T_1 and T_2 are respectively

[AIEEE 2011]

- (a) 372 K and 330 K
(b) 330 K and 268 K
(c) 310 K and 248 K
(d) 372 K and 310 K

Ans. (d)

$$\text{As efficiency, } \eta_1 = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{5}{6} \quad \dots(i)$$

$$\eta_2 = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$T_1 = 372 \text{ K and } T_2 = 310 \text{ K}$$

- 74** A Carnot engine, having an efficiency of $\eta = \frac{1}{10}$ as heat engine

is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

[AIEEE 2007]

- (a) 99 J (b) 90 J (c) 1 J (d) 100 J

Ans. (b)

For Carnot engine using as refrigerator,

$$W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

and $\eta = \frac{W_1}{Q_2} = \left(\frac{T_1}{T_2} - 1 \right)$

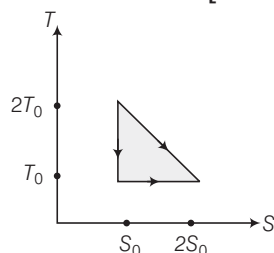
It is given that, $\eta = \frac{1}{10}$

$$\Rightarrow \eta = 1 - \frac{T_2}{T_1} \text{ or } \frac{T_2}{T_1} = \frac{9}{10}$$

So, $Q_2 = 90 \text{ J}$ [as $W = 10 \text{ J}$]

- 75** The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is

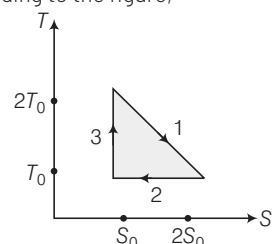
[AIEEE 2005]



- (a) 1/2 (b) 1/4 (c) 1/3 (d) 2/3

Ans. (c)

According to the figure,



For process 1,

$$Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0 = \frac{3}{2} T_0 S_0$$

For process 2,

$$Q_2 = T_0 (2S_0 - S_0) = T_0 S_0$$

For process 3, $Q_3 = 0$

$$\therefore \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

- 76** "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of

[AIEEE 2003]

- (a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics

Ans. (a)

Heat cannot flow itself from a body at lower temperature to a body at higher temperature. This corresponds to second law of thermodynamics.

- 77** A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C . The work done by the engine is

[AIEEE 2003]

- (a) $4.2 \times 10^6 \text{ J}$ (b) $8.4 \times 10^6 \text{ J}$
(c) $16.8 \times 10^6 \text{ J}$ (d) zero

Ans. (b)

$$T_1 = 627 + 273 = 900 \text{ K}, Q_1 = 3 \times 10^6 \text{ cal}$$

$$T_2 = 27 + 273 = 300 \text{ K}$$

Now, from Carnot theorem,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \text{ or } Q_2 = \frac{T_2}{T_1} \times Q_1$$

$$\Rightarrow Q_2 = \frac{300}{900} \times 3 \times 10^6 = 1 \times 10^6 \text{ cal}$$

$$\begin{aligned} \therefore \text{Work done} &= Q_1 - Q_2 \\ &= 3 \times 10^6 - 1 \times 10^6 \\ &= 2 \times 10^6 \text{ cal} \\ &= 2 \times 4.2 \times 10^6 \text{ J} \\ &= 8.4 \times 10^6 \text{ J} \end{aligned}$$

- 78** Even Carnot engine cannot give 100% efficiency, because we cannot

[AIEEE 2002]

- (a) prevent radiation
(b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction

Ans. (c)

The efficiency of Carnot engine is

$$\eta = 1 - (T_2 / T_1)$$

where, T_1 is the temperature of the source and T_2 that of sink.

Since, $\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$

So, $\eta = 1 - \frac{Q_2}{Q_1}$

To obtain 100% efficiency (i.e., $\eta = 1$), Q_2 must be zero i.e., if a sink at absolute zero would be available, all the heat taken from the source would have been converted into work.

The temperature of sink means a negative temperature on the absolute scale at which the efficiency of engine is greater than unity. This would be a violation of the 2nd law of thermodynamics. Hence, a negative temperature on the absolute scale is impossible. Hence, we cannot reach absolute zero temperature.

- 79** Which statement is incorrect?

[AIEEE 2002]

- (a) All reversible cycles have same efficiency
(b) Reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles

Ans. (a)

Efficiency of all reversible cycles depends upon temperature of source and sink which will be different. Rest of the statements are correct.

12

Kinetic Theory

TOPIC 1

Kinetic Theory of Gases and Gas Laws

- 01** For an ideal gas the instantaneous change in pressure p with volume V is given by the equation $\frac{dp}{dV} = -ap$.

If $p = p_0$ at $V = 0$ is the given boundary condition, then the maximum temperature one mole of gas can attain is

(Here R is the gas constant)

[2021, 31 Aug Shift-I]

- (a) $\frac{p_0}{aeR}$ (b) $\frac{ap_0}{cR}$
(c) infinity (d) 0°C

Ans. (a)

Given,

Change in pressure with volume

$$\frac{dp}{dV} = -ap$$

At $V = 0$, $p = p_0$

Number of mole $n = 1$

Gas constant, $R = 8.314 \text{ J K}^{-1}$

From given equation,

$$\frac{dp}{p} = -a dV$$

$$\Rightarrow \int_{p_0}^p \frac{dp}{p} = -a \int_0^V dV \Rightarrow [\ln p]_{p_0}^p = -a[V]_0^V$$

$$\Rightarrow \ln(p/p_0) = -a(V - 0)$$

Taking anti log on both side,

$$\Rightarrow p/p_0 = e^{-aV}$$

$$\Rightarrow p = p_0 e^{-aV}$$

By using ideal gas law,

$$pV = nRT$$

$$\Rightarrow (p_0 e^{-aV}) V = nRT$$

$$\Rightarrow \frac{1}{nR} (p_0 e^{-aV}) V = T$$

$$\Rightarrow \frac{1}{R} (p_0 e^{-aV}) V = T \quad [\because n=1] \dots(i)$$

For maximum temperature ($T_{\max}$),

On differentiating both side with respect to V , we get

$$\Rightarrow \frac{1}{nR} [p_0 \cdot e^{-aV} \cdot (-aV) + p_0 e^{-aV}]$$

$$= \frac{dT}{dV} = 0$$

$$\Rightarrow p_0 e^{-aV} (1 - aV) = 0$$

$$\Rightarrow 1 - aV = 0 \Rightarrow V = 1/a$$

$$\Rightarrow V = 1/a$$

Substituting the value in Eq. (i), we get

$$T = \frac{1}{R} (p_0 e^{-a \cdot 1/a}) (1/a) = \frac{p_0 e^{-1}}{R} (1/a)$$

$$= \frac{p_0}{eRa}$$

- 02** A mixture of hydrogen and oxygen has volume 500 cm^3 , temperature 300 K , pressure 400 kPa and mass 0.76 g . The ratio of masses of oxygen to hydrogen will be

[2021, 31 Aug Shift-II]

- (a) 3 : 8 (b) 3 : 16
(c) 16 : 3 (d) 8 : 3

Ans. (c)

Given,

$$\text{Volume, } V = 500 \text{ cm}^3 = 5 \times 10^2 \times 10^{-6} \text{ m}^3$$

$$= 5 \times 10^{-4} \text{ m}^3$$

$$\text{Pressure, } p = 400 \text{ kPa} = 4 \times 10^5 \text{ Pa}$$

$$\text{Temperature, } T = 300 \text{ K}$$

$$\text{Total mass, } m = 0.76 \text{ g}$$

We know that,

$$\text{Molar mass of hydrogen, } M_1 = 2 \text{ g}$$

$$\text{Molar mass of oxygen, } M_2 = 32 \text{ g}$$

Let n_1 and n_2 be the number of moles of hydrogen and oxygen, respectively.

Total number of moles of mixture,

$$n = n_1 + n_2$$

By using ideal gas equation, $pV = nRT$

Substituting the values, we get

$$4 \times 10^5 \times 5 \times 10^{-4} = n \times 8.314 \times 300$$

$$n = 0.08 \text{ mol}$$

$$\text{Then, } n_1 + n_2 = 0.08 \dots(i)$$

Let mass of hydrogen in mixture = m_1

and mass of oxygen in oxygen = m_2

$$n_1 M_1 + n_2 M_2 = m$$

$$[\because n = \frac{m}{M} \Rightarrow m = nM]$$

Substituting the values, we get

$$2n_1 + 32n_2 = 0.76$$

$$n_1 + 16n_2 = 0.38 \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$n_1 = 0.06$$

$$\text{and } n_2 = 0.02$$

$\therefore$ Ratio of masses,

$$\frac{m_2}{m_1} = \frac{n_2 M_2}{n_1 M_1} = \frac{0.02 \times 32}{0.06 \times 2} = \frac{16}{3}$$

$$\text{i.e. } m_2 : m_1 = 16 : 3$$

- 03** If the rms speed of oxygen molecules at 0°C is 160 m/s , find the rms speed of hydrogen molecules at 0°C .

[2021, 27 Aug Shift-II]

$$(a) 640 \text{ m/s}$$

$$(b) 40 \text{ m/s}$$

$$(c) 80 \text{ m/s}$$

$$(d) 332 \text{ m/s}$$

Ans. (a)

Given, rms speed of oxygen $(v_{\text{rms}})_{\text{O}_2} = 160 \text{ m/s}$

Let rms speed of hydrogen $= (v_{\text{rms}})_{\text{H}_2}$

$$\text{Since, } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where, R = universal gas constant,

T = temperature

and M = molecular mass.

$$\therefore \frac{(v_{\text{rms}})_{\text{O}_2}}{(v_{\text{rms}})_{\text{H}_2}} = \sqrt{\frac{M_{\text{H}_2}}{M_{\text{O}_2}}}$$

$$\Rightarrow \frac{160}{(v_{\text{rms}})_{\text{H}_2}} = \sqrt{\frac{1}{16}}$$

$$\Rightarrow \frac{160}{(v_{\text{rms}})_{\text{H}_2}} = \frac{1}{4}$$

$$\Rightarrow (v_{\text{rms}})_{\text{H}_2} = 160 \times 4 = 640 \text{ ms}^{-1}$$

- 04** An ideal gas is expanding such that $pT^3 = \text{constant}$. The coefficient of volume expansion of the gas is

[2021, 27 Aug Shift-I]

- (a) $1/T$ (b) $2/T$
(c) $4/T$ (d) $3/T$

Ans. (c)

Given, $pT^3 = \text{constant}$... (i)

From ideal gas equation, we have

$$\Rightarrow \frac{pV}{nRT} = 1$$

From Eq. (i), we get

$$\therefore \left(\frac{nRT}{V}\right)T^3 = \text{constant}$$

$$\Rightarrow nRT^4V^{-1} = \text{constant}$$

Differentiate above expression, we get

$$\Rightarrow 4nRT^3V^{-1}dT - \frac{nRT^4}{V^2}dV = 0$$

$$\Rightarrow 4dT = \frac{T}{V}dV \quad \dots (ii)$$

We know that, $dV = \gamma VdT$

Here, γ = coefficient of volume expansion.

$$\Rightarrow \gamma = \frac{dV}{VdT} \quad \dots (iii)$$

From Eq. (ii),

$$\frac{4}{T} = \frac{dV}{VdT} \quad \dots (iv)$$

On comparing Eqs. (iii) and (iv), we get

$$\gamma = \frac{4}{T}$$

Thus, the coefficient of volume expansion is $4/T$.

- 05** A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C . What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is -7°C ? [Assuming, the volume constant.]

[2021, 27 Aug Shift-I]

- (a) 181.46 kg (b) 214.15 kg
(c) 219.07 kg (d) 123.54 kg

Ans. (d)

Given, initial load, $M_1 = 185$ kg
Initial pressure, $p_1 = 76$ cm of Hg
Initial temperature, $T_1 = 27^\circ\text{C}$
 $= (273 + 27)\text{K} = 300\text{ K}$

Final pressure at height,

$$p_2 = 45 \text{ cm of Hg}$$

Final temperature, $T_2 = -7^\circ\text{C}$

$$= [273 + (-7)]\text{K} = 266\text{ K}$$

Volume of balloon is constant, $V_1 = V_2 = V$

We know that, for air,

$$p = \rho RT$$

Now, for initial condition, we can write

$$p_1 = \rho_1 RT_1 \quad \dots (i)$$

Similarly, for final condition, we can write

$$p_2 = \rho_2 RT_2 \quad \dots (ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{p_1}{p_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$$

Substituting the given values in above expression, we get

$$\frac{76 \text{ cm of Hg}}{45 \text{ cm of Hg}} = \frac{\rho_1 \left(\frac{300\text{K}}{266\text{K}}\right)}{\rho_2}$$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{76 \times 266}{45 \times 300}$$

We know that, density of balloon can be written as

$$\rho = \frac{M}{V}$$

As, volume is constant $\rho \propto M$

We can write,

$$\frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} = \frac{76 \times 266}{45 \times 300} \Rightarrow \frac{M_1}{M_2} = \frac{20216}{13500}$$

Substituting the value of M_1 in above expression, we get

$$M_2 = 185 \times \frac{13500}{20216} = 123.54 \text{ kg}$$

Thus, the load carried by balloon will be 123.54 kg.

- 06** A cylindrical container of volume $4.0 \times 10^{-3} \text{ m}^3$ contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is [Take, gas constant $= 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$]

[2021, 26 Aug Shift-II]

- (a) $249 \times 10^1 \text{ Pa}$ (b) $24.9 \times 10^3 \text{ Pa}$
(c) $24.9 \times 10^5 \text{ Pa}$ (d) 24.9 Pa

Ans. (c)

Given, volume of container,

$$V = 4.0 \times 10^{-3} \text{ m}^3$$

Gas constant, $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$

Number of moles of hydrogen, $n_1 = 1$

Number of moles of carbon dioxide,

$$n_2 = 2$$

Temperature of mixture, $T = 400 \text{ K}$

Using Dalton's law to calculate the pressure of mixture as follows

$$p = p_1 + p_2$$

$$\begin{aligned} &= \frac{n_1 RT}{V} + \frac{n_2 RT}{V} \\ &= \frac{RT}{V} (n_1 + n_2) = \frac{RT}{V} (1 + 2) = \frac{3RT}{V} \\ &= \frac{3 \times 8.3 \times 400}{4.0 \times 10^{-3}} = 24.9 \times 10^5 \text{ Pa} \end{aligned}$$

Thus, the pressure of mixture of gases is $24.9 \times 10^5 \text{ Pa}$.

- 07** The rms speeds of the molecules of hydrogen, oxygen and carbondioxide at the same temperature are v_H, v_O and v_{CO_2} respectively, then

[2021, 26 Aug Shift-I]

- (a) $v_H > v_O > v_{CO_2}$ (b) $v_{CO_2} > v_O > v_H$
(c) $v_H = v_O > v_{CO_2}$ (d) $v_H = v_O = v_{CO_2}$

Ans. (a)

The rms speed of the molecule of gas is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

According to the, question T is constant. Moreover R is also a constant. So, we get

$$v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$$

$$(M_H) M_{\text{hydrogen}} = 1.00784 \text{ u} \approx 1 \text{ u}$$

$$(M_O) M_{\text{oxygen}} = 15.999 \text{ u} \approx 16 \text{ u}$$

$$(M_{CO_2}) M_{\text{carbon dioxide}} = 44 \text{ u}$$

$$(v_{\text{rms}})_{\text{hydrogen}} > (v_{\text{rms}})_{\text{oxygen}} > (v_{\text{rms}})_{\text{carbon dioxide}} \Rightarrow v_H > v_O > v_{CO_2}$$

- 08** The number of molecules in 1 L of an ideal gas at 300 K and 2 atm pressure with mean kinetic energy $2 \times 10^{-9} \text{ J}$ per molecules is

[2021, 27 July Shift-I]

- (a) 0.75×10^{11} (b) 3×10^{11}
(c) 1.5×10^{11} (d) 6×10^{11}

Ans. (c)

Given, pressure of ideal gas, $p = 2 \text{ atm}$

$$= 2 \times 1 \times 10^5 \text{ N/m}^2$$

$$= 2 \times 10^5 \text{ N/m}^2$$

Volume of gas, $V = 1 \text{ L} = 10^{-3} \text{ m}^3$

We know that,

$$KE = \frac{3}{2} KT = \frac{3}{2} \frac{RT}{N_A} \quad \dots (i)$$

where, KE = mean kinetic energy,

K = Boltzmann constant

and T = temperature.

As per ideal gas equation,

$$pV = nRT$$

$$\Rightarrow pV = \frac{N}{N_A} RT$$

$$\Rightarrow pV = \frac{NRT}{N_A} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\therefore N = \frac{pV}{\left(\frac{2}{3}\right)KE}$$

$$\Rightarrow N = \frac{2 \times 10^5 \times 10^{-3}}{\frac{2}{3} \times 2 \times 10^{-9}}$$

$$\Rightarrow N = 1.5 \times 10^{11}$$

- 09** A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25} / \text{m}^3$. The diameter of A and B are $10 \text{ \AA}$ and $5 \text{ \AA}$, respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collisions is $\times 10^{-2}$. [2021, 25 July Shift-II]

Ans. (25)

Given, number density of gas A and B be n_A and n_B

$$\text{i.e. } n_A = n_B = 2 \times 10^{25} / \text{m}^3$$

Diameter of A and B, $d_A = 10 \text{ \AA}$, $d_B = 5 \text{ \AA}$

Let average distance covered by A and B be λ_A and λ_B .

As we know that,

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n}$$

For gas A,

$$\lambda_A = \frac{1}{\sqrt{2} \pi d_A^2 n_A} \quad \dots(i)$$

For gas B,

$$\text{and } \lambda_B = \frac{1}{\sqrt{2} \pi d_B^2 n_B} \quad \dots(ii)$$

On dividing Eqs. (i) by (ii), we get

$$\begin{aligned} \frac{\lambda_A}{\lambda_B} &= \frac{\sqrt{2} \pi d_B^2 n_B}{\sqrt{2} \pi d_A^2 n_A} \\ \Rightarrow \frac{\lambda_A}{\lambda_B} &= \frac{d_B^2 n_B}{d_A^2 n_A} = \frac{5^2}{10^2} \times \frac{2 \times 10^{25}}{2 \times 10^{25}} \\ &= \frac{25}{100} = \frac{1}{4} = 0.25 \\ &= 25 \times 10^{-2} \end{aligned}$$

- 10** For a gas $C_p - C_v = R$ in a state P and $C_p - C_v = 1.10 R$ in a state Q. T_P and T_Q are the temperatures in two different states P and Q, respectively. Then, [2021, 25 July Shift-I]
- (a) $T_P = T_Q$ (b) $T_P < T_Q$
(c) $T_P = 0.9 T_Q$ (d) $T_P > T_Q$

Ans. (d)

Given, $C_p - C_v = R$, for state P

and $C_p - C_v = 1.1 R$, for state Q

where, C_p and C_v are the specific heat at constant pressure and constant volume, respectively.

and R is the universal gas constant

Let n be the number of moles of gases.

Change in temperature in state P,

$$\Delta T_P = T_P - 0 = T_P$$

and change in temperature in state Q,

$$\Delta T_Q = T_Q - 0 = T_Q$$

As we know that,

Ideal gas obeys the Mayer's equation,

$$\text{i.e. } C_p - C_v = R$$

$\therefore$ P will be the ideal gas, but Q will not be the ideal gas.

Also, a gas behave like an ideal gas at high temperature and low pressure.

$$\therefore T_P > T_Q$$

- 11** Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is [2021, 20 July Shift-I]

$$(a) v_A = v_B = v_C = 0 \quad (b) \frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$$

$$(c) v_A = v_B \neq v_C \quad (d) \frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$$

Ans. (d)

rms velocity of gas molecules is given as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{m}} \quad \dots(i)$$

where, m = molar mass of the gas in kilograms per mole,

R = molar gas constant

and T = temperature in kelvin.

According to question,

$$m_A < m_B < m_C$$

From Eq. (i)

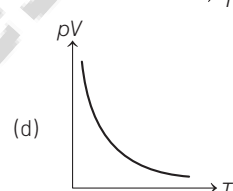
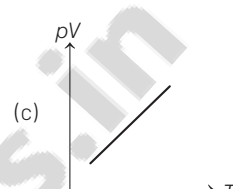
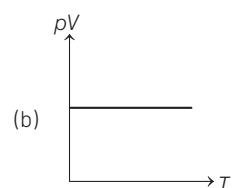
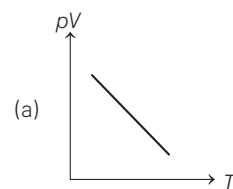
$$v_{\text{rms}} \propto \frac{1}{\sqrt{m}}$$

$\therefore$ We can write,

$$v_A > v_B > v_C$$

$$\text{or } \frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$$

- 12** Which of the following graphs represent the behaviour of an ideal gas? (Symbols have their usual meanings.) [2021, 20 July Shift-II]



Ans. (c)

According to ideal gas equation,

$$pV = nRT \quad \dots(i)$$

where, p = absolute pressure of gas,

V = volume the gas occupies,

n = number of atoms and molecules in the gas,

R = Rydberg constant

and T = absolute temperature of gas.

From Eq. (i) can be written as

$$pV = cT$$

where, c is any constant.

$$\Rightarrow pV \propto T$$

It means that pV versus T graph is a straight line with positive slope.

- 13** Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be (Molecular weight of oxygen is 32 g/mol; $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$) [2021, 18 March Shift-II]

(a) $\sqrt{\frac{3}{3}}$ (b) $\sqrt{\frac{8}{3}}$
(c) $\sqrt{\frac{3\pi}{8}}$ (d) $\sqrt{\frac{8\pi}{3}}$

Ans. (c)

The expression of the root mean square velocity of the gas molecules,

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

The expression of the average velocity of the gas molecules,

$$v_{av} = \sqrt{\frac{8RT}{\pi M}}$$

$$\frac{v_{rms}}{v_{av}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{3\pi}{8}}$$

- 14** The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T . Consider R as universal gas constant. The pressure of the mixture of gases is

[2021, 16 March Shift-I]

- (a) $\frac{88 RT}{V}$ (b) $\frac{3RT}{V}$
(c) $\frac{5 RT}{2 V}$ (d) $\frac{4RT}{V}$

Ans. (c)

From ideal gas equation,

$$pV = (n_1 + n_2 + n_3)RT$$

$$\text{where, } n_1 = \text{number of moles of oxygen} = \frac{16}{32}$$

$$n_2 = \text{number of moles of nitrogen} = \frac{28}{28}$$

$$n_3 = \text{number of moles of carbon dioxide} = \frac{44}{44}$$

$$\Rightarrow pV = \left[\frac{16}{32} + \frac{28}{28} + \frac{44}{44} \right] RT$$

$$= \left[\frac{1}{2} + 1 + 1 \right] RT$$

$$= \frac{5}{2} RT \Rightarrow p = \frac{5 RT}{2 V}$$

This is the required pressure of the mixture of the gases.

- 15** Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$)

[2021, 16 March Shift-II]

- (a) 58 nm (b) 32 nm
(c) 86 nm (d) 102 nm

Ans. (d)

Given,

$$\text{Pressure of the gas, } p = 1.01 \times 10^5 \text{ Pa}$$

$$\text{Absolute temperature of gas, } T = 27^\circ\text{C}$$

$$= 27 + 273 = 300 \text{ K}$$

$$\text{Molecular diameter, } d = 0.3 \text{ nm}$$

$$= 0.3 \times 10^{-9} \text{ m}$$

Boltzmann constant,

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{Mean free path, } \lambda = \frac{k_B T}{\sqrt{2} \pi d^2 p} \quad \dots (i)$$

Substituting the given values in Eq. (i), we get

$$\lambda = \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 3.14 \times (0.3 \times 10^{-9})^2 \times 1.01 \times 10^5}$$

$$= 102 \text{ nm}$$

- 16** The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$

where, α and β are positive constants. The equilibrium distance between two atoms will

be $\left(\frac{2\alpha}{\beta} \right)^{\frac{a}{b}}$, where $a = \dots\dots\dots$

[2021, 25 Feb Shift-I]**Ans. (1)**

$$\text{Given, potential energy, } U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

As we know for equilibrium, differentiation of potential energy with respect to distance $\left(\frac{du}{dr} \right) = 0$

$$\Rightarrow \frac{d}{dr} \left(\frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3 \right) = 0$$

$$\Rightarrow \frac{d}{dr} (\alpha r^{-10} - \beta r^{-5} - 3) = 0$$

$$\Rightarrow -10\alpha r^{-11} + 5\beta r^{-6} - 0 = 0$$

$$\Rightarrow 10\alpha r^{-11} = 5\beta r^{-6}$$

$$\Rightarrow 2\alpha r^{-11} = \beta r^{-6}$$

$$\Rightarrow \frac{2\alpha}{\beta} = \frac{r^{-6}}{r^{-11}} = r^5 \Rightarrow r = \left(\frac{2\alpha}{\beta} \right)^{1/5}$$

$$\therefore a = 1$$

- 17** The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1} . The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is

$$\frac{x}{\sqrt{3}} \text{ ms}^{-1}. \text{ The value of } x \text{ will be}$$

[2021, 24 Feb Shift-II]**Ans. (400)**

$$\text{Given, } T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K,}$$

$$p_1 = 1 \text{ atm, } v_1 = 200 \text{ ms}^{-1}, T_2 = 127^\circ\text{C}$$

$$= 400 \text{ K,}$$

$$p_2 = 2 \text{ atm, } v_2 = ?$$

As we know that,

$$\text{Root mean square speed, } v_{rms} = \sqrt{\frac{3RT}{m}}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow v_2 = \sqrt{\frac{4}{3}} v_1 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1}$$

$$\Rightarrow \frac{x}{\sqrt{3}} = \frac{400}{\sqrt{3}} \Rightarrow x = 400$$

- 18** On the basis of kinetic theory of gases, the gas exerts pressure because its molecules

[2021, 24 Feb Shift-II]

- (a) continuously lose their energy till it reaches wall
(b) are attracted by the walls of container
(c) continuously stick to the walls of container
(d) suffer change in momentum when impinge on the walls of container

Ans. (d)

On the basis of kinetic theory of gases, the gas exerts pressure because its molecules contain uniform speed, random motion and perform elastic collision with each other, as well as with the walls of container. As a result of which gaseous molecules suffer change in momentum when impinge on the walls of container.

- 19** A container is divided into two chambers by a partition. The volume of first chamber is 4.5 L and second chamber is 5.5 L. The first chamber contain 3.0 mol of gas at pressure 2.0 atm and second chamber contain 4.0 mol of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then the common equilibrium pressure existing in the mixture is $x \times 10^{-1}$ atm. Value of x is

[2021, 26 Feb Shift-I]

Ans. (25.5)Given, volume of 1st chamber, $V_1 = 4.5$ LVolume of 2nd chamber, $V_2 = 5.5$ L

$$n_1 = 3 \text{ mol}$$

$$p_1 = 2 \text{ atm,}$$

$$n_2 = 4 \text{ mol}$$

$$p_2 = 3 \text{ atm}$$

By using ideal gas equation, $pV = nRT$

$$\therefore p_1 V_1 + p_2 V_2 = p(V_1 + V_2)$$

$$\Rightarrow 2 \times 4.5 + 3 \times 5.5 = p \times 10$$

$$\Rightarrow 9 + 16.5 = 10p \Rightarrow \frac{25.5}{10} = p$$

$$\therefore p = 25.5 \times 10^{-1} \text{ atm}$$

Hence, $x = 25.5$ atm

- 20** In a dilute gas at pressure p and temperature T , the mean time between successive collisions of a molecule varies with T as

[2020, 6 Sep Shift-II]

- (a) T (b) $\frac{1}{\sqrt{T}}$
(c) $\frac{1}{T}$ (d) $\sqrt{T}$

Ans. (b)The expression of mean time (τ) between successive collisions of a molecule at pressure p and temperature T is given by

$$\tau = \frac{\lambda}{v_{\text{avg}}}$$

As, mean free path of gas molecule,

$$\lambda = \frac{1}{\sqrt{2} \pi d^2 n_v}$$

(where, n_v = number of molecules volume)

$$\Rightarrow \lambda \propto (T)^0$$

$$\text{Also, average speed, } v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}$$

$$\Rightarrow v_{\text{avg}} \propto \sqrt{T}$$

$$\therefore \tau \propto \frac{1}{\sqrt{T}}$$

Hence, correct option is (b).

- 21** A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to

[2020, 4 Sep Shift-I]**Ans. (266)**

Applying conservation of internal energy,

$$U_1 + U_2 = U_1' + U_2'$$

$$n_1 C_V T_1 + n_2 C_V T_2 = (n_1 + n_2) C_V T$$

$$\begin{aligned} n_1 T_1 + n_2 T_2 &= (n_1 + n_2) T \\ (0.1)(200) + (0.05)(400) &= (0.1 + 0.05) T \\ 20 + 20 &= (0.15) T \\ T &= \frac{40}{0.15} = 266.66 \text{ K} \approx 266 \text{ K} \end{aligned}$$

- 22** The change in the magnitude of the volume of an ideal gas when a small additional pressure Δp is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas are 300 K and 2 atm, respectively. If $|\Delta T| = C|\Delta p|$, then value of C (in K/atm) is

[2020, 4 Sep Shift-II]**Ans. (150)**

From ideal gas equation,

$$pV = nRT$$

At constant temperature, for small changes in p and V ,

$$\Delta(pV) = \Delta(nRT) \Rightarrow p\Delta V + V\Delta p = 0$$

$$\Rightarrow V\Delta p = -p\Delta V \Rightarrow \Delta p = -\frac{p\Delta V}{V}$$

$$\Rightarrow |\Delta p| = \left| -\frac{p\Delta V}{V} \right| \Rightarrow |\Delta p| = \frac{p\Delta V}{V} \quad \dots(i)$$

Again, from ideal gas equation,

$$pV = nRT$$

At constant pressure, for small changes in V and T .

$$\Delta(pV) = \Delta(nRT)$$

$$\Rightarrow p\Delta V = nR\Delta T$$

$$\Rightarrow \Delta T = \frac{p\Delta V}{nR}$$

$$\Rightarrow |\Delta T| = \left| \frac{p\Delta V}{nR} \right|$$

$$\Rightarrow |\Delta T| = \frac{p\Delta V}{nR} \quad \dots(ii)$$

If ΔV is same in both the cases (as given), then dividing eq. (ii) by eq. (i), we get

$$\frac{|\Delta T|}{|\Delta p|} = \frac{\left(\frac{p\Delta V}{nR} \right)}{\left(\frac{p\Delta V}{V} \right)} \Rightarrow \frac{|\Delta T|}{|\Delta p|} = \frac{V}{nR}$$

$$\Rightarrow |\Delta T| = \frac{V}{nR} |\Delta p|$$

$$\Rightarrow |\Delta T| = \frac{T}{p} |\Delta p| \quad (\because pV = nRT \Rightarrow \frac{V}{nR} = \frac{T}{p})$$

$$\Rightarrow |\Delta T| = \frac{300}{2} |\Delta p|$$

$$(\text{as } T = 300 \text{ K and } p = 2 \text{ atm})$$

$$\Rightarrow |\Delta T| = 150 |\Delta p|$$

Given that,

$$|\Delta T| = C |\Delta p|$$

On comparing both equations, we get

$$C = 150$$

- 23** Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to
(Given, mean kinetic energy of a molecule at T is 4×10^{-14} erg, $g = 980 \text{ cm/s}^2$, density of mercury = 13.6 g/cm^3)
- [2020, 5 Sep Shift-I]**
(a) 5.8×10^{16} (b) 4.0×10^{16}
(c) 4.0×10^{18} (d) 5.8×10^{18}

Ans. (c)Given, height, $h = 2$ cmKinetic energy, $U = 4 \times 10^{-14}$ ergDensity of mercury, $\rho = 13.6 \text{ g/cm}^3$ Volume, $V = 4 \text{ cm}^3$, $g = 980 \text{ cm/s}^2$

According to gas equation,

$$pV = NKT \Rightarrow N = \frac{pV}{KT}$$

For monoatomic gas,

$$\text{Energy, } U = \frac{3}{2} KT \Rightarrow KT = \frac{2}{3} U$$

$$\text{Now, } N = \frac{3pV}{2U}$$

$$\text{So, } N = \frac{3\rho ghV}{2U}$$

$$(\because \text{Pressure, } p = \rho gh) \dots(i)$$

Substituting the given values in eq. (i), we get

$$N = \frac{3 \times 980 \times 13.6 \times 2 \times 4}{2 \times 4 \times 10^{-14}} = 3.99 \times 10^{18}$$

$$\approx 4 \times 10^{18}$$

So, correct option is (c).

- 24** Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a N_2 molecule is

(Molar mass of N_2 gas is 28 g.)**[2020, 5 Sep Shift-II]****Ans. (41)**

The rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\text{For nitrogen molecule, } (v_{\text{rms}})_{\text{N}_2} = \sqrt{\frac{3RT_1}{M_1}}$$

and for hydrogen molecule,

$$(v_{rms})_{H_2} = \sqrt{\frac{3RT_2}{M_2}}$$

According to question,

$$(v_{rms})_{N_2} = (v_{rms})_{H_2}$$

$$\Rightarrow \sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{3RT_2}{M_2}} \Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{M_1}{M_2} \Rightarrow T_2 = \frac{T_1 M_2}{M_1} \quad \dots(i)$$

Given, $T_1 = 300^\circ\text{C} = 300 + 273 = 573\text{ K}$
 $M_1 = 28\text{ g}$ and $M_2 = 2\text{ g}$

Substituting all these values in Eq. (i), we get

$$T_2 = 573 \times \frac{2}{28} = 40.93\text{ K}$$

$$\approx 41\text{ K}$$

Hence, the required temperature is 41 K.

- 25** Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure p_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by p_2 . The ratio p_2/p_1 is [2020, 6 Sep Shift-I]

Ans. (5)



At 250 K

At 2000 K

Let $\mu_1 = \mu_0$, $V_2 = 2V_1$ (given)

Total number of moles at 2000 K
 = Number of moles of diatomic molecules + Number of moles of monoatomic

$$\mu_2 = 0.75\mu_0 + 2 \times 0.25\mu_0 = 1.25\mu_0$$

Ideal gas equation for situation 1,

$$p_1 V_1 = \mu_0 R T_1$$

$$p_1 V_1 = \mu_0 R (250) \quad \dots(i)$$

For situation 2,

$$p_2 V_2 = (1.25\mu_0) R (2000)$$

$$p_2 (2V_1) = (1.25\mu_0) R (2000) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{p_1 V_1}{p_2 (2V_1)} = \frac{\mu_0 R (250)}{(1.25\mu_0) R (2000)}$$

$$\frac{p_1}{p_2} = \frac{1}{5} \text{ or } \frac{p_2}{p_1} = 5$$

- 26** An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

[2020, 2 Sep Shift-II]

- A. The mean free path of the molecules decreases.
- B. The mean collision time between the molecules decreases.
- C. The mean free path remains unchanged.
- D. The mean collision time remains unchanged.

- (a) B and C (b) A and B
 (c) C and D (d) A and D

Ans. (a)

Mean free path,

$$\lambda = \frac{V}{\sqrt{2}\pi d^2 N} = \frac{kT}{\sqrt{2}\pi d^2 p}$$

where, V = volume of conductor,

N = number of molecules,

d = diameter of molecule,

k = Boltzmann's constant,

T = temperature

and p = pressure.

But from gas equation, $\frac{p}{T} = \frac{nR}{V}$

For a constant n and V ; p/T = constant.

So, mean free path does not depend on p/T or p and T .

So, in given condition (closed container), mean free path remains constant or unchanged.

$\therefore$ Statement A is incorrect, but C is correct.

Also, mean time between two collisions is

$$\tau = \frac{\text{mean free path}}{V_{\text{avg}}} = \frac{\lambda}{V_{\text{avg}}}$$

As, $V_{\text{avg}} \propto \sqrt{T}$

$$\Rightarrow \text{Average time, } \tau \propto \frac{1}{\sqrt{T}}$$

As, the temperature is increasing, so mean collision time (τ) decreases.

$\therefore$ Statement B is correct.

Hence, option (a) is correct.

- 27** Two moles of an ideal gas with

$$\frac{C_p}{C_v} = \frac{5}{3} \text{ are mixed with 3 mol of}$$

another ideal gas with $\frac{C_p}{C_v} = \frac{4}{3}$. The

value of $\frac{C_p}{C_v}$ for the mixture is

[2020, 7 Jan Shift-I]

- (a) 1.42 (b) 1.47
 (c) 1.50 (d) 1.45

Ans. (a)

When n_1 moles of an ideal gas having specific heats C_{p1} and C_{v1} is mixed with n_2 moles of another ideal gas with specific heats C_{p2} and C_{v2} , specific heats of mixture are given by

$$C_{p(\text{mix})} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 + n_2}$$

and $C_{v(\text{mix})} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$

So, ratio of specific heats of mixture will be

$$\frac{C_{p(\text{mix})}}{C_{v(\text{mix})}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}} \quad \dots(i)$$

Now, using $C_p = \left(\frac{\gamma}{\gamma-1}\right)R$

and $C_v = \frac{R}{\gamma-1}$

[Note] As $\frac{C_p}{C_v} = \gamma$ and $C_p - C_v = R$

$$\text{So, } C_p - \frac{C_p}{\gamma} = R \text{ or } C_p = \frac{\gamma}{\gamma-1} \text{ and } C_v = \frac{R}{\gamma-1}$$

We can write Eq. (i) as

$$\begin{aligned} \frac{C_{p(\text{mix})}}{C_{v(\text{mix})}} &= \frac{n_1 \left(\frac{\gamma_1}{\gamma_1-1}\right)R + n_2 \left(\frac{\gamma_2}{\gamma_2-1}\right)R}{n_1 \left(\frac{R}{\gamma_1-1}\right) + n_2 \left(\frac{R}{\gamma_2-1}\right)} \\ &= \frac{n_1 \gamma_1 (\gamma_2 - 1) + n_2 \gamma_2 (\gamma_1 - 1)}{n_1 (\gamma_2 - 1) + n_2 (\gamma_1 - 1)} \quad \dots(ii) \end{aligned}$$

Here, we are given

$$n_1 = 2, \gamma_1 = \frac{5}{3}, n_2 = 3$$

and $\gamma_2 = \frac{4}{3}$

Substituting the above values in Eq. (ii), we have

$$\frac{C_{p(\text{mix})}}{C_{v(\text{mix})}} = \frac{2 \times \frac{5}{3} \left(\frac{4}{3} - 1\right) + 3 \times \frac{4}{3} \left(\frac{5}{3} - 1\right)}{2 \times \left(\frac{4}{3} - 1\right) + 3 \times \left(\frac{5}{3} - 1\right)}$$

$$= \frac{\frac{10}{3} + \frac{24}{3}}{\frac{2}{3} + \frac{6}{3}} = \frac{34 \times 3}{9 \times 3}$$

$$= \frac{17}{12} = 1.42$$

- 28** Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to [2020, 9 Jan Shift-II]

(a) 4.67 (b) 2.3
(c) 3.67 (d) 1.83

Ans. (*)

Mean free path for a gas,

$$\lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

where, n = number density

and d = diameter of gas atom.

Mean speed of a gas atom,

$$v = \sqrt{\frac{8 k T}{\pi m}}$$

where, k = Boltzmann's constant,

T = temperature of sample

and m = atomic mass.

Hence, mean relaxation time,

$$\tau = \frac{\lambda}{v} = \frac{\left(\frac{1}{\sqrt{2} \pi n d^2} \right)}{\sqrt{\frac{8 k T}{\pi m}}}$$

$$\tau \propto \frac{\sqrt{m}}{d^2} \text{ when } T \text{ and } n \text{ are same.}$$

$$\text{Hence, } \frac{\tau_{Ar}}{\tau_{Xe}} = \frac{\sqrt{m_{Ar}}}{\sqrt{m_{Xe}}} \cdot \frac{d_{Xe}^2}{d_{Ar}^2} = \frac{\sqrt{m_{Ar}}}{\sqrt{m_{Xe}}} \cdot \frac{r_{Xe}^2}{r_{Ar}^2}$$

Here, $m_{Ar} = 40$, $m_{Xe} = 140$,

$r_{Ar} = 0.07 \text{ nm}$,

$r_{Xe} = 0.1 \text{ nm}$

$$\therefore \frac{\tau_{Ar}}{\tau_{Xe}} = \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07} \right)^2 = 1.09$$

- 29** Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecule changes from τ_1 to τ_2 . If $\frac{C_p}{C_v} = \gamma$ for this gas, then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by

[2020, 7 Jan Shift-II]

- (a) $\left(\frac{1}{2} \right)^{\frac{\gamma+1}{2}}$ (b) $\frac{1}{2}$
(c) 2 (d) $\left(\frac{1}{2} \right)^\gamma$

Ans. (a)

Mean free path (λ) of a gas molecule and mean relaxation time (or collision time) τ are related as

$$\begin{aligned} \tau &= \frac{\lambda}{v_{\text{mean}}} = \frac{\frac{1}{\sqrt{2}} \frac{k_B T}{\pi d^2 p}}{\sqrt{\frac{8}{\pi} \cdot \frac{k_B \cdot T}{m}}} \\ &= \frac{1}{4d^2} \left(\frac{k_B m}{\pi} \right)^{1/2} \cdot \left(\frac{T^{1/2}}{p} \right) \end{aligned}$$

Using $pV = nRT$, we have

$$\begin{aligned} \tau &= \frac{1}{4d^2} \cdot \left(\frac{k_B m}{\pi} \right)^{1/2} \cdot \frac{VT^{1/2}}{nRT} \\ &= \left\{ \frac{1}{4d^2} \cdot \left(\frac{k_B m}{\pi} \right)^{1/2} \cdot \frac{1}{nR} \right\} \cdot \frac{V}{\sqrt{T}} \end{aligned}$$

$\Rightarrow \tau \propto \frac{V}{\sqrt{T}}$, as other quantities remains

constant during the process.

$$\text{So, } \frac{\tau_1}{\tau_2} = \frac{V_1}{V_2} \cdot \left(\frac{T_2}{T_1} \right)^{1/2} \quad \dots (i)$$

As the process is adiabatic,

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ \Rightarrow \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2} \right)^{\gamma-1} \end{aligned}$$

So, Eq. (i) becomes

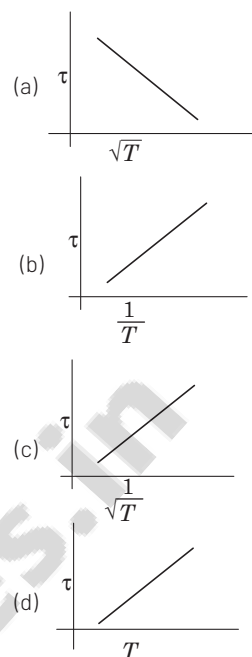
$$\begin{aligned} \frac{\tau_1}{\tau_2} &= \frac{V_1}{V_2} \cdot \left(\frac{V_1}{V_2} \right)^{\frac{\gamma-1}{2}} \\ \Rightarrow \frac{\tau_1}{\tau_2} &= \left(\frac{V_1}{V_2} \right)^{\frac{\gamma+1}{2}} \end{aligned}$$

Now, given $V_2 = 2V_1$, so

$$\frac{\tau_1}{\tau_2} = \left(\frac{1}{2} \right)^{\frac{\gamma+1}{2}}$$

- 30** The plot that depicts the behaviour of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively is (graphs are schematic and not drawn to scale)

[2020, 8 Jan Shift-I]



Ans. (c)

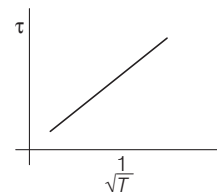
Mean free time (τ) for a gas molecule is time elapsed between two successive collisions.

$$\begin{aligned} \therefore \tau &= \frac{\text{Mean free path length } (\lambda)}{\text{Average speed } (v_{\text{av}})} \\ &= \frac{\left(\frac{1}{\sqrt{2} \cdot \pi d^2 \cdot n} \right)}{\left(\sqrt{\frac{8 R}{\pi M} \cdot T} \right)} \end{aligned}$$

So, for an ideal gas,

$$\tau \propto \frac{1}{\sqrt{T}}$$

Thus, graph of τ versus $\frac{1}{\sqrt{T}}$ is a straight line as shown in the figure.



- 31** Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_p / C_v value will be [2020, 8 Jan Shift-II]
(a) 40/27 (b) 23/15 (c) 19/13 (d) 67/45

Ans. (c)

For a mixture of two gases, ratio of specific heats is given by

$$\gamma_{\text{mixture}} = \frac{C_{p\text{mixture}}}{C_{V\text{mixture}}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}} \quad \dots(i)$$

Here, gas 1 is helium which is monoatomic, so

$$n_1 = n, C_{p1} = \frac{5}{2}R, C_{v1} = \frac{3}{2}R$$

and gas 2 is oxygen which is diatomic, so

$$n_2 = 2n, C_{p2} = \frac{7}{2}R, C_{v2} = \frac{5}{2}R$$

Hence, from Eq. (i), we get

$$\gamma_{\text{mix}} = \frac{n \times \frac{5R}{2} + 2n \times \frac{7R}{2}}{n \times \frac{3R}{2} + 2n \times \frac{5R}{2}} = \frac{19}{13}$$

- 32** Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? [Take, $R = 8.3 \text{ J/mol-K}$]

[2019, 12 April Shift-I]

- (a) 19.7 J/mol-K (b) 15.7 J/mol-K
(c) 17.4 J/mol-K (d) 21.6 J/mol-K

Ans. (c)

Let molar specific heat of the mixture is C_V .

Total number of molecules in the mixture
 $= 3 + 2 = 5$

$\therefore C_V$ can be determined using

$$nC_V dT = n_1 C_{V1} dT + n_2 C_{V2} dT$$

or $(n_1 + n_2)(C_V)_{\text{mix}} = n_1 C_{V1} + n_2 C_{V2}$
 [here, $n = n_1 + n_2$]

$$\text{Here, } C_{V1} = \frac{3R}{2} \text{ (for helium); } n_1 = 2$$

$$C_{V2} = \frac{5R}{2} \text{ (for hydrogen); } n_2 = 3$$

[For monoatomic gases, $C_V = \frac{3}{2}R$ and for diatomic gases, $C_V = \frac{5}{2}R$]

$$\therefore 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right)$$

$$\Rightarrow 5C_V = \frac{21R}{2}$$

$$\text{or } C_V = \frac{21R}{10} = \frac{21 \times 8.3}{10} = \frac{174.3}{10}$$

$$\text{or } C_V = 17.4 \text{ J/mol-K}$$

- 33** A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

[2019, 12 April Shift-II]

- (a) 25 J (b) 35 J (c) 30 J (d) 40 J

Ans. (b)

Work done by a gas is

$$W = p \Delta V = nR \Delta T = 10 \text{ J (given) } \dots(i)$$

As the gas is diatomic,

C_p = specific heat of gas at constant pressure

$$= \frac{7}{2}R$$

So, heat absorbed by gas at constant pressure is

$$\begin{aligned} \Delta Q &= nC_p \Delta T = n \cdot \frac{7}{2}R \Delta T \\ &= \frac{7}{2}(nR \Delta T) = \frac{7}{2} \times 10 \text{ [From Eq. (i)]} \\ &= 35 \text{ J} \end{aligned}$$

- 34** The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then, the total number of molecules is proportional to **[2019, 12 April Shift-II]**

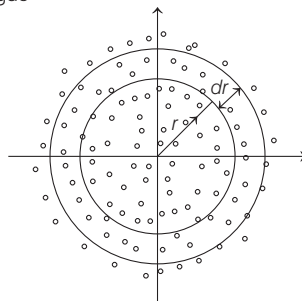
- (a) $n_0 \alpha^{-3/4}$ (b) $\sqrt{n_0} \alpha^{1/2}$
(c) $n_0 \alpha^{1/4}$ (d) $n_0 \alpha^{-3}$

Ans. (a)

Number density of gas molecules,

$$n = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

$\Rightarrow$ Number of molecules, $N = n \times \text{volume of gas}$



Now, consider a shell of radius r and thickness dr with centre at $r = 0$.

Volume of shell of differentiable thickness (dr), dV = surface area $\times$ thickness $= 4\pi r^2 dr$

Now, number of molecules in this shell is

$$dN = n(r) \cdot dV = n_0 e^{-\alpha r^4} \cdot 4\pi r^2 dr$$

So, total number of molecules present in given volume (extending from $r = 0$ to $r = \infty$) is

$$\begin{aligned} N &= \int_0^\infty n(r) \cdot dV \\ &= \int_0^\infty n_0 e^{-\alpha r^4} \cdot 4\pi r^2 dr \\ &= \int_0^\infty 4\pi n_0 e^{-\alpha r^4} \cdot r^2 dr \quad \dots(i) \end{aligned}$$

Here, we take

$$\alpha r^4 = t \Rightarrow r = t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}$$

$$\Rightarrow 4\alpha r^3 dr = dt$$

$$\Rightarrow r^2 dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\alpha^{\frac{3}{4}} t^{\frac{1}{4}}}$$

Also, when $r = 0$, $t = 0$ and when $r = \infty$, $t = \infty$ substituting in Eq. (i), we get

$$\begin{aligned} N &= \int_0^\infty 4\pi n_0 e^{-t} \cdot \frac{dt}{4\alpha^{\frac{3}{4}} t^{\frac{1}{4}}} \\ N &= \pi \alpha^{-\frac{3}{4}} \cdot n_0 \cdot \int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt \end{aligned}$$

As value of definite integral $\int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}} dt$ is a constant ($= k \text{ let}$),

we have

$$N = \pi k n_0 \alpha^{-\frac{3}{4}}$$

$$\Rightarrow N \propto n_0 \alpha^{-\frac{3}{4}}$$

- 35** A cylinder with fixed capacity of 67.2 L contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is [Take, $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$]

[2019, 10 April Shift-I]

- (a) 700 J (b) 748 J
(c) 374 J (d) 350 J

Ans. (b)

Given, capacity of cylinder is $= 67.2 \text{ L}$ and $\Delta T = 20^\circ\text{C}$

At STP volume $= 22.4 \text{ L}$

$$\therefore \text{Number of moles} = \frac{67.2}{22.4} = 3$$

Now, change in heat is given as

$$\Delta Q = nC_V \Delta T$$

Substituting the given values, we get

$$\begin{aligned} \Delta Q &= 3 \times \frac{3R}{2} \times 20 \quad (\because \text{for He gas, } C_V = \frac{3}{2}R) \\ &= 90 R \text{ J} \end{aligned}$$

Given, $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$

$$\begin{aligned} \therefore \Delta Q &= 90 \times 8.31 \\ &= 747.9 \text{ J} \\ &= 748 \text{ J} \end{aligned}$$

- 36** A $25 \times 10^{-3} \text{ m}^3$ volume cylinder is filled with 1 mole of O_2 gas at room temperature (300 K). The molecular diameter of O_2 and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?

[2019, 10 April Shift-I]

- (a) $\sim 10^{10}$ (b) $\sim 10^{12}$ (c) $\sim 10^{11}$ (d) $\sim 10^{13}$

Ans. (*)

Given,

Volume, $V = 25 \times 10^{-3} \text{ m}^3$

$N = 1 \text{ mole of } \text{O}_2 = 6.023 \times 10^{23} \text{ atoms of } \text{O}_2$,

$T = 300 \text{ K}$

Root mean square velocity of a gas molecule of

O_2 , $v_{\text{rms}} = 200 \text{ m/s}$

Radius, $r = \frac{0.3}{2} \text{ nm} = \frac{0.3}{2} \times 10^{-9} \text{ m}$

Now, average time, $\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda}$

where, $\lambda = \frac{RT}{\sqrt{2} N \pi r^2 p}$

As, $p = \frac{RT}{V}$

$\Rightarrow \lambda = \frac{V}{\sqrt{2} N \pi r^2}$

$\therefore$ Average collision per second,

$$\frac{1}{\tau} = \frac{v_{\text{av}}}{\lambda} = \frac{\sqrt{\frac{8}{3\pi}} \times v_{\text{rms}}}{\frac{V}{\sqrt{2} N \pi r^2}}$$

$$\frac{1}{\tau} = \sqrt{\frac{8}{3\pi}} \times$$

$$\frac{200 \times \sqrt{2} \times 6.023 \times 10^{23} \times \pi \times \frac{0.09}{4} \times 10^{-18}}{25 \times 10^{-3}}$$

$$\Rightarrow \frac{1}{\tau} = 4.4 \times 10^8 \text{ per second} \approx 10^8$$

$\therefore$ No option given is correct.

- 37** When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume, its temperature increases by ΔT . The heat required to produce the same change in temperature, at a constant pressure is

[2019, 10 April Shift-II]

- (a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$
(c) $\frac{3}{2}Q$ (d) $\frac{7}{5}Q$

Ans. (d)

Given, heat supplied at constant volume is Q .

$$\therefore Q = nC_V \Delta T$$

For same change in temperature, if heat supplied at constant pressure is Q' , then

$$Q' = nC_p \Delta T$$

So, we have

$$\frac{Q'}{Q} = \frac{nC_p \Delta T}{nC_V \Delta T} \Rightarrow \frac{Q'}{Q} = \frac{C_p}{C_V}$$

$$\Rightarrow Q' = Q \gamma \quad \left[\because \gamma = \frac{C_p}{C_V} \right]$$

As given gas is diatomic, so $\gamma = 7/5$

$$\therefore Q' = \frac{7}{5}Q$$

- 38** If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s , the pressure exerted by the gas molecules will be of the order of

[2019, 8 April Shift-I]

- (a) 10^4 N/m^2 (b) 10^8 N/m^2
(c) 10^3 N/m^2 (d) 10^{16} N/m^2

Ans. (*)

Momentum imparted to the surface in one collision,

$$\Delta p = (p_i - p_f) = mv - (-mv) = 2mv \quad \dots(i)$$

Force on the surface due to n collision

$$\text{per second, } F = \frac{n}{t} (\Delta p) = n \Delta p \quad (\because t = 1 \text{ s})$$

$$= 2mnv \quad [\text{from Eq. (i)}]$$

So, pressure on the surface,

$$p = \frac{F}{A} = \frac{2mnv}{A}$$

Here, $m = 10^{-26} \text{ kg}$, $n = 10^{22} \text{ s}^{-1}$,

$$v = 10^4 \text{ ms}^{-1}, A = 1 \text{ m}^2$$

$$\therefore \text{Pressure, } p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1}$$

$$= 2 \text{ N/m}^2$$

So, pressure exerted is of order of 10^0 .

- 39** The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to : [Boltzmann constant $k_B = 1.38 \times 10^{-23} \text{ J/K}$, Avogadro number $N_A = 6.02 \times 10^{26} / \text{kg}$, Radius of earth $= 6.4 \times 10^6 \text{ m}$, Gravitational acceleration on earth $= 10 \text{ ms}^{-2}$]

[2019, 8 April Shift-II]

- (a) 10^4 K (b) 650 K
(c) $3 \times 10^5 \text{ K}$ (d) 800 K

Ans. (a)

Root mean square velocity of hydrogen molecule is given as

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given, $v_{\text{rms}} = v_e$

$$\text{or } \sqrt{\frac{3k_B T}{m}} = \sqrt{2gR_e} \Rightarrow T = \frac{2gR_e m}{3 \times k_B}$$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^6 \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^4 \text{ K}$$

Alternate Solution

At rms speed, average thermal kinetic energy of a hydrogen gas molecule is

$$= \frac{3}{2} k_B T$$

and if v_e = escape velocity of the gas molecule from the earth, then its kinetic energy is $\text{KE} = \frac{1}{2} m v_e^2$, where m is the

mass of the gas molecule.

Equating the above thermal and kinetic energies, we have

$$\frac{3}{2} k_B T = \frac{1}{2} m v_e^2 \quad \text{or } T = \frac{m v_e^2}{3 k_B} \quad \dots(ii)$$

Here, $k_B = 1.38 \times 10^{-23} \text{ J/K}$,

$$v_e = 11.2 \times 10^3 \text{ ms}^{-1},$$

$$m = \frac{2}{6.02 \times 10^{26}} = 0.332 \times 10^{-26} \text{ kg}$$

Substituting these value in Eq. (ii), we get

$$T = \frac{0.332 \times 10^{-26} \times (11.2 \times 10^3)^2}{3 \times 1.38 \times 10^{-23}} \approx 10^4 \text{ K}$$

- 40** For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C . At 2 atm pressure and at 227°C , the rms speed of the molecules will be

[2019, 9 April Shift-I]

- (a) $100\sqrt{5} \text{ m/s}$ (b) 80 m/s
(c) 100 m/s (d) $80\sqrt{5} \text{ m/s}$

Ans. (a)

Key Idea For a gas molecule,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\therefore v_{\text{rms}} \propto \sqrt{T}$$

Let unknown rms speed be $v_{\text{rms},2}$
 at $T_2 = 227^\circ\text{C}$ (or 500 K)
 and at $T_1 = 127^\circ\text{C}$ (or 400 K)
 $v_{\text{rms},1} = 200 \text{ m/s}$

$\therefore$ Using the relation $v_{\text{rms}} \propto \sqrt{T}$, we can write

$$\frac{v_{\text{rms},2}}{v_{\text{rms},1}} = \sqrt{\frac{T_2}{T_1}} \quad \dots (i)$$

Substituting these given values in Eq. (i), we get

$$\begin{aligned} \therefore v_{\text{rms},2} &= \sqrt{\frac{500}{400}} \times 200 \text{ m/s} \\ &= \frac{1}{2}\sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s} \end{aligned}$$

- 41** One mole of an ideal gas passes through a process, where pressure and volume obey the relation

$$p = p_0 \left[1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right]. \text{ Here, } p_0 \text{ and } V_0 \text{ are constants. Calculate the change in the temperature of the gas, if its volume changes from } V_0 \text{ to } 2V_0. \quad \text{[2019, 10 April Shift-II]}$$

$$(a) \frac{1}{2} \frac{p_0 V_0}{R} \quad (b) \frac{1}{4} \frac{p_0 V_0}{R} \quad (c) \frac{3}{4} \frac{p_0 V_0}{R} \quad (d) \frac{5}{4} \frac{p_0 V_0}{R}$$

Ans. (d)

Given process equation for 1 mole of an ideal gas is

$$p = p_0 \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right) \quad \dots (i)$$

Also, for 1 mole of ideal gas,

$$\begin{aligned} pV &= RT \\ \therefore p &= \frac{RT}{V} \quad \dots (ii) \end{aligned}$$

So, from Eqs. (i) and (ii), we have

$$\begin{aligned} \frac{RT}{V} &= p_0 \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right) \\ \therefore T &= \frac{p_0 V}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{V} \right)^2 \right) \quad \dots (iii) \end{aligned}$$

When volume of gas is V_0 , then by substituting $V = V_0$ in Eq. (iii), we get
 Temperature of gas is

$$T_1 = \frac{p_0 V_0}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{V_0} \right)^2 \right) = \frac{p_0 V_0}{2R}$$

Similarly, at volume, $V = 2V_0$

Temperature of gas is

$$\begin{aligned} T_2 &= \frac{p_0 (2V_0)}{R} \left(1 - \frac{1}{2} \left(\frac{V_0}{2V_0} \right)^2 \right) \\ &= \frac{7}{4} \frac{p_0 V_0}{R} \end{aligned}$$

So, change in temperature as volume changes from V_0 to $2V_0$ is

$$\Delta T = T_2 - T_1 = \left(\frac{7}{4} - \frac{1}{2} \right) \frac{p_0 V_0}{R} = \frac{5}{4} \frac{p_0 V_0}{R}$$

- 42** An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is $6 \times 10^{-8} \text{ s}$. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

$$(a) 4 \times 10^{-8} \text{ s} \quad (b) 3 \times 10^{-6} \text{ s} \\ (c) 2 \times 10^{-7} \text{ s} \quad (d) 0.5 \times 10^{-8} \text{ s}$$

Ans. (a)

Mean time elapsed between two successive collisions is $t = \frac{\lambda}{v}$

where, λ = mean free path length and
 v = mean speed of gas molecule

$$\therefore t = \frac{\left(\frac{k_B T}{\sqrt{2} \pi d^2 p} \right)}{\sqrt{\frac{8}{\pi} \cdot \frac{k_B T}{M}}} \Rightarrow t = \frac{C \sqrt{T}}{p}$$

where, $C = \frac{1}{4d^2} \sqrt{\frac{k_B M}{\pi}}$ = a constant for a gas.

$$\text{So, } \frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2} \right) \quad \dots (i)$$

$$\text{Here given, } \frac{p_1}{p_2} = \frac{1}{2}, \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}} = \sqrt{\frac{5}{3}}$$

$$\text{and } t_1 = 6 \times 10^{-8} \text{ s}$$

Substituting these values in Eq. (i), we get

$$\begin{aligned} t_2 &= 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2} \\ &= 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s} \end{aligned}$$

- 43** In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = k$, where k is a constant. In this process, the temperature of the gas is increased by ΔT . The amount of heat absorbed by gas is (where, R is gas constant)

$$\begin{aligned} (a) \frac{1}{2} k R \Delta T \quad (b) \frac{2k}{3} \Delta T \\ (c) \frac{1}{2} R \Delta T \quad (d) \frac{3}{2} R \Delta T \end{aligned}$$

[2019, 11 Jan Shift-II]

Ans. (c)

Given, $VT = k$, (k is constant)

$$\text{or } T \propto \frac{1}{V} \quad \dots (i)$$

Using ideal gas equation,

$$\begin{aligned} pV &= nRT \\ pV &\propto T \\ \Rightarrow pV &\propto \frac{1}{V} \\ \text{or } pV^2 &= \text{constant} \quad \dots (ii) \end{aligned}$$

i.e. a polytropic process with $x = 2$.

(Polytropic process means, $pV^x = \text{constant}$)

We know that, work done in a polytropic process is given by

$$\Delta W = \frac{p_2 V_2 - p_1 V_1}{1-x} \quad (\text{for } x \neq 1) \quad \dots (iii)$$

$$\text{and, } \Delta W = pV \ln \left(\frac{V_2}{V_1} \right) \quad (\text{for } x = 1)$$

Here, $x = 2$,

$$\therefore \Delta W = \frac{p_2 V_2 - p_1 V_1}{1-x} = \frac{nR(T_2 - T_1)}{1-x}$$

$$\Rightarrow \Delta W = \frac{nR\Delta T}{1-2} = -nR\Delta T \quad \dots (iv)$$

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2} R \Delta T \quad \dots (v)$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2} R \Delta T - R \Delta T$$

$$\Rightarrow \Delta Q = \frac{1}{2} R \Delta T$$

- 44** Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C . Work done by gas is close to (Take, gas constant, $R = 8.31 \text{ J/mol-K}$)

[2019, 10 Jan Shift-II]

$$(a) 291 \text{ J} \quad (b) 581 \text{ J} \quad (c) 146 \text{ J} \quad (d) 73 \text{ J}$$

Ans. (a)

Work done by gas during heat process at constant pressure is given by

$$\begin{aligned} \Delta W &= p \Delta V \\ \text{Using ideal gas equation,} \\ pV &= nRT \\ \Rightarrow p \Delta V &= nR \Delta T \\ \text{So, } \Delta W &= nR \Delta T \quad \dots (i) \end{aligned}$$

Now, it is given that, $n = \frac{1}{2}$ and

$$\begin{aligned} \Delta T &= 90^\circ\text{C} - 20^\circ\text{C} \\ &= 363 \text{ K} - 293 \text{ K} = 70 \text{ K} \end{aligned}$$

and R (gas constant) = 8.31 J/mol-K

Substituting these values in Eq. (i), we get

$$\Delta W = \frac{1}{2} \times 8.31 \times 70 = 290.85 \text{ J}$$

$$\Delta W \approx 291 \text{ J}$$

- 45** 2 kg of a monoatomic gas is at a pressure of $4 \times 10^4 \text{ N/m}^2$. The density of the gas is 8 kg/m^3 . What is the order of energy of the gas due to its thermal motion?

[2019, 10 Jan Shift-II]

- (a) 10^6 J (b) 10^3 J
(c) 10^4 J (d) 10^5 J

Ans. (c)

Given, mass of gas, $m = 2 \text{ kg}$

Pressure on gas, $p = 4 \times 10^4 \text{ N/m}^2$

Density of gas, $\rho = 8 \text{ kg/m}^3$

$$\Rightarrow \text{Volume of gas} = \frac{\text{Mass}}{\text{Density}}$$

$$\Rightarrow V = \frac{2}{8} = \frac{1}{4} \text{ m}^3 \quad \dots (i)$$

Using ideal gas equation,

$$pV = nRT \Rightarrow nRT = 4 \times 10^4 \times \frac{1}{4}$$

$$nRT = 10^4 \quad \dots (ii)$$

Internal energy of n moles of a monoatomic gas is given by,

$$U = \frac{3}{2} nRT \Rightarrow U = \frac{3}{2} \times 10^4 \text{ J}$$

$$= 1.5 \times 10^4 \text{ J}$$

i.e. in order of 10^4 J .

- 46** A mixture of 2 moles of helium gas (atomic mass = $4u$) and 1 mole of argon gas (atomic mass = $40u$) is kept at 300 K in a container. The ratio of their rms speeds

$$\left[\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})} \right] \text{ is close to}$$

- (a) 0.32 (b) 2.24
(c) 3.16 (d) 0.45

Ans. (c)

Root mean square (rms) velocity of the molecules of a gas is given as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where, M is the atomic mass of the gas.

$$\Rightarrow v_{\text{rms}} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})} = \sqrt{\frac{M_{\text{argon}}}{M_{\text{helium}}}} \quad \dots (i)$$

Given, $M_{\text{argon}} = 40 u$

and $M_{\text{helium}} = 4u$

Substituting these values in Eq. (i), we get

$$\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

- 47** A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C . Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about (Take, $R = 8.3 \text{ J/K-mole}$)

[2019, 9 Jan Shift-II]

- (a) 10 kJ (b) 0.9 kJ (c) 14 kJ (d) 6 kJ

Ans. (a)

We know that from kinetic theory of gases,

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ or } v_{\text{rms}} \propto \sqrt{T}$$

where, R is gas constant, T is temperature and M is molecular mass of a gas.

Here, to double the v_{rms} , temperature must be 4 times of the initial temperature.

$\therefore$ New temperature,

$$T_2 = 4 \times T_1$$

As, $T_1 = 27^\circ\text{C} = 300 \text{ K}$

$$\Rightarrow T_2 = 4 \times 300 = 1200 \text{ K}$$

As, gas is kept inside a closed vessel.

$$\therefore Q = nC_V \cdot dT = \frac{15}{28} \times \frac{5R}{2} (1200 - 300)$$

$$\left[\text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen} \right]$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

[Given, $R = 8.3 \text{ J/K-mol}$]

$$\text{or } Q = 10004.4 \text{ J} \approx 10 \text{ kJ}$$

- 48** A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1 and that below the piston is l_2 , such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T . If the piston is stationary, its mass m , will be given by (where, R is universal gas constant and g is the acceleration due to gravity)

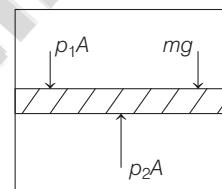
[2019, 12 Jan Shift-II]

- (a) $\frac{nRT}{g} \left[\frac{l_1 - l_2}{l_1 l_2} \right]$
(b) $\frac{nRT}{g} \left[\frac{1}{l_2} + \frac{1}{l_1} \right]$
(c) $\frac{RT}{g} \left[\frac{2l_1 + l_2}{l_1 l_2} \right]$
(d) $\frac{RT}{ng} \left[\frac{l_1 - 3l_2}{l_1 l_2} \right]$

Ans. (a)

Key Idea As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, i.e. on equilibrium as shown in the figure below,



then $p_1 A + mg = p_2 A$

$$\Rightarrow mg = p_2 A - p_1 A$$

$$\text{or } mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1} \right)$$

[$\because pV = nRT$ (ideal gas equation)]

$$= nRT \left(\frac{A}{Al_2} - \frac{A}{Al_1} \right) = nRT \left(\frac{l_1 - l_2}{l_1 l_2} \right)$$

$$\text{or } m = \frac{nRT}{g} \left(\frac{l_1 - l_2}{l_1 l_2} \right)$$

- 49** C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is

[JEE Main 2017]

- (a) $a = b$ (b) $a = 14b$
(c) $a = 28b$ (d) $a = \frac{1}{14}b$

Ans. (b)

By Mayor's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when n gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{ for } H_2 \quad \dots (i)$$

$$b = C_p - C_V = \frac{R}{28}; \text{ for } N_2 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$a = 14b$$

- 50** The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5 \text{ Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be [JEE Main 2017]

- (a) 1.38×10^{23} (b) 2.5×10^{25}
(c) -2.5×10^{25} (d) -1.61×10^{23}

Ans. (c)

$$\text{From } pV = nRT = \frac{N}{N_A} RT$$

$$\text{We have, } n_f - n_i = \frac{pV N_A}{RT_f} - \frac{pV N_A}{RT_i}$$

$$\Rightarrow n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \left(\frac{1}{300} - \frac{1}{290} \right)$$

$$= -2.5 \times 10^{25}$$

$$\therefore \Delta n = -2.5 \times 10^{25}$$

- 51** Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q , where V is the volume of the gas. The

value of q is $\left(\gamma = \frac{C_p}{C_v} \right)$ [JEE Main 2015]

- (a) $\frac{3\gamma+5}{6}$ (b) $\frac{3\gamma-5}{6}$
(c) $\frac{\gamma+1}{2}$ (d) $\frac{\gamma-1}{2}$

Ans. (c)

For an adiabatic process

$$TV^{\gamma-1} = \text{constant.}$$

We know that average time of collision between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}v_{\text{rms}}d^2}$$

where, n = number of molecules per unit volume

v_{rms} = rms velocity of molecules

$$\text{As } n \propto \frac{1}{V} \text{ and } v_{\text{rms}} \propto \sqrt{T}$$

$$\tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write

$$n = K_1 V^{-1} \text{ and } v_{\text{rms}} = K_2 T^{1/2}$$

where, K_1 and K_2 are constants.

For adiabatic process, $TV^{\gamma-1} = \text{constant}$. Thus, we can write

$$\tau \propto VT^{-1/2} \propto V(V^{1-\gamma})^{-1/2}$$

$$\text{or } \tau \propto V^{\frac{\gamma+1}{2}}$$

- 52** An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) [JEE Main 2014]

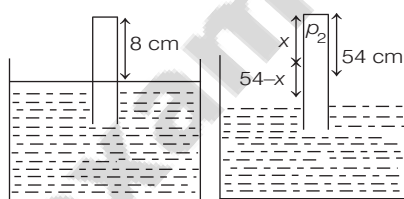
- (a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

Key Idea In this question, the system is accelerating horizontally, hence the pressure in the vertical direction will remain unaffected.

$$\text{i.e., } p_1 = p_0 + \rho gh$$

Again, we have to use the concept that the pressure in the same level will be same.

Ans. (a)



For air trapped in tube, $p_1 V_1 = p_2 V_2$

$$p_1 = p_{\text{atm}} = \rho g 76$$

$$V_1 = A \times 8$$

[$\because A$ = area of cross-section]

$$p_2 = p_{\text{atm}} - \rho g (54 - x) = \rho g (22 + x)$$

$$V_2 = A \cdot x$$

At constant temperature,

$$p_1 V_1 = p_2 V_2$$

$$\rho g 76 \cdot A \cdot 8 = \rho g (22 + x) A x$$

$$x^2 + 22x - 78 \times 8 = 0 \Rightarrow x = 16 \text{ cm}$$

- 53** A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure p and temperature T , whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be [AIEEE 2011]

- (a) $\frac{p}{2}, T$ (b) $\frac{p}{2}, \frac{T}{2}$ (c) p, T (d) $p, \frac{T}{2}$

Ans. (a)

Internal energy of the gas remains constant, hence

$$T_2 = T$$

Using at constant temperature,

$$p_1 V_1 = p_2 V_2 \Rightarrow p_2 = p/2$$

- 54** The potential energy function for the force between two atoms in a diatomic molecule is approximately

$$\text{given by } U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}, \text{ where } a$$

and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is

$$D = [U(x = \infty) - U_{\text{at equilibrium}}], D \text{ is [AIEEE 2010]}$$

- (a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

Ans. (c)

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$U(x = \infty) = 0$$

$$\text{As, } F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7} \right]$$

At equilibrium, $F = 0$

$$\therefore x^6 = 2a/b$$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

- 55** The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1} . The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)

- (a) 460 ms^{-1} (b) 500 ms^{-1}
(c) 650 ms^{-1} (d) 1420 ms^{-1}

[AIEEE 2008]

Ans. (d)

Speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v_{\text{O}_2} = \sqrt{\frac{7}{5} \frac{RT}{32}} \text{ and } v_{\text{He}} = \sqrt{\frac{5}{3} \frac{RT}{4}}$$

$$\therefore \frac{v_{\text{O}_2}}{v_{\text{He}}} = \sqrt{\frac{7 \times 3 \times 4}{5 \times 32 \times 5}}$$

$$\text{or } v_{\text{He}} = 460 \times \sqrt{\frac{5 \times 32 \times 5}{7 \times 3 \times 4}} \approx 1420 \text{ ms}^{-1}$$

56 Two rigid boxes $\frac{1}{2}$ containing

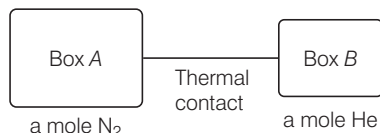
different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0 , while box B contains one mole of helium at temperature $(7/3)T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f , in terms of T_0 is

[AIEEE 2006]

- (a) $T_f = \frac{3}{7}T_0$ (b) $T_f = \frac{7}{3}T_0$
 (c) $T_f = \frac{3}{2}T_0$ (d) $T_f = \frac{5}{2}T_0$

Ans. (c)

Here, change in internal energy of the system is zero, i.e., increase in internal energy of one is equal to decrease in internal energy of other.



Change in internal energy in box A,

$$\Delta U_A = 1 \times \frac{5}{2} R (T_f - T_0)$$

Change in internal energy in box B

$$\Delta U_B = 1 \times \frac{3}{2} R \left(T_f - \frac{7}{3} T_0 \right)$$

Now, $\Delta U_A + \Delta U_B = 0$

$$\Rightarrow \frac{5R}{2} (T_f - T_0) + \frac{3R}{2} \left(T_f - \frac{7T_0}{3} \right) = 0$$

$$\text{or } 5T_f - 5T_0 + 3T_f - 7T_0 = 0$$

$$\text{or } 8T_f = 12T_0$$

$$\text{or } T_f = \frac{12}{8} T_0 = \frac{3}{2} T_0$$

57 A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_p / C_v of the mixture is

[AIEEE 2005]

- (a) 1.59 (b) 1.62
 (c) 1.4 (d) 1.54

Ans. (b)

$$\text{For mixture, } C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

For helium,

$$\text{Number of moles, } n_1 = \frac{16}{4} = 4 \text{ and } \gamma_1 = \frac{5}{3}$$

For oxygen,

$$\text{Number of moles, } n_2 = \frac{16}{32} = \frac{1}{2} \text{ and } \gamma_2 = \frac{7}{5}$$

$$\text{Then, } C_{v1} = \frac{R}{\gamma_1 - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2} R$$

$$\text{and } C_{v2} = \frac{R}{\gamma_2 - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2} R$$

$\therefore$ For mixture,

$$C_v = \frac{4 \times \frac{3}{2} R + \frac{1}{2} \times \frac{5}{2} R}{4 + \frac{1}{2}} = \frac{6R + \frac{5}{4} R}{\frac{9}{2}} = \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$$

$$\text{Now, } C_v = \frac{R}{\gamma - 1}$$

$$\Rightarrow \gamma - 1 = \frac{R}{C_v}$$

$$\text{or } \gamma = \frac{R}{C_v} + 1 = \frac{R}{\frac{29R}{18}} + 1$$

$$\Rightarrow \frac{C_p}{C_v} = \frac{18}{29} + 1 = \frac{18 + 29}{29} = 1.62$$

58 One mole of ideal monoatomic gas ($\gamma = \frac{5}{3}$) is mixed with one mole of

diatomic gas ($\gamma = \frac{7}{5}$). What is γ for

the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.

[AIEEE 2004]

- (a) $\frac{3}{2}$ (b) $\frac{23}{15}$
 (c) $\frac{35}{23}$ (d) $\frac{4}{3}$

Ans. (a)

Mayer's formula is

$$C_p - C_v = R \text{ and } \gamma = \frac{C_p}{C_v}$$

Therefore, using above two relations, we find

$$C_v = \frac{R}{\gamma - 1}$$

For a mole of monoatomic gas,

$$\gamma = \frac{5}{3}$$

$$\therefore C_v = \frac{R}{\left(\frac{5}{3}\right) - 1} = \frac{3}{2} R$$

For a mole of diatomic gas, $\gamma = \frac{7}{5}$

$$\therefore C_v = \frac{R}{\left(\frac{7}{5}\right) - 1} = \frac{5}{2} R$$

When these two moles are mixed, then heat required to raise the temperature to 1°C is

$$C_v = \frac{3}{2} R + \frac{5}{2} R = 4R$$

Hence, for one mole, heat required

$$= \frac{4R}{2} = 2R$$

$$\therefore C_v = 2R \Rightarrow \frac{R}{\gamma - 1} = 2R \text{ or } \gamma = \frac{3}{2}$$

Alternate Solution

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\text{Here, } n_1 = 1, n_2 = 1, \gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$$

$$\therefore \frac{1+1}{\gamma-1} = \frac{1}{\left(\frac{5}{3}\right)-1} + \frac{1}{\left(\frac{7}{5}\right)-1}$$

$$\Rightarrow \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2}$$

$$\text{or } \frac{2}{\gamma-1} = \frac{8}{2}$$

$$\text{or } \frac{2}{\gamma-1} = 4 \Rightarrow \gamma = \frac{2}{4} + 1$$

$$\text{Hence, } \gamma = \frac{3}{2}$$

59 1 mol of a gas with $\gamma = \frac{7}{5}$ is mixed

with 1 mol of a gas with $\gamma = \frac{5}{3}$, then

the value of γ for the resulting mixture is

[AIEEE 2002]

- (a) $\frac{7}{5}$ (b) $\frac{2}{5}$ (c) $\frac{24}{16}$ (d) $\frac{12}{7}$

Ans. (c)

Using the relation,

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \frac{1+1}{\gamma-1} = \frac{1}{\left(\frac{5}{3}\right)-1} + \frac{1}{\left(\frac{7}{5}\right)-1}$$

$$\text{or } \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma = \frac{3}{2} + 1 = \frac{24}{16}$$

60 At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ?

[AIEEE 2002]

- (a) 80 K (b) -73°C
 (c) 3 K (d) 20 K

Ans. (d)

The rms velocity of the molecule of a gas of molecular weight M at temperature T is given by

$$c_{rms} = \sqrt{(3RT/M)}$$

Let M_O and M_H be molecular weights of oxygen and hydrogen and T_O and T_H be the corresponding kelvin temperatures at which c_{rms} is same for both gases.

$$\text{i.e., } c_{rms}(O) = c_{rms}(H)$$

$$\sqrt{\left(\frac{3RT_O}{M_O}\right)} = \sqrt{\left(\frac{3RT_H}{M_H}\right)}$$

$$\text{Hence, } \frac{T_O}{M_O} = \frac{T_H}{M_H}$$

$$\text{Given, } T_O = 273 + 47 = 320 \text{ K}$$

$$\text{and } M_O = 32, M_H = 2$$

$$\therefore T_H = \frac{2}{32} \times 320 = 20 \text{ K}$$

- 61** Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will [AIEEE 2002]

- (a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others

Ans. (c)

Temperature of a gas is determined by the total translational kinetic energy measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.

TOPIC 2**Degree of Freedom and Law of Equipartition of Energy**

- 62** The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0°C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $x/10$, then the value of x (round off to the nearest integer) is
(Given, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$)

[2021, 1 Sep Shift-II]

Ans. (25)

Given, the number of diatomic moles, $n = 3 \text{ mol}$

The increase in temperature of the diatomic mole, $\Delta T = 40^\circ\text{C}$

Now, the degree of freedom,

$f = \text{linear} + \text{rotational} + \text{no oscillation}$

$$f = 3 + 2 + 0 \Rightarrow f = 5$$

Change in internal energy,

$$\Delta U = nC_V\Delta T \quad \dots(i)$$

$$\text{where, } C_V = \frac{f}{2}R = \frac{5}{2}R$$

Substituting the value in Eq. (i), we get

$$\Delta U = \frac{5R}{2} n\Delta T$$

Now, work done by the gas for isobaric process,

$$W = p\Delta V = nR\Delta T$$

The ratio of the change in internal energy to the work done by the gas,

$$\frac{\Delta U}{W} = \frac{\frac{5}{2}nR\Delta T}{nR\Delta T} \Rightarrow \frac{\Delta U}{W} = \frac{5}{2}$$

Multiply and divide the above equation with 5, we get

$$\frac{\Delta U}{W} = \frac{5 \times 5}{2 \times 5} = \frac{25}{10}$$

$$\text{Comparing with given equation, } \frac{\Delta U}{W} = \frac{x}{10}$$

The value of the $x = 25$.

- 63** What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T ? [2021, 22 July Shift-II]

- (a) $\frac{2}{3}K_B T$ (b) $K_B T$
(c) $\frac{3}{2}K_B T$ (d) $\frac{1}{2}K_B T$

Ans. (c)

Given, nature of gas is monoatomic.

Let degree of freedom of monoatomic gas be f i.e. $f = 3$.

As we know that,

Average energy,

$$E_{av} = \frac{1}{2}fK_B T$$

$$\text{For monoatomic gas, } E_{av} = \frac{3}{2}K_B T$$

- 64** The correct relation between the degrees of freedom f and the ratio of specific heat γ is

[2021, 20 July Shift-II]

$$\begin{aligned} \text{(a)} f &= \frac{2}{\gamma - 1} & \text{(b)} f &= \frac{2}{\gamma + 1} \\ \text{(c)} f &= \frac{\gamma + 1}{2} & \text{(d)} f &= \frac{1}{\gamma + 1} \end{aligned}$$

Ans. (a)

Since, we know that,

$$\begin{aligned} \gamma &= 1 + \frac{2}{f} \\ \frac{2}{f} &= \gamma - 1 \Rightarrow f = \frac{2}{\gamma - 1} \end{aligned}$$

- 65** What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T ? (k_B is Boltzmann constant)

[2021, 18 March Shift-I]

$$\text{(a)} \frac{1}{2}k_B T \quad \text{(b)} \frac{2}{3}k_B T \quad \text{(c)} \frac{3}{2}k_B T \quad \text{(d)} k_B T$$

Ans. (a)

We know that,

The average value of energy of an ideal gas in thermal equilibrium temperature T ,

$$E = \frac{f}{2}k_B T$$

Given, degree of freedom, $f = 1$

$$\text{Energy per degree of freedom, } E = \frac{1}{2}k_B T$$

- 66** If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_p}{C_v}\right)$, then

the value of β is

[2021, 17 March Shift-II]

- (a) 1.02 (b) 1.2
(c) 1.25 (d) 1.35

Ans. (b)

For polyatomic gas molecule has 3 rotational degrees of freedom, 3 translational degrees of freedom, and 2 vibrational modes.

So, number of vibrational degrees of freedom $= 2(2) = 4$

$\therefore$ Total number of degrees of freedom,

$$f = 3 + 3 + 4 = 10$$

Here, ratio of molar specific heats is given as

$$\beta = \frac{C_p}{C_v}$$

$$\text{In terms of degrees of freedom, } \beta = 1 + \frac{2}{f}$$

Substituting the value of f in above equation, we get

$$\beta = 1 + \frac{2}{10}$$

$$\Rightarrow \beta = 1.2$$

- 67** Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If f_1 and f_2 , m_1 and m_2 , n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is [2021, 17 March Shift-I]

(a) $\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$ (b) $\frac{n_1 f_1 T_1 + n_2 f_2 T_2}{n_1 f_1 + n_2 f_2}$

(c) $\frac{n_1 f_1 T_1 + n_2 f_2 T_2}{f_1 + f_2}$ (d) $\frac{n_1 f_1 T_1 + n_2 f_2 T_2}{n_1 + n_2}$

Ans. (b)

According to question, two ideal atomic gases at temperatures T_1 and T_2 are mixed.

Let the final temperature of this mixture be T .

As per question there is no loss of energy, it means

$$\Delta U = 0 \quad \dots(i)$$

As, we know,

$$\Delta U = \frac{f_1 n_1 R \Delta T}{2} + \frac{f_2 n_2 R \Delta T}{2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{f_1 n_1 R \Delta T}{2} + \frac{f_2 n_2 R \Delta T}{2} = 0$$

$$\Rightarrow f_1 n_1 R \Delta T + f_2 n_2 R \Delta T = 0$$

$$\Rightarrow f_1 n_1 R(T_1 - T) + f_2 n_2 R(T_2 - T) = 0$$

$$\Rightarrow f_1 n_1 (T_1 - T) + f_2 n_2 (T_2 - T) = 0$$

$$\Rightarrow f_1 n_1 T_1 - f_1 n_1 T + f_2 n_2 T_2 - f_2 n_2 T = 0$$

$$\Rightarrow f_1 n_1 T + f_2 n_2 T = f_1 n_1 T_1 + f_2 n_2 T_2$$

$$\Rightarrow T(f_1 n_1 + f_2 n_2) = f_1 n_1 T_1 + f_2 n_2 T_2$$

$$T = \frac{f_1 n_1 T_1 + f_2 n_2 T_2}{f_1 n_1 + f_2 n_2}$$

- 68** A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ? [2021, 17 March Shift-I]

- (a) 1.03 (b) 1.30
(c) 1.37 (d) 10.3

Ans. (a)

Since we know that, the heat capacity ratio γ for an ideal gas can be related to the degrees of freedom of gas molecules (f) by formula

$$\gamma = 1 + \frac{2}{f} \quad \dots(i)$$

As each vibrational mode has 2 degrees of freedom, hence total vibrational degrees of freedom = $2 \times 24 = 48$

$$\Rightarrow f = 3(\text{rotational}) + 3(\text{translational}) + 48(\text{vibrational})$$

$$\Rightarrow f = 3 + 3 + 48$$

$$\Rightarrow f = 54$$

Now, put the value of f in Eq. (i), we get

$$\gamma = 1 + \frac{2}{54} \Rightarrow \gamma = 1 + \frac{1}{27}$$

$$\Rightarrow \gamma = \frac{28}{27} \text{ or } \gamma = 1.03$$

- 69** The internal energy (U), pressure (p) and volume (V) of an ideal gas are related as $U = 3pV + 4$. The gas is [2021, 26 Feb Shift II]

- (a) diatomic only
(b) polyatomic only
(c) Either monoatomic or diatomic
(d) monoatomic only

Ans. (b)

Given, ideal gas equation, $U = 3pV + 4$

Here, U = internal energy,

p = pressure

and V = volume.

By ideal gas equation, $pV = nRT$

$$= nRT \quad (\because n = 1) \dots(i)$$

$$\Delta U = nC_V \Delta T = 3pV + 4$$

$$= n \frac{f}{2} R \Delta T = 3pV + 4$$

$$\Rightarrow fR \Delta T = 6pV + 8$$

$$\Rightarrow f \cdot pV = 6pV + 8 \quad [\text{From Eq. (i)}]$$

$$\Rightarrow f = 6 + \frac{8}{pV}$$

As, degree of freedom is more than 6.

$\therefore$ Gas is polyatomic.

- 70** Given below are two statements:

Statement I In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below. [2021, 25 Feb Shift-II]

- (a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

Ans. (c)

According to Statement 1 : In diatomic molecule the rotational energy at a given temperature obeys Maxwell's distribution is correct.

But, according to Statement 2 : In diatomic molecule, the rotational energy at a given temperature equals translational energy for each molecule is false.

Because kinetic energy (KE) of gaseous molecule is $\frac{f}{2} K_B T$.

where, f is the degree of freedom, and $f(\text{diatomic atom}) = 5 = [3(\text{translational}) + 2(\text{rotational})]$

Therefore, translational KE of gas

$$= \frac{3}{2} K_B T \quad \dots(i)$$

$$\text{and rotational KE of gas} = \frac{2}{2} K_B T = K_B T \quad \dots(ii)$$

$\therefore$ Eq. (i) is not equal to Eq. (ii)

Hence, option (c) is the correct.

- 71** A diatomic gas having $C_p = \frac{7}{2} R$ and

$C_V = \frac{5}{2} R$, is heated at constant

pressure. The ratio $dU : dQ : dW$ is [2021, 25 Feb Shift I]

- (a) 5 : 7 : 3 (b) 5 : 7 : 2
(c) 3 : 7 : 2 (d) 3 : 5 : 2

Ans. (b)

Given, $C_p = \frac{7}{2} R$, $C_V = \frac{5}{2} R$

Since, change in internal energy

$$(dU) = nC_V dT$$

$$\text{Heat change } (dQ) = nC_p dT$$

$$\text{Work done } (dW) = nRdT$$

$$\therefore dU : dQ : dW = nC_V dT : nC_p dT : nRdT$$

$$= C_V : C_p : R = \frac{5}{2} R : \frac{7}{2} R : R$$

$$= 5R : 7R : 2R = 5 : 7 : 2$$

- 72** An engine takes in 5 moles of air at 20°C and 1 atm and compresses it adiabatically to $1/10$ th of the original volume. Assuming air to be a diatomic ideal gas made up of

rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is

[2020, 2 Sep Shift-I]

Ans. (4)

Given, air is diatomic gas

$\therefore$ Degrees of freedom, $f = 5$

So, ratio of specific heats of gas,

$$\gamma = 1 + \frac{2}{f} = \frac{7}{5}$$

Also, given that $V_i = V$ and $V_f = \frac{V}{10}$

$$T_i = 20^\circ\text{C} = 273 + 20 = 293\text{ K}$$

As for adiabatic change,

$$TV^{\gamma-1} = \text{constant}$$

$$\Rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$\Rightarrow 293 \cdot V^{\frac{7}{5}-1} = T_f \cdot \left(\frac{V}{10}\right)^{\frac{7}{5}-1}$$

$$\Rightarrow T_f = 293 \times 10^{2/5} \text{ K}$$

Change in internal energy is

$$\begin{aligned} \Delta U &= \frac{n \cdot f \cdot R (T_f - T_i)}{2} \\ &= \frac{5 \times 5 \times \frac{25}{3} (293 \times 10^{2/5} - 293)}{2} \\ &= 4.195 \times 10^3 \text{ J} \approx 4 \text{ kJ} \end{aligned}$$

73 A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T . Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is

[2020, 2 Sep Shift-I]

- (a) 15 (b) 13
(c) 11 (d) 20

Ans. (a)

Internal energy of a gas mixture = Sum of internal energies of its components.

Also, internal energy of a gas is given by

$$U = \frac{f}{2} (nRT)$$

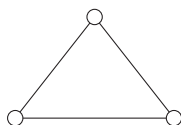
Here, gases are oxygen (diatomic, $f = 5$) and argon (monoatomic, $f = 3$).

So, total internal energy of the mixture is

$$\begin{aligned} U &= \left(\frac{f}{2} nRT\right)_{\text{O}_2} + \left(\frac{f}{2} nRT\right)_{\text{Ar}} \\ &= \frac{5}{2} \times 3 \times RT + \frac{3}{2} \times 5 \times RT \\ &= 15RT \end{aligned}$$

Hence, correct option is (a).

74



Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is

[2020, 3 Sep Shift-I]

- (a) $\frac{5}{2} RT$ (b) $\frac{3}{2} RT$ (c) $\frac{9}{2} RT$ (d) $3 RT$

Ans. (d)

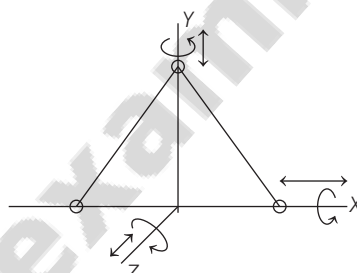
Internal energy U of a mole of the gas at temperature T is given by,

$$U = \frac{f}{2} RT$$

where, f = degree of freedom of gas molecule

and R = gas constant.

A triatomic triangular gas molecules can have



three-translational motions (translation along X , Y or Z -axis) and three-rotational motions (rotation along X , Y or Z -axis). So, it has total 6 degrees of freedom.

So, internal energy,

$$U = \frac{f}{2} RT = \frac{6}{2} RT = 3RT$$

Hence, option (d) is correct.

75

To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 cal of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 cal of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

[2020, 3 Sep Shift-II]

- (a) 5 (b) 7
(c) 6 (d) 3

Ans. (c)

Given, in constant pressure process,

$$\Delta Q = 160 \text{ cal}$$

$$\Delta T = 50^\circ\text{C}$$

$$\text{So, } \Delta Q = nC_p \Delta T$$

$$\Rightarrow 160 = nC_p (50) \quad \dots(i)$$

Also, in constant volume process,

$$\Delta Q = 240 \text{ cal}$$

$$\Delta T = 100^\circ\text{C} \Rightarrow \Delta Q = nC_v \Delta T$$

$$\Rightarrow 240 = nC_v (100) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{160}{240} = \frac{nC_p(50)}{nC_v(100)} \Rightarrow \frac{C_p}{C_v} = \frac{2 \times 160}{240} = \frac{4}{3}$$

Now, $\frac{C_p}{C_v} = \gamma$ and degrees of freedom

of a gas,

$$f = \frac{2}{\gamma - 1} \Rightarrow f = \frac{2}{\frac{4}{3} - 1} = \frac{2}{\frac{1}{3}} \Rightarrow f = 6$$

Hence, correct option is (c).

76 Match the C_p/C_v ratio for ideal

gases with different type of molecules:

[2020, 4 Sep Shift-I]

Molecule type	C_p/C_v
(A) Monatomic molecules	I. 7/5
(B) Diatomic rigid molecules	II. 9/7
(C) Diatomic non-rigid molecules	III. 4/3
(D) Triatomic rigid molecules	IV. 5/3

- A B C D
(a) IV I II III
(b) III IV II I
(c) II III I IV
(d) IV II I III

Ans. (a)

$$\therefore \gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

where, f = degree of freedom.

Now, (A) for monatomic gas, $f = 3$

$$\text{so, } \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) for diatomic rigid gas, $f = 5$

$$\text{so, } \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) for diatomic non-rigid gas, $f = 7$

$$\text{so, } \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) for triatomic rigid gas, $f = 6$

$$\text{so, } \gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

Therefore, A-IV, B-I, C-II, D-III.

Hence, correct option is (a).

77 Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T . The total internal energy U of a mole of this gas,

and the value of $\gamma \left(= \frac{C_p}{C_v} \right)$ are given

respectively, by [2020, 6 Sep Shift-I]

(a) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$

(b) $U = 5RT$ and $\gamma = \frac{7}{5}$

(c) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$

(d) $U = 5RT$ and $\gamma = \frac{6}{5}$

Ans. (c)

Given that, translational degree of freedom, $f_{\text{trans}} = 3$

Rotational degree of freedom, $f_{\text{rot}} = 2$

Total degree of freedom, $f = f_{\text{trans}} + f_{\text{rot}}$
 $= 3 + 2 = 5$

At T temperature, total internal energy of an ideal gas is given by $U = \frac{f}{2}\mu RT$

$$U = 5/2RT \quad (\because \mu = 1)$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

78 Consider two ideal diatomic gases A and B at some temperature T . Molecules of the gas A are rigid and have a mass m . Molecules of the gas B have an additional vibrational mode and have a mass $m/4$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B respectively is [2020, 9 Jan Shift-I]

(a) 5 : 9 (b) 7 : 9 (c) 3 : 5 (d) 5 : 7

Ans. (d)

For a gas value of specific heat at constant volume, $C_V = 1/2 fR$ where, f = degrees of freedom and R = gas constant.

For a diatomic gas molecule, degrees of freedom is in general 5 but when vibration occurs, then degrees of freedom increases to 7.

So, for gas A, specific heat is $C_V^A = \frac{1}{2} 5R$

and for gas B, specific heat is $C_V^B = \frac{1}{2} 7R$

$\therefore$ Ratio of specific heats of gas A and B is,
 $C_V^A / C_V^B = 5/7$

79 The specific heats, C_p and C_v of a gas of diatomic molecules, A are given (in units of $\text{J mol}^{-1} \text{K}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal gases, then [2019, 9 April Shift-II]

(a) A has a vibrational mode but B has none
 (b) Both A and B have a vibrational mode each

(c) A has one vibrational mode and B has two

(d) A is rigid but B has a vibrational mode

Ans. (a)

Key Idea A diatomic gas molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational, at low temperature ranges ($\sim 250 \text{ K}$ to 750 K). At temperatures above 750 K , molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case,

For gas A, $C_p = 29$, $C_v = 22$

For gas B, $C_p = 30$, $C_v = 21$

By using $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

We have,

$$\text{For gas A, } 1 + \frac{2}{f} = \frac{29}{22} \Rightarrow f = 6.67 \approx 7$$

So, gas A has vibrational mode of degree of freedom.

$$\text{For gas B, } 1 + \frac{2}{f} = \frac{30}{21} \Rightarrow f = 5$$

Hence, gas B does not have any vibrational mode of degree of freedom.

80 An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be [2019, 9 April Shift-I]

(a) $\frac{m\bar{v}^2}{3k_B}$ (b) $\frac{m\bar{v}^2}{7k_B}$

(c) $\frac{m\bar{v}^2}{5k_B}$ (d) $\frac{m\bar{v}^2}{6k_B}$

Ans. (b)

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

$$\text{So, } n = 7 \Rightarrow \frac{1}{2} m v_{\text{rms}}^2 = \frac{7}{2} k_B T$$

$$\text{Here, } v_{\text{rms}}^2 = \bar{v}^2 \Rightarrow T = \frac{m\bar{v}^2}{7k_B}$$

81 A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is $TV^x = \text{constant}$, then x is [2019, 11 Jan Shift-I]

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

Ans. (a)

For a diatomic gas, degree of freedom, $f = 5$

$$\therefore \gamma = 1 + 2/f = 1 + \frac{2}{5} = \frac{7}{5}$$

As for adiabatic process,

$$TV^{\gamma-1} = \text{constant} \quad \dots(i)$$

and it is given that, here

$$TV^x = \text{constant} \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\gamma - 1 = x$$

$$\Rightarrow \frac{7}{5} - 1 = x$$

$$\text{or } x = 2/5$$

82 A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T . Considering only translational and rotational modes, the total internal energy of the system is [2019, 11 Jan Shift-I]

(a) $12 RT$ (b) $15 RT$ (c) $20 RT$ (d) $4 RT$

Ans. (b)

Internal energy of a gas with f degree of freedom is $U = \frac{n f R T}{2}$, where n is the number of moles.

Internal energy due to O_2 gas which is a

$$\text{diatomic gas is } U_1 = \frac{n_1 f_1 R T}{2} = 3 \times \frac{5}{2} RT$$

($\because n_1 = 3$ moles, degree of

freedom for a diatomic gas $f_1 = 5$)

Internal energy due to Ar gas which is a monoatomic gas is

$$U_2 = \frac{n_2 f_2 R T}{2} = 5 \times \frac{3}{2} RT$$

($\because n_2 = 5$ moles, degree of freedom for a monoatomic gas $f_2 = 3$)

$\therefore$ Total internal energy = $U_1 + U_2$

$$\Rightarrow U = 15 RT$$

83 An ideal gas occupies a volume of 2 m^3 at a pressure of $3 \times 10^6 \text{ Pa}$. The energy of the gas is [2019, 12 Jan Shift-I]

(a) $6 \times 10^4 \text{ J}$

(b) 10^8 J

(c) $9 \times 10^6 \text{ J}$

(d) $3 \times 10^2 \text{ J}$

Ans. (c)

Key Idea Internal energy of 'n' moles of a gas with degree of freedom f ($= 3$ for an ideal gas), at temperature T is

$$E = \frac{f}{2} \cdot nRT = \frac{3}{2} nRT$$

For an ideal gas, internal energy, $E = \frac{3}{2} nRT$

$$= \frac{3}{2} p \cdot V \quad (\because \text{from } pV = nRT)$$

Substituting the given values, we get

$$\begin{aligned} &= \frac{3}{2} \times 3 \times 10^6 \times 2 \\ &= 9 \times 10^6 \text{ J} \end{aligned}$$

- 84** Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is **[AIEEE 2011]**

- (a) $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$
 (b) $\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$
 (c) $\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$
 (d) $\frac{(T_1 + T_2 + T_3)}{3}$

Ans. (a)

$$\begin{aligned} \frac{F}{2} n_1 k T_1 + \frac{F}{2} n_2 k T_2 + \frac{F}{2} n_3 k T_3 \\ = \frac{F}{2} (n_1 + n_2 + n_3) k T \end{aligned}$$

[$\because F$ is degree of freedom]

Temperature of mixture,

$$T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

- 85** One kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ Nm}^{-2}$. The density of the gas is 4 kgm^{-3} . What is the energy of the gas due to its thermal motion? **[AIEEE 2009]**

- (a) $3 \times 10^4 \text{ J}$ (b) $5 \times 10^4 \text{ J}$
 (c) $6 \times 10^4 \text{ J}$ (d) $7 \times 10^4 \text{ J}$

Ans. (b)

Thermal energy corresponds to internal energy

$$\text{Mass} = 1 \text{ kg}$$

$$\text{Density} = 4 \text{ kg m}^{-3}$$

$$\Rightarrow \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4} \text{ m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{ Nm}^{-2}$$

$$\text{As internal energy} = \frac{f}{2} nRT = \frac{f}{2} pV$$

$$\text{Degree of freedom for diatomic} = 5$$

$$\therefore \text{Internal energy} = \frac{5}{2} p \times V$$

$$\text{Putting the values of } p = 8 \times 10^4 \text{ Nm}^{-2}$$

$$\text{and } V = \frac{1}{4} \text{ m}^3, \text{ we get}$$

$$\begin{aligned} \text{Internal energy} &= \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} \\ &= 5 \times 10^4 \text{ J} \end{aligned}$$

- 86** An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1 . The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2 . If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **[AIEEE 2008]**

- (a) $\frac{T_1 T_2 (p_1 V_1 + p_2 V_2)}{p_1 V_1 T_2 + p_2 V_2 T_1}$ (b) $\frac{p_1 V_1 T_1 + p_2 V_2 T_2}{p_1 V_1 + p_2 V_2}$

$$(c) \frac{p_1 V_1 T_2 + p_2 V_2 T_1}{p_1 V_1 + p_2 V_2} \quad (d) \frac{T_1 T_2 (p_1 V_1 + p_2 V_2)}{p_1 V_1 T_1 + p_2 V_2 T_2}$$

Ans. (a)

As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant i.e., $U = U_1 + U_2$. Assuming both gases have same degree of freedom, then

$$\text{Internal energy, } U = \frac{f(n_1 + n_2)RT}{2}$$

$$\text{and } U_1 = \frac{f n_1 R T_1}{2}, U_2 = \frac{f n_2 R T_2}{2}$$

Solving, we get

Common temperature,

$$T = \frac{(p_1 V_1 + p_2 V_2) T_1 T_2}{p_1 V_1 T_2 + p_2 V_2 T_1}$$

- 87** The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process, the temperature of the gas increases by 7°C . The gas is ($R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$) **[AIEEE 2006]**

- (a) diatomic
 (b) triatomic
 (c) a mixture of monatomic and diatomic
 (d) monatomic

Ans. (a)

For adiabatic process, no exchange of heat.

$$\therefore dQ = 0$$

$$\text{So, } dU = -\Delta W$$

$$\Rightarrow n C_V dT = +146 \times 10^3 \text{ J}$$

$$\Rightarrow \frac{n f R}{2} \times 7 = 146 \times 10^3$$

[where, f is degree of freedom]

$$\Rightarrow \frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$$

$$\therefore f = 5.02 \approx 5$$

So, it is a diatomic gas.

Oscillations

TOPIC 1

Simple Harmonic Motion

01 For a body executing SHM

- potential energy is always equal to its kinetic energy.
- average potential and kinetic energy over any given time interval are always equal.
- sum of the kinetic and potential energy at any point of time is constant.
- average kinetic energy in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below.

[2021, 31 Aug Shift-II]

- (C) and (D)
- Only (C)
- (B) and (C)
- Only (B)

Ans. (a)

As we know that, for a body executing SHM the average kinetic energy is equal to average potential energy over a complete time period

$$\text{i.e. } KE_{av} = PE_{av} = \frac{1}{4} m \omega^2 a^2$$

$\therefore$ Total mechanical energy at any time = Sum of kinetic and potential energy = $\frac{1}{2} m \omega^2 a^2 = \text{constant}$.

where, m = mass of body executing SHM,
 ω = angular frequency
 and a = amplitude of SHM.
 Hence, (c) and (d) are true.

02 Two simple harmonic motion, are represented by the equations

$$y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$\text{and } y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

Ratio of amplitude of y_1 to

$y_2 = x:1$. The value of x is

[2021, 27 Aug Shift-II]

Ans. (1)

$$\text{Given, } y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$A_1 = 10$$

$$y_2 = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

$$= 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t$$

Amplitude of SHM y_2 is

$$A_2 = \sqrt{5^2 + (5\sqrt{3})^2}$$

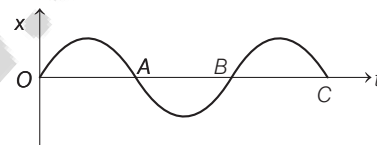
$$= \sqrt{25 + 75}$$

$$= \sqrt{100} = 10$$

$$\therefore \text{Ratio of amplitudes, } \frac{A_1}{A_2} = \frac{10}{10} = 1:1$$

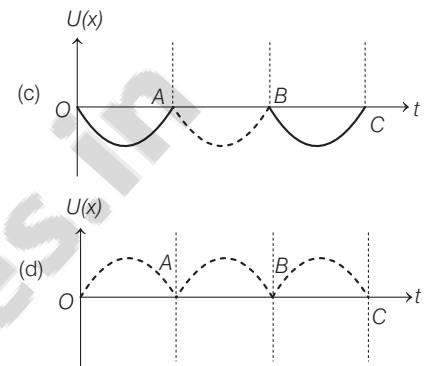
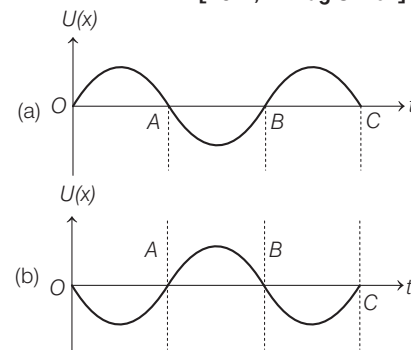
$$\therefore x = 1$$

03 The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.



The potential energy $U(x)$ versus time (t) plot of the particle is correctly shown in figure :

[2021, 27 Aug Shift-I]



Ans. (d)

It is given that motion is free simple harmonic motion. Consider a spring mass system performing free simple harmonic motion.

Displacement of mass is given as

$$x = x_0 \sin \omega t$$

The potential energy stored in spring for displacement x can be given as

$$PE = \frac{1}{2} kx^2$$

Substituting the value of x , we get

$$PE = \frac{1}{2} kx_0^2 \sin^2 \omega t$$

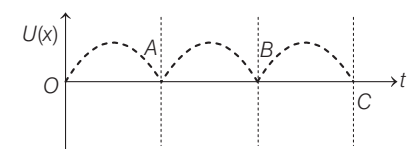
Here, k is spring constant and x_0 is also constant.

$$PE = C \sin^2 \omega t \quad \left(\because C = \frac{1}{2} kx_0^2 \right)$$

Simplify the value of $\sin^2 \omega t$, we get

$$PE = C \left[\frac{1 - \cos 2\omega t}{2} \right]$$

The value of function $\cos 2\omega t$ will always lies between -1 and 1 . Thus, the value of potential energy will be greater than or equal to zero.



$$0 \leq PE$$

This condition is only satisfies by graph (d).

- 04** Two simple harmonic motions are represented by the equations

$$x_1 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right) \text{ and}$$

$$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$$

The amplitude of second motion is times the amplitude in first motion.

[2021, 26 Aug Shift-II]

Ans. (2)

Equation of first simple harmonic motion,

$$x_1 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right).$$

Equation of second simple harmonic motion, $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$.

Simplifying the second equation of simple harmonic motion as follows

$$x_2 = 5\sqrt{2}\left(\sin 2\pi t \times \frac{1}{\sqrt{2}} + \cos 2\pi t \times \frac{1}{\sqrt{2}}\right)\sqrt{2}$$

$$= 10\left(\sin 2\pi t \cos \frac{\pi}{4} + \cos 2\pi t \sin \frac{\pi}{4}\right)$$

$$= 10 \sin\left(2\pi t + \frac{\pi}{4}\right)$$

The ratio of amplitudes of two simple harmonic motions is

$$\frac{A_2}{A_1} = \frac{10}{5} = 2$$

Thus, amplitude of second motion is 2 times the amplitude in first motion.

- 05** A particle executes simple harmonic motion represented by displacement function as

$$x(t) = A \sin(\omega t + \phi)$$

If the position and velocity of the particle at $t = 0$ s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm, where the value of x is

[2021, 27 July Shift-II]

Ans. (2)

A particle executes simple harmonic motion represented by displacement function,

$$x(t) = A \sin(\omega t + \phi) \quad \dots(i)$$

Differentiating equation with respect to t , we get

$$v(t) = A\omega \cos(\omega t + \phi) \quad \dots(ii)$$

As per question, position of particle at $t = 0$ is 2 cm

Using Eq. (i), we get

$$2 = A \sin[\omega(0) + \phi]$$

$$\Rightarrow A \sin \phi = 2 \quad \dots(iii)$$

and velocity of particle at $t = 0$ is 2ω cm s^{-1}

Using Eq. (ii), we get

$$2\omega = A\omega \cos[\omega(0) + \phi]$$

$$\Rightarrow A \cos \phi = 2 \quad \dots(iv)$$

Divide Eq. (i) by Eq. (ii), we get

$$\frac{A \sin \phi}{A \cos \phi} = \frac{2}{2}$$

$$\Rightarrow \tan \phi = 1 \Rightarrow \tan \phi = \tan 45^\circ$$

$$\Rightarrow \phi = 45^\circ \quad [\because \tan 45^\circ = 1]$$

Putting the value of ϕ in Eq. (iii), we get

$$A \sin 45^\circ = 2$$

$$\Rightarrow A \left(\frac{1}{\sqrt{2}}\right) = 2 \quad \left[\because \sin 45^\circ = \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow A = 2\sqrt{2} \quad \dots(v)$$

According to question, amplitude is $x\sqrt{2}$ cm. So, on comparing it with Eq. (v), we can write $x = 2$.

- 06** A particle starts executing simple harmonic motion (SHM) of amplitude a and total energy E . At any instant, its kinetic energy is $\frac{3E}{4}$,

then its displacement y is given by

[2021, 27 July Shift-I]

$$(a) y = a \quad (b) y = \frac{a}{\sqrt{2}}$$

$$(c) y = \frac{a\sqrt{3}}{2} \quad (d) y = \frac{a}{2}$$

Ans. (d)

In first scenario, a particle is executing SHM of amplitude a and energy E .

$$\therefore \text{Energy, } E = \frac{1}{2} K a^2 \quad \dots(i)$$

In second scenario, it is given that the kinetic energy of particle becomes $\frac{3E}{4}$.

$$\therefore \text{Energy, } \frac{3E}{4} = \frac{1}{2} K (a^2 - y^2) \quad \dots(ii)$$

where, y is the displacement.

On dividing Eqs. (i) and (ii), we get

$$\frac{E}{\frac{3E}{4}} = \frac{\frac{1}{2} K a^2}{\frac{1}{2} K (a^2 - y^2)}$$

$$\Rightarrow \frac{4}{3} = \frac{a^2}{a^2 - y^2} \Rightarrow 4a^2 - 4y^2 = 3a^2$$

$$\Rightarrow -4y^2 = -a^2$$

$$\Rightarrow y^2 = \frac{a^2}{4} \Rightarrow y = \frac{a}{2}$$

- 07** In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is

midway between mean and extreme position.

[2021, 25 July Shift-II]

$$(a) \frac{1}{2} \quad (b) \frac{3}{4} \\ (c) \frac{1}{3} \quad (d) \frac{1}{4}$$

Ans. (b)

Let, total mechanical energy = ME

Kinetic energy = KE

Potential energy = PE

Given, wave displacement, $x = \frac{A}{2}$

Here, A is amplitude of oscillation.

As we know that,

$$ME = KE + PE$$

$$\text{In SHM, } KE = \frac{1}{2} m \omega^2 (A^2 - x^2)$$

where, m is mass of particle, ω is angular frequency

and A is amplitude.

$$= \frac{1}{2} m \omega^2 \left(A^2 - \frac{A^2}{4} \right) = \frac{3}{8} m \omega^2 A^2 \quad \dots(i)$$

$$PE = \frac{1}{2} k x^2$$

$$\text{As, } \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow k = m \omega^2$$

$$\therefore PE = \frac{1}{2} m \omega^2 \frac{A^2}{4} = \frac{1}{8} m \omega^2 A^2$$

Now, ME = KE + PE

$$= \frac{3}{8} m \omega^2 A^2 + \frac{1}{8} m \omega^2 A^2$$

$$= \frac{4 m \omega^2 A^2}{8} = \frac{m \omega^2 A^2}{2} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{KE}{ME} = \frac{\frac{3}{8} m \omega^2 A^2}{\frac{1}{2} m \omega^2 A^2} = \frac{3}{4}$$

- 08** A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position, the velocities of the particle are v_1 and v_2 respectively, then the time period of its oscillation is given as

[2021, 20 July Shift-II]

$$(a) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}} \quad (b) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$$

$$(c) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}} \quad (d) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

Ans. (d)

Velocity of particle performing simple harmonic motion,

$$v = \pm \omega \sqrt{A^2 - x^2} \quad \dots(i)$$

where, A = amplitude, ω = angular frequency,and x = displacement covered by the particle from its mean position.

According to given situation,

$$v_1 = \omega \sqrt{A^2 - x_1^2} \quad \dots(ii)$$

$$\text{and } v_2 = \omega \sqrt{A^2 - x_2^2} \quad \dots(iii)$$

On squaring Eq. (ii), we get

$$v_1^2 = \omega^2 (A^2 - x_1^2) \quad \dots(iv)$$

On squaring Eq. (iii), we get

$$v_2^2 = \omega^2 (A^2 - x_2^2) \quad \dots(v)$$

Subtracting Eq. (iv) from Eq. (v), we get

$$v_1^2 - v_2^2 = \omega^2 (x_2^2 - x_1^2)$$

$$\Rightarrow v_1^2 - v_2^2 = \left(\frac{2\pi}{T}\right)^2 (x_2^2 - x_1^2)$$

$$\Rightarrow T = 2\pi \sqrt{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}}$$

- 09** The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is

[2021, 18 March Shift-II]

(a) $\sin(\omega t) = \cos(\omega t)$

(b) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

(c) $\sin^2(\omega t)$

(d) $3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$

Ans. (d)

Time period, $T = \frac{2\pi}{\omega'}$

Given, time period, $T = \frac{\pi}{\omega}$

$$\frac{2\pi}{\omega'} = \frac{\pi}{\omega}$$

$$\Rightarrow \omega' = 2\omega$$

Options (a) and (b) are incorrect.

Option (c),

$$\sin^2 \omega t = \frac{1}{2} (2 \sin^2 \omega t)$$

$$= \frac{1}{2} (1 - \cos 2\omega t)$$

Hence, the angular frequency is 2ω .

Option (d),

$$3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$$

Angular frequency of SHM is 2ω .

Option (d) is the correct answer.

- 10** A particle performs simple harmonic motion with a period of 2 s. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $1/a$ s. The value of a to the nearest integer is

[2021, 18 March Shift-I]**Ans. (6)**Given, the time period of the simple harmonic motion, $T = 2$ sDisplacement covered by the particle, $x = A/2$ Here, A is the amplitude of the particle.

The general equation of the simple harmonic motion,

$$x = A \sin \omega t$$

$$\Rightarrow \frac{A}{2} = A \sin\left(\frac{2\pi}{T} t\right) \Rightarrow \frac{1}{2} = \sin\left(\frac{2\pi}{2} t\right)$$

$$\Rightarrow \sin \frac{\pi}{6} = \sin(\pi t) \Rightarrow \frac{\pi}{6} = \pi t$$

$$\Rightarrow t = \frac{1}{6}$$

The time taken by the particle to cover a displacement equal to half of its

amplitude from the mean position is $1/6$ s.Comparing with $t = 1/a$ Hence, the value of the a is 6.

- 11** For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal?

[2021, 17 March Shift-I]

(a) $x = 0$

(b) $x = \pm A$

(c) $x = \pm \frac{A}{\sqrt{2}}$

(d) $x = \frac{A}{2}$

Ans. (c)

Potential energy of SHM

$$= \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$$

where m = mass of particle, ω = angular velocityand x = displacement.

$$\text{Kinetic energy of SHM} = \frac{1}{2} m(\omega \sqrt{A^2 - x^2})^2$$

Here, A is the amplitude of SHM.

According to question,

Potential energy of SHM = Kinetic energy of SHM

$$\frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m(\omega \sqrt{A^2 - x^2})^2$$

$$\Rightarrow \omega^2 x^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow \omega^2 x^2 = \omega^2 (A^2 - x^2)$$

$$\Rightarrow \omega^2 x^2 = \omega^2 A^2 - \omega^2 x^2$$

$$\Rightarrow 2\omega^2 x^2 = \omega^2 A^2$$

$$\Rightarrow 2x^2 = A^2 \Rightarrow x^2 = \frac{A^2}{2}$$

$$\Rightarrow x = \pm \sqrt{\frac{A^2}{2}} \Rightarrow x = \pm \frac{A}{\sqrt{2}}$$

- 12** A particle executes SHM, the graph of velocity as a function of displacement is **[2021, 26 Feb Shift-II]**

(a) a circle

(b) a parabola

(c) an ellipse

(d) a helix

Ans. (c)Velocity of particle in SHM in terms of displacement x is given as

$$v = \omega \sqrt{A^2 - x^2}$$

$$\Rightarrow v = A\omega \sqrt{1 - x^2/A^2}$$

$$\Rightarrow \frac{v}{A\omega} = \sqrt{1 - x^2/A^2}$$

$$\Rightarrow \frac{v^2}{A^2\omega^2} = 1 - \frac{x^2}{A^2}$$

$$\Rightarrow \frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1 \quad \dots(i)$$

As equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Eq. (i) is similar to the equation of ellipse. So, the graph between velocity and displacement is an ellipse.

- 13** A particle executes SHM with amplitude a and time period T . The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is

[2021, 26 Feb Shift-II]**Ans. (3)**Given, amplitude of particle in SHM = a Maximum velocity, $v = A\omega$ Let displacement be y when speed,

$$v_2 = v/2 = \frac{A\omega}{2}$$

By using equation of wave velocity,

$$v_2^2 = \omega^2 (A^2 - y^2)$$

$$\therefore \left(\frac{\omega A}{2}\right)^2 = \omega^2 (A^2 - y^2)$$

$$\Rightarrow \frac{\omega^2 A^2}{4} = \omega^2 (A^2 - y^2) \Rightarrow \frac{A^2}{4} = A^2 - y^2$$

$$\Rightarrow y = \frac{\sqrt{3}}{2} A = \frac{a\sqrt{x}}{2}$$

Hence, $x = 3$

- 14** $Y = A \sin(\omega t + \phi_0)$ is the time-displacement equation of SHM. At $t = 0$, the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then, the initial phase angle ϕ_0 will be

[2021, 25 Feb Shift-II]

- (a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

Ans. (a)

Given, displacement-time equation,

$$Y = A \sin(\omega t + \phi_0)$$

Here, A is amplitude, ω is angular frequency, t is time taken and ϕ_0 is the phase constant.

At $t = 0$, $Y = A/2$

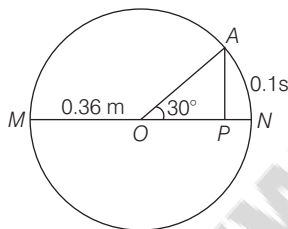
$$\therefore Y = A/2 = A \sin(0 + \phi_0) = A \sin \phi_0$$

$$\Rightarrow \sin \phi_0 = 1/2$$

$$\Rightarrow \phi_0 = \pi/6$$

- 15** The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection P from A on the diameter MN represents the simple harmonic motion of P . The restoration force per unit mass when P touches M will be

[2021, 25 Feb Shift-II]



- (a) 100 N (b) 9.87 N
(c) 50 N (d) 0.49 N

Ans. (b)Given, radius of circle, $R = 0.36$ mAngular distance, $\theta = 30^\circ = \pi/6$ radLet l be the arc length.

$$\therefore l = R\theta$$

$$\Rightarrow l = \frac{36}{100} \times \frac{\pi}{6} = \frac{6\pi}{100} \text{ m}$$

As, speed on circular track (v) = Arc length (l) / Time (t)

$$\Rightarrow v = \frac{6\pi}{100 \times 0.1} = \frac{6\pi}{10} \text{ ms}^{-1}$$

If F be the restoration force and a_r be the radial acceleration ($= v^2/R$), then

$$F = ma_r$$

$$a_r = \frac{F}{m} = \frac{v^2}{R} = \left(\frac{6\pi}{10}\right)^2 \times \frac{100}{36}$$

$$= \frac{36 \times 9.87}{100} \times \frac{100}{36} = 9.87 \text{ N}$$

- 16** When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is [2021, 24 Feb Shift-II]

- (a) circular (b) elliptical
(c) parabolic (d) straight line

Ans. (b)

Since, the particle is executing SHM.

Therefore, displacement equation of wave will be

$$y = A \sin \omega t$$

$$\Rightarrow y/A = \sin \omega t$$

and wave velocity equation will be

$$v_y = \frac{dy}{dt} = A\omega \cos \omega t$$

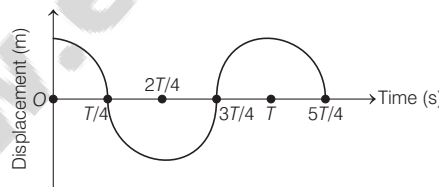
$$\Rightarrow v_y/A\omega = \cos \omega t$$

$$\text{Now, } \sin^2 \omega t + \cos^2 \omega t = 1$$

$$\therefore (y/A)^2 + (v_y/A\omega)^2 = 1$$

This equation is similar to the equation of ellipse.

- 17** The displacement-time graph of a particle executing SHM is given in figure (sketch is schematic and not to scale) [2020, 2 Sep Shift-II]

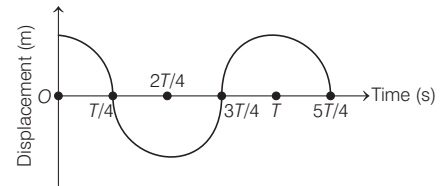


Which of the following statement(s) is/are true for this motion?

- A. The force is zero at $t = \frac{3T}{4}$.
B. The acceleration is maximum at $t = T$.
C. The speed is maximum at $t = \frac{T}{4}$.
D. The potential energy is equal to kinetic energy of the oscillation at $t = \frac{T}{2}$.
- (a) A, B and D (b) B, C and D
(c) A, B and C (d) A and D

Ans. (c)

The displacement - time graph is as shown below,



At $t = \frac{3T}{4}$, particle is at mean position. At

mean position of SHM, acceleration of particle is zero. i.e. force is zero.

Statement (A) is correct.

At $t = T$, particle is at extreme position.

As direction is changing, so its acceleration is maximum. Statement (B) is correct.

At $t = \frac{T}{4}$, particle is at mean position, so

whole of energy is kinetic. i.e. its speed is maximum. Statement (C) is correct.

PE = KE at $t = \frac{T}{8}$, so statement (D) is

incorrect.

Hence, option (c) is correct.

- 18** A particle is executing simple harmonic motion (SHM) of amplitude A , along the X-axis, about $x = 0$. when its potential energy (PE) equals kinetic energy (KE), the position of the particle will be [2019, 9 Jan Shift-II]

- (a) A (b) $\frac{A}{2}$
(c) $\frac{A}{2\sqrt{2}}$ (d) $\frac{A}{\sqrt{2}}$

Ans. (d)Here, A = amplitude of particle in SHM.

We know that in SHM potential energy (U) of a particle is given by the relation at distance x from the mean position is

$$U = \frac{1}{2} kx^2$$

and at the same point kinetic energy (K)

= Total energy - Potential energy

$$= \frac{1}{2} kA^2 - \frac{1}{2} kx^2$$

According to the question,

potential energy = kinetic energy

$$\therefore \frac{1}{2} kx^2 = \frac{1}{2} kA^2 - \frac{1}{2} kx^2$$

$$\text{or } kx^2 = \frac{kA^2}{2} \text{ or } x = \pm \frac{A}{\sqrt{2}}$$

- 19** A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time (in seconds) is

[2019, 10 Jan Shift-II]

- (a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{7\pi}{3}$ (d) $\frac{3\pi}{8}$

Ans. (b)

In simple harmonic motion, position (x), velocity (v) and acceleration (a) of the particle are given by

$$x = A \sin \omega t$$

$$v = \omega \sqrt{A^2 - x^2} \text{ or } v = A \omega \cos \omega t$$

and $a = -\omega^2 x$ or $a = -\omega^2 A \sin \omega t$

Given, amplitude $A = 5$ cm and displacement $x = 4$ cm.

At this time (when $x = 4$ cm), velocity and acceleration have same magnitude.

$$\Rightarrow |v_{x=4}| = |a_{x=4}|$$

$$\text{or } |\omega \sqrt{5^2 - 4^2}| = | -4\omega^2 |$$

$$\Rightarrow 3\omega = 4\omega^2 \Rightarrow \omega = (3/4) \text{ rad/s}$$

$$\text{So, time period, } T = \frac{2\pi}{\omega}$$

$$\Rightarrow T = \frac{2\pi}{3/4} \times 4 = \frac{8\pi}{3} \text{ s.}$$

- 20** A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin \frac{\pi t}{90}$. The ratio of kinetic to

potential energy of this particle at $t = 210$ s will be

[2019, 11 Jan Shift-I]

- (a) 2 (b) 1 (c) $\frac{1}{9}$ (d) 3

Ans. (*)

Here given, displacement, $x(t) = A \sin \frac{\pi t}{90}$

where A is amplitude of SHM, t is time taken by particle to reach a point where its potential energy $U = \frac{1}{2} kx^2$ and kinetic

energy $= \frac{1}{2} k(A^2 - x^2)$ here k is force

constant and x is position of the particle.

Potential energy (U) at $t = 210$ s is

$$U = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \sin^2 \left(\frac{210}{90} \pi \right)$$

$$= \frac{1}{2} kA^2 \sin^2 \left(2\pi + \frac{3}{9} \pi \right)$$

$$= \frac{1}{2} kA^2 \sin^2 \left(\frac{\pi}{3} \right)$$

Kinetic energy at $t = 210$ s, is

$$K = \frac{1}{2} k(A^2 - x^2)$$

$$= \frac{1}{2} kA^2 \left[1 - \sin^2 \left(\frac{210\pi}{90} \right) \right]$$

$$= \frac{1}{2} kA^2 \cos^2 (210\pi/90)$$

$$\Rightarrow K = \frac{1}{2} kA^2 \cos^2 (\pi/3)$$

So, ratio of kinetic energy to potential energy is

$$\frac{K}{U} = \frac{\frac{1}{2} kA^2 \cos^2 (\pi/3)}{\frac{1}{2} kA^2 \sin^2 (\pi/3)} = \cot^2 (\pi/3) = \frac{1}{3}$$

$\therefore$ No option given is correct.

- 21** A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$ cm. The amplitude and time period of the motion are

[2019, 12 Jan Shift-II]

- (a) 10 cm, $\frac{3}{2}$ s (b) 5 cm, $\frac{2}{3}$ s
(c) 5 cm, $\frac{3}{2}$ s (d) 10 cm, $\frac{2}{3}$ s

Ans. (d)

Equation for SHM is given as

$$y = 5(\sin 3\pi t + \sqrt{3} \cos 3\pi t)$$

$$= 5 \times 2 \left(\frac{1}{2} \times \sin 3\pi t + \frac{\sqrt{3}}{2} \cos 3\pi t \right)$$

$$= 5 \times 2 \left(\cos \frac{\pi}{3} \cdot \sin 3\pi t + \sin \frac{\pi}{3} \cdot \cos 3\pi t \right)$$

$$= 5 \times 2 \sin \left(3\pi t + \frac{\pi}{3} \right)$$

[using, $\sin(a+b) = \sin a \cos b + \cos a \sin b$]

$$\text{or } y = 10 \sin \left(3\pi t + \frac{\pi}{3} \right)$$

Comparing this equation with the general equation of SHM, i.e.

$$y = A \sin \left(\frac{2\pi t}{T} + \phi \right),$$

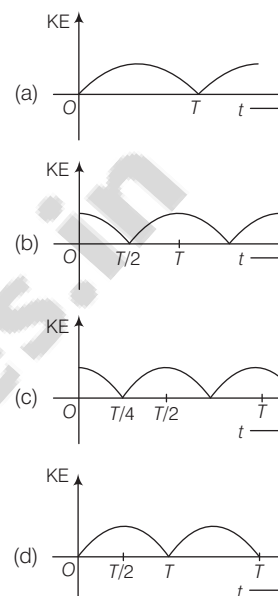
We get, amplitude, $A = 10$ cm

$$\text{and } 3\pi = 2\pi/T$$

$$\text{or Time period, } T = \frac{2}{3} \text{ s}$$

- 22** A particle is executing simple harmonic motion with a time period T . At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like

[JEE Main 2017 (Offline)]

**Ans. (c)**

KE is maximum at mean position and minimum at extreme position (at $t = \frac{T}{4}$).

- 23** A particle performs simple harmonic motion with amplitude A . Its speed is trebled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is

[JEE Main 2016]

- (a) $\frac{A}{3} \sqrt{41}$ (b) $3A$
(c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

Ans. (d)

Key Idea The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$v = \omega \sqrt{A^2 - x^2}$$

where, ω is angular frequency, A is amplitude and x is displacement of a particle.

Suppose that the new amplitude of the motion be A' .

Initial velocity of a particle performs SHM,

$$v^2 = \omega^2 \left[A^2 - \left(\frac{2A}{3} \right)^2 \right] \quad \dots (i)$$

where, A is initial amplitude and ω is angular frequency.

Final velocity,

$$(3v)^2 = \omega^2 \left[A'^2 - \left(\frac{2A}{3} \right)^2 \right] \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{9} = \frac{A^2 - \frac{4A^2}{9}}{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

- 24** A particle moves with simple harmonic motion in a straight line. In first t sec, after starting from rest, it travels a distance a and in next t sec, it travels $2a$ in same direction, then **[JEE Main 2014]**

- (a) amplitude of motion is $3a$
 (b) time period of oscillations is 5π
 (c) amplitude of motion is $4a$
 (d) time period of oscillations is 6π

Ans. (d)

In SHM, a particle starts from rest, we have

$$x = A \cos \omega t, \text{ at } t=0, x=A$$

$$\text{When } t=t, \text{ then } x=A-a$$

$$\text{When } t=2t, \text{ then } x=A-3a$$

$$\text{So, } A-a = A \cos \omega t$$

$$A-3a = A \cos 2\omega t$$

$$\text{As, } \cos 2\omega t = 2 \cos^2 \omega t - 1$$

$$\Rightarrow \frac{A-3a}{A} = 2 \left(\frac{A-a}{A} \right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA \Rightarrow A = 2a$$

$$\text{Now, } A-a = A \cos \omega t \Rightarrow \cos \omega t = 1/2$$

$$\frac{2\pi}{T} = \frac{\pi}{3}$$

$$T = 6\pi$$

- 25** Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x -axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between

them is $(X_0 + A)$, the phase difference between their motion is **[AIEEE 2011]**

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
 (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Ans. (a)

$$\text{Let } x_1 = A \sin(\omega t + \phi_1) \text{ and}$$

$$x_2 = A \sin(\omega t + \phi_2)$$

$$x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t + \phi_1)]$$

$$= 2A \cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right) \sin\left(\frac{\phi_2 - \phi_1}{2}\right)$$

The resultant motion can be treated as a simple harmonic motion with amplitude

$$2A \sin\left(\frac{\phi_2 - \phi_1}{2}\right)$$

Given, maximum distance between the particles

$$= X_0 + A$$

$\therefore$ Amplitude of resultant SHM

$$= X_0 + A - X_0 = A$$

$$\therefore 2A \sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A$$

$$\Rightarrow \text{Phase difference, } \phi_2 - \phi_1 = \pi/3$$

- 26** The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is

$$D = [U(x=\infty) - U_{\text{at equilibrium}}], \text{ then } D \text{ is}$$

- (a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

Ans. (c)

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$\Rightarrow U(x=\infty) = 0$$

$$\text{As, } F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

$$\text{At equilibrium, } F = 0$$

$$\therefore x^6 = \frac{2a}{b}$$

$$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$$

$$\therefore D = [U(x=\infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

- 27** If x , v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T , then which of the following does not change with time? **[AIEEE 2009]**

- (a) $a^2 T^2 + 4\pi^2 v^2$ (b) $\frac{aT}{x}$
 (c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

Ans. (b)

$$\frac{aT}{x} = \frac{\omega^2 x T}{x} = \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T} = \text{constant}$$

Hence, aT/x does not change with time.

- 28** A point mass oscillates along the x -axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then **[AIEEE 2007]**

- (a) $A = x_0, \delta = -\frac{\pi}{4}$ (b) $A = x_0 \omega^2, \delta = \frac{\pi}{4}$
 (c) $A = x_0 \omega^2, \delta = -\frac{\pi}{4}$ (d) $A = x_0 \omega^2, \delta = \frac{3\pi}{4}$

Ans. (d)

$$x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$\text{Acceleration, } a = \frac{d^2 x}{dt^2}$$

$$= -\omega^2 x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= \omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$$

$$\text{So, } A = \omega^2 x_0 \text{ and } \delta = \frac{3\pi}{4}$$

- 29** A particle of mass m executes simple harmonic motion with amplitude a and frequency ν . The average kinetic energy during its motion from the position of equilibrium to the end is **[AIEEE 2007]**

- (a) $\pi^2 m a^2 \nu^2$ (b) $\frac{1}{4} m a^2 \nu^2$
 (c) $4\pi^2 m a^2 \nu^2$ (d) $2\pi^2 m a^2 \nu^2$

Ans. (a)

Average kinetic energy of particle

$$= \frac{1}{4} m a^2 \omega^2 \quad [\text{as } \omega = 2\pi\nu]$$

$$= \frac{1}{4} m a^2 (2\pi\nu)^2 = \pi^2 \nu^2 m a^2$$

- 30** The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is [AIEEE 2007]

- (a) 0.5 s (b) 0.75 s
(c) 0.125 s (d) 0.25 s

Ans. (a)

$$x = (2 \times 10^{-2}) \cos \pi t$$

Here, $a = 2 \times 10^{-2} \text{ m} = 2 \text{ cm}$

At $t = 0$, $x = 2 \text{ cm}$, i.e., the object is at positive extreme, so to acquire maximum speed (i.e., to reach mean position), it takes $\frac{1}{4}$ th of time period.

$$\therefore \text{Required time} = \frac{T}{4}$$

As $x = a \cos \omega t$

where, $\omega = \frac{2\pi}{T} = \pi$

$$\Rightarrow T = 2 \text{ s}$$

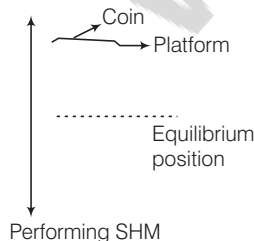
So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5 \text{ s}$

- 31** A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω . The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time [AIEEE 2006]

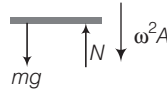
- (a) at the mean position of the platform
(b) for an amplitude of g/ω^2
(c) for an amplitude of g^2/ω^2
(d) at the highest position of the platform

Ans. (b)

As the amplitude is increased, the maximum acceleration of the platform (along with coin as long as they do not get separated) increases.



If we draw the free body diagram of coin at one of the extreme positions as shown, then from Newton's law,



$$mg - N = m\omega^2 A$$

For losing contact with the platform,

$$N = 0$$

So, $A = \frac{g}{\omega^2}$

- 32** The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1} . The period of oscillation is [AIEEE 2006]

- (a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

Ans. (a)

The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$$\therefore 4.4 = (7 \times 10^{-3}) \times 2\pi/T$$

$$\text{or } T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01 \text{ s}$$

- 33** Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? [AIEEE 2006]

- (a) $\frac{1}{6} \text{ s}$ (b) $\frac{1}{4} \text{ s}$ (c) $\frac{1}{3} \text{ s}$ (d) $\frac{1}{12} \text{ s}$

Ans. (a)

KE of a body undergoing SHM is given by

$$KE = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t \text{ and}$$

$$KE_{\max} = \frac{m\omega^2 A^2}{2}$$

[symbols represent standard quantities]

From given information,

$$KE = (KE_{\max}) \times \frac{75}{100}$$

$$\Rightarrow \frac{m\omega^2 A^2}{2} \cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$$

$$\text{or } \cos \omega t = \pm \frac{\sqrt{3}}{2} \text{ or } \omega t = \frac{\pi}{6}$$

$$\text{or } \frac{2\pi}{T} \times t = \frac{\pi}{6} \text{ or } t = \frac{T}{12} = \frac{1}{6} \text{ s}$$

- 34** If a simple harmonic motion is represented by $\frac{d^2 x}{dt^2} + \alpha x = 0$, its time period is [AIEEE 2005]

- (a) $\frac{2\pi}{\alpha}$ (b) $\frac{2\pi}{\sqrt{\alpha}}$ (c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$

Ans. (b)

$$\frac{d^2 x}{dt^2} = -\alpha x \quad \dots(i)$$

We know that, for SHM

$$a = \frac{d^2 x}{dt^2} = -\omega^2 x \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\omega^2 = \alpha \text{ or } \omega = \sqrt{\alpha}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\alpha}$$

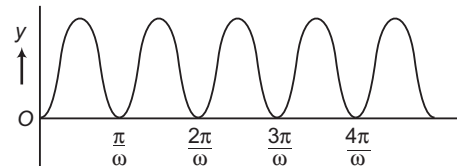
$$\text{or } T = \frac{2\pi}{\sqrt{\alpha}}$$

- 35** The function $\sin^2(\omega t)$ represents [AIEEE 2005]

- (a) a periodic but not simple harmonic motion with a period $2\pi/\omega$
(b) a periodic but not simple harmonic motion with a period π/ω
(c) a simple harmonic motion with a period $2\pi/\omega$
(d) a simple harmonic motion with a period π/ω

Ans. (b)

Here, $y = \sin^2 \omega t$



$$\frac{dy}{dt} = 2\omega \sin \omega t \cos \omega t = \omega \sin 2\omega t$$

$$\frac{d^2 y}{dt^2} = 2\omega^2 \cos 2\omega t$$

For SHM, $\frac{d^2 y}{dt^2} \propto -y$

Hence, function is not SHM but periodic.

From the y - t graph, time period is

$$T = \pi/\omega.$$

$$\left[\because \frac{2\pi}{T} = 2\omega \right]$$

- 36** Two simple harmonic motions are represented by the equations

$$y_1 = 0.1 \sin \left(100\pi t + \frac{\pi}{3} \right) \text{ and}$$

$$y_2 = 0.1 \cos \pi t. \text{ The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is [AIEEE 2005]}$$

- (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Ans. (a)

Given, $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$

 $\Rightarrow$ Velocity of particle 1,

$$\frac{dy_1}{dt} = v_1 = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$$

or $v_1 = 10\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

or $v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right)$

and $y_2 = 0.1 \cos \pi t$ $\Rightarrow$ Velocity of particle 2,

$$\frac{dy_2}{dt} = v_2 = -0.1 \sin \pi t$$

or $v_2 = 0.1 \sin(\pi t + \pi)$

Hence, phase difference

$$\begin{aligned} \Delta\phi &= \phi_1 - \phi_2 \\ &= \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi) \\ &= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \quad [\text{at } t=0] \end{aligned}$$

- 37** The total energy of a particle, executing simple harmonic motion is [AIEEE 2004]

- (a) $\propto x$
 (b) $\propto x^2$
 (c) independent of x
 (d) $\propto x^{1/2}$

where, x is the displacement from the mean position.

Ans. (c)

In simple harmonic motion, when a particle is displaced to a position from its mean position, then its kinetic energy gets converted into potential energy.

Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x .

- 38** A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as function of displacement x . Which of the following statements is true? [AIEEE 2003]

- (a) KE is maximum when $x=0$
 (b) TE is zero when $x=0$
 (c) KE is maximum when x is maximum
 (d) PE is maximum when $x=0$

Ans. (a)

At $x=0$, kinetic energy is maximum and potential energy is minimum.

$$\therefore \text{Kinetic energy, } K = \frac{1}{2}m\omega^2(a^2 - x^2)$$

 $K_{\max}$ when $x=0$

$$K_{\max} = \frac{1}{2}m\omega^2 a^2 \text{ and potential energy,}$$

$$U = \frac{1}{2}m\omega^2 x^2,$$

at $x=0$, $U=0$

- 39** The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is [AIEEE 2003]

- (a) 4 (b) 4 (c) $4\sqrt{2}$ (d) 8

Ans. (c)

$$\begin{aligned} x &= 4(\cos \pi t + \sin \pi t) \\ &= \frac{4}{\sqrt{2}} \times \sqrt{2} [\cos \pi t + \sin \pi t] \end{aligned}$$

$$x = 4\sqrt{2} \left[\sin \pi t \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cos \pi t \right]$$

$$x = 4\sqrt{2} \left[\sin \pi t + \cos \frac{\pi}{4} + \cos \pi t \sin \frac{\pi}{4} \right]$$

$$x = 4\sqrt{2} \sin \left[\pi t + \frac{\pi}{4} \right]$$

So, amplitude $= 4\sqrt{2}$

- 40** In a simple harmonic oscillator, at the mean position [AIEEE 2002]

- (a) kinetic energy is minimum, potential energy is maximum
 (b) both kinetic and potential energies are maximum
 (c) kinetic energy is maximum, potential energy is minimum
 (d) both kinetic and potential energies are minimum

Ans. (c)

Kinetic energy of particle of mass m in SHM at any point

$$= \frac{1}{2}m\omega^2(a^2 - x^2)$$

$$\text{and potential energy} = \frac{1}{2}m\omega^2 x^2$$

where, a is amplitude of particle and x is the distance from mean position.

So, at mean position, $x=0$

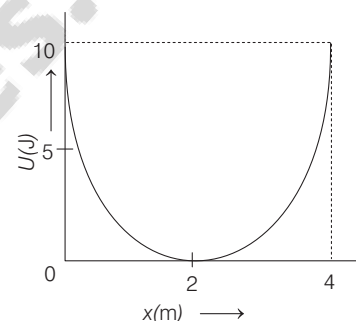
$$\therefore \text{KE} = \frac{1}{2}m\omega^2 a^2 \quad [\text{maximum}]$$

$$\text{PE} = 0 \quad [\text{minimum}]$$

TOPIC 2

Some Systems Executing SHM

- 41** A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed? [2021, 1 Sep Shift-II]



- (a) 10 m/s^2 (b) 5 m/s^2
 (c) 4 m/s^2 (d) 9.8 m/s^2

Ans. (a)

(c) From the potential energy curve,

$$U_{\max} = \frac{1}{2}kA^2 \Rightarrow 10 = \frac{1}{2}k(2)^2$$

$$\Rightarrow k = 5 \text{ N/m}$$

The length of the simple pendulum, $L = 4 \text{ m}$

Time period of spring,

$$T = 2\pi\sqrt{\frac{k}{m}}$$

Time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The time period of simple pendulum is same as the time period of the spring oscillation.

$$\Rightarrow 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{k}{m}}$$

Substituting the values in the above equations, we get

$$2\pi\sqrt{\frac{4}{g}} = 2\pi\sqrt{\frac{5}{5}} \Rightarrow g = 4 \text{ m/s}^2$$

- 42** A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1} . The mass is pulled slightly downward and released, so that it executes free simple harmonic motion with time period T . The time when the kinetic energy and potential energy of the system will become equal, is T/x . The value of x is. [2021, 31 Aug Shift-I]

Ans. (8)

Given, mass of particle, $m = 1 \text{ kg}$

Spring constant, $k = 100 \text{ Nm}^{-1}$

Let time period is T .

Kinetic energy (KE) = Potential energy (PE) = E

Amplitude = A

Angular velocity = ω

Wave displacement = x

$$\begin{aligned} \text{KE in SHM} &= \text{PE in SHM} \\ \Rightarrow \frac{1}{2} m \omega^2 (A^2 - x^2) &= \frac{1}{2} k x^2 \\ \Rightarrow m \omega^2 (A^2 - x^2) &= k x^2 \quad \dots (i) \end{aligned}$$

Since, force, $F = m \omega^2 x = kx$

$$\Rightarrow k = m \omega^2$$

Substituting the value in Eq. (i), we get

$$\Rightarrow m \omega^2 (A^2 - x^2) = m \omega^2 x^2$$

$$\Rightarrow A^2 - x^2 = x^2 \Rightarrow A^2 = 2x^2$$

$$\Rightarrow x = A / \sqrt{2}$$

Since, wave displacement, $x = A \sin \omega t$

$$\Rightarrow \frac{A}{\sqrt{2}} = A \sin \omega t$$

$$\Rightarrow \sin(\pi/4) = \sin \omega t$$

$$\Rightarrow \omega t = \pi/4$$

$$\Rightarrow \frac{2\pi}{T} \cdot t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{T}{8}$$

Comparing the given value in the question i.e. $\frac{T}{x}$, the value of $x = 8$.

- 43** A bob of mass m suspended by a thread of length l undergoes simple harmonic oscillations with time period T . If the bob is immersed in a liquid that has density $1/4$ times that of the bob and the length of the thread is increased by $1/3$ rd of the original length, then the time period of the simple harmonic oscillations will be

[2021, 31 Aug Shift-II]

- (a) T (b) $\frac{3}{2}T$
(c) $\frac{3}{4}T$ (d) $\frac{4}{3}T$

Ans. (d)

Given that, a bob of mass m suspended by a thread of length l undergoes SHM with time period,

$$T_1 = T = 2\pi \sqrt{\frac{l}{g}} \quad \dots (i)$$

In liquid, effective gravity,

$$g_{\text{eff}} = g \left(\frac{\sigma - \rho}{\sigma} \right)$$

where, ρ = density of liquid
and σ = density of body (bob).

$$\text{Given, } \rho = \frac{\sigma}{4} \Rightarrow \sigma = 4\rho$$

Substituting this value, we get

$$g_{\text{eff}} = g \left(\frac{4\rho - \rho}{4\rho} \right) = \frac{3g}{4}$$

Now, the new time period of pendulum in liquid is

$$T' = 2\pi \sqrt{\frac{l'}{g_{\text{eff}}}} \quad \dots (ii)$$

$$\text{Here, } l' = l + \frac{l}{3} = \frac{4l}{3}$$

Substituting the values in Eq. (ii), we get

$$T' = 2\pi \sqrt{\frac{4l}{3} \times \frac{4}{3g}} = \left(2\pi \sqrt{\frac{l}{g}} \right) \frac{4}{3}$$

$$\Rightarrow T' = \frac{4T}{3} \quad [\text{from Eq. (i)}]$$

- 44** The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass m to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as%.

[2021, 26 Aug Shift-II]

Ans. (14)

Given, accuracy of acceleration due to gravity = 4%

Accuracy of time period = 3%

Energy stored in pendulum at any instant is given as

$$E = \frac{mgL\theta^2}{2} \quad \dots (i)$$

Time period of pendulum is given by expression

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Rearrange the above expression for L , we get

$$L = g \left(\frac{T}{2\pi} \right)^2 \quad \dots (ii)$$

Substituting the value of L in Eq. (i), we get

$$E = \frac{mg^2\theta^2}{2} \left(\frac{T}{2\pi} \right)^2 = \frac{mg^2\theta^2 T^2}{8\pi^2}$$

Now, the accuracy in measurement of energy can be calculated as

$$\frac{\Delta E}{E} \times 100 = 2 \frac{\Delta g}{g} \times 100 + 2 \frac{\Delta T}{T} \times 100$$

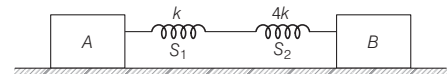
($\because m, \theta$ and π are constant)

$$\Rightarrow \frac{\Delta E}{E} \times 100 = 2 \times 4\% + 2 \times 3\% = 14\%$$

Thus, the accuracy to which E is known is 14%.

- 45** In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be rad/s when $k = 20 \text{ N/m}$.

[2021, 25 July Shift-I]



Ans. (10)

Given, spring constants of springs k_1 and k_2 are k and $4k$.

where, $k = 20 \text{ N/m}$

$$m_A = 200 \text{ g} = 200 \times 10^{-3} \text{ kg}$$

$$m_B = 800 \text{ g} = 800 \times 10^{-3} \text{ kg}$$

As we know that,

$$T = 2\pi \sqrt{\frac{m_{\text{eq}}}{k_{\text{eq}}}} \quad \dots (i)$$

where, equivalent or reduced mass,

$$m_{\text{eq}} = \frac{m_A m_B}{m_A + m_B}$$

$$\begin{aligned} \Rightarrow m_{\text{eq}} &= \frac{(200 \times 10^{-3})(800 \times 10^{-3})}{200 \times 10^{-3} + 800 \times 10^{-3}} \\ &= \frac{16 \times 10^{-6}}{1000 \times 10^{-3}} = 16 \times 10^{-2} \text{ kg} \end{aligned}$$

and equivalent spring constant,

$$\begin{aligned} k_{\text{eq}} &= \frac{k_1 k_2}{k_1 + k_2} = \frac{k \times 4k}{k + 4k} = \frac{4k}{5} \\ &= \frac{4}{5} \times 20 = 16 \text{ N/m} \end{aligned}$$

Putting these values in Eq. (i), we get

$$T = 2\pi\sqrt{\frac{16 \times 10^{-2}}{16}} = 2\pi \times 10^{-1} = \frac{2\pi}{10}$$

As, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} \times 10$
 $= 10 \text{ rads}^{-1}$

- 46** T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$

times of its initial value, then the modified time period is

[2021, 22 July Shift-II]

- (a) T_0 (b) $8\pi T_0$
 (c) $4T_0$ (d) $\frac{1}{4}T_0$

Ans. (d)

Given, time period of simple pendulum be T_0 .

Initial length of the pendulum string be l for time period T_0 and final length of the pendulum string be $\frac{l}{16}$.

As we know that,

$$T_0 = 2\pi\sqrt{\frac{l}{g}} \quad \dots(i)$$

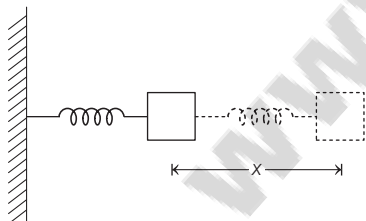
where, g is the acceleration due to gravity.

Therefore, final time be

$$T' = 2\pi\sqrt{\frac{l}{16 \times g}} = \frac{2\pi}{4}\sqrt{\frac{l}{g}} = \frac{T_0}{4}$$

[From Eq. (i)]

- 47** The motion of a mass on a spring, with spring constant k is as shown in figure.



The equation of motion is given by

$$x(t) = A \sin \omega t + B \cos \omega t \text{ with } \omega = \sqrt{\frac{k}{m}}$$

Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C \cos(\omega t - \phi)$, where C and ϕ are

[2021, 22 July Shift-II]

$$(a) C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$$

$$(b) C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$$

$$(c) C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$$

$$(d) C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$$

Ans. (d)

According to given figure,

$$\text{Angular frequency, } \omega = \sqrt{\frac{k}{m}}$$

Equation of motion be

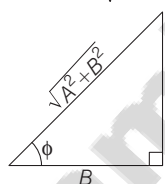
$$x = A \sin \omega t + B \cos \omega t \quad \dots(i)$$

On multiplying and dividing Eq. (i) by

$$\sqrt{A^2 + B^2}$$

$$x = \sqrt{A^2 + B^2}$$

$$\left[\frac{A \sin \omega t}{\sqrt{A^2 + B^2}} + \frac{B \cos \omega t}{\sqrt{A^2 + B^2}} \right] \quad \dots(ii)$$



$$\therefore \sin \phi = \frac{A}{\sqrt{A^2 + B^2}}$$

$$\cos \phi = \frac{B}{\sqrt{A^2 + B^2}}$$

$$\text{and } \tan \phi = \frac{A}{B}$$

Substituting these values of in Eq. (ii), we get

$$x(t) = \sqrt{A^2 + B^2} (\sin \phi \sin \omega t + \cos \phi \cos \omega t)$$

$$\therefore x(t) = \sqrt{A^2 + B^2} \cos(\omega t - \phi)$$

$$\text{and } x(0) = \sqrt{A^2 + B^2} \cos \phi \text{ along X-axis}$$

$$v(t) = \frac{dx(t)}{dt} = -\omega \sqrt{A^2 + B^2} \sin(\omega t - \phi)$$

$$\Rightarrow \frac{v(0)}{\omega} = \sqrt{A^2 + B^2} \sin(-\phi) \text{ along Y-axis}$$

$$\therefore C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2} \text{ and } \tan \phi = \frac{v(0)}{\omega x(0)}$$

$$\Rightarrow \phi = \tan^{-1}\left[\frac{v(0)}{\omega x(0)}\right]$$

- 48.** Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2 , respectively.

If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is

[2021, 17 March Shift-II]

- (a) $\frac{k_2}{k_1}$ (b) $\frac{k_1}{k_2}$
 (c) $\sqrt{\frac{k_1}{k_2}}$ (d) $\sqrt{\frac{k_2}{k_1}}$

Ans. (d)

(d) We know that, the expression of maximum velocity during oscillation,

$$V_{\max} = A\omega$$

Given, $V_{\max}(A) = V_{\max}(B)$

$$\Rightarrow A_1\omega_1 = A_2\omega_2$$

Now, the ratio of the amplitude during oscillation,

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} \quad \dots(i)$$

We know that, $\omega = \sqrt{k/m}$

where, k is the spring constant, m is the mass of the object.

Substituting the value of ω in Eq. (i), we get

$$\frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

- 49** Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig.2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is

[2021, 17 March Shift-I]

(Round off to the nearest integer)

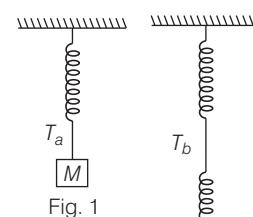


Fig. 1

Fig. 2

Ans. (2)

Time period of Fig. 1 can be given as

$$T_a = 2\pi\sqrt{\frac{M}{k}}$$

where, M is mass of the suspended object and k is the force constant.

In Fig. 2, both the springs are in series combination, therefore its time period can be given as

$$T_b = 2\pi \sqrt{\frac{M}{k_{eq}}} = 2\pi \sqrt{\frac{M}{k/2}} \left(\because k_{eq} = \frac{k \times k}{k + k} \right)$$

$$\text{Now, } \frac{T_b}{T_a} = \frac{2\pi \sqrt{\frac{M}{k/2}}}{2\pi \sqrt{\frac{M}{k}}}$$

$$\Rightarrow \frac{T_b}{T_a} = \sqrt{2} \quad \dots(i)$$

According to question, the ratio of time period of oscillation of two SHM is $T_b/T_a = \sqrt{x}$, so on comparing it with Eq. (i) we can say, $x = 2$

- 50** Time period of a simple pendulum is T inside a lift, when the lift is stationary. If the lift moves upwards with an acceleration $g/2$, then the time period of pendulum will be [2021, 16 March Shift-I]

- (a) $\sqrt{3}T$ (b) $\frac{T}{\sqrt{3}}$
(c) $\sqrt{\frac{3}{2}}T$ (d) $\sqrt{\frac{2}{3}}T$

Ans. (d)

Time period of a simple pendulum can be given as

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots(i)$$

When the lift moves upwards, then effective acceleration is

$$\Rightarrow g_{eff} = g + a = g + \frac{g}{2} = \frac{3g}{2}$$

$\therefore$ New time period,

$$T_1 = 2\pi \sqrt{\frac{l}{g_{eff}}} = 2\pi \sqrt{\frac{2l}{3g}}$$

$$\Rightarrow T_1 = \sqrt{\frac{2}{3}}T \quad [\text{From Eq. (i)}]$$

- 51** Given below are two statements:
Statement I A second's pendulum has a time period of 1 s.
Statement II It takes precisely one second to move between the two extreme positions.
In the light of the above statements, choose the correct answer from the options given below. [2021, 26 Feb Shift-II]

Ans. (b)

Statement I is false because time period of second's pendulum is always 2 s.

Therefore, time taken to move between two extreme positions will be $T/2 = 2/2 = 1$ s

Hence, option (b) is the correct.

- (a) Both Statement I and Statement II are false.
(b) Statement I is false but Statement II is true.
(c) Statement I is true but Statement II is false.
(d) Both Statement I and Statement II are true.

- 52** Time period of a simple pendulum is T . The time taken to complete $5/8$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is

..... [2021, 26 Feb Shift-II]

Ans. (7)

Given, angular displacement to complete

$$\frac{5}{8} \left(\frac{1}{2} + \frac{1}{8} \right) \text{ rev} = \left(\pi + \frac{\pi}{6} \right) \text{ rad} = \left(\frac{7\pi}{6} \right) \text{ rad}$$

$$\text{Since, } \omega = \frac{2\pi}{T} = \frac{\theta}{t}$$

$$\Rightarrow \theta = \frac{2\pi}{T} \cdot t \Rightarrow \frac{7\pi}{6} = \frac{2\pi}{T} t$$

$$\Rightarrow t = \frac{7T}{12} = \frac{\alpha}{\beta}T$$

$$\text{Hence, } \alpha = 7$$

- 53** If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor [2021, 26 Feb Shift-I]

- (a) $\frac{1}{2}, \sqrt{2}$ (b) $\frac{1}{4}, \sqrt{2}$
(c) $\frac{1}{4}, 2\sqrt{2}$ (d) $\frac{1}{2}, 2\sqrt{2}$

Ans. (a)

Let series equivalent of spring constant = k_{eq} and T be the time period.

$$\text{In series arrangement, } \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_1} = \frac{2}{k_1} \Rightarrow k_{eq} = \frac{k_1}{2}$$

$$\text{As, } T = 2\pi \sqrt{\frac{m}{k_1}}$$

where, m is mass of body connected with spring.

$$\Rightarrow T \propto \sqrt{\frac{1}{k_1}}$$

$$\text{and } T' \propto \sqrt{\frac{2}{k_1}} \Rightarrow T' = \sqrt{2}T$$

- 54** Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $(R/2)$ from the earth's centre, where R is the radius of the earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period?

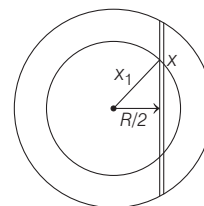
[2021, 26 Feb Shift-I]

- (a) $\frac{2\pi R}{g}$ (b) $\frac{g}{2\pi R}$
(c) $\frac{1}{2\pi} \sqrt{\frac{g}{R}}$ (d) $2\pi \sqrt{\frac{R}{g}}$

Ans. (d)

Given, radius of earth = R

Distance of chord from centre of earth = $R/2$



Let x_1 be the radius of inner circle and M be the mass of earth.

$\therefore m'$ (effective mass of earth)

$$= \frac{M}{4} \cdot \frac{4}{\pi R^3} \cdot \frac{4}{3} \pi x_1^3$$

$$\Rightarrow m' = \frac{M}{R^3} x_1^3$$

If F is the gravitational force exerted by earth on particle at position x and ω be the angular velocity in time period T , then

$$F = \frac{Gm'm}{x_1^2} = \frac{Gm}{x_1^2} \cdot \frac{M}{R^3} x_1^3$$

$$\Rightarrow m\omega^2 x_1 = \frac{GMmx_1}{R^3}$$

$$\Rightarrow \omega = \sqrt{GM/R^3} \quad \dots(i)$$

$$\text{Since, } \omega = \frac{2\pi}{T} \text{ and } GM = gR^2$$

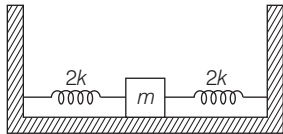
Substituting the above values in Eq. (i), we get

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{gR^2}{R^3}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{g}}$$

- 55** Two identical springs of spring constant $2k$ are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is

[2021, 25 Feb Shift-II]



- (a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{m}{k}}$
 (c) $\pi\sqrt{\frac{m}{k}}$ (d) $\pi\sqrt{\frac{m}{2k}}$

Ans. (c)

Let spring constants of two springs be k_1 and k_2 . Since, two springs are connected in parallel connection and parallel equivalent spring constant,

$$k_{eq} = k_1 + k_2$$

$$\Rightarrow k_{eq} = 2k + 2k = 4k$$

As, time period, $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{4k}} = \frac{2\pi}{2}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}$$

- 56** If the time period of a 2 m long simple pendulum is 2 s, the acceleration due to gravity at the place, where pendulum is executing SHM is

[2021, 25 Feb Shift-I]

- (a) $\pi^2 \text{ ms}^{-2}$ (b) 9.8 ms^{-2}
 (c) $2\pi^2 \text{ ms}^{-2}$ (d) 16 ms^{-2}

Ans. (c)

Given, length of simple pendulum, $l = 2 \text{ m}$

Time period, $T = 2 \text{ s}$

Let g_{eff} be the acceleration due to gravity.

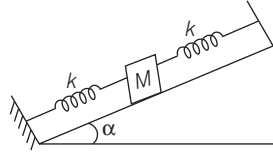
$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$\Rightarrow g_{eff} = 4\pi^2 \frac{l}{T^2}$$

$$= 4\pi^2 \cdot \frac{2}{4} = 2\pi^2 \text{ ms}^{-2}$$

- 57** In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If

each spring has spring constant k , then the frequency of oscillation of given body is [2021, 24 Feb Shift-II]



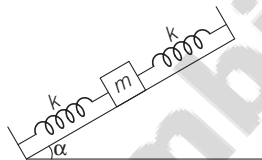
- (a) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg \sin \alpha}}$
 (c) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg \sin \alpha}}$

Ans. (c)

Let T be the time period of oscillation, then

$$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$$

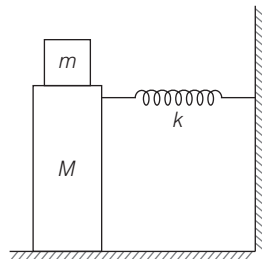
$$\therefore T = 2\pi\sqrt{\frac{M}{2k}} \quad [\because k_{eq} = k + k]$$



and frequency $(f) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

- 58** In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k . The mass oscillates on a frictionless surface with time period T and amplitude A . When the mass is in equilibrium position as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be

[2021, 24 Feb Shift-I]



- (a) $A\sqrt{\frac{M+m}{M}}$ (b) $A\sqrt{\frac{M}{M+m}}$
 (c) $A\sqrt{\frac{M-m}{M}}$ (d) $A\sqrt{\frac{M}{M-m}}$

Ans. (b)

Given, initial amplitude = A

Velocity at mean position, $v = A\omega$

Applying conservation of momentum at mean position, we get

$$M_1 v_1 = M_2 v_2$$

$$MA\omega = (M+m)v'$$

$$\Rightarrow v' = \frac{MA\omega}{M+m} = \frac{MA\sqrt{\frac{k}{M}}}{M+m}$$

$$\therefore v' = A'\omega' = A'\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow A' = \frac{MA\sqrt{\frac{k}{M}}}{M+m} \times \sqrt{\frac{M+m}{k}}$$

$$A' = \sqrt{\frac{M}{M+m}} A$$

- 59** A block of mass m attached to a massless spring is performing oscillatory motion of amplitude A on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system becomes fA . The value of f is [2020, 3 Sep Shift-II]

- (a) $\sqrt{2}$ (b) $\frac{1}{2}$
 (c) $\frac{1}{\sqrt{2}}$ (d) 1

Ans. (c)

In the equilibrium position, the velocity is maximum and it is equal to $v_{max} = \omega A$

where, $\omega = \sqrt{\frac{k}{m}}$ = angular frequency and

k = spring constant.

Now, kinetic energy in equilibrium position,

$$E = \frac{1}{2} m \omega^2 A^2 = \frac{1}{2} m \left(\frac{k}{m} \right) A^2 \quad \dots(i)$$

When half of the mass of the block breaks off, also the kinetic energy of the system will become half, i.e. $E/2$.

The angular frequency will become,

$$\omega' = \sqrt{\frac{k}{(m/2)}}$$

Now, the amplitude will become fA .

$\therefore$ The new kinetic energy,

$$E' = \frac{E}{2} = \frac{1}{2} \left(\frac{m}{2} \right) \omega'^2 (fA)^2$$

$$\Rightarrow \frac{E}{2} = \frac{1}{2} \left(\frac{m}{2} \right) \left(\frac{2k}{m} \right) (fA)^2$$

$$\Rightarrow \frac{1}{2} \left[\frac{1}{2} m \left(\frac{k}{m} \right) A^2 \right] = \frac{1}{2} \cdot \frac{m}{2} \cdot \frac{2k}{m} \cdot f^2 A^2$$

[Using Eq. (i)]

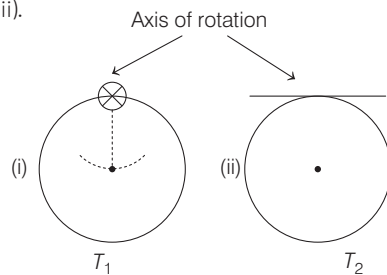
$$\Rightarrow 1 = 2f^2 \Rightarrow f = \frac{1}{\sqrt{2}}$$

Hence, correct option is (c).

- 60** A ring is hung on a nail. It can oscillate without slipping or sliding
- in its plane with a time period T_1
 - back and forth in a direction perpendicular to its plane, with a period T_2 .
- The ratio T_1/T_2 will be
- [2020, 5 Sep Shift-II]
- (a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

Ans. (a)

Let I_1 be the moment of inertia in case (i) and I_2 be the moment of inertia in case (ii).



The time period of physical pendulum is given by

$$T = 2\pi \sqrt{\frac{I}{mgd}} \quad (\text{where, } d = R)$$

$$\text{Here, } I_1 = 2mR^2 \Rightarrow I_2 = \frac{3}{2}mR^2$$

$$\therefore T_1 = 2\pi \sqrt{\frac{I_1}{mgd}} \text{ and } T_2 = 2\pi \sqrt{\frac{I_2}{mgd}}$$

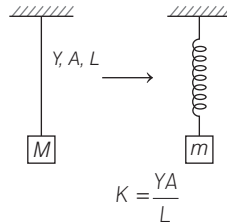
$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2mR^2}{\frac{3}{2}mR^2}} = \frac{2}{\sqrt{3}}$$

Hence, correct option is (a).

- 61** An object of mass m is suspended at the end of a massless wire of length L and area of cross-section A . Young modulus of the material of the wire is Y . If the mass is pulled down slightly its frequency of oscillation along the vertical direction is
- [2020, 6 Sep Shift-I]
- (a) $f = \frac{1}{2\pi} \sqrt{\frac{mL}{YA}}$ (b) $f = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$
- (c) $f = \frac{1}{2\pi} \sqrt{\frac{mA}{YL}}$ (d) $f = \frac{1}{2\pi} \sqrt{\frac{YL}{mA}}$

Ans. (b)

As we know that an elastic wire behaves as an elastic spring with spring constant K given by,



This block-wire system can be taken as spring block system with time period of small oscillation given by

$$T = 2\pi \sqrt{\frac{m}{K}} = 2\pi \sqrt{\frac{mL}{YA}}$$

$$\therefore \text{Frequency of oscillation, } f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$$

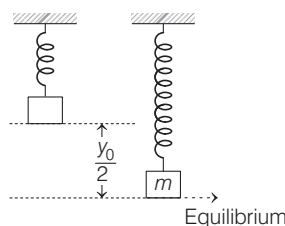
- 62** When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where y is measured from the lower end of unstretched spring. Then ω is
- [2020, 6 Sep Shift-II]
- (a) $\frac{1}{2} \sqrt{\frac{g}{y_0}}$ (b) $\sqrt{\frac{g}{y_0}}$ (c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{2g}{y_0}}$

Ans. (c)

Given that, displacement of particle at any instant

$$y(t) = y_0 \sin^2 \omega t, \quad y = y_0 \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$y - \frac{y_0}{2} = -\frac{y_0}{2} \cos 2\omega t \quad \text{or} \quad Y = A \cos 2\omega t$$



where, amplitude (displacement), $A = \frac{y_0}{2}$

and $2\omega = \sqrt{k/m}$... (i)

Maximum displacement, $\frac{y_0}{2} = \frac{mg}{k}$

(see figure)

or $k = \frac{2mg}{y_0}$... (ii)

Putting the value of k from Eq. (ii) in Eq. (i), we get

$$2\omega = \sqrt{\frac{2mg}{y_0} \times \frac{1}{m}}$$

or $\omega = \sqrt{\frac{g}{2y_0}}$

Hence, correct option is (c).

- 63** A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l . The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then, the stretch in the spring is
- [2020, 8 Jan Shift-I]

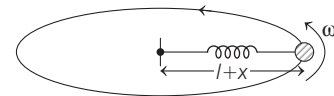
(a) $\frac{m\omega^2}{k + m\omega^2}$ (b) $\frac{m\omega^2}{k - m\omega^2}$

(c) $\frac{m\omega^2}{k - m\omega^2}$ (d) $\frac{m\omega^2}{k + m\omega^2}$

Ans. (c)

Initially, it is given that the unstretched length of the spring is l . When it is given an angular speed ω , then let x = stretched length of spring.

Then, total length of the spring system while rotating will be $(l + x)$ as shown in the figure.



As we know, spring force will give the necessary centripetal force for rotation.

So, $kx = m(l + x)\omega^2$

$$\Rightarrow (k - m\omega^2)x = m\omega^2 \Rightarrow x = \frac{m\omega^2}{(k - m\omega^2)}$$

- 64** A spring mass system (mass m , spring constant k and natural length l) rests in equilibrium on horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about its axis with an angular velocity ω , ($k \gg m\omega^2$), the relative change in the length of the spring is best given by the option
- [2020, 9 Jan Shift-II]

(a) $\frac{m\omega^2}{3k}$ (b) $\frac{2m\omega^2}{k}$

(c) $\sqrt{\frac{2}{3}} \left(\frac{m\omega^2}{k} \right)$ (d) $\frac{m\omega^2}{k}$

Ans. (d)

When disc and spring mass system rotates about central axis, spring force provides necessary centripetal pull for rotation of spring mass system.

Let extension in spring = x .

Then, total length of rotating spring = $l_0 + x$. Equating spring force and centripetal pull, we have

$$kx = m\omega^2(l_0 + x)$$

$$\Rightarrow \frac{x}{l_0} = \frac{m\omega^2}{k - m\omega^2} \Rightarrow \frac{x}{l_0} = \frac{m\omega^2}{k \left(1 - \frac{m\omega^2}{k}\right)}$$

As $k \gg m\omega^2$, so $1 - \frac{m\omega^2}{k} \approx 1$

Hence, $\frac{x}{l_0} = \frac{m\omega^2}{k}$

- 65** A simple pendulum oscillating in air has period T . The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material

of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is [2019, 9 April Shift-I]

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$ (c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

Ans. (d)

We know that,

Time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} \quad \dots(i)$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}} \quad \dots(ii)$

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad (\text{given})$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V \cdot \frac{\rho}{16} \cdot g \quad \dots(iii)$$

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff} , then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \dots(iv)$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g/16$$

$$\Rightarrow g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \dots(v)$$

Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

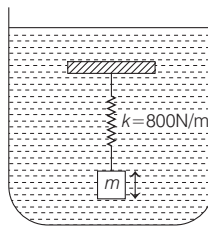
$$= \frac{4}{\sqrt{15}} \times T \quad [\text{using Eq. (ii)}]$$

- 66** A massless spring ($k = 800 \text{ N/m}$), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released, so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K , specific heat of water = 4184 J/kg K) [2019, 9 April Shift-II]

- (a) 10^{-4} K (b) 10^{-3} K
(c) 10^{-1} K (d) 10^{-5} K

Ans. (d)

The given situation is shown in the figure given below



When vibrations of mass are suddenly stopped, oscillation energy (or stored energy of spring) is dissipated as heat, causing rise of temperature.

So, conservation of energy gives

$$\frac{1}{2}kx_m^2 = (m_1s_1 + m_2s_2)\Delta T$$

where, x_m = amplitude of oscillation,

s_1 = specific heat of mass,

s_2 = specific heat of water

and ΔT = rise in temperature.

Substituting values given in question, we have

$$\frac{1}{2} \times 800 \times (2 \times 10^{-2})^2$$

$$= \left(\left(\frac{500}{1000} \right) \times 400 + 1 \times 4184 \right) \Delta T$$

$$\Rightarrow \Delta T = \frac{16 \times 10^{-2}}{4384}$$

$$= 3.65 \times 10^{-5} \text{ K}$$

So, the order of magnitude of change in temperature is approx 10^{-5} .

- 67** A spring whose unstretched length is l has a force constant k . The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants k_1 and k_2 will be [2019, 12 April Shift-II]

- (a) n (b) $1/n^2$ (c) $1/n$ (d) n^2

Ans. (c)

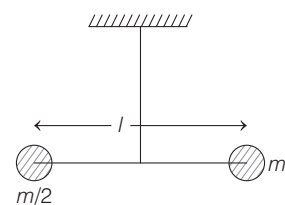
If parameters like material, number of loops per unit length, area of cross-section, etc., are kept same, then force constant of spring is inversely proportional to its length.

In given case, all other parameters are same for both parts of spring.

So, $k_1 \propto \frac{1}{l_1}$ and $k_2 \propto \frac{1}{l_2} \therefore \frac{k_1}{k_2} = \frac{l_2}{l_1}$

$$= \frac{l_2}{nl_2} = \frac{1}{n} \quad [\because l_1 = nl_2]$$

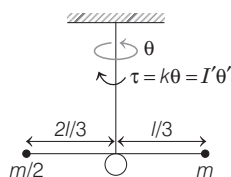
- 68** Two masses m and $m/2$ are connected at the two ends of a massless rigid rod of length l . The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k , the restoring torque is $\tau = k\theta$ for angular displacement θ . If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be [2019, 9 Jan Shift-I]



- (a) $\frac{2k\theta_0^2}{l}$ (b) $\frac{k\theta_0^2}{l}$ (c) $\frac{3k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

Ans. (b)

Since in the given question, rotational torque, $\tau \propto$ angular displacement.

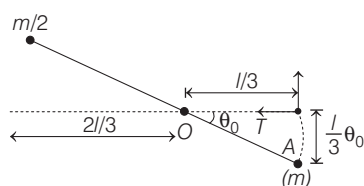


Thus, when it will be released, the system will execute SHM with a time period, $T = 2\pi\sqrt{I/k}$

(Where I is moment of inertia and k is torsional constant)

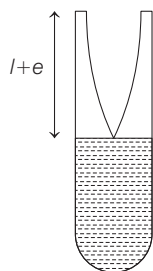
and the angular frequency is given as, $\omega = \sqrt{k/I}$.

If we know look at the top view of the above figure, we have



At some angular displacement θ_0 , at point 'A' the maximum velocity will be

$$v_{\max} = \frac{L}{3} \theta_0 \omega = \frac{L}{3} \theta_0 \sqrt{\frac{k}{I}} \quad \dots(i)$$



Then, tension in the rod when it passes through mean position will be

$$T = \frac{m \times v_{\max}^2}{\frac{L}{3}} = \frac{m l^2 \theta_0^2 k \times 3}{9 \times L \times I} \quad [\text{using Eq. (i)}]$$

$$= \frac{m l \theta_0^2 k}{3I}$$

The moment of inertia I at point O,

$$= \frac{m}{2} \left(\frac{2L}{3} \right)^2 + m \left(\frac{L}{3} \right)^2 = \frac{2L^2 m}{9} + \frac{mL^2}{9}$$

$$= \frac{3mL^2}{9} = \frac{mL^2}{3}$$

$$\Rightarrow T = \frac{m l \theta_0^2 k \times 3}{3 \times mL^2} = \frac{\theta_0^2 k}{L} = \frac{k \theta_0^2}{L}$$

69 A rod of mass ' M ' and length ' $2L$ ' is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of ' m ' are attached at distance ' $L/2$ ' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to

[2019, 9 Jan Shift-II]

- (a) 0.57 (b) 0.37 (c) 0.77 (d) 0.17

Ans. (b)

We know that in case of torsional

oscillation frequency $\nu = \frac{k}{\sqrt{I}}$

where, I is moment of inertia and k is torsional constant.

$\therefore$ According to question, $\nu_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$

(As, moment of inertia of a bar is $I = \frac{ML^2}{12}$)

$$\text{or } \nu_1 = \frac{k}{\sqrt{\frac{ML^2}{3}}} \quad \dots(i)$$

When two masses are attached at ends of rod. Then its moment of inertia is

$$\frac{M(2L)^2}{12} + 2m \left(\frac{L}{2} \right)^2$$

So, new frequency of oscillations is,

$$\nu_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m \left(\frac{L}{2} \right)^2}}$$

$$\nu_2 = \frac{k}{\sqrt{\frac{ML^2}{3} + \frac{mL^2}{2}}} \quad \dots(ii)$$

As, $\nu_2 = 80\% \text{ of } \nu_1 = 0.8 \nu_1$

So,

$$\frac{k}{\sqrt{\frac{ML^2}{3} + \frac{mL^2}{2}}} = \frac{0.8 \times k}{\sqrt{\frac{ML^2}{3}}}$$

After solving it, we get,

$$\frac{m}{M} = 0.37$$

70 A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω . If the

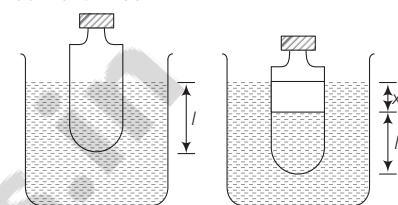
radius of the bottle is 2.5 cm, then ω is close to (Take, density of water $= 10^3 \text{ kg/m}^3$)

[2019, 10 Jan Shift-II]

- (a) 2.50 rad s^{-1} (b) 5.00 rad s^{-1}
(c) 1.25 rad s^{-1} (d) 3.75 rad s^{-1}

Ans. (*)

In equilibrium condition bottle floats in water and its length ' l ' inside water is same as the height of water upto which bottle is filled.



$$\text{So, } l = \frac{\text{Volume of water in bottle}}{\text{Area}} = \frac{310}{\pi \times (2.5)^2} = 15.8 \text{ cm} = 0.158 \text{ m}$$

When bottle is slightly pushed inside by an amount x then, restoring force acting on the bottle is the upthrust of fluid displaced when bottle goes into liquid by amount x .

So, restoring force is;

$$F = -(\rho A x) g \quad \dots(i)$$

where ρ = density of water,

A = area of cross-section of bottle and

x = displacement from equilibrium position

But $F = ma$ $\dots(ii)$

where, m = mass of water and bottle system $= A l \rho$

From (i) and (ii) we have,

$$A l \rho a = -\rho A x g \text{ or } a = -\frac{g}{l} x$$

As for SHM, $a = -\omega^2 x$

$$\text{We have } \omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.158}}$$

$$= \sqrt{63.29}$$

$$\approx 8 \text{ rad s}^{-1}$$

$\therefore$ No option is correct.

71 The mass and the diameter of a planet are three times the respective values for the earth. The period of oscillation of a simple pendulum on the earth is 2 s. The period of oscillation of the same pendulum on the planet would be

[2019, 11 Jan Shift-II]

- (a) $\frac{2}{\sqrt{3}}$ s (b) $\frac{3}{2}$ s
 (c) $2\sqrt{3}$ s (d) $\frac{\sqrt{3}}{2}$ s

Ans. (c)

Period of motion of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \dots(i)$$

On the surface of earth, let period of motion is T_e and acceleration due to gravity is g_e

$$\therefore T_e = 2\pi\sqrt{\frac{l}{g_e}} \quad \dots(ii)$$

On the another planet, let period of motion is T_p and gravitational acceleration is g_p

$$\therefore T_p = 2\pi\sqrt{\frac{l}{g_p}} \quad \dots(iii)$$

($\because$ Pendulum is same, so l will be same)

From Eqs. (ii) and (iii), we get

$$\frac{T_e}{T_p} = \frac{2\pi\sqrt{\frac{l}{g_e}}}{2\pi\sqrt{\frac{l}{g_p}}} = \sqrt{\frac{g_p}{g_e}} \quad \dots(iv)$$

$$\text{Now, } g_e = \frac{GM_e}{R_e^2} \text{ and } g_p = \frac{GM_p}{R_p^2}$$

$$\text{Given, } M_p = 3M_e \text{ and } R_p = 3R_e$$

$$\therefore g_p = \frac{G \times 3M_e}{9R_e^2} = \frac{1}{3} \cdot \frac{GM_e}{R_e^2} = \frac{1}{3} g_e$$

$$\Rightarrow \frac{g_p}{g_e} = \frac{1}{3} \text{ or } \sqrt{\frac{g_p}{g_e}} = \frac{1}{\sqrt{3}} \quad \dots(v)$$

$$\text{From Eqs. (iv) and (v), } T_p = \sqrt{3} T_e$$

$$\text{or } T_p = 2\sqrt{3} \text{ s } (\because T_e = 2 \text{ s})$$

- 72** A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

[2019, 11 Jan Shift-II]

- (a) 1 rad/s (b) 10^{-5} rad/s
 (c) 10^{-3} rad/s (d) 10^{-1} rad/s

Ans. (c)

We know that time period of a pendulum is given by $T = 2\pi\sqrt{l/g}$

$$\text{So, angular frequency } \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}} \quad \dots(i)$$

Now, differentiate both side w.r.t g

$$\therefore \frac{d\omega}{dg} = \frac{1}{2\sqrt{g}l} \quad \dots(ii)$$

$$d\omega = \frac{dg}{2\sqrt{g}l} \quad \dots(ii)$$

By dividing Eq. (ii) by Eq. (i), we get

$$d\omega/\omega = dg/2g$$

Or we can write

$$\frac{\Delta\omega}{\omega} = \frac{\Delta g}{2g} \quad \dots(iii)$$

As Δg is due to oscillation of support.

$$\therefore \Delta g = 2\omega^2 A \quad (\omega_1 \rightarrow 1 \text{ rad/s, support})$$

Putting value of Δg in Eq. (iii) we get

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \cdot \frac{2\omega_1^2 A}{g} = \frac{\omega_1^2 A}{g}; (A = 10^{-2} \text{ m}^2)$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{1 \times 10^{-2}}{10} = 10^{-3} \text{ rad/s}$$

- 73** A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1 . If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2 . Then

[2019, 11 Jan Shift-II]

- (a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$
 (c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

Ans. (b)

Kinetic energy of a pendulum is maximum at its mean position. Also, maximum kinetic energy of pendulum

$$K_{\max} = \frac{1}{2} m \omega^2 a^2$$

where, angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}} \text{ or } \omega = \sqrt{\frac{g}{l}} \text{ or } \omega^2 = \frac{g}{l}$$

and a = amplitude.

As amplitude is same in both cases so;

$$K_{\max} \propto \omega^2$$

$$\text{or } K_{\max} \propto \frac{1}{l} \quad [\because g \text{ is constant}]$$

According to given data, $K_1 \propto \frac{1}{l}$ and

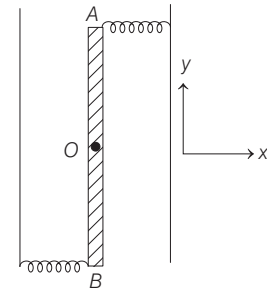
$$K_2 \propto \frac{1}{2l}$$

$$\therefore \frac{K_1}{K_2} = \left(\frac{1/l}{1/2l} \right) = 2$$

$$\text{or } K_1 = 2K_2$$

$$\Rightarrow K_2 = \frac{K_1}{2}$$

- 74** Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m . The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure.



The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is

[2019, 12 Jan Shift-I]

- (a) $\frac{1}{2\pi} \sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{3k}{m}}$
 (c) $\frac{1}{2\pi} \sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Ans. (c)

When a system oscillates, the magnitude of restoring torque of system is given by

$$\tau = C\theta \quad \dots(i)$$

where, C = constant that depends on system.

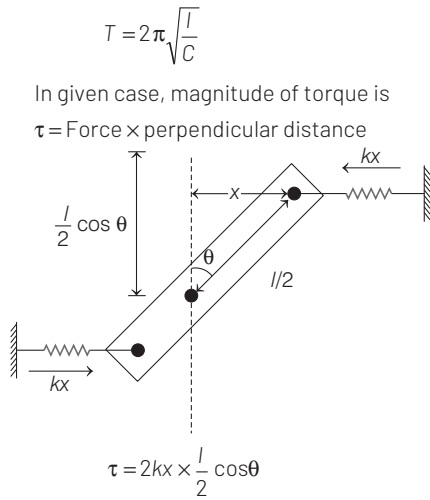
$$\text{Also, } \tau = I\alpha \quad \dots(ii)$$

where, I = moment of inertia and α = angular acceleration

From Eqs. (i) and (ii),

$$\alpha = \frac{C}{I} \theta \quad \dots(iii)$$

and time period of oscillation of system will be



For small deflection, $\tau = \left(\frac{kl^2}{2}\right)\theta$... (iv)

$\therefore$ For small deflections, $\sin\theta = \frac{x}{(l/2)} \approx \theta$

$\Rightarrow x = \frac{l\theta}{2}$

Also, $\cos\theta \approx 1$
 comparing Eqs. (iv) and (i), we get

$C = \frac{kl^2}{2}$

$\Rightarrow \alpha = \frac{(kl^2/2)}{\left(\frac{1}{12}ml^2\right)} \cdot \theta$

$\Rightarrow \alpha = \frac{6k}{m} \cdot \theta$

Hence, time period of oscillation is

$T = 2\pi\sqrt{\frac{m}{6k}}$

Frequency of oscillation is given by

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

- 75** A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number = 6.02×10^{23} g mol⁻¹) [JEE Main 2018]

- (a) 6.4 N/m
 (b) 7.1 N/m
 (c) 2.2 N/m
 (d) 5.5 N/m

Ans. (b)

For a harmonic oscillator,

$T = 2\pi\sqrt{\frac{m}{k}}$

where, k = force constant and $T = \frac{1}{f}$

$\therefore k = 4\pi^2 f^2 m$

$= 4 \times \left(\frac{22}{7}\right)^2 \times (10^{12})^2 \times \frac{108 \times 10^{-3}}{6.02 \times 10^{23}}$

$\Rightarrow k = 7.1 \text{ N/m}$

- 76** A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s in a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion (α) of the metal of the pendulum shaft are, respectively [JEE Main 2016]

- (a) 25°C, $\alpha = 1.85 \times 10^{-5} / ^\circ\text{C}$
 (b) 60°C, $\alpha = 1.85 \times 10^{-4} / ^\circ\text{C}$
 (c) 30°C, $\alpha = 1.85 \times 10^{-3} / ^\circ\text{C}$
 (d) 55°C, $\alpha = 1.85 \times 10^{-2} / ^\circ\text{C}$

Ans. (d)

Key Idea Time period of a pendulum,

$T = 2\pi\sqrt{\frac{l}{g}}$

where l is length of pendulum and g is acceleration due to gravity.

Such as change in time period of a pendulum,

$\frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l}$

When clock gains 12 s, we get

$\frac{12}{T} = \frac{1}{2} \alpha (40 - \theta)$... (i)

When clock loses 4 s, we get

$\frac{4}{T} = \frac{1}{2} \alpha (\theta - 20)$... (ii)

Comparing Eqs. (i) and (ii), we get

$3 = \frac{40 - \theta}{\theta - 20}$

$\Rightarrow 3\theta - 60 = 40 - \theta \Rightarrow 4\theta = 100 \Rightarrow \theta = 25^\circ\text{C}$

Substituting the value of θ in Eq. (i), we have

$\frac{12}{T} = \frac{1}{2} \alpha (40 - 25)$

$\Rightarrow \frac{12}{24 \times 3600} = \frac{1}{2} \alpha (15)$

$\Rightarrow \alpha = \frac{24}{24 \times 3600 \times 15}$

$\Rightarrow \alpha = 1.85 \times 10^{-5} / ^\circ\text{C}$

Thus, the coefficient of linear expansion in a pendulum clock = $1.85 \times 10^{-5} / ^\circ\text{C}$

- 77** A pendulum made of a uniform wire of cross-sectional area A has time period T . When an additional mass M is added to its bob, the time period changes T_M . If the Young's modulus of the material of the wire is Y , then $\frac{1}{Y}$ is equal to

(g = gravitational acceleration)

[JEE Main 2015]

(a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{Mg}{A}$
 (c) $\left[1 - \left(\frac{T_M}{T}\right)^2\right] \frac{A}{Mg}$ (d) $\left[1 - \left(\frac{T}{T_M}\right)^2\right] \frac{A}{Mg}$

Ans. (a)

We know that time period, $T = 2\pi\sqrt{\frac{L}{g}}$
 When additional mass M is added to its bob

$T_M = 2\pi\sqrt{\frac{L + \Delta L}{g}}$

where, ΔL is increase in length.

We know that

$Y = \frac{Mg/A}{\Delta L/L} = \frac{MgL}{A\Delta L} \Rightarrow \Delta L = \frac{MgL}{AY}$

$\Rightarrow \therefore T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}}$

$\Rightarrow \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$ or $\frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$

or $\frac{1}{Y} = \frac{A}{Mg} \left[\left(\frac{T_M}{T}\right)^2 - 1\right]$

- 78** The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured

value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is [JEE Main 2015]

- (a) 2% (b) 3% (c) 1% (d) 5%

Ans. (b)**Key Idea** Given time period

$$T = 2\pi \sqrt{L/g}$$

Thus, changes can be expressed as

$$\pm \frac{2\Delta T}{T} = \pm \frac{\Delta L}{L} \pm \frac{\Delta g}{g}$$

According to the question, we can write

$$\frac{\Delta L}{L} = \frac{0.1 \text{ cm}}{20.0 \text{ cm}} = \frac{1}{200}$$

Again time period

$$T = \frac{90}{100} \text{ s and } \Delta T = \frac{1}{100} \text{ s}$$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{90}$$

Now,

$$\therefore T = 2\pi \sqrt{\frac{L}{g}}$$

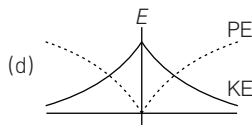
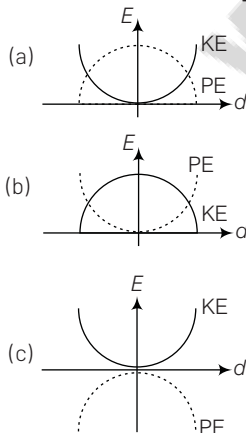
$$\therefore g = 4\pi^2 \frac{L}{T^2}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$$\begin{aligned} \text{or } \frac{\Delta g}{g} \times 100\% &= \left(\frac{\Delta L}{L} \right) \times 100\% \\ &\quad + \left(\frac{2\Delta T}{T} \right) \times 100\% \\ &= \left(\frac{1}{200} \times 100 \right) \% + 2 \times \frac{1}{90} \times 100\% \\ &= 2.72\% \approx 3\% \end{aligned}$$

Thus, accuracy in the determination of g is approx 3%.

- 79** For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement (d) Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

[JEE Main 2015]**Ans. (b)**

During oscillation, motion of a simple pendulum KE is maximum of mean position where PE is minimum. At extreme position, KE is minimum and PE is maximum. Thus, correct graph is depicted in option (b).

- 80** An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and the cylinder have equal cross-sectional area A . When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency

[JEE Main 2013]

$$\begin{aligned} \text{(a)} \quad & \frac{1}{2\pi} \frac{A_\gamma p_0}{V_0 M} & \text{(b)} \quad & \frac{1}{2\pi} \frac{V_0 M p_0}{A^2 \gamma} \\ \text{(c)} \quad & \frac{1}{2\pi} \sqrt{\frac{A^2 \gamma p_0}{M V_0}} & \text{(d)} \quad & \frac{1}{2\pi} \sqrt{\frac{M V_0}{A_\gamma p_0}} \end{aligned}$$

Ans. (c)

Pressure applied by piston,

$$\frac{Mg}{A} = p_0$$

$$Mg = p_0 A$$

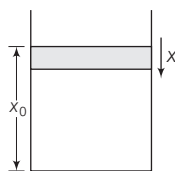
...(i)

As no exchange of heat, so process is adiabatic.

$$p_0 V_0^\gamma = p V^\gamma,$$

$$p_0 A x_0^\gamma = p A (x_0 - x)^\gamma$$

$$p = \frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma}$$

Let piston be displaced by x .

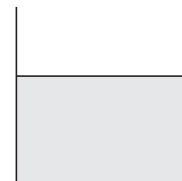
$$Mg - \left(\frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma} \right) A = f_{\text{restoring}}$$

$$\begin{aligned} p_0 A \left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma} \right) &= F_{\text{restoring}} \\ [\because x_0 - x \approx x_0] \\ F &= - \frac{\gamma p_0 A x}{x_0} \\ \therefore f &= \frac{1}{2\pi} \sqrt{\frac{\gamma p_0 A}{x_0 M}} = \frac{1}{2\pi} \sqrt{\frac{\gamma p_0 A^2}{M V_0}} \end{aligned}$$

- 81** A wooden cube (density of wood d) of side l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period, T . Then, T is equal to

[AIIEE 2011]

$$\begin{aligned} \text{(a)} \quad & 2\pi \sqrt{\frac{l\rho}{(\rho-d)g}} & \text{(b)} \quad & 2\pi \sqrt{\frac{ld}{\rho g}} \\ \text{(c)} \quad & 2\pi \sqrt{\frac{l\rho}{dg}} & \text{(d)} \quad & 2\pi \sqrt{\frac{ld}{(\rho-d)g}} \end{aligned}$$

Ans. (b)Let at any instant, cube be at a depth x from the equilibrium position, thenNet force acting on the cube = Upthrust on the portion of length x 

$$\therefore F = -\rho l^2 x g = -\rho l^2 g x \quad \dots(i)$$

$$F \propto -x \text{ (SHM)}$$

Negative sign shows that force is opposite to x . Hence, equation of SHM

$$F = -kx \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$k = \rho l^2 g$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi \sqrt{\frac{ld}{\rho g}}$$

- 82** If a spring of stiffness k is cut into two parts A and B of length $l_A : l_B = 2:3$, then the stiffness of spring A is given by

[AIIEE 2011]

$$\begin{aligned} \text{(a)} \quad & \frac{5}{2} k & \text{(b)} \quad & \frac{3}{5} k \\ \text{(c)} \quad & \frac{2}{5} k & \text{(d)} \quad & k \end{aligned}$$

Ans. (a)

$$\text{As } F = -kx \Rightarrow k \propto \frac{1}{x} \propto \frac{1}{l}$$

For spring, $k \propto \frac{1}{l}$

$$\therefore \frac{k_A}{k} = \frac{l}{l_A} \Rightarrow k_A = \frac{l_A + l_B}{l_A} k = \frac{5}{2} k$$

- 83** A mass M , attached to a horizontal spring, executes SHM with amplitude A_1 . When the mass M passes through its mean position, then a smaller mass m is placed over it and both of them move together with amplitude A_2 . The

ratio of $\left(\frac{A_1}{A_2}\right)$ is

[AIEEE 2011]

- (a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{1/2}$
 (c) $\left(\frac{M+m}{M}\right)^{1/2}$ (d) $\frac{M}{M+m}$

Ans. (c)

At mean position, $F_{\text{net}} = 0$

$\therefore$ By conservation of linear momentum,

$$Mv_1 = (M+m)v_2$$

$$M\omega_1 A_1 = (M+m)\omega_2 A_2$$

But angular velocity,

$$\omega_1 = \sqrt{\frac{k}{M}} \text{ and } \omega_2 = \sqrt{\frac{k}{M+m}}$$

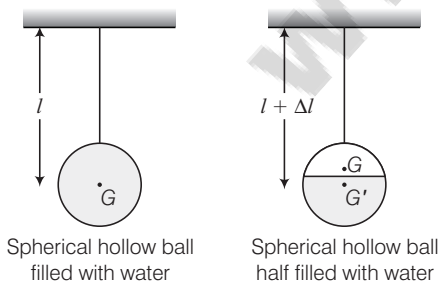
$$\text{On solving, } \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$$

- 84** The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

[AIEEE 2005]

- (a) first increase and then decrease to the original value
 (b) first decrease and then increase to the original value
 (c) remain unchanged
 (d) increase towards a saturation value

Ans. (a)



$$T = 2\pi \sqrt{\frac{l}{g}}$$

As water is coming out, CG of the system goes down, effective length increase.

$$T_1 = 2\pi \sqrt{\frac{l + \Delta l}{g}}, T_1 > T_2$$

So, time period also increases but upto half empty. As water is coming out after half empty, CG goes up.

Effective length decreases and for complete hollow sphere, CG is also at the initial position, so effective length as well as time period decreases and same as initial at last.

$$T_2 = 2\pi \sqrt{\frac{l - \Delta l}{g}} \text{ and } T_1 < T_2$$

Hence, time period first increases and then decreases to the original value.

- 85** The bob of a simple pendulum executes simple harmonic motion in water with a period t , while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000 \text{ kg m}^{-3}$. What relationship between t and t_0 is true?

[AIEEE 2004]

- (a) $t = t_0$ (b) $t = \frac{t_0}{2}$
 (c) $t = 2t_0$ (d) $t = 4t_0$

Ans. (c)

The time period of simple pendulum in air,

$$T = t_0 = 2\pi \sqrt{\frac{l}{g}} \quad \dots(i)$$

Here, l being the length of simple pendulum,

In water, effective weight of bob,

$$w' = \text{Weight of bob in air} - \text{Upthrust}$$

$$\Rightarrow \rho V g_{\text{eff}} = mg - m'g = \rho V g - \rho' V g$$

$$= (\rho - \rho') V g$$

where, $\rho' = \text{density of water,}$

$\rho = \text{density of bob.}$

$$\therefore g_{\text{eff}} = \left(\frac{\rho - \rho'}{\rho}\right) g = \left(1 - \frac{\rho'}{\rho}\right) g$$

$$\therefore t = 2\pi \sqrt{\frac{l}{\left(1 - \frac{\rho'}{\rho}\right) g}} \quad \dots(ii)$$

$$\text{Thus, } \frac{t}{t_0} = \sqrt{\frac{1}{\left(1 - \frac{\rho'}{\rho}\right)}}$$

$$\begin{aligned} &= \sqrt{\frac{1}{1 - \frac{1000}{(4/3) \times 1000}}} \\ &= \sqrt{\frac{4}{4-3}} = 2 \text{ or } t = 2t_0 \end{aligned}$$

- 86** A particle at the end of a spring executes simple harmonic motion with a period t_1 , while the corresponding period for another spring is t_2 . If the period of oscillation with the two springs in series is T , then

[AIEEE 2004]

- (a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
 (c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

Ans. (b)

Time period of spring,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Here, k being the force constant of spring.

For first spring,

$$t_1 = 2\pi \sqrt{\frac{m}{k_1}} \quad \dots(i)$$

For second spring,

$$t_2 = 2\pi \sqrt{\frac{m}{k_2}} \quad \dots(ii)$$

In series, f is same.

$\therefore$

$$\frac{-F}{k} = \frac{-F}{k_1} - \frac{F}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

So, the effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Time period of combination,

$$T = 2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

$$\Rightarrow T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \quad \dots(iii)$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2 m \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$\text{or } t_1^2 + t_2^2 = 4\pi^2 m \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$\text{or } t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow t_1^2 + t_2^2 = T^2 \quad [\text{from Eq. (iii)}]$$

- 87** A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0 . An external force $F(t)$ proportional to $\cos \omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to

[AIEEE 2004]

- (a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
 (c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

Ans. (b)

Initial angular velocity of particle = ω_0
 and at any instant t , angular velocity = ω

Therefore, for a displacement x , the resultant acceleration

$$f = (\omega_0^2 - \omega^2)x \quad \dots(i)$$

$$\text{External force, } F = m(\omega_0^2 - \omega^2)x \quad \dots(ii)$$

$$\text{Since, } F \propto \cos \omega t \quad [\text{given}]$$

From Eq. (ii), we get

$$m(\omega_0^2 - \omega^2)x \propto \cos \omega t \quad \dots(iii)$$

Now, from equation of simple harmonic motion,

$$x = A \sin(\omega t + \phi) \quad \dots(iv)$$

$$\text{At } t = 0; x = A$$

$$\therefore A = A \sin(0 + \phi)$$

$$\Rightarrow \phi = \frac{\pi}{2}$$

$$\therefore x = A \sin\left(\omega t + \frac{\pi}{2}\right) = A \cos \omega t \quad \dots(v)$$

Hence, from Eqs. (iii) and (v), we get

$$m(\omega_0^2 - \omega^2)A \cos \omega t \propto \cos \omega t$$

$$\Rightarrow A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

- 88** A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T . If the mass is increased by m , the time period becomes $5T/3$, then the ratio of m/M is

[AIEEE 2003]

- (a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

Ans. (c)

$$\text{As time period, } T = 2\pi \sqrt{\frac{M}{k}} \quad \dots(i)$$

$$T' = 2\pi \sqrt{\frac{M+m}{k}}$$

$$\Rightarrow \frac{5T}{3} = 2\pi \sqrt{\frac{M+m}{k}} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{3}{5} = \sqrt{\frac{M}{M+m}} \quad \text{or} \quad \frac{9}{25} = \frac{M}{M+m}$$

$$\text{or } 9M + 9m = 25M \quad \text{or } 16M = 9m$$

$$\text{or } \frac{m}{M} = \frac{16}{9}$$

- 89** Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2 , respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is [AIEEE 2003]

- (a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_1}{k_2}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_2}{k_1}$

Ans. (c)

$$v_{\max} = a\omega = a \frac{2\pi}{T} = \frac{2\pi a}{2\pi \sqrt{\frac{m}{k}}} = a \sqrt{\frac{k}{m}}$$

Hence,

$$\frac{v_{\max 1}}{v_{\max 2}} = \frac{a_1}{a_2} \sqrt{\frac{k_1}{k_2}}$$

$\therefore$

$$v_{\max 1} = v_{\max 2} \quad [\text{given}]$$

$\Rightarrow$

$$\frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

- 90** The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is [AIEEE 2003]

- (a) 11% (b) 21% (c) 42% (d) 10%

Ans. (d)

$$T = 2\pi \sqrt{\frac{l}{g}}, T^2 = 4\pi^2 \frac{l}{g}, l = \left(\frac{g}{4\pi^2}\right) T^2$$

$$\therefore \% \text{ Change} = x + y + \frac{xy}{100}$$

Valid only for two variables in terms of percentage.

$$x \rightarrow \% \text{ change in first variable}$$

$$y \rightarrow \% \text{ change in second variable}$$

$$\% \text{ Increase in length} = x + x + \frac{x^2}{100}$$

$$21 = 2x + \frac{x^2}{100}$$

On solving, $x = 10\%$

[by cross check method]

- 91** A child swinging on a swing in sitting position, stands up, then the time period of the swing will

[AIEEE 2002]

- (a) increase
 (b) decrease
 (c) remain same
 (d) increase, if the child is long and decrease, if the child is short

Ans. (b)

As the child stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to $T = 2\pi \sqrt{l/g}$, T will decrease.

- 92** If a spring has time period T and is cut into n equal parts, then the time period of each part will be

[AIEEE 2002]

- (a) $T\sqrt{n}$ (b) $T/\sqrt{n}$
 (c) nT (d) T

Ans. (b)

As we know that spring constant of spring is inversely proportional to length of spring, so new spring constant for each part is given by $k' = nk$, where nk is the spring constant of whole spring.

From the theory of spring pendulum, we know that time period of spring pendulum is inversely proportional to square root of spring constant i.e.,

$$T \propto \frac{1}{\sqrt{k}} \quad \text{and} \quad T' \propto \frac{1}{\sqrt{nk}}$$

$$\text{So, } T' = \frac{T}{\sqrt{n}}$$

TOPIC 3

Forced, Damped Oscillations and Resonance

- 93** A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 min, the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (Take, $\ln 2 = 0.693$)

[2021, 17 March Shift-II]

- (a) 0.69×10^2 kg/s (b) 3.3×10^2 kg/s
 (c) 1.16×10^{-2} kg/s (d) 5.7×10^{-3} kg/s

Ans. (c)

Given, mass block, $m = 1$ kg

Initial amplitude, $A_0 = 12$ cm

Final amplitude, $A = 6$ cm

The time taken to reduce the amplitude, $t = 2$ min = 120 s

Using the expression of damped

oscillation, $A = A_0 e^{-\frac{b}{2m}t}$

Substituting the values in the above equation, we get

$$6 = 12e^{-\frac{b(120)}{2(1)}} \Rightarrow e^{60b} = 2$$

or $60b = 10 \log 2$

$$\Rightarrow b = 0.693/60 = 1.16 \times 10^{-2} \text{ kg/s}$$

Hence, the value of damping constant is $1.16 \times 10^{-2} \text{ kg/s}$.

- 94** A particle is projected with velocity v_0 along X-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin, i.e. $ma = -\alpha x^2$. The distance at which the particle stops is

[2021, 24 Feb Shift-II]

$$(a) \left(\frac{3mv_0^2}{2\alpha} \right)^{1/2} \quad (b) \left(\frac{2mv_0}{3\alpha} \right)^{1/3}$$

$$(c) \left(\frac{2mv_0^2}{3\alpha} \right)^{1/2} \quad (d) \left(\frac{3mv_0^2}{2\alpha} \right)^{1/3}$$

Ans. (d)

Given, speed of projection = v_0

Damping force, $F = ma = -\alpha x^2$

$$\Rightarrow a = -\alpha x^2 / m$$

$$\text{Also, } a = v \frac{dv}{dx} \Rightarrow v dv = a dx = -\frac{\alpha}{m} x^2 dx$$

Integrating both sides, we get

$$\int_{v_0}^0 v dv = \int_0^x -\frac{\alpha}{m} x^2 dx$$

$$\Rightarrow (v^2/2)_{v_0}^0 = -\frac{\alpha}{m} (x^3/3)_{0}^x$$

$$\Rightarrow 0 - v_0^2/2 = -\frac{\alpha}{m} \frac{x^3}{3} \Rightarrow x = \left(\frac{3m v_0^2}{2\alpha} \right)^{1/3}$$

- 95** A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to 1/1000 of the original amplitude is close to

[2019, 8 April Shift-II]

$$(a) 20 \text{ s} \quad (b) 50 \text{ s} \quad (c) 100 \text{ s} \quad (d) 10 \text{ s}$$

Ans. (a)

Given, frequency of oscillations is

$$f = 5 \text{ osc s}^{-1}$$

$\Rightarrow$ Time period of oscillations is

$$T = \frac{1}{f} = \frac{1}{5} \text{ s}$$

So, time for 10 oscillations is $= 10/5 = 2 \text{ s}$

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped

oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$$\therefore \text{After } 2 \text{ s, } \frac{A_0}{2} = A_0 e^{-\gamma(2)} \Rightarrow 2 = e^{2\gamma}$$

$$\Rightarrow \gamma = \frac{\log 2}{2} \quad \dots(i)$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, i.e. $\frac{A_0}{1000} = A_0 e^{-\gamma t}$

$$\Rightarrow \log(1000) = \gamma t \Rightarrow \log(10^3) = \gamma t$$

$$3 \log 10 = \gamma t$$

$$\Rightarrow t = \frac{2 \times 3 \log 10}{\log 2} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow t = 19.93 \text{ s} \quad \text{or } t \approx 20 \text{ s}$$

- 96** The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t} \cos(10\pi t + \phi)$. Here, t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to

[2019, 10 April Shift-I]

$$(a) 27 \text{ s} \quad (b) 13 \text{ s} \quad (c) 4 \text{ s} \quad (d) 7 \text{ s}$$

Ans. (d)

Given, displacement is

$$x(t) = e^{-0.1t} \cos(10\pi t + \phi)$$

Here, amplitude of the oscillator is

$$A = e^{-0.1t} \quad \dots(i)$$

Let it takes t seconds for amplitude to be dropped by half.

$$\text{At } t=0 \Rightarrow A = 1 \quad [\text{from Eq. (i)}]$$

$$\text{At } t=t \Rightarrow A' = A/2 = 1/2$$

So, Eq. (i) can be written as

$$e^{-0.1t} = 1/2$$

$$\text{or } e^{0.1t} = 2 \text{ or } 0.1t = \ln(2)$$

$$\text{or } t = \frac{1}{0.1} \ln(2) = 10 \ln(2)$$

$$\text{Now, } \ln(2) = 0.693$$

$$\therefore t = 10 \times 0.693 = 6.93 \text{ s} \quad \text{or } t \approx 7 \text{ s}$$

- 97** If a simple pendulum has significant amplitude (upto a factor of $1/e$ of original) only in the period between $t=0$ s to $t=\tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum (assuming damping is small) in seconds is

[AIEEE 2012]

$$(a) \frac{0.693}{b} \quad (b) b \quad (c) \frac{1}{b} \quad (d) \frac{2}{b}$$

Ans. (d)

For damped harmonic motion, $ma = -kx - mbv$ or $ma + mbv + kx = 0$

Solution of above equation is

$$x = A_0 e^{-\frac{bt}{2}} \sin \omega t; \quad \text{with } \omega^2 = \frac{k}{m} - \frac{b^2}{4}$$

where, amplitude drops exponentially with time.

$$\text{i.e., } A_\tau = A_0 e^{-\frac{b\tau}{2}}$$

Average time τ is that duration when amplitude drops by 63%, i.e., becomes A_0/e .

$$\text{Thus, } A_\tau = A_0/e = A_0 e^{-\frac{b\tau}{2}}$$

$$\text{or } b\tau/2 = 1 \quad \text{or } \tau = 2/b$$

- 98** The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s, it will decrease to α times its original magnitude, where α equals

[JEE Main 2013]

$$(a) 0.7 \quad (b) 0.81 \quad (c) 0.729 \quad (d) 0.6$$

Ans. (c)

Amplitude of damped oscillator is given

$$\text{by } A = A_0 e^{-\frac{bt}{2m}}$$

$$\text{After } 5 \text{ s, } 0.9 A_0 = A_0 e^{-\frac{b(5)}{2m}}$$

$$\Rightarrow 0.9 = e^{-\frac{b(5)}{2m}} \quad \dots(i)$$

$$\text{After } 10 \text{ s, } A = A_0 e^{-\frac{b(10)}{2m}}$$

$$\Rightarrow A = A_0 \left(e^{-\frac{5b}{2m}} \right)^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$A = 0.729 A_0$$

$$\text{Hence, } \alpha = 0.729$$

- 99** In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then

$$(a) \omega_1 = \omega_2 \quad [\text{AIEEE 2004}]$$

$$(b) \omega_1 > \omega_2$$

$$(c) \omega_1 < \omega_2, \text{ when damping is small and } \omega_1 > \omega_2, \text{ when damping is large}$$

$$(d) \omega_1 < \omega_2$$

Ans. (a)

For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance. In resonance state, $\omega_1 = \omega_2$.

14

Waves

TOPIC 1

Basic of Waves and Progressive Waves

- 01** The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is mm.

[Take, refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000 Å]

[2021, 27 July Shift-II]

Ans. (2)

Let the thickness of air column be t .

∴ We know that,

Thickness, $t = n\lambda$

[where, n is the number of waves, λ is the wavelength of wave]

$$\Rightarrow n = \frac{t}{\lambda} \quad \dots(i)$$

∴ As per question, the difference in the number of waves when yellow light propagates through air and vacuum column of same thickness is 1.

$$\Rightarrow n_{\text{air}} - n_{\text{vacuum}} = 1 \quad \dots(ii)$$

Using Eq. (i), with Eq. (ii) can be written as

$$\frac{t}{\lambda_{\text{air}}} - \frac{t}{\lambda_{\text{vacuum}}} = 1$$

$$\Rightarrow t \left(\frac{1}{\lambda_{\text{air}}} - \frac{1}{\lambda_{\text{vacuum}}} \right) = 1$$

$$\Rightarrow t \left(\frac{1.0003}{6000} - \frac{1}{6000} \right) = 1$$

$$\Rightarrow t = 2 \text{ mm}$$

- 02** The amplitude of wave disturbance propagating in the positive

x -direction is given by $y = \frac{1}{(1+x)^2}$ at

time $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at

$t = 1$ s, where x and y are in metre.

The shape of wave does not

change during the propagation.

The velocity of the wave will be m/s.

[2021, 20 July Shift-I]

Ans. (2)

As per question,

$$\text{at } t = 0, y = \frac{1}{(1+x)^2}$$

$$\text{and at } t = 1 \text{ s}, y = \frac{1}{1+(x-2)^2} \quad \dots(i)$$

As we know,

$$\text{At } t = t \text{ s}, y = \frac{1}{1+(x-vt)^2}$$

$$\text{So, at } t = 1 \text{ s}, y = \frac{1}{1+(x-v)^2} \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

$$v = 2 \text{ ms}^{-1}$$

Hence, the velocity of the wave will be 2 m/s.

- 03** A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive X -axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

[2021, 17 March Shift-II]

$$(a) y(x, t) = 0.03 [\sin 5.1x - (0.2 \times 10^3)t]$$

$$(b) y(x, t) = 0.06 [\sin 5.1x - (1.5 \times 10^3)t]$$

$$(c) y(x, t) = 0.06 [\sin 0.8x - (0.5 \times 10^3)t]$$

$$(d) y(x, t) = 0.03 [\sin 5.1x - (1.5 \times 10^3)t]$$

Ans. (d)

Given, The frequency of the sound wave, $f = 245 \text{ Hz}$

The speed of the travelling wave, $v = 300 \text{ m/s}$

As, total distance of to and fro motion is 6 cm.

Hence, the amplitude of the wave,

$$A = 6/2 = 3 \text{ cm} = 0.03 \text{ m}$$

As we know,

Angular frequency is given as

$$\omega = 2\pi f$$

Substituting the value of f , we get

$$\omega = 2\pi(245)$$

$$\omega = 1.54 \times 10^3 \text{ rad/s}$$

Propagation constant is given as

$$k = \frac{\omega}{v}$$

Substituting the value of v and ω , we get

$$k = \frac{1.54 \times 10^3}{300} \Rightarrow k = 5.1 \text{ m}^{-1}$$

General mathematical expression for a travelling wave is given as

$$y = A \sin(kx - \omega t)$$

Substituting the values in the above equation, we get

$$y = 0.03 \sin(5.1x - 1.5 \times 10^3 t)$$

- 04** The mass per unit length of a uniform wire is 0.135 g/cm . A transverse wave of the form $y = -0.21 \sin(x + 30t)$ is produced in it, where x is in metre and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2} \text{ N}$. Value of x is (Round-off to the nearest integer)

[2021, 26 Feb Shift-I]

Ans. (1215)

Given, mass per unit length,

$$\mu = 0.135 \text{ g/cm}$$

Transverse wave equation,

$$y = -0.21 \sin(x + 30t)$$

From given equation, $\omega = 30 \text{ rad/s}$, $k = 1$

$$\text{Speed of wave, } v = \frac{\omega}{k} = \frac{30}{1} = 30 \text{ ms}^{-1}$$

Also,

$$v = \sqrt{\frac{T}{\mu}}$$

$\Rightarrow$

$$T = v^2 \mu$$

$$T = (30)^2 \times \frac{0.135 \times 10^{-3}}{10^{-2}}$$

$$= 900 \times 0.0135 = 12.15 \text{ N}$$

$$= 1215 \times 10^{-2} \text{ N}$$

Hence, $x = 1215$

- 05** Which of the following equations represents a travelling wave?

[2021, 24 Feb Shift-II]

- (a) $y = A \sin(15x - 2t)$
 (b) $y = Ae^{-x^2} (vt + \theta)$
 (c) $y = Ae^x \cos(\omega t - \theta)$
 (d) $y = A \sin x \cos \omega t$

Ans. (a)

The equation of a travelling wave in standard form,

$$y = A \sin(\omega t \pm kx)$$

Only option (a), i.e. $y = A \sin(15x - 2t)$ satisfies this equation.

- 06** A wire of density $9 \times 10^{-3} \text{ kg cm}^{-3}$ is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4} . The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire, $Y = 9 \times 10^{10} \text{ Nm}^{-2}$), (to the nearest integer)

[2020, 2 Sep Shift-II]

Ans. (35)

Density of wire, $d = 9 \times 10^{-3} \text{ kg cm}^{-3}$
 $= 9 \times 10^3 \text{ kg m}^{-3}$

Strain in the wire, $\epsilon = 4.9 \times 10^{-4}$

Young's modulus of wire is

$$Y = 9 \times 10^{10} \frac{\text{N}}{\text{m}^2}$$

Lowest frequency of vibration in wire will be

$$f = \frac{1}{2L} \sqrt{\frac{T}{M/L}} \quad \dots(i)$$

$$\text{Now, } \frac{T}{M/L} = \frac{T}{V\rho/L} = \frac{T}{\left(\frac{LA \cdot \rho}{L}\right)} = \frac{T}{A\rho}$$

$$\text{But } \frac{T}{A} = \text{stress} = Y \times \text{strain} = Y \times \epsilon$$

$$\Rightarrow \frac{T}{M/L} = \frac{Y \times \epsilon}{\rho}$$

So, from Eq (i), frequency will be

$$f = \frac{1}{2L} \sqrt{\frac{T}{M/L}} = \frac{1}{2L} \sqrt{\frac{Y \times \epsilon}{\rho}}$$

$$= \frac{1}{2 \times 1} \times \sqrt{\frac{9 \times 10^{10} \times 4.9 \times 10^{-4}}{9 \times 10^3}} = 35 \text{ Hz}$$

- 07** Two identical strings X and Z made of same material have tensions T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz respectively, then the ratio of T_X / T_Z is

[2020, 2 Sep Shift-I]

- (a) 1.25 (b) 2.25
(c) 0.44 (d) 1.5

Ans. (b)

Fundamental frequency of oscillation of a taut string is

$$f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$$

where, l = length, T = tension and μ = mass per unit length.

Now, given X and Z are identical strings.

$\therefore l$ and μ are same for both X and Z.

$$\Rightarrow f \propto \sqrt{T} \Rightarrow \frac{f_X}{f_Z} = \sqrt{\frac{T_X}{T_Z}}$$

$$\Rightarrow \frac{T_X}{T_Z} = \frac{f_X^2}{f_Z^2} = \left(\frac{450}{300}\right)^2 = \frac{9}{4} = 2.25$$

Hence, correct option is (b).

- 08** A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wavetrain of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope?

[2020, 3 Sep Shift-I]

- (a) 3 (b) 6
(c) 12 (d) 9

Ans. (c)

As wavetrain moves up over string, tension and hence wave speed also changes. Also, frequency (f) of wavetrain remains constant as it depends only on the source.

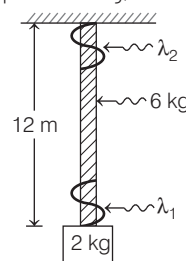
$$\text{Now, using } v = \sqrt{\frac{T}{\mu}}$$

where, v = wave speed, T = tension in string and μ = mass per unit length of string.

Tension T_1 at bottom = 2 g

Tension T_2 at top = (2 + 6) g = 8 g

If λ_1 and λ_2 are wavelengths at bottom and top, then



$$\frac{v_1}{v_2} = \frac{f \lambda_1}{f \lambda_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \lambda_2 = \lambda_1 \sqrt{\frac{T_2}{T_1}}$$

$$= 6 \times \sqrt{\frac{8g}{2g}} = 12 \text{ cm}$$

Hence, option (c) is correct.

- 09** For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in metre) of the waves are

[2020, 4 Sep Shift-I]

- (a) 1, 2, 3, ... (b) $\frac{1}{3}, \frac{1}{5}, \dots$
(c) 1, 3, 5, ... (d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots$

Ans. (b)

Distance between any two crests

$$n\lambda = 5 \quad (n \text{ is an integer}) \quad \dots(i)$$

Distance between any crest and any trough

$$(2m + 1) \frac{\lambda}{2} = 1.5 \quad (m \text{ is an integer})$$

$$\Rightarrow (2m + 1)\lambda = 3 \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \frac{n\lambda}{(2m + 1)\lambda} = \frac{5}{3}$$

$$3n = 5(2m + 1)$$

$$3n = 10m + 5 \quad \dots(iii)$$

Now, putting the values of m and n in Eq.

(iii), which satisfy the equation (only integral values of m and n are acceptable).

$$(I) \quad m = 1 \text{ and } n = 5$$

Now, on putting these values in Eqs. (i) and (ii), we get

$$\lambda = 1$$

$$(II) \quad m = 4 \text{ and } n = 15$$

Now, on putting these values in Eqs. (i) and (ii), we get

$$\lambda = \frac{1}{3}$$

$$(III) \quad m = 7 \text{ and } n = 25$$

Now, on putting these values in Eqs. (i) and (ii), we get $\lambda = \frac{1}{5}$ and so

on.

So, possible values of $\lambda = 1, \frac{1}{3}, \frac{1}{5}, \dots$

Hence, correct option is (b).

- 10** Assume that the displacement of air is proportional to the pressure difference Δp created by a sound wave. Displacement further depends on the speed of sound v , density of air ρ and the frequency f . If $\Delta p \sim 10 \text{ Pa}$, $v \sim 300 \text{ m/s}$, $\rho \sim 1 \text{ kg/m}^3$ and $f \sim 1000 \text{ Hz}$, then s will be of the order of (Take the multiplicative constant to be 1.) [2020, 5 Sep Shift-I]

- (a) 10 mm (b) 1 mm
(c) $\frac{1}{10}$ mm (d) $\frac{3}{100}$ mm

Ans. (d)

The pressure difference is given by,

$$\Delta p = \frac{B\omega}{v} \times s$$

$$\therefore \text{Displacement of air, } s = \frac{\Delta p \times v}{B\omega} = \frac{\Delta p v}{\rho v^2 \omega}$$

$$= \frac{\Delta p}{\rho v \omega} = \frac{10}{1 \times 300 \times 1000} \approx \frac{3}{100} \text{ mm}$$

Hence, correct option is (d).

- 11** Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1} . If the Young's modulus of wire is $16 \times 10^{11} \text{ Nm}^{-2}$, the extension of wire over its natural length is [2020, 7 Jan Shift-I]

- (a) 0.01 mm (b) 0.04 mm
(c) 0.03 mm (d) 0.02 mm

Ans. (c)

Speed of transverse wave over a string,

$$v = \sqrt{\frac{T}{\mu}} \quad \dots(i)$$

where, T = tension or force on string
and $\mu = \frac{m}{l}$ = mass per unit length.

Also, Young's modulus of string,

$$Y = \frac{TI}{A\Delta l} \Rightarrow T = \frac{YA\Delta l}{l} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$v^2 = \frac{YA\Delta l}{\mu l} \text{ or } \Delta l = \frac{mv^2}{YA} \quad \dots(iii)$$

Here, $m = 6 \text{ g} = 6 \times 10^{-3} \text{ kg}$, $l = 60 \text{ cm}$

$$= 60 \times 10^{-2} \text{ m}, A = 1 \text{ mm}^2$$

$$= 1 \times 10^{-6} \text{ m}^2,$$

$$Y = 16 \times 10^{11} \text{ Nm}^{-2} \text{ and } v = 90 \text{ ms}^{-1}$$

Substituting these given values in Eq. (iii), we get

$$\Delta l = \frac{6 \times 10^{-3} \times (90)^2}{16 \times 10^{11} \times 1 \times 10^{-6}} = 3.03 \times 10^{-5} \text{ m}$$

$$\approx 30 \times 10^{-6} \text{ m} = 0.03 \text{ mm}$$

- 12** A transverse wave travels on a taut steel wire with a velocity of v when tension in it is $2.06 \times 10^4 \text{ N}$. When the tension is changed to T , the velocity changed to $v/2$. The value of T is close to [2020, 8 Jan Shift-II]

- (a) $10.2 \times 10^2 \text{ N}$ (b) $5.15 \times 10^3 \text{ N}$
(c) $2.50 \times 10^4 \text{ N}$ (d) $30.5 \times 10^4 \text{ N}$

Ans. (b)

Transverse wave speed over a string is given by

$$v = \sqrt{\frac{T}{\mu}} \quad \dots(i)$$

where, T = tension in string

and μ = mass per unit length of string.

Here, when velocity is v , then tension,

$$T_1 = 2.06 \times 10^4 \text{ N}$$

Let when velocity is $\frac{v}{2}$, then tension is T ,

hence from Eq. (i), we get

$$\frac{v}{2} = \sqrt{\frac{T_1}{T}} \Rightarrow T = \frac{T_1}{4} = \frac{2.06 \times 10^4}{4}$$

$$\text{or } T = 5.15 \times 10^3 \text{ N}$$

- 13** The pressure wave $p = 0.01 \sin [1000t - 3x] \text{ Nm}^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C . On some other day when temperature is T , the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1} . Approximate value of T is [2019, 9 April Shift-I]

- (a) 15°C (b) 11°C
(c) 12°C (d) 4°C

Ans. (d)

$$\text{Given, } p = 0.01 \sin(1000t - 3x) \text{ N/m}^2$$

Comparing with the general equation of pressure wave of sound, i.e.

$$p_0 \sin(\omega t - kx),$$

we get $\omega = 1000$ and $k = 3$

$$\text{Also, } k = \frac{\omega}{v} \Rightarrow v = \omega/k$$

$$\therefore \text{Velocity of sound is } |v_1| = \frac{1000}{3}$$

Or

Speed of sound wave can also be calculated as

$$v = - \frac{(\text{coefficient of } t)}{(\text{coefficient of } x)} \\ = - \frac{1000}{(-3)} = \frac{1000}{3} \text{ m/s}$$

Now, relation between velocity of sound and temperature is

$$v = \sqrt{\frac{\gamma RT}{m}} \Rightarrow v \propto \sqrt{T}$$

$$\text{or } \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow T_2 = \frac{v_2^2}{v_1^2} \cdot T_1$$

Here, $v_2 = 336 \text{ m/s}$, $v_1 = 1000/3 \text{ m/s}$,

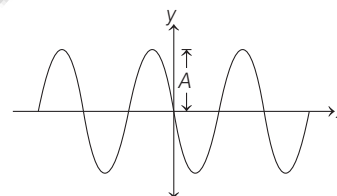
$$T_1 = 0^\circ \text{C} = 273 \text{ K}$$

$$\therefore T_2 = \frac{(336)^2}{(1000/3)^2} \times 273 = 277.38 \text{ K}$$

$$\therefore T_2 = 4.38^\circ \text{C} \approx 4^\circ \text{C}$$

- 14** A progressive wave travelling along the positive x -direction is represented by

$y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. [2019, 12 April Shift-I]



For this wave, the phase ϕ is

- (a) $-\frac{\pi}{2}$ (b) π
(c) 0 (d) $\frac{\pi}{2}$

Ans. (b)

From the given snapshot at $t = 0$,

$$y = 0 \text{ at } x = 0$$

and $y = -ve$ when x increases from zero.

Standard expression of any progressive wave is given by $y = A \sin(kx - \omega t + \phi)$

Here, ϕ is the phase difference, we need to get

$$\text{at } t = 0 \quad y = A \sin(kx + \phi)$$

Clearly $\phi = \pi$, so that

$$y = A \sin(kx + \pi)$$

$$y = -A \sin(kx)$$

$$\Rightarrow y = 0 \text{ at } x = 0$$

$$\text{and } y = -ve \text{ at } x > 0$$

Which satisfies the given snapshot.

- 15** A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Take, reference intensity of sound as 10^{-12} W/m^2] [2019, 12 April Shift-II]

- (a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

Ans. (a)

Loudness of sound in decibel is given by

$$\beta = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

where, I = intensity of sound in W/m^2 ,
 I_0 = reference intensity ($= 10^{-12} \text{ W/m}^2$),
 chosen because it is near the lower limit of the human hearing range.

Here, $\beta = 120 \text{ dB}$

$$\text{So, we have } 120 = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

$$\Rightarrow 12 = \log_{10} \left(\frac{I}{10^{-12}} \right)$$

Taking antilog, we have

$$\Rightarrow 10^{12} = \frac{I}{10^{-12}} \Rightarrow I = 1 \text{ W/m}^2$$

This is the intensity of sound reaching the observer.

$$\text{Now, intensity, } I = \frac{P}{4\pi r^2}$$

where, r = distance from source,

P = power of output source.

Here, $P = 2 \text{ W}$, we have

$$1 = \frac{2}{4\pi r^2} \Rightarrow r^2 = \frac{1}{2\pi}$$

$$\Rightarrow r = \sqrt{\frac{1}{2\pi}} \text{ m} = 0.398 \text{ m} \approx 40 \text{ cm}$$

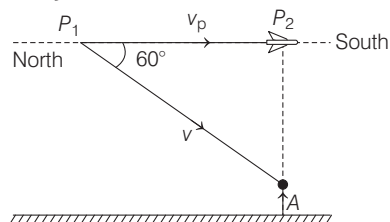
- 16** A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, then speed of the plane is

[2019, 12 Jan Shift-I]

- (a) $\frac{\sqrt{3}}{2}v$ (b) v
 (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$

Ans. (d)

Let P_1 be the position of plane at $t = 0$, when sound waves started towards person A and P_2 is the position of plane observed at time instant t as shown in the figure below.



In triangle P_1P_2A ,

$$P_1P_2 = \text{speed of plane} \times \text{time} = v_p \times t$$

$$P_1A = \text{speed of sound} \times \text{time} = v \times t$$

Now, from ΔP_1P_2A

$$\cos \theta = \frac{\text{base}}{\text{hypotenues}}$$

$$\cos 60^\circ = \frac{P_1P_2}{P_1A} = \frac{v_p \times t}{v \times t}$$

$$\frac{1}{2} = \frac{v_p}{v} \Rightarrow v_p = \frac{v}{2}$$

- 17** A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

[2019, 10 Jan Shift-I]

- (a) 16.6 cm (b) 33.3 cm
 (c) 10.0 cm (d) 20.0 cm

Ans. (d)

Velocity V of the wave on the string

$$= \sqrt{\frac{T}{\mu}}$$

where, T = tension and μ = mass per unit length. Substituting the given values, we get

$$v = \sqrt{\frac{8}{5}} \times 1000 = 40 \text{ ms}^{-1}$$

Wavelength of the wave on the string,

$$\lambda = \frac{v}{f}$$

where, f = frequency of wave.

$$\Rightarrow \lambda = \frac{40}{100} \text{ m} = 40 \text{ cm}$$

$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0 \text{ cm}.$$

- 18** Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is

[2019, 11 Jan Shift-I]

- (a) 5 N (b) 12.5 N
 (c) 7.5 N (d) 10 N

Ans. (b)

Given, equation can be rewritten as,

$$y = 0.03 \sin 450 \left(t - \frac{9x}{450} \right) \quad \dots(i)$$

We know that the general equation of a travelling wave is given as,

$$y = A \sin \omega(t - x/v) \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\text{velocity, } v = \frac{450}{9} = 50 \text{ m/s}$$

and angular velocity, $\omega = 450 \text{ rad/s}$

As, the velocity of wave on stretched string with tension (T) is given as

$$v = \sqrt{\frac{T}{\mu}}$$

where, μ is linear density

$$\therefore T = \mu v^2 = 5 \times 10^{-3} \times 50 \times 50 = 12.5 \text{ N}$$

($\because$ given, $\mu = 5 \text{ g/m} = 5 \times 10^{-3} \text{ kg/m}$)

- 19** A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50t + 2x)$, where x and y are in metre and t is in second. Which of the following is a correct statement about the wave?

[2019, 12 Jan Shift-I]

- (a) The wave is propagating along the negative X-axis with speed 25 ms^{-1} .
 (b) The wave is propagating along the positive X-axis with speed 25 ms^{-1} .
 (c) The wave is propagating along the positive X-axis with speed 100 ms^{-1} .
 (d) The wave is propagating along the negative X-axis with speed 100 ms^{-1}

Ans. (a)

Wave equation is given by,

$$y = 10^{-3} \sin(50t + 2x)$$

Speed of wave is obtained by differentiating phase of wave.

Now, phase of wave from given equation is

$$\phi = 50t + 2x = \text{constant}$$

Differentiating ϕ w.r.t. 't', we get

$$\frac{d}{dx}(50t + 2x) = \frac{d}{dt} \quad (\text{constant})$$

$$\Rightarrow 50 + 2 \left(\frac{dx}{dt} \right) = 0$$

$$\Rightarrow \frac{dx}{dt} = \frac{-50}{2} = -25 \text{ ms}^{-1}$$

So, wave is propagating in negative x-direction with a speed of 25 ms^{-1} .

Alternate Solution

The general equation of a wave travelling in negative x direction is given as

$$y = a \sin(\omega t + kx) \quad \dots(i)$$

Given equation of wave is

$$y = 10^{-3} \sin(50 + 2x) \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\omega = 50 \text{ and } k = 2$$

Velocity of the wave,

$$v = \frac{\omega}{k} = \frac{50}{2} = 25 \text{ m/s}$$

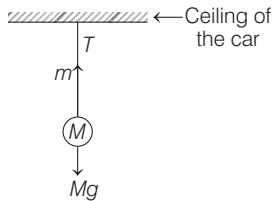
- 20** A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m (m \ll M)$. When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1} . When the car has acceleration a , the wave speed increases to 60.5 ms^{-1} . The value of a , in terms of gravitational acceleration g is closest to

[2019, 9 Jan Shift-I]

- (a) $\frac{g}{20}$ (b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

Ans. (b)

When the car is at rest, then the situation can be shown in the figure below.

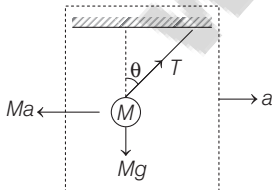


Since, $m \ll M$, then we can neglect the mass of the string. So, the initial velocity of the wave in the string can be given as,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\frac{m}{l}}} = 60 \text{ m/s} \quad \dots(i)$$

where, T is the tension in the string and μ is the mass of the block per unit length (l).

Now, when the car has acceleration a , then the situation can be shown in the figure given below,



Resolving the components of T' along X-and Y-axis, we get

$$T \cos \theta = Mg \quad \dots(ii)$$

$$T \sin \theta = Ma \quad \dots(iii)$$

Squaring both sides of Eqs. (ii) and (iii) and adding them, we get

$$T^2 (\sin^2 \theta + \cos^2 \theta) = M^2 (g^2 + a^2)$$

$$\Rightarrow T = M(g^2 + a^2)^{\frac{1}{2}} \quad \dots(iv)$$

Now, the velocity of the wave in the string would be given as,

$$v' = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{m}{l}}} = 60.5 \text{ m/s} \quad \dots(v)$$

Dividing Eq. (i) and Eq. (v), we get

$$\frac{v}{v'} = \frac{\sqrt{\frac{Mg}{\frac{m}{l}}}}{\sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{m}{l}}}} = \sqrt{\frac{g}{(g^2 + a^2)^{\frac{1}{2}}}}$$

$$= \frac{60}{60.5}$$

$$\text{or } \frac{60.5}{60} = \sqrt{\frac{(g^2 + a^2)^{\frac{1}{2}}}{g}}$$

Squaring both the sides, we get

$$\frac{(60.5)^2}{(60)^2} = \frac{(g^2 + a^2)^{\frac{1}{2}}}{g}$$

$$\text{or } \left(1 + \frac{0.5}{60}\right)^2 = \sqrt{1 + \left(\frac{a}{g}\right)^2}$$

Using binomial expansion,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \dots$$

On both sides, we get

$$1 + 2 \times \frac{0.5}{60} = 1 + \frac{1}{2} \cdot \frac{a^2}{g^2}$$

$$\Rightarrow 1 + \frac{1}{60} = 1 + \frac{1}{2} \cdot \frac{a^2}{g^2}$$

$$\text{or } \frac{a}{g} = \frac{1}{\sqrt{30}}$$

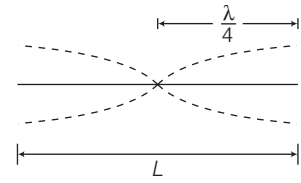
$$\text{or } a = \frac{g}{\sqrt{30}} = \frac{g}{5.4} \approx \frac{g}{5}$$

- 21** A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \text{ kg/m}^3$ and its Young's modulus is $9.27 \times 10^{10} \text{ Pa}$. What will be the fundamental frequency of the longitudinal vibrations?

[JEE Main 2018]

- (a) 5 kHz (b) 2.5 kHz
(c) 10 kHz (d) 7.5 kHz

Ans. (a)



From vibration mode,

$$\frac{\lambda}{2} = L$$

$$\Rightarrow \lambda = 2L$$

$$\therefore \text{Wave speed, } v = \sqrt{\frac{Y}{\rho}}$$

$$\text{So, frequency } f = \frac{v}{\lambda}$$

$$\Rightarrow f = \frac{1}{2L} \sqrt{\frac{Y}{\rho}}$$

$$= \frac{1}{2 \times 60 \times 10^{-2}} \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$$\approx 5000 \text{ Hz}$$

$$f = 5 \text{ kHz}$$

- 22** A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10 \text{ ms}^{-2}$)

[JEE Main 2016]

- (a) $2\pi\sqrt{2} \text{ s}$
(b) 2 s
(c) $2\sqrt{2} \text{ s}$
(d) $\sqrt{2} \text{ s}$

Ans. (c)

A uniform string of length 20 m is suspended from a rigid support. Such that the time taken to reach the support,

$$T = \frac{mgx}{1}$$

So, velocity at point

$$v = \sqrt{\frac{mgx}{\frac{m}{l}}}$$

i.e.

$$v = \sqrt{gx}$$

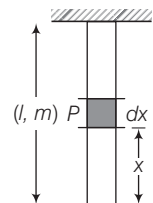
$$\frac{dx}{dt} = \sqrt{gx}$$

$$\Rightarrow \int_0^{20} \frac{dx}{\sqrt{x}} = \int_0^t \sqrt{g} dt$$

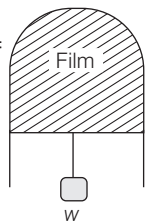
$$[2\sqrt{x}]_0^{20} = \sqrt{10} t$$

$$\Rightarrow 2\sqrt{20} = \sqrt{10} t$$

$$t = 2\sqrt{2} \text{ s}$$



- 23** A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel, if density and elasticity of steel are $7.7 \times 10^3 \text{ kg/m}^3$ and $2.2 \times 10^{11} \text{ N/m}^2$, respectively?



[JEE Main 2013]

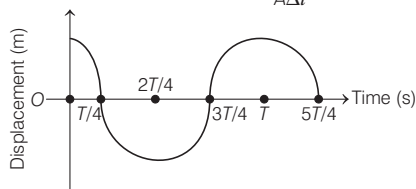
- (a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

Ans. (b)

$$\text{Frequency, } f = \frac{v}{2l} = \frac{1}{2l} \sqrt{\frac{T}{\mu}} = \frac{1}{2l} \sqrt{\frac{T}{Ad}}$$

where, T is tension in the wire and μ is the mass per unit length of wire.

$$\text{Also, Young's modulus, } Y = \frac{Tl}{A\Delta l}$$



$$\Rightarrow \frac{T}{A} = \frac{Y\Delta l}{l}$$

Putting this value in expression of frequency, we have

$$f = \frac{1}{2l} \sqrt{\frac{Y\Delta l}{ld}}$$

$$\text{Given, } l = 1.5 \text{ m, } \frac{\Delta l}{l} = 0.01$$

$$d = 7.7 \times 10^3 \text{ kg/m}^3, \\ Y = 2.2 \times 10^{11} \text{ N/m}^2$$

Putting these values we, have

$$f = \frac{1}{2l} \sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}}$$

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3} \Rightarrow f \approx 178.2 \text{ Hz}$$

- 24 Statement I** Two longitudinal waves given by equations— $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statement II Intensity of waves of given frequency in same medium is proportional to the square of amplitude only. [AIEEE 2011]

- (a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false

- (c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not correct explanation of Statement I

Ans. (d)

$$\text{As Intensity, } I = \frac{1}{2} \rho \omega^2 A^2 v$$

Here, ρ = density of medium,
 A = amplitude,

ω = angular frequency

and v = velocity of wave.

$\therefore$ Intensity depend upon amplitude, frequency as well as velocity of wave.

$$\text{Also, } I_1 = I_2$$

- 25** The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$.

This represents a [AIEEE 2011]

- (a) wave moving in $-x$ direction with

$$\text{speed } \sqrt{\frac{b}{a}}$$

- (b) standing wave of frequency $\sqrt{b}$

- (c) standing wave of frequency $1/\sqrt{b}$

- (d) wave moving in $+x$ direction with speed

$$\sqrt{\frac{a}{b}}$$

Ans. (a)

$$y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)} \\ = e^{-(\sqrt{a}x + \sqrt{b}t)^2}$$

It is a function of type $y = f(\omega t + kx)$
So, $y(x, t)$ represents wave travelling along $-x$ direction.

$$\therefore \text{Speed of wave} = \frac{\omega}{k} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$$

- 26** The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02 \text{ (m) sin}$

$$\left[2\pi \left(\frac{t}{0.04 \text{ (s)}} - \frac{x}{0.50 \text{ (m)}} \right) \right]$$

The tension in the string is

[AIEEE 2010]

- (a) 4.0 N (b) 12.5 N
(c) 0.5 N (d) 6.25 N

Ans. (d)

$$\text{As } v = \sqrt{\frac{T}{\mu}}$$

$$\text{So, } T = \mu v^2 = \mu \frac{\omega^2}{k^2} = 0.04 \frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} \\ = 6.25 \text{ N}$$

- 27** A wave travelling along the x -axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s respectively, then α and β in appropriate units are [AIEEE 2008]

$$(a) \alpha = 25.00\pi, \beta = \pi$$

$$(b) \alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$$

$$(c) \alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$$

$$(d) \alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$$

Ans. (a)

As general equation is $y(x, t) = a \cos(kx - \omega t)$

$$\text{where, } k = \frac{2\pi}{\lambda} \text{ and } \omega = \frac{2\pi}{T}$$

$$\therefore y(x, t) = 0.005 \cos(\alpha x - \beta t)$$

$$\Rightarrow \frac{2\pi}{\lambda} = \alpha \text{ and } \frac{2\pi}{T} = \beta$$

$$\text{So, } \alpha = \frac{2\pi}{0.08} = 25\pi \text{ m}^{-1}$$

$$\text{and } \beta = \frac{2\pi}{2} = \pi \text{ s}^{-1}$$

- 28** A sound absorber attenuates the sound level by 20 db. The intensity decreases by a factor of [AIEEE 2007]

- (a) 1000 (b) 10000 (c) 10 (d) 100

Ans. (d)

Let intensity of sound be I and I' .

Loudness of sound initially,

$$\beta_1 = 10 \log \left(\frac{I}{I_0} \right)$$

$$\text{Later, } \beta_2 = 10 \log \left(\frac{I'}{I_0} \right)$$

$$\text{Given, } \beta_1 - \beta_2 = 20$$

$$\therefore 20 = 10 \log \left(\frac{I}{I'} \right)$$

$$\text{or } I' = \frac{I}{100}$$

Therefore, intensity decreases by a factor of 100.

- 29** The intensity of gamma radiation from a given source is I . On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead, which will reduce the intensity to $I/2$ will be [AIEEE 2005]

- (a) 6 mm (b) 9 mm (c) 18 mm (d) 12 mm

Ans. (d)As intensity, $I' = Ie^{-\mu x}$

Taking log on both sides, we get

$$-\mu x = \log \frac{I'}{I}$$

$$\text{or } -\mu \cdot 36 = \log \frac{I}{8I} \quad \dots(i)$$

$$\text{and } -\mu x' = \log \frac{I}{2I} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{36}{x'} = \frac{3 \log(1/2)}{\log \frac{1}{2}} \text{ or } x' = 12 \text{ mm}$$

30 The displacement y of a particle in a medium can be expressed as

$$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right) \text{ m}$$

where, t is in second and x in metre. The speed of the wave is**[AIEEE 2004]**

- (a) 2000 ms^{-1} (b) 5 ms^{-1}
(c) 20 ms^{-1} (d) $5\pi \text{ ms}^{-1}$

Ans. (b)

As given

$$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right) \quad \dots(i)$$

Comparing it with

$$y = a \sin(\omega t + kx + \phi), \text{ we obtain } \dots(ii)$$

$$\omega = 100 \text{ rad s}^{-1}, k = 20 \text{ m}^{-1}$$

$$\therefore v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ ms}^{-1}$$

31 The displacement y of a wave travelling in the x -direction is given by

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$$

metre

where, x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is**[AIEEE 2003]**

- (a) 300 (b) 600 (c) 1200 (d) 200

Ans. (a)

The given equation of wave,

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right) \quad \dots(i)$$

Standard equation of wave

$$y = a \sin(\omega t - kx + \phi) \quad \dots(ii)$$

Now, comparing Eqs. (i) and (ii), we get

$$\omega = 600 \text{ rad s}^{-1} \text{ and } k = 2 \text{ m}^{-1}$$

$$\therefore \text{Velocity of wave} = \frac{\omega}{k} = \frac{600}{2} = 300 \text{ ms}^{-1}$$

32 When temperature increases, the frequency of a tuning fork**[AIEEE 2002]**

- (a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material

Ans. (b)

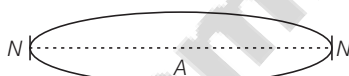
$$\text{As, } f = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

When temperature increases, then length increases and frequency $\left(f \propto \frac{1}{l}\right)$ decreases.**33** Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is**[AIEEE 2002]**

- (a) 20 (b) 80 (c) 40 (d) 120

Ans. (b)

When the string is plucked in the middle, it vibrates in one loop with nodes at fixed ends and an antinode in the middle.



$$\text{So that, } \frac{\lambda_1}{2} = l$$

$$\text{or } \lambda_1 = 2 \times 40 = 80 \text{ cm}$$

TOPIC 2

Superposition and Reflection of Waves

34 A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively.

The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The

length of the open pipe is $\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}}$ where x is

(Round off to the nearest integer)

[2021, 16 March Shift-II]**Ans. (4)**First overtone of open pipe $= v_2 / L_2$

First overtone of closed pipe at one end

$$= \frac{3v}{4L}$$

As per question,

$$\begin{aligned} \frac{3v}{4L} &= \frac{v_2}{L} \\ \Rightarrow \sqrt{\frac{B}{\rho_1}} \cdot \frac{3}{4L} &= \sqrt{\frac{B}{\rho_2}} \cdot \frac{1}{L_2} \quad \left(\because v = \sqrt{\frac{B}{\rho}}\right) \\ \Rightarrow L_2 &= \frac{4L}{3} \sqrt{\frac{\rho_1}{\rho_2}} \quad \dots(i) \end{aligned}$$

According to question, the length of the open pipe is

$$\frac{x}{3} L \sqrt{\frac{\rho_1}{\rho_2}} \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$x = 4$$

35 A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filled, the beat frequency decreases to 2 beats/s. What is the frequency of fork A?**[2021, 26 Feb Shift-II]**

- (a) 342 Hz (b) 345 Hz
(c) 335 Hz (d) 338 Hz

Ans. (c)Given, $f = 340 \text{ Hz}$ Let beat frequency of tuning fork A is f_A and f'_A be the frequency of tuning fork A after filling.

In first case, beat frequency = 5

$$\Rightarrow f_A = 340 \pm 5$$

$$\Rightarrow f_A = 345 \text{ Hz}$$

$$\text{or } 335 \text{ Hz}$$

After filling tuning fork A,

$$f'_A = 340 \pm 2$$

$$\Rightarrow f'_A = 342 \text{ or } 338 \text{ Hz}$$

As on filling, frequency of A increases.

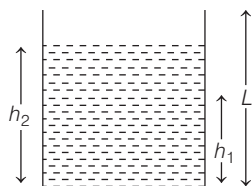
Hence, original frequency of tuning fork A is 335 Hz.

36 In a resonance tube experiment, when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised, the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is**[2020, 5 Sep Shift-I]**

- (a) 1100 Hz
(b) 3300 Hz
(c) 2200 Hz
(d) 550 Hz

Ans. (c)

Let λ be the wavelength L be the length of the tube.



The resonance conditions for the given two cases are:

$$L - h_1 = \frac{n\lambda}{2} + \frac{\lambda}{4} \quad \dots(i)$$

$$L - h_2 = (n-1)\frac{\lambda}{2} + \frac{\lambda}{4} \quad \dots(ii)$$

Subtracting eq. (i) from eq. (ii), we get

$$(L - h_1) - (L - h_2) = \frac{n\lambda}{2} + \frac{\lambda}{4} - (n-1)\frac{\lambda}{2} - \frac{\lambda}{4}$$

$$h_2 - h_1 = \frac{\lambda}{2}$$

Here, $h_1 = 17.0$ cm and $h_2 = 24.5$ cm

Substituting these values in above equation, we get

$$24.5 - 17.0 = \frac{\lambda}{2} \Rightarrow \frac{\lambda}{2} = 7.5 \text{ cm}$$

$$\Rightarrow \lambda = 15.0 \text{ cm}$$

Now, the tuning fork frequency,

$$f = \frac{v}{\lambda} = \frac{330}{15 \times 10^{-2}} = 2200 \text{ Hz}$$

Hence, correct option is (c).

- 37** A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is Hz.

[2020, 8 Jan Shift-I]

Ans. (106.06)

Speed of sound in air by Laplace formula,

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

So, ratio of speed of sound in a gas to that of air,

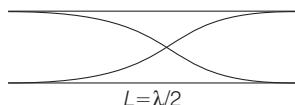
$$\frac{v_{\text{gas}}}{v_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}}$$

$$\text{Here, } v_{\text{air}} = 300 \text{ ms}^{-1} \text{ and } \frac{\rho_{\text{air}}}{\rho_{\text{gas}}} = \frac{1}{2}$$

So, we have

$$\frac{v_{\text{gas}}}{300} = \frac{1}{\sqrt{2}} \Rightarrow v_{\text{gas}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ ms}^{-1}$$

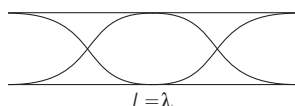
Now, for a tube open at both ends.



$\therefore$ Fundamental frequency,

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

Second harmonic frequency f_2 in tube,



$$f_2 = \frac{v}{\lambda/2}, f_2 = v/L$$

So, frequency difference,

$$\Delta f = f_2 - f_1 = \frac{v}{L} - \frac{v}{2L} = \frac{v}{2L} = \frac{150\sqrt{2}}{2 \times 1}$$

$$= 75\sqrt{2} = 106.06 \text{ Hz}$$

($\therefore L = 1\text{m}$)

- 38** Three harmonic waves having equal frequency v and same intensity I_0 , have phase angles $0, \frac{\pi}{4}$

and $-\frac{\pi}{4}$, respectively. When they

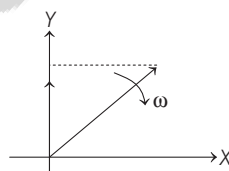
are superimposed, the intensity of the resultant wave is close to

[2020, 9 Jan Shift-I]

- (a) $0.2I_0$ (b) I_0
(c) $3I_0$ (d) $5.8I_0$

Ans. (d)

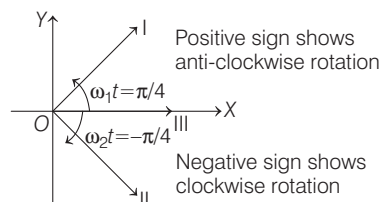
We can represent a wave vectorially with a phasor.



(Note Length of phasor = Amplitude of wave and its projection on Y-axis gives variation in displacement.)

To find resultant amplitude obtained due to superposition of two or more waves, we can add phasors vectorially at any instant during their rotation.

Here, we are given with following phasors



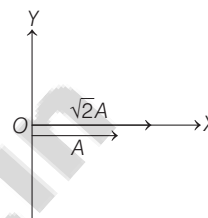
Each phasor is of same amplitude, $A^2 = I_0$.

From phasor diagram, amplitude of resultant of phasors I and II,

$$A' = A \cos \frac{\pi}{4} + A \cos \frac{\pi}{4}$$

$$= 2A \times \frac{1}{\sqrt{2}} = \sqrt{2} A$$

Now, this resultant phasor is along the direction of IIIrd phasor.



$\therefore$ Resultant amplitude of these two will be

$$A_R = \sqrt{2}A + 1A = (\sqrt{2} + 1)A$$

As intensity is proportional to square of amplitude, resultant intensity is given by

$$I_R = A_R^2 = (\sqrt{2} + 1)^2 A^2$$

$$= (\sqrt{2} + 1)^2 I_0 = (2 + 1 + 2\sqrt{2}) I_0$$

$$= (3 + 2\sqrt{2}) I_0$$

$$= (3 + 2 \times 1.4) I_0 = 5.8 I_0$$

- 39** A wire of length L and mass per unit length $6.0 \times 10^{-3} \text{ kg m}^{-1}$ is put under tension of 540 N. Two consecutive frequencies that it resonates at are :
420 Hz and 490 Hz. Then, L in metres is [2020, 9 Jan Shift-II]

- (a) 8.1 m (b) 2.1 m (c) 5.1 m (d) 1.1 m

Ans. (b)

Resonant frequency for a stretched string is given by

$$f_n = n f_1 = \frac{n}{2l} \sqrt{\frac{T}{\mu}}$$

Given, two consecutive resonant frequencies are 420 Hz and 490 Hz.

$$\text{So, } \frac{n}{2l} \sqrt{\frac{T}{\mu}} = 420 \quad \dots(i)$$

$$\text{and } \frac{n+1}{2l} \sqrt{\frac{T}{\mu}} = 490 \quad \dots(ii)$$

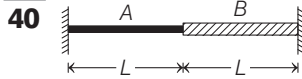
Subtracting Eq. (ii) from Eq. (i), we get

$$\frac{1}{2l} \sqrt{\frac{T}{\mu}} = 490 - 420 = 70$$

Here, $T = 540 \text{ N}$, $\mu = 6 \times 10^{-3} \text{ kg m}^{-1}$

$$\text{So, } l = \frac{1}{2 \times 70} \sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 70} \times \sqrt{\frac{540}{6 \times 10^{-3}}}$$

$$= 2.14 \text{ m} \approx 2.1 \text{ m}$$

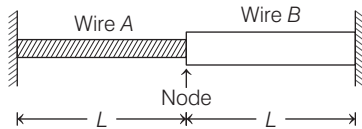


A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q , then the ratio $p:q$ is

[2019, 8 April Shift-I]

- (a) 3 : 5 (b) 4 : 9
(c) 1 : 2 (d) 1 : 4

Ans. (c)



Let mass per unit length of wires are μ_A and μ_B , respectively.

$\therefore$ For same material, density is also same.

$$\text{So, } \mu_A = \frac{\rho \pi r^2 L}{L} = \mu$$

$$\text{and } \mu_B = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension (T) in both connected wires are same.

So, speed of wave in wires are

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{T}{\mu}}$$

$$[\because \mu_A = \mu \text{ and } \mu_B = 4\mu]$$

$$\text{and } v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{T}{4\mu}}$$

So, n th harmonic in such wires system is

$$f_{nth} = \frac{pv}{2L}$$

$$\Rightarrow f_A = \frac{pv_A}{2L} = \frac{p}{2L} \sqrt{\frac{T}{\mu}} \quad (\text{for } p \text{ antinodes})$$

Similarly,

$$f_B = \frac{qv_B}{2L} = \frac{q}{2L} \sqrt{\frac{T}{4\mu}} = \frac{1}{2} \left(\frac{q}{2L} \sqrt{\frac{T}{\mu}} \right)$$

(for q antinodes)

As frequencies f_A and f_B are given equal.

$$\text{So, } f_A = f_B \Rightarrow \frac{p}{2L} \sqrt{\frac{T}{\mu}} = \frac{q}{2} \left[\frac{1}{2L} \sqrt{\frac{T}{\mu}} \right]$$

$$\frac{p}{q} = \frac{1}{2}$$

$$\Rightarrow p:q = 1:2$$

- 41** A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is (All quantities are in SI units) [2019, 9 April Shift-I]
- (a) 60 m (b) 40 m (c) 80 m (d) 20 m

Ans. (c)

Given equation of stationary wave is

$$Y = 0.3 \sin(0.157x) \cos(200\pi t)$$

Comparing it with general equation of stationary wave, i.e. $Y = a \sin kx \cos \omega t$, we get

$$k = (2\pi/\lambda) = 0.157$$

$$\Rightarrow \lambda = \frac{2\pi}{0.157} = 4\pi^2 \left(\because \frac{1}{2\pi} \approx 0.157 \right) \dots (i)$$

$$\text{and } \omega = 200\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{20} \text{ s}$$

As the possible wavelength associated with n th harmonic of a vibrating string, i.e. fixed at both ends is given as

$$\lambda = \frac{2l}{n} \text{ or } l = n \left(\frac{\lambda}{2} \right)$$

Now, according to question, string is fixed from both ends and oscillates in 4th harmonic, so

$$4 \left(\frac{\lambda}{2} \right) = l \Rightarrow 2\lambda = l$$

$$\text{or } l = 2 \times 4\pi^2 = 8\pi^2 \text{ [using Eq. (i)]}$$

$$\text{Now, } \pi^2 \approx 10 \Rightarrow l \approx 80 \text{ m}$$

- 42** A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is [2019, 9 April Shift-II]
- (a) 180 m/s, 80 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 120 Hz

Ans. (b)

Frequency of vibration of a string in n th harmonic is given by

$$f_n = n \cdot \frac{v}{2l} \dots (i)$$

where, v = speed of sound and l = length of string.

Here, $f_3 = 240 \text{ Hz}$, $l = 2 \text{ m}$ and $n = 3$

Substituting these values in Eq. (i), we get

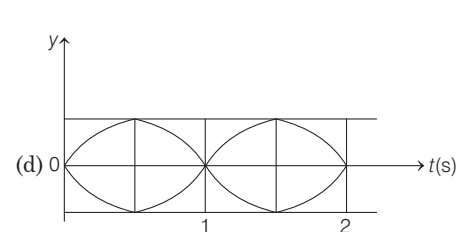
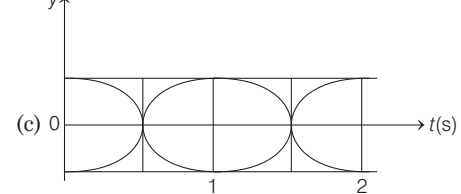
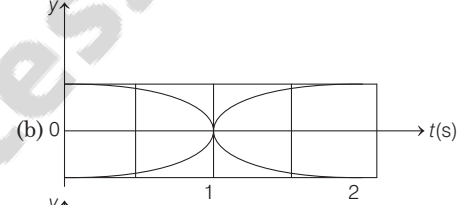
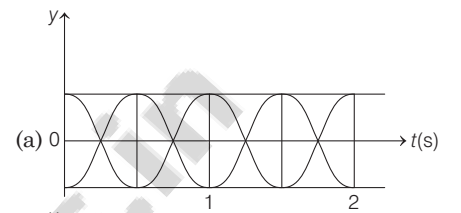
$$\therefore 240 = 3 \times \frac{v}{2 \times 2}$$

$$\Rightarrow v = \frac{4 \times 240}{3} = 320 \text{ ms}^{-1}$$

Also, fundamental frequency is

$$f = \frac{f_n}{n} = \frac{f_3}{3} = \frac{240}{3} = 80 \text{ Hz}$$

- 43** The correct figure that shows schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is [2019, 10 April Shift-II]



Ans. (a)

When waves of nearby frequencies overlaps, beats are produced.

Beat frequency is given by,

$$f_{\text{beat}} = f_1 - f_2$$

$$\text{Here, } f_1 = 11 \text{ Hz and } f_2 = 9 \text{ Hz}$$

$\Rightarrow$ Beat frequency is,

$$f_{\text{beat}} = 11 - 9 = 2 \text{ Hz}$$

Hence, time period of beats or time interval between beats,

$$T = \frac{1}{f_{\text{beat}}}$$

$$\Rightarrow T = \frac{1}{2} = 0.5 \text{ s}$$

So, resultant wave has a time period of 0.5 s which is correctly depicted in option (a) only.

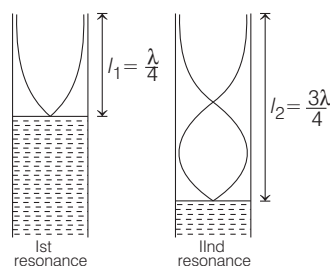
- 44** A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column $l_1 = 30$ cm and $l_2 = 70$ cm. Then, v is equal to

[2019, 12 April Shift-II]

- (a) 332 ms^{-1} (b) 384 ms^{-1}
(c) 338 ms^{-1} (d) 379 ms^{-1}

Ans. (b)

In a resonance tube apparatus, first and second resonance occur as shown



As in a stationary wave, distance between two successive nodes is $\frac{\lambda}{2}$

and distance of a node and an antinode is $\frac{\lambda}{4}$.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

So, speed of sound, $v = f\lambda$
 $= f \times 2(l_2 - l_1)$
 $= 480 \times 2 \times (70 - 30) \times 10^{-2}$
 $= 384 \text{ ms}^{-1}$

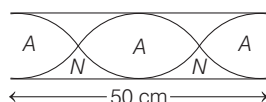
- 45** A musician produce the sound of second harmonics from open end flute of 50 cm. The other person moves toward the musician with speed 10 km/h from the second end of room. If the speed of sound 330 m/s, the frequency heard by running person will be

[2019, 9 Jan Shift-II]

- (a) 666 Hz (b) 500 Hz
(c) 753 Hz (d) 333 Hz

Ans. (a)

According to the question, the musician uses a open flute of length 50 cm and produce second harmonic sound waves



When the flute is open from both ends and produce second harmonic, then

$$L = \lambda_2 \Rightarrow f_2 = \frac{v}{L}$$

where, λ_2 = wavelength for second harmonic,

f_2 = frequency for second harmonic

and v = speed of wave.

$$\text{For given question } \Rightarrow f_2 = \frac{v}{L} = \frac{330}{50 \times 10^{-2}}$$

$f_2 = 660$ Hz (frequency produce by source)

Now, a person runs towards the musician from another end of a hall

$v_{\text{observer}} = 10$ km/h (towards source)

There is apparant change in frequency, which heard by person and given by Doppler's effect formula

$$\Rightarrow v' = f' = v \left[\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}} \right]$$

$$f' = f_2 \left[\frac{v_s + v_o}{v_{\text{sound}}} \right]$$

$$f' = 660 \left[\frac{330 + \frac{50}{18}}{330} \right]$$

$$f' = 666 \text{ Hz}$$

- 46** A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)

[2019, 10 Jan Shift-II]

- (a) 7 (b) 4 (c) 5 (d) 6

Ans. (d)

Fundamental frequency of closed organ pipe is given by $f_0 = v/4L$, where v is the velocity of sound in it and L is the length of the pipe.

Also, overtone frequencies are given by

$$f = (2n + 1) \frac{v}{4L} \text{ or } f = (2n + 1) f_0$$

Given that, $f_0 = 1500$ Hz and

$f_{\text{max}} = 20000$ Hz

This means, $f_{\text{max}} > f$

So, $f_{\text{max}} > (2n + 1) f_0$

$$\Rightarrow 20000 > (2n + 1) 1500$$

$$\Rightarrow 2n + 1 < 13.33 \Rightarrow 2n < 13.33 - 1$$

$$\Rightarrow 2n < 12.33 \text{ or } n < 6.16$$

or $n = 6$ (integer number)

Hence, total six overtones will be heard.

- 47** A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark. near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment is close to

[2019, 12 Jan Shift-II]

- (a) 328 ms^{-1}
(b) 341 ms^{-1}
(c) 322 ms^{-1}
(d) 335 ms^{-1}

Ans. (a)

Key idea To overcome the error occurred in measurement of resonant length. We introduce end correction factor e in length

In first resonance, length of air column
 $= \frac{\lambda}{4}$

$$\text{So, } l_1 + e = \frac{\lambda}{4}$$

$$\text{or } 11 \times 4 + 4e = \lambda$$

So, speed of sound is

$$\Rightarrow v = f_1 \lambda = 512(44 + 4e) \dots (i)$$

And in second case,

$$l_1' + e = \frac{\lambda'}{4}$$

$$\text{or } 27 \times 4 + 4e = \lambda'$$

$$\Rightarrow v = f_2 \lambda' = 256(108 + 4e) \dots (ii)$$

Dividing both Eqs. (i) and (ii), we get

$$1 = \frac{512(44 + 4e)}{256(108 + 4e)}$$

$$\Rightarrow e = 5 \text{ cm}$$

Substituting value of e in Eq. (i), we get

$$\text{Speed of sound } v = 512(44 + 4e)$$

$$= 512(44 + 4 \times 5)$$

$$= 512 \times 64 \text{ cm s}^{-1}$$

$$= 327.68 \text{ ms}^{-1}$$

$$\approx 328 \text{ ms}^{-1}$$

- 48** A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now [JEE Main 2016]

- (a) $\frac{f}{2}$ (b) $\frac{3f}{4}$
(c) $2f$ (d) f

Ans. (d)

For open ends, fundamental frequency f in air, we have

$$\frac{\lambda}{2} = 1$$

$$\Rightarrow \lambda = 2l$$

$$v = f\lambda$$

$$\Rightarrow f = \frac{v}{\lambda} = \frac{v}{2l}$$

When a pipe is dipped vertically in water, so that half of it is in water, we have

$$\frac{\lambda}{4} = \frac{l}{2}$$

$$\Rightarrow \lambda = 2l \Rightarrow v = f'\lambda$$

$$\Rightarrow f' = \frac{v}{\lambda} = \frac{v}{2l} = f$$

Thus, the fundamental frequency of the air column is now,
 $f = f'$

- 49** A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. [JEE Main 2014]

- (a) 12 (b) 8
(c) 6 (d) 4

Ans. (c)

$$\text{For closed organ pipe} \\ = \frac{(2n+1)v}{4l} \quad (n=0, 1, 2, \dots)$$

$$\frac{(2n+1)v}{4l} < 1250$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

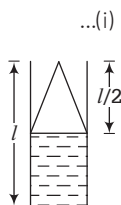
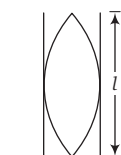
$$(2n+1) < 12.5$$

$$\Rightarrow 2n < 11.50$$

$$\Rightarrow n < 5.25$$

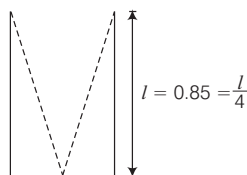
$$\text{So, } n = 0, 1, 2, 3, \dots, 5$$

So, we have 6 possibilities.



Alternate Method

In closed organ pipe, fundamental node



$$\text{i.e., } \frac{\lambda}{4} = 0.85 \Rightarrow \lambda = 4 \times 0.85$$

$$\text{As we know, } v = \frac{c}{\lambda} = \frac{340}{4 \times 0.85} = 100 \text{ Hz}$$

$\therefore$ Possible frequencies = 10 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

- 50** A cylindrical tube open at both ends, has a fundamental frequency f , in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now [AIEEE 2012]

- (a) f (b) $f/2$ (c) $3f/4$ (d) $2f$

Ans. (d)

Initially for open organ pipe, fundamental frequency,

$$v_o = \frac{v}{2l} = f \quad [\text{given}]$$

But when it is half dipped in water, then it becomes closed organ pipe of length $\frac{l}{2}$. In this case, fundamental frequency,

$$v_c = \frac{v}{4l'} = \frac{v}{4 \cdot \frac{l}{2}} = \frac{v}{2l} = f$$

- 51** Three sound waves of equal amplitudes have frequencies $(v-1)$, v , $(v+1)$. They superpose to give beat. The number of beats produced per second will be [AIEEE 2009]

- (a) 4 (b) 3 (c) 2 (d) 1

Ans. (c)

Maximum number of beats

$$= v + 1 - (v - 1) = 2$$

- 52** While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same

experiment during summer, she measures the column length to be x cm for the second resonance.

Then,

[AIEEE 2008]

- (a) $18 > x$
(b) $x > 54$
(c) $54 > x > 36$
(d) $36 > x > 18$

Ans. (b)

$$l_1 = 18 \text{ cm}$$

$$f = \frac{v_1}{4l_1} \quad [\text{for first resonances}]$$

$$f = \frac{3v_2}{4l_2} \quad [\text{for second resonances}]$$

where, $l_2 = x$ according to given situation and also $v_1 < v_2$ as during summer temperature would be higher.

$$\Rightarrow \frac{3v_2}{4l_2} = \frac{v_1}{4l_1} \quad \text{or } l_2 = 3l_1 \times \frac{v_2}{v_1}$$

$$\Rightarrow x = 54 \times (\text{A quantity greater than 1})$$

$$\text{So, } x > 54$$

- 53** A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is [AIEEE 2006]

- (a) 105 Hz
(b) 1.05 Hz
(c) 1050 Hz
(d) 10.5 Hz

Ans. (a)

For string fixed at both the ends, resonant frequency are given by $f = \frac{nv}{2L}$,

where symbols have their usual meanings. It is given that 315 Hz and 420 Hz are two consecutive resonant frequencies, let these be n th and $(n+1)$ th harmonics.

$$\text{Then, } 315 = \frac{nv}{2L} \quad \dots(i)$$

$$\text{and } 420 = \frac{(n+1)v}{2L} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{315}{420} = \frac{n}{n+1} \quad \text{or } n = 3$$

From Eq. (i), lowest resonant frequency

$$f_0 = \frac{v}{2L} = \frac{315}{3} = 105 \text{ Hz}$$

- 54** When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard.

If the frequency of fork 1 is 200 Hz, then what will be the original frequency of fork 2? [AIEEE 2005]

- (a) 200 Hz (b) 202 Hz
(c) 196 Hz (d) 204 Hz

Ans. (b)

The frequency of fork 2 = 200 ± 4
= 196 or 204 Hz

Since, on attaching the tape on the prong of fork 2, its frequency decreases but now the number of beats per second is 6 i.e., the frequency difference now increases.

It is possible only when before attaching the tape, the frequency of fork 2 is less than the frequency of tuning fork 1. Hence, the frequency of fork 2 is 196 Hz.

- 55** A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n . The frequency n of the alternating source is [AIEEE 2003]

- (a) 50 Hz (b) 100 Hz
(c) 200 Hz (d) 25 Hz

Ans. (a)

The wire will vibrate with the same frequency as that of source. This can be considered as an example of forced vibration.

$$T = 10 \times 9.8 \text{ N} = 98 \text{ N},$$

$$m = 9.8 \times 10^{-3} \text{ kg m}^{-1}$$

$$\text{Frequency of wire, } f = \frac{1}{2L} \sqrt{\left(\frac{T}{m}\right)}$$

$$= \frac{1}{2 \times 1} \sqrt{\left(\frac{98}{9.8 \times 10^{-3}}\right)} = 50 \text{ Hz}$$

- 56** A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the

tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was [AIEEE 2003]

- (a) $(256 + 2)$ Hz (b) $(256 - 2)$ Hz
(c) $(256 - 5)$ Hz (d) $(256 + 5)$ Hz

Ans. (c)

$$f_1 = 256 \text{ Hz}$$

For tuning fork, where $f_2 - f_1 = \pm 5$

$$f_2 = \text{frequency of piano}$$

$$f_2 = (256 + 5) \text{ Hz or } (256 - 5) \text{ Hz}$$

When tension is increased, the beat frequency decreases to 2 beats per second.

If we assume that the frequency of piano string is 261 Hz, then on increasing tension, frequency, more than 261 Hz. But it is given that beat frequency decreases to 2, therefore 261 Hz is not possible.

Hence, 251 Hz i.e., $(256 - 5)$ was the frequency of piano string before increasing tension.

- 57** A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then, the equation of the unknown wave is

- (a) $y = a \sin(\omega t + kx)$ [AIEEE 2002]
(b) $y = -a \sin(\omega t + kx)$
(c) $y = a \sin(\omega t - kx)$
(d) $y = -a \sin(\omega t - kx)$

Ans. (b)

Equation of a wave,

$$y = a \sin(\omega t - kx) \quad \dots(i)$$

Let equations of another wave may be

$$y = a \sin(\omega t + kx) \quad \dots(ii)$$

$$\text{and } y = -a \sin(\omega t + kx) \quad \dots(iii)$$

If Eq. (i) propagates with Eq. (ii), then we get

$$y = 2a \cos kx \sin \omega t \quad \dots(iv)$$

If Eq. (i), propagates with Eq. (iii), then we get

$$y = -2a \sin kx \cos \omega t \quad \dots(v)$$

After putting $x = 0$ in Eqs. (iv) and (v) respectively, we get

$$y = 2a \sin \omega t \quad \text{and} \quad y = 0$$

Hence, Eq. (iii) is an equation of unknown wave.

- 58** Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is [AIEEE 2002]

- (a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1

Ans. (c)

Let the tubes A and B have equal length called as l . Since, tube A is opened at both the ends, therefore, its fundamental frequency,

$$n_A = \frac{v}{2l} \quad \dots(i)$$

Since, tube B is closed at one end, therefore its fundamental frequency,

$$n_B = \frac{v}{4l} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{n_A}{n_B} = \frac{v/2l}{v/4l} = \frac{4}{2} = 2:1$$

- 59** A tuning fork arrangement (pair) produces 4 beats per second with one fork of frequency 288 cps. A little wax is placed on the unknown fork and then it produces 2 beats per second. The frequency of the unknown fork is [AIEEE 2002]

- (a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps

Ans. (b)

The tuning fork of frequency 288 Hz is producing 4 beats per second with the unknown tuning fork i.e., the frequency difference between them is 4. Therefore, the frequency of unknown tuning fork.

$$= 288 \pm 4 = 292 \text{ or } 284$$

On placing a little wax on unknown tuning fork, its frequency decreases but now the number of beats produced per second is 2 i.e., the frequency difference now decreases.

It is possible only when before placing the wax, the frequency of unknown fork is greater than the frequency of given tuning fork. Hence, the frequency of unknown tuning fork = 292 Hz.

TOPIC 3 Doppler Effect

- 60** The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s, then the speed of car is km/h.

[2021, 20 July Shift-I]

Ans. (132)

∴ Since, frequency received by the wall,

$$f' = \left(\frac{v_s}{v_s - v} \right) f_0 \quad \dots(i)$$

where, v_s = velocity of sound in air,

v = velocity of car

and f_0 = observed frequency of sound.

Reflected frequency received by man is

$$f'' = \left(\frac{v_s + v}{v_s} \right) f' \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$f'' = \left(\frac{v_s + v}{v_s} \right) \left(\frac{v_s}{v_s - v} \right) f_0$$

$$\Rightarrow f'' = \left(\frac{v_s + v}{v_s - v} \right) f_0$$

$$\Rightarrow 500 = \left(\frac{330 + v}{330 - v} \right) \times 400$$

$$\Rightarrow \frac{500}{400} = \frac{(330 + v)}{(330 - v)}$$

$$\Rightarrow 5(330 - v) = 4(330 + v)$$

$$\Rightarrow 1650 - 5v = 1320 + 4v$$

$$\Rightarrow 9v = 330 \Rightarrow v = \frac{330}{9} \text{ m/s}$$

$$\text{or } v = \frac{330}{9} \times \frac{18}{5} = 132 \text{ km/h}$$

- 61** Two cars are approaching each other at an equal speed of 7.2 km/h. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be Hz. [Velocity of sound in air is 340 m/s.] **[2021, 24 Feb Shift-II]**

Ans. (8)

Given, $v_A = v_B = 7.2 \text{ kmh}^{-1}$

$$= \frac{72}{10} \times \frac{5}{18} = 2 \text{ ms}^{-1}$$

Frequency of source, $f_s = 676 \text{ Hz}$

Speed of sound in air, $v = 340 \text{ ms}^{-1}$

Let f_0 be the frequency heard by each driver.

By using Doppler effect for A,

$$\begin{aligned} (v - v_A)f_s &= (v + v_B)f_0 \\ \Rightarrow f_0 &= \left(\frac{v + v_A}{v - v_B} \right) f_s = \left(\frac{340 + 2}{340 - 2} \right) 676 \\ &= \frac{342}{338} \times 676 = 684 \text{ Hz} \end{aligned}$$

Now, beat frequency = $f_0 - f_s = 684 - 676 = 8 \text{ Hz}$

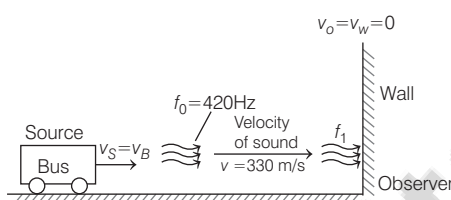
- 62** The driver of a bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz, when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1} .

[2020, 4 Sep Shift-II]

- (a) 81 kmh^{-1} (b) 91 kmh^{-1}
(c) 71 kmh^{-1} (d) 61 kmh^{-1}

Ans. (b)

Let v_B be the speed of the bus. Before reflection, the frequency of the horn is 420 Hz.

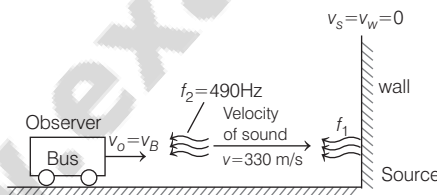


Using Doppler's effect,

$$f_1 = \left(\frac{v}{v - v_s} \right) f_0$$

$$\Rightarrow f_1 = \left(\frac{330}{330 - v_B} \right) \times 420 \quad \dots(i)$$

After reflection from the wall, the frequency of the horn becomes 490 Hz.



Again using Doppler's effect,

$$f_2 = \left(\frac{v + v_o}{v} \right) f_1$$

$$\Rightarrow 490 = \left(\frac{330 + v_B}{330} \right) \times \left(\frac{330}{330 - v_B} \right) \times 420$$

$$\Rightarrow \frac{490}{420} = \frac{330 + v_B}{330 - v_B} \Rightarrow \frac{7}{6} = \frac{330 + v_B}{330 - v_B}$$

On applying Componendo-Dividendo rule, we get

$$\frac{7+6}{7-6} = \frac{(330 + v_B) + (330 - v_B)}{(330 + v_B) - (330 - v_B)}$$

$$\Rightarrow \frac{13}{1} = \frac{2 \times 330}{2v_B}$$

$$\Rightarrow v_B = \frac{330}{13} \text{ m/s} = \frac{330}{13} \times \frac{18}{5} \text{ km/h} = 91.38 \text{ km/h} \approx 91 \text{ km/h}$$

Hence, option (b) is correct.

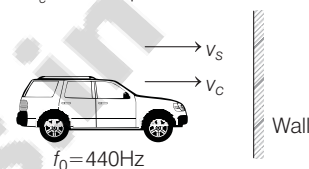
- 63** A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m/s, then the speed of the car is

[2020, 5 Sep Shift-II]

- (a) 54 km/h (b) 36 km/h
(c) 18 km/h (d) 24 km/h

Ans. (a)

Let v_c be the speed of the car.



Using Doppler's effect, Frequency received by wall,

$$\begin{aligned} f_1 &= f_0 \left(\frac{v_s}{v_s - v_c} \right) \\ \Rightarrow f_1 &= f_0 \left(\frac{345}{345 - v_c} \right) \quad \dots(i) \end{aligned}$$

Now, wall will act as a source of frequency f_1 .

Frequency received by driver after reflection from wall,

$$f_2 = f_1 \left(\frac{v_s + v_c}{v_s} \right) = f_1 \left(\frac{345 + v_c}{345} \right)$$

Substituting value of f_1 from eq. (i), we get

$$\begin{aligned} f_2 &= f_0 \left(\frac{345 + v_c}{345} \right) \left(\frac{345}{345 - v_c} \right) \\ \Rightarrow f_2 &= f_0 \left(\frac{345 + v_c}{345 - v_c} \right) \end{aligned}$$

Given, $f_0 = 440 \text{ Hz}$ and $f_2 = 480 \text{ Hz}$

$$\therefore 480 = 440 \left(\frac{345 + v_c}{345 - v_c} \right)$$

$$\Rightarrow 12 = 11 \left(\frac{345 + v_c}{345 - v_c} \right)$$

$$\Rightarrow 12(345 - v_c) = 11(345 + v_c)$$

$$\Rightarrow 345 = 23v_c \Rightarrow v_c = 15 \text{ m/s}$$

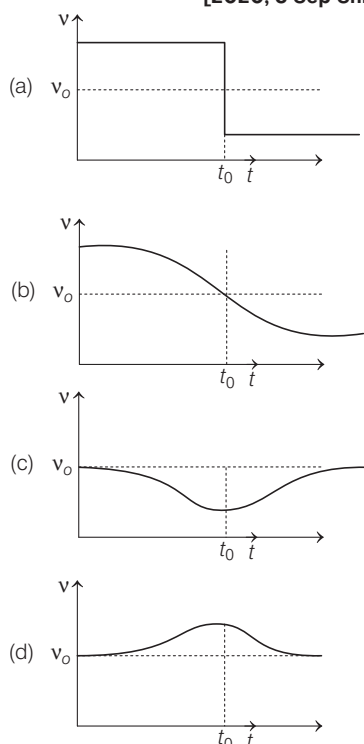
$$\Rightarrow v_c = \frac{15 \times 18}{5} \text{ km/h} = 54 \text{ km/h}$$

Hence, correct option is (a).

- 64** A sound source S is moving along a straight track with speed v and is emitting sound of frequency v_0 (see figure). An observer is standing at a finite distance, at the

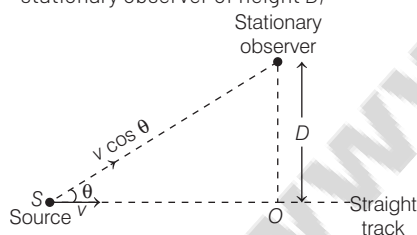
point O , from the track. The time variation of frequency heard by the observer is best represented by (Here, t_0 represents the instant when the distance between the source and observer is minimum.)

[2020, 6 Sep Shift-I]



Ans. (b)

The situation can be shown for a stationary observer of height D ,



Source frequency = ν_0

Using the concept of Doppler's effect,
Observed frequency,

$$\nu_{\text{observed}} = \left(\frac{\nu_{\text{sound}}}{\nu_{\text{sound}} - \nu \cos \theta} \right) \nu_0$$

When $t < t_0$ ($0^\circ < \theta < 90^\circ$)

Initially, θ is less but increasing with time, so $\cos \theta$ decreases continuously and ν_{observed} also decreases.

When $t = t_0$ ($\theta = 90^\circ$)

$$\cos \theta = \cos 90^\circ = 0, \nu_{\text{observed}} = \nu_0$$

When $t > t_0$ ($0^\circ < \theta < 90^\circ$)

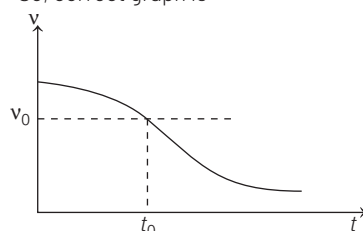
i.e. sound source is moving away from observer.

So, in this case expression of observed frequency will be

$$\nu_{\text{observed}} = \frac{\nu_{\text{sound}}}{\nu_{\text{sound}} + \nu \cos \theta} \cdot \nu_0$$

With time θ decrease and $\cos \theta$ increases, so ν_{observed} decreases continuously.

So, correct graph is



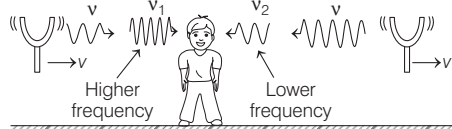
- 65 A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/s. The oscillation frequency of each tuning fork is $\nu_0 = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to

[2020, 7 Jan Shift-II]

- (a) $\frac{1}{8}$ m/s (b) $\frac{1}{2}$ m/s
(c) 1 m/s (d) $\frac{1}{4}$ m/s

Ans. (d)

Due to Doppler's effect, frequency of approaching tuning fork is higher and receding tuning fork is lower than emitted frequency.



Frequency received from tuning fork moving towards observer,

$$\nu_1 = \nu \left(\frac{\nu}{\nu - \nu_s} \right)$$

where, ν = frequency emitted by tuning fork, ν = speed of sound and ν_s = speed of tuning fork.

Frequency received from receding tuning fork,

$$\nu_2 = \nu \left(\frac{\nu}{\nu + \nu_s} \right)$$

Beat frequency is the difference of both received frequencies.

$$\text{So, } \nu_{\text{beat}} = \nu_1 - \nu_2$$

$$\Rightarrow \nu_{\text{beat}} = \nu \left(\frac{\nu}{\nu - \nu_s} \right) - \nu \left(\frac{\nu}{\nu + \nu_s} \right) \\ = \frac{2\nu\nu_s\nu}{\nu^2 - \nu_s^2} = \frac{2\nu\nu_s}{\nu \left(\frac{1 - \nu_s^2}{\nu^2} \right)}$$

$$\text{If } \nu_s \ll \nu, \text{ then } \nu_{\text{beat}} = \frac{2f\nu_s}{\nu}$$

$$\text{Here, } \nu = 1400 \text{ Hz, } \nu_s = 350 \text{ ms}^{-1}$$

$$\text{and } \nu_{\text{beat}} = 2 \text{ s}^{-1}$$

$$\text{So, } 2 = \frac{2 \times 1400 \times \nu_s}{350}$$

$$\Rightarrow \nu_s = \frac{1}{4} \text{ ms}^{-1}$$

- 66 Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B?

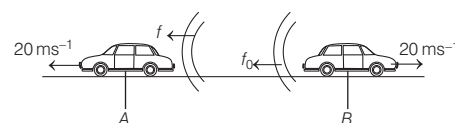
(speed of sound in air = 340 ms^{-1})

[2019, 9 April Shift-II]

- (a) 2060 Hz (b) 2250 Hz
(c) 2300 Hz (d) 2150 Hz

Ans. (b)

The given condition can be shown below as,



Here, source and observer both are moving away from each other. So, by Doppler's effect, observed frequency is given by

$$f = f_0 \left(\frac{\nu + \nu_o}{\nu - \nu_s} \right) \quad \dots (i)$$

where, ν = speed of sound = 340 ms^{-1} ,

ν_o = speed of observer
= -20 ms^{-1} ,

ν_s = speed of source = -20 ms^{-1} ,

f_0 = true frequency

and f = apparent frequency
= 2000 Hz

Substituting the given values in Eq. (i), we get

$$2000 = \left(\frac{340 - 20}{340 + 20} \right) \times f_0$$

$$\Rightarrow f_0 = \frac{2000 \times 360}{320} = 2250 \text{ Hz}$$

- 67** A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are in ms^{-1} , (Take, speed of sound = 300 m/s)

[2019, 10 April Shift-I]

- (a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

Ans. (b)

Given,

Frequency of sound source (f_0) = 500 Hz
 Apparent frequency heard by observer 1, $f_1 = 480$ Hz and apparent frequency heard by observer 2, $f_2 = 530$ Hz.

Let v_0 be the speed of sound.

When observer moves away from the source, Apparent frequency,

$$f_1 = f_0 \left(\frac{v - v_0'}{v} \right) \quad \dots (i)$$

When observer moves towards the source,

$$\text{Apparent frequency, } f_2 = f_0 \left(\frac{v + v_0''}{v} \right) \quad \dots (ii)$$

Substituting values in Eq. (i), we get

$$480 = 500 \left(\frac{300 - v_0'}{300} \right)$$

$$\Rightarrow 96 \times 3 = 300 - v_0'$$

$$\Rightarrow v_0' = 12 \text{ m/s}$$

Substituting values in Eq. (ii), we get

$$530 = 500 \left(\frac{330 + v_0''}{300} \right)$$

$$\Rightarrow 106 \times 3 = 300 + v_0''$$

$$\Rightarrow v_0'' = 18 \text{ m/s}$$

Thus, their respective speeds (in m/s) is 12 and 18.

- 68** A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the

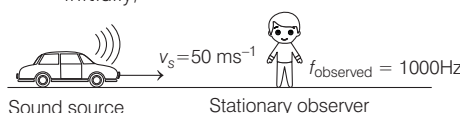
observer after crossing him? (Take, velocity of sound in air is 350 m/s)

[2019, 10 April Shift-II]

- (a) 807 Hz (b) 1143 Hz
 (c) 750 Hz (d) 857 Hz

Ans. (c)

Initially,



After sometime,



When source is moving towards stationary observer, frequency observed is more than source frequency due to Doppler's effect, it is given by

$$f_{\text{observed}} = f \left(\frac{v}{v - v_s} \right)$$

where, f = source frequency,
 f_0 = observed frequency = 1000 Hz,
 v = speed of sound in air = 350 ms^{-1}
 and v_s = speed of source = 50 ms^{-1}

$$\text{So, } f = \frac{f_{\text{obs}}(v - v_s)}{v} = \frac{1000(350 - 50)}{350}$$

$$= \frac{6000}{7} \text{ Hz}$$

When source moves away from stationary observer, observed frequency will be lower due to Doppler's effect and it is given by

$$f_0 = f \left(\frac{v}{v + v_s} \right) = \frac{6000 \times 350}{7 \times (350 + 50)}$$

$$= \frac{6000 \times 350}{7 \times 400} = 750 \text{ Hz}$$

- 69** A submarine A travelling at 18 km/h is being chased along the line of its velocity by another submarine B travelling at 27 km/h. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν . The value of ν is close to (Speed of sound in water = 1500 ms^{-1})

[2019, 12 April Shift-I]

- (a) 504 Hz (b) 507 Hz
 (c) 499 Hz (d) 502 Hz

Ans. (d)

Given, velocity of submarine (A),

$$v_A = 18 \text{ km/h} = \frac{18000}{3600} \text{ m/s}$$

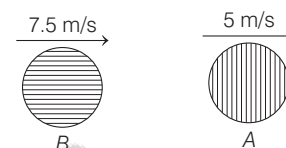
or $v_A = 5 \text{ m/s}$... (i)

and velocity of submarine (B),

$$v_B = 27 \text{ km/h} = \frac{27000}{3600} \text{ m/s}$$

or $v_B = 7.5 \text{ m/s}$... (ii)

Signal sent by submarine (B) is detected by submarine (A) can be shown as



Frequency of the signal, $f_0 = 500$ Hz

So, in this relative motion, frequency received by submarine (A) is

$$f_1 = \left(\frac{v_s + v_A}{v_s - v_B} \right) f_0 = \left(\frac{1500 + 5}{1500 - 7.5} \right) 500 \text{ Hz}$$

$$\Rightarrow f_1 = \frac{1495}{1492.5} \times 500 \text{ Hz}$$

The reflected frequency f_1 is now received back by submarine (B).

So, frequency received at submarine (B) is

$$f_2 = \left(\frac{v_s + v_B}{v_s + v_A} \right) f_1$$

$$= \left(\frac{1500 + 7.5}{1500 + 5} \right) \left(\frac{1495}{1492.5} \right) 500 \text{ Hz}$$

$$\Rightarrow f_2 = \left(\frac{1507.5}{1505} \right) \left(\frac{1495}{1492.5} \right) 500 \text{ Hz}$$

$$\Rightarrow f_2 = 1.00166 \times 1.00167 \times 500$$

$$\Rightarrow f_2 = 501.67 \text{ Hz} \approx 502 \text{ Hz}$$

- 70** Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz.

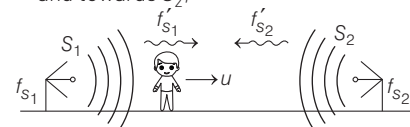
A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equal to

[2019, 12 April Shift-II]

- (a) 5.5 m/s (b) 15.0 m/s
 (c) 2.5 m/s (d) 10.0 m/s

Ans. (c)

When observer moves away from S_1 and towards S_2 ,



then due to Doppler's effect observed frequencies of sources by observer are

$$f'_{S_1} = \frac{v - v_o}{v} \cdot f_{S_1}$$

(observer moving away from source)

$$\text{and } f'_{S_2} = \left(\frac{v + v_o}{v} \right) \cdot f_{S_2}$$

(observer moving towards source)

(where, v = speed of sound, v_o = speed of observer)

So, beat frequency heard by observer is

$$f_b = f'_{S_2} - f'_{S_1}$$

$$\text{Here, } v_o = u, v = 330 \text{ ms}^{-1}$$

$$f_b = 10 \text{ Hz}, f_{S_1} = f_{S_2} = 660 \text{ Hz}$$

On putting the values, we get

$$\begin{aligned} f_b &= f'_{S_2} - f'_{S_1} \\ &= \left(\frac{v + v_o}{v} \right) \cdot f_{S_2} - \left(\frac{v - v_o}{v} \right) \cdot f_{S_1} \\ &= f_{S_1} \left\{ \frac{v + v_o}{v} - \frac{v - v_o}{v} \right\} = f_{S_1} \cdot \frac{2v_o}{v} \end{aligned}$$

$$\Rightarrow 10 = \frac{660 \times 2u}{330} \quad [\because v_o = u]$$

$$\Rightarrow u = \frac{330 \times 10}{2 \times 660}$$

$$\Rightarrow u = 2.5 \text{ ms}^{-1}$$

- 71** A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 . If the speed of the train is reduced to 17 m/s, the frequency registered is f_2 . If speed of sound is 340 m/s, then the ratio f_1 / f_2 is **[2019, 10 Jan Shift-I]**

- (a) $\frac{19}{18}$ (b) $\frac{21}{20}$
(c) $\frac{20}{19}$ (d) $\frac{18}{17}$

Ans. (a)

When a source is moving towards a stationary observer, observed frequency is given by

$$f_{\text{observed}} = f \left(\frac{v}{v + v_s} \right)$$

where, f = frequency of sound from the source,

v = speed of sound and v_s = speed of source.

Now applying above formula to two different conditions given in problem, we get

$$f_1 = \text{Observed frequency initially}$$

$$= f \left(\frac{340}{340 - 34} \right) = f \left(\frac{340}{306} \right)$$

and f_2 = Observed frequency when speed of source is reduced

$$= f \left(\frac{340}{340 - 17} \right) = \frac{340}{323}$$

$$\text{So, the ratio } f_1 : f_2 \text{ is } \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

- 72** An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = $3 \times 10^8 \text{ ms}^{-1}$) **[JEE Main 2017]**

- (a) 12.1 GHz
(b) 17.3 GHz
(c) 15.3 GHz
(d) 10.1 GHz

Ans. (b)

As the observer is moving towards the source, so frequency of waves emitted by the source will be given by the formula

$$f_{\text{observed}} = f_{\text{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c} \right)^{1/2}$$

$$\text{Here, frequency } \frac{v}{c} = \frac{1}{2}$$

$$\text{So, } f_{\text{observed}} = f_{\text{actual}} \cdot \left(\frac{3/2}{1/2} \right)^{1/2}$$

$$\therefore f_{\text{observed}} = 10 \times \sqrt{3} = 17.3 \text{ GHz}$$

- 73** A train is moving on a straight track with speed 20 ms^{-1} . It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound = 320 ms^{-1}) **[JEE Main 2015]**

- (a) 6% (b) 12%
(c) 18% (d) 24%

Ans. (b)

Apparent frequency heard by the person before crossing the train.

$$f_1 = \left(\frac{c}{c - v_s} \right) f_0 = \left(\frac{320}{320 - 20} \right) 1000$$

Similarly, apparent frequency heard, after crossing the trains

$$f_2 = \left(\frac{c}{c + v_s} \right)$$

$$f_0 = \left(\frac{320}{320 + 20} \right) 1000$$

[c = speed of sound]

$$\Delta f = f_1 - f_2 = \left(\frac{2cv_s}{c^2 - v_s^2} \right) f_0$$

$$\begin{aligned} \text{or } \frac{\Delta f}{f_0} \times 100 &= \left(\frac{2cv_s}{c^2 - v_s^2} \right) \times 100 \\ &= \frac{2 \times 320 \times 20}{300 \times 340} \times 100 \\ &= \frac{2 \times 32 \times 20}{3 \times 34} \\ &= 12.54\% \approx 12\% \end{aligned}$$

- 74** A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is **[AIEEE 2011]**

(a) a standing wave having nodes at

$$x = \left(n + \frac{1}{2} \right) \frac{\lambda}{2}, n = 0, 1, 2$$

(b) a wave travelling along + x direction

(c) a wave travelling along -x direction

(d) a standing wave having nodes at

$$x = \frac{n\lambda}{2}; n = 0, 1, 2$$

Ans. (a)

By superposition principle,

$$y = y_1 + y_2 = A \sin(\omega t - kx) + A \sin(\omega t + kx)$$

$$y = 2A \sin \omega t \cos kx$$

Clearly, it is equation of standing wave for position of nodes $y = 0$.

$$\text{i.e., } x = (2n + 1) \frac{\lambda}{4}$$

$$\Rightarrow \left(n + \frac{1}{2} \right) \frac{\lambda}{2}, n = 0, 1, 2, 3$$

- 75** A motor cycle starts from rest and accelerates along a straight path at 2 ms^{-2} . At the starting point of the motor cycle, there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}) **[AIEEE 2009]**

- (a) 49 m (b) 98 m
(c) 147 m (d) 196 m

Ans. (b)For motor cycle, $u = 0, a = 2 \text{ ms}^{-2}$

Observer is in motion and source is at rest, then apparent frequency,

$$n' = n \frac{v - v_o}{v + v_s}$$

$$\Rightarrow \frac{94}{100} n = n \frac{330 - v_o}{330}$$

$$\Rightarrow 330 - v_o = \frac{330 \times 94}{100}$$

$$\Rightarrow v_o = 330 - \frac{94 \times 33}{10}$$

$$= \frac{33 \times 6}{10} \text{ ms}^{-1}$$

$$s = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100}$$

$$= \frac{9 \times 1089}{100} \approx 98 \text{ m}$$

- 76** A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed $v \text{ ms}^{-1}$. The velocity of sound in air is 300 ms^{-1} . If the person can hear frequencies upto a

maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is [AIEEE 2006]

- (a) $15\sqrt{2} \text{ ms}^{-1}$ (b) $15/\sqrt{2} \text{ ms}^{-1}$
(c) 15 ms^{-1} (d) 30 ms^{-1}

Ans. (c)

Given, velocity of sound in air = 300 m/s

If a source of sound is moving towards a stationary listener, the frequency heard by the listener would be different from the actual frequency of the source, this apparent frequency is given by

$$f_{\text{app}} = \left(\frac{v_{\text{sound in air}}}{v_{\text{sound in air}} \pm v_{\text{source}}} \right) f$$

symbols have their usual meanings.

In the denominator, +ve sign would be taken when source is receding away from the listener, while -ve sign would be taken when source is approaching the listener.

Let v be the maximum value of source velocity for which the person is able to hear the sound, then

$$10000 = f_{\text{app}} = \left(\frac{300}{300 - v} \right) \times 9500$$

$$\text{or } v = 15 \text{ ms}^{-1}$$

- 77** An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

[AIEEE 2005]

- (a) Zero (b) 0.5%
(c) 5% (d) 20%

Ans. (d)

$$v_o = \frac{v}{5} \Rightarrow v_o = \frac{320}{5} = 64 \text{ ms}^{-1}$$

When observer moves towards the stationary source, then

$$n' = \left(\frac{v + v_o}{v} \right) n$$

$$\Rightarrow n' = \left(\frac{320 + 64}{320} \right) n$$

$$\text{or } n' = \left(\frac{384}{320} \right) n \text{ or } \frac{n'}{n} = \frac{384}{320}$$

Hence, percentage increase

$$\left(\frac{n' - n}{n} \right) = \left(\frac{384 - 320}{320} \times 100 \right)$$

$$= \left(\frac{64}{320} \times 100 \right) = 20\%$$

15

Electric Charges and Fields

TOPIC 1

Electric Charges and Coulomb's Law

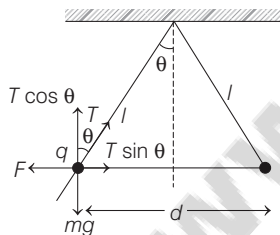
- 01** Two identical tennis balls each having mass m and charge q are suspended from a fixed point by threads of length l . What is the equilibrium separation when each thread makes a small angle θ with the vertical? [2021, 27 July Shift-I]

$$(a) d = \left(\frac{q^2 l}{2\pi\epsilon_0 mg} \right)^{\frac{1}{2}} \quad (b) d = \left(\frac{q^2 l}{2\pi\epsilon_0 mg} \right)^{\frac{1}{3}}$$

$$(c) d = \left(\frac{q^2 l^2}{2\pi\epsilon_0 m^2 g} \right)^{\frac{1}{3}} \quad (d) d = \left(\frac{q^2 l^2}{2\pi\epsilon_0 m^2 g^2} \right)^{\frac{1}{3}}$$

Ans. (b)

The given situation is shown below



$\therefore$ According to Coulomb's law,

$$F = \frac{kq_1 q_2}{r^2}$$

Here, $q_1 = q_2 = q$

$\therefore$ Force due to charge,

$$F = \frac{k \times q \times q}{d^2} \quad \{\because r = d \text{ here}\}$$

$$\Rightarrow F = \frac{kq^2}{d^2} \quad \dots(i)$$

Using above diagram, we can write

$$T \cos \theta = mg \text{ and } T \sin \theta = F$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{F}{mg} = \frac{kq^2}{\frac{d^2}{mg}}$$

{using Eq (i)}

$$\Rightarrow \tan \theta = \frac{kq^2}{d^2 mg} \quad \dots(ii)$$

$$\therefore \text{We know that, } \tan \theta \approx \sin \theta \approx \frac{d}{2l} \quad \dots(iii)$$

$\therefore$ From Eqs. (ii) and (iii), we get

$$\frac{d}{2l} = \frac{kq^2}{d^2 mg}$$

$$\Rightarrow d^3 mg = kq^2 2l \Rightarrow d^3 = \frac{kq^2 2l}{mg}$$

$$\therefore k = \frac{1}{4\pi\epsilon_0}$$

$$\Rightarrow d^3 = \frac{q^2 2l}{4\pi\epsilon_0 mg}$$

$$\Rightarrow d^3 = \frac{q^2 l}{2\pi\epsilon_0 mg}$$

$$\Rightarrow d = \left(\frac{q^2 l}{2\pi\epsilon_0 mg} \right)^{\frac{1}{3}}$$

- 02** A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance 2 m when each is carrying same charge q . If the free charged particle is displaced from its equilibrium position through distance x ($x \ll 1$ m), the particle executes SHM. Its angular frequency of oscillation will be $\times 10^5$ rad/s, if $q^2 = 10C^2$.

[2021, 25 July Shift-I]

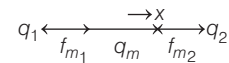
Ans. (6000)

Given,

$$\text{Mass, } m = 1 \text{ mg} = 1 \times 10^{-6} \text{ kg,}$$

Separation between two charges, $d = 2$ m and $q^2 = 10C^2$

When free charge is displaced from its position, then from given diagram,



$$F_{\text{net}} = F_{m1} - f_{m2} = -m\omega^2 x$$

where, F_{net} = net electrostatic force,

F_{m1}, F_{m2} = force on m due to q_1 and q_2 respectively,

ω and x = angular frequency and equilibrium distance, respectively.

By using Coulomb's law,

$$F = \frac{kq_1 q_2}{r^2}$$

where, k = Coulomb's constant

$$= 9 \times 10^9 \text{ N-m}^2 \text{C}^{-2}$$

r = distance between two charges

$$= \frac{d}{2} = 1 \text{ m}$$

$$\therefore F_{\text{net}} = \frac{kq_m q_1}{(r+x)^2} - \frac{kq_m q_2}{(r-x)^2} = -m\omega^2 x$$

$$\Rightarrow \frac{kq^2}{m} \left[\frac{(r-x)^2 - (r+x)^2}{(r^2 - x^2)^2} \right] = -\omega^2 x$$

$$(\because q_1 = q_2 = q_m = q)$$

$$\Rightarrow \frac{9 \times 10^9 \times 10}{10^{-6}} \left[\frac{-4rx}{r^4} \right] = -\omega^2 x$$

$$(\because x \ll 1 \text{ m, } \therefore r^2 - x^2 = r^2)$$

$$\Rightarrow \omega = \sqrt{36 \times 10^{16}} = 6 \times 10^8 \text{ rads}^{-1}$$

From question $\omega = x \times 10^5 \text{ rad/s}$

$$\therefore x = 6000$$

- 03** An electric dipole is placed on X-axis in proximity to a line charge of linear charge density $3.0 \times 10^{-6} \text{ C/m}$. Line charge is placed on

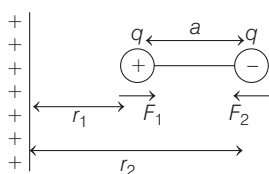
Z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin, respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

[2021, 22 July Shift-II]

- (a) 815.1 nC (b) 8.8 μC
(c) 0.485 nC (d) 4.44 μC

Ans. (d)

Given, linear charge density,
 $\lambda = 3 \times 10^{-6} \text{ C/m}$



$$r_1 = 10 \text{ mm} = 10 \times 10^{-3} \text{ m},$$

$$r_2 = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

and the separation between charges
 $= r_2 - r_1 = 12 - 10 = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

Electric field due to line charge, $E = \frac{2k\lambda}{x}$

Now, force on dipole due to line charge be F_1 and F_2 from +ve and -ve charge respectively, then the net force.

$$F_{\text{net}} = F_1 - F_2 = qE_1 - qE_2$$

$$\Rightarrow 4 = q \left(\frac{2k\lambda}{r_1} - \frac{2k\lambda}{r_2} \right)$$

$$\Rightarrow 4 = 2k\lambda q \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$\Rightarrow 4 = 2 \times 9 \times 10^9 \times 3 \times 10^{-6} \times q$$

$$\times 10^3 \left(\frac{1}{10} - \frac{1}{12} \right)$$

$$\Rightarrow 4 = 54 \times 10^5 \left(\frac{2}{12} \right) q$$

$$\Rightarrow 4 = 9 \times 10^5 q$$

$$\Rightarrow q = \frac{4}{9} \times 10^{-5}$$

$$= 0.44 \times 10^{-5}$$

$$= 4.44 \times 10^{-6}$$

$$= 4.44 \mu\text{C}$$

- 04 A certain charge Q is divided into two parts q and $(Q - q)$. How should the charges Q and q be divided, so

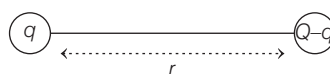
that q and $(Q - q)$ placed at a certain distance apart experience maximum electrostatic repulsion?

[2021, 20 July Shift-I]

- (a) $Q = \frac{q}{2}$ (b) $Q = 2q$
(c) $Q = 4q$ (d) $Q = 3q$

Ans. (d)

(b) Let's say the charge q and $(Q - q)$ are at r distance from each other. This can be shown as



According to Coulomb's law, force between both the parts can be given as

$$F = \frac{kq(Q - q)}{r^2}$$

$$F = \frac{k}{r^2} (qQ - q^2)$$

As we know that $\frac{dF}{dq} = 0$, for maximum

force.

$$\Rightarrow \frac{dF}{dq} = \frac{d}{dq} \left[\frac{k}{r^2} (qQ - q^2) \right] = 0$$

$$\Rightarrow \frac{k}{r^2} (Q - 2q) = 0 \Rightarrow Q = 2q$$

- 05 An infinite number of point charges, each carrying $1 \mu\text{C}$ charge, are placed along the Y-axis at $y = 1 \text{ m}, 2 \text{ m}, 4 \text{ m}, 8 \text{ m}$.

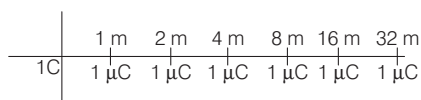
The total force on a 1 C point charge, placed at the origin, is $x \times 10^3 \text{ N}$. The value of x to the nearest integer, is

(Take, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$)

[2021, 18 March Shift-II]

Ans. (12)

The 1 C charge present in the origin and other $1 \mu\text{C}$ charge are placed at 1 m, 2 m, 4 m, 8 m ...



Using the Coulomb's law,

$$F = \frac{kq_1q_2}{r^2}$$

$$F = 9 \times 10^9 \times (1) \times 10^{-6} \left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \dots \right]$$

$$= 9 \times 10^3 \times \left[\frac{1}{1 - \frac{1}{4}} \right] = 12 \times 10^3 \text{ N}$$

The value of x to the nearest integer is 12.

- 06 Find out the surface charge density at the intersection of point $X = 3 \text{ m}$ plane and X-axis, in the region of uniform line charge of 8 nC/m lying along the Z-axis in free space.

[2021, 16 March Shift-II]

- (a) 0.424 nCm^{-2} (b) 47.88 nCm^{-2}
(c) 0.07 nCm^{-2} (d) 4.0 nCm^{-2}

Ans. (a)

Given,

Linear charge density, $\lambda = 8 \text{ nC/m}$
 $= 8 \times 10^{-9} \text{ C/m}$

The relation between surface charge density and linear charge density can be given as

$$\frac{2k\lambda}{r} = \frac{\sigma}{\epsilon_0} \quad \dots(i)$$

where,

k = Coulomb's constant,

λ = linear charge density,

σ = surface charge density,

ϵ_0 = absolute electrical permittivity of free space

and r = distance.

Substituting the values in Eq. (i), we get

$$\sigma = \frac{2k\lambda\epsilon_0}{r}$$

$$= \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-9} \times 8.85 \times 10^{-12}}{3}$$

$$= 0.424 \times 10^{-9} \text{ Cm}^{-2} = 0.424 \text{ nCm}^{-2}$$

- 07 Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21} \times 10^{-8} \text{ C}$. The value of a will be

[Given, $g = 10 \text{ ms}^{-2}$]

[2021, 25 Feb Shift-II]

Ans. (630)

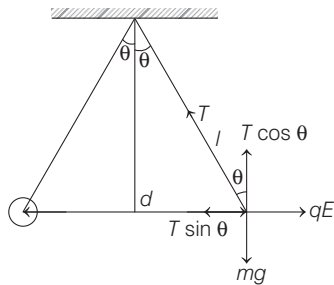
Given, mass of each spheres, $m = 10 \text{ mg}$
 $= 10 \times 10^{-3} \text{ g}$

Length of thread (l) = 0.5 m

Separation between charges, $d = 0.2 \text{ m}$

Charge of each sphere, $q = \frac{a}{21} \times 10^{-8} \text{ C}$

Acceleration due to gravity (g) = 10 ms^{-2}
The situation can be shown as below,



Taking component of tension (T)

$$T \cos \theta = mg \quad \dots(i)$$

$$T \sin \theta = qE = \frac{kq^2}{d^2} \quad \dots(ii)$$

$$\sin \theta = \frac{d/2}{l} = \frac{0.1}{0.5} = \frac{1}{5}$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - 1/25} = \frac{\sqrt{24}}{5}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{kq^2}{d^2 mg}$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \frac{1/5}{\sqrt{24}/5} = \frac{kq^2}{d^2 mg}$$

$$\Rightarrow \frac{1}{\sqrt{24}} = \frac{9 \times 10^9 \times q^2}{(0.2)^2 \times 10 \times 10^{-3} \times 10}$$

$$\Rightarrow q = \sqrt{\frac{(0.2)^2 \times 10^{-1}}{\sqrt{24} \times 9 \times 10^9}}$$

$$\Rightarrow q = 3 \times 10^{-7} \text{ C}$$

$$\Rightarrow \frac{a}{21} \times 10^{-8} = 30 \times 10^{-8}$$

$$a = 630$$

- 08** Two identical conducting spheres with negligible volume have 2.1 nC and -0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m . The electrostatic force acting between the spheres is $\times 10^{-9} \text{ N}$.

[Given, $4\pi\epsilon_0 = \frac{1}{9 \times 10^9} \text{ SI unit}$]

[2021, 25 Feb Shift-II]

Ans. (7.56)

Given, $q_1 = 2.1 \text{ nC} = 2.1 \times 10^{-9} \text{ C}$,

$q_2 = -0.1 \text{ nC} = -0.1 \times 10^{-9} \text{ C}$

Separation (d) = 0.5 m

By Coulomb's law,

$$\text{Force (F)} = \frac{kq_1 q_2}{d^2}$$

where, $k = 9 \times 10^9 \text{ N-m}^2 \text{C}^{-2}$

= Coulomb's constant

$$\therefore F = \frac{9 \times 10^9 \times 2.1 \times 10^{-9} \times (-0.1 \times 10^{-9})}{(0.5)^2}$$

$$= -7.56 \times 10^{-9} \text{ N}$$

- 09** Two electrons each are fixed at a distance $2d$. A third charge proton placed at the mid-point is displaced slightly by a distance x ($x \ll d$) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency? (m = mass of charged particle)

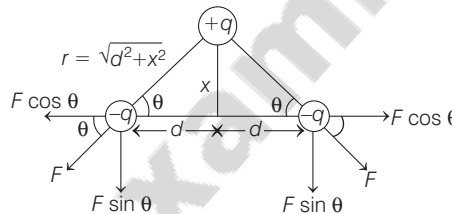
[2021, 24 Feb Shift-II]

(a) $\left(\frac{2q^2}{\pi\epsilon_0 md^3} \right)^{1/2}$ (b) $\left(\frac{\pi\epsilon_0 md^3}{2q^2} \right)^{1/2}$

(c) $\left(\frac{q^2}{2\pi\epsilon_0 md^3} \right)^{1/2}$ (d) $\left(\frac{2\pi\epsilon_0 md^3}{q^2} \right)^{1/2}$

Ans. (c)

The arrangement of charges is shown below



As we know that,

Coulomb's force between two charges.

i.e. q_1 and q_2 ,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(d^2 + x^2)} \quad \dots(i)$$

Here, $q_1 = q_2 = q$

Force in SHM, $F = m\omega^2 x$ $\dots(ii)$

Since, in order to have SHM $+q$ should move downwards and force responsible for this will be only

$$F' = F \sin \theta + F \sin \theta = 2F \sin \theta \quad \dots(iii)$$

Using Eqs. (ii) and (iii), we get

$$2F \sin \theta = m\omega^2 x$$

$$\Rightarrow \frac{2}{4\pi\epsilon_0} \frac{q^2}{(d^2 + x^2)} \sin \theta = m\omega^2 x$$

$$\Rightarrow \frac{2}{4\pi\epsilon_0} \frac{q^2}{(d^2 + x^2)} \cdot \frac{x}{(d^2 + x^2)^{1/2}} = m\omega^2 x$$

$$\Rightarrow \omega = \left(\frac{1}{2\pi\epsilon_0} \frac{q^2}{(d^2 + x^2)^{3/2} m} \right)^{1/2}$$

As, $x \ll d$

$$\therefore \omega = \left(\frac{1}{2\pi\epsilon_0} \frac{q^2}{md^3} \right)^{1/2}$$

- 10** A charge Q is distributed over three concentric spherical shells of radii a, b, c ($a < b < c$) such that their surface charge densities are equal to one another.

The total potential at a point at distance r from their common centre, where $r < a$ would be

[2019, 10 Jan Shift-I]

(a) $\frac{Q(a^2 + b^2 + c^2)}{4\pi\epsilon_0(a^3 + b^3 + c^3)}$

(b) $\frac{Q(a + b + c)}{4\pi\epsilon_0(a^2 + b^2 + c^2)}$

(c) $\frac{Q}{4\pi\epsilon_0(a + b + c)}$

(d) $\frac{Q}{12\pi\epsilon_0} \cdot \frac{ab + bc + ca}{abc}$

Ans. (b)

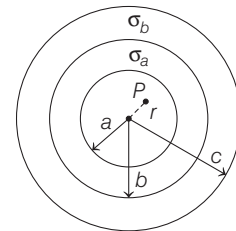
Given charge distribution is shown in the figure below

Given surface charge densities of each shell are same.

$$\therefore \sigma_a = \sigma_b = \sigma_c \quad \dots(i)$$

As, surface charge density of shell of radius ' r ' and having charge ' Q ' is given as

$$\sigma = \frac{Q}{4\pi r^2}$$



So relation (i) can be rewritten as

$$\frac{Q_a}{4\pi a^2} = \frac{Q_b}{4\pi b^2} = \frac{Q_c}{4\pi c^2}$$

$$\Rightarrow Q_a : Q_b : Q_c = a^2 : b^2 : c^2$$

where Q_a , Q_b and Q_c are charges on shell of radius a, b and c , respectively.

Also, $Q_a + Q_b + Q_c = Q$

Hence, $Q_a = \frac{a^2}{a^2 + b^2 + c^2} \cdot Q$

$$Q_b = \frac{b^2}{a^2 + b^2 + c^2} \cdot Q$$

$$\Rightarrow Q_c = \frac{c^2}{a^2 + b^2 + c^2} \cdot Q$$

As we know for charged spherical shell with charge Q of radius ' R ', the potential

at a point

'P' at distance r such that $r < R$ is,

$$V_P = \frac{kQ}{R}$$

$\therefore$ potential at point P at a distance ' r ' = Potential due to Q_a + Potential due to Q_b + Potential due to Q_c

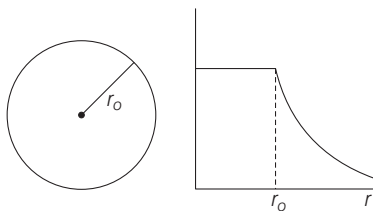
$$= \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

Substituting values of Q_a , Q_b and Q_c , we get

$$V = \frac{Q(a+b+c)}{4\pi\epsilon_0(a^2+b^2+c^2)}$$

- 11** The given graph shows variation (with distance r from centre) of

[2019, 11 Jan Shift-I]



- (a) electric field of a uniformly charged spherical shell
(b) potential of a uniformly charged spherical shell
(c) electric field of a uniformly charged sphere
(d) potential of a uniformly charged sphere

Ans. (b)

For a uniformly charged spherical shell, electric potential inside it is given by

$$V_{\text{inside}} = V_{\text{surface}} = kq/r_0 = \text{constant},$$

(where r_0 = radius of the shell).

and electric potential outside the shell at a distance r is

$$V_{\text{outside}} = \frac{kq}{r} \Rightarrow V \propto 1/r$$

$\therefore$ The given graph represents the variation of r and potential of a uniformly charged spherical shell.

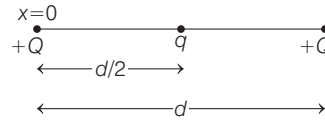
- 12** Three charges $+Q, q, +Q$ are placed respectively at distance $0, d/2$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x=0$ is zero, then value of q is

[2019, 9 Jan Shift-I]

- (a) $\frac{+Q}{2}$ (b) $\frac{+Q}{4}$
(c) $\frac{-Q}{2}$ (d) $\frac{-Q}{4}$

Ans. (d)

The given condition is shown in the figure given below,



Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as,

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$\therefore$ Net force on charge ' Q ' placed at origin i.e. at $x=0$ in accordance with the principle of superposition can be given as

$$F_{\text{net}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

Since, it has been given that, $F_{\text{net}} = 0$.

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times Q}{(d)^2} = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

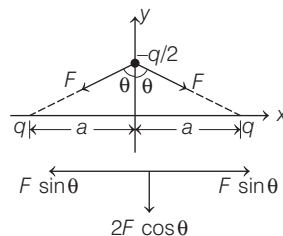
$$\text{or } q = -\frac{Q}{4}$$

- 13** Two charges each equal to q , are kept at $x = -a$ and $x = a$ on the x -axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given, a small displacement ($y \ll a$) along the y -axis, the net force acting on the particle is proportional to

[JEE Main 2013]

- (a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

Ans. (b)



$$2F \cos \theta$$

$$F_{\text{net}} = 2F \cos \theta$$

$$\therefore F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2+a^2})^2} \cdot \frac{y}{\sqrt{y^2+a^2}}$$

[negative sign indicate the net force is towards the mean position]

$$F_{\text{net}} = -\frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \Rightarrow \frac{kq^2 y}{a^3} \propto -y$$

- 14** Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d , ($d \ll l$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result, charges approach each other with a velocity v . Then, as a function of distance x between them, is

[AIEEE 2011]

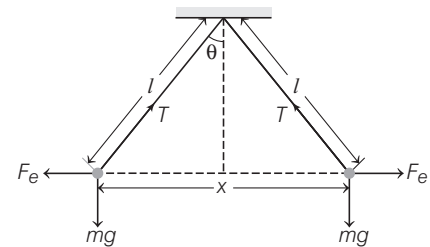
- (a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

Ans. (d)

At any instant,

$$T \cos \theta = mg \quad \dots(i)$$

$$T \sin \theta = F_e = \frac{kq^2}{x^2} \quad \dots(ii)$$



From Eqs. (i) and (ii), we get

$$\frac{kq^2}{x^2} = mg \tan \theta$$

$$\Rightarrow q^2 = \frac{mg}{k} \cdot \frac{x}{2l} \cdot x^2 \quad \left[\because \tan \theta \approx \frac{x}{2l} \right]$$

$$\Rightarrow q^2 = \frac{mg}{2kl} x^3 \quad \dots(iii)$$

$$\Rightarrow 2q \frac{dq}{dt} = \frac{3mg}{2kl} x^2 \frac{dx}{dt}$$

$$\Rightarrow 2 \left(\frac{mg}{2kl} x^3 \right)^{1/2} \frac{dq}{dt} = \frac{3mg}{2kl} x^2 v \quad \left[\because q = \left(\frac{mg}{2kl} x^3 \right)^{1/2} \right]$$

$$\Rightarrow vx^{1/2} = \text{constant} \Rightarrow v \propto x^{-1/2}$$

- 15** Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3} , the angle remains the

same. If density of the material of the sphere is 1.6 g cm^{-3} , then dielectric constant of the liquid is

[AIEEE 2010]

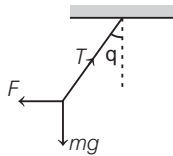
- (a) 4 (b) 3 (c) 2 (d) 1

Ans. (c)

From FBD of sphere, using Lami's theorem,

$$\frac{F}{mg} = \tan \theta \quad \dots(i)$$

When suspended in liquid, as θ remains same.



$$\therefore \frac{F'}{mg \left(1 - \frac{\rho}{d}\right)} = \tan \theta \quad \dots(ii)$$

Using Eqs. (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg \left(1 - \frac{\rho}{d}\right)}, \text{ where } F' = \frac{F}{K}$$

$$\therefore \frac{F}{mg} = \frac{F}{mgK \left(1 - \frac{\rho}{d}\right)}$$

$$\text{or } K = \frac{1}{1 - \frac{\rho}{d}} = \frac{1}{\left(1 - \frac{0.8}{1.6}\right)} = 2$$

- 16** The question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. [AIEEE 2009]

Statement I For a charged particle moving from point P to point Q , the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q .

Statement II The net work done by a conservative force on an object moving along a closed loop is zero.

- (a) Statement I is true, Statement II is false
 (b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
 (c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
 (d) Statement I is false, Statement II is true

Ans. (b)

Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

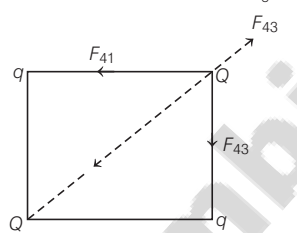
- 17** A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals [AIEEE 2009]

- (a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$

Ans. (a)

Three forces F_{41} , F_{42} and F_{43} acting on Q as shown.

$$\text{Resultant of } F_{41} + F_{43} = \sqrt{2} F_{\text{each}} = \sqrt{2} \frac{1}{4\pi\epsilon_0} \frac{Qq}{d^2}$$



Resultant on Q becomes zero only when q charges are of negative nature.

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{Q \times Q}{(\sqrt{2}d)^2} \Rightarrow \sqrt{2} \frac{dQ}{d^2} = \frac{Q \times Q}{2d^2} \Rightarrow \sqrt{2} \times q = \frac{Q \times Q}{2} \Rightarrow q = \frac{Q}{2\sqrt{2}} \text{ or } \frac{Q}{q} = -2\sqrt{2}$$

- 18** Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with C , then brought in contact with B , and finally removed away from both. The new force of repulsion between B and C is [AIEEE 2004]

- (a) $\frac{F}{4}$ (b) $\frac{3F}{4}$ (c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

Ans. (d)

Let the spherical conductors B and C have same charge as q . The electric force between them is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

Here, r being the distance between them.

When third uncharged conductor A is brought in contact with B , then charge on each conductor

$$q_A = q_B = \frac{q_A + q_B}{2} = \frac{0 + q}{2} = \frac{q}{2}$$

When this conductor A is now brought in contact with C , then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2} = \frac{(q/2) + q}{2} = \frac{3q}{4}$$

Hence, electric force acting between B and C is

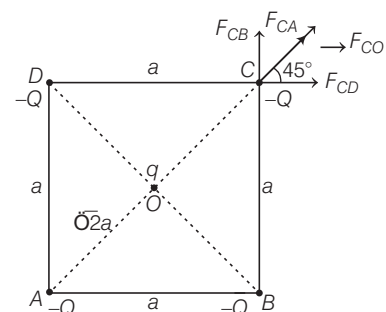
$$F' = \frac{1}{4\pi\epsilon_0} \frac{q_B q_C}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{(q/2)(3q/4)}{r^2} = \frac{3}{8} \left[\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \right] = \frac{3F}{8}$$

- 19** Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is [AIEEE 2004]

- (a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$
 (c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

Ans. (b)

The system is in equilibrium means the force experienced by each charge is zero. It is clear that charge placed at centre would be in equilibrium for any value of q , so we are considering the equilibrium of charge placed at any corner.



$$F_{CD} + F_{CA} \cos 45^\circ + F_{CO} \cos 45^\circ = 0 \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\epsilon_0} \frac{(-Q)(-Q)}{(\sqrt{2}a)^2} \times \frac{1}{\sqrt{2}} + \frac{1}{4\pi\epsilon_0} \frac{(-Q)q}{(\sqrt{2}a/2)^2} \times \frac{1}{\sqrt{2}} = 0$$

$$\text{or } \frac{1}{4\pi\epsilon_0} \frac{Q^2}{a^2} + \frac{1}{4\pi\epsilon_0} \frac{Q^2}{2a^2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{4\pi\epsilon_0} \frac{2Qq}{a^2} \times \frac{1}{\sqrt{2}} = 0$$

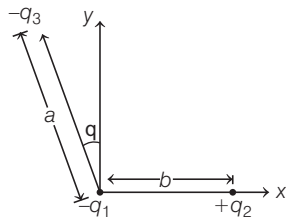
$$\text{or } Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0$$

$$\text{or } 2\sqrt{2}Q + Q - 4q = 0$$

$$\text{or } 4q = (2\sqrt{2} + 1)Q$$

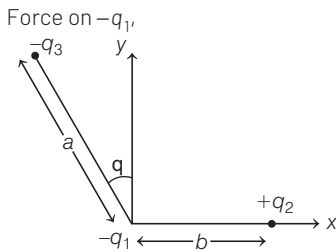
$$\text{or } q = (2\sqrt{2} + 1) \frac{Q}{4}$$

- 20** Three charges $-q_1, +q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to [AIEEE 2003]



- (a) $\frac{q_2}{b^2} - \frac{q_3}{a^2} \cos \theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2} \sin \theta$
 (c) $\frac{q_2}{b^2} + \frac{q_3}{a^2} \cos \theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2} \sin \theta$

Ans. (b)



$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{b^2} \hat{i} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{a^2} [\sin \theta \hat{i} - \cos \theta \hat{j}]$$

From above, x' component of force is

$$F_x = \frac{q_1}{4\pi\epsilon_0} \left[\frac{q_2}{b^2} + \frac{q_3}{a^2} \sin \theta \right]$$

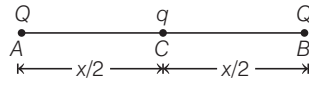
$$F_x \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2} \sin \theta \right]$$

- 21** If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is [AIEEE 2002]

- (a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$
 (c) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$

Ans. (d)

Let charge q be placed at mid-point of line AB as shown below.



Also, $AB = x$ [say]

$$\therefore AC = \frac{x}{2}, BC = \frac{x}{2}$$

For the system to be in equilibrium,

$$F_{Qq} + F_{Qq} = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{Qq}{(x/2)^2} + \frac{1}{4\pi\epsilon_0} \frac{Qq}{x^2} = 0$$

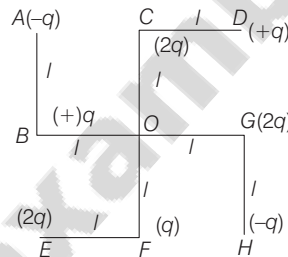
$$\text{or } q = -\frac{Q}{4}$$

TOPIC 2

Electric and Field Lines

- 22** What will be the magnitude of electric field at point O as shown in figure? Each side of the figure is l and perpendicular to each other.

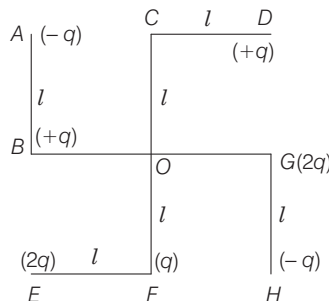
[2021, 27 July Shift-II]



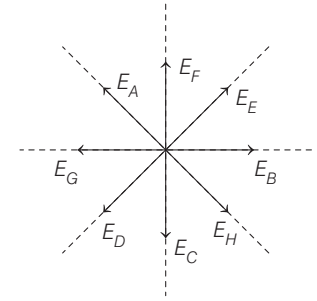
- (a) $\frac{1}{4\pi\epsilon_0} \frac{q}{l^2}$
 (b) $\frac{1}{4\pi\epsilon_0} \frac{q}{(2l)^2} (2\sqrt{2} - 1)$
 (c) $\frac{q}{4\pi\epsilon_0 (2l)^2}$
 (d) $\frac{1}{4\pi\epsilon_0} \frac{2q}{2l^2} (\sqrt{2})$

Ans. (d)

Consider the given figure,



At centre O, different electric fields, will be acting. These can be shown as follows



The electric field in different directions can be given as

$$E_A = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2}; E_B = \frac{kq}{(l)^2} = \frac{kq}{l^2};$$

$$E_C = \frac{2kq}{l^2}; E_D = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2}$$

$$E_E = \frac{2kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2};$$

$$E_F = \frac{kq}{l^2}; E_G = \frac{2kq}{l^2};$$

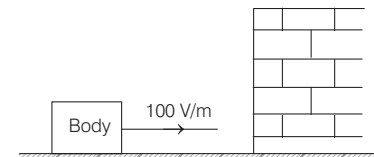
$$E_H = \frac{kq}{(\sqrt{2}l)^2} = \frac{kq}{2l^2}$$

$$\therefore E_{\text{net}} = \sqrt{(E_C - E_F)^2 + (E_G - E_D)^2}$$

$$= \sqrt{\left(\frac{kq}{l^2}\right)^2 + \left(\frac{kq}{l^2}\right)^2}$$

$$= \frac{kq\sqrt{2}}{l^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q(\sqrt{2})}{2l^2}$$

- 23** A body having specific charge $8 \mu\text{C/g}$ is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be s. [2021, 20 July Shift-I]



Ans. (1)

$$\text{Given, } q = 8 \mu\text{C/g} = 8 \times 10^{-6} \text{ C/g}$$

$$= 8 \times 10^{-3} \text{ C/kg}$$

$$s = 10 \text{ cm} = 0.1 \text{ m}$$

$$\Rightarrow E = 100 \text{ V/m}$$

We know that, acceleration,

$$a = \frac{\text{force}(F)}{\text{mass}(m)}$$

$$\Rightarrow a = \frac{qE}{m} \quad [\because F = qE]$$

$$= \frac{8 \times 10^{-6} \times 100}{10^{-3}} = 0.8 \text{ ms}^{-2}$$

As per question, when electric field is switched on, the body strikes to the wall and then returns back.

For one oscillation,

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow 0.1 = \frac{1}{2} \times 0.8t^2 \quad [\because u = 0]$$

$$\Rightarrow 0.2 = 0.8t^2$$

$$\Rightarrow \frac{2}{8} = t^2$$

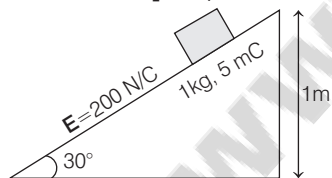
$$\Rightarrow t^2 = \frac{1}{4} \Rightarrow t = \frac{1}{2}$$

$$\therefore \text{Time period} = 2 \times \frac{1}{2} = 1 \text{ s}$$

Therefore, if the collision of the body is perfectly elastic, the time period of motion will be 1s.

- 24** An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field 200 N/C as shown in the figure. A body of mass 1kg and charge 5 mC is allowed to slide down from rest at a height of 1m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom. [Take, $g = 9.8 \text{ m/s}^2$, $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$]

[2021, 26 Feb Shift-II]



- (a) 0.92 s (b) 0.46 s (c) 2.3 s (d) 1.3 s

Ans. (d)

Given, mass of block, $m = 1 \text{ kg}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Inclination, $\theta = 30^\circ$

Electric field, $E = 200 \text{ N/C}$

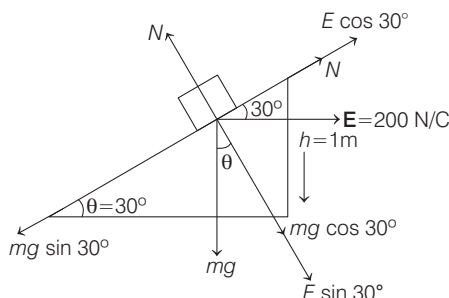
Coefficient of friction, $\mu = 0.2$

Charge, $q = 5 \text{ mC} = 5 \times 10^{-3} \text{ C}$

Let friction force, $f = \mu N$

where, N be the normal reaction.

Since, net force is zero along the perpendicular direction of incline.



Therefore, force along Y-axis will be zero.

$$\Rightarrow N = mg \cos 30^\circ + qE \sin 30^\circ$$

$$\Rightarrow N = 1 \times 9.8 \times \frac{\sqrt{3}}{2} + 5 \times 10^{-3} \times 200 \times \frac{1}{2}$$

$$= 8.49 + 0.5$$

$$= 8.99 \text{ N} \approx 9 \text{ N}$$

$$\therefore f = \mu N = \frac{2}{10} \times 9$$

$$= \frac{18}{10} = 1.8 \text{ N}$$

Now, total force along the plane of incline, $mg \sin 30^\circ - f - qE \cos 30^\circ = ma$

$$\Rightarrow 1 \times 9.8 \times \frac{1}{2} - 1.8 - 5 \times 10^{-3} \times \frac{200\sqrt{3}}{2} = a$$

$$\Rightarrow 5 - 1.8 - 1.732/2 = a$$

$$\Rightarrow a = 2.34 \text{ ms}^{-2}$$

Since, initial velocity of body, $u = 0 \text{ ms}^{-1}$

and distance along incline, $s = h / \sin 30^\circ = 1 / \sin 30^\circ = 2$

By using second equation of motion,

$$s = ut + \frac{1}{2}at^2$$

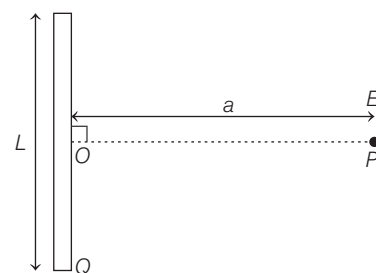
$$\Rightarrow 2 = 0 + \frac{1}{2} \times 2.34 \times t^2$$

$$\Rightarrow t^2 = \frac{4}{2.34}$$

$$\Rightarrow t = \frac{2}{\sqrt{2.34}} = 0.65 \times 2 = 1.3 \text{ s}$$

- 25** Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q . The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2} L$.

[2021, 26 Feb Shift I]



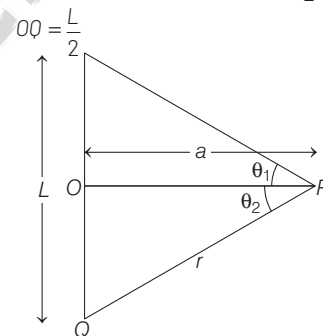
- (a) $\frac{\sqrt{3} Q}{4\pi\epsilon_0 L^2}$ (b) $\frac{Q}{3\pi\epsilon_0 L^2}$
(c) $\frac{Q}{2\sqrt{3}\pi\epsilon_0 L^2}$ (d) $\frac{Q}{4\pi\epsilon_0 L^2}$

Ans. (c)

Given, length of conductor = L

Charge on conductor = Q

According to figure, $OP = a = \frac{\sqrt{3}}{2} L$,



$$\text{Let } PQ = r = \sqrt{OP^2 + OQ^2}$$

$$\Rightarrow PQ = \sqrt{\left(\frac{\sqrt{3}L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}$$

$$= \sqrt{\frac{3L^2}{4} + \frac{L^2}{4}} = L$$

and E be the electric field at point P.

Since, E (due to finite wire)

$$= \frac{k\lambda}{a} (\sin \phi_1 + \sin \phi_2) \quad \dots (i)$$

where, k = Coulomb's constant = $\frac{1}{4\pi\epsilon_0}$

$$\lambda = \text{linear charge density} = \frac{Q}{L}$$

$$\text{and } \sin \phi_1 = \sin \phi_2 = \frac{L/2}{L} = \frac{1}{2}$$

Substituting the above value in Eq. (i), we get

$$E = \frac{k\lambda}{a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \frac{1}{\frac{\sqrt{3}L}{2}}$$

$$= \frac{1}{2\sqrt{3}\pi\epsilon_0} \frac{Q}{L^2}$$



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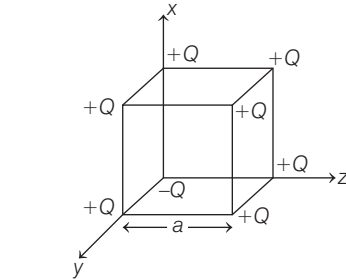


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- 26** A cube of side a has point charges $+Q$ located at each of its vertices except at the origin, where the charge is $-Q$. The electric field at the centre of cube is

[2021, 24 Feb Shift-I]



- (a) $\frac{-Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (b) $\frac{Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (c) $\frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$
 (d) $\frac{2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$

Ans. (c)

We can replace $-Q$ charge at origin by $+Q$ and $-2Q$. Now, due to $+Q$ charge at every corner of cube, electric field at centre of cube is zero. So, net electric field at centre is only due to $-2Q$ charge at origin. Vector form of electric field strength,

$$\mathbf{E} = \frac{Kqr}{r^3}$$

Here, position vector, $\mathbf{r} = \frac{a}{2}(\hat{x} + \hat{y} + \hat{z})$

$$\Rightarrow |\mathbf{r}| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{\sqrt{3}a}{2}$$

$$\Rightarrow \mathbf{E} = \frac{1}{4\pi\epsilon_0} \times \frac{(-2Q)(a/2)}{\left(\frac{a\sqrt{3}}{2}\right)^3} (\hat{x} + \hat{y} + \hat{z})$$

$$\mathbf{E} = \frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2}(\hat{x} + \hat{y} + \hat{z})$$

- 27** An oil drop of radius 2 mm with a density 3 g cm^{-3} is held stationary under a constant electric field $3.55 \times 10^5 \text{ Vm}^{-1}$ in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess? (Take, $g = 9.81 \text{ m/s}^2$) [2021, 18 March Shift-I]

- (a) 48.8×10^{11} (b) 1.73×10^{10}
 (c) 17.3×10^{10} (d) 1.73×10^{12}

Ans. (b)

Given, the radius of oil drop, $r = 2 \text{ mm}$
 $= 0.002 \text{ m}$

The density of oil drop, $\rho = 3 \text{ g/cm}^3$
 $= 3 \times 10^3 \text{ kg/m}^3$

The constant electric field,
 $E = 3.55 \times 10^5 \text{ Vm}^{-1}$

Under stationary condition of oil drop,

$$mg = qE$$

$$\Rightarrow \rho Vg = qE \quad [\because m = \rho V]$$

$$\Rightarrow \rho \left(\frac{4}{3} \pi r^3 \right) g = neE$$

$$\left[\because V = \frac{4}{3} \pi r^3 \text{ and } q = ne \right]$$

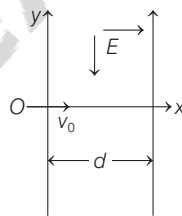
$$n = \frac{\rho \left(\frac{4}{3} \pi r^3 \right) g}{eE}$$

$$\Rightarrow n = \frac{3 \times 10^3 \times \left(\frac{4}{3} \pi (0.002)^3 \right) 9.81}{1.6 \times 10^{-19} \times (3.55 \times 10^5)}$$

$$\Rightarrow n = 1.73 \times 10^{10}$$

- 28** A charged particle (mass m and charge q) moves along X-axis with velocity v_0 . When it passes through the origin it enters a region having uniform electric field $\mathbf{E} = -E\hat{j}$ which extends upto $x = d$. Equation of path of electron in the region ($x > d$) is

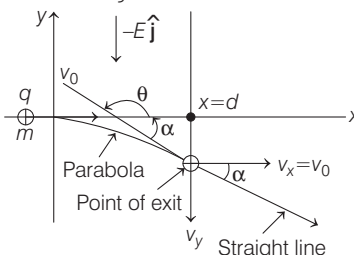
[2020, 2 Sep Shift-I]



- (a) $y = \frac{qEd}{mv_0^2}x$ (b) $y = \frac{qEd}{mv_0^2}(x-d)$
 (c) $y = \frac{qEd}{mv_0^2}\left(\frac{d}{2} - x\right)$ (d) $y = \frac{qEd^2}{mv_0^2}x$

Ans. (c)

The path followed by charged particle is shown in figure below,



When particle is in the region of electric field ($0 < x \leq d$); it has two velocity components

$$\text{Along X-axis, } v_x = v_0 \hat{i} \quad \dots(i)$$

$$\text{Along Y-axis, } v_y = a_y \cdot t \hat{j}$$

where, t = time in which particle crosses region of field.

$$\Rightarrow v_y = \frac{F}{m} \cdot t \hat{j} \Rightarrow v_y = \frac{-qEt}{m} \hat{j} \quad \dots(ii)$$

Now, if particle crosses region of field in time t , then

$$d = v_x t \Rightarrow t = \frac{d}{v_0} \quad \dots(iii)$$

So, from Eqs. (ii) and (iii), we get

$$v_y = \frac{-qEd}{mv_0} \hat{j}$$

Hence, angle α is given by

$$\tan \alpha = \frac{|v_y|}{|v_x|} = \frac{qEd}{mv_0^2}$$

So, slope of path (straight line) of particle when it comes out of region of field is

$$m' = \tan \theta = \tan (180^\circ - \alpha) = -\tan \alpha$$

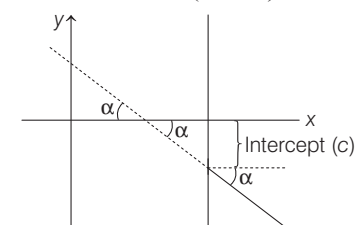
$$= \frac{-qEd}{mv_0^2}$$

Now, y-coordinate at point of exit is

$$y = \frac{1}{2} a_y t^2 \Rightarrow y = \frac{-1}{2} \left(\frac{qE}{m} \right) \cdot t^2 = -\frac{1}{2} \frac{qEd^2}{mv_0^2}$$

So, the intercept length c is $-y$ as shown in figure.

$$c = \left(\frac{1}{2} \frac{qEd^2}{mv_0^2} \right)$$



$\therefore$ Equation of path will be

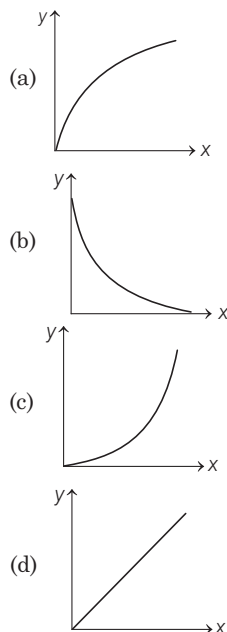
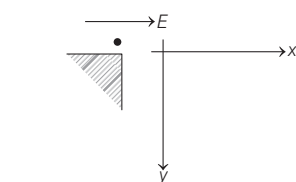
$$y = m'x + c$$

$$\Rightarrow y = \frac{-qEdx}{mv_0^2} + \frac{1}{2} \frac{qEd^2}{mv_0^2} = \frac{qEd}{mv_0^2} \left(\frac{d}{2} - x \right)$$

Hence, correct option is (c).

- 29** A small point mass carrying some positive charge on it, is released from the edge of a table. There is an uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale)

[2020, 2 Sep Shift-II]



Ans. (d)

As gravitational force is considerably smaller than electric force. So, there is only one prominent acceleration in the direction of electric field.

Hence, path of charged particle of small mass is nearly a straight line.

Hence, the graph in option (d) is correct.

- 30** A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x -direction, where a and E_0 are constants. Initially, the particle was at rest at $x = 0$. Other than the initial position, the kinetic energy of the particle becomes zero when the distance of the particle from the origin is

[2020, 4 Sep Shift-II]

- (a) $\sqrt{\frac{1}{a}}$ (b) a (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{2}{a}}$

Ans. (c)

Here, initial kinetic energy, $K_i = 0$

[at $x = 0$]

Final kinetic energy, $K_f = 0$ [at $x = x_0$ (say)]

So, change in kinetic energy,

$$\Delta K = K_f - K_i = 0$$

From work-energy theorem,

Work done = Change in kinetic energy

i.e., $W = \Delta K \therefore W = 0$

$$\Rightarrow \int F dx = 0 \Rightarrow \int qE dx = 0 \quad (\because F = qE)$$

$$\Rightarrow \int q[E_0(1 - ax^2)] dx = 0$$

[given that, $E = E_0(1 - ax^2)$]

$$\Rightarrow qE_0 \int_0^{x_0} (1 - ax^2) dx = 0$$

$$\Rightarrow qE_0 \left[x - \frac{ax^3}{3} \right]_0^{x_0} = 0$$

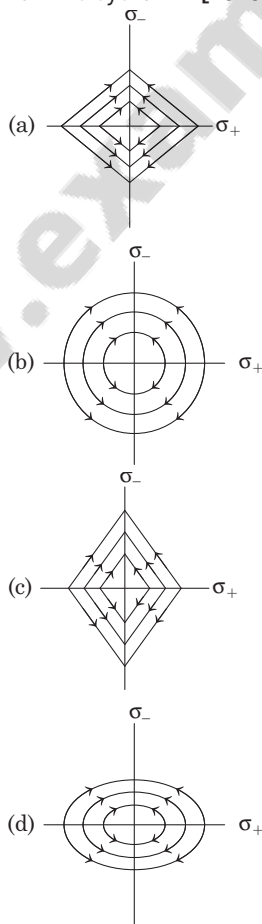
$$\Rightarrow \left[x_0 - \frac{ax_0^3}{3} \right] - [0 - 0] = 0$$

$$\Rightarrow x_0 - \frac{ax_0^3}{3} = 0 \Rightarrow x_0 = \frac{ax_0^3}{3}$$

$$\Rightarrow 1 = \frac{ax_0^2}{3} \Rightarrow x_0^2 = \frac{3}{a} \Rightarrow x_0 = \sqrt{\frac{3}{a}}$$

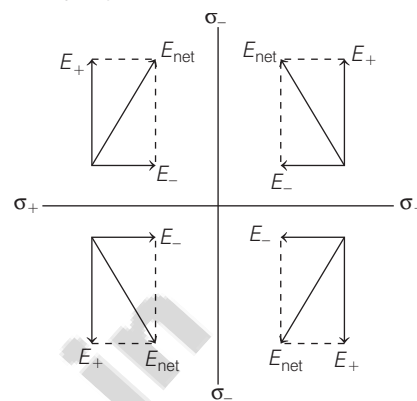
Hence, option (c) is correct.

- 31** Two charged thin infinite plane sheets of uniform surface charge densities σ_+ and σ_- , where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system? [2020, 4 Sep Shift-I]



Ans. (c)

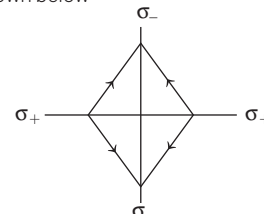
Electric field in each quadrant will look like this



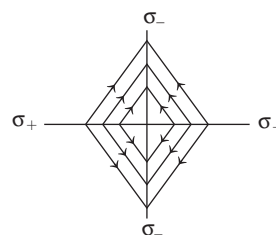
$$\therefore |\sigma_+| > |\sigma_-|$$

$$\therefore |E_+| > |E_-|$$

So, the final electric field will become as shown below



So, the nearest matching option is



Hence, correct option is (c).

- 32** Consider the force F on a charge q due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statement is true for F , if q is placed at distance r from the centre of the shell?

[2020, 6 Sep Shift-II]

(a) $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{R^2}$ (for $r < R$)

(b) $\frac{1}{4\pi\epsilon_0} \frac{Qq}{R^2} > F > 0$ (for $r < R$)

(c) $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$ (for $r > R$)

(d) $F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{R^2}$ (for all r)

Ans. (c)

To calculate force on a point charge q , we need to find electric field due to uniformly charged spherical shell at various points.

If $r < R$, i.e., inside the shell, then $E = 0$

$$\Rightarrow F = qE = 0$$

If $r > R$, i.e., outside the shell, then

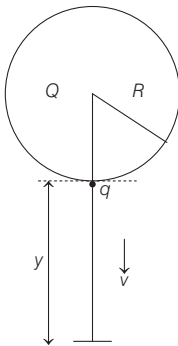
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\Rightarrow F = qE \Rightarrow F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2}$$

If $r = R$, i.e., at surface of shell, then

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \Rightarrow F = \frac{Qq}{4\pi\epsilon_0 R^2}$$

- 33** A solid sphere of radius R carries a charge $Q + q$ distributed uniformly over its volume. A very small point-like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q . If it acquires a speed v when it has fallen through a vertical height y (see figure), then (Assume the remaining portion to be spherical.) [2020, 5 Sep Shift-I]



- (a) $v^2 = y \left[\frac{qQ}{4\pi\epsilon_0 R^2 y m} + g \right]$
 (b) $v^2 = 2y \left[\frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$
 (c) $v^2 = y \left[\frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$
 (d) $v^2 = 2y \left[\frac{qQR}{4\pi\epsilon_0 (R+y)^3 m} + g \right]$

Ans. (b)

Using law of conservation of total energy,

$$\frac{1}{2}mv^2 = mgy + (\Delta PE)$$

$$\Rightarrow \frac{1}{2}mv^2 = mgy + kQq \left[\frac{1}{R} - \frac{1}{(R+y)} \right]$$

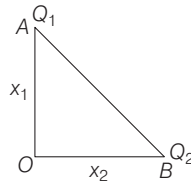
$$\Rightarrow v^2 = 2gy + \frac{2kQq}{m} \frac{y}{R(R+y)}$$

$$\Rightarrow v^2 = 2y \left[\frac{qQ}{4\pi\epsilon_0 R(R+y)m} + g \right]$$

$$\left[\because k = \frac{1}{4\pi\epsilon_0} \right]$$

Hence, correct option is (b).

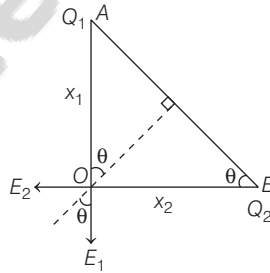
- 34** Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1 / Q_2 is proportional to [2020, 6 Sep Shift-I]



- (a) $\frac{x_1^3}{x_2^3}$ (b) $\frac{x_2}{x_1}$
 (c) $\frac{x_1}{x_2}$ (d) $\frac{x_2^2}{x_1^2}$

Ans. (c)

Let electric field produced by charges Q_1 and Q_2 at point O be E_1 and E_2 , respectively. The direction of fields are shown in the figure below and a perpendicular is also drawn on side AB, that passes through point O.



If the resultant electric field at point O is perpendicular to hypotenuse this means resultant of E_1 and E_2 must be along it.

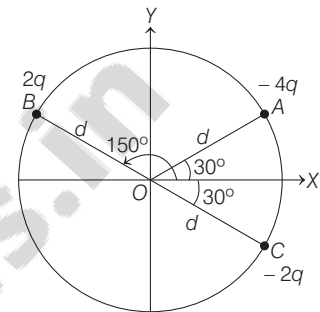
$$\therefore \tan\theta = \frac{E_2}{E_1} = \frac{\frac{KQ_2}{x_1^2}}{\frac{KQ_1}{x_2^2}} = \frac{Q_2}{Q_1} \frac{x_1^2}{x_2^2} \quad \dots (i)$$

$$\text{In } \triangle OAB, \quad \tan\theta = \frac{x_1}{x_2} \quad \dots (ii)$$

Equating Eqs. (i) and (ii), we get

$$\frac{Q_2}{Q_1} \cdot \frac{x_1^2}{x_2^2} = \frac{x_1}{x_2} \quad \text{or} \quad \frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

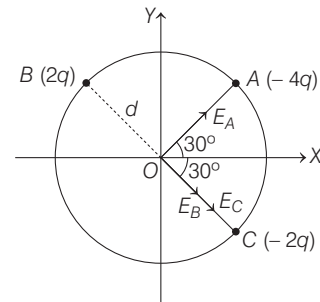
- 35** Three charged particles A, B and C with charges $-4q$, $2q$ and $-2q$ are present on the circumference of a circle of radius d . The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is [2020, 8 Jan Shift-I]



- (a) $\frac{2\sqrt{3}q}{\pi\epsilon_0 d^2}$ (b) $\frac{\sqrt{3}q}{4\pi\epsilon_0 d^2}$
 (c) $\frac{3\sqrt{3}q}{4\pi\epsilon_0 d^2}$ (d) $\frac{\sqrt{3}q}{\pi\epsilon_0 d^2}$

Ans. (d)

Fields of charges at A, B and C are as shown below.



Magnitude of resultant field component directed along positive X-axis,

$$E_x = E_A \cos 30^\circ + E_B \cos 30^\circ + E_C \cos 30^\circ$$

$$= (E_A + E_B + E_C) \cos 30^\circ$$

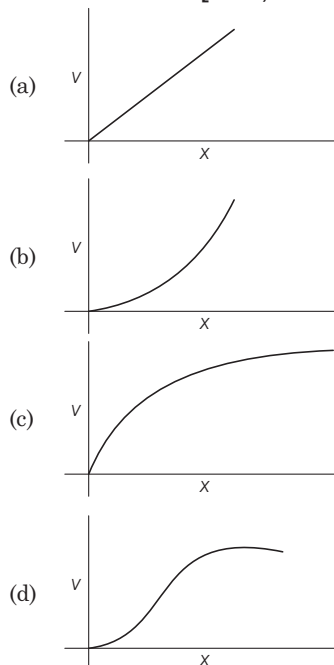
$$= \left(\frac{k(4q)}{d^2} + \frac{k(2q)}{d^2} + \frac{k(2q)}{d^2} \right) \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{8q}{d^2} \times \frac{\sqrt{3}}{2} \Rightarrow E_x = \frac{\sqrt{3}q}{\pi\epsilon_0 d^2}$$

- 36** A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the

distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale)

[2020, 8 Jan Shift-II]



Ans. (c)

Acceleration of the particle,

$$a = \frac{F}{m} = \frac{q}{m} \cdot E$$

Velocity v and distance x can be related using

$$v^2 - u^2 = 2ax$$

$$\Rightarrow v = \sqrt{2\left(\frac{q}{m}E\right)x} \quad (\because u=0)$$

or $v^2 = 2\left(\frac{q}{m}E\right)x$

This equation resembles a parabola $y^2 = 4ax$. So, the graph between v and x will be as shown in option (c).

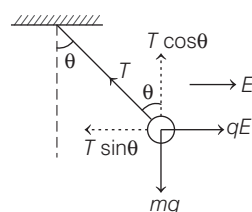
- 37** The bob of a simple pendulum has mass $2g$ and a charge of $5.0\mu\text{C}$. It is at rest in a uniform horizontal electric field of intensity 2000 V/m . At equilibrium, the angle that the pendulum makes with the vertical is (take $g = 10\text{ m/s}^2$)

[2019, 8 April Shift-I]

- (a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$
(c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

Ans. (d)

Forces on the bob are as shown



For equilibrium,

$$T \cos \theta = mg \quad \dots(i)$$

and $T \sin \theta = qE \quad \dots(ii)$

Dividing Eq. (ii) by Eq. (i), we get

$$\tan \theta = \frac{qE}{mg}$$

Here, $q = 5\mu\text{C} = 5 \times 10^{-6}\text{ C}$,

$$E = 2000\text{ V/m}$$

$$m = 2g = 2 \times 10^{-3}\text{ kg}, g = 10\text{ ms}^{-2}$$

$$\therefore \tan \theta = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10}$$

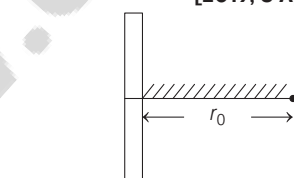
$$= \frac{1}{2} = 0.5$$

So, the angle made by the string of the pendulum with the vertical is

$$\theta = \tan^{-1}(0.5)$$

- 38** A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to

[2019, 8 April Shift-II]



(a) $v \propto \left(\frac{r}{r_0}\right)$

(b) $v \propto e^{+\frac{r}{r_0}}$

(c) $v \propto \ln\left(\frac{r}{r_0}\right)$

(d) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

Ans. (d)

For a positive line charge or charged wire with uniform density λ , electric field at distance x is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\epsilon_0 x} \quad \dots(i)$$

So, force on charge q which is at a distance r_0 due to this line charge is

$$F = qE = \frac{2kq\lambda}{x} \quad \dots(ii) \quad [\text{using Eq. (i)}]$$

Now, work done when charge is pushed by field by a small displacement dx is

$$dW = F \cdot dx = \frac{2kq\lambda}{x} \cdot dx \quad [\text{using Eq. (ii)}]$$

$\therefore$ Total work done by field of wire in taking charge q from distance r_0 to distance r will be

$$W = \int_{r_0}^r dW = \int_{r_0}^r \frac{2kq\lambda}{x} \cdot dx$$

$$= 2kq\lambda [\log x]_{r_0}^r$$

$$= 2kq\lambda (\log r - \log r_0)$$

$$= 2kq\lambda \log \left| \frac{r}{r_0} \right| \quad \dots(iii)$$

As we know, from work-kinetic energy theorem,

$$K_{\text{final}} - K_{\text{initial}} = W$$

$$\Rightarrow \frac{1}{2}mv^2 - 0 = 2kq\lambda \log \left| \frac{r}{r_0} \right|$$

[using Eq. (iii)]

$$\Rightarrow v = \left(\frac{4kq\lambda \log \left| \frac{r}{r_0} \right|}{m} \right)^{\frac{1}{2}}$$

$$\therefore v \propto \left(\log \left| \frac{r}{r_0} \right| \right)^{\frac{1}{2}}$$

- 39** Four point charges $-q, +q, +q$ and $-q$ are placed on Y -axis at $y = -2d, y = -d, y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the X -axis at $x = D$, with $D \gg d$, will behave as

[2019, 9 April Shift-II]

(a) $E \propto \frac{1}{D}$

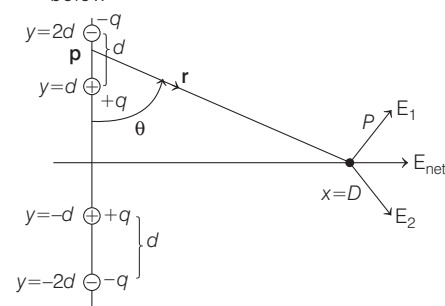
(b) $E \propto \frac{1}{D^3}$

(c) $E \propto \frac{1}{D^2}$

(d) $E \propto \frac{1}{D^4}$

Ans. (d)

Given charge distribution is as shown below



So, we can view above point charges as combination of pair of dipoles or a quadrupole.

By symmetry, the field components parallel to quadrupole cancels and the resultant perpendicular field is

$$E = \frac{2q}{4\pi\epsilon_0} \left(\frac{1}{D^2} - \frac{D}{(D^2 + d^2)^{3/2}} \right)$$

$$= \frac{2q}{4\pi\epsilon_0 D^2} \left(1 - \left(1 + \frac{d^2}{D^2} \right)^{-3/2} \right)$$

As, $\left(1 + \frac{d^2}{D^2} \right)^{-3/2} \approx \left(1 - \frac{3}{2} \frac{d^2}{D^2} \right)$
(using binomial expansion)

We have,

$$E = \frac{3qd^2}{4\pi\epsilon_0 D^4}$$

$$\Rightarrow E \propto \frac{1}{D^4}$$

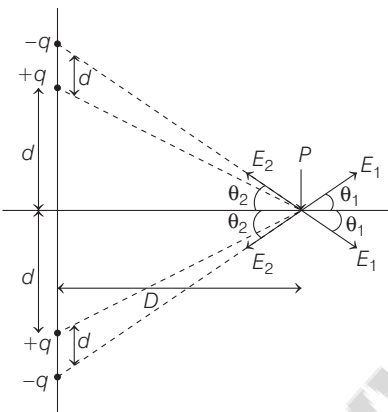
Note Dependence of field for a point charge is

$$E \propto \frac{1}{r^2}$$

For a dipole, it is, $E \propto \frac{1}{r^3}$

For a quadrupole, it is, $E \propto \frac{1}{r^4}$ etc.

Alternate Solution The given distribution of charges can be shown as the figure below



Electric field at point P,

$$E = E_1 \cos\theta_1 + E_1 \cos\theta_1 - E_2 \cos\theta_2 - E_2 \cos\theta_2$$

$$= 2E_1 \cos\theta_1 - 2E_2 \cos\theta_2$$

$$= \frac{2kq}{(d^2 + D^2)} \cos\theta_1 - \frac{2kq}{(2d)^2 + D^2} \cos\theta_2$$

$$\text{As, } \cos\theta_1 = \frac{D}{(d^2 + D^2)^{1/2}}$$

$$\text{Similarly, } \cos\theta_2 = \frac{D}{[(2d)^2 + D^2]^{1/2}}$$

$$= \frac{2kqD}{(d^2 + D^2)^{3/2}} - \frac{2kqD}{(4d^2 + D^2)^{3/2}}$$

$$= \frac{2kqD}{D^3} \left[\left(1 + \frac{d^2}{D^2} \right)^{-3/2} - \left(1 + \frac{4d^2}{D^2} \right)^{-3/2} \right]$$

As $D \gg d$, then by applying binomial approximation, we get

$$= \frac{2kq}{D^2} \left[1 - \frac{3}{2} \frac{d^2}{D^2} - \left(1 - \frac{3}{2} \frac{4d^2}{D^2} \right) \right]$$

$$= \frac{2kq}{D^2} \left[\frac{9d^2}{2D^2} \right] = \frac{9kqd^2}{D^4} \Rightarrow E \propto \frac{1}{D^4}$$

40 Two point charges $q_1 (\sqrt{10} \mu\text{C})$ and $q_2 (-25 \mu\text{C})$ are placed on the x-axis at $x=1\text{m}$ and $x=4\text{m}$, respectively. The electric field (in V/m) at a point $y=3\text{m}$ on Y-axis is

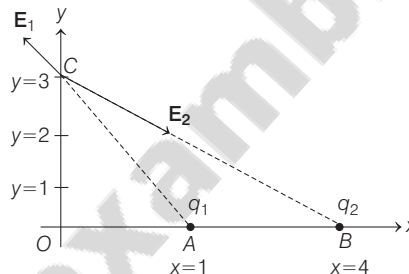
(Take, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N}\cdot\text{m}^2\text{C}^{-2}$)

[2019, 9 Jan Shift-II]

- (a) $(63\hat{i} - 27\hat{j}) \times 10^2$
(b) $(81\hat{i} - 81\hat{j}) \times 10^2$
(c) $(-81\hat{i} + 81\hat{j}) \times 10^2$
(d) $(-63\hat{i} + 27\hat{j}) \times 10^2$

Ans. (a)

Here, $q_1 = \sqrt{10} \mu\text{C} = \sqrt{10} \times 10^{-6} \text{ C}$
 $q_2 = -25 \mu\text{C} = -25 \times 10^{-6} \text{ C}$



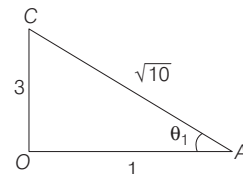
Let E_1 and E_2 are the values of electric field due to q_1 and q_2 respectively.

$$\text{Here, } E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{AC^2} = \frac{1}{4\pi\epsilon_0} \times \frac{\sqrt{10} \times 10^{-6}}{(1^2 + 3^2)}$$

$$= 9 \times 10^9 \times \sqrt{10} \times 10^{-7} = 9\sqrt{10} \times 10^2$$

$$\therefore E_1 = 9\sqrt{10} \times 10^2 [\cos\theta_1(-\hat{i}) + \sin\theta_1\hat{j}]$$

From ΔOAC ,



$$\sin\theta_1 = \frac{3}{\sqrt{10}}$$

$$\text{and } \cos\theta_1 = \frac{1}{\sqrt{10}}$$

$$\therefore E_1 = 9\sqrt{10} \times 10^2 \left[\frac{1}{\sqrt{10}}(-\hat{i}) + \frac{3}{\sqrt{10}}\hat{j} \right]$$

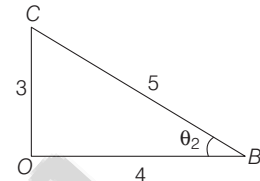
$$= 9 \times 10^2 [-\hat{i} + 3\hat{j}]$$

$$= (-9\hat{i} + 27\hat{j}) \times 10^2 \text{ V/m}$$

$$\text{and } E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{-25 \times 10^{-6}}{(4^2 + 3^2)}$$

$$= 9 \times 10^3 \text{ V/m}$$

From ΔOBC ,



$$\sin\theta_2 = \frac{3}{5}$$

$$\cos\theta_2 = \frac{4}{5}$$

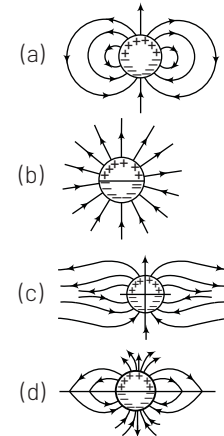
$$\therefore E_2 = 9 \times 10^3 [\cos\theta_2\hat{i} - \sin\theta_2\hat{j}]$$

$$E_2 = 9 \times 10^3 \left[\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j} \right]$$

$$= (72\hat{i} - 54\hat{j}) \times 10^2$$

$$\therefore E = E_1 + E_2 = (63\hat{i} - 27\hat{j}) \times 10^2 \text{ V/m}$$

41 A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) [JEE Main 2015]



Ans. (a)

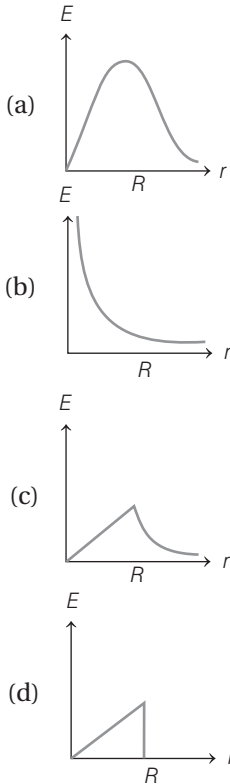
Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible.

Electric field lines cannot form corners as shown in (d).

Thus, correct option is (a).

- 42** In a uniformly charged sphere of total charge Q and radius R , the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be

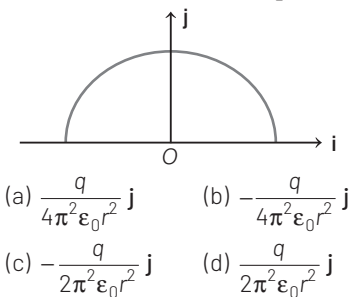
[AIEEE 2012]

**Ans. (c)**

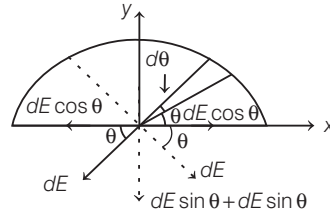
Electric field inside the uniformly charged sphere varies linearly, $E = \frac{kQ}{R^3} \cdot r$, ($r \leq R$), while outside the sphere, it varies as inverse square of distance, $E = \frac{kQ}{r^2}$; ($r \geq R$) which is correctly represented in option (c).

- 43** A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is

[AIEEE 2010]

**Ans. (c)**

Linear charge density, $\lambda = \left(\frac{q}{\pi r}\right)$



$$E = \int dE \sin \theta (-\mathbf{j}) = \int_0^\pi \int \frac{K \cdot dq}{r^2} \sin \theta (-\mathbf{j})$$

$$E = \frac{K}{r^2} \int_0^\pi \frac{qr}{\pi r} d\theta \sin \theta (-\mathbf{j})$$

$$= \frac{K}{r^2} \frac{q}{\pi} \int_0^\pi \sin \theta d\theta (-\mathbf{j})$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{\pi r^2} [-\cos \theta]_0^\pi (-\mathbf{j})$$

$$= \frac{q}{2\pi^2\epsilon_0 r^2} (-\mathbf{j})$$

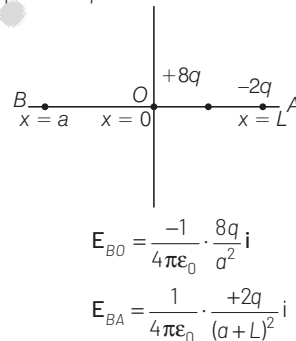
- 44** Two point charges $+8q$ and $-2q$ are located at $x=0$ and $x=L$, respectively. The location of a point on the x -axis at which the net electric field due to these two point charges is zero, is

[AIEEE 2005]

- (a) $2L$ (b) $\frac{L}{4}$
 (c) $8L$ (d) $4L$

Ans. (a)

Suppose that a point B , where net electric field is zero due to charges $8q$ and $-2q$.



According to condition,

$$E_{B0} + E_{BA} = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{8q}{a^2} = \frac{1}{4\pi\epsilon_0} \frac{2q}{(a+L)^2}$$

$$\text{or } \frac{2}{a} = \frac{1}{a+L} \text{ or } 2a + 2L = a \text{ or } 2L = -a$$

Thus, at distance $2L$ from origin, net electric field will be zero.

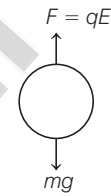
- 45** A charged oil drop is suspended in uniform field of $3 \times 10^4 \text{ Vm}^{-1}$ so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge $= 9.9 \times 10^{-15} \text{ kg}$ and $g = 10 \text{ ms}^{-2}$)

[AIEEE 2004]

- (a) $3.3 \times 10^{-18} \text{ C}$ (b) $3.2 \times 10^{-18} \text{ C}$
 (c) $1.6 \times 10^{-18} \text{ C}$ (d) $4.8 \times 10^{-18} \text{ C}$

Ans. (a)

In steady state,



Electric force on drop = Weight of drop

$$\text{i.e., } qE = mg$$

$$\Rightarrow q = \frac{mg}{E} = \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4}$$

$$= 3.3 \times 10^{-18} \text{ C}$$

TOPIC 3 Electric Dipole

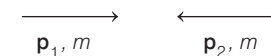
- 46** Two identical electric point dipoles have dipole moments $\mathbf{p}_1 = p\hat{\mathbf{i}}$ and $\mathbf{p}_2 = -p\hat{\mathbf{i}}$ are held on the X -axis at distance a from each other. When released, they move along the X -axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is m , their speed when they are infinitely far apart is

[2020, 6 Sep Shift-II]

- (a) $\frac{p}{a} \sqrt{\frac{1}{\pi\epsilon_0 m a}}$ (b) $\frac{p}{a} \sqrt{\frac{1}{2\pi\epsilon_0 m a}}$
 (c) $\frac{p}{a} \sqrt{\frac{2}{\pi\epsilon_0 m a}}$ (d) $\frac{p}{a} \sqrt{\frac{3}{2\pi\epsilon_0 m a}}$

Ans. (b)

When two electric dipoles of opposite dipole moments are placed on a line, they experience force of attraction along the same line as shown below.



Considering both dipoles as a system, we find that net external force on system is zero, i.e., $\mathbf{F}_{\text{ext}} = 0$

So, total mechanical energy = constant

$$(ME)_i = (ME)_f$$

$$\text{or } (KE)_i + (PE)_i = (KE)_f + (PE)_f$$

As, initially they are released from rest, so initial KE is zero and finally they are infinite apart, so final PE is zero.

$$0 + \left(-\frac{2kp_1p_2}{r^3} \cos 180^\circ \right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

$$\text{Here, } m_1 = m_2 = m, p_1 = p_2 = p, r = a$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{2kp^2}{a^3} \quad \dots(i)$$

Using conservation of momentum,

$$\mathbf{p}_i = \mathbf{p}_f \Rightarrow 0 = m\mathbf{v}_1 + m\mathbf{v}_2$$

$$\text{or } \mathbf{v}_1 = -\mathbf{v}_2 \Rightarrow v_1 = v_2 = v \quad \dots(ii)$$

Putting this value in Eq. (i), we get

$$mv^2 = \frac{2kp^2}{a^3} \quad \text{or } v = \frac{p}{a} \sqrt{\frac{2k}{ma}}$$

$$\text{or } v = \frac{p}{a} \sqrt{\frac{1}{2\pi\epsilon_0 ma}} \quad \left(\because k = \frac{1}{4\pi\epsilon_0} \right)$$

Hence, correct option is (b).

- 47** An electric dipole of moment $\mathbf{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C-m is at the origin (0, 0, 0). The electric field due to this dipole at $\mathbf{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\mathbf{r} \cdot \mathbf{p} = 0$) is parallel to

[2020, 9 Jan Shift-I]

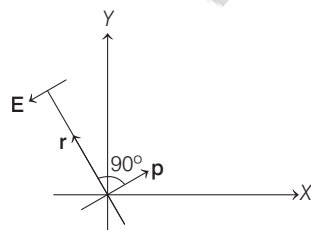
- (a) $(+\hat{i} - 3\hat{j} - 2\hat{k})$
 (b) $(-\hat{i} - 3\hat{j} + 2\hat{k})$
 (c) $(+\hat{i} + 3\hat{j} - 2\hat{k})$
 (d) $(-\hat{i} + 3\hat{j} - 2\hat{k})$

Ans. (c)

Given, $\mathbf{r} \cdot \mathbf{p} = 0$

So, $\mathbf{r} \perp \mathbf{p}$, i.e. we have following situation,

So, we have to find direction of electric field at equatorial line. As $\mathbf{E}$ is directed opposite to $\mathbf{p}$ at all equatorial points, direction of $\mathbf{E}$ is along $-\mathbf{p}$.



$$\begin{aligned} \text{So, } \mathbf{E} &= \lambda(-\mathbf{p}) \\ &= \lambda[-(-\hat{i} - 3\hat{j} + 2\hat{k})] \\ &= \lambda(\hat{i} + 3\hat{j} - 2\hat{k}) \end{aligned}$$

- 48** An electric dipole is formed by two equal and opposite charges q with separation d . The charges have same mass m . It is kept in a uniform electric field E . If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is [2019, 8 April Shift-II]

- (a) $\sqrt{\frac{2qE}{md}}$ (b) $2\sqrt{\frac{qE}{md}}$ (c) $\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

Ans. (a)

Key Idea When an electric dipole is placed in an electric field E at some angle θ , then two forces equal in magnitude but opposite in direction acts on the +ve and -ve charges, respectively. These forces forms a couple which exert a torque, which is given as

$$\tau = \mathbf{p} \times \mathbf{E}$$

where, $\mathbf{p}$ is dipole moment.

$\therefore$ Torque on the dipole can also be given as

$$\tau = I\alpha = -pE \sin\theta$$

where, I is the moment of inertia and α is angular acceleration. For small angles, $\sin\theta \approx \theta$

$$\therefore \alpha = -\left(\frac{pE}{I}\right)\theta \quad \dots(i)$$

Moment of inertia of the given system is

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$$\Rightarrow \alpha = -\left(\frac{2pE}{md^2}\right)\theta \quad \dots(ii)$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii), with the general equation of angular SHM, i.e.

$$\alpha = -\omega^2\theta$$

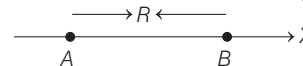
where, ω is the angular frequency, we get

$$\omega^2 = \frac{2pE}{md^2} \quad \text{or } \omega = \sqrt{\frac{2pE}{md^2}}$$

As $p = qd$

$$\therefore \omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

- 49** Two electric dipoles, A, B with respective dipole moments $\mathbf{d}_A = -4qa\hat{i}$ and $\mathbf{d}_B = -2qa\hat{i}$ are placed on the X-axis with a separation R , as shown in the figure

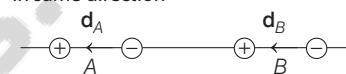


The distance from A at which both of them produce the same potential is [2019, 10 Jan Shift-I]

- (a) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (c) $\frac{R}{\sqrt{2}+1}$ (d) $\frac{R}{\sqrt{2}-1}$

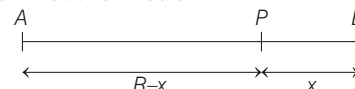
Ans. (a)

Key Idea As, dipole moments points in same direction



So, potential of both dipoles can be same at some point between A and B.

Let potentials are same at P, distant x from B as shown below



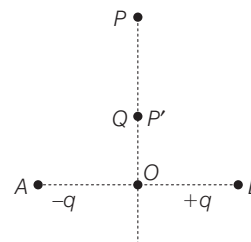
$$\text{Then, } \frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$$

$$\begin{aligned} 2x^2 &= (R-x)^2 \\ \Rightarrow \sqrt{2}x &= R-x \Rightarrow x = \frac{R}{\sqrt{2}+1} \end{aligned}$$

Distance from A is

$$\Rightarrow R-x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

- 50** Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y \gg 2a$. The charge Q experiences an electrostatic force F .



If Q is now moved along the equatorial line to P' such that

$OP' = \left(\frac{y}{3}\right)$, the force on

Q will be close to $\left(\frac{y}{3} \gg 2a\right)$

[2019, 10 Jan Shift-II]

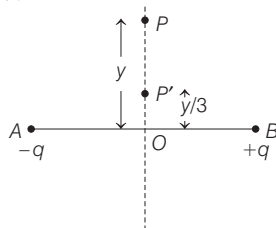
- (a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

Ans. (d)

Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \quad \dots (i)$$

where, P is the dipole moment of the charges.



In first case $r = y$

$$\Rightarrow E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

Here, $y^2 \gg a^2$

$$\Rightarrow y^2 + a^2 \approx y^2 \text{ or } E_1 = \frac{2kP}{y^3} \quad \dots (ii)$$

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kPQ}{y^3} \quad \dots (iii)$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $\frac{y}{3} \gg a$

$$\Rightarrow \left(\frac{y}{3}\right)^2 + a^2 \approx \left(\frac{y}{3}\right)^2$$

$$\Rightarrow E_2 = \frac{2kP}{(y/3)^3}$$

$$\Rightarrow E_2 = 27 \times \frac{2kP}{y^3}$$

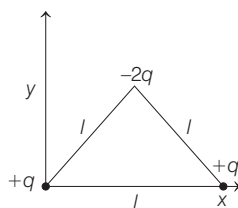
Force on charge in this position,

$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \quad \dots (iv)$$

From Eqs. (iii) and (iv), we get

$$F' = 27F$$

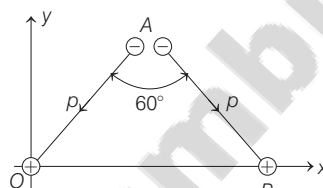
- 51** Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. [2019, 12 Jan Shift-I]



- (a) $\sqrt{3}ql \frac{\hat{j} - \hat{i}}{\sqrt{2}}$ (b) $2ql \hat{j}$
(c) $-\sqrt{3}ql \hat{j}$ (d) $(ql) \frac{\hat{i} + \hat{j}}{\sqrt{2}}$

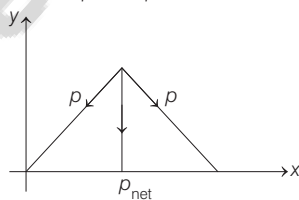
Ans. (c)

Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below



Now, magnitude of resultant of these dipole moments is

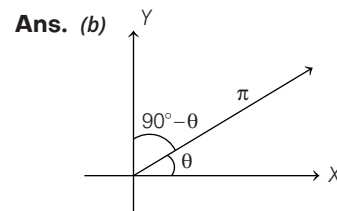
$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p \cdot p \cos 60^\circ} \\ = \sqrt{3}p = \sqrt{3}ql$$



As, resultant is directed along negative y-direction $p_{\text{net}} = -\sqrt{3}p \hat{j} = -\sqrt{3}ql \hat{j}$

- 52** An electric dipole has a fixed dipole moment $\mathbf{p}$, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{i}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{k}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E\hat{j}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is [JEE Main 2017]

- (a) 45° (b) 60° (c) 90° (d) 30°



Torque applied on a dipole $\tau = pE \sin \theta$ where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{i}$

it means field is directed along positive X direction, so angle between dipole and field will remain θ , therefore torque in this direction

$$E_1 = pE_1 \sin \theta$$

In electric field $E_2 = \sqrt{3}E\hat{j}$,

it means field is directed along positive Y-axis, so angle between dipole and field will be $90^\circ - \theta$.

Torque in this direction

$$\tau_2 = pE \sin(90^\circ - \theta) = p\sqrt{3}E_1 \cos \theta$$

According to question

$$\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$$

$$\therefore pE_1 \sin \theta = p\sqrt{3}E_1 \cos \theta$$

$$\tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ$$

$$\therefore \theta = 60^\circ$$

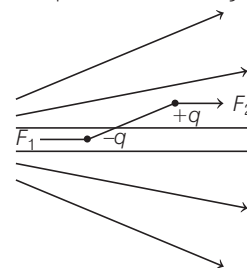
- 53** An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience [AIEEE 2006]

- (a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

Ans. (c)

In a non-uniform electric field, the dipole may experience both non-zero torque as well as translational force.

For example, as shown in figure



$F_1 \neq F_2$ as E is non-uniform.

Torque would also be non-zero.

TOPIC 4

Electric Flux and Gauss Laws

- 54** The total charge enclosed in an incremental volume of $2 \times 10^{-9} \text{ m}^3$ located at the origin is nC, if electric flux density of its field is found as $D = e^{-x} \sin y \hat{i} - e^{-x} \cos y \hat{j} + 2z \hat{k} \text{ C/m}^2$. [2021, 22 July Shift-II]

Ans. (4)

Given, incremental volume,
 $dV = 2 \times 10^{-9} \text{ m}^3$

Electric flux density,

$$D = e^{-x} \sin y \hat{i} - e^{-x} \cos y \hat{j} + 2z \hat{k} \text{ C/m}^2$$

As we know that,

$$\begin{aligned} \rho(x, y, z) &= \nabla \cdot D = \frac{d}{dx}(e^{-x} \sin y) \\ &\quad - \frac{d}{dy}(e^{-x} \cos y) + \frac{d}{dz} 2z \\ &= -e^{-x} \sin y + e^{-x} \sin y + 2 \end{aligned}$$

$$\rho(0, 0, 0) = 2 = \frac{Q}{dV}$$

$$\Rightarrow Q = 2dV = 2 \times 2 \times 10^{-9} = 4 \times 10^{-9} = 4 \text{ nC}$$

- 55** The electric field in a region is given by $\mathbf{E} = \frac{2}{5} E_0 \hat{i} + \frac{3}{5} E_0 \hat{j}$ with $E_0 = 4.0 \times 10^3 \text{ N/C}$. The flux of this field through a rectangular surface area 0.4 m^2 parallel to the yz-plane is $\text{N-m}^2 \text{ C}^{-1}$.

[2021, 17 March Shift-II]

Ans. (640)

Given,

The electric field in the region,

$$\mathbf{E} = \frac{2}{5} E_0 \hat{i} + \frac{3}{5} E_0 \hat{j}$$

$$\text{Here, } E_0 = 4 \times 10^3 \frac{\text{N}}{\text{C}}$$

Area of the rectangular surface,

$$A = 0.4 \text{ m}^2$$

The direction of electric field vector and area vector is same, so the angle between the electric field vector and area vector is 0° .

As we know the expression of electric flux,

$$\phi = E \cdot A \cos \theta \quad \dots(i)$$

Here, E is the electric field vector, and A is the surface area of the surfaces.

Consider the surface parallel to the Y-Z plane, so the area vector, $A = 0.4 \hat{i} \text{ m}^2$

Substituting the values in Eq. (i), we get

$$\phi = E \cdot A \cos 0^\circ \Rightarrow \phi = \frac{2}{5} E_0 (0.4)$$

$$\Rightarrow \phi = \frac{2}{5} (4 \times 10^3) (0.4) = 640 \text{ Nm}^2 \text{ C}^{-1}$$

Hence, the electric flux of the surface parallel to the Y-Z plane is $640 \text{ Nm}^2 \text{ C}^{-1}$.

- 56** Given below are two statements:

Statement I An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r ($< R$) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of the above statements, choose the correct answer from the options given below.

[2021, 26 Feb Shift-II]

- (a) Both Statement I and Statement II are true.
 (b) Statement I is true but Statement II is false.
 (c) Both Statement I and Statement II are false.
 (d) Statement I is false but Statement II is true.

Ans. (b)

Net charge on electric dipole
 $= +q - q = 0$

Hence, according to Gauss's law,

$$\text{Electric flux, } \phi = \frac{q_{\text{net}}}{\epsilon_0} = \frac{0}{\epsilon_0} = 0$$

Electric field due to electric dipole is non-zero and varies at point to point.

Hence, statement I is true.

Electric field due to charged solid sphere at a distance r from centre.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3} \quad \left[\begin{array}{l} \text{when } r < R \\ R \rightarrow \text{radius} \end{array} \right]$$

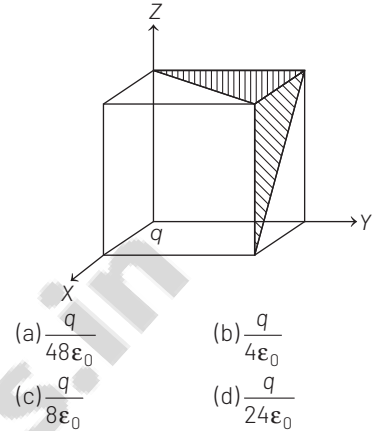
which is non-zero.

Hence, statement II is false.

Hence, option (b) is the correct.

- 57** A charge q is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\mathbf{E}$ through the shaded area is

[25 Feb 2021 Shift-II]



- (a) $\frac{q}{48\epsilon_0}$ (b) $\frac{q}{4\epsilon_0}$
 (c) $\frac{q}{8\epsilon_0}$ (d) $\frac{q}{24\epsilon_0}$

Ans. (d)

Given, charge q is at one of the corner of the cube.

$\therefore$ Contribution of q in cube will be
 $q_{\text{enclosed}} = q/8$

As, only 3 faces of cube is allowing the flux lines to pass through it.

$$\therefore \text{Flux } (\phi) = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{1}{3} \frac{q/8}{\epsilon_0} = \frac{q}{24\epsilon_0}$$

- 58** The electric field in a region is given

$$\mathbf{E} = \left(\frac{3}{5} E_0 \hat{i} + \frac{4}{5} E_0 \hat{j} \right) \frac{\text{N}}{\text{C}}. \text{ The ratio of}$$

flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to YZ-plane) to that of the surface of area 0.3 m^2 (parallel to XZ-plane) is $a:b$, where $a = \dots\dots\dots$

[Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors

along X, Y and Z-axes,

respectively] [2021, 25 Feb Shift-I]

Ans. (1)

$$\text{Given, } \mathbf{E} = \frac{3E_0}{5} \hat{i} + \frac{4}{5} E_0 \hat{j},$$

$$\mathbf{A}_1 = 0.2 \text{ m}^2 \hat{i} \text{ and } \mathbf{A}_2 = 0.3 \text{ m}^2 \hat{j}$$

Let ϕ_1 and ϕ_2 be the flux linked with area A_1 and A_2 , respectively.

As we know that,

$$\begin{aligned} \phi &= \oint \mathbf{E} \cdot d\mathbf{S} = \mathbf{E} \cdot \mathbf{A} \\ \Rightarrow \phi_1 &= \left(\frac{3}{5} E_0 \hat{i} + \frac{4}{5} E_0 \hat{j} \right) \cdot 0.2 \hat{i} \\ &= \frac{3}{5} E_0 \times 0.2 \end{aligned}$$

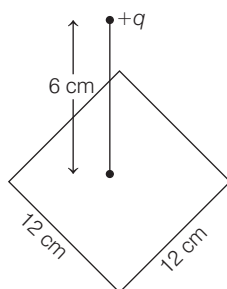
$$\text{and similarly, } \phi_2 = \frac{4}{5} E_0 \times 0.3$$

$$\text{Now, } \frac{\phi_1}{\phi_2} = \frac{3/5 E_0 \times 0.2}{4/5 E_0 \times 0.3} = \frac{0.6}{1.2} = \frac{1}{2}$$

$$\therefore a = 1$$

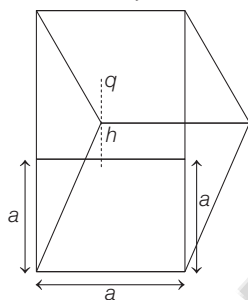
- 59** A point charge of $+12\mu\text{C}$ is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be $\times 10^3 \text{ N-m}^2/\text{C}$.

[2021, 24 Feb Shift-II]



Ans. (226)

Given, charge, $q = 12\mu\text{C} = 12 \times 10^{-6} \text{ C}$
 Height of charge from surface, $h = 6 \text{ cm}$
 $= 6 \times 10^{-2} \text{ m}$
 and side of square, $a = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$
 From figure, it is clear that the given square is one of the face of a cube of side 12 cm and $+12\mu\text{C}$ charge is placed at its centre. Then, by Gauss's theorem,

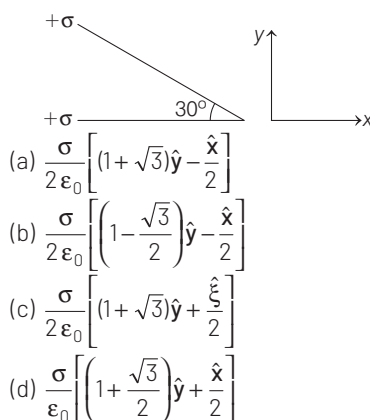


$$\text{Flux through any face, } \phi = \frac{q}{6\epsilon_0}$$

$$\Rightarrow \phi = \frac{12 \times 10^{-6}}{6 \times 8.854 \times 10^{-12}} \\ = 0.226 \times 10^6 \text{ N-m}^2/\text{C} \\ = 226 \times 10^3 \text{ N-m}^2/\text{C}$$

- 60** Two infinite planes each with uniform surface charged density $+\sigma$ are kept in such a way that the angle between them is 30° . The electric field in the region shown between them is given by

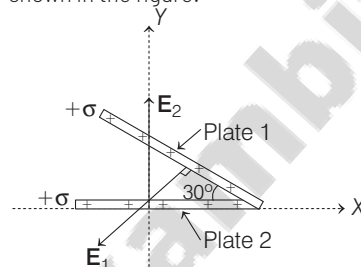
[2020, 7 Jan Shift-I]



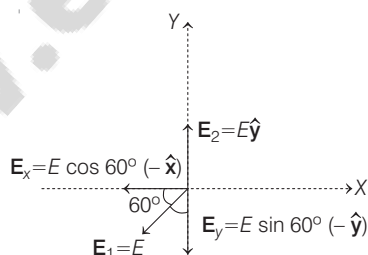
Ans(b)

Electric field of an infinite plate is perpendicular to the plane of plate and its magnitude is $E = \frac{\sigma}{2\epsilon_0}$.

We are given with two positively charged plates with a set of coordinate axes as shown in the figure.



From geometry of figure, net electric field in region between plates is resultant of fields of both plates,



Now, field of plate 1 can be resolved along X and Y-axes as shown in above figure.

$$\text{Now, } \mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 \\ = E \cos 60^\circ (-\hat{x}) + E \sin 60^\circ (-\hat{y}) + E \hat{y}$$

$$\text{Here, } E = \frac{\sigma}{2\epsilon_0} \text{ and } \hat{x} \text{ and } \hat{y} \text{ are unit}$$

vectors along X and Y-axes.

$$= \frac{\sigma}{2\epsilon_0} \left[-\frac{\hat{x}}{2} - \frac{\sqrt{3}}{2} \hat{y} + \hat{y} \right] \\ = \frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2} \right) \hat{y} - \frac{\hat{x}}{2} \right]$$

- 61** In finding the electric field using Gauss law the formula $|\mathbf{E}| = \frac{q_{\text{enc}}}{\epsilon_0 |\mathbf{A}|}$ is

applicable. In the formula, ϵ_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation?

[2020, 8 Jan Shift-I]

- Only when the Gaussian surface is an equipotential surface and $|\mathbf{E}|$ is constant on the surface.
- Only when the Gaussian surface is an equipotential surface.
- For any choice of Gaussian surface.
- Only when $|\mathbf{E}|$ is constant on the surface.

Ans. (a)

$$\text{Equation } |\mathbf{E}| = \frac{q_{\text{en}}}{\epsilon_0 |\mathbf{A}|} \text{ Gives } \int \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{en}}}{\epsilon_0}$$

Now, in finding the electric field by above equation, the integral is easy to evaluate, if $|\mathbf{E}|$ is constant. Also, if $|\mathbf{E}|$ is constant for the surface, then surface is equipotential.

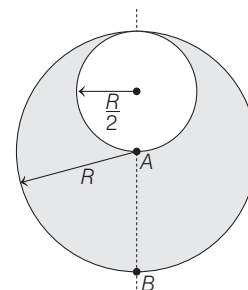
- 62** Consider a sphere of radius R which carries a uniform charge density ρ . If a sphere of radius $\frac{R}{2}$ is carved out

of it, as shown in the figure the

ratio $\frac{|\mathbf{E}_A|}{|\mathbf{E}_B|}$ of magnitude of electric

field $\mathbf{E}_A$ and $\mathbf{E}_B$ respectively, at points A and B due to the remaining portion is

[2020, 9 Jan Shift-I]



- $\frac{21}{34}$
- $\frac{18}{54}$
- $\frac{17}{54}$
- $\frac{18}{34}$

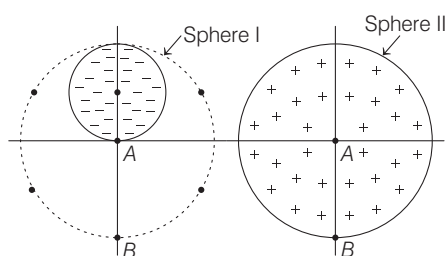
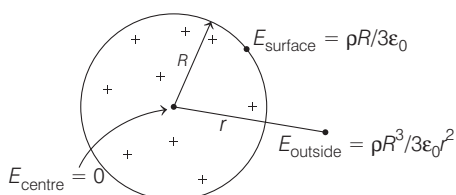
Ans. (d)

Electric field at points A and B can be viewed as a superposition of

- electric field due to complete solid sphere of radius R and charge density ρ .

- (ii) electric field due to sphere of radius $\frac{R}{2}$ and charge density $-\rho$.

Now, we use standard result of electric field due to a solid sphere which are



Now, electric field due to sphere I at points A and B are

$$E_{IA} = \frac{\rho \left(\frac{R}{2} \right)}{3\epsilon_0} = \frac{\rho R}{6\epsilon_0}$$

$$E_{IB} = \frac{\rho \left(\frac{R}{2} \right)^3}{3\epsilon_0 \left(\frac{3R}{2} \right)^2} = \frac{\rho R}{54\epsilon_0}$$

and similarly due to sphere II,

$$E_{IIA} = 0$$

$$E_{IIB} = \frac{\rho R}{3\epsilon_0}$$

So, net field magnitude at point A,

$$E_A = E_{IA} + E_{IIA} = \frac{\rho R}{6\epsilon_0} + 0 = \frac{\rho R}{6\epsilon_0}$$

and net field magnitude to point B,

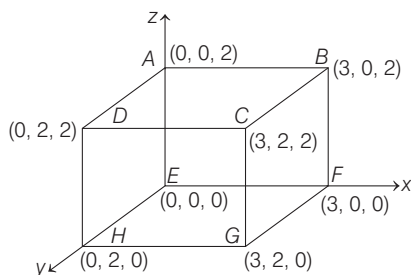
$$E_B = \frac{\rho R}{3\epsilon_0} - \frac{\rho R}{54\epsilon_0} = \frac{17\rho R}{54\epsilon_0}$$

Hence, ratio

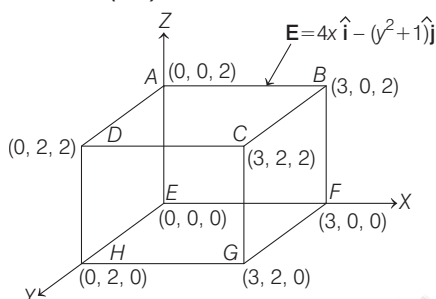
$$\frac{E_A}{E_B} = \frac{\rho R}{6\epsilon_0} \times \frac{54\epsilon_0}{17\rho R} = \frac{18}{34}$$

- 63** An electric field $\mathbf{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_I and ϕ_{II} respectively. The difference between $(\phi_I - \phi_{II})$ is (in $\text{N} \cdot \text{m}^2/\text{C}$)

[2020, 9 Jan Shift-II]



Ans. (-48)



Area vector of face ABCD,

$$\mathbf{A}_1 = 2 \times 3 \hat{k} = 6\hat{k}$$

Area vector of face BCGF,

$$\mathbf{A}_2 = 2 \times 2 \hat{i} = 4\hat{i}$$

So, flux through face ABCD,

$$\phi_1 = \mathbf{E} \cdot \mathbf{A}_1 = (4x\hat{i} - (y^2 + 1)\hat{j}) \cdot 6\hat{k} = 0$$

Flux through face BCGF,

$$\phi_2 = \mathbf{E} \cdot \mathbf{A}_2 = (4x\hat{i} - (y^2 + 1)\hat{j}) \cdot 4\hat{i} = 16x$$

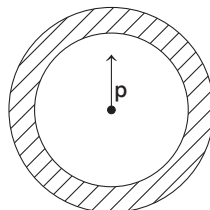
At face BCGF, $x = 3$

So, $\phi_2 = 16 \times 3 = 48$ units

$$\therefore \phi_1 - \phi_2 = 0 - 48 = -48 \text{ N} \cdot \text{m}^2 \text{ C}^{-1}$$

- 64** Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carries charge Q . At its centre is a dipole $\mathbf{p}$ as shown.

In this case, [2019, 12 April Shift-I]



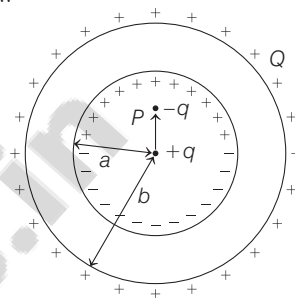
- (a) surface charge density on the inner surface is uniform and equal to $\frac{Q}{4\pi a^2}$
- (b) electric field outside the shell is the same as that of a point charge at the centre of the shell

- (c) surface charge density on the outer surface depends on $|\mathbf{p}|$

- (d) surface charge density on the inner surface of the shell is zero everywhere

Ans. (b)

Electric charge distribution at inner and outer surface of spherical shell due to the electric dipole can be shown as below



Here, we need to consider two different factors

- (i) charge on the spherical shell is $+Q$ which will be distributed on its outer surface as shown in figure.

- (ii) Electric dipole will create non-uniform electric field inside the shell which will distribute the charges on inner surface as shown in figure. But its net contribution to the outer side of the shell will be zero as net charge of a dipole is zero.

$\therefore$ Net charge on outer surface of shell will be $+Q$.

Hence, using (ii), option (a) is incorrect as field inside shell is not uniform. Option (b) is correct, as net charge on outer surface is $+Q$ even in the presence of dipole.

Option (c) is incorrect, as surface charge density at outer surface is uniform

$$\left(= \frac{Q}{A} = \frac{Q}{4\pi b^2} \right)$$

Option (d) is incorrect, as surface charge density at inner surface is non-zero.

So, option (b) is correct.

Alternate Solution Using Gauss' law at outer surface, let charge on dipole is q ,

$$\phi = \frac{\Sigma q}{\epsilon_0} = E \cdot A \text{ or } E = \frac{1}{A\epsilon_0} \Sigma q$$

$$= \frac{(+Q + q - q)}{A\epsilon_0} = \frac{Q}{A\epsilon_0} = \frac{\sigma}{\epsilon_0} = \text{constant}$$

- 65** Let a total charge $2Q$ be distributed in a sphere of radius R , with the charge density given by $\rho(r) = kr$, where r is the distance from the

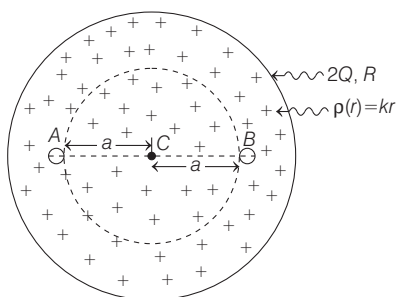
centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a , from the centre. If A and B do not experience any force, then

[2019, 12 April Shift-II]

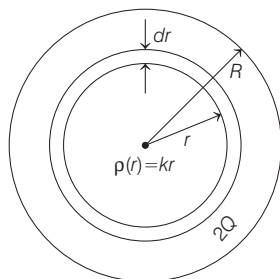
- (a) $a = 8^{-1/4} R$ (b) $a = \frac{3R}{2^{1/4}}$
 (c) $a = 2^{-1/4} R$ (d) $a = R/\sqrt{3}$

Ans. (a)

Key Idea Force on A is zero only when repulsion of A and B = attraction of positive charge distribution of radius a and charge A .



In given charge distribution, let r is radius of a shell of thickness dr .



Charge dQ present in shell of thickness dr
 = Volume of shell $\times$ Volumetric charge density

$$\Rightarrow dQ = (4\pi r^2 \times dr) \times (kr) = 4\pi k r^3 dr$$

Total charge in sphere is

$$2Q = \int_0^R dQ = \int_0^R 4\pi k r^3 dr$$

$$\Rightarrow 2Q = 4\pi k \left[\frac{r^4}{4} \right]_0^R \Rightarrow k = \frac{2Q}{\pi R^4}$$

Now, using Gauss' law, electric field on the surface of sphere of radius a is

$$E \int_0^a dA = \frac{1}{\epsilon_0} \cdot \int_0^a (kr \cdot 4\pi r^2 dr)$$

$$\Rightarrow E \cdot 4\pi a^2 = \frac{1}{\epsilon_0} \cdot 4\pi k \left(\frac{a^4}{4} \right)$$

$$\Rightarrow E = \frac{k a^2}{4\epsilon_0} = \frac{2Q a^2}{4\pi \epsilon_0 R^4}$$

Force of attraction on charge A (or B) due to this field is

$$F_1 = QE = \frac{2Q^2 a^2}{4\pi \epsilon_0 R^4}$$

Force of repulsion on charge A due to B is

$$F_2 = \frac{1}{4\pi \epsilon_0} \frac{Q^2}{(2a)^2} = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q^2}{4a^2}$$

If charge A (or B) does not feel any force, then

$$F_1 = F_2$$

$$\Rightarrow \frac{2Q^2 a^2}{4\pi \epsilon_0 R^4} = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q^2}{4a^2}$$

$$\Rightarrow 8a^4 = R^4$$

$$\Rightarrow a = 8^{-1/4} R$$

66 Charge is distributed within a sphere of radius R with a volume

$$\text{charge density } \rho(r) = \frac{A}{r^2} e^{-\frac{2r}{a}},$$

where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

[2019, 9 Jan Shift-II]

- (a) $a \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$ (b) $a \log \left(1 - \frac{Q}{2\pi a A} \right)$
 (c) $\frac{a}{2} \log \left(1 - \frac{Q}{2\pi a A} \right)$ (d) $\frac{a}{2} \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$

Ans. (d)

Here, volume charge density,

$$\rho(r) = \frac{A}{r^2} \cdot e^{-\frac{2r}{a}}$$

where, a and A are constant.

Let a spherical region of small element of radius r .

If Q is total charge distribution upto radius R , then

$$Q = \int_0^R \rho \cdot dV = \int_0^R \frac{A}{r^2} e^{-2r/a} (4\pi r^2 dr)$$

(From figure, we observe

$$dV = A \cdot dr = 4\pi r^2 \cdot dr$$

$$= 4\pi A \int_0^R e^{-2r/a} dr$$

$$= 4\pi A \left(\frac{e^{-2r/a}}{-2/a} \right)_0^R$$

$$= 4\pi A \times \left(\frac{-a}{2} \right) (e^{-2R/a} - e^0)$$

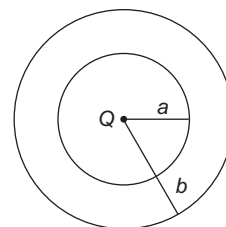
$$= 2\pi A(-a) [e^{-2R/a} - 1]$$

$$\text{or } Q = 2\pi a A (1 - e^{-2R/a})$$

$$\text{or } R = \frac{a}{2} \log \left(\frac{1}{1 - \frac{Q}{2\pi a A}} \right)$$

67 The region between two concentric spheres of radii a and b , respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where

A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q . The value of A such that the electric field in the region between the spheres will be constant is [JEE Main 2016]



- (a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$
 (c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

Ans. (a)

As, Gaussian surface at distance r from centre,

$$Q + \int_a^r 4\pi r^2 dr = E 4\pi r^2 \epsilon_0$$

$$E 4\pi \epsilon_0 r^2 = Q + A \frac{4\pi}{r^2} \left(\frac{r^2 - a^2}{2} \right)$$

$$E = \frac{1}{4\pi \epsilon_0} \left[\frac{Q}{r^2} + A 2\pi \left(\frac{r^2 - a^2}{r^2} \right) \right]$$

$$E = \frac{1}{4\pi \epsilon_0} \left(\frac{Q}{r^2} + A 2\pi - \frac{A 2\pi a^2}{r^2} \right)$$

$$E = \frac{1}{4\pi \epsilon_0} \times A \times 2\pi$$

At the centre of the spheres is a point charge Q . The value of A such that the electric field in the region between the spheres will be constant is

$$\text{As, } Q = 2\pi A a^2 \text{ i.e. } A = \frac{Q}{2\pi a^2}$$

- 68** This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. [AIEEE 2012]

An insulating solid sphere of radius R has a uniform positive charge density ρ . As a result of this uniform charge distribution, there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement I When a charge q is taken from the centre of the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\epsilon_0}$.

Statement II The electric field at a distance r ($r < R$) from the centre of the sphere is $\frac{\rho r}{3\epsilon_0}$.

- (a) Statement I is false, Statement II is true
 (b) Statement I is true, Statement II is false
 (c) Statement I is true, Statement II is true; Statement II the correct explanation for Statement I
 (d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. (a)

Statement I is dimensionally wrong while from Gauss' law,

$$E(4\pi r^2) = \frac{\rho \cdot \frac{4}{3}\pi r^3}{\epsilon_0} \Rightarrow E = \frac{\rho r}{3\epsilon_0}$$

gives Statement II is correct.

- 69** The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre and a, b are constants. Then, the charge density inside the ball is [AIEEE 2011]

- (a) $-6a\epsilon_0 r$ (b) $-24\pi a\epsilon_0$
 (c) $-6a\epsilon_0$ (d) $-24\pi a\epsilon_0 r$

Ans. (c)

Electric field, $E = -\frac{d\phi}{dr} = -2ar$

By Gauss' theorem,

$$E(4\pi r^2) = \frac{q}{\epsilon_0} \Rightarrow q = -8\pi\epsilon_0 ar^3$$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr} \times \frac{dr}{dV}$$

$$= (-24\pi\epsilon_0 ar^2) \left(\frac{1}{4\pi r^2} \right)$$

$$= -6\epsilon_0 a$$

- 70** Let there be a spherically symmetric charge distribution with charge density varying as

$$\rho(r) = \rho_0 \left(\frac{5}{4} - \frac{r}{R} \right) \text{ upto } r = R \text{ and}$$

$\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r , ($r < R$) from the origin is given by

[AIEEE 2010]

- (a) $\frac{4\pi\rho_0 r}{3\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right)$ (b) $\frac{\rho_0 r}{4\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right)$
 (c) $\frac{4\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right)$ (d) $\frac{\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right)$

Ans. (b)

Apply Shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2 \cdot dr \cdot \rho$$

$$= 4\pi r^2 \cdot dr \cdot \rho_0 \left[\frac{5}{4} - \frac{r}{R} \right]$$

$$= 4\pi\rho_0 \left[\frac{5}{4} r^2 dr - \frac{r^3}{R} dr \right]$$

$$\int dq = q = 4\pi\rho_0 \int_0^r \left[\frac{5}{4} r^2 dr - \frac{r^3}{R} dr \right]$$

$$q = 4\pi\rho_0 \left[\frac{5}{4} \frac{r^3}{3} - \frac{1}{R} \frac{r^4}{4} \right]$$

As electric field intensity,

$$E = \frac{kq}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{r^2} \cdot 4\pi\rho_0 \left[\frac{5}{4} \left(\frac{r^3}{3} \right) - \frac{r^4}{4R} \right]$$

$$E = \frac{\rho_0 r}{4\epsilon_0} \left[\frac{5}{3} - \frac{r}{R} \right]$$

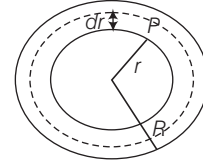
- 71** Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge

density distribution for a solid sphere of radius R and total charge Q . For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

[AIEEE 2009]

- (a) zero (b) $\frac{Q}{4\pi\epsilon_0 r_1^2}$
 (c) $\frac{Qr_1^2}{4\pi\epsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\epsilon_0 R^4}$

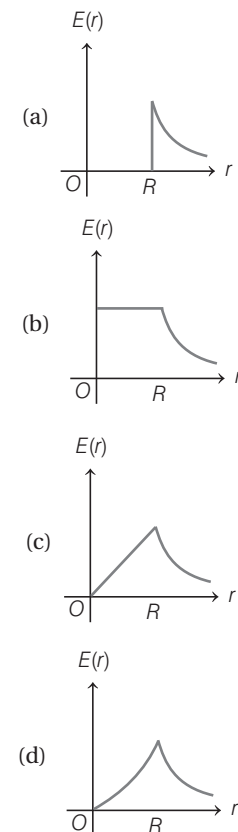
Ans. (c)



$$E4\pi r^2 = \frac{\int_0^r \frac{Q}{\pi R^4} r^4 \pi r^2 dr}{\epsilon_0}$$

$$\Rightarrow E = \frac{Qr_1^2}{4\pi\epsilon_0 R^4}$$

- 72** A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \leq r < \infty$, where r is the distance from the centre of the shell? [AIEEE 2008]



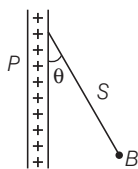
Ans. (a)

For uniformly charged spherical shell,

$$E = 0, r < R$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}, r \geq R$$

- 73** A charged ball B hangs from a silk thread S , which makes an angle θ with a large charged conducting sheet P , as shown in the figure. The surface charge density σ of the sheet is proportional to [AIEEE 2005]



- (a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

Ans. (d)

Electric field due to a charged conducting sheet of surface charge density σ is given by $E = \frac{\sigma}{\epsilon_0\epsilon_r}$.

where, ϵ_0 is the permittivity in vacuum and ϵ_r is the relative permittivity of medium.

Here, electrostatic force on B ,

$$QE = \frac{Q\sigma}{\epsilon_0\epsilon_r}$$

FBD of B is shown in figure.

In equilibrium, $T \cos\theta = mg$

$$\text{and } T \sin\theta = \frac{Q\sigma}{\epsilon_0\epsilon_r}$$

$$\text{Thus, } \tan\theta = \frac{Q\sigma}{\epsilon_0\epsilon_r mg} \Rightarrow \tan\theta \propto \sigma$$

- 74** If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2 , the electric charge inside the surface will be [AIEEE 2003]

- (a) $(\phi_2 - \phi_1)\epsilon_0$ (b) $\frac{(\phi_1 + \phi_2)}{\epsilon_0}$
 (c) $\frac{(\phi_2 - \phi_1)}{\epsilon_0}$ (d) $(\phi_1 + \phi_2)\epsilon_0$

Ans. (a)

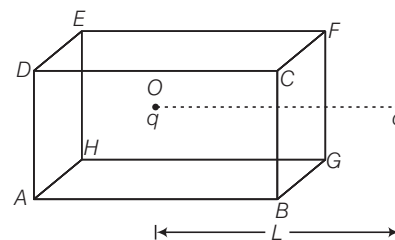
From Gauss' law, $\frac{\text{Charge enclosed}}{\epsilon_0}$

= Net flux

$$\Rightarrow \frac{q}{\epsilon_0} = \phi_2 - \phi_1$$

$$\text{or } q = (\phi_2 - \phi_1)\epsilon_0$$

- 75** A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O . Then, the electric flux through $ABCD$ is [AIEEE 2002]

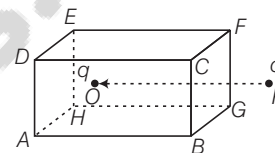


- (a) $\frac{q}{4\pi\epsilon_0 L}$ (b) $\frac{q}{2\pi\epsilon_0 L}$
 (c) $\frac{q}{2\pi\epsilon_0}$ (d) None of these

Ans. (d)

Electric flux for any surface is defined as

$$\phi = \int \mathbf{E} \cdot d\mathbf{s}$$



$$\text{As flux on the cube} = \frac{q}{\epsilon_0}$$

$$\text{Flux on each face} = \frac{1}{6} \frac{q}{\epsilon_0}$$

$$\text{So, flux on the face } ABCD = \frac{1}{6} \frac{q}{\epsilon_0}$$

The options (a), (c) and (d) are dimensionally incorrect, so they cannot be answers.

16

Electrostatics Potential and Capacitance

TOPIC 1

Electrostatic Potential and Potential Energy

- 01** The average translational kinetic energy of N_2 gas molecules at°C becomes equal to the KE of an electron accelerated from rest through a potential difference of 0.1 V.

[Given, $K_B = 1.38 \times 10^{-23}$ J/K]

[2021, 1 Sep Shift-II]

Ans. (500)

Given, the average translational kinetic energy of dinitrogen (N_2) = Kinetic energy of an electron ... (i)

Translational kinetic energy of dinitrogen (N_2)

$$KE = \frac{3}{2} K_B T$$

Here, T = temperature of the gas, and K_B = Boltzmann constant.

Kinetic energy of an electron = eV

Given, the potential difference of an electron, $V = 0.1$ V

Substituting the values in the Eq. (i), we get

$$\frac{3}{2} K_B T = eV \Rightarrow \frac{3}{2} \times 1.38 \times 10^{-23} \times T$$

$$= 1.6 \times 10^{-19} \times (0.1)$$

$$T = 773 \text{ K} = 773 - 273^\circ \text{C} = 500^\circ \text{C}$$

- 02** 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drop is times that of a smaller drop.

[2021, 26 Feb Shift-II]

Ans. (243)

Given, $n = 27$

$$V_1 = 10 \text{ V}$$

Let q_1 be the charge of one drop, r_1 be its radius, r be the radius of bigger drop and q be its charge.

As, volume remains constant.

$$\therefore \frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_1^3 \times 27$$

$$\Rightarrow r^3 = 27 r_1^3$$

$$\Rightarrow r = 3r_1$$

$$\text{Electric potential } (V) = \frac{3kq_1^2}{5r}$$

where, k is Coulomb constant.

Therefore, potential energy of small drop,

$$U_s = \frac{3}{5} \frac{kq_1^2}{r_1} \quad \dots (i)$$

and potential energy of bigger drop,

$$U_B = \frac{3}{5} \frac{k(q_1^2 \times 27)^2}{r}$$

$$U_B = \frac{3}{5} \frac{k(27)^2 q_1^2}{3r_1} \quad \dots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{U_B}{U_s} = \frac{27 \times 27}{3}$$

$$= 243$$

$$\Rightarrow U_B = 243 U_s$$

Hence, potential energy of big drop is 243 times of small drop.

- 03** 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is V.

[2021, 25 Feb Shift-I]

Ans. (128)

Given, number of mercury drops, $n = 512$

Voltage of each drop, $V = 2$ V

Let r , R be the radius of drop small and combined spherical drop, respectively.

Now, when all drops are joined into single drop, volume remains constant,

$$\text{i.e. } 512 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R^3 = (512 r)^3 \Rightarrow R = 8r$$

$$\therefore V = \frac{kq}{r} = 2 \quad \dots (i)$$

$$\therefore V_{\text{net}} = \frac{kQ}{R}$$

where, Q be the charge of bigger sphere.

$$\Rightarrow V_{\text{net}} = \frac{k \times 512q}{8r} \quad \dots (ii)$$

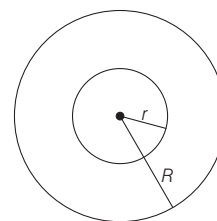
On dividing Eq. (ii) by Eq. (i), we get

$$\frac{V_{\text{net}}}{V} = \frac{k \times 512q \times r}{8r \times kq} = \frac{512}{8}$$

$$\Rightarrow V_{\text{net}} = 2 \times \frac{512}{8} = 128 \text{ V}$$

- 04** A charge Q is distributed over two concentric conducting thin spherical shells radii r and R ($R > r$). If the surface charge densities on the two shells are equal, the electric potential at the common centre is

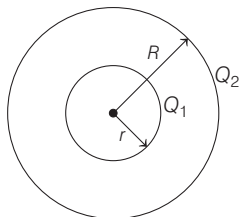
[2020, 2 Sep Shift-II]



- (a) $\frac{1}{4\pi\epsilon_0} \frac{(R+r)}{2(R^2+r^2)} Q$ (b) $\frac{1}{4\pi\epsilon_0} \frac{(2R+r)}{(R^2+r^2)} Q$
 (c) $\frac{1}{4\pi\epsilon_0} \frac{(R+2r)Q}{2(R^2+r^2)}$ (d) $\frac{1}{4\pi\epsilon_0} \frac{(R+r)}{(R^2+r^2)} Q$

Ans. (d)

Given, total charge Q is uniformly distributed over concentric shells of radii r and R .



Let Q_1 and Q_2 be the charges over inner and outer shells, such that charge densities are equal on both shells.

$$\text{Charge density of inner shell, } \sigma = \frac{Q_1}{4\pi r^2}$$

$$\text{Charge density of outer shell, } \sigma = \frac{Q_2}{4\pi R^2}$$

$$\Rightarrow \frac{Q_1}{4\pi r^2} = \frac{Q_2}{4\pi R^2} \Rightarrow \frac{Q_1}{r^2} = \frac{Q_2}{R^2} \Rightarrow Q_2 = \frac{Q_1 R^2}{r^2}$$

$$\text{Also, } Q_1 + Q_2 = Q$$

$$\Rightarrow Q_1 + \frac{Q_1 R^2}{r^2} = Q \text{ or } Q_1 = \frac{Q r^2}{(r^2 + R^2)}$$

$$\text{and } Q_2 = \frac{Q R^2}{(r^2 + R^2)}$$

Potential due to Q_1 and Q_2 at common centre of shells is

$$\begin{aligned} V &= V_{Q_1} + V_{Q_2} = \frac{kQ_1}{r} + \frac{kQ_2}{R} \\ &= \frac{kQr}{r^2 + R^2} + \frac{kQR}{r^2 + R^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q(r+R)}{r^2 + R^2} \end{aligned}$$

Hence, correct option is (d).

- 05** Concentric metallic hollow spheres of radii R and $4R$ hold charges Q_1 and Q_2 , respectively. Given that, surface charge densities of the concentric spheres are equal. The potential difference $V(R) - V(4R)$ is

[2020, 3 Sep Shift-II]

- (a) $\frac{3Q_2}{4\pi\epsilon_0 R}$ (b) $\frac{3Q_1}{16\pi\epsilon_0 R}$
(c) $\frac{3Q_1}{4\pi\epsilon_0 R}$ (d) $\frac{Q_2}{4\pi\epsilon_0 R}$

Ans. (b)

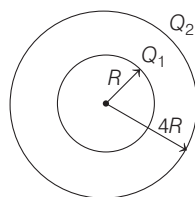
Potential due to a hollow spherical charge distribution is

$$V = \begin{cases} \frac{kq}{r}, & r > R \text{ (outside points)} \\ \frac{kq}{R}, & r = R \text{ (at surface)} \\ \frac{kq}{R}, & r < R \text{ (inside points)} \end{cases}$$

Given arrangement is

Potential on the surface of inner sphere,
 $V(R)$ = potential due to charge Q_1 +
potential due to charge Q_2

$$= \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$



Potential on the surface of outer sphere,

$V(4R)$ = potential due to charge Q_1 +
potential due to charge Q_2

$$= \frac{kQ_1}{4R} + \frac{kQ_2}{4R}$$

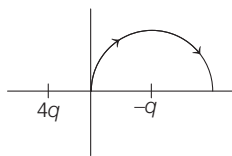
The potential difference $V(R) - V(4R)$

$$\begin{aligned} &= \left(\frac{kQ_1}{R} + \frac{kQ_2}{4R} \right) - \left(\frac{kQ_1}{4R} + \frac{kQ_2}{4R} \right) \\ &= \frac{kQ_1}{R} \left(\frac{3}{4} \right) \\ &= \frac{3}{16\pi\epsilon_0} \cdot \frac{Q_1}{R} \end{aligned}$$

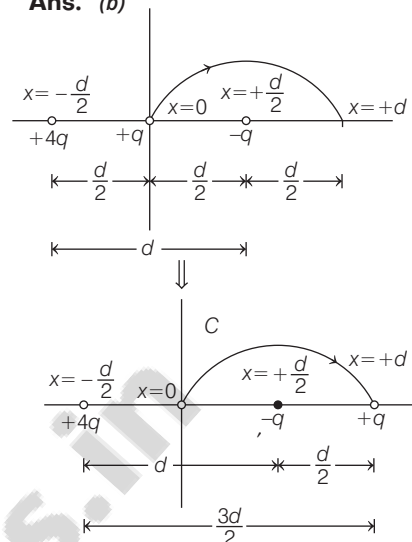
Hence, correct option is (b).

- 06** Two point charges $4q$ and $-q$ are fixed on the X-axis at $x = \frac{-d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point

charge q is taken from the origin to $x = d$ along the semi-circle as shown in the figure, the energy of the charge will [2020, 4 Sep Shift-I]



- (a) increase by $\frac{2q^2}{3\pi\epsilon_0 d}$
(b) decrease by $\frac{4q^2}{3\pi\epsilon_0 d}$
(c) increase by $\frac{3q^2}{4\pi\epsilon_0 d}$
(d) decrease by $\frac{q^2}{4\pi\epsilon_0 d}$

Ans. (b)

Change in potential energy, $\Delta U = U_f - U_i$

$$\begin{aligned} \Delta U &= \left[\frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{3d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}} \right] \\ &\quad - \left[\frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}} \right] \\ &= \frac{k(4q)(-q)}{d} + \frac{k(4q)(q)}{\frac{3d}{2}} + \frac{k(-q)(q)}{\frac{d}{2}} \\ &\quad - \frac{k(4q)(-q)}{d} - \frac{k(4q)(q)}{\frac{d}{2}} - \frac{k(-q)(q)}{\frac{d}{2}} \\ &= \frac{k(4q)(q)}{\frac{3d}{2}} - \frac{k(4q)(q)}{\frac{d}{2}} = \frac{8kq^2}{3d} - \frac{8kq^2}{d} \\ &= -\frac{16kq^2}{3d} = -\frac{16q^2}{3d \times 4\pi\epsilon_0} = -\frac{4q^2}{3\pi\epsilon_0 d} \end{aligned}$$

So, ΔU will decrease by $\frac{4q^2}{3\pi\epsilon_0 d}$.

Hence, correct option is (b).

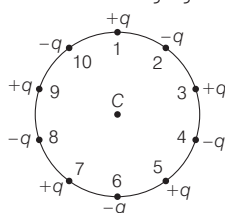
- 07** Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $+q$ each, while 2, 4, 6, 8, 10 have charge $-q$ each. The potential V and the electric field E at the centre of the circle respectively, are (Take, $V = 0$ at infinity)

[2020, 5 Sep Shift-II]

- (a) $V = \frac{10q}{4\pi\epsilon_0 R}$; $E = 0$
 (b) $V = 0$; $E = \frac{10q}{4\pi\epsilon_0 R^2}$
 (c) $V = 0$; $E = 0$
 (d) $V = \frac{10q}{4\pi\epsilon_0 R}$; $E = \frac{10q}{4\pi\epsilon_0 R^2}$

Ans. (c)

The arrangement of all the charges is shown in the following figure,



Potential at the centre of the circle C,

$$V = \frac{kq}{R} + \frac{k(-q)}{R} + \frac{kq}{R} + \frac{k(-q)}{R} + \dots$$

$$= \sum \frac{kq}{R} = k \frac{\sum(q)}{R} = \frac{k(0)}{R} = 0$$

Electric field at centre of the circle C,
 $E = (E \text{ due to 5 +ve charges})$

$+ (E \text{ due to 5 -ve charges})$

As, all the charges are equidistant from centre. So, the electric field of one charge will get cancelled due to another symmetrical charge in front of it.

$\therefore E = 0$

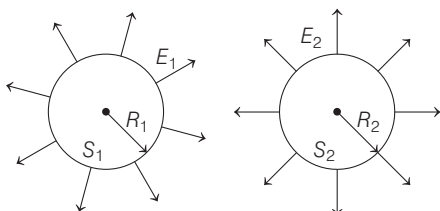
Hence, correct option is (c).

- 08** Consider two charged metallic spheres S_1 and S_2 of radii R_1 and R_2 , respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1/E_2 = R_1/R_2$. Then the ratio V_1 (on S_1)/ V_2 (on S_2) of the electrostatic potentials on each sphere is **[2020, 8 Jan Shift-II]**

- (a) $\left(\frac{R_1}{R_2}\right)^3$ (b) R_2/R_1
 (c) R_1/R_2 (d) $(R_1/R_2)^2$

Ans. (d)

Given, $\frac{E_1}{E_2} = \frac{R_1}{R_2}$... (i)



For a charged sphere, potential is given by

$$V = \frac{kQ}{R} = \frac{kQ}{R^2} \cdot R \text{ or } V = E \cdot R$$

So, $\frac{V_1}{V_2} = \frac{E_1 R_1}{E_2 R_2} = \frac{R_1}{R_2} \times \frac{R_1}{R_2}$ [from Eq. (i)]

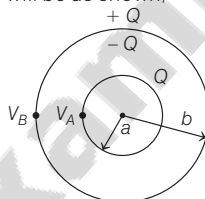
$$\Rightarrow \frac{V_1}{V_2} = \frac{R_1^2}{R_2^2} = \left(\frac{R_1}{R_2}\right)^2$$

- 09** A solid conducting sphere, having a charge Q , is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V . If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is **[2019, 8 April Shift-I]**

- (a) $-2V$ (b) $2V$ (c) $4V$ (d) V

Ans. (d)

Initially when uncharged shell encloses charge Q , charge distribution due to induction will be as shown,



The potential on surface of inner shell is

$$V_A = \frac{kQ}{a} + \frac{k(-Q)}{b} + \frac{kQ}{b} \dots (i)$$

where, k = proportionality constant.

Potential on surface of outer shell is

$$V_B = \frac{kQ}{b} + \frac{k(-Q)}{b} + \frac{kQ}{b} \dots (ii)$$

Then, potential difference is

$$\Delta V_{AB} = V_A - V_B = kQ \left(\frac{1}{a} - \frac{1}{b} \right)$$

Given, $\Delta V_{AB} = V$

$$\text{So, } kQ \left(\frac{1}{a} - \frac{1}{b} \right) = V \dots (iii)$$

Finally after giving charge $-4Q$ to outer shell, potential difference will be

$$\Delta V_{AB} = V_A - V_B = \left(\frac{kQ}{a} + \frac{k(-4Q)}{b} \right) - \left(\frac{kQ}{b} + \frac{k(-4Q)}{b} \right)$$

$$= kQ \left(\frac{1}{a} - \frac{1}{b} \right) = V \quad [\text{from Eq. (iii)}]$$

Hence, we obtain that potential difference does not depend on the charge of outer sphere, hence potential difference remains same.

- 10** The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20 \text{ SI unit}$ and $B = 10 \text{ SI unit}$. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2 , then $V_1 - V_2$ is **[2019, 8 April Shift-II]**

- (a) -48 V (b) -520 V
 (c) 180 V (d) 320 V

Ans. (c)

Given, $\mathbf{E} = (Ax + B)\hat{i} \text{ NC}^{-1}$

The relation between electric field and potential is given as $dV = -\mathbf{E} \cdot d\mathbf{x}$

Integrating both sides within the specified limits, we get

$$\therefore \int_1^2 dV = V_2 - V_1 = - \int_{x_1}^{x_2} \mathbf{E} \cdot d\mathbf{x}$$

$$\Rightarrow V_1 - V_2 = \int_{x_1}^{x_2} \mathbf{E} \cdot d\mathbf{x}$$

$$= \int_{x_1}^{x_2} (Ax + B)\hat{i} \cdot (dx\hat{i}) = \int_{x_1}^{x_2} (Ax + B) \cdot dx$$

Here, $A = 20 \text{ SI unit}$, $B = 10 \text{ SI unit}$,

$x_1 = 1$ and $x_2 = -5$

$$\Rightarrow V_1 - V_2 = \int_1^{-5} (20x + 10) \cdot dx$$

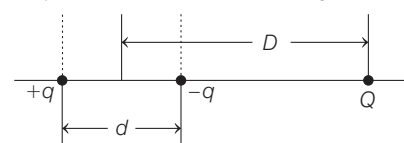
$$= \left[\frac{20x^2}{2} + 10x \right]_1^{-5}$$

$$= 10 [x^2 + x]_1^{-5}$$

$$= 10 [(-5)^2 + (-5) - (1)^2 - (1)]$$

$$= 10 (25 - 5 - 2) = 180 \text{ V}$$

- 11** A system of three charges are placed as shown in the figure



If $D \gg d$, the potential energy of the system is best given by

[2019, 9 April Shift-I]

(a) $\frac{1}{4\pi\epsilon_0} \left[-\frac{q^2}{d} + \frac{2qQd}{D^2} \right]$

(b) $\frac{1}{4\pi\epsilon_0} \left[+\frac{q^2}{d} + \frac{qQd}{D^2} \right]$

(c) $\frac{1}{4\pi\epsilon_0} \left[-\frac{q^2}{d} - \frac{qQd}{2D^2} \right]$

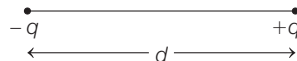
(d) $\frac{1}{4\pi\epsilon_0} \left[-\frac{q^2}{d} - \frac{qQd}{D^2} \right]$

Ans. (d)

The system of two charges, i.e. $+q$ and $-q$ that are separated by distance d can be considered as a dipole. Thus, the charge Q would be at D distance from the centre of an electric dipole on its axial line.

So, the total potential energy of the system will be due to two components.

(1) Potential energy of dipole's own system

$$(PE)_1 = \frac{Kq_1q_2}{d} = -\frac{Kq^2}{d} \quad \dots(i)$$


(2) Potential energy of charge Q and dipole system

$$(PE)_2 = -\frac{KQq}{D^2} \cdot d \quad \dots(ii)$$

Hence, total potential energy of the system

$$\begin{aligned} (PE)_{\text{total}} &= (PE)_1 + (PE)_2 \\ &= -\frac{Kq^2}{d} - \frac{KQq}{D^2} \cdot d \\ \Rightarrow (PE)_{\text{total}} &= -\frac{1}{4\pi\epsilon_0} \left[\frac{q^2}{d} + \frac{Qqd}{D^2} \right] \end{aligned}$$

- 12** In free space, a particle A of charge $1\mu\text{C}$ is held fixed at a point P. Another particle B of the same charge and mass $4\mu\text{g}$ is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is

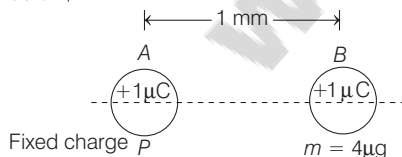
$$\left[\text{Take, } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2} \right]$$

[2019, 10 April Shift-II]

- (a) $15 \times 10^2 \text{ m/s}$ (b) $3.0 \times 10^4 \text{ m/s}$
(c) 1.0 m/s (d) $2.0 \times 10^3 \text{ m/s}$

Ans. (d)

Given situation is shown in the figure below,



When charged particle B is released due to mutual repulsion, it moves away from A. In this process, potential energy of system of charges reduces and this change of potential energy appears as kinetic energy of B.

Now, potential energy of system of charges at separation of 1 mm is

$$\begin{aligned} U_1 &= \frac{Kq_1q_2}{r} \\ \text{Here, } q_1 &= q_2 = 1 \times 10^{-6} \text{ C} \\ r &= 1 \text{ mm} = 1 \times 10^{-3} \text{ m} \\ \therefore U_1 &= \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1 \times 10^{-3}} = 9 \text{ J} \end{aligned}$$

Potential energy of given system of charges at separation of 9 mm is

$$U_2 = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{9 \times 10^{-3}} = 1 \text{ J}$$

By energy conservation,

Change in potential energy of system of A and B

= Kinetic energy of charged particle B

$$\Rightarrow U_1 - U_2 = \frac{1}{2} m_B v_B^2$$

where, m_B = mass of particle B = $4\mu\text{g}$
 $= 4 \times 10^{-6} \times 10^{-3} \text{ kg} = 4 \times 10^{-9} \text{ kg}$

and v_B = velocity of particle B at separation of 9 mm

$$\Rightarrow 9 - 1 = \frac{1}{2} \times 4 \times 10^{-9} \times v_B^2$$

$$\Rightarrow v_B^2 = 4 \times 10^9 \Rightarrow v_B = 2 \times 10^3 \text{ ms}^{-1}$$

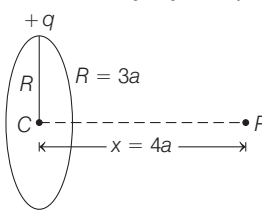
- 13** A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the Z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is

[2019, 10 April Shift-I]

- (a) $\sqrt{\frac{2}{m} \left(\frac{1}{5} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}}$
 (b) $\sqrt{\frac{2}{m} \left(\frac{4}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}}$
 (c) $\sqrt{\frac{2}{m} \left(\frac{1}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}}$
 (d) $\sqrt{\frac{2}{m} \left(\frac{2}{15} \frac{q^2}{4\pi\epsilon_0 a} \right)^{1/2}}$

Ans. (d)

Potential at any point at distance x from the centre of the ring is given by



$$V_p = \frac{Kq}{\sqrt{R^2 + x^2}}$$

Given, $R = 3a$ and $x = 4a$

$$\therefore V_p = \frac{Kq}{\sqrt{9a^2 + 16a^2}} = \frac{Kq}{5a} \quad \dots(i)$$

At centre, $x = 0$

So, potential at centre is

$$V_c = \frac{Kq}{R} = \frac{Kq}{3a} \quad \dots(ii)$$

Now, energy required to get this charge from $x = 4a$ to the centre is

$$\Delta U = q \Delta V = q [V_c - V_p]$$

$$= q \left[\frac{Kq}{3a} - \frac{Kq}{5a} \right]$$

$$= \frac{Kq^2}{a} \left[\frac{1}{3} - \frac{1}{5} \right]$$

$$\Delta U = \frac{2}{15} \frac{Kq^2}{a} \quad \dots(iii)$$

This energy must be equal to (or less than) the kinetic energy of the charge, i.e.

$$\frac{1}{2} mv^2 \geq \frac{2}{15} \frac{Kq^2}{a}$$

So, minimum energy required is

$$\frac{1}{2} mv^2 = \frac{2}{15} \times \frac{1}{4\pi\epsilon_0} \times \frac{q^2}{a} \quad (\text{put } K = 1/4\pi\epsilon_0)$$

$\therefore$ Minimum velocity,

$$v^2 = \frac{2}{m} \times \frac{2}{4\pi\epsilon_0} \times \frac{q^2}{15a}$$

$$\text{or } v = \sqrt{\frac{2}{m}} \times \sqrt{\frac{2q^2}{4\pi\epsilon_0 a \times 15}}$$

- 14** A point dipole $\mathbf{p} = -p_0 \hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are, respectively [Take, $V = 0$ at infinity]

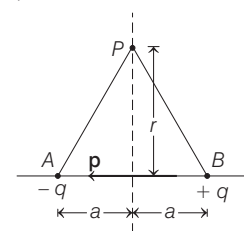
[2019, 12 April Shift-I]

(a) $\frac{|\mathbf{p}|}{4\pi\epsilon_0 d^2}, \frac{\mathbf{p}}{4\pi\epsilon_0 d^3}$ (b) $0, \frac{-\mathbf{p}}{4\pi\epsilon_0 d^3}$

(c) $0, \frac{\mathbf{p}}{4\pi\epsilon_0 d^3}$ (d) $\frac{|\mathbf{p}|}{4\pi\epsilon_0 d^2}, \frac{-\mathbf{p}}{4\pi\epsilon_0 d^3}$

Ans. (b)

The given problem can be shown as clearly potential difference at point P due to dipole is



$$V = V_{AP} + V_{BP} \quad (\text{scalar addition})$$

$$\Rightarrow V = \frac{k(-q)}{AP} + \frac{k(q)}{BP} \quad \dots(i)$$

Here, $AP = BP = \sqrt{a^2 + r^2}$

$$\therefore V = -\frac{kq}{\sqrt{a^2 + r^2}} + \frac{kq}{\sqrt{a^2 + r^2}} = 0 \quad \dots(ii)$$

Now, electric field at any point on Y-axis, i.e. equatorial line of the dipole can be given by

$$E = -\frac{k p}{r^3} \quad (\text{standard expression})$$

$$\Rightarrow E = -\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

Given, $r = d$

$$\therefore E = -\frac{1}{4\pi\epsilon_0} \frac{p}{d^3} \quad \dots(iii)$$

From Eqs. (ii) and (iii), correct option is (b).

Alternate Solution

Electric field at any point at θ angle from axial line of dipole is given by

$$E = -\frac{k p}{r^3} \sqrt{3 \cos^2 \theta + 1}$$

Here, $\theta = 90^\circ$

$$\Rightarrow \cos \theta = \cos 90^\circ = 0 \text{ and } r = d$$

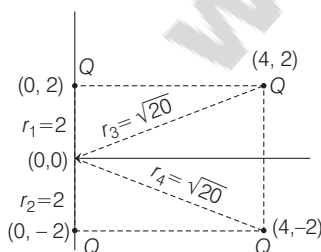
$$\therefore E = -\frac{k p}{d^3} = -\frac{p}{4\pi\epsilon_0 d^3}$$

- 15** Four equal point charges Q each are placed in the xy-plane at $(0, 2)$, $(4, 2)$, $(4, -2)$ and $(0, -2)$. The work required to put a fifth charge Q at the origin of the coordinate system (in joule) will be [2019, 10 Jan Shift-II]

(a) $\frac{Q^2}{4\pi\epsilon_0}$ (b) $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{3}}\right)$
 (c) $\frac{Q^2}{2\sqrt{2}\pi\epsilon_0}$ (d) $\frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{5}}\right)$

Ans. (d)

The four charges are shown in the figure below



Electric potential at origin $(0, 0)$ due to these charges can be found by scalar addition of electric potentials due to each charge.

$$\therefore V = \frac{KQ}{r_1} + \frac{KQ}{r_2} + \frac{KQ}{r_3} + \frac{KQ}{r_4} \quad \dots(i)$$

$$\Rightarrow V = KQ \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{20}} \right] = KQ \left[1 + \frac{1}{\sqrt{5}} \right]$$

$$\Rightarrow V = KQ \frac{(\sqrt{5} + 1)}{\sqrt{5}} \text{ volt} \quad \dots(ii)$$

Now, if another charge Q is placed at origin, then work done to get the charge at origin

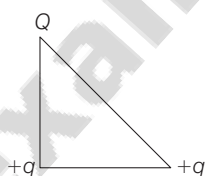
$$W = QV \quad \dots(iii)$$

By putting value of V from Eq. (ii) in Eq. (iii), we get

$$W = KQ^2 \frac{(\sqrt{5} + 1)}{\sqrt{5}} \text{ joule}$$

$$\text{or } W = \frac{Q^2}{4\pi\epsilon_0} \left(1 + \frac{1}{\sqrt{5}} \right) \text{ joule}$$

- 16** Three charges Q , $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is [2019, 11 Jan Shift-I]



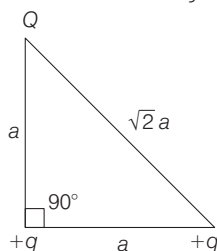
(a) $-2q$ (b) $\frac{-q}{1 + \sqrt{2}}$ (c) $+q$ (d) $\frac{-\sqrt{2}q}{\sqrt{2} + 1}$

Ans. (d)

Electrostatic energy between two charges q_1 and q_2 such that the distance between them is r is given as

$$U = \frac{K q_1 q_2}{r}$$

In accordance to the principle of superposition, total energy of the charge system as shown in the figure below is



$$U = \frac{Kq^2}{a} + \frac{KQq}{a} + \frac{KQq}{\sqrt{2}a}$$

It is given that, $U = 0$

$$\therefore \frac{Kq}{a} \left[q + Q + \frac{Q}{\sqrt{2}} \right] = 0$$

$$\Rightarrow Q = \frac{-\sqrt{2} \times q}{(\sqrt{2} + 1)}$$

- 17** An electric field of 1000 V/m is applied to an electric dipole at angle of 45° . The value of electric dipole moment is 10^{-29} C-m . What is the potential energy of the electric dipole? [2019, 11 Jan Shift-II]

- (a) $-9 \times 10^{-20} \text{ J}$
 (b) $-10 \times 10^{-29} \text{ J}$
 (c) $-20 \times 10^{-18} \text{ J}$
 (d) $-7 \times 10^{-27} \text{ J}$

Ans. (d)

Given, $E = 1000 \text{ V/m}$

$$\theta = 45^\circ$$

and $p = 10^{-29} \text{ C-m}$

We know that, electric potential energy stored in an electric dipole kept in uniform electric field is given by the relation

$$U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$$

$$= -10^{-29} \times 1000 \times \cos 45^\circ$$

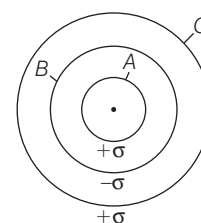
$$\Rightarrow U \approx -7 \times 10^{-27} \text{ J}$$

- 18** Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is [JEE Main 2018]

(a) $\frac{\sigma}{\epsilon_0} \left[\frac{a^2 - b^2}{a} + c \right]$ (b) $\frac{\sigma}{\epsilon_0} \left[\frac{a^2 - b^2}{b} + c \right]$
 (c) $\frac{\sigma}{\epsilon_0} \left[\frac{b^2 - c^2}{b} + a \right]$ (d) $\frac{\sigma}{\epsilon_0} \left[\frac{b^2 - c^2}{c} + a \right]$

Ans. (b)

Key Idea Potential of B = Potential due to charge on A + Potential due to charge on B + Potential due to charge on C.



$$\therefore V_B = \frac{k(Q_A + Q_B)}{b} + \frac{kQ_C}{c}$$

$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c} \right] \\
 &= \frac{\sigma\epsilon}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + \frac{c^2}{c} \right) \\
 &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + c \right) \\
 V_B &= \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + c \right)
 \end{aligned}$$

- 19** A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with

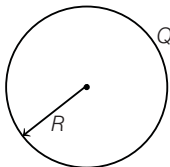
potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$

have radius R_1, R_2, R_3 , and R_4 respectively. Then, [JEE Main 2015]

- (a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
 (b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
 (c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
 (d) $2R < R_4$

Ans. (c, d)

Potential at the surface of the charged sphere



Charged sphere

$$V_0 = \frac{KQ}{R}$$

$$V = \frac{KQ}{r}, r \geq R = \frac{KQ}{2R^3} (3R^2 - r^2);$$

$$r \leq R$$

$$V_{\text{centre}} = V_c = \frac{KQ}{2R^3} \times 3R^2 = \frac{3KQ}{2R} = \frac{3V_0}{2}$$

$$\Rightarrow R_1 = 0$$

As potential decreases for outside points.

Thus, according to the question, we can write

$$V_{R_2} = \frac{5V_0}{4} = \frac{KQ}{2R^3} (3R^2 - R_2^2)$$

$$\frac{5V_0}{4} = \frac{V_0}{2R^2} (3R^2 - R_2^2) \text{ or } \frac{5}{2} = 3 - \left(\frac{R_2}{R}\right)^2$$

$$\text{or } \left(\frac{R_2}{R}\right)^2 = 3 - \frac{5}{2} = \frac{1}{2} \text{ or } R_2 = \frac{R}{\sqrt{2}}$$

Similarly,

$$V_{R_3} = \frac{3V_0}{4} \Rightarrow \frac{KQ}{R_3} = \frac{3}{4} \times \frac{KQ}{R} \text{ or } R_3 = \frac{4}{3} R$$

$$V_{R_4} = \frac{KQ}{R_4} = \frac{V_0}{4} \Rightarrow \frac{KQ}{R_4} = \frac{1}{4} \times \frac{KQ}{R} \text{ or } R_4 = 4R$$

- 20** Assume that an electric field $\mathbf{E} = 30x^2 \mathbf{i}$ exists in space. Then, the potential difference $V_A - V_0$, where V_0 is the potential at the origin and V_A is the potential at $x = 2\text{m}$, is

[JEE Main 2014]

- (a) 120 J (b) -120 J (c) -80 J (d) 80 J

Ans. (c)

As we know, potential difference

$$V_A - V_0 \text{ is}$$

$$dV = -E dx$$

$$\int_{V_0}^{V_A} dV = - \int_0^2 30x^2 dx$$

$$\begin{aligned}
 V_A - V_0 &= -30 \times \left[\frac{x^3}{3} \right]_0^2 \\
 &= -10 \times [2^3 - (0)^3] \\
 &= -10 \times 8 = -80 \text{ J}
 \end{aligned}$$

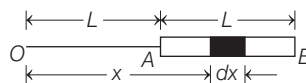
- 21** A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is

[JEE Main 2013]



- (a) $\frac{Q}{8\pi\epsilon_0 L}$ (b) $\frac{3Q}{4\pi\epsilon_0 L}$
 (c) $\frac{Q}{4\pi\epsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\epsilon_0 L}$

Ans. (d)



Let an element of length dx , charge dq , at distance x from point O .

$$dV = k \frac{dq}{x}$$

$$\text{where, } dq = \frac{Q}{L} dx$$

$$\begin{aligned}
 \therefore V &= \int_L^{2L} \frac{k dq}{x} = \int_L^{2L} \left(\frac{Q}{L} \right) \frac{dx}{x} \\
 &= \frac{Q}{4\pi\epsilon_0 L} \int_L^{2L} \left(\frac{1}{x} \right) dx
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{Q}{4\pi\epsilon_0 L} [\log_e x]_L^{2L} \\
 &= \frac{Q}{4\pi\epsilon_0 L} [\log_e 2L - \log_e L] \\
 &= \frac{Q}{4\pi\epsilon_0 L} \left[\log_e^2 \frac{2L}{L} \right] = \frac{Q}{4\pi\epsilon_0 L} \ln(2)
 \end{aligned}$$

- 22** Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is

[AIEEE 2011]

- (a) $\frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left(1 - \frac{1}{\sqrt{5}} \right)$
 (b) zero
 (c) $\frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left(1 + \frac{1}{\sqrt{5}} \right)$
 (d) $\frac{1}{4\pi\epsilon_0} \frac{2qQ}{a} \left(1 - \frac{2}{\sqrt{5}} \right)$

Ans. (a)

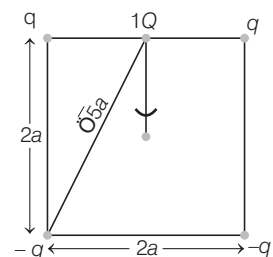
Initial potential energy of the system,

$$\begin{aligned}
 U_i &= \frac{2kqQ}{a} + \frac{2k(-q)Q}{\sqrt{5}a} \\
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{2qQ}{a} \left[1 - \frac{1}{\sqrt{5}} \right]
 \end{aligned}$$

Final potential energy of the system,

$$U_f = 0$$

By conservation of energy,



Gain in KE = Loss in PE

$$K = \frac{1}{4\pi\epsilon_0} \cdot \frac{2qQ}{a} \left[1 - \frac{1}{\sqrt{5}} \right]$$

- 23** Two points P and Q are maintained at the potentials of 10 V and -4 V, respectively. The work done in moving 100 electrons from P to Q is

[AIEEE 2009]

- (a) $-19 \times 10^{-17} \text{ J}$ (b) $9.60 \times 10^{-17} \text{ J}$
 (c) $-2.24 \times 10^{-16} \text{ J}$ (d) $2.24 \times 10^{-16} \text{ J}$

Ans. (d)

$$\begin{aligned}
 W &= QdV = Q(V_q - V_p) \\
 &= -100 \times (1.6 \times 10^{-19}) \times (-4 - 10) \\
 &= +100 \times 1.6 \times 10^{-19} \times 14 \\
 &= +2.24 \times 10^{-16} \text{ J}
 \end{aligned}$$

- 24** The potential at a point x (measured in μm) due to some charges situated on the x -axis is given by $V(x) = 20/(x^2 - 4)$ volt.

The electric field E at $x = 4\mu\text{m}$ is given by [AIEEE 2007]

- (a) $\frac{5}{3} \text{ V}/\mu\text{m}$ and in the -ve x -direction
 (b) $\frac{5}{3} \text{ V}/\mu\text{m}$ and in the +ve x -direction
 (c) $\frac{10}{9} \text{ V}/\mu\text{m}$ and in the -ve x -direction
 (d) $\frac{10}{9} \text{ V}/\mu\text{m}$ and in the +ve x -direction

Ans. (d)

$$\begin{aligned}
 E &= -\frac{\partial V}{\partial x} \mathbf{i} - \frac{\partial V}{\partial y} \mathbf{j} - \frac{\partial V}{\partial z} \mathbf{k} \\
 \Rightarrow E_x &= -\frac{\partial V}{\partial x} = -\frac{d}{dx} \left[\frac{20}{x^2 - 4} \right] \\
 &= \frac{40x}{(x^2 - 4)^2} \\
 \Rightarrow E_x \text{ at } x = 4\mu\text{m} &= \frac{10}{9} \text{ V}/\mu\text{m}
 \end{aligned}$$

and is along positive x -direction.

- 25** Charges are placed on the vertices of a square as shown. Let $\mathbf{E}$ be the electric field and V be the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then [AIEEE 2007]

- (a) $\mathbf{E}$ remains unchanged, V changes
 (b) both $\mathbf{E}$ and V change
 (c) $\mathbf{E}$ and V remain unchanged
 (d) $\mathbf{E}$ changes, V remains unchanged

Ans. (d)

Direction of $\mathbf{E}$ reverses while magnitude remains same and V remains unchanged.

- 26** An electric charge $10^{-3} \mu\text{C}$ is placed at the origin $(0, 0)$ of xy -coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and $(2, 0)$ respectively. The potential difference between the points A and B will be [AIEEE 2007]

- (a) 9 V (b) zero (c) 2 V (d) 4.5 V

Ans. (b)

Potential at A due to charge at O ,

$$\begin{aligned}
 V_A &= \frac{1}{4\pi\epsilon_0} \frac{(10^{-3})}{OA} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10^{-3})}{\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}} \\
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{(10^{-3})}{2}
 \end{aligned}$$

Potential at B due to charge at O ,

$$V_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10^{-3})}{OB} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10^{-3})}{2}$$

$$\text{So, } V_A - V_B = 0$$

- 27** Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is [AIEEE 2006]

- (a) 4 : 1 (b) 1 : 2
 (c) 2 : 1 (d) 1 : 4

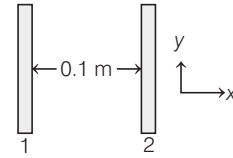
Ans. (c)

When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

$$\text{Therefore, } E = \frac{V}{r} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2}{1} = 2:1$$

- 28** Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20 \text{ V}$ (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1 \text{ m}$ and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2?

$$(e = 1.6 \times 10^{-19} \text{ C}, m_0 = 9.11 \times 10^{-31} \text{ kg}) \quad [\text{AIEEE 2006}]$$



- (a) $2.65 \times 10^6 \text{ ms}^{-1}$ (b) $7.02 \times 10^{12} \text{ ms}^{-1}$
 (c) $1.87 \times 10^6 \text{ ms}^{-1}$ (d) $32 \times 10^{-19} \text{ ms}^{-1}$

Ans. (a)

Since, $V_2 > V_1$, so electric field will point from plate 2 to plate 1.

The electron will experience an electric force, opposite to the direction of electric field and hence move towards the plate 2.

Use work-energy theorem to find speed of electron when it strikes the plate 2.

$$\therefore \frac{m_e v^2}{2} - 0 = e(V_2 - V_1)$$

where, v is the required speed.

$$\Rightarrow \frac{9.11 \times 10^{-31}}{2} v^2 = 1.6 \times 10^{-19} \times 20$$

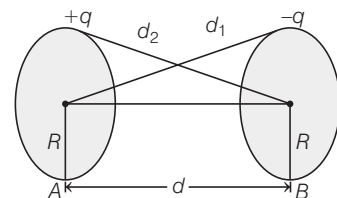
$$\text{or } v = \sqrt{\frac{1.6 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}} = 2.65 \times 10^6 \text{ ms}^{-1}$$

- 29** Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is [AIEEE 2005]

- (a) $\frac{qR}{4\pi\epsilon_0 d^2}$
 (b) $\frac{q}{2\pi\epsilon_0} \left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}} \right]$
 (c) zero
 (d) $\frac{q}{4\pi\epsilon_0} \left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}} \right]$

Ans. (b)

V_A = Potential due to charge $+q$ on ring A
 + Potential due to charge $-q$ on ring B



$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{R} - \frac{q}{d_1} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{R} - \frac{q}{\sqrt{R^2 + d^2}} \right)$$

$$[\because d_1 = \sqrt{R^2 + d^2}] \dots (i)$$

Similarly, $V_B = \frac{1}{4\pi\epsilon_0} \left(-\frac{q}{R} + \frac{q}{\sqrt{R^2 + d^2}} \right)$

Potential difference, $V_A - V_B$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{R} - \frac{q}{\sqrt{R^2 + d^2}} \right) - \frac{1}{4\pi\epsilon_0} \left(-\frac{q}{R} + \frac{q}{\sqrt{R^2 + d^2}} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{R} + \frac{1}{4\pi\epsilon_0} \frac{q}{R} - \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{R^2 + d^2}} - \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{R^2 + d^2}}$$

$$= \frac{q}{2\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}} \right)$$

- 30** A charged particle q is shot towards another charged particle Q which is fixed with a speed v . It approaches Q upto a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be



[AIEEE 2004]

- (a) r (b) $2r$
(c) $\frac{r}{2}$ (d) $\frac{r}{4}$

Ans. (d)

Let a particle of charge q having velocity v approaches Q upto a closest distance r and if the velocity becomes $2v$, the closest distance will be r' .

The law of conservation of energy yields, kinetic energy of particle = electric potential energy between them at closest distance of approach.

$$\text{or } \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r}$$

$$\text{or } \frac{1}{2}m(2v)^2 = \frac{kQq}{r'} \dots (i)$$

$$[\because k = \text{constant} = \frac{1}{4\pi\epsilon_0}]$$

$$\text{and } \frac{1}{2}m(2v)^2 = \frac{kQq}{r'} \dots (ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}m(2v)^2} = \frac{\frac{kQq}{r}}{\frac{kQq}{r'}} \Rightarrow \frac{1}{4} = \frac{r'}{r}$$

$$\Rightarrow r' = \frac{r}{4}$$

- 31** A thin spherical conducting shell of radius R has a charge q . Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

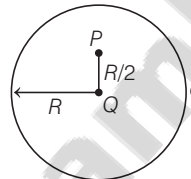
[AIEEE 2003]

- (a) $\frac{2Q}{4\pi\epsilon_0 R}$ (b) $\frac{2Q}{4\pi\epsilon_0 R} - \frac{2q}{4\pi\epsilon_0 R}$
(c) $\frac{2Q}{4\pi\epsilon_0 R} + \frac{q}{4\pi\epsilon_0 R}$ (d) $\frac{(q+Q)2}{4\pi\epsilon_0 R}$

Ans. (c)

At P due to shell, potential

$$V_1 = \frac{q}{4\pi\epsilon_0 R}$$



At P due to Q , potential

$$V_2 = \frac{Q}{4\pi\epsilon_0 \frac{R}{2}} = \frac{2Q}{4\pi\epsilon_0 R}$$

$\therefore$ Net potential at P ,

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\epsilon_0 R} + \frac{2Q}{4\pi\epsilon_0 R}$$

- 32** On moving a charge of 20 C by 2 cm, 2 J of work is done, then the potential difference between the points is

[AIEEE 2002]

- (a) 0.1 V (b) 8 V
(c) 2 V (d) 0.5 V

Ans. (a)

Potential difference between two points in an electric field is $V_A - V_B = \frac{W}{q_0}$

where, W is work done by moving charge q_0 from point A to B.

$$\text{Here, } W = 2 \text{ J}$$

$$q_0 = 20 \text{ C}$$

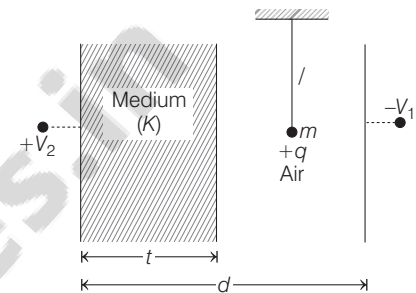
$$\text{So, } V_A - V_B = \frac{2}{20} = 0.1 \text{ V}$$

TOPIC 2

Capacitors and Capacitance

- 33** A simple pendulum of mass m , length l and charge $+q$ suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be

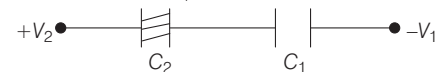
[2021, 27 July Shift-II]



- (a) $\tan^{-1} \left[\frac{q}{mg} \times \frac{C_1(V_2 - V_1)}{(C_1 + C_2)(d - t)} \right]$
(b) $\tan^{-1} \left[\frac{q}{mg} \times \frac{C_2(V_2 - V_1)}{(C_1 + C_2)(d - t)} \right]$
(c) $\tan^{-1} \left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)} \right]$
(d) $\tan^{-1} \left[\frac{q}{mg} \times \frac{C_1(V_1 + V_2)}{(C_1 + C_2)(d - t)} \right]$

Ans. (c)

Considering one part of capacitance with dielectric as C_2 and remaining with pendulum as C_1 .



$\therefore$ Charge on each capacitor will be

$$Q = \Delta V \times C_{eq} = [V_2 - (-V_1)] \times \left[\frac{C_1 C_2}{C_1 + C_2} \right]$$

$[\because C_1 \text{ and } C_2 \text{ are in series}]$

$$Q = \frac{C_1 C_2}{C_1 + C_2} (V_1 + V_2)$$

$\therefore$ Potential difference across C_1 , $V_1 = \frac{Q}{C_1}$

$$\Rightarrow V_1 = \frac{C_2(V_1 + V_2)}{(C_1 + C_2)} \dots (i)$$

$\therefore$ Electric field will be given as

$$E = \frac{V_1}{(d - t)} \dots (ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$E = \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)} \dots (iii)$$

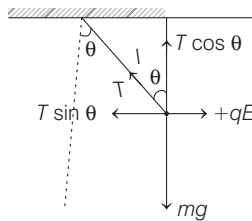
$\therefore$ Force, $F = Eq$... (iv)

where, E = electric field and q = charge.

From Eqs. (iii) and (iv), we get

$$F = \frac{qC_2(V_1 + V_2)}{(C_1 + C_2)(d - t)} \quad \dots (v)$$

If we consider the motion of pendulum inside capacitor, then it will be shown as follows



$$\Rightarrow \tan \theta = \frac{qE}{mg} \quad \left\{ \begin{array}{l} T \sin \theta = qE = F \\ T \cos \theta = mg \end{array} \right. \dots (vi)$$

$$\Rightarrow \tan \theta = \frac{F}{mg} \quad \left\{ \begin{array}{l} \therefore \tan \theta = \frac{\sin \theta}{\cos \theta} \end{array} \right. \dots (vi)$$

From Eqs. (v) and (vi), we get

$$\theta = \tan^{-1} \left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)} \right]$$

- 34** Two capacitors of capacities $2C$ and C are joined in parallel and charged upto potential V . The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K . The potential difference across the capacitors will now be [2021, 27 July Shift-I]

(a) $\frac{V}{K+2}$ (b) $\frac{V}{K}$ (c) $\frac{3V}{K+2}$ (d) $\frac{3V}{K}$

Ans. (c)

Total charge on both capacitors,

$$Q = Q_1 + Q_2 = C_1V + C_2V = 2CV + CV = 3CV$$

$$\Rightarrow Q = 3CV$$

When dielectric material is filled within the capacitor of capacitance C after removing the battery, then its charge will remain same but its capacitance becomes KC .

Now, potential difference across the capacitors will be common potential given as

$$V_C = \frac{Q}{KC + 2C}$$

$$= \frac{3CV}{C(K + 2)}$$

$$= \frac{3V}{K + 2}$$

- 35** If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant K placed between the capacitor plates, then bound charge q_b can be expressed as [2021, 25 July Shift-II]

(a) $q_b = q_f \left(1 - \frac{1}{\sqrt{K}} \right)$ (b) $q_b = q_f \left(1 - \frac{1}{K} \right)$

(c) $q_b = q_f \left(1 + \frac{1}{\sqrt{K}} \right)$ (d) $q_b = q_f \left(1 + \frac{1}{K} \right)$

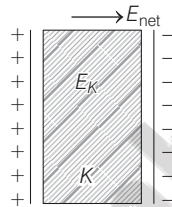
Ans. (b)

As we know that,

$$E_0 = \frac{\sigma_f}{\epsilon_0}$$

And

$$E_K = \frac{\sigma}{K\epsilon_0}$$



where, E_0 is electric field without dielectric and E_K is electric field with dielectric.

$$\sigma = \text{surface charge density} = \frac{q}{A}$$

ϵ_0 = free space permittivity

$$\therefore E_{\text{net}} = E_0 - E_K$$

$$\Rightarrow \left(\frac{\sigma}{\epsilon_0} \right)_b = \frac{\sigma_f}{\epsilon_0} - \frac{\sigma_f}{K\epsilon_0}$$

$$\Rightarrow \frac{q_b}{A\epsilon_0} = \frac{q_f}{A\epsilon_0} - \frac{q_f}{K\epsilon_0 A}$$

$$\Rightarrow q_b = q_f \left(1 - \frac{1}{K} \right)$$

- 36** A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m . The space between the plates is filled upto a thickness 5 m with a material of dielectric constant of 10 . The resultant capacitance of the system is $x\text{ pF}$. The value of $\epsilon_0 = 8.85 \times 10^{-12}\text{ fm}^{-1}$. The value of x to the nearest integer is [2021, 18 March Shift-I]

Ans. (161)

Given,

The area of the parallel plate capacitor, $A = 100\text{ m}^2$

The distance between the parallel plate of the capacitor, $d = 10\text{ m}$

The dielectric constant of the material, $K = 10$

The thickness of the space between the plates, $t = 5\text{ m}$

As we know, the expression of the capacitor with dielectric constant

$$C = \frac{A\epsilon_0}{\left(d - t + \frac{t}{K} \right)}$$

$$\Rightarrow C = \frac{100 \times 8.85 \times 10^{-12}}{\left(10 - 5 + \frac{5}{10} \right)}$$

$$\Rightarrow C = 161 \times 10^{-12}\text{ F} = 161\text{ pF}$$

Hence, the resultant capacitance of the system is 161 pF .

So, the value of x to the nearest integer is 161 .

- 37** Two isolated conducting spheres S_1 and S_2 of radii $\frac{2}{3}R$ and $\frac{1}{3}R$, have charges $12\mu\text{C}$ and $-3\mu\text{C}$ respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done, the charges on S_1 and S_2 respectively, are [2020, 3 Sep Shift-I]

- (a) $4.5\mu\text{C}$ on both
(b) $+4.5\mu\text{C}$ and $-4.5\mu\text{C}$
(c) $3\mu\text{C}$ and $6\mu\text{C}$
(d) $6\mu\text{C}$ and $3\mu\text{C}$

Ans. (d)

When both spheres are connected by a conducting wire, charge from higher potential sphere flows to lower potential sphere till

both spheres reach a common potential V_C which is given by,

$$V_C = \frac{Q_1 + Q_2}{C_1 + C_2}$$

For a sphere, capacitance, $C = 4\pi\epsilon_0 R$

$$\text{so, } V_C = \frac{12 \times 10^{-6} + (-3 \times 10^{-6})}{4\pi\epsilon_0 \left(\frac{2}{3}R + \frac{1}{3}R \right)}$$

$$= \frac{9 \times 10^{-6}}{4\pi\epsilon_0 \left(\frac{2}{3}R + \frac{1}{3}R \right)} = \frac{9 \times 10^{-6}}{4\pi\epsilon_0 R}$$

Values of charges on spheres after redistribution are :

Q_1 = charge on first sphere

$$Q_1 = C_1 V_C = 4\pi\epsilon_0 R_1 \cdot \frac{(9 \times 10^{-6})}{4\pi\epsilon_0 R}$$

$$= \frac{\left(4\pi\epsilon_0 \frac{2}{3}R\right)(9 \times 10^{-6})}{4\pi\epsilon_0 R}$$

$$= \frac{2}{3} \times 9 \times 10^{-6} \text{ C}$$

$$= \frac{2}{3} \times 9 \mu\text{C} = 6 \mu\text{C}$$

And, $Q_2 =$ charge on second sphere

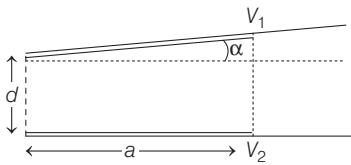
$$Q_2 = C_2 V_c$$

$$= \frac{4\pi\epsilon_0 \left(\frac{1}{3}R\right) \times 9 \times 10^{-6}}{4\pi\epsilon_0 R}$$

$$= 3 \times 10^{-6} \text{ C} = 3 \mu\text{C}$$

Hence, option (d) is correct.

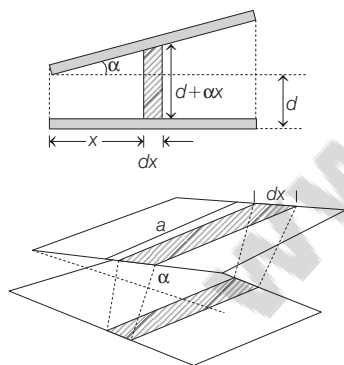
- 38** A capacitor is made of two square plates each of side a making a very small angle α between them, as shown in figure. The capacitance will be close to [2020, 8 Jan Shift-II]



- (a) $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{4d}\right)$ (b) $\frac{\epsilon_0 a^2}{d} \left(1 + \frac{\alpha a}{d}\right)$
- (c) $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{2d}\right)$ (d) $\frac{\epsilon_0 a^2}{d} \left(1 - \frac{3\alpha a}{2d}\right)$

Ans. (c)

For given capacitor, consider a small width dx at a distance x from origin O .



For this elemental capacitor, we have

Area of each plate $= a \times dx$

Distance between plates $= d + \alpha x$

Capacity of this elemental capacitor,

$$dC = \frac{\epsilon_0 a dx}{d + \alpha x}$$

Total capacity of arrangement is sum of capacitances of all such elemental capacitors because they all are in parallel.

Hence,

$$C_{\text{system}} = \int_0^a dC = \int_0^a \frac{\epsilon_0 a}{d + \alpha x} \cdot dx$$

$$= \frac{\epsilon_0 a}{\alpha} \ln \left(1 + \frac{\alpha a}{d}\right)$$

As, for small values of $\frac{\alpha a}{d}$,

$$\ln \left(1 + \frac{\alpha a}{d}\right) \approx \frac{\alpha a}{d} - \frac{1}{2} \left(\frac{\alpha a}{d}\right)^2$$

$$\text{So, } C_{\text{system}} = \frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{2d}\right)$$

- 39** Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is 10^{-4} m^2 . What is the dielectric constant, if the capacitance is 15 pF?

(Take, $\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$)

[2019, 8 April Shift-I]

- (a) 3.8 (b) 8.5 (c) 4.5 (d) 6.2

Ans. (b)

As we know, capacitance of a capacitor filled with dielectric medium,

$$C = \frac{\epsilon_0 K A}{d} \quad \dots(i)$$

and potential difference between plates is

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E} \quad \dots(ii)$$

So, by combining both Eqs. (i) and (ii), we get

$$K = \frac{CV}{\epsilon_0 A E} \quad \dots(iii)$$

Given, $C = 15 \text{ pF} = 15 \times 10^{-12} \text{ F}$,

$V = 500 \text{ V}$, $E = 10^6 \text{ Vm}^{-1}$,

$A = 10^{-4} \text{ m}^2$

and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

Substituting values in Eq. (iii), we get

$$K = \frac{15 \times 10^{-12} \times 500}{8.85 \times 10^{-12} \times 10^{-4} \times 10^6}$$

$$= 8.47 \approx 8.5$$

- 40** A parallel plate capacitor has $1 \mu\text{F}$ capacitance. One of its two plates is given $+2 \mu\text{C}$ charge and the other plate $+4 \mu\text{C}$ charge. The potential difference developed across the capacitor is [2019, 8 April Shift-II]

- (a) 1 V (b) 5 V
(c) 2 V (d) 3 V

Ans. (a)

Net value of charge on plates of capacitor after steady state is reached is

$$q_{\text{net}} = \frac{q_2 - q_1}{2}$$

where, q_2 and q_1 are the charges given to plates.

(Note that this formula is valid for any polarity of charge.)

Here, $q_2 = 4 \mu\text{C}$, $q_1 = 2 \mu\text{C}$

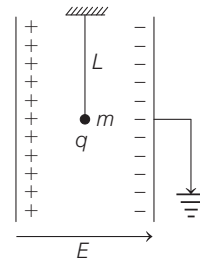
$\therefore$ Charge of capacitor is

$$q = \Delta q_{\text{net}} = \frac{4 - 2}{2} = 1 \mu\text{C}$$

Potential difference between capacitor plates is

$$V = \frac{Q}{C} = \frac{1 \mu\text{C}}{1 \mu\text{F}} = 1 \text{ V}$$

- 41** A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E , as shown in figure. Its bob has mass m and charge q . The time period of the pendulum is given by [2019, 10 April Shift-II]



- (a) $2\pi \sqrt{\frac{L}{g^2 + \left(\frac{qE}{m}\right)^2}}$
- (b) $2\pi \sqrt{\frac{L}{g^2 - \frac{q^2 E^2}{m^2}}}$
- (c) $2\pi \sqrt{\frac{L}{\left(g + \frac{qE}{m}\right)^2}}$
- (d) $2\pi \sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)^2}}$

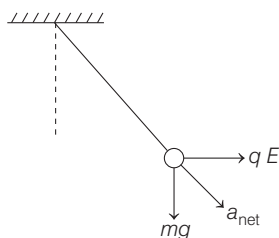
Ans. (a)

When pendulum is oscillating between capacitor plates, it is subjected to two forces;

(i) Weight downwards $= mg$

(ii) Electrostatic force acting horizontally $= qE$

So, net acceleration of pendulum bob is resultant of accelerations produced by these two perpendicular forces.



Net acceleration is, $a_{\text{net}} = \sqrt{a_1^2 + a_2^2}$

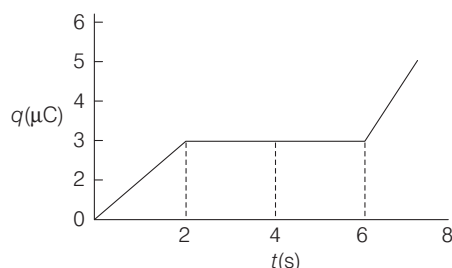
$$= \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

So, time period of oscillations of pendulum is

$$T = 2\pi \sqrt{\frac{l}{a_{\text{net}}}} = 2\pi \sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

- 42** The charge on a capacitor plate in a circuit as a function of time is shown in the figure.

[2019, 12 Jan Shift-II]



What is the value of current at $t = 4$ s?

- (a) $2\mu\text{A}$ (b) $1.5\mu\text{A}$
 (c) Zero (d) $3\mu\text{A}$

Ans. (c)

As we know, current, $I = \frac{dq}{dt}$

= Slope of q versus t graph

= Zero at $t = 4$ s; (as graph is a line parallel to time axis at $t = 4$ s)

- 43** A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m . If the electric field between the plates is 100 N/C , the magnitude of charge on each plate is

(Take, $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N-m}^2}$)

[2019, 12 Jan Shift-II]

- (a) $9.85 \times 10^{-10}\text{ C}$ (b) $8.85 \times 10^{-10}\text{ C}$
 (c) $7.85 \times 10^{-10}\text{ C}$ (d) $6.85 \times 10^{-10}\text{ C}$

Ans. (b)

If Q = charge on each plate, then

$$Q = CV = \frac{\epsilon_0 A}{d} \cdot Ed = \epsilon_0 AE$$

Here, $A = 1\text{ m}^2$, $E = 100\text{ N/C}$

and $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N-m}^2}$

So, by substituting given values, we get

$$Q = 8.85 \times 10^{-12} \times 1 \times 100$$

$$= 8.85 \times 10^{-10}\text{ C}$$

- 44** A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V . If a dielectric material of dielectric constant $K = \frac{5}{3}$

is inserted between the plates, the magnitude of the induced charge will be [JEE Main 2018]

- (a) 1.2 nC (b) 0.3 nC
 (c) 2.4 nC (d) 0.9 nC

Ans. (a)

Magnitude of induced charge is given by

$$Q' = (K - 1) CV_0 = \left(\frac{5}{3} - 1\right) 90 \times 10^{-12} \times 20$$

$$= 1.2 \times 10^{-9}\text{ C} \Rightarrow Q' = 1.2\text{ nC}$$

- 45** A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is $3 \times 10^4\text{ V/m}$, the charge density of the positive plate will be close to [JEE Main 2014]

- (a) $6 \times 10^{-7}\text{ C/m}^2$ (b) $3 \times 10^{-7}\text{ C/m}^2$
 (c) $3 \times 10^4\text{ C/m}^2$ (d) $6 \times 10^4\text{ C/m}^2$

Ans. (a)

Electric field inside dielectric,

$$\frac{\sigma}{K\epsilon_0} = 3 \times 10^4$$

$$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

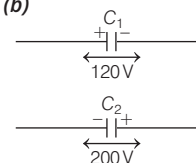
$$= 6.6 \times 8.85 \times 10^{-8}$$

$$= 5.841 \times 10^{-7} = 6 \times 10^{-7}\text{ C/m}^2$$

- 46** Two capacitors C_1 and C_2 are charged to 120 V and 200 V , respectively. It is found that by connecting them together, the potential on each one can be made zero. Then, [JEE Main 2013]

- (a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
 (c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

Ans. (b)



For potential to be made zero after connection, the charge of both capacitors are equal.

$$\therefore q_1 = q_2 \Rightarrow C_1 V_1 = C_2 V_2$$

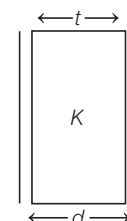
$$\Rightarrow 120 C_1 = 200 C_2 \Rightarrow 3C_1 = 5C_2$$

- 47** A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

[AIEEE 2003]

- (a) decreases
 (b) remains unchanged
 (c) becomes infinite
 (d) increases

Ans. (b)



Aluminium is a metal, so when we insert an aluminium foil, equal and opposite charges appear on its two surfaces. Since, it is of negligible thickness, it will not affect the capacitance.

Alternate Solution From the formula,

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

Here, $K = \infty$ and $t \rightarrow 0$

So, $C = \frac{\epsilon_0 A}{d + 0} = \frac{\epsilon_0 A}{d} = C_0$

- 48** Capacitance (in F) of a spherical conductor having radius 1 m , is [AIEEE 2002]

- (a) 1.1×10^{-10} (b) 10^{-6}
 (c) 9×10^{-9} (d) 10^{-3}

Ans. (a)

Capacitance of spherical conductor $= 4\pi\epsilon_0 a$

where, a is radius of conductor.

Therefore, $C = \frac{1}{9 \times 10^9} \times 1$

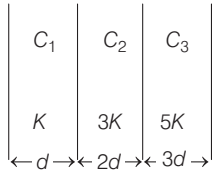
$$= \frac{1}{9} \times 10^{-9} = 0.11 \times 10^{-9}\text{ F}$$

$$= 1.1 \times 10^{-10}\text{ F}$$

TOPIC 3

Combination of Capacitors and Energy Stored in a Capacitor

- 49** In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be (Take, area of plate = A) **[2021, 27 July Shift-II]**



- (a) $\frac{15}{34} \frac{K \epsilon_0 A}{d}$ (b) $\frac{15}{6} \frac{K \epsilon_0 A}{d}$
 (c) $\frac{25}{6} \frac{K \epsilon_0 A}{d}$ (d) $\frac{9}{6} \frac{K \epsilon_0 A}{d}$

Ans. (a)

The value of C_1 , C_2 and C_3 can be given as

$$C_1 = \frac{K \epsilon_0 A}{d}, C_2 = \frac{3K \epsilon_0 A}{2d}, C_3 = \frac{5K \epsilon_0 A}{3d}$$

$\therefore$ In the above figure, we can see that C_1 , C_2 and C_3 are in series combination.

$$\begin{aligned} \Rightarrow \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ \Rightarrow \frac{1}{C_{eq}} &= \frac{1}{\frac{K \epsilon_0 A}{d}} + \frac{1}{\frac{3K \epsilon_0 A}{2d}} + \frac{1}{\frac{5K \epsilon_0 A}{3d}} \\ \Rightarrow \frac{1}{C_{eq}} &= \frac{d}{K \epsilon_0 A} + \frac{2d}{3K \epsilon_0 A} + \frac{3d}{5K \epsilon_0 A} \\ \Rightarrow \frac{1}{C_{eq}} &= \frac{d}{K \epsilon_0 A} \left(1 + \frac{2}{3} + \frac{3}{5} \right) \\ \Rightarrow \frac{1}{C_{eq}} &= \frac{d}{K \epsilon_0 A} \left(\frac{15 + 10 + 9}{15} \right) \\ \Rightarrow \frac{1}{C_{eq}} &= \frac{34d}{15K \epsilon_0 A} \\ \Rightarrow C_{eq} &= \frac{15}{34} \left(\frac{K \epsilon_0 A}{d} \right) \end{aligned}$$

- 50** A parallel plate capacitor with plate area A and distance of separation d is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as **[2021, 25 July Shift-I]**

$$\epsilon(x) = \epsilon_0 + kx, \text{ for } \left(0 < x \leq \frac{d}{2} \right)$$

$$\epsilon(x) = \epsilon_0 + k(d - x), \text{ for } \left(\frac{d}{2} \leq x \leq d \right)$$

- (a) $\left(\epsilon_0 + \frac{kd}{2} \right)^{2/KA}$ (b) $\frac{kA}{2 \ln \left(\frac{2\epsilon_0 + kd}{2\epsilon_0} \right)}$
 (c) zero (d) $\frac{kA}{2} \ln \left(\frac{2\epsilon_0}{2\epsilon_0 - kd} \right)$

Ans. (b)

Given, plate area = A

Separation between plates = d

Permittivity of dielectric,

$$\epsilon(x) = \epsilon_0 + kx, \text{ for } \left(0 < x \leq \frac{d}{2} \right)$$

$$\epsilon(x) = \epsilon_0 + k(d - x), \text{ for } \left(\frac{d}{2} \leq x \leq d \right)$$

As we know that,

Capacitance with dielectric

$$(C_k) = kC_0 = \frac{k\epsilon_0 A}{d}$$

where, ϵ_0 = free space permittivity

and $k\epsilon_0 = \epsilon$ = permittivity of medium.

Since, series capacitance adds up in reciprocal manner.

$$\frac{1}{C_{eq}} = \frac{dx}{A\epsilon}$$

where, C_{eq} = equivalent capacitance

and dx = distance between plates.

$$\begin{aligned} \Rightarrow \frac{1}{C_{eq}} &= \int_0^{\frac{d}{2}} \frac{dx}{A(\epsilon_0 + kx)} + \int_{\frac{d}{2}}^d \frac{dx}{A[\epsilon_0 + k(d - x)]} \\ &= \frac{1}{Ak} [\ln(\epsilon_0 + kx)]_0^{d/2} - \frac{1}{Ak} [\ln[\epsilon_0 + k(d - x)]]_{d/2}^d \\ &= \frac{1}{Ak} [\ln(\epsilon_0 + kd/2) - \ln \epsilon_0] - \frac{1}{Ak} [\ln \epsilon_0 + k(d - d)] \\ &\quad + \ln[\epsilon_0 + k(d - d/2)] \\ &= \frac{1}{Ak} \left[\ln \left(\epsilon_0 + \frac{kd}{2} \right) - \ln \epsilon_0 - \ln \epsilon_0 + \ln \left(\epsilon_0 + \frac{kd}{2} \right) \right] \\ &= \frac{2}{Ak} \left[\ln \left(\epsilon_0 + \frac{kd}{2} \right) - \ln \epsilon_0 \right] \\ &= \frac{1}{Ak} 2 \ln \left[\frac{\left(\epsilon_0 + \frac{kd}{2} \right)}{\epsilon_0} \right] \end{aligned}$$

$$\Rightarrow C_{eq} = \frac{kA}{2 \ln \left[\frac{\left(\epsilon_0 + \frac{kd}{2} \right)}{\epsilon_0} \right]} = \frac{kA}{2 \ln \left[\frac{2\epsilon_0 + kd}{2\epsilon_0} \right]}$$

- 51** A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference $V = 12$ V between its plates. The charging battery is now disconnected and a porcelain plate with $K = 7$ is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of pJ.

(Assume no friction)

[2021, 17 March Shift-I]

Ans. (864)

Given, capacitance of capacitor, $C = 14$ pF

Potential difference, $V = 12$ V

$$\text{Energy of capacitor, } E_i = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 14 \times 12 \times 12 = 1008 \text{ pJ}$$

Now, the charging battery is disconnected and a porcelain plate with $K = 7$ is inserted between the plates

$$\therefore E_f = \frac{E_i}{7} \Rightarrow f = \frac{1008}{7} \text{ pJ} \Rightarrow f = 144 \text{ pJ}$$

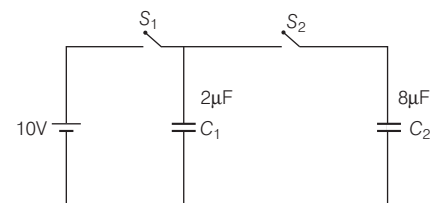
Mechanical energy with which the plate would oscillate back and forth between the plates will be

$$= (1008 - 144) \text{ pJ} = 864 \text{ pJ}$$

- 52** A $2 \mu\text{F}$ capacitor C_1 is first charged to a potential difference of 10 V using a battery. Then, the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of $8 \mu\text{F}$. The charge in C_2 on equilibrium condition is μC .

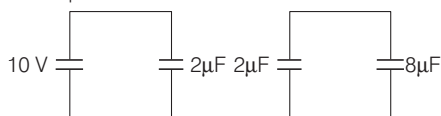
(Round off to the nearest integer)

[2021, 17 March Shift-II]



Ans. (16)

Let V is the voltage across each capacitor,



Now, after removing the battery the capacitor is connected.

So, using the law of conservation of charge,

$$2V + 8V = 2 \times 10 = 20$$

$$\Rightarrow 10V = 20 \Rightarrow V = 2V$$

As we know, $Q = CV = 8 \times 2 = 16 \mu C$

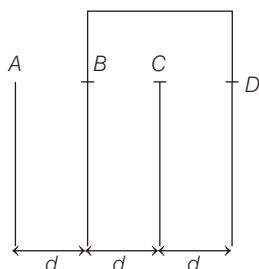
Hence, the magnitude of the charge in C_2 on equilibrium condition is $16 \mu C$.

- 53** Four identical rectangular plates with length, $l = 2$ cm and breadth, $b = \frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\epsilon_0}{d}$. The value

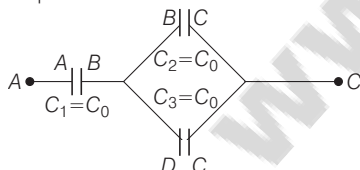
of x is

(Round off to the nearest integer)

[2021, 17 March Shift-I]

**Ans. (2)**

The given figure can be shown with capacitors as



Let C_0 be the capacitance of each capacitor.

In the above figure capacitance C_1 is in series combination with equivalent of parallel combination of capacitance C_2 and C_3 .

$$C'_{eq} = C_2 + C_3 \Rightarrow C'_{eq} = C_0 + C_0 = 2C_0$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C'_{eq}} = \frac{1}{C_0} + \frac{1}{2C_0} = \frac{3}{2C_0}$$

$$\Rightarrow C_{eq} = \frac{2C_0}{3} \quad \dots(i)$$

$$\text{As, } C_0 = \frac{\epsilon_0 A}{d} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$C_{eq} = \frac{2}{3} \left(\frac{\epsilon_0 A}{d} \right)$$

$$\Rightarrow C_{eq} = \frac{2\epsilon_0}{3d} (l \times b) = \frac{2\epsilon_0}{3d} \left(2 \times \frac{3}{2} \right)$$

$$\Rightarrow C_{eq} = \frac{2\epsilon_0}{d} \quad \dots(iii)$$

According to question, the equivalent capacitance between A and C is $\frac{x\epsilon_0}{d}$.

So, comparing it with Eq. (iii), we get

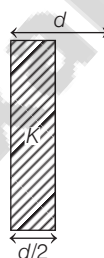
$$x = 2$$

- 54** In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m . If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see figure) the capacitance of the set-up will be ϵ_0 .

(Dielectric constant of the material = 3.2)

(Round off to the nearest integer)

[2021, 16 March Shift-II]

**Ans. (3)**

The equivalent capacitance of the capacitor when dielectric material is partially filled, is given as

$$C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}} = \frac{\epsilon_0 A}{\left(d - \frac{d}{2}\right) + \frac{d/2}{K}} = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d}{2K}}$$

where,

ϵ_0 = absolute electrical permittivity of free space,

A = area of the plates of capacitor = 2 m^2

K = dielectric constant = 3.2

t = thickness of dielectric material = $\frac{d}{2}$

and d = distance between the plates = 2 m .

$$\therefore C = \frac{2\epsilon_0 A}{\frac{d}{K} + d} = \frac{2 \times 2 \epsilon_0}{\frac{1}{3.2} + 1} = \frac{4 \times 3.2}{4.2} \epsilon_0$$

$$\Rightarrow C = 3.04 \epsilon_0$$

The required value after rounding off to the nearest integer is 3 .

- 55** For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where d is the

separation between the plates of parallel plate capacitor. The new capacitance

(C') in terms of original capacitance (C_0) is given by the following relation

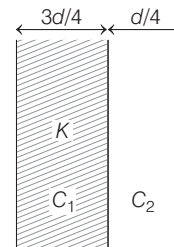
[2021, 16 March Shift-I]

$$(a) C' = \frac{3+K}{4K} C_0 \quad (b) C' = \frac{4+K}{3} C_0$$

$$(c) C' = \frac{4K}{K+3} C_0 \quad (d) C' = \frac{4}{3+K} C_0$$

Ans. (c)

As per question, the figure can be shown for a parallel plate capacitors,



Initially, capacitance,

$$C_0 = \frac{\epsilon_0 A}{d} \quad \dots(i)$$

where, ϵ_0 = permittivity of free space, A = area of plates and d = distance between plates.

In presence of a dielectric medium between the plates, the capacitance will be

$$C = \frac{\epsilon_0 K A}{d} \quad \dots(ii)$$

Also, from the figure capacitors C_1 and C_2 are in series.

$\therefore$ Equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_{eq}} = \left(\frac{3d/4}{\epsilon_0 K A} \right) + \left(\frac{d/4}{\epsilon_0 A} \right)$$

[using Eqs. (i) and (ii)]

$$\Rightarrow \frac{1}{C_{eq}} = \frac{d}{4\epsilon_0 A} \left(\frac{3+K}{K} \right)$$

$$\Rightarrow C_{eq} = \frac{4KC_0}{(3+K)} \quad [\text{using Eq.(i)}]$$

- 56** Consider the combination of two capacitors C_1 and C_2 , with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $15/4$ time the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors $\frac{C_2}{C_1}$.

[2021, 26 Feb Shift-I]

- (a) $\frac{15}{11}$ (b) $\frac{11}{80}$
(c) $\frac{29}{15}$ (d) $\frac{15}{4}$

Ans. (*)

Let, equivalent capacitance of two capacitors C_1 and C_2 connected in parallel be C_a and equivalent capacitance of same, when connected in series be C_b .

According to given data,

$$C_a = \frac{15}{4} C_b \quad \dots(i)$$

Since, equivalent capacitance in parallel combination,

$$C_{eq} = C_1 + C_2$$

$$\therefore C_a = C_1 + C_2 \quad \dots(ii)$$

and equivalent capacitance in series combination,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \frac{1}{C_b} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_b = \frac{C_1 C_2}{C_1 + C_2} \quad \dots(iii)$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$C_1 + C_2 = \frac{15}{4} \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow 4(C_1 + C_2)^2 = 15 C_1 C_2$$

$$\Rightarrow 4C_1^2 + 4C_2^2 + 8C_1 C_2 = 15 C_1 C_2$$

$$\Rightarrow 4C_1^2 + 4C_2^2 - 7C_1 C_2 = 0$$

On dividing both sides by C_1^2 , we get

$$4 + 4\left(\frac{C_2}{C_1}\right)^2 - 7\frac{C_2}{C_1} = 0$$

or $4\left(\frac{C_2}{C_1}\right)^2 - 7\left(\frac{C_2}{C_1}\right) + 4 = 0$

If $\frac{C_2}{C_1} = x$, then $4x^2 - 7x + 4 = 0$

By using the concept of quadratic equation,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{7 \pm \sqrt{49 - 64}}{8}$$

$$\Rightarrow x = \frac{C_2}{C_1} = \frac{7 \pm \sqrt{-15}}{8} = \frac{7 \pm \sqrt{15}i}{8}$$

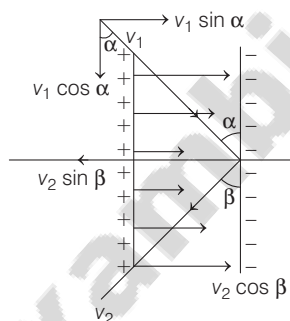
- 57** An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle α with the plates. It leaves the plates at angle β with kinetic energy K_2 . Then, the ratio of kinetic energies $K_1 : K_2$ will be

[2021, 25 Feb Shift-II]

- (a) $\frac{\cos \beta}{\cos \alpha}$ (b) $\frac{\cos \beta}{\sin \alpha}$ (c) $\frac{\sin^2 \beta}{\cos^2 \alpha}$ (d) $\frac{\cos^2 \beta}{\cos^2 \alpha}$

Ans. (b)

The given situation can be shown as below



Let, v_1 and v_2 be the incoming and outgoing velocities of electron into the capacitor and out of the capacitor, respectively.

Since, electric field is along X-axis, hence electric force on electron along Y-axis, $(F_y) = 0$

$\therefore$ Change in momentum along Y-axis,

$$\Delta p_y = 0$$

i.e. $p_1 = p_2$ (along Y-axis)

$$\Rightarrow m_1 v_1 \cos \alpha = m_2 v_2 \cos \beta$$

$$\Rightarrow v_1 / v_2 = \cos \beta / \cos \alpha$$

$\therefore$ Kinetic energy $(K) = 1/2 mv^2$

If mass is same, $K \propto v^2$

$$\therefore \frac{K_1}{K_2} = \left(\frac{v_1}{v_2} \right)^2 = \left(\frac{\cos \beta}{\cos \alpha} \right)^2 = \frac{\cos^2 \beta}{\cos^2 \alpha}$$

- 58** Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be

[2021, 24 Feb Shift-I]

- (a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

Ans. (d)

Given, $C_1 = C_2 = C$

When both capacitors are connected in series, their equivalent capacitance will be

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_s = \frac{C}{2}$$

When both capacitors are connected in parallel, their equivalent capacitance will be $C_p = C + C = 2C$

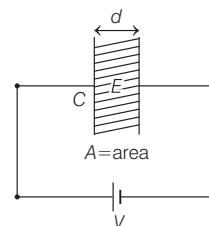
$\therefore$ The ratio of equivalent capacitance in series and parallel combination is

$$\frac{C_s}{C_p} = \frac{C/2}{2C} = \frac{1}{4} \Rightarrow C_s : C_p = 1 : 4$$

- 59** A $5 \mu\text{F}$ capacitor is charged fully by a 220V supply. It is then disconnected from the supply and is connected in series to another uncharged $2.5 \mu\text{F}$ capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J, then value of

X to the nearest integer is

[2020, 2 Sep Shift-I]

**Ans. (4)**

Initial energy of charged capacitor,

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (220)^2$$

$$= 121 \times 10^{-3} \text{ J}$$

Common potential after redistribution of charges is

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{C_1 V_1}{C_1 + C_2}$$

$$= \frac{5 \times 10^{-6} \times 220}{5 \times 10^{-6} + 2.5 \times 10^{-6}} = 220 \times \frac{2}{3} \text{ V}$$

Final stored energy is

$$U_2 = \frac{1}{2} (C_1 + C_2) V^2$$

$$= \frac{1}{2} \times 7.5 \times 10^{-6} \times \left(220 \times \frac{2}{3} \right)^2$$

$$\approx 80 \times 10^{-3} \text{ J}$$

Loss of energy

$$\Delta U = (121 - 80) \times 10^{-3} \text{ J} = 41 \times 10^{-3} \text{ J}$$

$$= \frac{4.1}{100} \text{ J} \approx \frac{4}{100} \text{ J}$$

Given, $\Delta U = \frac{X}{100} \text{ J} \therefore X = 4$

- 60** A $10\ \mu\text{F}$ capacitor is fully charged to a potential difference of $50\ \text{V}$. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now, the potential difference across them becomes $20\ \text{V}$. The capacitance of the second capacitor is

[2020, 2 Sep Shift-II]

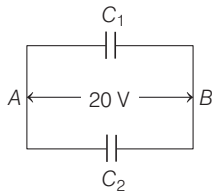
- (a) $15\ \mu\text{F}$ (b) $30\ \mu\text{F}$
(c) $20\ \mu\text{F}$ (d) $10\ \mu\text{F}$

Ans. (a)

Initially we have a capacitor of capacitance, $C_1 = 10\ \mu\text{F}$ and $V_1 = 50\ \text{V}$
Charge on capacitor is

$$Q = C_1 V_1 = 10\ \mu\text{F} \times 50\text{V} = 500\ \mu\text{C}$$

Now, another uncharged capacitor of capacity C_2 is connected in parallel with C_1 as shown in figure.



Given that, after redistribution, voltage or common potential is $20\ \text{V}$.

Final charge on C_1 is

$$Q_1 = C_1 V = 10\ \mu\text{F} \times 20\text{V} = 200\ \mu\text{C}$$

So, another capacitor has a charge of

$$Q_2 = 500 - 200 = 300\ \mu\text{F}$$

Potential, $V = 20\ \text{V}$

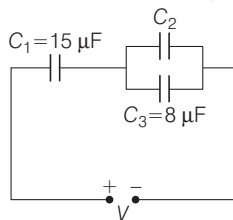
$$\therefore \text{Capacity, } C_2 = \frac{Q_2}{V} = \frac{300\ \mu\text{F}}{20\text{V}}$$

$$= 15\ \mu\text{F}$$

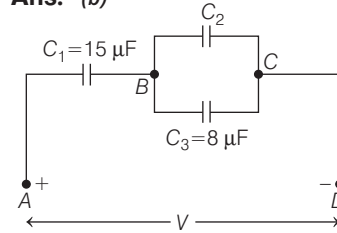
Hence, correct option is (a).

- 61** In the circuit shown in the figure, the total charge is $750\ \mu\text{C}$ and the voltage across capacitor C_2 is $20\ \text{V}$. Then, the charge on capacitor C_2 is

[2020, 3 Sep Shift-I]



- (a) $450\ \mu\text{C}$ (b) $590\ \mu\text{C}$
(c) $160\ \mu\text{C}$ (d) $650\ \mu\text{C}$

Ans. (b)

In given arrangement,

$$V_{BC} = \text{potential drop across } C_2 = 20\text{V}$$

As combination of capacitors C_2 and C_3 is in parallel, so potential drop across C_3 is also $20\ \text{V}$.

$$\text{So, } Q_3 = \text{charge on } C_3 = C_3 \times V = 8 \times 10^{-6} \times 20 = 160\ \mu\text{C}$$

Also, total charge given by cell is

$$Q = 750\ \mu\text{C}$$

As combination of capacitors C_2 and C_3 is in series with capacitor C_1 , so charge on C_1 = charge on combination of capacitors C_2 and C_3 = total charge given by cell.

$$\text{Hence, } Q_2 + Q_3 = 750\ \mu\text{C}$$

$$\Rightarrow Q_2 + 160 = 750$$

$$\Rightarrow Q_2 = 750 - 160 = 590\ \mu\text{C}$$

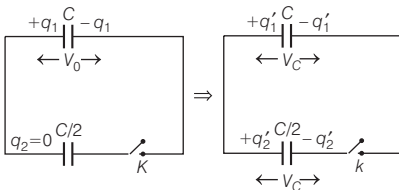
Hence, option (b) is correct.

- 62** A capacitor C is fully charged with voltage V_0 . After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is

[2020, 4 Sep Shift-II]

(a) $\frac{1}{3}CV_0^2$ (b) $\frac{1}{6}CV_0^2$

(c) $\frac{1}{2}CV_0^2$ (d) $\frac{1}{4}CV_0^2$

Ans. (b)

Before charge distribution

After charge distribution

Using law of conservation of charge, we have

$$q_1 + q_2 = q_1' + q_2'$$

$$CV_0 + \frac{C}{2}(0) = CV_C + \frac{C}{2}(V_C)$$

$$CV_0 + 0 = \frac{3}{2}CV_C \Rightarrow CV_0 = \frac{3}{2}CV_C$$

$$V_C = \frac{2}{3}V_0 \quad \dots(i)$$

Here, V_C = common potential.

Now, initial electrical energy,

$$U_i = \frac{1}{2}CV_0^2 + \frac{1}{2}\left(\frac{C}{2}\right)(0)^2$$

$$= \frac{1}{2}CV_0^2 + 0 = \frac{1}{2}CV_0^2$$

And, final electrical energy,

$$U_f = \frac{1}{2}\left(C + \frac{C}{2}\right)V_C^2 = \frac{1}{2}\left(\frac{3C}{2}\right)\left(\frac{2}{3}V_0\right)^2$$

$$= \frac{1}{2} \times \frac{3C}{2} \times \frac{4}{9}V_0^2 = \frac{1}{3}CV_0^2$$

Loss in electrical energy,

$$\Delta U_{\text{loss}} = U_i - U_f = \frac{1}{2}CV_0^2 - \frac{1}{3}CV_0^2$$

$$= \frac{3CV_0^2 - 2CV_0^2}{6} = \frac{1}{6}CV_0^2$$

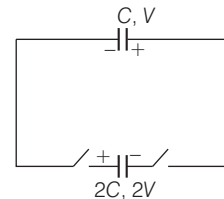
Hence, option (b) is correct.

- 63** Two capacitors of capacitances C and $2C$ are charged to potential differences V and $2V$ respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is [2020, 5 Sep Shift-I]

- (a) $\frac{9}{2}CV^2$ (b) $\frac{3}{2}CV^2$
(c) $\frac{25}{6}CV^2$ (d) zero

Ans. (b)

Before connecting the two capacitors in parallel,



Charge on capacitor C , $Q_1 = CV$ and charge on capacitor

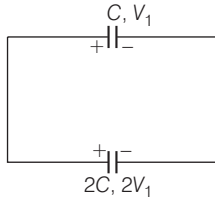
$$2C, Q_2 = 2C \times 2V = 4CV$$

After connecting the two capacitors in parallel,

Using law of conservation of charge,

$$4CV - CV = (C + 2C)V_{\text{common}}$$

$$V_{\text{common}} = \frac{4CV - CV}{3C} = V$$



$$V_{\text{common}} = V$$

Now, final energy of the configuration,

$$U_f = \frac{1}{2}CV^2 + \frac{1}{2} \times 2CV^2 = \frac{3}{2}CV^2$$

Hence, correct option is (b).

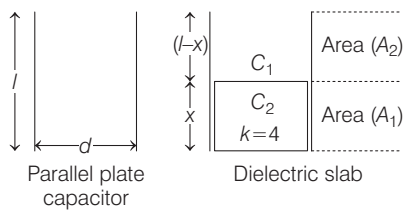
- 64** A parallel plate capacitor has plate of length l , width w and separation of plates is d . It is connected to a battery of emf V . A dielectric slab of the same thickness d and of dielectric constant $k=4$ is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?

[2020, 5 Sep Shift-II]

- (a) $\frac{2l}{3}$ (b) $\frac{l}{3}$ (c) $\frac{l}{4}$ (d) $\frac{l}{2}$

Ans. (b)

Let x be the required length of the slab.



The capacitance of a parallel plate capacitor is given by

$$C = \frac{\epsilon_0 A}{d}$$

Capacitance of parallel plate capacitor before inserting the dielectric slab,

$$C_{\text{initial}} = \frac{\epsilon_0 A}{d} \Rightarrow C_{\text{initial}} = \frac{\epsilon_0 lw}{d}$$

Now, capacitance of parallel plate capacitor after inserting the dielectric slab,

$$\begin{aligned} C_{\text{final}} &= C_1 + C_2 \\ &= \frac{k\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d} = \frac{k\epsilon_0 wx}{d} + \frac{\epsilon_0 w(l-x)}{d} \\ &= \frac{\epsilon_0 w}{d} [kx + (l-x)] = \frac{\epsilon_0 w}{d} [l + (k-1)x] \end{aligned}$$

According to question,

Energy stored in capacitor = 2(initial energy stored)

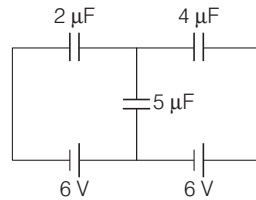
$$\Rightarrow \frac{1}{2}C_{\text{final}}V^2 = 2\left(\frac{1}{2}C_{\text{initial}}V^2\right)$$

$$\begin{aligned} \Rightarrow C_{\text{final}} &= 2C_{\text{initial}} \\ \Rightarrow \frac{\epsilon_0 w}{d} [l + (k-1)x] &= \frac{2\epsilon_0 wl}{d} \\ \Rightarrow \frac{\epsilon_0 w}{d} [l + 3x] &= \frac{2\epsilon_0 wl}{d} \\ \Rightarrow l + 3x &= 2l \Rightarrow 3x = l \Rightarrow x = \frac{l}{3} \end{aligned}$$

Hence, correct option is (b).

- 65** In the circuit shown, charge on the $5\mu\text{F}$ capacitor is

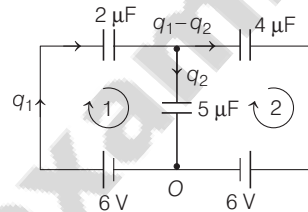
[2020, 5 Sep Shift-II]



- (a) $18.00\mu\text{C}$ (b) $10.90\mu\text{C}$
(c) $16.36\mu\text{C}$ (d) $5.45\mu\text{C}$

Ans. (c)

Let q_2 be the required charge on $5\mu\text{F}$ capacitor. The given circuit is shown below,



Applying KVL in loop 1,

$$6 - \frac{q_1}{2} - \frac{q_2}{5} = 0 \quad \dots(i)$$

Now, applying KVL in loop 2,

$$-\left(\frac{q_1 - q_2}{4}\right) - 6 + \frac{q_2}{5} = 0 \quad \dots(ii)$$

Adding Eqs. (i) and (ii), we get

$$-\left(\frac{q_1 - q_2}{4}\right) - \frac{q_1}{2} = 0 \Rightarrow -\frac{q_1}{4} + \frac{q_2}{4} - \frac{q_1}{2} = 0$$

$$\frac{q_2}{4} = \frac{q_1}{2} + \frac{q_1}{4}$$

$$\Rightarrow \frac{q_2}{4} = \frac{3q_1}{4} \Rightarrow q_1 = \frac{q_2}{3} \quad \dots(iii)$$

Substituting the value of q_1 in Eq. (i), we get

$$6 - \frac{q_2}{6} - \frac{q_2}{5} = 0 \Rightarrow 6 = \frac{5q_2 + 6q_2}{30}$$

$$\frac{11}{30}q_2 = 6$$

$$\Rightarrow q_2 = \frac{180}{11}\mu\text{C} = 16.36\mu\text{C}$$

Hence, correct option is (c).

- 66** A parallel plate capacitor has plates of area A separated by distance d between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$, where x is the distance measured from one of the plates. If $(\alpha d) \ll 1$, the total capacitance of the system is best given by the expression

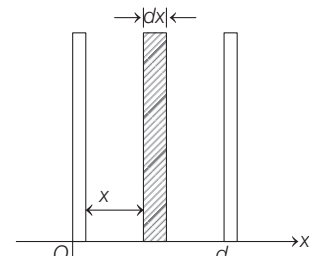
[2020, 7 Jan Shift-I]



- (a) $\frac{AK\epsilon_0}{d}(1 + \alpha d)$
(b) $\frac{A\epsilon_0 K}{d}\left(1 + \left(\frac{\alpha d}{2}\right)^2\right)$
(c) $\frac{AK\epsilon_0}{d}\left(1 + \frac{\alpha d}{2}\right)$
(d) $\frac{A\epsilon_0 K}{d}\left(1 + \frac{\alpha^2 d^2}{2}\right)$

Ans. (c)

Consider a dielectric slab of differentiable thickness dx at a distance x from left hand side plate.



Capacity of this differential thickness capacitor,

$$\begin{aligned} dC &= \frac{\epsilon_0 KA}{dx} \\ &= \frac{\epsilon_0 K(1 + \alpha x)A}{dx} \end{aligned}$$

Total capacity of system is equivalent capacitance of series combination of these differential capacitors.

$$\text{As in series, } \frac{1}{C_{\text{eq}}} = \sum_{i=1}^n \frac{1}{C_i}$$

$$\text{So, } \frac{1}{C_{\text{eq}}} = \int_0^d \frac{1}{dC} = \int_0^d \frac{dx}{\epsilon_0 K(1 + \alpha x)A}$$

$$\begin{aligned} \text{or } \frac{1}{C_{\text{eq}}} &= \frac{1}{\epsilon_0 AK} \int_0^d \frac{dx}{1 + \alpha x} \\ &= \frac{1}{\epsilon_0 AK} [\ln(1 + \alpha x)]_0^d \end{aligned}$$

$$= \frac{1}{\epsilon_0 A \alpha K} [\ln(1 + d\alpha) - \ln(1)]$$

$$= \frac{1}{\epsilon_0 A \alpha K} \ln(1 + d\alpha)$$

Now, we use expansion,

$$\ln(1 + y) = y - \frac{y^2}{2} + \frac{y^3}{3} \dots, \text{ we get}$$

$$\frac{1}{C_{eq}} = \frac{1}{\epsilon_0 A \alpha K} \left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} \dots \right)$$

$$= \frac{\alpha d}{\epsilon_0 A \alpha K} \left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} \dots \right)$$

$$\text{or } C_{eq} = \frac{\epsilon_0 A K}{d} \left(1 - \frac{\alpha d}{2} \right)^{-1}$$

From binomial approximation $(1 + x)^n = 1 + nx$, we have

$$C_{eq} = \frac{\epsilon_0 A K}{d} \left(1 + \frac{\alpha d}{2} \right)$$

- 67** Effective capacitance of parallel combination of two capacitors C_1 and C_2 is $10 \mu\text{F}$. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1 . If these capacitors are connected in series, then their effective capacitance will be

[2020, 8 Jan Shift-I]

- (a) $4.2 \mu\text{F}$ (b) $3.2 \mu\text{F}$
(c) $1.6 \mu\text{F}$ (d) $8.4 \mu\text{F}$

Ans. (c)

Let capacitances of capacitors are C_1 and C_2 .

Given, energy stored in capacitor 1 = 4 × energy stored in capacitor 2.

$$\Rightarrow \frac{1}{2} C_1 \times V^2 = 4 \times \frac{1}{2} C_2 \times V^2 \Rightarrow C_1 = 4C_2$$

Also, equivalent capacity in parallel combination is $10 \mu\text{F}$.

$$\Rightarrow C_1 + C_2 = 10$$

$$\Rightarrow 5C_2 = 10 \quad (\because C_1 = 4C_2)$$

$$\Rightarrow C_2 = 2 \mu\text{F}$$

$$\text{So, } C_1 = 4 \times 2 = 8 \mu\text{F}$$

Equivalent capacity in series combination will be

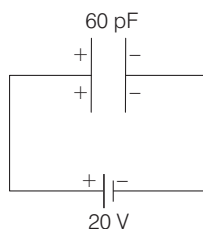
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 2}{8 + 2} = 1.6 \mu\text{F}$$

- 68** A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged

60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time, the charge is redistributed between them is (in nJ) [2020, 7 Jan Shift-II]

Ans. (6)

Initially a 60 pF capacitor is fully charged by a 20 V supply as shown in the figure.



Its energy is, $U_1 = \frac{1}{2} CV^2$

$$\Rightarrow U_1 = \frac{1}{2} \times 60 \times 10^{-12} \times (20)^2$$

$$= 12 \times 10^{-9} \text{ J}$$

Then, it is connected to another uncharged 60 pF capacitor as shown in the figure.



Transfer of charge from charged to uncharged capacitor occurs till both reaches a common potential.

If V = common potential, then

$$V = \frac{\text{total charge}}{\text{total capacitance}}$$

$$\Rightarrow V = \frac{Q_1}{C_1 + C_2} = \frac{C_1 V_1}{C_1 + C_2}$$

$$= \frac{60 \times 10^{-12} \times 20}{60 \times 10^{-12} + 60 \times 10^{-12}} = 10 \text{ V}$$

Final energy of system,

$$U_2 = \frac{1}{2} (C_1 + C_2) \cdot V^2$$

$$= \frac{1}{2} \times 2 \times 60 \times 10^{-12} \times (10)^2$$

$$= 60 \times 10^{-10} = 6 \times 10^{-9} \text{ J}$$

Loss of energy,

$$\Delta U = U_1 - U_2 = 12 \times 10^{-9} - 6 \times 10^{-9}$$

$$= 6 \times 10^{-9} \text{ J} = 6 \text{ nJ}$$

- 69** A capacitor with capacitance $5 \mu\text{F}$ is charged to $5 \mu\text{C}$. If the plates are pulled apart to reduce the capacitance to $2 \mu\text{F}$, how much work is done? [2019, 9 April Shift-II]
- (a) $6.25 \times 10^{-6} \text{ J}$ (b) $2.16 \times 10^{-6} \text{ J}$
(c) $2.55 \times 10^{-6} \text{ J}$ (d) $3.75 \times 10^{-6} \text{ J}$

Ans. (d)

Potential energy stored in a capacitor is

$$U = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C}$$

So, initial energy of the capacitor,

$$U_i = \frac{1}{2} Q^2 / C_1$$

Final energy of the capacitor,

$$U_f = \frac{1}{2} Q^2 / C_2$$

As we know, work done,

$$W = \Delta U = U_f - U_i$$

$$= \frac{1}{2} Q^2 \left[\frac{1}{C_2} - \frac{1}{C_1} \right]$$

Here,

$$Q = 5 \mu\text{C} = 5 \times 10^{-6} \text{ C},$$

$$C_1 = 5 \mu\text{F} = 5 \times 10^{-6} \text{ F},$$

$$C_2 = 2 \mu\text{F} = 2 \times 10^{-6} \text{ F}$$

$$\Delta U = \frac{1}{2} \times (5 \times 10^{-6})^2$$

$$\left[\frac{1}{2 \times 10^{-6}} - \frac{1}{5 \times 10^{-6}} \right]$$

$$= \frac{1}{2} \times \frac{5 \times 5 \times 10^{-12}}{10^{-6}} \times \frac{3}{10}$$

$$= \frac{25 \times 3}{20} \times 10^{-6} \text{ J}$$

$$\Rightarrow \Delta U = 3.75 \times 10^{-6} \text{ J}$$

$\therefore$ Work done in reducing the capacitance from $5 \mu\text{F}$ to $2 \mu\text{F}$ by pulling plates of capacitor apart is $3.75 \times 10^{-6} \text{ J}$.

- 70** The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V . When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is

[2019, 9 April Shift-II]

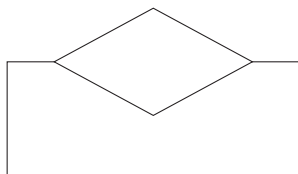
(a) $\frac{(n+1)V}{(K+n)}$ (b) $\frac{nV}{K+n}$

(c) V (d) $\frac{V}{K+n}$

Ans. (a)

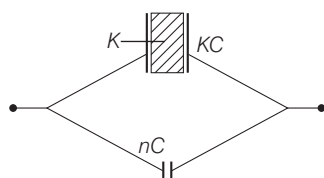
Key Idea When a dielectric slab is inserted to fill the space between the plates, then the charge on the capacitor plates remains same. However, the capacitance increase, i.e. $C = KC_0$, where C_0 is the capacitance of the capacitor without slab and K is the dielectric constant of slab.

When parallel combination is fully charged, charge on the combination is



$$Q = C_{eq}V = C(1+n)V$$

When battery is removed and a dielectric slab is placed between two plates of first capacitor, then charge on the system remains same. Now, equivalent capacitance after insertion of dielectric is



$$C_{eq} = KC + nC = (n+K)C$$

If potential value after insertion of dielectric is V' , then charge on system is

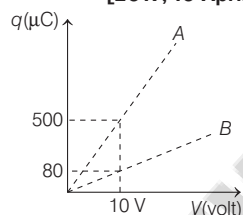
$$Q' = C_{eq}V' = (n+K)CV'$$

As $Q = Q'$, we have

$$C(1+n)V = (n+K)CV'$$

$$\therefore V' = \frac{(1+n)V}{(n+K)}$$

- 71** Figure shows charge (q) versus voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are [2019, 10 April Shift-I]



- (a) $60\mu\text{F}$ and $40\mu\text{F}$ (b) $50\mu\text{F}$ and $30\mu\text{F}$
(c) $20\mu\text{F}$ and $30\mu\text{F}$ (d) $40\mu\text{F}$ and $10\mu\text{F}$

Ans. (d)

Key Idea For a capacitor,

$$Q = CV \Rightarrow C = Q/V$$

Hence, slope of the Q - V curve should represent the capacitance.

In the given figure,

Slope of $OA >$ Slope of OB

Since, we know that,

net capacitance of parallel combination $>$ net capacitance of series combination

$\therefore$ Parallel combination's capacitance,

$$C_P = C_1 + C_2 = \frac{500\mu\text{C}}{10\text{V}} = 50\mu\text{F} \quad \dots (i)$$

Series combination's capacitance,

$$C_S = \frac{C_1 C_2}{C_1 + C_2} = \frac{80\mu\text{C}}{10\text{V}} = 8\mu\text{F} \dots (ii)$$

$$\text{or } C_1 C_2 = 8 \times (C_1 + C_2)$$

$$= 8 \times 50\mu\text{F}$$

$$= 400\mu\text{F} \quad [\text{using Eq. (i)}] \dots (iii)$$

From Eqs. (i) and (iii), we get

$$C_1 = 50 - C_2 \text{ and } C_1 C_2 = 400$$

$$\Rightarrow C_2 (50 - C_2) = 400$$

$$\Rightarrow 50 C_2 - C_2^2 = 400$$

$$\text{or } C_2^2 - 50 C_2 + 400 = 0$$

$$\Rightarrow C_2 = \frac{+50 \pm \sqrt{2500 - 1600}}{2}$$

$$= \frac{+50 \pm 30}{2}$$

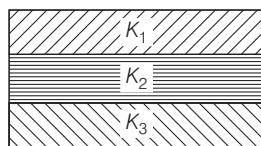
$$\Rightarrow C_2 = +40\mu\text{F} \text{ or } +10\mu\text{F}$$

$$\text{Also, } C_1 = 50 - C_2$$

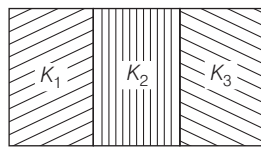
$$\Rightarrow C_1 = +10\mu\text{F} \text{ or } +40\mu\text{F}$$

Hence, capacitance of two given capacitors is $10\mu\text{F}$ and $40\mu\text{F}$.

- 72** Two identical parallel plate capacitors of capacitance C each, have plates of area A , separated by a distance d . The space between the plates of the two capacitors, is filled with three dielectrics of equal thickness and dielectric constants K_1, K_2 and K_3 . The first capacitor is filled as shown in Fig. I, and the second one is filled as shown in Fig. II. If these two modified capacitors are charged by the same potential V , the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)): [2019, 12 April Shift-I]



(I)



(II)

$$(a) \frac{E_1}{E_2} = \frac{K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}$$

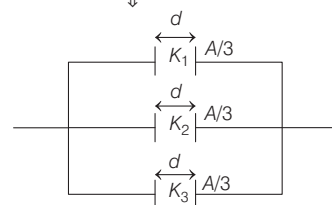
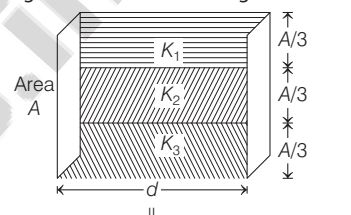
$$(b) \frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}{K_1 K_2 K_3}$$

$$(c) \frac{E_1}{E_2} = \frac{9K_1 K_2 K_3}{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}$$

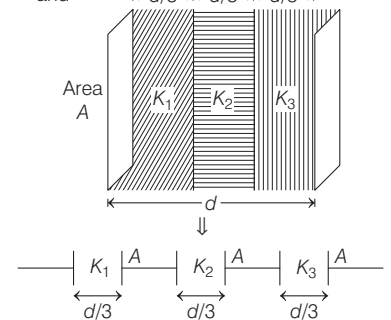
$$(d) \frac{E_1}{E_2} = \frac{(K_1 + K_2 + K_3)(K_2 K_3 + K_3 K_1 + K_1 K_2)}{9K_1 K_2 K_3}$$

Ans. (d)

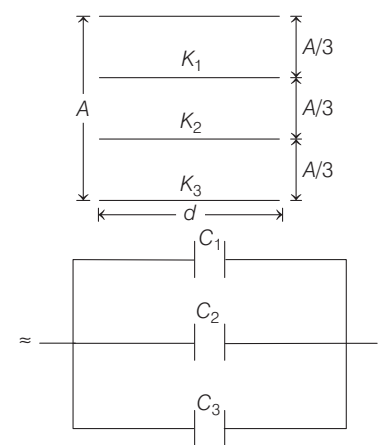
Key Idea A capacitor filled with dielectrics can be treated/compared as series/parallel combinations of capacitor having individual dielectric. e.g.



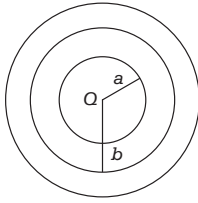
and



Case I



Capacitance in the equivalent circuit are



$$C_1 = \frac{\epsilon_0 \left(\frac{A}{3}\right)}{d} K_1 = \frac{\epsilon_0 A}{3d} K_1$$

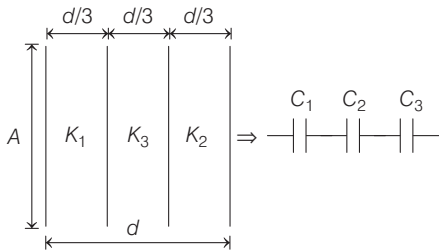
$$C_2 = \frac{\epsilon_0 \left(\frac{A}{3}\right)}{d} K_2 = \frac{\epsilon_0 A}{3d} K_2$$

and $C_3 = \frac{\epsilon_0 \left(\frac{A}{3}\right)}{d} K_3 = \frac{\epsilon_0 A}{3d} K_3$

So, equivalent capacitance,

$$\begin{aligned} C_I &= C_1 + C_2 + C_3 \\ &= \frac{\epsilon_0 A}{3d} K_1 + \frac{\epsilon_0 A}{3d} K_2 + \frac{\epsilon_0 A}{3d} K_3 \\ C_I &= \frac{\epsilon_0 A}{3d} (K_1 + K_2 + K_3) \quad \dots(i) \end{aligned}$$

Case II



Capacitance of equivalent circuit are

$$C_1 = \frac{\epsilon_0 A}{\left(\frac{d}{3}\right)} K_1 = \frac{3\epsilon_0 A}{d} K_1$$

$$C_2 = \frac{\epsilon_0 A}{\left(\frac{d}{3}\right)} K_2 = \frac{3\epsilon_0 A}{d} K_2$$

and $C_3 = \frac{\epsilon_0 A}{\left(\frac{d}{3}\right)} K_3 = \frac{3\epsilon_0 A}{d} K_3$

So, equivalent capacitance,

$$\begin{aligned} \frac{1}{C_{II}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ &= \frac{d}{3\epsilon_0 A K_1} + \frac{d}{3\epsilon_0 A K_2} + \frac{d}{3\epsilon_0 A K_3} \\ \Rightarrow \frac{1}{C_{II}} &= \frac{d}{3\epsilon_0 A} \left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \right] \\ &= \frac{d}{3\epsilon_0 A} \left[\frac{K_2 K_3 + K_1 K_3 + K_1 K_2}{K_1 K_2 K_3} \right] \end{aligned}$$

$$C_{II} = \frac{3\epsilon_0 A}{d} \left[\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_3 K_1} \right] \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

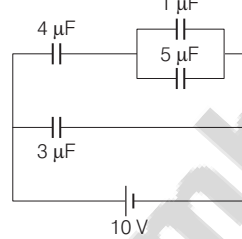
$$\begin{aligned} \frac{C_I}{C_{II}} &= \frac{\epsilon_0 A}{3d} (K_1 + K_2 + K_3) \\ &\quad \times \frac{d(K_1 K_2 + K_2 K_3 + K_3 K_1)}{3\epsilon_0 A (K_1 K_2 K_3)} \\ &= \frac{(K_1 + K_2 + K_3)(K_1 K_2 + K_2 K_3 + K_3 K_1)}{9K_1 K_2 K_3} \end{aligned}$$

Now, energy stored in capacitor,

$$\begin{aligned} E &= \frac{1}{2} CV^2 \\ \Rightarrow E &\propto C \\ \therefore \frac{E_I}{E_{II}} &= \frac{C_I}{C_{II}} \end{aligned}$$

73 In the given circuit, the charge on $4 \mu\text{F}$ capacitor will be

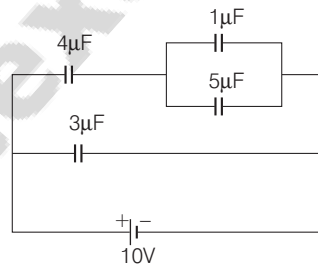
[2019, 12 April Shift-II]



(a) $5.4 \mu\text{C}$ (b) $9.6 \mu\text{C}$ (c) $13.4 \mu\text{C}$ (d) $24 \mu\text{C}$

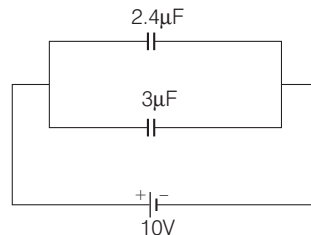
Ans. (d)

Given circuit is



In parallel, $C_{eq} = 5 + 1 = 6 \mu\text{F}$
and in series, $C'_{eq} = \frac{6 \times 4}{6 + 4} = 2.4 \mu\text{F}$

This is equivalent to



So, potential difference across upper branch
 $= 10 \text{ V}$

Using $Q = C \times V$, charge delivered to upper branch is

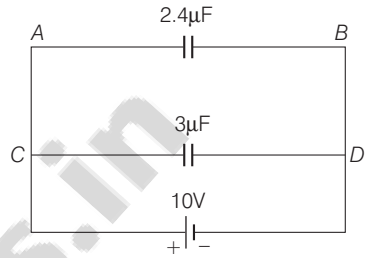
$$Q = C'_{eq} \cdot V = 2.4 \mu\text{F} \times 10\text{V} = 24 \mu\text{C}$$

As we know, in series connection, same charge is shared by capacitors, so charge on $4 \mu\text{F}$ capacitor and $6 \mu\text{F}$ capacitor would be same,

$$\text{i.e. } Q_{4\mu\text{F}} = 24 \mu\text{C}$$

Alternate Solution

The circuit obtained,



This can be further simplified as, $2.4 \mu\text{F}$ and $3 \mu\text{F}$ are in parallel.

So, net capacitance,

$$C_{net} = 2.4 + 3 = 5.4 \mu\text{F}$$

Net charge flow through circuit,

$$Q = C_{net} V = 5.4 \times 10 = 54 \mu\text{C}$$

$\therefore$ This charge will be distributed in the ratio of capacitance in the two branches AB and CD as

$$\frac{Q_1}{Q_2} = \frac{2.4}{3} = \frac{4}{5}$$

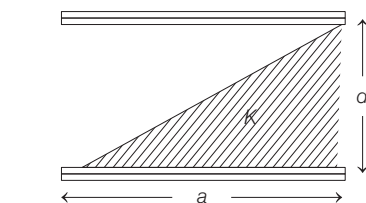
$$\Rightarrow 9x = 54 \mu\text{C} \text{ or } x = 6 \mu\text{C}$$

$\therefore$ Charge on $4 \mu\text{F}$ capacitor is

$$= 4 \times 6 \mu\text{C} = 24 \mu\text{C}$$

74 A parallel plate capacitor is made of two square plates of side 'a' separated by a distance d ($d \ll a$). The lower triangular portions is filled with a dielectric of dielectric constant k , as shown in the figure. Capacitance of this capacitor is

[2019, 9 Jan Shift-I]



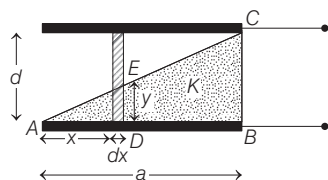
- (a) $\frac{K\epsilon_0 a^2}{d} \ln K$ (b) $\frac{K\epsilon_0 a^2}{d(K-1)} \ln K$
(c) $\frac{K\epsilon_0 a^2}{2d(K+1)}$ (d) $\frac{1}{2} \cdot \frac{K\epsilon_0 a^2}{d}$

Ans. (b)

Let's consider a strip of thickness ' dx ' at a distance of ' x ' from the left end as shown

in figure. From the figure, $\triangle ABC$ and $\triangle ADE$ are similar triangles,

$$\Rightarrow \frac{y}{x} = \frac{d}{a} \Rightarrow y = \left(\frac{d}{a}\right)x \quad \dots(i)$$



We know that, the capacitance of parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

$$C_1 = \frac{\epsilon_0 (adx)}{(d-y)}$$

and $C_2 = \frac{K\epsilon_0 (adx)}{y}$

Here, two capacitor are placed in series with variable thickness, therefore

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow C_{eq} = \frac{K\epsilon_0 adx}{Kd + (1-K)y} \quad \dots(ii)$$

Now, integrate it from 0 to a

$$C = \int_0^a \frac{K\epsilon_0 adx}{Kd + (1-K)y}$$

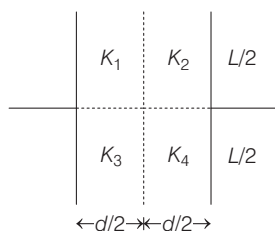
Using Eq. (i), $y = \left(\frac{d}{a}\right)x$, we get

$$C = \epsilon_0 a \int_0^a \frac{dx}{d + \left(\frac{1}{K} - 1\right)\frac{d}{a}x}$$

$$\Rightarrow C = \frac{\epsilon_0 a}{\left(\frac{1-K}{K}\right)\frac{d}{a}} \ln \left[\frac{1}{K} \right] \Rightarrow C = \frac{\epsilon_0 a^2 K \ln K}{(K-1)d}$$

- 75** A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be:

[2019, 9 Jan Shift-II]



$$(a) K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$$

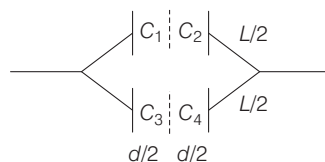
$$(b) K = \frac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$$

$$(c) K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$$

$$(d) K = \frac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$$

Ans. (*)

This capacitor system can be converted into two parts as shown in the figure



where C_1, C_2, C_3 and C_4 are capacitance of the capacitor having dielectric constants K_1, K_2, K_3 and K_4 respectively.

Here, $C_1 = \frac{K_1 \epsilon_0 A/2}{d/2} = \frac{K_1 \epsilon_0 A}{d}$

Similarly, $C_2 = \frac{K_2 \epsilon_0 A}{d}, C_3 = \frac{K_3 \epsilon_0 A}{d}$

and $C_4 = \frac{K_4 \epsilon_0 A}{d}$

Since, equivalent capacitance in series combination is

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Here, C_1, C_2 and C_3, C_4 are in series combination.

$$\begin{aligned} \therefore (C_{eq})_{12} &= \frac{C_1 \cdot C_2}{C_1 + C_2} \\ &= \frac{\frac{K_1 \epsilon_0 A}{d} \cdot \frac{K_2 \epsilon_0 A}{d}}{\frac{K_1 \epsilon_0 A}{d} + \frac{K_2 \epsilon_0 A}{d}} \\ &= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\epsilon_0 A}{d} \end{aligned}$$

$$\text{Similarly, } (C_{eq})_{34} = \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\epsilon_0 A}{d}$$

Now, $(C_{eq})_{12}$ and $(C_{eq})_{34}$ are in parallel combination.

$$\begin{aligned} \therefore C_{net} &= (C_{eq})_{12} + (C_{eq})_{34} \\ &= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\epsilon_0 A}{d} + \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\epsilon_0 A}{d} \\ \Rightarrow C_{net} &= \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4} \right) \frac{\epsilon_0 A}{d} \quad \dots(i) \end{aligned}$$

If K is effective dielectric constant, then

$$C_{net} = \frac{K \epsilon_0 A}{d} \quad \dots(ii)$$

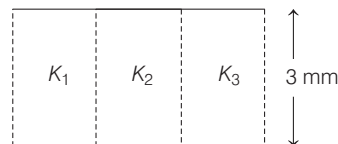
From Eqs. (i) and (ii),

$$\frac{K \epsilon_0 A}{d} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4} \right) \frac{\epsilon_0 A}{d}$$

$$\text{or } K = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4} \right)$$

- 76** A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm .

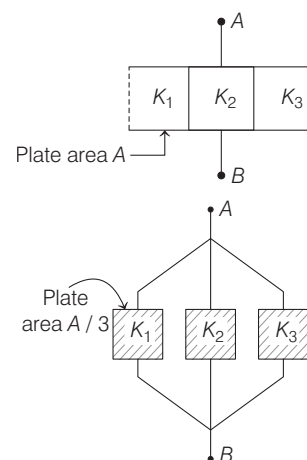
The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10, K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which give same capacitance when fully inserted in above capacitor, would be [2019, 10 Jan Shift-I]



- (a) 4 (b) 36
(c) 12 (d) 14

Ans. (c)

In the given arrangement, capacitor can be viewed as three-different capacitors connected in parallel as shown below,



So, equivalent capacity of the system is

$$\begin{aligned} C_{eq} &= C_1 + C_2 + C_3 \\ \Rightarrow \frac{K \epsilon_0 A}{d} &= \frac{K_1 \epsilon_0 A/3}{d} + \frac{K_2 \epsilon_0 A/3}{d} + \frac{K_3 \epsilon_0 A/3}{d} \end{aligned}$$

$$\Rightarrow K = \frac{K_1}{3} + \frac{K_2}{3} + \frac{K_3}{3}$$

Here $K_1 = 10, K_2 = 12$ and $K_3 = 14$

$$\text{So, } K = \frac{10 + 12 + 14}{3} \Rightarrow K = 12$$

- 77** A parallel plate capacitor having capacitance $12 \mu\text{F}$ is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is

[2019, 10 Jan Shift-II]

- (a) $560 \mu\text{J}$ (b) $508 \mu\text{J}$
(c) $692 \mu\text{J}$ (d) $600 \mu\text{J}$

Ans. (b)

Energy stored in a charged capacitor is given by

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{Q^2}{C} \quad \dots (i)$$

Here, $C = 12 \times 10^{-12} \text{ F}$ and $V = 10 \text{ V}$.

$$\Rightarrow U = \frac{1}{2} \times 12 \times 10^{-12} \times 100$$

$$U = 6 \times 10^{-10} \text{ J} \quad \dots (ii)$$

After insertion of slab, capacitance will be

$C' = KC$ and final energy,

$$U' = \frac{1}{2} \cdot \frac{Q^2}{C'} = \frac{1}{2} \cdot \frac{Q^2}{KC}$$

$$\Rightarrow U' = \frac{1}{K} U = \frac{1}{6.5} \times 6 \times 10^{-10} \text{ J} \quad \dots (iv)$$

($\because$ given, $K = 6.5$)

So, energy dissipated in the process will be equal to work done on the slab, i.e.

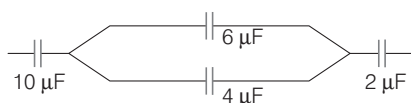
$$\Delta U = U - U' = \left(1 - \frac{1}{6.5}\right) \times 6 \times 10^{-10} \text{ J}$$

$$\Rightarrow \Delta U = \frac{5.5}{6.5} \times 6 \times 10^{-10} \text{ J}$$

$$\approx 5.08 \times 10^{-10} \text{ J or } 508 \mu\text{J}$$

- 78** In the figure shown below, the charge on the left plate of the $10 \mu\text{F}$ capacitor is $-30 \mu\text{C}$. The charge on the right plate of the $6 \mu\text{F}$ capacitor is

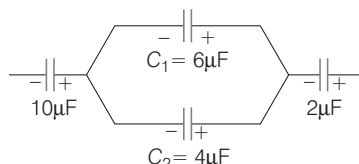
[2019, 11 Jan Shift-I]



- (a) $+12 \mu\text{C}$ (b) $+18 \mu\text{C}$
(c) $-12 \mu\text{C}$ (d) $-18 \mu\text{C}$

Ans. (b)

Applying the concept of charge conservation on isolated plates of $10 \mu\text{F}$, $6 \mu\text{F}$ and $4 \mu\text{F}$. Since, $6 \mu\text{F}$ and $4 \mu\text{F}$ are in parallel, so total charge on this combination will be $30 \mu\text{C}$



$\therefore$ Charge on $6 \mu\text{F}$ capacitor

$$= \left(\frac{C_1}{C_1 + C_2} \right) q = \frac{6}{6 + 4} \times 30$$

$$= 18 \mu\text{C}$$

Since, the charge has been asked on the right plate of the capacitor. Thus, it would be $+18 \mu\text{C}$.

Alternate Solution

Let charge on $6 \mu\text{F}$ capacitor is $q \mu\text{C}$.

Now, V at $6 \mu\text{F} = V$ at $4 \mu\text{F}$

$$\therefore \frac{q}{6 \mu\text{F}} = \frac{30 - q}{4 \mu\text{F}} \quad (\because V = q/C)$$

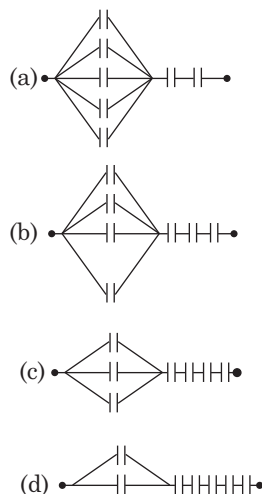
$$\Rightarrow 4q = -6q + 180$$

$$\Rightarrow q = 18 \mu\text{C}$$

- 79** Seven capacitors, each of capacitance $2 \mu\text{F}$ are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right) \mu\text{F}$. Which of the combinations

shown in figures below will achieve the desired value?

[2019, 11 Jan Shift-II]



Ans. (c)

Here, effective capacitance $= \left(\frac{6}{13}\right) \mu\text{F}$

In the given options some capacitors are joined in parallel and some are in series.

For parallel combination,

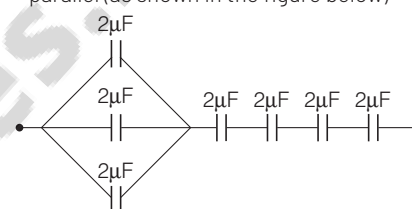
$$C_P = C_1 + C_2 + C_3 + \dots$$

For series combination,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots$$

Here, capacity of each capacitor is $2 \mu\text{F}$.

$\therefore$ Three capacitors must be in parallel to get $6 \mu\text{F}$. Now, we consider the combinations with three capacitors in parallel (as shown in the figure below)

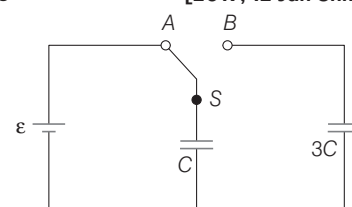


$$\therefore \frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{13}{6}$$

$$\text{or } C_{eq} = \frac{6}{13} \mu\text{F}$$

- 80** In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is

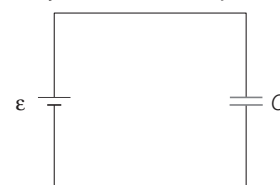
[2019, 12 Jan Shift-I]



- (a) $\frac{3}{4} \cdot \frac{Q^2}{C}$ (b) $\frac{5}{8} \cdot \frac{Q^2}{C}$
(c) $\frac{1}{8} \cdot \frac{Q^2}{C}$ (d) $\frac{3}{8} \cdot \frac{Q^2}{C}$

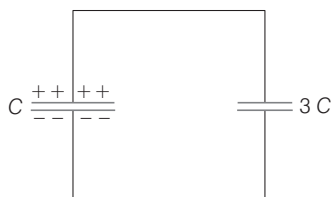
Ans. (d)

In position 'A' of switch, we have a capacitor joined with battery.



So, energy stored in position, $U_1 = \frac{1}{2} C \epsilon^2$

When switch is turned to position B, we have a charged capacitor joined to an uncharged capacitor.



Common potential in steady state will be

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{C\epsilon}{4C} = \frac{\epsilon}{4}$$

Now, energy stored will be

$$U_2 = \frac{1}{2} (C_{eq}) (V_{common})^2$$

$$= \frac{1}{2} 4C \times \left(\frac{\epsilon}{4}\right)^2 = \frac{1}{8} C \epsilon^2$$

So, energy dissipated is

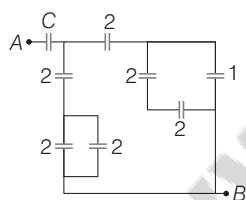
$$\Delta U = U_1 - U_2 = \frac{1}{2} C \epsilon^2 - \frac{1}{8} C \epsilon^2$$

$$= \frac{3}{8} C \epsilon^2 = \frac{3}{8} C \left(\frac{Q}{C}\right)^2$$

$$= \frac{3}{8} \cdot \frac{Q^2}{C}$$

- 81** In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5 \mu\text{F}$. All values in the circuit are in μF .

[2019, 12 Jan Shift-II]



- (a) $\frac{6}{5} \mu\text{F}$ (b) $4 \mu\text{F}$
(c) $\frac{7}{10} \mu\text{F}$ (d) $\frac{7}{11} \mu\text{F}$

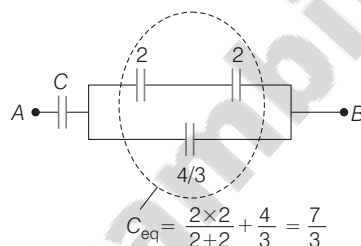
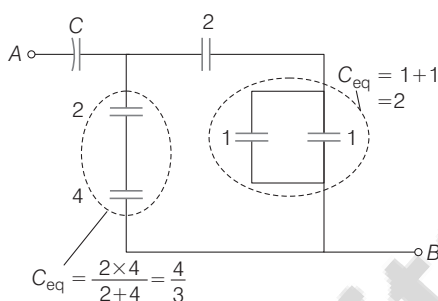
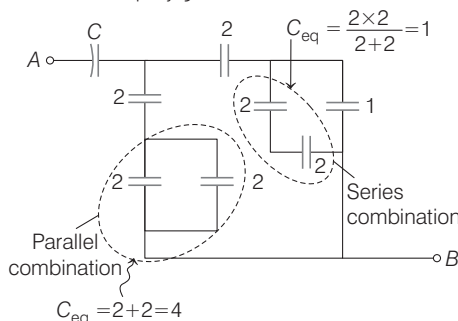
Ans. (d)

Key Idea Two capacitors C_1 and C_2 , if connected in series, then their equivalent capacitance is

$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

If they are connected in parallel, then their equivalent capacitance is,
 $C_{eq} = C_1 + C_2$.

We simplify given circuit as



So, $C_{AB} = \frac{C \times 7/3}{C + 7/3} = \frac{1}{2}$ (given)

$$\Rightarrow \frac{7C}{3} = \frac{C}{2} + \frac{7}{6}$$

$$\Rightarrow \left(\frac{14-3}{6}\right)C = \frac{7}{6}$$

$$\Rightarrow C = \frac{7}{11} \mu\text{F}$$

- 82** A capacitance of $2 \mu\text{F}$ is required in an electrical circuit across a potential difference of 1 kV . A large number of $1 \mu\text{F}$ capacitors are available which can withstand a potential difference of not more than 300 V . The minimum number of capacitors required to achieve this is

[JEE Main 2017]

- (a) 16 (b) 24 (c) 32 (d) 2

Ans. (c)

As each capacitors cannot withstand more than 300 V , so there should be four capacitors in each row became in this condition 1 kV i.e. 1000 V will be divided by 4 (i.e. 250 not more than 300 V).

Now, equivalent capacitance of one row

$$= \frac{1}{4} \times 1 \mu\text{F} = 0.25 \mu\text{F}$$

[$\therefore$ in series combination, $C_{eq} = C/n$]

Now, we need equivalent of $2 \mu\text{F}$, so let we need n such rows

$$\therefore n \times 0.25 = 2 \mu\text{F}$$

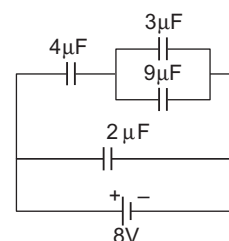
[$\therefore$ in parallel combination $C_{eq} = nc$]

$$n = \frac{2}{0.25} = 8$$

$\therefore$ Total number of capacitors = number of rows $\times$ number of capacitors in each row = $8 \times 4 = 32$

- 83** A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4 \mu\text{F}$ and $9 \mu\text{F}$ capacitors), at a point distance 30 m from it, would equal to

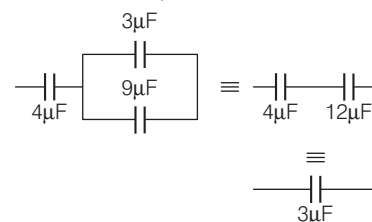
[JEE Main 2016]



- (a) 240 N/C (b) 360 N/C
(c) 420 N/C (d) 480 N/C

Ans. (c)

Resultant circuit,



As, charge on $3 \mu\text{F} = 3 \mu\text{F} \times 8 \text{ V} = 24 \mu\text{C}$

$\therefore$ Charge on $4 \mu\text{F} =$ Charge on $12 \mu\text{F} = 24 \mu\text{C}$

Charge on $3 \mu\text{F} = 3 \mu\text{F} \times 2 \text{ V} = 6 \mu\text{C}$

Charge on $9 \mu\text{F} = 9 \mu\text{F} \times 2 \text{ V} = 18 \mu\text{C}$

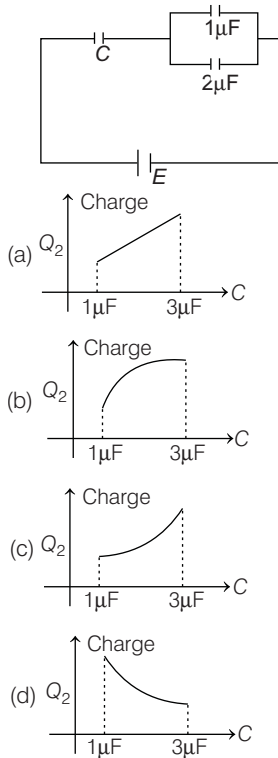
Charge on $4 \mu\text{F} +$ Charge on $9 \mu\text{F}$
 $= (24 + 18) \mu\text{C} = 42 \mu\text{C}$

$\therefore$ Electric field at a point distant 30 m

$$= \frac{9 \times 10^3 \times 42 \times 10^{-6}}{30 \times 30}$$

$$= 420 \text{ N/C}$$

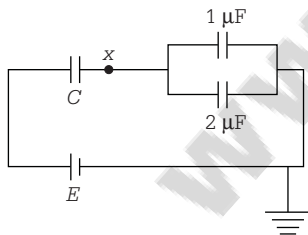
- 84** In the given circuit, charge Q_2 on the $2\mu\text{F}$ capacitor changes as C is varied from $1\mu\text{F}$ to $3\mu\text{F}$. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale) [JEE Main 2015]



Ans. (b)

Key Idea Assume negative terminal of the battery as grounded (0 V).

Suppose, potential of point x is V.



From the circuit diagram, we can write

$$Q_C = Q_1 + Q_2 \text{ or } C(E - V) = 1 \times V + 2 \times V$$

$$\text{or } V[C + 3] = CE \text{ or } V = \frac{CE}{3 + C}$$

$$\therefore Q_2 = C_2(V) = \frac{2CE}{3 + C} = \frac{2E}{1 + 3/C}$$

As C_1 varied from $1\mu\text{F}$ to $3\mu\text{F}$, charge increases with decreasing slope.

Note As $C \rightarrow \infty$, $Q_2 \rightarrow 2E = \text{constant}$

- 85** Combination of two identical capacitors, a resistor R and a DC voltage source of voltage 6 V is used in an experiment on C - R circuit. It is found that for a parallel combination of the capacitor, the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 s . For series combination, the time needed for reducing the voltage of the fully charged series combination by half is [AIEEE 2011]
- (a) 20 s (b) 10 s (c) 5 s (d) 2.5 s

Ans. (d)

If C_e is the effective capacitance, then

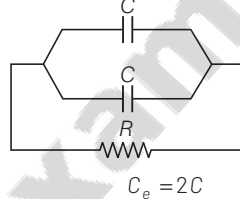
$$V_C = \frac{1}{2}V_0, \quad \frac{q}{C_e} = \frac{q_0}{2C_e}$$

$$\text{As } q = q_0(1 - e^{-t/RC_e})$$

$$\Rightarrow q_0(1 - e^{-t/RC_e}) = \frac{q_0}{2}$$

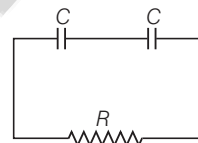
$$\Rightarrow t = RC_e \ln 2$$

For parallel grouping,



$$\therefore t_2 = 2RC \ln 2$$

For series grouping,



$$\therefore t_1 = \frac{RC}{2} \ln 2$$

$$\therefore \frac{t_2}{t_1} = \frac{1}{4} \Rightarrow t_2 = 2.5\text{ s}$$

- 86** A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is [AIEEE 2011]

- (a) $\frac{\pi}{4} \sqrt{LC}$ (b) $2\pi \sqrt{LC}$
(c) $\sqrt{LC}$ (d) $\pi \sqrt{LC}$

Ans. (a)

As initially charge is maximum.

$$\therefore q = q_0 \cos \omega t$$

$$\Rightarrow i = \frac{dq}{dt} = -\omega q_0 \sin \omega t$$

Given, energy stored in inductor $\left(U = \frac{1}{2} Li^2\right)$ is equal to energy stored in capacitor $\left(U = \frac{q^2}{2C}\right)$.

$$\therefore \frac{1}{2} Li^2 = \frac{q^2}{2C}$$

$$\Rightarrow \frac{1}{2} L (\omega q_0 \sin \omega t)^2 = \frac{(q_0 \cos \omega t)^2}{2C}$$

$$\text{But } \omega = \frac{1}{\sqrt{LC}} \Rightarrow \tan \omega t = 1 \Rightarrow \omega t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4} \sqrt{LC}$$

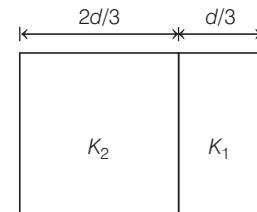
- 87** A parallel plate capacitor with air between the plates has a capacitance of 9 pF . The separation between its plates is d . The space between the plate is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now [AIEEE 2008]

- (a) 1.8 pF (b) 45 pF
(c) 40.5 pF (d) 20.25 pF

Ans. (c)

As both capacitors are connected in series.

$$\therefore C = \frac{C_1 C_2}{C_1 + C_2} \quad \dots(i)$$



$$\text{where, } C_1 = \frac{K_1 \epsilon_0 A}{d/3} \quad \dots(ii)$$

$$\text{and } C_2 = \frac{K_2 \epsilon_0 A}{2d/3} \quad \dots(iii)$$

It is given that, $\frac{\epsilon_0 A}{d} = 9\text{ pF}$

On substituting Eqs. (ii) and (iii) in Eq. (i), we get the result $C = 40.5\text{ pF}$.

- 88** A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is [AIEEE 2007]

- (a) $\frac{1}{2}(K-1)CV^2$ (b) $CV^2(K-1)/K$
 (c) $(K-1)CV^2$ (d) zero

Ans. (d)

On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process,

$$W = U_f - U_i = \frac{1}{2}CV^2 - \frac{1}{2}CV^2 = 0$$

- 89** A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be [AIEEE 2007]

- (a) 1 (b) 2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Ans. (d)

Ratio of energy stored in the capacitor and the work done by the battery

$$= \frac{\frac{1}{2}qV}{qV} = \frac{1}{2}$$

- 90** A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two

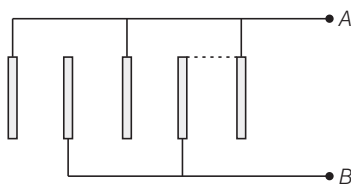
adjacent plates is C , then the resultant capacitance is

[AIEEE 2005]

- (a) $(n-1)C$ (b) $(n+1)C$
 (c) C (d) nC

Ans. (a)

Each plate is taking part in the formation of two capacitors except the plates at the ends.



These capacitors are in parallel and n plates form $(n-1)$ capacitors.

Thus, equivalent capacitance between A and B = $(n-1)C$

- 91** A fully charged capacitor has a capacitance C . It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m . If the temperature of the block is raised by ΔT , the potential difference V across the capacitance is [AIEEE 2005]

- (a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$
 (c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$

Ans. (d)

$$E = \left(\frac{1}{2}\right)CV^2 \quad \dots(i)$$

The energy stored in capacitor is lost in form of heat energy.

$$\therefore H = ms\Delta T \quad \dots(ii)$$

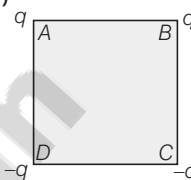
From Eqs. (i) and (ii), we have

$$ms\Delta T = \left(\frac{1}{2}\right)CV^2 \text{ or } V = \sqrt{\frac{2ms\Delta T}{C}}$$

- 92** The work done in placing a charge of $8 \times 10^{-18} \text{ C}$ on a condenser of capacity $100 \mu\text{F}$ is [AIEEE 2003]

- (a) $16 \times 10^{-32} \text{ J}$ (b) $3.1 \times 10^{-26} \text{ J}$
 (c) $4 \times 10^{-10} \text{ J}$ (d) $32 \times 10^{-32} \text{ J}$

Ans. (d)



$$W = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \times \frac{(8 \times 10^{-18})^2}{100 \times 10^{-6}} \\ = \frac{1}{2} \times \frac{64 \times 10^{-36}}{100 \times 10^{-6}} \\ = 32 \times 10^{-32} \text{ J}$$

- 93** If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to [AIEEE 2002]

- (a) CV (b) $\frac{1}{2}nC V^2$
 (c) CV^2 (d) $\frac{1}{2n}CV^2$

Ans. (b)

Energy stored by any system of capacitors

$$= \frac{1}{2}C_{\text{net}}V^2$$

where, V is source voltage.

Thus, n capacitors are connected in parallel.

$$\text{So, } C_{\text{net}} = nC$$

$$\therefore E_{\text{net}} = \frac{1}{2}nC V^2$$

Current Electricity

TOPIC 1

Ohm's Law and Resistance

- 01** A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be

[2021, 31 Aug Shift-I]

Ans. (3)

Let the sides of square be a .

$\therefore$ Total length or perimeter of square $= 4a$

If the radius of shape of circle be r , then

$$\therefore 2\pi r = 4a$$

$$\Rightarrow r = \frac{4a}{2\pi}$$

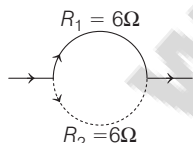
$$\Rightarrow r = \frac{2a}{\pi}$$

Since, resistance of each side of square $= 3\Omega$

$\therefore$ Total resistance of square $= 4 \times 3 = 12\Omega$

i.e. resistance of length $2\pi r = 12\Omega$

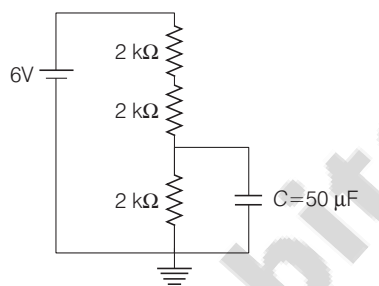
$\Rightarrow$ Resistance of $\pi r = 6\Omega$



Now, equivalent resistance of circle diametrically opposite

$$(R_{eq}) = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 6}{6 + 6} = 3\Omega$$

- 02** A capacitor of $50\mu F$ is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is μC .



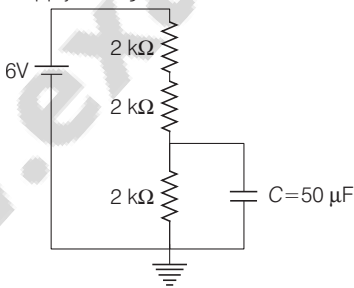
[2021, 31 Aug Shift-I]

Ans. (100)

According to given circuit diagram,

Capacitance $= 50\mu F = 50 \times 10^{-6} F$

Supply voltage, $V = 6V$



In steady state, capacitor will act as open circuit,

$\therefore$ Equivalent resistance

$$R_{eq} = (2 + 2 + 2) k\Omega = 6 k\Omega$$

$$\text{Circuit current, } I = \frac{V}{R_{eq}}$$

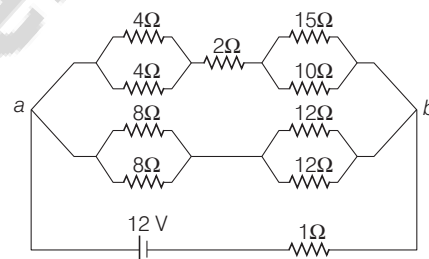
$$= \frac{6}{6 \times 1000} = 10^{-3} A$$

$$\begin{aligned} \therefore \text{Voltage across } 2k\Omega &= I \times 2 \\ &= 10^{-3} \times 2 \times 10^3 \\ &= 2V \end{aligned}$$

Now, charge on capacitor,

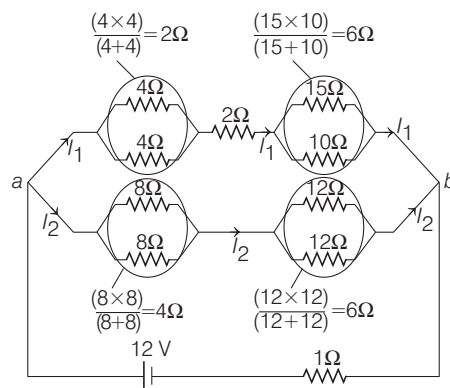
$$\begin{aligned} q &= CV = 50 \times 10^{-6} \times 2 \\ &= 100 \times 10^{-6} C \\ &= 100 \mu C \end{aligned}$$

- 03** The voltage drop across 15Ω resistance in the given figure will be V. [2021, 31 Aug Shift-I]



Ans. (6)

According to given circuit diagram,



As we know that, parallel equivalent resistance,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

and series equivalent resistance,

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

Let the net resistance across a and b be R'

$$\therefore \frac{1}{R'} = \frac{1}{2 + 2 + 6} + \frac{1}{4 + 6}$$

$$a = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$R' = 5\Omega$$

Hence, total resistance,

$$R = R' + 1 = 5 + 1 = 6 \Omega$$

According to current division rule,
current in upper branch,

$$I_1 = I \cdot \frac{4+6}{(2+2+6)+(4+6)}$$

$$= I \cdot \frac{10}{20} = \frac{I}{2} = \frac{1}{2} \cdot \frac{V}{R} = \frac{1}{2} \times \frac{12}{6} = 1 \text{ A}$$

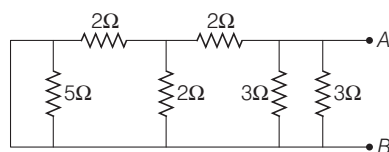
Again, according to current division rule,
current in 15Ω resistor,

$$I_{15} = I_1 \cdot \frac{10}{10+15} = 1 \times \frac{2}{5} = 0.4 \text{ A}$$

$\therefore$ Voltage drop across 15Ω resistor,

$$V_{15} = I_{15} \times 15 = 0.4 \times 15 = 6 \text{ V}$$

- 04** The equivalent resistance of the given circuit between the terminals A and B is [2021, 31 Aug Shift-II]

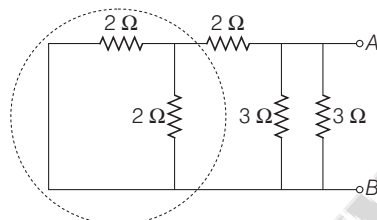


- (a) 0 (b) 3Ω
(c) $9/2 \Omega$ (d) 1Ω

Ans. (d)

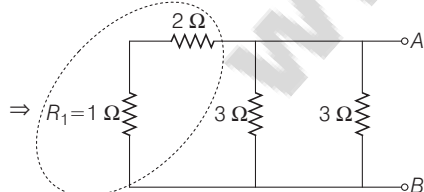
In the given circuit, 5Ω resistance is shorted. So, it can be discarded.

Now, we get a resolved circuit as shown below



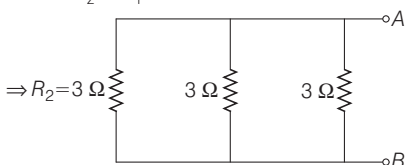
In parallel,

$$R_1 = \frac{2 \times 2}{2+2} = 1 \Omega$$



In series,

$$R_2 = R_1 + 2 = 1 + 2 = 3 \Omega$$

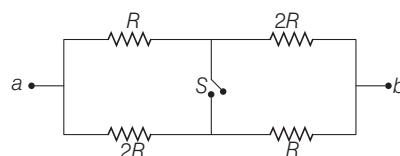


Here, all the three resistances (3Ω) are parallel.

$\therefore$ Equivalent resistance across A and B,

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \Rightarrow R_{eq} = 1 \Omega$$

- 05** The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is $x : 8$. The value of x is [2021, 27 Aug Shift-II]



Ans. (9)

According to given circuit diagram

When switch is open, then combination R_1 and R_2 will be in series and also combination R_3 and R_4 will be in series and these branches will be in parallel.

$\therefore$ Equivalent resistance

$$(R_{eq}) = \frac{3R \cdot 3R}{3R + 3R} = \frac{9R^2}{6R} = \frac{3}{2} R \quad \dots(i)$$

When switch is closed, then combination R_1 and R_3 will be in parallel and also, combination R_2 and R_4 will be in parallel. After that, both will be in series.

$\therefore$ Equivalent resistance,

$$(R'_{eq}) = \frac{R \cdot 2R}{R + 2R} + \frac{R \cdot 2R}{R + 2R} = \frac{2R^2}{3R} + \frac{2R^2}{3R}$$

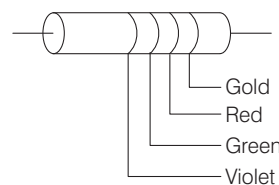
$$= \frac{2R}{3} + \frac{2R}{3} = \frac{4R}{3} \quad \dots(ii)$$

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_{eq}}{R'_{eq}} = \frac{\frac{3}{2} R}{\frac{4}{3} R} = \frac{9}{8}$$

$$x = 9$$

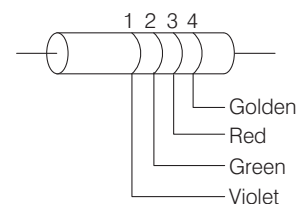
- 06** The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is [2021, 27 Aug Shift-II]



- (a) $(5700 \pm 285) \Omega$ (b) $(7500 \pm 750) \Omega$
(c) $(5700 \pm 375) \Omega$ (d) $(7500 \pm 375) \Omega$

Ans. (d)

According to given colour coded resistor



According to resistor colour chart,

Violet	Green	Red	Golden
↓	↓	↓	↓
7	5	2	5%

$$\therefore \text{Resistance of resistor} = 75 \times 10^2 \pm 5\%$$

$$= 7500 \pm 5\% = (7500 \pm 375) \Omega$$

- 07** First, a set of n equal resistors of 10Ω each are connected in series to a battery of emf 20 V and internal resistance 10Ω . A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is

..... [2021, 27 Aug Shift-I]

Ans. (20)

Given, value of each resistance, $R = 10 \Omega$
Emf of battery, $e = 20 \text{ V}$

Internal resistance of battery, $r = 10 \Omega$

Current in parallel connection is 20 times current in series combination, $i_p = 20i_s$.

Net resistance in parallel combination will be given as

$$R_p = r + \left[\frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} \dots + \frac{1}{R}} \right] = r + \frac{R}{n}$$

$$R_p = 10 + \frac{10}{n}$$

$$(\because r = 10 \Omega \text{ and } R = 10 \Omega) \dots(i)$$

In series combination,

The net resistance of circuit will be equivalent to sum of all resistances as all are connected in series.

$$R_s = [R + R + \dots + n] + r = nR + r$$

$$R_s = 10n + 10$$

$$(\because r = 10 \Omega \text{ and } R = 10 \Omega) \dots(ii)$$

By Ohm's law, current flowing in circuit is given as

$$i = V / R$$

As, $i_p = 20i_s$

$$\frac{V_p}{R_p} = 20 \frac{V_s}{R_s}$$

$$\Rightarrow \frac{20}{10 + \frac{10}{n}} = \frac{20 \times 20}{10n + 10} \quad [\because V_p = V_s = e = 20 \text{ V}]$$

$$\Rightarrow \frac{20n}{10n + 10} = \frac{400}{10n + 10} \Rightarrow 20n = 400$$

$$n = 20$$

Thus, the value of number of resistances n is 20.

- 08** If you are provided a set of resistances 2Ω , 4Ω , 6Ω and 8Ω . Connect these resistances, so as to obtain an equivalent resistance of $\frac{46}{3}\Omega$. **[2021, 26 Aug Shift-II]**

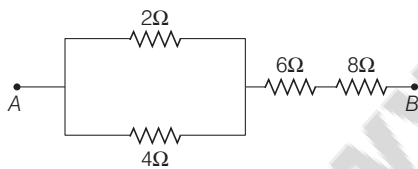
- (a) 4Ω and 6Ω are in parallel with 2Ω and 8Ω in series.
 (b) 6Ω and 8Ω are in parallel with 2Ω and 4Ω in series.
 (c) 2Ω and 6Ω are in parallel with 4Ω and 8Ω in series.
 (d) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series.

Ans. (d)

The given value of resistances are 2Ω , 4Ω , 6Ω and 8Ω . The required value of combination is $46/3\Omega$.

In order to achieve the above mentioned values of resistance from given resistances, we will connect 2Ω and 4Ω resistance in parallel, then join 6Ω and 8Ω resistance in series with the combination.

The circuit diagram for connection is shown below.



$$R_{eq} = (2 \parallel 4) + 6 + 8 = \frac{2 \times 4}{2 + 4} + 14 = \frac{46}{3}\Omega$$

Thus, resistance of 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series combination.

- 09** What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω ?

(Given, resistivities of iron and copper-nickel alloy wire are $12\mu\Omega \text{ cm}$ and $51\mu\Omega \text{ cm}$ respectively)

[2021, 26 Aug Shift-I]

- (a) 82 m (b) 97 m (c) 110 m (d) 90 m

Ans.

(b) Let the resistance of iron wire be R_1 and that of copper nickel alloy wire be R_2

$$r_1 = r_2 = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\begin{aligned} \rho_1 &= 12\mu\Omega \text{ cm} \\ &= 12 \times 10^{-6} \Omega \text{ cm} \\ &= 12 \times 10^{-8} \Omega \text{ m} \\ \rho_2 &= 51\mu\Omega \text{ cm} \\ &= 51 \times 10^{-6} \Omega \text{ cm} \\ &= 51 \times 10^{-8} \Omega \text{ m} \end{aligned}$$

For parallel combination,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\begin{aligned} 3 &= \frac{\frac{\rho_1 l}{\pi r_1^2} \frac{\rho_2 l}{\pi r_2^2}}{\frac{\rho_1 l}{\pi r_1^2} + \frac{\rho_2 l}{\pi r_2^2}} \\ &= \frac{(12 \times 10^{-8}) l \times 51 \times 10^{-8} l}{\pi \times 10^{-6} l + \pi \times 10^{-6} l} \end{aligned}$$

$\Rightarrow$

$$3 = \frac{12 \times 10^{-8} l \times 51 \times 10^{-8} l}{\pi \times 10^{-6} l + \pi \times 10^{-6} l}$$

On solving, $l = 97 \text{ m}$

- 10** The resistance of a conductor at 15°C is 16Ω and at 100°C is 20Ω . What will be the temperature coefficient of resistance of the conductor? **[2021, 27 July Shift-II]**

- (a) 0.010°C^{-1} (b) 0.033°C^{-1}
 (c) 0.003°C^{-1} (d) 0.042°C^{-1}

Ans. (c)

Given,

Resistance of conductor at $15^\circ\text{C} = 16\Omega$
 and resistance of conductor at $100^\circ\text{C} = 20\Omega$

$\therefore$ We know that,

$$R_t = R_0 [1 + \alpha(\Delta T)] \quad \dots(i)$$

where, R_t = resistance of conductor at $t^\circ\text{C}$,

R_0 = resistance of conductor at 0°C ,

α = temperature coefficient of resistance

and ΔT = temperature difference.

$\therefore$ Using Eq. (i), we can write

$$16 = R_0 [1 + \alpha(15 - T_0)] \quad \dots(ii)$$

$$\text{and } 20 = R_0 [1 + \alpha(100 - T_0)] \quad \dots(iii)$$

Assume that T_0 to be 0°C as per general convention and divide Eq. (ii) by Eq. (iii), we get

$$\frac{16}{20} = \frac{R_0 (1 + \alpha \times 15)}{R_0 (1 + \alpha \times 100)}$$

$$\Rightarrow \frac{4}{5} = \frac{1 + 15\alpha}{1 + 100\alpha}$$

$$\Rightarrow 4 + 400\alpha = 5 + 75\alpha$$

$$\Rightarrow 400\alpha - 75\alpha = 5 - 4 \Rightarrow 325\alpha = 1$$

$$\Rightarrow \alpha = \frac{1}{325} = 0.003^\circ\text{C}^{-1}$$

- 11** In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius $0.5\text{ \AA}$. If the speed of electron is $2.2 \times 10^6 \text{ m/s}$, then the current associated with the electron will be $\times 10^{-2} \text{ mA}$. [Take, π as $\frac{22}{7}$]

[2021, 27 July Shift-I]

Ans. (112)

Radius of circular orbit,

$$r = 0.5\text{ \AA} = 0.5 \times 10^{-10} \text{ m}$$

Velocity of electron, $v = 2.2 \times 10^6 \text{ ms}^{-1}$

$\therefore$ We know that,

$$\text{Current, } I = \frac{q}{t}$$

where, q = charge and t = time taken.

$$\Rightarrow I = \frac{e}{t} \quad \dots(i) \quad [\because q = e]$$

As electron is moving in circular orbit, so it means electron is tracing a distance of $2\pi r$.

$$\therefore \text{Time} = \frac{\text{Distance}}{\text{Speed}} \Rightarrow t = \frac{2\pi r}{v} \quad \dots(ii)$$

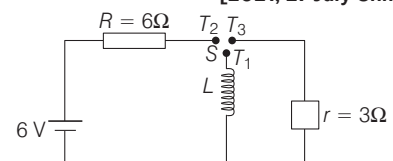
From Eqs. (i) and (ii), we get

$$I = \frac{e}{\frac{2\pi r}{v}} \Rightarrow I = \frac{ev}{2\pi r}$$

$$\Rightarrow I = \frac{1.6 \times 10^{-19} \times 2.2 \times 10^6}{2 \times \frac{22}{7} \times 0.5 \times 10^{-10}} = 112 \times 10^{-2} \text{ mA}$$

- 12** Consider an electrical circuit containing a two way switch S . Initially S is open and then T_1 is connected to T_2 . As the current in $R = 6\Omega$ attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3 . Potential drop across $r = 3\Omega$ resistor immediately after T_1 is connected to T_3 is V. (Round off to the nearest integer)

[2021, 27 July Shift-I]



Ans. (3)

According to given circuit diagram in question, when T_1 and T_2 are connected, the steady state current in the inductor,

$$I = \frac{V}{R} = \frac{6}{6} = 1 \text{ A}$$

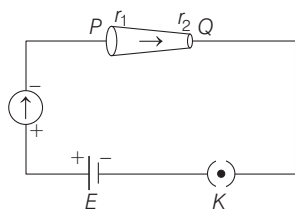
When T_1 and T_3 are connected, the steady state current through inductor remains same.

Therefore, potential difference,

$$V = Ir = 1 \times 3 = 3 \text{ V}$$

- 13** In the given figure, a battery of emf E is connected across a conductor PQ of length l and different area of cross-sections having radii r_1 and r_2 ($r_2 < r_1$).

[2021, 27 July Shift-I]

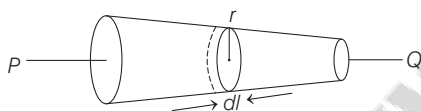


Choose the correct option as one moves from P to Q .

- (a) Drift velocity of electron increases
- (b) Electric field decreases
- (c) Electron current decreases
- (d) All of the above

Ans. (a)

If we consider only conductor PQ as follows



Let dl be the small element of conductor PQ .

$\therefore$ Current passing through PQ is constant.

So, resistance of this small element dl will be given as

$$dR = \frac{\rho dl}{A} \quad \dots(i)$$

where, ρ = resistivity,

dl = small element of conductor PQ
and A = area of cross-section.

$$\therefore \text{Current, } I = \frac{dV}{dR}$$

$$\Rightarrow dV = IdR \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$dV = \frac{\rho dl}{A} \Rightarrow dV = \frac{\rho dl}{\pi r^2} \quad \dots(iii) \{ \because A = \pi r^2 \}$$

$\therefore$ We know that,

$$\text{Electric field, } E = \frac{dV}{dl} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\Rightarrow E = \frac{\rho dl}{\pi r^2 dl}$$

$$\Rightarrow E = \frac{\rho}{\pi r^2}$$

$$\Rightarrow E \propto \frac{1}{r^2} \quad \dots(v)$$

Also, we know that, drift velocity,

$$v_d = \frac{eE\tau}{m}$$

$$\Rightarrow v_d \propto E \quad \dots(vi)$$

$\therefore$ From Eqs. (v) and (vi), we can say

If r decreases, E will increase and due to this, v_d will increase.

- 14** For the circuit shown, the value of current at time $t = 3.2 \text{ s}$ will be

[2021, 27 July Shift-II]

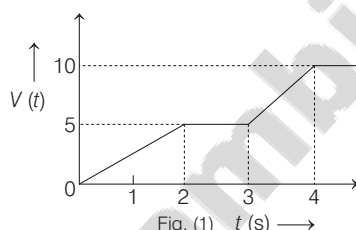
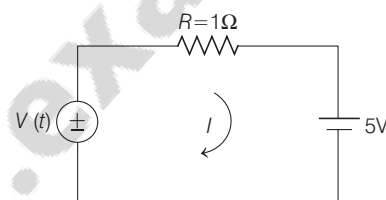
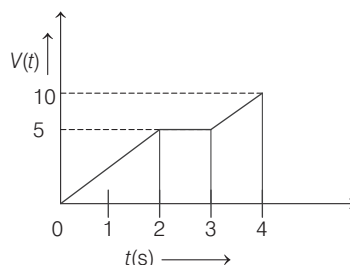
Fig. (1) t (s) $\rightarrow$ 

Fig. (2)

[Voltage distribution $V(t)$ is shown by Fig. (1) and the circuit is shown in Fig. (2).]

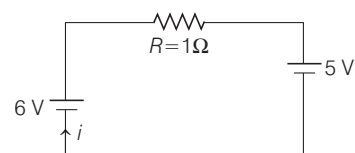
Ans. (1)

Consider the given figure 1,



From the above graph, we can say that voltage at $t = 3.2 \text{ s}$ is 6 V .

Now, consider figure 2,



$\therefore$ The value of I in the above figure will be

$$I = \frac{6\text{V} - 5\text{V}}{1\Omega} \Rightarrow I = \frac{1\text{V}}{1\Omega} \Rightarrow I = 1 \text{ A}$$

- 15** A 16Ω wire is bent to form a square loop. A 9 V supply having internal resistance of 1Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is $\times 10^{-1} \text{ V}$.

[2021, 25 July Shift-II]

Ans.

(45) Given, resistance of wire = 16Ω .

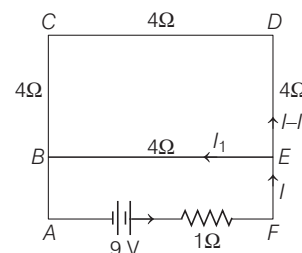
Let, length of wire = $4L$

Hence, side of square $a = L$

$$\text{Resistance of one side} = \frac{16}{4} = 4 \Omega$$

Supply voltage $V = 9 \text{ V}$ and internal resistance = 1Ω

Current flowing through circuit be I and voltage across diagonal be V' .



There $4L$ resistance are in series with 4Ω and 1Ω resistance in parallel.

$$\begin{aligned} \text{Equivalent resistance } (R_{eq}) &= \frac{12 \times 4}{12 + 4} + 1 \\ &= \frac{48}{16} + 1 = 4 \Omega \end{aligned}$$

As we know that,

By Ohm's law, $V = IR$

$$\Rightarrow I = \frac{V}{R} \Rightarrow I = \frac{9}{4} \text{ A}$$

Now current through BE be I_1 and $EDCB$ is $(I - I_1)$.

$\therefore$ In parallel connection,

Voltage across BE (V_1) = Voltage across $EDCB$ (V_2)

$$\begin{aligned} \therefore I_1 \times 4 &= \left(\frac{9}{4} - I_1 \right) 12 \\ I_1 &= \left(\frac{9}{4} - I_1 \right) 3 \end{aligned}$$

$$\Rightarrow 4I_1 = (9 - 4I_1)3$$

$$\Rightarrow 16I_1 = 27$$

$$\Rightarrow I_1 = \frac{27}{16} \text{ A}$$

Equivalent resistance across diagonal,

$$\frac{1}{R_d} = \frac{1}{4+4} + \frac{1}{4+4} = \frac{1}{8} + \frac{1}{8}$$

$$R_d = \frac{8}{2} = 4\Omega$$

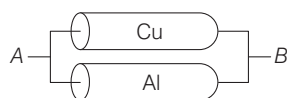
and $I_2 = I - I_1 = \frac{9}{4} - \frac{27}{16} = \frac{9}{16} \text{ A}$

$$\therefore V' = (4+4)I_2 = 8 \times \frac{9}{16} = 4.5 \text{ V}$$

$$= 45 \times 10^{-1} \text{ V}$$

- 16** A copper (Cu) rod of length 25 cm and cross-sectional area 3 mm^2 is joined with a similar aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.
(Take, resistivity of copper $= 1.7 \times 10^{-8} \Omega\text{-m}$, resistivity of aluminium $= 2.6 \times 10^{-8} \Omega\text{-m}$)

[2021, 22 July Shift-II]



- (a) $2.170 \text{ m}\Omega$ (b) $1.420 \text{ m}\Omega$
(c) $0.0858 \text{ m}\Omega$ (d) $0.858 \text{ m}\Omega$

Ans. (d)

Given, length of copper and aluminium,
 $l = 25 \text{ cm} = 25 \times 10^{-2} \text{ m}$

Area of cross-section,

$$A = 3 \text{ mm}^2 = 3 \times 10^{-6} \text{ m}^2$$

and their resistivities of copper (ρ_{Cu}) and aluminium (ρ_{Al}) be $1.7 \times 10^{-8} \Omega\text{-m}$ and $2.6 \times 10^{-8} \Omega\text{-m}$ respectively.

According to given diagram, both copper rod and aluminium rods are in parallel and we know that,

in parallel combination, equivalent resistance $\left(\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \right)$

We know that, $R = \rho l / A$

$$\therefore \text{Resistance of copper } (R_{\text{Cu}}) = \frac{1.7 \times 10^{-8} \times 25 \times 10^{-2}}{3 \times 10^{-6}}$$

$$= 14.167 \times 10^{-4} \Omega$$

Resistance of aluminium

$$(R_{\text{Al}}) = \frac{2.6 \times 10^{-8} \times 25 \times 10^{-2}}{3 \times 10^{-6}}$$

$$= 21.67 \times 10^{-4} \Omega$$

Hence,

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_{\text{Cu}}} + \frac{1}{R_{\text{Al}}}$$

$$\Rightarrow R_{\text{eq}} = \frac{R_{\text{Cu}} \cdot R_{\text{Al}}}{R_{\text{Cu}} + R_{\text{Al}}}$$

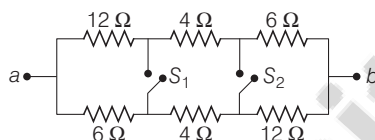
$$\Rightarrow R_{\text{eq}} = \frac{14.167 \times 10^{-4} \times 21.67 \times 10^{-4}}{10^{-4}(14.167 + 21.67)}$$

$$= \frac{14.167 \times 21.67 \times 10^{-4}}{35.837}$$

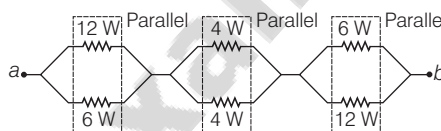
$$= 0.857 \times 10^{-3} = 0.857 \text{ m}\Omega$$

$$\approx 0.858 \text{ m}\Omega$$

- 17** In the given figure, switches S_1 and S_2 are in open condition. The resistance across ab when the switches S_1 and S_2 are closed is Ω . [2021, 20 July Shift-II]

**Ans. (10)**

When switches S_1 and S_2 are closed, then the given figure will be represented as follows



$\therefore$ The resistance across ab will be given as

$$R_{ab} = \frac{12 \times 6}{12 + 6} + \frac{4 \times 4}{4 + 4} + \frac{6 \times 12}{6 + 12}$$

$$\Rightarrow R_{ab} = \frac{72}{18} + 2 + \frac{72}{18}$$

$$\Rightarrow R_{ab} = 4 + 2 + 4 = 10 \Omega$$

- 18** A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 m^2 . At every point, the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is
(Take, resistivity of magnesium, $\rho = 44 \times 10^{-8} \Omega\text{-m}$)

[2021, 20 July Shift-I]

- (a) $11 \times 10^{-2} \text{ V/m}$ (b) $11 \times 10^{-7} \text{ V/m}$
(c) $11 \times 10^{-5} \text{ V/m}$ (d) $11 \times 10^{-3} \text{ V/m}$

Ans. (c)

Given, current, $I = 5 \text{ A}$

Area of cross-section of wire, $A = 0.04 \text{ m}^2$

We know that, $J = \frac{I}{A}$

$$\Rightarrow I = JA$$

$$\text{or } I = J \cdot A \text{ or } I = JA \cos \theta$$

where, $J =$ current density.

$$\Rightarrow 5 = J \left(\frac{4}{100} \right) \times \cos(60^\circ)$$

[$\because$ Given, $\theta = 60^\circ$]

$$J = 500 \times \frac{1}{2} \left[\because \cos 60^\circ = \frac{1}{2} \right]$$

$$\Rightarrow J = 250 \text{ Am}^{-2}$$

The relation between electric field, current density and resistivity can be given as

$$E = \rho \cdot J$$

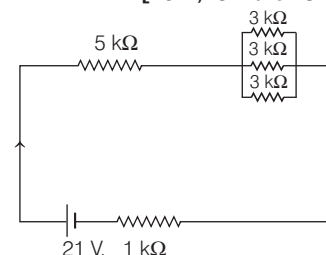
$$= 44 \times 10^{-8} \times 250$$

$$[\because \text{Resistivity, } \rho = 44 \times 10^{-8} \Omega\text{-m}]$$

$$= 11 \times 10^{-5} \text{ V/m}$$

- 19** In the figure given, the electric current flowing through the $5 \text{ k}\Omega$ resistor is $x \text{ mA}$.

[2021, 16 March Shift-I]



The value of x to the nearest Integer is

Ans. (3)

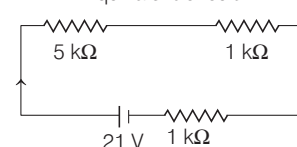
According to the figure given in question, all 3Ω resistances are in parallel combination. So, their equivalent resistance is

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{R_{\text{parallel}}} = \frac{3}{3}$$

$$\Rightarrow R_{\text{parallel}} = 1 \text{ k}\Omega$$

Equivalent circuit



$5 \text{ k}\Omega$ and $1 \text{ k}\Omega$ resistance are in series to the equivalent of all 3Ω resistances.

$$\therefore R_{\text{net}} = 5 + 1 + R_{\text{parallel}} \quad \dots(i)$$

$$\Rightarrow R_{\text{net}} = (5 + 1 + 1)\text{k}\Omega = 7\text{k}\Omega = 7 \times 10^3 \Omega$$

$\therefore$ The value of electric current flowing through $5\text{k}\Omega$ resistor will be

$$I = \frac{V}{R_{\text{net}}} = \frac{21}{7 \times 10^3} = 3 \times 10^{-3} \text{ A} = 3 \text{ mA}$$

Comparing with the given value in the question i.e., $x \text{ mA}$, the value of $x = 3$.

- 20** Two wires of same length and thickness having specific resistances $6\Omega\text{-cm}$ and $3\Omega\text{-cm}$ respectively are connected in parallel. The effective resistivity is $\rho\Omega\text{-cm}$. The value of ρ to the nearest integer, is

[2021, 18 March Shift-II]

Ans. (4)

Given, specific resistance for wire 1,

$$\rho_1 = 6\Omega\text{-cm}$$

Specific resistance for wire 2,

$$\rho_2 = 3\Omega\text{-cm}$$

Resistance,

$$R = \frac{\rho l}{A}$$

For parallel connections,

$$R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow \frac{\rho l}{2A} = \frac{\frac{\rho_1 l}{A} \times \frac{\rho_2 l}{A}}{\frac{\rho_1 l}{A} + \frac{\rho_2 l}{A}}$$

$$\Rightarrow \frac{\rho}{2} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2}$$

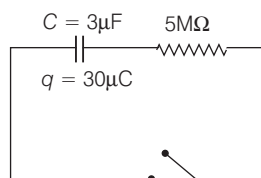
$$\Rightarrow \frac{\rho}{2} = \frac{6 \times 3}{6 + 3}$$

$$\rho = 4\Omega\text{-cm}$$

Hence, the value of ρ to the nearest integer is 4.

- 21** The circuit shown in the figure consists of a charged capacitor of capacity $3\mu\text{F}$ and a charge of $30\mu\text{C}$. At time $t = 0$, when the key is closed, the value of current flowing through the $5\text{M}\Omega$ resistor is $x\mu\text{A}$. The value of x to the nearest integer is

[2021, 18 March Shift-I]



Ans. (2)

According to given circuit diagram, At $t = 0$, the key is in closed position. Current through the resistor will be maximum.

Using Ohm's law,

$$I_{\text{max}} = \frac{V}{R}$$

$$\Rightarrow I_{\text{max}} = \left(\frac{Q}{C}\right) \times \frac{1}{R}$$

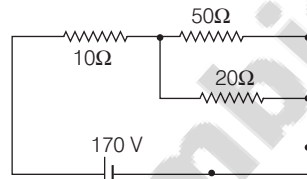
$$\Rightarrow I_{\text{max}} = \left(\frac{30 \times 10^{-6}}{3 \times 10^{-6}}\right) \times \frac{1}{5 \times 10^6}$$

$$I_{\text{max}} = 2 \times 10^{-6} \text{ A}$$

$$I_{\text{max}} = 2\mu\text{A}$$

The value of the current flowing through the 5Ω resistor is $2\mu\text{A}$. Hence, the value of the x to the nearest integer is 2.

- 22** The voltage across the 10Ω resistor in the given circuit is x volt.

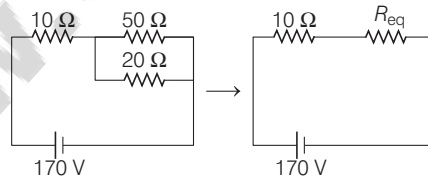


The value of x to the nearest integer is

[2021, 18 March Shift-I]

Ans. (70)

Electrical circuit is shown in the diagram. Now, let's draw the equivalent circuit.



Equivalent resistance of the circuit,

$$R_{\text{eq}} = \frac{50 \times 20}{50 + 20}$$

$$R_{\text{eq}} = \frac{100}{7} \Omega$$

According to voltage division rule, voltage across the 10Ω resistance of the circuit,

$$V_{10\Omega} = 170 \times \left(\frac{10}{10 + \frac{100}{7}} \right)$$

$$V_{10\Omega} = 70 \text{ V}$$

The value of the x to the nearest integer is 70.

- 23** A current of 10 A exists in a wire of cross sectional area of 5 mm^2 with a drift velocity of $2 \times 10^{-3} \text{ ms}^{-1}$. The number of free electrons in each cubic metre of the wire is

[2021, 17 March Shift-I]

$$(a) 2 \times 10^6$$

$$(b) 625 \times 10^{25}$$

$$(c) 2 \times 10^{25}$$

$$(d) 1 \times 10^{23}$$

Ans. (b)

Given, current, $I = 10 \text{ A}$

Cross-sectional area,

$$A = 5 \text{ mm}^2 = 5 \times 10^{-6} \text{ m}^2$$

Drift velocity, $v_d = 2 \times 10^{-3} \text{ ms}^{-1}$

The value of current flowing through a conductor can be given by

$$I = neAv_d \quad \dots(i)$$

where, n = number of free electrons

and e = charge on an electron

Putting all the given values in Eq. (i) we get

$$10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$$

$$\Rightarrow n = \frac{10}{1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}}$$

$$= 0.625 \times 10^{28} = 625 \times 10^{25}$$

- 24** The equivalent resistance of series combination of two resistors is s . When they are connected in parallel, the equivalent resistance is p . If $s = np$, then the minimum value for n is

(Round off to the nearest integer)

[2021, 17 March Shift-I]

Ans. (4)

Let two resistors have resistances R_1 and R_2 , respectively.

As per question, equivalent resistance of series combination is s

$$\Rightarrow s = R_1 + R_2 \quad \dots(i)$$

and equivalent resistance of parallel combination is p

$$\Rightarrow p = \frac{R_1 R_2}{R_1 + R_2} \quad \dots(ii)$$

According to the question,

$$s = np \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$R_1 + R_2 = n \frac{R_1 R_2}{(R_1 + R_2)}$$

$$\Rightarrow n(R_1 R_2) = (R_1 + R_2)^2 \Rightarrow n = \frac{(R_1 + R_2)^2}{R_1 R_2}$$

For n to be minimum, $R_1 = R_2 = R$

$$\Rightarrow n = \frac{(R + R)^2}{R \cdot R} = \frac{(2R)^2}{R^2} = \frac{4R^2}{R^2}$$

$$\Rightarrow n = 4$$

- 25** A conducting wire of length l , area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current. If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be

[2021, 16 March Shift-I]

- (a) $\frac{1}{4} \frac{VA}{\rho l}$ (b) $\frac{3}{4} \frac{VA}{\rho l}$
 (c) $\frac{1}{4} \frac{\rho l}{VA}$ (d) $4 \frac{VA}{\rho l}$

Ans. (a)

Initially, the resistance of wire is

$$R_1 = \rho L / A$$

In second case,

$$\text{Length, } l' = 2l$$

$$\text{Area, } A' = \frac{A}{2}$$

$$\therefore R_2 = \frac{\rho l'}{A'} = \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$$

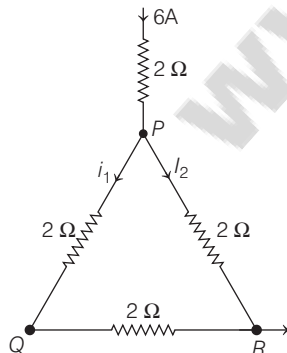
$$\text{According to Ohm's law, } I = \frac{V}{R_2}$$

$$\Rightarrow I = \frac{V}{4\rho l / A} = \frac{1}{4} \frac{VA}{\rho l}$$

This is the required value of resultant current.

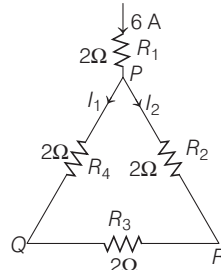
- 26** A current of 6 A enters one corner P of an equilateral triangle PQR having three wires of resistance $2\ \Omega$ each and leaves by the corner R . The currents i_1 in ampere is

[2021, 25 Feb Shift-II]

**Ans. (2)**

Let resistances be R_1, R_2, R_3 and R_4 and i_1 current is passing through R_4 as shown in figure

$\therefore i_2 = (6 - i_1)$ is passing through R_2
 As, same current is flowing through R_4 and R_3 .
 $\therefore R_4$ and R_3 are in series.



and series equivalent resistance,

$$R_{eq} = R_4 + R_3$$

$$R_{eq} = 2 + 2 = 4\ \Omega$$

Voltage through R_{eq} and R_2 will be same.

$$\Rightarrow i_1 R_{eq} = i_2 R_2 \Rightarrow i_1 4 = (6 - i_1) 2$$

$$\Rightarrow 2i_1 = 6 - i_1 \Rightarrow i_1 = 2\text{ A}$$

- 27** A cylindrical wire of radius 0.5 mm and conductivity $5 \times 10^7\text{ S/m}$ is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3 \pi\text{ mA}$. The value of x is

[2021, 24 Feb Shift-II]

Ans. (5)Given, radius of cylindrical wire, $r = 0.5\text{ mm} = 0.5 \times 10^{-3}\text{ m}$ Conductivity, $\sigma = 5 \times 10^7\text{ S/m}$ Electric field, $E = 10\text{ mV/m} = 10 \times 10^{-3}\text{ V/m}$

We know that current density,

$$\begin{aligned} \therefore J &= \sigma E \\ &= 5 \times 10^7 \times 10 \times 10^{-3} \\ &= 5 \times 10^5\text{ A/m}^2 \end{aligned}$$

$$\text{Also, } J = I / A \Rightarrow I = JA$$

$$\begin{aligned} \Rightarrow I &= 5 \times 10^5 \times \pi \times (0.5 \times 10^{-3})^2 \\ &= 5 \times 10^5 \times \pi \times 25 \times 10^{-8} \\ &= 125 \pi \times 10^{-3} \\ \Rightarrow x^3 \pi \text{ mA} &= 125 \pi \text{ mA} \\ \Rightarrow x^3 &= 5^3 \\ \Rightarrow x &= 5 \end{aligned}$$

- 28** A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$, where $\alpha_0 = 20\text{ A/s}$ and $\beta = 8\text{ A/s}^2$. Find the charge crossed through a section of the wire in 15 s.

[2021, 24 Feb Shift-I]

- (a) 260 C (b) 2100 C
 (c) 11250 C (d) 2250 C

Ans. (c)

Given, $i = \alpha_0 t + \beta t^2$
 where, $\alpha_0 = 20\text{ A/s}$, $\beta = 8\text{ A/s}^2$

We know that, $i = \frac{dq}{dt}$

$$\Rightarrow \frac{dq}{dt} = i = \alpha_0 t + \beta t^2 = 20t + 8t^2$$

$$\Rightarrow dq = (20t + 8t^2) dt$$

On integrating both sides, we get

$$\int_0^q dq = \int_0^{15} (20t + 8t^2) dt$$

$$q = \left[\frac{20t^2}{2} + \frac{8t^3}{3} \right]_0^{15} = 10 \times (15)^2 + \frac{8}{3} \times (15)^3$$

$$\therefore q = 11250\text{ C}$$

- 29** Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity ρ_C, ρ_T, ρ_M and ρ_A , respectively.

Then, [2020, 2 Sep Shift-I]

- (a) $\rho_A > \rho_T > \rho_C$
 (b) $\rho_A > \rho_M > \rho_C$
 (c) $\rho_C > \rho_A > \rho_T$
 (d) $\rho_M > \rho_A > \rho_C$

Ans. (d)

Aluminium is more resistive than copper and mercury is most resistive of all.

So, $\rho_M > \rho_A > \rho_C$

Hence, correct option is (d).

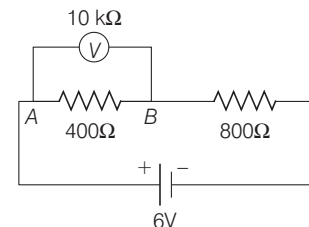
- 30** Two resistors $400\ \Omega$ and $800\ \Omega$ are connected in series across a 6 V battery. The potential difference measured by a voltmeter of $10\text{ k}\Omega$ across $400\ \Omega$ resistor is close to

[2020, 3 Sep, Shift-II]

- (a) 1.95 V (b) 2 V
 (c) 1.8 V (d) 2.05 V

Ans. (a)

The circuit diagram is

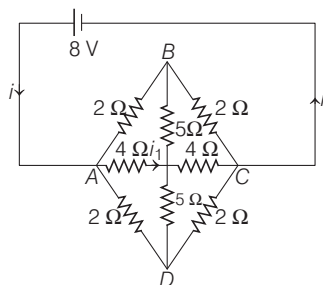


The equivalent resistance across AB,

$$\begin{aligned} R_{AB} &= \frac{10000 \times 400}{10000 + 400} \\ &= \frac{10000 \times 400}{10400} \end{aligned}$$

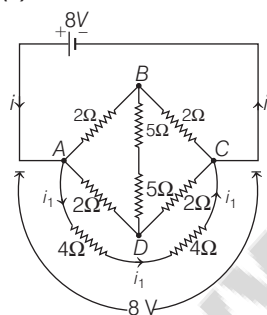
$$\begin{aligned}
 &= \frac{40000}{104} \\
 &= 384.6 \Omega \\
 &\approx 385 \Omega \\
 \therefore \text{Total circuit resistance} \\
 &= 385 + 800 = 1185 \Omega \\
 \text{Hence, circuit current is} \\
 I &= \frac{V}{R_{eq}} = \frac{6}{1185} \\
 &= 5.06 \times 10^{-3} \text{ A} \\
 \text{Now, potential drop across points A and B is} \\
 V_{AB} &= I \cdot R_{AB} \\
 &= 5.06 \times 10^{-3} \times 385 \\
 &= 1.946 \text{ V} \approx 1.95 \text{ V}
 \end{aligned}$$

- 31** The value of current i_1 flowing from A to C in the circuit diagram is
[2020, 4 Sep Shift-II]



- (a) 1A (b) 5A (c) 2A (d) 4A

Ans. (a)



From the given circuit diagram,
Potential drop across AC, $V = 8 \text{ V}$
Resistance of mentioned wire,
 $R = 4 + 4 = 8 \Omega$

So, the current flowing from A to C,

$$i_1 = \frac{V}{R} = \frac{8 \text{ V}}{8 \Omega} = 1 \text{ A}$$

Hence, option (a) is correct.

- 32** A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected correctly to the resistor. In the circuit,
[2020, 6 Sep Shift-II]

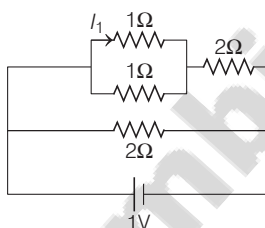
- (a) ammeter is always used in parallel and voltmeter in series
(b) Both ammeter and voltmeter must be connected in parallel
(c) ammeter is always connected in series and voltmeter in parallel
(d) Both ammeter and voltmeter must be connected in series

Ans. (c)

In the circuit to verify Ohm's law, the ammeter is connected in series as it measures the current through the resistance, while the voltmeter is connected in parallel as it measures the potential difference across the ends of resistor.

Hence, correct option is (c).

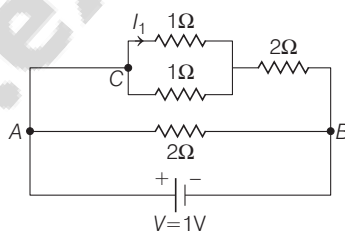
- 33** The current I_1 (in ampere) flowing through 1Ω resistor in the following circuit is
[2020, 7 Jan Shift-I]



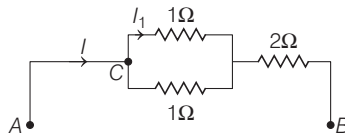
- (a) 0.25 (b) 0.4
(c) 0.5 (d) 0.2

Ans. (d)

Given circuit is



In given circuit $V_{AB} = 1 \text{ V}$, so upper branch of circuit is as shown below figure.



$$\begin{aligned}
 \text{Equivalent resistance of upper branch,} \\
 R_{eq} &= (1 \Omega \parallel 1 \Omega) + 2 \Omega \\
 &= \frac{1}{2} + 2 = \frac{5}{2} \Omega
 \end{aligned}$$

So, current in upper branch,

$$\begin{aligned}
 I &= \frac{V}{R} = \frac{V_{AB}}{R_{eq}} \\
 &= \frac{1}{5/2} = \frac{2}{5} \text{ A}
 \end{aligned}$$

At point C, this current is equally divided into two parts.

$$\text{So, } I_1 = \frac{1}{2} \left(\frac{2}{5} \right) = 0.2 \text{ A}$$

- 34** A 200Ω resistor has a certain colour code. If one replaces the red colour by green in the code, the new resistance will be

- (a) 100Ω (b) 400Ω
(c) 300Ω (d) 500Ω

[2019, 8 April Shift-I]

Ans. (d)

Given, resistance is $200 \Omega = 20 \times 10^1 \Omega$

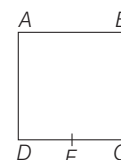


So, colour scheme will be red, black and brown.

Significant figure of red band is 2 and for green is 5. When red (2) is replaced with green (5), new resistance will be

$$200 \text{ ohm} \longrightarrow 500 \text{ ohm.}$$

- 35** A wire of resistance R is bent to form a square $ABCD$ as shown in the figure. The effective resistance between E and C is [E is mid-point of arm CD]
[2019, 9 April Shift-I]

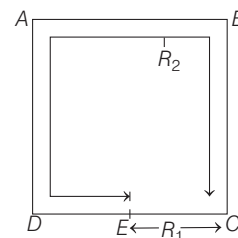


- (a) $\frac{7}{64} R$ (b) $\frac{3}{4} R$
(c) R (d) $\frac{1}{16} R$

Ans. (a)

Let the length of each side of square $ABCD$ is a .

$$\begin{aligned}
 \therefore \text{Resistance per unit length of each side} \\
 &= \frac{R}{4a}
 \end{aligned}$$

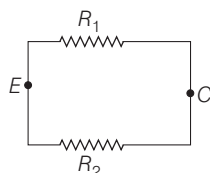


$$\text{Now, } R_1 = \frac{R}{4a} [EC] = \frac{R}{4a} \times \frac{a}{2} = \frac{R}{8}$$

Similarly,

$$R_2 = \frac{R}{4a} [EDABC] \\ = \frac{R}{4a} \times \frac{7a}{2} = \frac{7R}{8}$$

Now, effective resistance between E and C is the equivalent resistance of R_1 and R_2 that are connected in parallel as shown below.



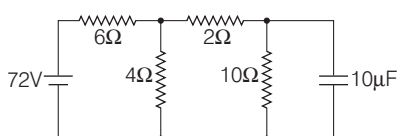
$$R_{EC} = \frac{R_1 R_2}{R_1 + R_2} \\ = \frac{(R/8) \times (7R/8)}{(R/8) + (7R/8)} \\ = \frac{7R^2}{64} \times \frac{8}{8R} = \frac{7}{64} R$$

- 36** Determine the charge on the capacitor in the following circuit [2019, 9 April Shift-I]

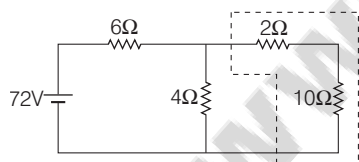
- (a) $2\mu\text{C}$ (b) $200\mu\text{C}$
(c) $10\mu\text{C}$ (d) $60\mu\text{C}$

Ans. (b)

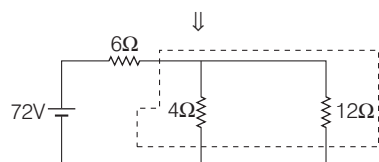
Given circuit is



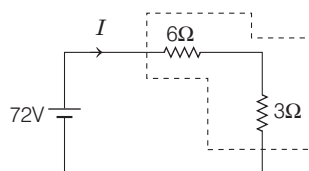
To find charge on capacitor, we need to determine voltage across it. In steady state, capacitor will act as open circuit and circuit can be reduced as



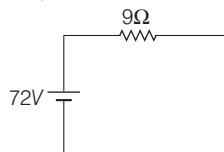
In series, $R_{eq} = 2\Omega + 10\Omega = 12\Omega$



In parallel, $R_{eq} = \frac{4 \times 12}{4 + 12} = 3\Omega$



In series, $R_{eq} = 6\Omega + 3\Omega = 9\Omega$

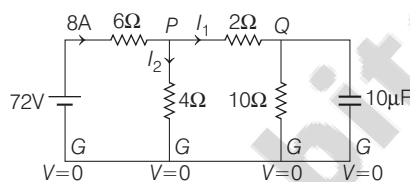


So, current in steady state,

$$I = \frac{V}{R} = \frac{72}{9} = 8\text{A}$$

Now, by using current division, at point P, current in 6Ω branch is

$$\frac{72 - V_P}{6\Omega} = 8\text{A} \Rightarrow V_P = 72 - 48 = 24\text{V}$$



Current in 4Ω branch is,

$$I_2 = \frac{V_P - 0}{4} = \frac{24 - 0}{4} = 6\text{A}$$

So, current in 2Ω resistance is

$$I_1 = 8 - I_2 \quad [\because I = I_1 + I_2] \\ = 8 - 6 = 2\text{A}$$

$\therefore$ Potential difference across 10Ω resistor is

$$V_{06} = 2\text{A} \times 10\Omega = 20\text{V}$$

Same potential difference will be applicable over the capacitor (parallel combination).

So, charge stored in the capacitor will be

$$Q = CV = 10 \times 10^{-6} \times 20 \\ \Rightarrow Q = 2 \times 10^{-4} \text{C} = 200\mu\text{C}$$

- 37** In a conductor, if the number of conduction electrons per unit volume is $8.5 \times 10^{28} \text{ m}^{-3}$ and mean free time is 25 fs (femto second), its approximate resistivity is (Take, $m_e = 9.1 \times 10^{-31} \text{ kg}$)

[2019, 9 April Shift-II]

- (a) $10^{-7} \Omega\text{-m}$ (b) $10^{-5} \Omega\text{-m}$
(c) $10^{-6} \Omega\text{-m}$ (d) $10^{-8} \Omega\text{-m}$

Ans. (d)

Resistivity of a conductor is

$$\rho = \frac{m_e}{ne^2\tau} \quad \dots (i)$$

where, m_e = mass of electron

$$= 9.1 \times 10^{-31} \text{ kg},$$

$$n = \text{free charge density} = 8.5 \times 10^{28} \text{ m}^{-3},$$

$$\tau = \text{mean free time} = 25 \text{ fs} = 25 \times 10^{-15} \text{ s}$$

and e = charge of electron = $1.6 \times 10^{-19} \text{ C}$

Substituting values in Eq. (i), we get

$$\rho = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}} \\ = \frac{9.1 \times 10^{-6}}{8.5 \times 2.56 \times 25} = 0.016 \times 10^{-6} \\ = 1.6 \times 10^{-8} \Omega\text{-m} \approx 10^{-8} \Omega\text{-m}$$

- 38** A metal wire of resistance 3Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be [2019, 9 April Shift-II]

- (a) $\frac{7}{2} \Omega$ (b) $\frac{5}{2} \Omega$
(c) $\frac{12}{5} \Omega$ (d) $\frac{5}{3} \Omega$

Ans. (d)

Initial resistance of wire is 3Ω . Let its length is l and area is A .

$$\text{Then, } R_{\text{initial}} = \rho \frac{l}{A} = 3\Omega \quad \dots (i)$$

When wire is stretched twice its length, then its area becomes A' , on equating volume, we have

$$Al = A'2l \Rightarrow A' = \frac{A}{2}$$

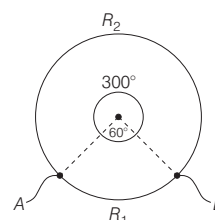
So, after stretching, resistance of wire will be

$$R' = R_{\text{final}} = \rho \frac{l'}{A'} = 4\rho \frac{l}{A} = 12\Omega$$

[using Eq. (i)]

Now, this wire is made into a circle and connected across two points A and B (making 60° angle at centre) as

Now, above arrangement is a combination of two resistances in parallel,



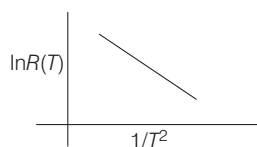
$$R_1 = \frac{60 \times R'}{360} = \frac{1}{6} \times 12 = 2 \Omega$$

and $R_2 = \frac{300}{360} \times R' = \frac{5}{6} \times 12 = 10 \Omega$

Since, R_1 and R_2 are connected in parallel.

$$\text{So, } R_{AB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 2}{12} = \frac{5}{3} \Omega$$

- 39** In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line.

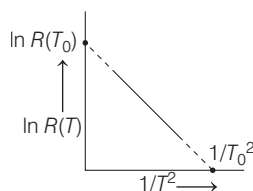


One may conclude that

- (a) $R(T) = R_0 e^{-T^2/T_0^2}$
 (b) $R(T) = R_0 e^{T^2/T_0^2}$
 (c) $R(T) = R_0 e^{-T_0^2/T^2}$
 (d) $R(T) = \frac{R_0}{T^2}$

Ans. (c)

From the given graph,



We can say that, $\ln R(T) \propto -\frac{1}{T^2}$

Negative sign implies that the slope of the graph is negative.

$$\text{or } \ln R(T) = \text{constant} \left(-\frac{1}{T^2} \right)$$

$$\Rightarrow R(T) = \frac{\exp(\text{const.})}{\exp\left(\frac{1}{T^2}\right)}$$

$$\Rightarrow R(T) = R_0 \exp\left(-\frac{T_0^2}{T^2}\right)$$

Alternate Solution

$$\text{From graph, } \frac{1}{T_0^2} + \frac{\ln R(T)}{\ln R(T_0)} = 1$$

$$\Rightarrow \ln R(T) = [\ln R(T_0)] \cdot \left[1 - \frac{T_0^2}{T^2} \right]$$

$$\text{or } R(T) = R_0 \exp\left(\frac{-T_0^2}{T^2}\right)$$

- 40** A current of 5 A passes through a copper conductor (resistivity $= 1.7 \times 10^{-8} \Omega\text{-m}$) of radius of cross-section 5 mm. Find the mobility of the charges, if their drift velocity is $1.1 \times 10^{-3} \text{ m/s}$.

[2019, 10 April Shift-I]

- (a) $1.5 \text{ m}^2/\text{V-s}$ (b) $1.3 \text{ m}^2/\text{V-s}$
 (c) $1.0 \text{ m}^2/\text{V-s}$ (d) $1.8 \text{ m}^2/\text{V-s}$

Ans. (c)

Given, $I = 5 \text{ A}$,

$$\rho = 1.7 \times 10^{-8} \Omega\text{-m},$$

$$r = 5 \text{ mm} = 5 \times 10^{-3} \text{ m},$$

$$v_d = 1.1 \times 10^{-3} \text{ m/s}$$

Mobility of charges in a conductor is given by

$$\mu = \frac{v_d}{E} \quad \dots (i)$$

and resistivity is given by

$$\rho = \frac{E}{J} = \frac{E}{I/A} \quad (\because J = \sigma E = \frac{1}{\rho} \times E)$$

$$\Rightarrow \rho = \frac{EA}{I}$$

$$\text{or } E = \frac{\rho I}{A} \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\mu = \frac{v_d A}{\rho I}$$

Substituting the given values, we get

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{1.7 \times 10^{-8} \times 5}$$

$$= \frac{86.35 \times 10^{-9}}{8.5 \times 10^{-8}} = 10.1 \times 10^{-1}$$

$$\Rightarrow \mu \approx 1 \text{ m}^2/\text{V-s}$$

- 41** Space between two concentric conducting spheres of radii a and b ($b > a$) is filled with a medium of resistivity ρ . The resistance between the two spheres (in ohm) will be

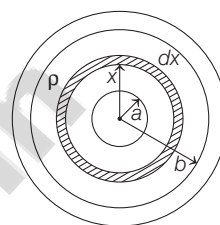
[2019, 10 April Shift-II]

- (a) $\frac{\rho}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$ (b) $\frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right)$
 (c) $\frac{\rho}{2\pi} \left(\frac{1}{a} - \frac{1}{b} \right)$ (d) $\frac{\rho}{4\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$

Ans. (b)

Key Idea Resistance between surface of inner shell and a circumferential point of outer shell can be formed by finding resistance of a thin (differentially thin) shell in between these two shells. Then, this result can be integrated (summed up) to get resistance of the complete arrangement.

For a elemental shell of radius x and thickness dx ,



$$\text{Resistance, } dR = \rho \frac{l}{A}$$

$$\Rightarrow dR = \rho \frac{dx}{4\pi x^2}$$

So, resistance of complete arrangement is

$$R = \int_a^b dR = \int_a^b \rho \frac{dx}{4\pi x^2} = \frac{\rho}{4\pi} \int_a^b x^{-2} dx$$

$$\Rightarrow R = \frac{\rho}{4\pi} \left(\frac{x^{-1}}{-1} \right)_a^b = \frac{\rho}{4\pi} \left(-\frac{1}{x} \right)_a^b$$

$$= \frac{\rho}{4\pi} \left(-\frac{1}{b} - \left(-\frac{1}{a} \right) \right)$$

$$= \frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b} \right) \text{ ohm}$$

- 42** Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross-section 5 mm^2 is v . If the electron density in copper is $9 \times 10^{28} / \text{m}^3$, the value of v (in mm/s) is close to (Take, charge of electron to be $= 1.6 \times 10^{-19} \text{ C}$)

[2019, 9 Jan Shift-I]

- (a) 0.02 (b) 0.2 (c) 2 (d) 3

Ans. (a)

Relation between current (I) flowing through a conducting wire and drift velocity of electrons (v_d) is given as

$$I = neAv_d$$

where, n is the electron density and A is the area of cross-section of wire.

$$\Rightarrow v_d = \frac{I}{neA}$$

Substituting the given values, we get

$$v = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$$

$$v = \frac{1.5 \times 10^{-3}}{72} \text{ m/s} = 0.2 \times 10^{-4} \text{ m/s}$$

or $v = 0.02 \text{ mm/s}$

- 43** A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance, if its volume remains unchanged is

[2019, 9 Jan Shift-I]

(a) 2.0% (b) 1.0% (c) 0.5% (d) 2.5%

Ans. (b)

Electrical resistance of wire of length l , area of cross-section A and resistivity ρ is given as

$$R = \rho \frac{l}{A} \quad \dots(i)$$

Since we know, volume of the wire is

$$V = A \times l \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$R = \rho \frac{l^2}{V} \quad \dots(iii)$$

As, the length has been increased to 0.5%.

$\therefore$ New length of the wire,

$$l' = l + 0.5\% \text{ of } l = l + 0.005l = 1.005l$$

But V and ρ remains unchanged.

So, new resistance,

$$R' = \frac{\rho [(1.005)l]^2}{V} \quad \dots(iv)$$

Dividing Eq. (iv) and Eq. (iii), we get

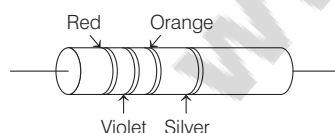
$$\frac{R'}{R} = (1.005)^2$$

$\Rightarrow$ % change in the resistance

$$= \left(\frac{R'}{R} - 1 \right) \times 100$$

$$= [(1.005)^2 - 1] \times 100 = 1.0025\% \approx 1\%$$

- 44** A resistance is shown in the figure. Its value and tolerance are given respectively by [2019, 9 Jan Shift-I]



- (a) 270Ω , 5%
 (b) $27 \text{ k}\Omega$, 20%
 (c) $27 \text{ k}\Omega$, 10%
 (d) $270 \text{ k}\Omega$, 10%

Ans. (c)

The value of a carbon resistor is given as,

$$R = AB \times C \pm D\% \quad \dots(i)$$

where, color band A and B (first two colors) indicate the first two significant figures of resistance in ohms, C (third

color) indicates the decimal multiplier and D (fourth color) indicates the tolerance in % as per the indicated value. The table for color code of carbon resistor is given below,

Color Codes	Values (A, B)	Multiplier (C)	Tolerance (D)(%)
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold			$\pm 5\%$
Silver			$\pm 10\%$
No colour			$\pm 20\%$

$\therefore$ Comparing the colors given in the question and the table and writing in the manner of

Eq. (i), we get

$$R = 27 \times 10^3 \Omega, \pm 10\% = 27 \text{ k}\Omega, \pm 10\%$$

- 45** A carbon resistance has a following color code. What is the value of the resistance? [2019, 9 Jan Shift-II]



- (a) $5.3 \text{ M}\Omega \pm 5\%$ (b) $64 \text{ k}\Omega \pm 10\%$
 (c) $6.4 \text{ M}\Omega \pm 5\%$ (d) $530 \text{ k}\Omega \pm 5\%$

Ans. (d)

The value of a carbon resistor is given as

$$R = AB \times C \pm D\% \quad \dots(i)$$

where, colour band A and B (first two colour) indicates the first two significant figures of resistance in Ohms.

C (third colour) indicate the decimal multiplies and D (fourth colour) indicates the tolerance in % as per the indicated value.

The table for colour code,

Colour code	Values (AB)	Multiplier (C)	Tolerance
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	

Colour code	Values (AB)	Multiplier (C)	Tolerance
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold	-	-	$\pm 5\%$
Silver	-	-	$\pm 10\%$
No colour	-	-	$\pm 20\%$

$\therefore$ Comparing the colours given in the question and table and writing in the manner of Eq. (i), we get

$$R = 53 \times 10^4 \pm 5\% \Omega$$

$$\text{or } R = 530 \times 10^3 \Omega \pm 5\%$$

$$\text{or } R = 530 \text{ k}\Omega \pm 5\%$$

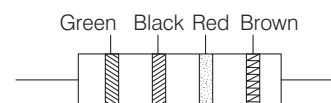
- 46** A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is

[2019, 10 Jan Shift-I]

- (a) 0.4 mA (b) 63 mA
 (c) 20 mA (d) 100 mA

Ans. (c)

Colour code of carbon resistance is shown in the figure below



So, resistance value of resistor using colour code is

$$R = 502 \times 10 = 502 \times 10^2 \Omega$$

Here, we must know that for given carbon resistor first three colours give value of resistance and fourth colour gives multiplier value.

Now using power, $P = i^2 R$

$$\text{we get, } i = \sqrt{\frac{P}{R}} = \sqrt{\frac{2}{502 \times 10^2}}$$

$$\approx 20 \times 10^{-3} \text{ A}$$

$$= 20 \text{ mA.}$$

- 47** A uniform metallic wire has a resistance of 18Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

[2019, 10 Jan Shift-I]

- (a) 12Ω (b) 8Ω (c) 2Ω (d) 4Ω

Ans. (d)

Resistance of each arm of equilateral triangle will be

$$R = \frac{18}{3} = 6 \Omega$$

So we have following combination will be Equivalent resistance is

$$\therefore R_{AB} = \frac{12 \times 6}{12 + 6} = \frac{12 \times 6}{18} = 4 \Omega$$

- 48** The temperature dependence of resistances of Cu and undoped Si in the temperature range 300–400 K, is best described by

[JEE Main 2016]

- (a) linear increase for Cu, linear increase for Si
 (b) linear increase for Cu, exponential increase for Si
 (c) linear increase for Cu, exponential decrease for Si
 (d) linear decrease for Cu, linear decrease for Si

Ans. (c)

As, we know Cu is a conductor, so increase in temperature, resistance will increase. Then, Si is semiconductor, so with increase in temperature, resistance will decrease.

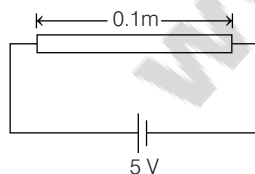
- 49** When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5 \times 10^{-4} \text{ ms}^{-1}$. If the electron density in the wire is $8 \times 10^{28} \text{ m}^{-3}$ the resistivity of the material is close to

[JEE Main 2015]

- (a) $1.6 \times 10^{-8} \Omega \text{ m}$ (b) $1.6 \times 10^{-7} \Omega \text{ m}$
 (c) $1.6 \times 10^{-6} \Omega \text{ m}$ (d) $1.6 \times 10^{-5} \Omega \text{ m}$

Ans. (d)

According to the question,



$$v_d = 2.5 \times 10^{-4} \text{ m/s}$$

$$\Rightarrow n = 8 \times 10^{28} / \text{m}^3$$

We know that

$$J = nev_d \text{ or } I = nev_d A$$

where, symbols have their usual meaning.

$$\Rightarrow \frac{V}{R} = nev_d A \text{ or } \frac{V}{\frac{\rho L}{A}} = nev_d A$$

$$\text{or } \frac{V}{\rho L} = nev_d \text{ or } \rho = \frac{V}{nev_d L}$$

$$= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$$\text{or } \rho = 1.6 \times 10^{-5} \Omega \text{ m}$$

- 50** If 400Ω of resistance is made by adding four 100Ω resistance of tolerance 5%, then the tolerance of the combination is **[AIEEE 2011]**

- (a) 20% (b) 5%
 (c) 10% (d) 15%

Ans. (b)

Resistance of combination, $R_e = 4R$

$$\Rightarrow \frac{\Delta R_e}{R_e} = \frac{\Delta R}{R} = \frac{5 \times 100}{100} = 5\%$$

- 51** If a wire is stretched to make it 0.1% longer, its resistance will **[AIEEE 2011]**

- (a) increase by 0.2%
 (b) decrease by 0.2%
 (c) decrease by 0.05%
 (d) increase by 0.05%

Ans. (d)

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{V} \quad [\because V = \text{volume}]$$

$$\therefore \frac{\Delta R}{R} = 2 \frac{\Delta l}{l} = +0.2\%$$

- 52** Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2 . The respective temperature coefficients of their series and parallel combinations are nearly **[AIEEE 2010]**

- (a) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$
 (b) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$
 (c) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$
 (d) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

Ans. (d)

Let R_0 be the initial resistance of both conductors.

$\therefore$ At temperature θ , their resistances will be

$$R_1 = R_0(1 + \alpha_1 \theta)$$

$$\text{and } R_2 = R_0(1 + \alpha_2 \theta)$$

For series combination,

$$R_s = R_1 + R_2$$

$$R_{s0}(1 + \alpha_s \theta) = R_0(1 + \alpha_1 \theta) + R_0(1 + \alpha_2 \theta)$$

where, $R_{s0} = R_0 + R_0 = 2R_0$

$$\therefore 2R_0(1 + \alpha_s \theta) = 2R_0 + R_0 \theta (\alpha_1 + \alpha_2)$$

or $\alpha_s = \frac{\alpha_1 + \alpha_2}{2}$

For parallel combination,

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{p0}(1 + \alpha_p \theta) = \frac{R_0(1 + \alpha_1 \theta) R_0(1 + \alpha_2 \theta)}{R_0(1 + \alpha_1 \theta) + R_0(1 + \alpha_2 \theta)}$$

where, $R_{p0} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$

$$\therefore \frac{R_0}{2}(1 + \alpha_p \theta) = \frac{R_0^2(1 + \alpha_1 \theta + \alpha_2 \theta + \alpha_1 \alpha_2 \theta^2)}{R_0(2 + \alpha_1 \theta + \alpha_2 \theta)}$$

As α_1 and α_2 are small quantities.

$\therefore \alpha_1 \alpha_2$ is negligible.

$$\text{or } \alpha_p = \frac{\alpha_1 + \alpha_2}{2 + (\alpha_1 + \alpha_2)\theta}$$

$$= \frac{\alpha_1 + \alpha_2}{2} \left[1 - \left(\frac{\alpha_1 + \alpha_2}{2} \right) \theta \right]$$

As $(\alpha_1 + \alpha_2)^2$ is negligible.

$$\therefore \alpha_p = \frac{\alpha_1 + \alpha_2}{2}$$

- 53** This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha \Delta T)$. The resistance of a wire changes from 100Ω to 150Ω when its temperature is increased from 27°C to 227°C . This implies that $\alpha = 2.5 \times 10^{-3} / ^\circ \text{C}$.

Statement II $R = R_i(1 + \alpha \Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) \ll R_0$.

- (a) Statement I is true, Statement II is false
 (b) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
 (c) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
 (d) Statement I is false, Statement II is true

Ans.(d)

$$R = R_t (1 + \alpha \Delta t)$$

Given, $R_t = 100 \, \Omega$, $R = 150 \, \Omega$, $\Delta t = 200^\circ \text{C}$

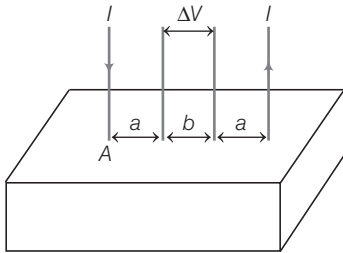
$$\therefore 150 = 100 (1 + \alpha 200)$$

$$\Rightarrow 0.5 = 200 \alpha$$

$$\Rightarrow \alpha = \frac{0.5}{200} = \frac{1}{400} \\ = 25 \times 10^{-4} / ^\circ \text{C}$$

Directions (Q. Nos. 54 to 55) are based on the following paragraph.

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps



- Take current I entering from A and assume it to spread over a hemispherical surface on the block.
- Calculate field $E(r)$ at distance r from A by using Ohm's law $= \rho J$, where J is the current per unit area at r .
- From the r dependence of $E(r)$, obtain the potential $V(r)$ at r .
- Repeat steps (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

54 ΔV measured between B and C is

[AIEEE 2008]

- $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$
- $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$
- $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$
- $\frac{\rho I}{2\pi(a-b)}$

Ans. (c)

Electric field at a distance r from A is

$$E = \rho \times \frac{I}{2\pi r^2}$$

$$\Rightarrow \int dV = - \int E dr$$

$$\Rightarrow V_C - V_B = \Delta V$$

$$= - \int_a^{a+b} \frac{\rho I}{2\pi} \times \frac{dr}{r^2}$$

$$\text{or } \Delta V = \frac{\rho I}{2\pi} \left[\frac{1}{a} - \frac{1}{a+b} \right]$$

55 For current entering at A, the electric field at a distance r from A is

[AIEEE 2008]

- $\frac{\rho I}{8\pi r^2}$
- $\frac{\rho I}{r^2}$
- $\frac{\rho I}{2\pi r^2}$
- $\frac{\rho I}{4\pi r^2}$

Ans. (c)

$$\text{As } E = \frac{V}{l} = \frac{IR}{l} = \frac{I \rho l}{l A} = \rho J \\ [\because \text{current density, } J = I/A]$$

$$\text{From } E = \rho J = \frac{\rho I}{2\pi r^2}$$

56 The resistance of a wire is $5 \, \Omega$ at 50°C and $6 \, \Omega$ at 100°C . The resistance of the wire at 0°C will be

[AIEEE 2007]

- $2 \, \Omega$
- $1 \, \Omega$
- $4 \, \Omega$
- $3 \, \Omega$

Ans. (c)

$$\text{From } R_t = R_0 (1 + \alpha t)$$

$$\therefore 5 = R_0 (1 + 50\alpha) \quad \dots(i)$$

$$\text{and } 6 = R_0 (1 + 100\alpha) \quad \dots(ii)$$

$$\therefore \frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha}$$

$$\text{or } \alpha = \frac{1}{200}$$

Putting the value of α in Eq. (i), we get

$$5 = R_0 \left(1 + 50 \times \frac{1}{200} \right) \text{ or } R_0 = 4 \, \Omega$$

57 A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then, for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be

[AIEEE 2006]

- 1
- $1/2$
- $1/4$
- 2

Ans.(d)

Let (ρ_A, l_A, r_A, A_A) and (ρ_B, l_B, r_B, A_B) be specific resistances, lengths, radii and areas of wires A and B, respectively.

$$\text{Resistance of A} = R_A = \frac{\rho_A l_A}{A_A} = \frac{\rho_A l_A}{\pi r_A^2}$$

$$\text{Resistance of B} = R_B = \frac{\rho_B l_B}{A_B} = \frac{\rho_B l_B}{\pi r_B^2}$$

From given information,

$$\rho_B = 2\rho_A$$

$$r_B = 2r_A$$

and

$$R_A = R_B$$

 $\therefore$

$$\frac{\rho_A l_A}{\pi r_A^2} = \frac{\rho_B l_B}{\pi r_B^2}$$

or

$$\frac{\rho_A l_A}{\pi r_A^2} = \frac{2\rho_A \times l_B}{\pi (2r_A)^2}$$

or

$$\frac{l_B}{l_A} = \frac{2}{1} = 2:1$$

58 The resistance of a bulb filament is $100 \, \Omega$ at a temperature of 100°C . If its temperature coefficient of resistance is $0.005 \text{ per } ^\circ \text{C}$, its resistance will become $200 \, \Omega$ at a temperature of

[AIEEE 2006]

- 300°C
- 400°C
- 500°C
- 200°C

Ans. (b)

Let resistance of bulb filament be R_0 at 0°C , then from expression,

$$R_\theta = R_0 [1 + \alpha \Delta \theta]$$

$$\text{We have, } 100 = R_0 [1 + 0.005 \times 100]$$

$$\text{and } 200 = R_0 [1 + 0.005 \times x]$$

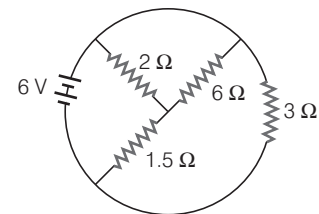
where, x is temperature in $^\circ \text{C}$ at which resistance become $200 \, \Omega$.

Dividing the above two equations, we get

$$\frac{200}{100} = \frac{1 + 0.005x}{1 + 0.005 \times 100} \Rightarrow x = 400^\circ \text{C}$$

59 The total current supplied to the circuit by the battery is

[AIEEE 2004]

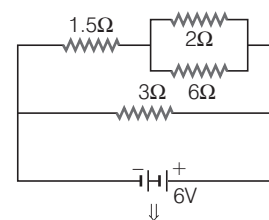


- 1 A
- 2 A
- 4 A
- 6 A

Ans. (c)

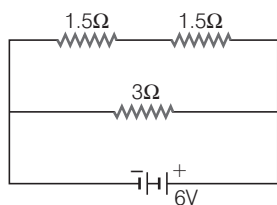
The equivalent circuit can be drawn as $6 \, \Omega$ and $2 \, \Omega$ are in parallel.

$$R' = \frac{2 \times 6}{8} = 1.5 \, \Omega$$



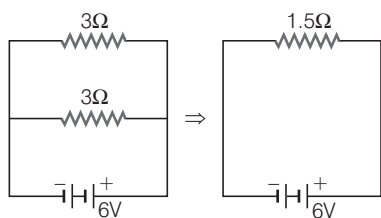
As $1.5 \, \Omega$ and $1.5 \, \Omega$ are in series.

$$R'' = 1.5 + 1.5 = 3 \, \Omega$$



As 3Ω and 3Ω are in parallel.

$$R_{eq} = \frac{3 \times 3}{6} = 1.5\Omega$$



Hence, current supplied by the battery is

$$I = \frac{V}{R_{eq}} = \frac{6}{1.5} = 4\text{ A}$$

- 60** The resistance of the series combination of two resistances is S . When they are joined in parallel, the total resistance is P . If $S = nP$, then the minimum possible value of n is [AIEEE 2004]

- (a) 4 (b) 3
(c) 2 (d) 1

Ans. (a)

Let resistances be R_1 and R_2 . Then,

$$S = R_1 + R_2$$

and $P = \frac{R_1 R_2}{R_1 + R_2}$

$$\therefore (R_1 + R_2) = \frac{n \times R_1 R_2}{R_1 + R_2} \quad [\because S = nP]$$

$$\text{or } (R_1 + R_2)^2 = n R_1 R_2$$

$$\Rightarrow n = \left[\frac{R_1^2 + R_2^2 + 2R_1 R_2}{R_1 R_2} \right] = \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} + 2 \right]$$

We know,

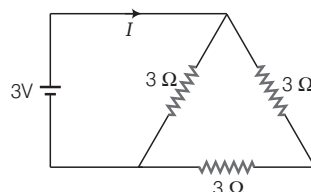
Arithmetic Mean $\geq$ Geometric Mean

$$\frac{\frac{R_1}{R_2} + \frac{R_2}{R_1}}{2} \geq \sqrt{\frac{R_1}{R_2} \times \frac{R_2}{R_1}}$$

$$\Rightarrow \frac{R_1}{R_2} + \frac{R_2}{R_1} \geq 2$$

So, $n(\text{minimum value}) = 2 + 2 = 4$

- 61** A 3 V battery with negligible internal resistance is connected in a circuit as shown in the figure. [AIEEE 2003]

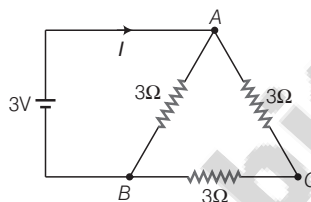


The current I in the circuit will be

- (a) 1 A (b) 1.5 A
(c) 2 A (d) $\frac{1}{3}$ A

Ans. (b)

Resistance in the arms AC and BC are in series.



$$\therefore R' = 3 + 3 = 6\Omega$$

Now, R' and 3Ω are in parallel.

$$\therefore R_{eq} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

$$\text{Now, } V = IR \Rightarrow I = \frac{3}{2} = 1.5\text{ A}$$

- 62** The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter, the change in the resistance of the wire will be [AIEEE 2003]

- (a) 200% (b) 100%
(c) 50% (d) 300%

Ans. (d)

$$\text{Given, } l' = l + 100\% l = 2l$$

$$\text{Initial volume} = \text{Final volume}$$

$$\text{i.e., } \pi r^2 l = \pi r'^2 l'$$

$$\text{or } r'^2 = \frac{r^2 l}{l'} = r^2 \times \frac{l}{2l}$$

$$\text{or } r'^2 = \frac{r^2}{2}$$

$$\therefore R' = \rho \frac{l'}{A'} = \rho \frac{2l}{\pi r'^2} \left[\because R = \frac{\rho l}{A} \right]$$

$$= \frac{\rho \cdot 4l}{\pi r^2} = 4R$$

$$\text{Thus, } \Delta R = R' - R = 4R - R = 3R$$

$$\therefore \% \Delta R = \frac{3R}{R} \times 100\% = 300\%$$

Alternate Solution

By the shortcut formula,

$$\% \text{ Change} = x + y + \frac{xy}{100}$$

Valid only for 2 variables in terms of percentage.

$$\text{As } R \propto l^2\%$$

Increase in resistance

$$= 100 + 100 + \frac{100 \times 100}{100} = 300\%$$

TOPIC 2

Heating Effect of Current

- 63** A uniform heating wire of resistance 36Ω is connected across a potential difference of 240 V. The wire is then cut into half and potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be 1: x , where x is [2021, 1 Sep Shift-II]

Ans. (4)

For case I,

The potential difference of the uniform wire, $V = 240\text{ V}$

The resistance of the uniform wire, $R_1 = 36\Omega$

The power dissipation in the first case,

$$P_1 = \frac{V^2}{R_1} = \frac{(240)^2}{36}$$

For case II,

The resistance of each half,

$$R_2 = \frac{R_1}{2} = \frac{36}{2} = 18\Omega$$

$$P_2 = \frac{V^2}{R_2} + \frac{V^2}{R_2} = \frac{(240)^2}{18} + \frac{(240)^2}{18} = \frac{(240)^2}{9}$$

Thus, the ratio of the total power dissipation in the first case to the second case

$$\frac{P_1}{P_2} = \frac{(240)^2 / 36}{(240)^2 / 9}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{4}$$

$$\text{Comparing with, } \frac{P_1}{P_2} = \frac{1}{x}$$

The value of the $x = 4$.

- 64** A resistor dissipates 192 J of energy in 1 s when a current of 4 A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s is J. [2021, 31 Aug Shift-II]

Ans. (3840)

Given that, initial current, $I_1 = 4\text{ A}$

Final current, $I_2 = 2I_1 = 8\text{ A}$

Initial heat dissipated, $H_1 = 192\text{ J}$

Initial time, $t_1 = 1\text{ s}$

Final time, $t_2 = 5\text{ s}$

Let final heat dissipated = H_2

By Joule's law of heating, $H \propto I^2 R T$

Since resistance remains same at initial and final condition,

$$\therefore \frac{H_2}{H_1} = \frac{I_2^2 R t_2}{I_1^2 R t_1} = \frac{I_2^2 t_2}{I_1^2 t_1}$$

Substituting the given values, we get

$$\frac{H_2}{192} = \left(\frac{8}{4}\right)^2 \times \frac{5}{1}$$

$\Rightarrow$

$$H_2 = 3840\text{ J}$$

- 65** An electric bulb of 500 W at 100 V is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb, so that the power delivered by the bulb is 500 W. [2021, 26 Aug Shift-II]

(a) 20 Ω (b) 30 Ω (c) 5 Ω (d) 10 Ω

Ans. (a)

Given, power rating of bulb, $P_B = 500\text{ W}$

Voltage across bulb, $V_B = 100\text{ V}$

Supply voltage, $V_S = 200\text{ V}$

If a resistance R is attached in series with the bulb, then the voltage across resistance will be 100 V.

Now, current flowing in circuit when bulb delivers power of 500 W is given as

$$\begin{aligned} P_B &= V_B I \\ \Rightarrow 500 &= 100 \times I \\ \Rightarrow I &= 5\text{ A} \end{aligned}$$

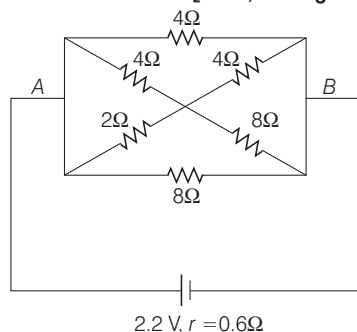
Same amount of current will flow from the resistance as it is connected in series.

Using Ohm's law,

$$\begin{aligned} V &= IR \\ \Rightarrow 100 &= 5 \times R \\ \Rightarrow R &= 20\Omega \end{aligned}$$

Thus, the resistance connected in series is 20 Ω .

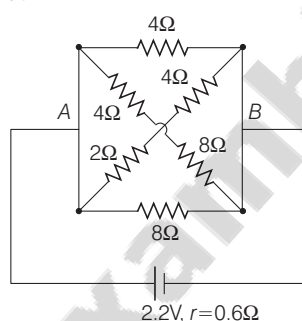
- 66** In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6 Ω . Calculate the power dissipated in the whole circuit [2021, 26 Aug Shift-I]



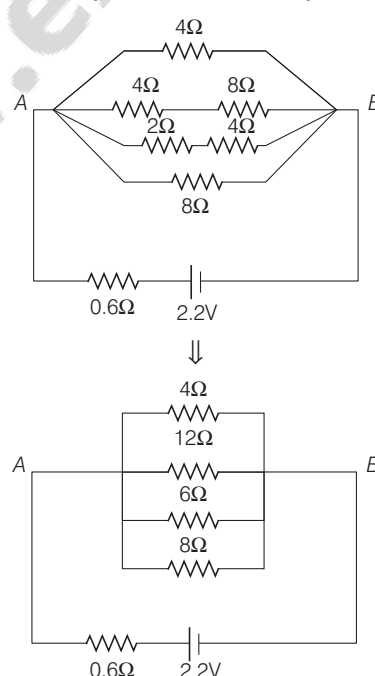
- (a) 1.32 W (b) 0.65 W
(c) 2.2 W (d) 4.4 W

Ans. (c)

The given circuit diagram can be drawn as

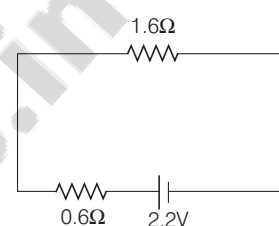


Redrawing the above circuit diagram as



Equivalent resistance of the circuit between point A and B is given as

$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \\ &= \frac{1}{4} + \frac{1}{12} + \frac{1}{6} + \frac{1}{8} \\ &= \frac{6+2+4+3}{24} \\ &= \frac{15}{24} \\ &= \frac{5}{8} \\ R_{eq} &= \frac{8}{5} = 1.6\Omega \end{aligned}$$



$$\begin{aligned} R_{total} &= 1.6 + 0.6 = 2.2\Omega \\ P &= \frac{V^2}{R_{total}} = \frac{2.2 \times 2.2}{2.2} \\ &= 2.2\text{ W} \end{aligned}$$

- 67** The energy dissipated by a resistor is 10 mJ in 1s when an electric current of 2 mA flows through it. The resistance is Ω .

(Round off to the nearest integer)

[2021, 16 March Shift-II]

Ans. (2500)

Given, energy dissipated by a resistor,

$$\begin{aligned} H &= 10\text{ mJ} \\ &= 10 \times 10^{-3}\text{ J} \end{aligned}$$

Time, $t = 1\text{ s}$

$$\begin{aligned} \text{Electric current, } I &= 2\text{ mA} \\ &= 2 \times 10^{-3}\text{ A} \end{aligned}$$

Resistance, $R = ?$

According to Joule's law of heating,

$$H = I^2 R t$$

$$\Rightarrow R = \frac{H}{I^2 t} \quad \dots (i)$$

Substituting the given values in Eq. (i), we get

$$\begin{aligned} R &= \frac{10 \times 10^{-3}}{(2 \times 10^{-3})^2 \times 1} \\ \Rightarrow R &= \frac{10^{-2}}{4 \times 10^{-6}} \\ \Rightarrow R &= 0.25 \times 10^4 \\ \Rightarrow R &= 2500\Omega \end{aligned}$$

- 68** A resistor develops 500 J of thermal energy in 20 s, when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3 A, what will be the energy developed in 20 s?

[2021, 16 March Shift-II]

(a) 1500 J (b) 1000 J (c) 500 J (d) 2000 J

Ans. (d)

Given,

Heat energy, $H_1 = 500$ J

Initial current, $I_1 = 1.5$ A, final current,

$$I_2 = 3 \text{ A}$$

and time, $t = 20$ s

According to Joule's law of heating,

$$H = I^2 R t$$

$$\Rightarrow H_1 = I_1^2 R t \quad \dots(i)$$

$$\text{and } H_2 = I_2^2 R t \quad \dots(ii)$$

[$\because$ Resistance and time being the same in both cases]

Dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \frac{H_1}{H_2} = \frac{I_1^2 R t}{I_2^2 R t} \Rightarrow \frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2$$

$$\Rightarrow \frac{H_1}{H_2} = \left(\frac{1.5}{3}\right)^2 = \left(\frac{15}{30}\right)^2 \Rightarrow \frac{H_1}{H_2} = \left(\frac{1}{2}\right)^2$$

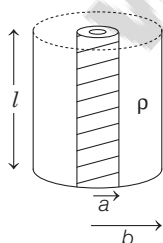
$$\Rightarrow \frac{H_1}{H_2} = \frac{1}{4} \Rightarrow \frac{500}{H_2} = \frac{1}{4}$$

$$\Rightarrow H_2 = 500 \times 4$$

$$\Rightarrow H_2 = 2000 \text{ J}$$

- 69** A torch battery of length l is to be made up of a thin cylindrical bar of radius a and a concentric thin cylindrical shell of radius b is filled in between with an electrolyte of resistivity ρ (see figure). If the battery is connected to a resistance R , the maximum joule's heating in R will take place for

[2020, 3 Sep Shift-I]

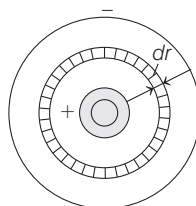
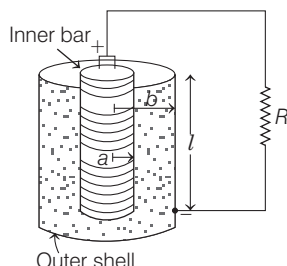


(a) $R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right)$ (b) $R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right)$

(c) $R = \frac{\rho}{\pi l} \ln\left(\frac{b}{a}\right)$ (d) $R = \frac{2\rho}{\pi l} \ln\left(\frac{b}{a}\right)$

Ans. (b)

By maximum power theorem, maximum joule's heating in external resistance R takes place when internal resistance of battery is equal to external resistance R .



Now, resistance of a elemental cylinder of radius r and thickness dr is

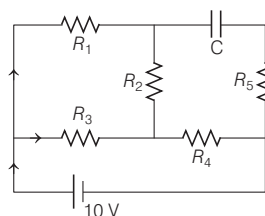
$$\begin{aligned} (dR)_{\text{internal}} &= \frac{\rho \cdot dr}{2\pi r l} \\ \Rightarrow R_{\text{internal}} &= \int (dR)_{\text{internal}} \\ &= \int_{r=a}^{r=b} \frac{\rho \cdot dr}{2\pi r l} = \frac{\rho}{2\pi l} \int_a^b \frac{dr}{r} \\ &= \frac{\rho}{2\pi l} [\ln r]_a^b \\ &= \frac{\rho}{2\pi l} (\ln b - \ln a) = \frac{\rho}{2\pi l} \cdot \ln\left(\frac{b}{a}\right) \end{aligned}$$

So, the maximum joule's heating in R will take place when its value is

$$= R_{\text{internal}} = \frac{\rho}{2\pi l} \cdot \ln\left(\frac{b}{a}\right)$$

Hence, option (b) is correct.

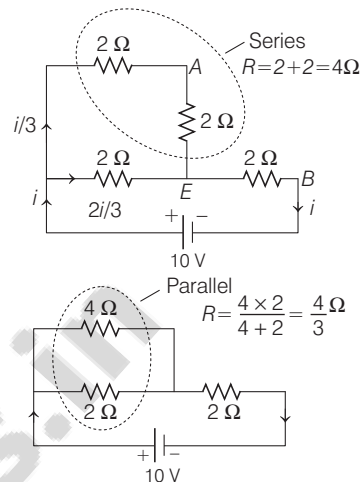
- 70** An ideal cell of emf 10 V is connected in circuit shown in figure. Each resistance is 2Ω . The potential difference (in V) across the capacitor when it is fully charged is



[2020, 2 Sep Shift-II]

Ans. (8)

When capacitor is fully charged, circuit is reduced to as shown below,



So,

$$\text{Total resistance is } R_{\text{eq}} = \frac{4}{3} + 2 = \frac{10}{3} \Omega$$

$$\text{Current in circuit, } i = \frac{V}{R_{\text{eq}}} = \frac{10}{10/3} = 3 \text{ A}$$

Hence, potential difference across capacitor
= potential difference across AEB

$$= 2i/3 + 2 \times i = 2 \times \frac{3}{3} + 2 \times 3 = 8 \text{ V}$$

- 71** A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is

[2020, 4 Sep Shift-I]

(a) 0.072 W (b) 0.125 W
(c) 0.50 W (d) 0.10 W

Ans. (d)

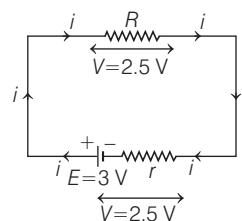
Using Kirchhoff's loop law,

$$V = E - ir$$

$$2.5 = 3 - ir$$

$$ir = 0.5$$

...(i)



Now, from Ohm's law,

$$V = iR, 2.5 = iR, iR = 2.5$$

...(ii)

and given that

$$P_R = 0.5, i^2 R = 0.5$$

...(iii)

Dividing eq (i) by eq (ii), we get

$$\frac{ir}{iR} = \frac{0.5}{2.5} \Rightarrow \frac{r}{R} = \frac{1}{5} \quad \dots(\text{iv})$$

$$\text{Now, } \frac{P_r}{P_R} = \frac{i^2 r}{i^2 R} \Rightarrow \frac{P_r}{0.5} = \frac{r}{R} \Rightarrow \frac{P_r}{0.5} = \frac{1}{5}$$

$$\Rightarrow P_r = 0.1 \text{ W} = 0.10 \text{ W}$$

Hence, correct option is (d).

- 72** In a building, there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be

[2020, 7 Jan Shift-II]

- (a) 25 A (b) 10 A (c) 20 A (d) 15 A

Ans. (c)

Power of 15 bulbs (each of 45 W),

$$P_1 = 15 \times 45 = 675 \text{ W}$$

Power of 15 bulbs (each of 100 W),

$$P_2 = 15 \times 100 = 1500 \text{ W}$$

Power of 15 fans (each of 10 W),

$$P_3 = 15 \times 10 = 150 \text{ W}$$

Power of 2 heaters (each of 1 kW),

$$P_4 = 2 \times 1000 = 2000 \text{ W}$$

Total power usage of building,

$$P = P_1 + P_2 + P_3 + P_4 = 4325 \text{ W}$$

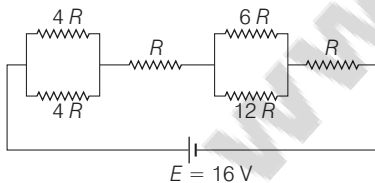
Using $P = VI$, current drawn from mains supply of 220 V,

$$I = \frac{P}{V} = \frac{4325}{220} = 19.66 \text{ A}$$

$\therefore$ Minimum fuse capacity required is 20 A.

- 73** The resistive network shown below is connected to a DC source of 16 V. The power consumed by the network is 4 W. The value of R is

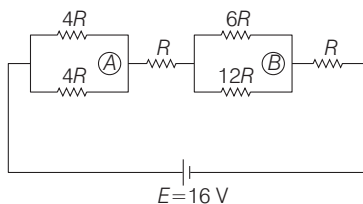
[2019, 12 April Shift-I]



- (a) 6 Ω (b) 8 Ω (c) 1 Ω (d) 16 Ω

Ans. (b)

Given circuit is



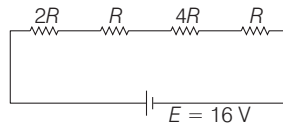
Equivalent resistance of part A,

$$R_A = \frac{4R \times 4R}{4R + 4R} = 2R$$

Equivalent resistance of part B,

$$R_B = \frac{6R \times 12R}{6R + 12R} = \frac{72}{18} R = 4R$$

$\therefore$ Equivalent circuit is



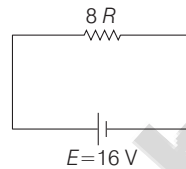
$\therefore$ Total resistance of the given network is

$$R_s = 2R + R + 4R + R = 8R$$

As we know, power of the circuit,

$$P = \frac{E^2}{R_s} = \frac{(16)^2}{8R} = \frac{16 \times 16}{8R} \quad \dots(\text{i})$$

According to question, power consumed by the network, $P = 4 \text{ W}$



From Eq. (i), we get

$$\therefore \frac{16 \times 16}{8R} = 4$$

$$\Rightarrow R = \frac{16 \times 16}{8 \times 4} = 8 \Omega$$

- 74** A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is

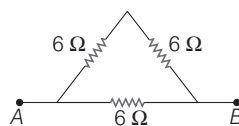
[2019, 10 Jan Shift-II]

- (a) $11 \times 10^{-4} \text{ W}$ (b) $11 \times 10^{-5} \text{ W}$
(c) $11 \times 10^5 \text{ W}$ (d) $11 \times 10^{-3} \text{ W}$

Ans. (b)

Power dissipated by any resistor R , when I current flows through it is,

$$P = I^2 R \quad \dots(\text{i})$$



Given $I = 2 \text{ mA} = 2 \times 10^{-3} \text{ A}$ and $P = 4.4 \text{ W}$

Using Eq. (i), we get

$$4.4 = (2 \times 10^{-3})^2 \times R$$

$$\text{or } R = \frac{4}{4 \times 10^{-6}} \Omega \quad \dots(\text{ii})$$

When this resistance R is connected with 11 V

supply then power dissipated is

$$P = \frac{V^2}{R}$$

$$\text{or } P = \frac{(11)^2}{4.4} \times 4 \times 10^{-6}$$

[$\therefore$ Using Eq. (ii)]

$$\Rightarrow P = \frac{11 \times 11 \times 4 \times 10^{-6}}{4.4 \times 10^{-1}} \text{ W}$$

$$\text{or } P = 11 \times 10^{-5} \text{ W}$$

- 75** Two equal resistances when connected in series to a battery consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be

[2019, 11 Jan Shift-I]

- (a) 60 W (b) 30 W
(c) 240 W (d) 120 W

Ans. (c)

Let P_1 and P_2 be the individual electric powers of the two resistances, respectively.

In series combination, power is

$$P_0 = \frac{P_1 P_2}{P_1 + P_2} = 60 \text{ W}$$

Since, the resistances are equal and the current through each resistor in series combination is also same. Then,

$$P_1 = P_2 = 120 \text{ W}$$

In parallel combination, power is

$$P = P_1 + P_2 = 120 + 120 = 240 \text{ W}$$

Alternate Solution

Let R be the resistance.

$\therefore$ Net resistance in series = $R + R = 2R$

$$P = \frac{V^2}{2R} = 60 \text{ W} \Rightarrow \frac{V^2}{R} = 120 \text{ W}$$

New resistance in parallel = $\frac{R \times R}{R + R} = R/2$

$$P' = \frac{V^2}{R/2} = 2 \left(\frac{V^2}{R} \right) = 240 \text{ W}$$

- 76** Two electric bulbs rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then

[2019, 12 Jan Shift-I]

- (a) $P_1 = 16 \text{ W}, P_2 = 4 \text{ W}$
(b) $P_1 = 4 \text{ W}, P_2 = 16 \text{ W}$
(c) $P_1 = 9 \text{ W}, P_2 = 16 \text{ W}$
(d) $P_1 = 16 \text{ W}, P_2 = 9 \text{ W}$

Ans. (a)

Resistance of a bulb of power P and with a voltage source V is given by

$$R = \frac{V^2}{P}$$

Resistance of the given two bulbs are

$$R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{25}$$

and $R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{100}$

Since, bulbs are connected in series. This means same amount of current flows through them.

$\therefore$ Current in circuit is

$$i = \frac{V}{R_{\text{total}}} = \frac{220}{\frac{(220)^2}{25} + \frac{(220)^2}{100}} = \frac{1}{11} \text{ A}$$

Power drawn by bulbs are respectively,

$$P_1 = i^2 R_1 = \left(\frac{1}{11}\right)^2 \times \frac{220 \times 220}{25} = 16 \text{ W.}$$

and $P_2 = i^2 R_2 = \left(\frac{1}{11}\right)^2 \times \frac{220 \times 220}{100} = 4 \text{ W.}$

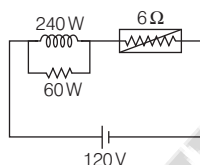
- 77** In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. Then, the minimum current should be **[JEE Main 2014]**

(a) 8 A (b) 10 A (c) 12 A (d) 14 A

Ans. (c)

Total power (P) consumed

$$= (15 \times 40) + (5 \times 100) + (5 \times 80) + (1 \times 1000) = 2500 \text{ W}$$



As, we know that

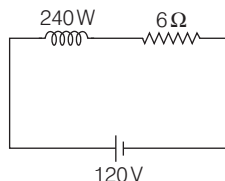
$$P = VI \Rightarrow I = \frac{2500}{200} = \frac{125}{11} = 11.3 \text{ A}$$

Hence, minimum capacity should be 12 A.

- 78** The supply voltage to room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **[JEE Main 2013]**
- (a) Zero (b) 2.9 V (c) 13.3 V (d) 10.04 V

Ans. (d)

$$P = \frac{V^2}{R}$$



Resistance of the bulb,

$$R = \frac{120 \times 120}{240} = 60 \Omega$$

$$R_{\text{eq}} = 240 + 6 = 246 \Omega$$

$$\Rightarrow I_1 = \frac{V}{R_{\text{eq}}} = \frac{120}{246}$$

$$V_1 = I_1 R_1 = \frac{120}{246} \times 240 = 117.073 \text{ V}$$

Resistance of the heater

$$= \frac{V^2}{P} = \frac{120 \times 120}{240} = 60 \Omega$$

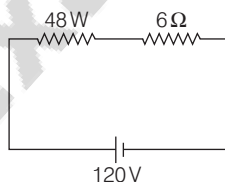
As bulb and heater are connected in parallel.

$$\text{Net resistance} = \frac{240 \times 60}{300} = 48 \Omega$$

$$\text{Total resistance, } R_2 = 48 + 6 = 54 \Omega$$

$$\text{Total current, } I_2 = V / R_2 = 120 / 54$$

Potential across heater = Potential across bulb



$$V_2 = \frac{120}{54} \times 48 = 106.66 \text{ V}$$

$$V_1 - V_2 = 117.073 - 106.66 = 10.04 \text{ V}$$

- 79** Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse? **[AIEEE 2012]**

(a) Both (b) 100 W
(c) 25 W (d) None of these

Ans. (c)

As the rated power of 25 W is less than 100 W, it implies that 25 W bulb has higher resistance. As in series connection, current through both the bulbs is same but heating in 25 W bulb is more than that of 100 W bulb. So, 25 W bulb will get fused.

- 80** An electric bulb is rated 220 V-100 W. The power consumed by it when operated on 110 V will be **[AIEEE 2006]**

(a) 75 W (b) 40 W (c) 25 W (d) 50 W

Ans. (c)

Resistance of electric bulb, $R = \frac{V^2}{P}$, where subscripts denote for rated parameters.

$$\Rightarrow R = \frac{(220)^2}{100}$$

Power consumed at 110 V,

$$P_{\text{consumed}} = \frac{V^2}{R}$$

$$\therefore P_{\text{consumed}} = \frac{(110)^2}{(220)^2 / 100} = 25 \text{ W}$$

- 81** A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **[AIEEE 2005]**

(a) doubled (b) four times
(c) one-fourth (d) halved

Ans. (a)

Is heat generated

$$H_1 = \frac{V^2}{R} t \quad \text{and} \quad H_2 = \frac{V^2}{(R/2)} t$$

$$\therefore \frac{H_2}{H_1} = 2 \quad \text{or} \quad H_2 = 2H_1$$

- 82** The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp, when not in use? **[AIEEE 2005]**
- (a) 40 Ω (b) 20 Ω (c) 400 Ω (d) 200 Ω

Ans. (a)

$$P = V^2 / R$$

$$\therefore R_{\text{hot}} = \frac{V^2}{P} = \frac{200 \times 200}{100}$$

$$= 400 \Omega$$

$$R_{\text{cold}} = \frac{400}{10} = 40 \Omega$$

- 83** The thermistors are usually made of **[AIEEE 2004]**

(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

Ans.(c)

Thermistors are the resistors made up of semiconductors whose resistance decreases with the increase in temperature. This implies that they have negative and high temperature coefficient of resistivity.

They are usually made of metal oxides with high temperature coefficient of resistivity.

- 84** Time taken by a 836 W heater to heat 1 L of water from 10°C to 40°C is [AIEEE 2004]

- (a) 50 s (b) 100 s
(c) 150 s (d) 200 s

Ans.(c)

Let time taken in boiling the water by the heater be t second. Then,

$$Q = ms \Delta T$$

$$\Rightarrow \frac{Pt}{J} = ms \Delta T$$

$$\therefore \frac{836}{4.2} t = 1 \times 1000(40 - 10)$$

$$\text{or } \frac{836}{4.2} t = 1000 \times 30$$

$$\text{or } t = \frac{1000 \times 30 \times 4.2}{836} = 150 \text{ s}$$

- 85** A 220 V-1000 W bulb is connected across a 110 V mains supply. The power consumed will be [AIEEE 2003]

- (a) 750 W (b) 500 W
(c) 250 W (d) 1000 W

Ans. (c)

$$R = \frac{V^2}{P} = \frac{(220)^2}{1000}$$

where, V and P are denoting rated voltage and power, respectively.

$$\therefore P_{\text{consumed}} = \frac{V_{\text{applied}}^2}{R}$$

$$= \frac{110 \times 110}{220 \times 220} \times 1000$$

$$= 250 \text{ W}$$

- 86** A wire when connected to 220 V mains supply has power dissipation P_1 . Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2 . Then, $P_2 : P_1$ is [AIEEE 2002]

- (a) 1 (b) 4
(c) 2 (d) 3

Ans. (b)**Case I**

Using the formula, $P = \frac{V^2}{R}$... (i)

where, R is resistance of wire, V is voltage across wire and P is power dissipation in wire

$$\text{and } R = \frac{\rho l}{A} \quad \dots (ii)$$

From Eqs. (i) and (ii), we have

$$P_1 = \frac{V^2}{\rho l / A}$$

$$\text{or } P_1 = \frac{V^2}{\rho l} \cdot A \quad \dots (iii)$$

Case II

Let R_2 be net resistance of two wires in parallel, then

$$R_2 = \frac{R \times R}{R + R} = \frac{R}{2}$$

where, R is the resistance of half wire.

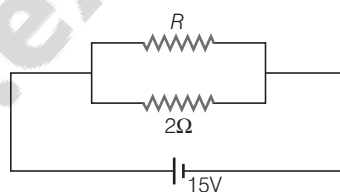
$$\Rightarrow R_2 = \frac{\rho \cdot \left(\frac{l}{2}\right)}{A \cdot 2} = \frac{\rho l}{4A}$$

$$\text{or } P_2 = \frac{V^2}{\rho l} \cdot 4A \quad \dots (iv)$$

Hence, from Eqs. (iii) and (iv), we get

$$\frac{P_1}{P_2} = \frac{1}{4} \Rightarrow \frac{P_2}{P_1} = 4$$

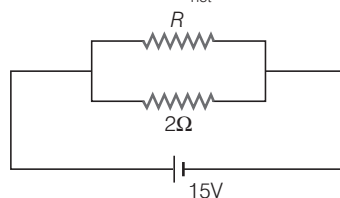
- 87** If in the circuit, power dissipation is 150 W, then R is [AIEEE 2002]



- (a) 2 Ω (b) 6 Ω
(c) 5 Ω (d) 4 Ω

Ans. (b)

$$P = \frac{V^2}{R_{\text{net}}}$$



$$\therefore 150 = \frac{(15)^2}{R \cdot 2}$$

$$\Rightarrow R = 6 \Omega$$

TOPIC 3

Cells and its Combination and Kirchhoff's Rules

- 88** Five identical cells each of internal resistance 1Ω and emf $5V$ are connected in series and in parallel with an external resistance R . For what value of R , current in series and parallel combination will remain the same?

[2021, 27 Aug Shift-I]

- (a) 1Ω (b) 25Ω
(c) 5Ω (d) 10Ω

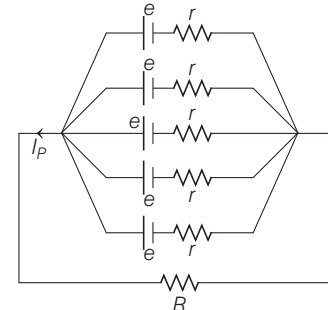
Ans. (a)

Given, number of cells, $n = 5$

Internal resistance of each cell, $r = 1\Omega$

Emf of each cell, $e = 5V$

When all cells are connected in parallel as shown below.



Potential will remain same as, $V_p = 5V$

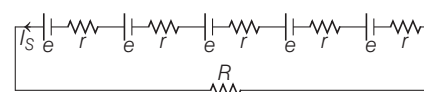
Net resistance in parallel combination will be given as

$$R_p = R + \left[\frac{1}{\frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r}} \right]$$

$$= R + \frac{r}{5}$$

$$R_p = R + \frac{1}{5} \quad (\because r = 1\Omega) \dots (i)$$

When all cells are connected in series as shown below



The net potential will increase as cells are connected in series,

$$V_s = 5 + 5 + 5 + 5 + 5 = 25V$$

The net resistance of circuit will be equivalent of sum of all resistances as all are connected in series.

$$R_s = r + r + r + r + r + R = 5r + R$$

$$R_s = 5 + R \quad (\because r = 1\Omega) \dots (ii)$$

By Ohm's law, current flowing in circuit is given as

$$I = V/R$$

As current in both series and parallel combination is same,

$$\frac{V_p}{R_p} = \frac{V_s}{R_s}$$

$$\Rightarrow \frac{5}{R + \frac{1}{5}} = \frac{25}{5 + R} \quad [\text{From Eqs. (i) and (ii)}]$$

$$\Rightarrow 25 + 5R = 25R + 5 \Rightarrow R = 1 \Omega$$

- 89** In an electric circuit, a cell of certain EMF provides a potential difference of 1.25 V across a load resistance of 5 Ω . However, it provides a potential difference of 1 V across a load resistance of 2 Ω . The emf of the cell is given by $\frac{x}{10}$ V.

Then, the value of x is

[2021, 22 July Shift-II]

Ans. (15)

Let emf of cell = ϵ

Potential difference across the terminal of cell

$$V_1 = 1.25 \text{ V}$$

Load resistance, $R_{L_1} = 5 \Omega$

when load resistance, $R_{L_2} = 2 \Omega$,

then $V_2 = 1 \text{ V}$

I_1, I_2 be the current through load in the two above mentioned cases and r be internal resistance of cell.

As we know that,

By using Kirchhoff's voltage law,

$$\epsilon - V = I_1 R_{L_1}$$

$$\epsilon = I_1 R_{L_1} + V$$

$$\epsilon = I_1 (R_{L_1} + r) \quad [\because V = I_1 r]$$

$$\Rightarrow I_1 = \frac{\epsilon}{R_{L_1} + r} = \frac{\epsilon}{5 + r}$$

$$\therefore V_1 = I_1 R_{L_1}$$

$$\therefore 1.25 = \frac{\epsilon}{5 + r} \times 5$$

$$\Rightarrow \frac{\epsilon}{5 + r} = 0.25 = \frac{25}{100} = \frac{1}{4}$$

$$\Rightarrow 4\epsilon = 5 + r \quad \dots(i)$$

$$\text{and } V_2 = I_2 R_{L_2} = \frac{\epsilon}{2 + r} \times 2 = 1$$

$$\Rightarrow 2\epsilon = 2 + r \quad \dots(ii)$$

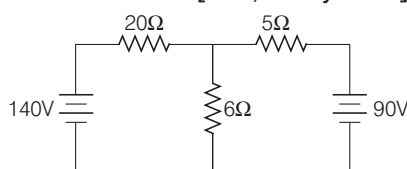
On subtracting Eq. (i) and Eq. (ii), we get

$$2\epsilon = 3$$

$$\epsilon = \frac{3}{2} = 1.5 \text{ V} = \frac{15}{10} \text{ V}$$

Hence, $x = 15$

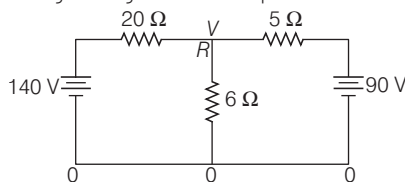
- 90** The value of current in the 6 Ω resistance is [2021, 20 July Shift-I]



- (a) 4 A (b) 8 A
(c) 10 A (d) 6 A

Ans. (c)

The given figure can be represented as



From the above figure, it can be clearly seen that the voltage across point R is assumed as V.

Therefore, applying Kirchhoff's current law at point R, we can write

$$\frac{V - 0}{6} + \frac{V - 90}{5} + \frac{V - 140}{20} = 0$$

$$\Rightarrow \frac{V}{6} + \frac{V - 90}{5} + \frac{V - 140}{20} = 0$$

$$\Rightarrow \frac{10V + 12V - 1080 + 3V - 420}{60} = 0$$

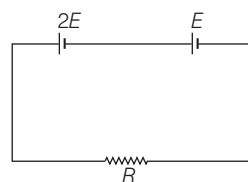
$$\Rightarrow 25V = 1500$$

$$\Rightarrow V = 60 \text{ V}$$

Therefore, current through 6 Ω resistor is

$$I = \frac{V}{R} = \frac{60}{6} = 10 \text{ A}$$

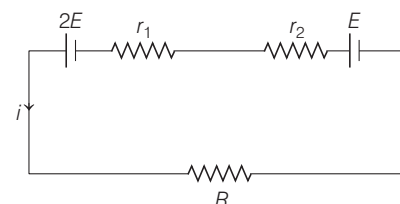
- 91** Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is [2021, 17 March Shift-II]



- (a) $r_1 + r_2$ (b) $\frac{r_1}{2} - r_2$
(c) $\frac{r_1}{2} + r_2$ (d) $r_1 - r_2$

Ans. (b)

The given circuit can be drawn as



Since in series combination, the current through each resistance remains same. So, equivalent resistance of the circuit is given as

$$R_{\text{equivalent}} = R + r_1 + r_2$$

and equivalent emf,

$$E_{\text{equivalent}} = 2E + E = 3E$$

$$\text{From Ohm's law, } I = \frac{E_{\text{equivalent}}}{R_{\text{equivalent}}}$$

$$\Rightarrow I = \frac{3E}{R + r_1 + r_2}$$

When potential difference is zero across the first cell, then potential positive terminal is equal to the potential at negative terminal.

$$V_P = V_N$$

$$2E = I r_1$$

Substituting the values in the above equation, we get

$$2E = \frac{3E}{R + r_1 + r_2} r_1$$

$$2R + 2r_1 + 2r_2 = 3r_1$$

$$\Rightarrow R = \frac{r_1 - 2r_2}{2}$$

$$\Rightarrow R = \frac{r_1}{2} - r_2$$

- 92** In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is J. [26 Feb 2021 Shift-I]

Ans. (300)

Given, charge passing through circuit, $q = 20 \text{ C}$

Potential difference between two plates,

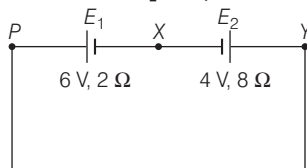
$$V = 15 \text{ V}$$

Let W be the amount of work done by battery.

$$\therefore W = qV = 20 \times 15 = 300 \text{ J}$$

- 93** A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is

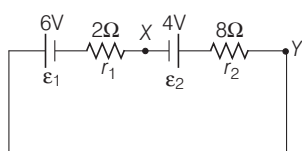
[2021, 24 Feb Shift-I]



- (a) 2.0 V (b) 3.6 V (c) 5.6 V (d) 10.0 V

Ans. (c)

The circuit can be shown as below



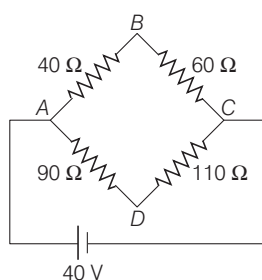
The current through the circuit,

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2} = \frac{6 - 4}{10} = \frac{1}{5} \text{ A}$$

∴ Potential difference across points X and Y is

$$V_{XY} = E_2 + Ir_2 = 4 + \frac{1}{5} \times 8 = 5.6 \text{ V}$$

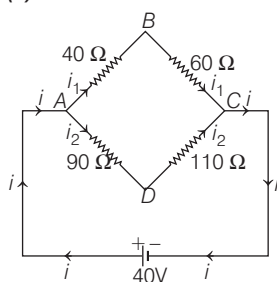
94



Four resistances 40Ω , 60Ω , 90Ω and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40 V and internal resistance negligible. The potential difference across BD (in volt) is

..... [2020, 4 Sep Shift-II]

Ans. (2)



The resistors 40Ω and 60Ω are connected in series combination. Similarly, the resistors 90Ω and 110Ω are also connected in series combination.

$$\text{So, } i_1 = \frac{V_{AC}}{R_{AB} + R_{BC}} = \frac{40}{40 + 60} = \frac{40}{100} = \frac{2}{5} \text{ A}$$

$$\text{and } i_2 = \frac{V_{AC}}{R_{AD} + R_{DC}} = \frac{40}{90 + 110} = \frac{40}{200} = \frac{1}{5} \text{ A}$$

For path BAD, using KVL (Kirchhoff's voltage law),

$$V_B + i_1 \times 40 - i_2 \times 90 = V_D$$

$$V_B + \frac{2}{5} \times 40 - \frac{1}{5} \times 90 = V_D$$

$$V_B + 16 - 18 = V_D$$

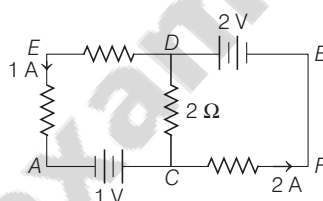
$$V_B - 2 = V_D$$

$$V_B - V_D = 2 \text{ V}$$

So, the potential difference across BD (in volt) is 2.

- 95** In the given circuit, currents in different branches and value of one resistor are shown. Then, potential at point B with respect to the point A is

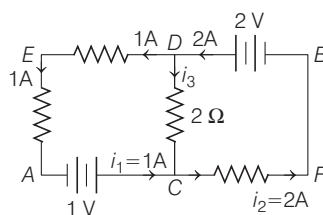
[2020, 5 Sep Shift-II]



- (a) +2 V (b) -2 V
(c) -1 V (d) +1 V

Ans. (d)

The given circuit can be drawn as,



Current in branch DC (using KCL at point C),

$$i_1 + i_3 = i_2$$

$$2 - 1 - i_3 = 0$$

$$i_3 = 1 \text{ A}$$

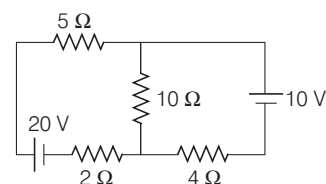
Now, while moving from A to B via C and D, the potential along ACDB,

$$V_A + 1 + 2 \times i_3 - 2 = V_B$$

$$\Rightarrow V_B - V_A = 1 \text{ V}$$

Hence, correct option is (d).

96



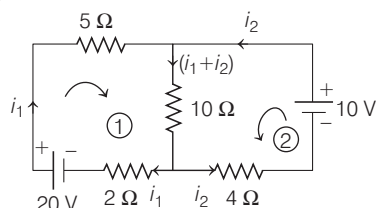
In above figure shown, the current in the 10 V battery is close to

[2020, 6 Sep Shift-II]

- (a) 0.71 A from positive to negative terminal
(b) 0.42 A from positive to negative terminal
(c) 0.21 A from positive to negative terminal
(d) 0.36 A from negative to positive terminal

Ans. (c)

Assume the current in branches as shown in figure,



Applying KVL in loop 1,
 $+20 - 5i_1 - 10(i_1 + i_2) - 2i_1 = 0$

$$17i_1 + 10i_2 = 20 \quad \dots(i)$$

Applying KVL in loop 2,
 $+10 - 10(i_1 + i_2) - 4i_2 = 0$

$$5i_1 + 7i_2 = 5 \quad \dots(ii)$$

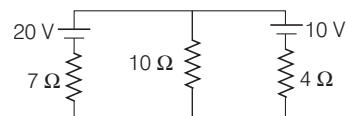
On solving Eqs. (i) and (ii), we get

$$i_2 = -0.214 \text{ A or } i_2 \approx -0.21 \text{ A}$$

It means the direction of current is from positive to negative terminal of battery inside it.

Alternate solution

The circuit can be reduced to

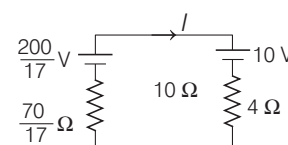


Using voltage division rule, voltage across 10Ω resistor,

$$V' = \frac{20 \times 10}{10 + 7} = \frac{200}{17} \text{ V}$$

and resistance, $R' = \frac{7 \times 10}{7 + 10} = \frac{70}{17} \Omega$

Now, circuit becomes



$$\therefore I = \frac{V_{eq}}{R_{eq}} = \frac{\frac{200}{17} - 10}{\frac{70}{17} + 4} = \frac{30}{138} \text{ A} = 0.21 \text{ A}$$

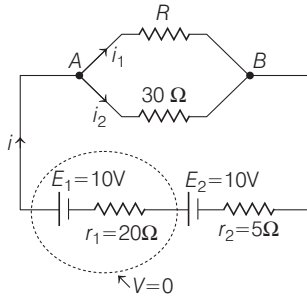
Hence, correct option is (c).

- 97** The series combination of two batteries, both of the same emf 10 V, but different internal resistance of 20Ω and 5Ω , is connected to the parallel combination of two resistors 30Ω and $R\Omega$. The voltage difference across the battery of internal resistance 20Ω is zero, the value of R (in Ω) is

[2020, 8 Jan Shift-II]

Ans. (30)

Given arrangement of batteries and resistances is shown below.



Let i = circuit current, then it is given that potential difference across battery of 10V and 20Ω is zero.

$$\text{i.e. } E_1 - ir_1 = 0 \Rightarrow 10 - i(20) = 0 \\ \Rightarrow i = \frac{1}{2} = 0.5 \text{ A}$$

Now, potential drop across combination of resistors,

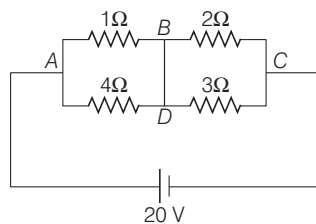
$$V_{AB} = (E_1 + E_2) - i(r_1 + r_2) \\ = 20 - 0.5 \times (25) = 7.5 \text{ V}$$

Now, at junction B,

$$i = i_1 + i_2 \Rightarrow 0.5 = \frac{7.5}{R} + \frac{7.5}{30}$$

On solving, we get, $R = 30\Omega$

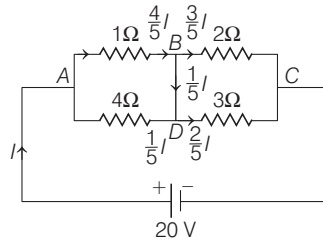
- 98** In the given circuit diagram, a wire is joining points B and D. The current in this wire is [2020, 9 Jan Shift-I]



- (a) 0.4 A (b) zero (c) 2 A (d) 4 A

Ans. (c)

In given circuit, current distribution following Kirchhoff's law will be as shown in the figure.



Since, the current flows in inverse ratio of the resistance of branch.

Now, total circuit resistance,

$$R_{eq} = (1\Omega \parallel 4\Omega) + (2\Omega \parallel 3\Omega) \\ = \left(\frac{1 \times 4}{1+4} \right) + \left(\frac{2 \times 3}{2+3} \right) \\ = \frac{4}{5} + \frac{6}{5} \\ = \frac{10}{5} \Omega = 2\Omega$$

So, current drawn from cell,

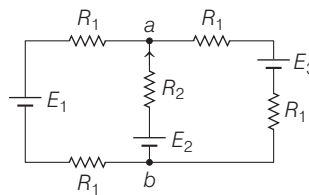
$$I = \frac{V}{R_{eq}} = \frac{20\text{V}}{2\Omega} = 10 \text{ A}$$

Hence, current through BD arm is (refer to circuit diagram),

$$I_{BD} = \frac{I}{5} = \frac{10}{5} \\ = 2 \text{ A}$$

- 99** For the circuit shown with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2 \text{ V}$ and $E_2 = E_3 = 4 \text{ V}$, the potential difference between the points a and b is approximately (in volt)

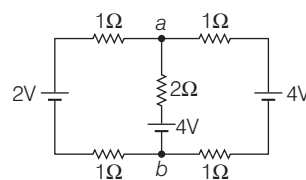
[2019, 8 April Shift-I]



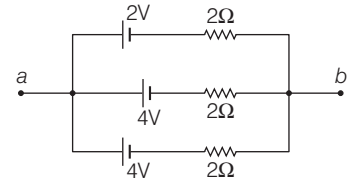
- (a) 2.7 (b) 2.3
(c) 3.7 (d) 3.3

Ans. (d)

Given circuit is



Above circuit can be viewed as



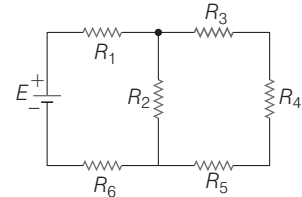
This is a parallel combination of three cells or in other words, a parallel grouping of three cells with internal resistances.

$$\text{So, } V_{ab} = E_{eq} = \frac{E_1 + E_2 + E_3}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} \\ = \frac{4 + 4 + 4}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{12}{\frac{3}{2}} = 8 \text{ V}$$

- 100** In the figure shown, what is the current (in ampere) drawn from the battery? You are given

$$R_1 = 15\Omega, R_2 = 10\Omega, R_3 = 20\Omega, \\ R_4 = 5\Omega, R_5 = 25\Omega, R_6 = 30\Omega, \\ E = 15 \text{ V}$$

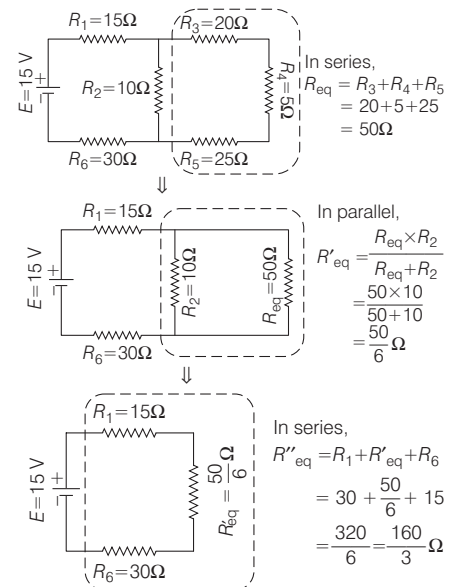
[2019, 8 April Shift-II]



- (a) 13/24 (b) 7/18 (c) 20/3 (d) 9/32

Ans. (d)

Given circuit is redrawn and can be simplified as



So, current drawn through cell is

$$i = \frac{\text{Voltage}}{\text{Net resistance of the circuit}}$$

$$= \frac{V}{R'_{eq}}$$

$$= \frac{15}{(160/3)}$$

$$= \frac{9}{32} \text{ A}$$

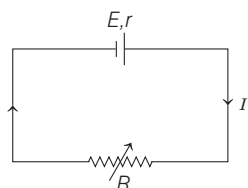
- 101** A cell of internal resistance r drives current through an external resistance R . The power delivered by the cell to the external resistance will be maximum when

[2019, 8 April Shift-II]

- (a) $R = 2r$ (b) $R = r$
(c) $R = 0.001r$ (d) $R = 1000r$

Ans. (b)

Given circuit is shown in the figure below



Net current, $I = \frac{E}{R+r}$... (i)

Power across R is given as

$$P = I^2 R = \left(\frac{E}{R+r} \right)^2 \cdot R \quad [\text{using Eq. (i)}]$$

For the maximum power,

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \frac{dP}{dR} = \frac{d}{dR} \left(\left(\frac{E}{R+r} \right)^2 \cdot R \right)$$

$$= E^2 \frac{d}{dR} \left(\frac{R}{(R+r)^2} \right)$$

$$= E^2 \left[\frac{(R+r)^2 \times 1 - 2R \times (R+r)}{(R+r)^4} \right] = 0$$

$$\Rightarrow (R+r)^2 = 2R(R+r)$$

$$\text{or } R+r = 2R$$

$$\Rightarrow r = R$$

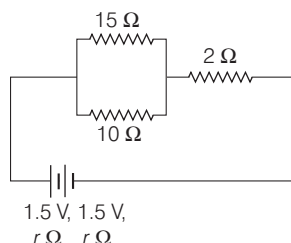
$\therefore$ The power delivered by the cell to the external resistance is maximum when $R = r$.

Alternate Solution

From maximum power theorem, power dissipated will be maximum when internal resistance of source will be equals to external load resistance, i.e. $r = R$.

- 102** In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2 V. The internal resistance r , of each cell is

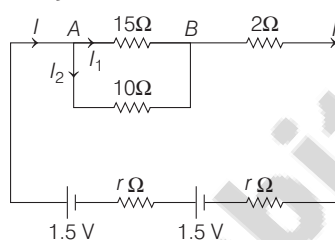
[2019, 10 April Shift-I]



- (a) 1.5Ω (b) 0.5Ω (c) 1Ω (d) 0Ω

Ans. (b)

For the given circuit



Given, $V_{AB} = 2V$

$\therefore$ Current in circuit,

$$I = I_1 + I_2$$

$$= \frac{2}{15} + \frac{2}{10} \quad [\because V = IR \text{ or } I = V/R]$$

$$= \frac{4+6}{30} = \frac{1}{3} \text{ A} \quad \dots (i)$$

Also, voltage drop across $(r+r)$ resistors is

$$= \text{voltage of the cell}$$

$$- \text{voltage drop across } AB$$

$$= 3 - 2 = 1V$$

Using $V = IR$ over the entire circuit

$$\Rightarrow 1 = I(2+2r)$$

$$= \frac{1}{3}(2+2r) \quad [\text{using Eq. (i)}]$$

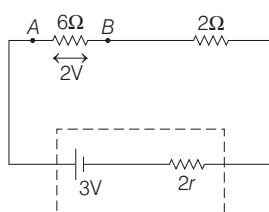
$$\Rightarrow 3 = 2+2r \text{ or } 2r = 1 \Omega$$

$$\text{or } r = \frac{1}{2} \Omega = 0.5 \Omega$$

Alternate Solution

Equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6} \Rightarrow R_{AB} = 6 \Omega$$



$\therefore$ Equivalent resistance of the entire circuit is, $R_{eq} = 6 \Omega + 2 \Omega + 2r = 8 + 2r$

Now, current passing through the circuit is given

$$\text{as, } I = \frac{E_{net}}{R + r_{eq}} = \frac{E_{net}}{R_{eq}}$$

where, R is external resistance, r_{eq} is net internal resistance and E_{net} is the emf of the cells.

$$\text{Here, } E_{net} = 1.5 + 1.5 = 3V$$

$$r_{eq} = r + r = 2r \Rightarrow I = \frac{3}{8+2r}$$

Also, reading of the voltmeter,

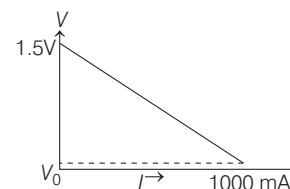
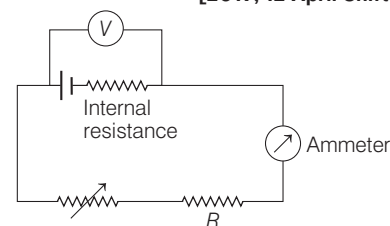
$$V = 2V = I \cdot R_{AB}$$

$$2 = \left(\frac{3}{8+2r} \right) \times 6 \Rightarrow 8+2r = 9$$

$$\text{or } r = \frac{1}{2} = 0.5 \Omega$$

- 103** To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current and the following graph is obtained

[2019, 12 April Shift-I]



If V_0 is almost zero, then identify the correct statement.

- (a) The emf of the battery is 1.5 V and its internal resistance is 1.5Ω
(b) The value of the resistance R is 1.5Ω
(c) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA
(d) The emf of the battery is 1.5 V and the value of R is 1.5Ω

Ans. (a)

Given circuit in a series combination of internal resistance of cell (r) and external resistance R .

$\therefore$ Effective resistance in the circuit,

$$R_{eff} = r + R$$

∴ Current in the circuit,

$$I = \frac{E}{R+r} \text{ or } E = IR + Ir$$

Voltage difference across resistance R is V , so

$$E = V + Ir \quad \dots(i)$$

Now, from graph at $I = 0$, $V = 1.5$ V

From Eq. (i) at $I = 0$, $E = V = 1.5$ V $\dots(ii)$

At $I = 1000$ mA (or 1 A), $V = 0$

From Eq. (i) at $I = 1$ A and $V = 0$

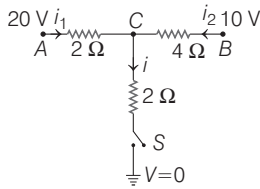
$$\Rightarrow E = I \times r = r \quad \dots(iii)$$

From Eqs. (ii) and (iii), we can get

$$r = E = V = 1.5 \text{ V}$$

$$\therefore r = 1.5 \Omega$$

- 104** When the switch S in the circuit shown is closed, then the value of current i will be



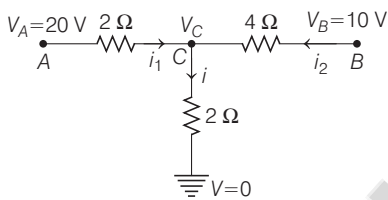
[2019, 9 Jan Shift-I]

- (a) 4 A
(c) 2 A

- (b) 3 A
(d) 5 A

Ans. (d)

When the switch 'S' is closed the circuit, hence formed is given in the figure below,



Then, according to Kirchhoff's current law, which states that the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point. Since, in the above circuit, that point is 'C',

$$\therefore i_1 + i_2 = i$$

$$\Rightarrow \frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - 0}{2}$$

(∵ using Ohm's law, $V = iR$)

$$\text{or } \frac{20 - V_C}{2} + \frac{10 - V_C}{4} = \frac{V_C - 0}{2}$$

$$\Rightarrow 20 - V_C + (10 - V_C)2 = V_C$$

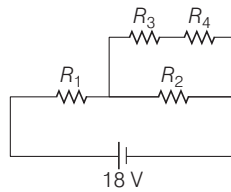
$$\Rightarrow 40 = V_C + 3V_C$$

$$40 = 4V_C \text{ or } V_C = 10 \text{ V}$$

$$\therefore \text{The current, } i = \frac{V_C}{2} = \frac{10}{2} = 5 \text{ A}$$

- 105** In the given circuit, the internal resistance of the 18 V cell is negligible. If $R_1 = 400 \Omega$, $R_3 = 100 \Omega$ and $R_4 = 500 \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be

[2019, 9 Jan Shift-II]

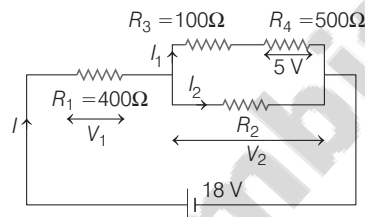


- (a) 550 Ω
(c) 300 Ω

- (b) 230 Ω
(d) 450 Ω

Ans. (c)

According to question, the voltage across R_4 is 5 volt, then the current across it



According to Ohm's law,

$$\Rightarrow V = IR$$

$$\Rightarrow 5 = I_4 \times R_4$$

$$\Rightarrow 5 = I_4 \times 500$$

$$I_4 = \frac{5}{500} = \frac{1}{100} \text{ A}$$

The potential difference across series combination of R_3 and R_4

$$\Rightarrow V_2 = (R_3 + R_4)I = 600 \times \frac{1}{100} = 6 \text{ V}$$

So, potential difference (across R_1)

$$V_1 = 18 - 6 = 12 \text{ V}$$

Current through R_1 is,

$$I = \frac{V_1}{R_1} = \frac{12}{400} = \frac{3}{100} \text{ A}$$

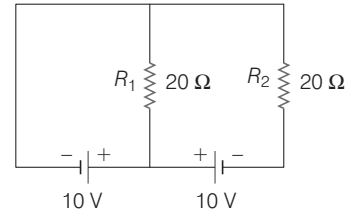
So current through R_2 is,

$$I_2 = I - I_1 = \frac{3}{100} - \frac{1}{100} \text{ A} = \frac{2}{100} \text{ A}$$

Now, from $V = IR$, we have,

$$R_2 = \frac{V_2}{I_2} = \frac{6}{(2/100)} = 300 \Omega.$$

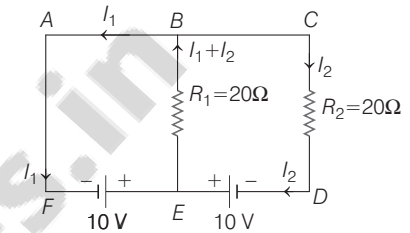
- 106** In the given circuit, the cells have zero internal resistance. The currents (in Ampere) passing through resistances R_1 and R_2 respectively are [2019, 10 Jan Shift-I]



- (a) 0.5, 0 (b) 1, 2 (c) 2, 2 (d) 0, 1

Ans. (a)

By Kirchhoff's loop rule in the given loop ABEFA, we get



$$10 - (I_1 + I_2)R_1 = 0$$

$$\Rightarrow 10 - (I_1 + I_2)20 = 0 \text{ or } I_1 + I_2 = \frac{1}{2} \dots(i)$$

and from loop BCDEB, we get

$$10 - (I_1 + I_2)R_1 - I_2R_2 = 0$$

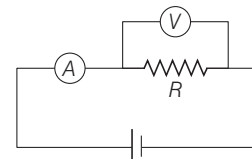
$$\Rightarrow 10 - (I_1 + I_2)20 = 0$$

$$\Rightarrow I_1 + 2I_2 = \frac{1}{2} \dots(ii)$$

From Eqs. (i) and (ii), we get

$$I_2 = 0 \text{ and } I_1 = 0.5 \text{ A}$$

- 107** The actual value of resistance R , shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is [2019, 10 Jan Shift-II]



- (a) 600 Ω (b) 570 Ω (c) 350 Ω (d) 35 Ω

Ans. (b)

Measured value of $R = 5\%$ less than actual value of R .

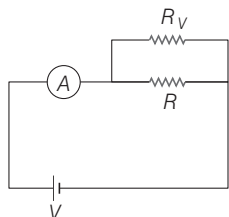
Actual values of $R = 30 \Omega$

So, measured value of R is

$$R' = 30 - (5\% \text{ of } 30) = 30 - \frac{5}{100} \times 30$$

$$\Rightarrow R' = 28.5 \Omega \quad \dots(i)$$

Now, let us assume that internal resistance of voltmeter R_V . Replacing voltmeter with its internal resistance, we get following circuit.



It is clear that the measured value, R' should be equal to parallel combination of R and R_V . Mathematically,

$$R' = \frac{RR_V}{R + R_V} = 28.5 \Omega$$

Given, $R = 30 \Omega \Rightarrow \frac{30R_V}{30 + R_V} = 28.5$

$$\Rightarrow 30R_V = (28.5 \times 30) + 28.5 R_V$$

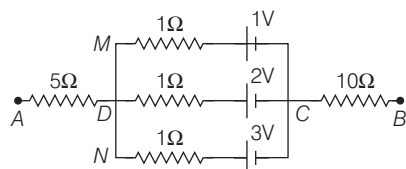
$$\Rightarrow 1.5R_V = 28.5 \times 30$$

$$\Rightarrow R_V = \frac{28.5 \times 30}{1.5} = 19 \times 30$$

or $R_V = 570 \Omega$

108 In the circuit shown, the potential difference between A and B is

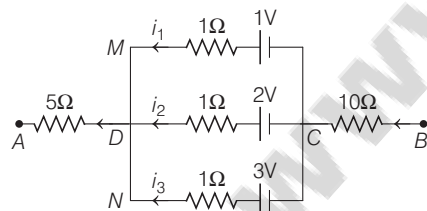
[2019, 11 Jan Shift-II]



- (a) 3 V (b) 1 V (c) 6 V (d) 2 V

Ans. (d)

In the given circuit, let's assume currents in the arms are i_1, i_2 and i_3 , respectively.



Now, $i_1 = \frac{V_1}{R_1} = \frac{1}{1} = 1 \text{ A}$

Similarly, $i_2 = \frac{2}{1} = 2 \text{ A}$ and $i_3 = \frac{3}{1} = 3 \text{ A}$

Total current in the arm DA is

$$i = i_1 + i_2 + i_3 = 6 \text{ A}$$

As all three resistors between D and C are in parallel.

$\therefore$ Equivalent resistance between terminals D and C is

$$\frac{1}{R_{DC}} = \left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1} \right)$$

$$\therefore R_{DC} = \frac{1}{3} \Omega$$

So, potential difference across D and C is

$$V_{DC} = iR_{DC} = 6 \times \frac{1}{3} \Rightarrow V_{DC} = 2 \text{ V}$$

Now, V_{AD} and $V_{CB} = 0$

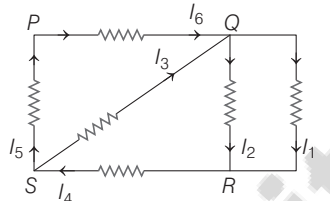
(In case of open circuits, $I = 0$)

So, $V_{AB} = V_{AD} + V_{DC} + V_{CB} = V_{DC}$

So, $V_{AB} = 2 \text{ V}$

109 In the given circuit diagram, the currents $I_1 = -0.3 \text{ A}$, $I_4 = 0.8 \text{ A}$ and $I_5 = 0.4 \text{ A}$, are flowing as shown. The currents I_2, I_3 and I_6 respectively, are

[2019, 12 Jan Shift-II]



(a) 1.1 A, 0.4 A, 0.4 A

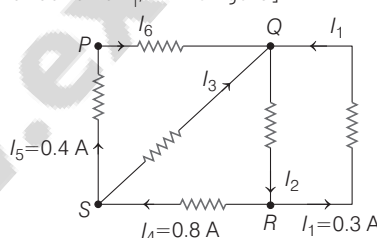
(b) 1.1 A, -0.4 A, 0.4 A

(c) 0.4 A, 1.1 A, 0.4 A

(d) -0.4 A, 0.4 A, 1.1 A

Ans. (a)

Given circuit with currents as shown in the figure below, [In the question $I_1 = 0.3 \text{ A}$ is given, due to it we change the direction of I_1 , in this figure]



From Kirchoff's junction rule, $\Sigma I = 0$

At junction S, $I_4 = I_5 + I_3$

$$\Rightarrow 0.8 = 0.4 + I_3 \Rightarrow I_3 = 0.4 \text{ A}$$

At junction P, $I_5 = I_6 \Rightarrow I_6 = 0.4 \text{ A}$

$$\text{At junction Q, } I_2 = I_1 + I_3 + I_6 = 0.3 + 0.4 + 0.4 = 1.1 \text{ A}$$

110 Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10Ω . The internal resistances of the two batteries are 1Ω and 2Ω , respectively. The voltage across the load lies between

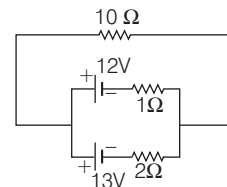
[JEE Main 2018]

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V

(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

Ans. (b)

For parallel combination of cells,



$$E_{eq} = \frac{E_1 + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} \therefore E_{eq} = \frac{12 + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3} \text{ V}$$

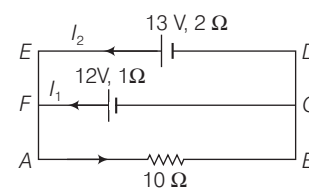
Potential drop across 10Ω resistance,

$$V = \left(\frac{E}{R_{total}} \right) \times 10 = \left(\frac{37/3}{10 + \frac{2}{3}} \right) \times 10$$

$$= 11.56 \text{ V}$$

$$\therefore V = 11.56 \text{ V}$$

Alternate Solution



Applying KVL, in loop ABCFA,

$$-12 + 10(I_1 + I_2) + 1 \times I_1 = 0$$

$$\Rightarrow 12 = 11I_1 + 10I_2 \quad \dots(i)$$

Similarly,

In loop ABDEA,

$$-13 + 10(I_1 + I_2) + 2 \times I_2 = 0$$

$$\Rightarrow 13 = 10I_1 + 12I_2 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

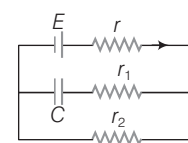
$$I_1 = \frac{7}{16} \text{ A, } I_2 = \frac{23}{32} \text{ A}$$

$\therefore$ Voltage drop across 10Ω resistance is

$$V = 10 \left(\frac{7}{16} + \frac{23}{32} \right) = 11.56 \text{ V}$$

111 In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be

[JEE Main 2017]



(a) $CE \frac{r_1}{(r_2 + r)}$

(b) $CE \frac{r_2}{(r + r_2)}$

(c) $CE \frac{r_1}{(r_1 + r)}$

(d) CE

Ans. (b)

In steady state, no current flows through the capacitor. So, resistance r_p becomes ineffective.

So, the current in circuit,

$$I = \frac{E}{r + r_2 \text{ (Total Resistance)}}$$

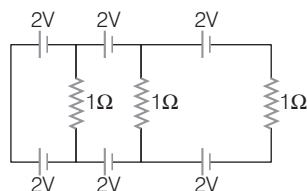
∴ Potential drop across capacitor = Potential drop

$$\text{across } r_2 = Ir_2 = \frac{Er_2}{r + r_2}$$

∴ Stored charge of capacitor,

$$Q = CV = CE \frac{r_2}{r + r_2}$$

112 In the below circuit, the current in each resistance is [JEE Main 2017]

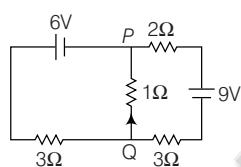


- (a) 0.25 A (b) 0.5 A (c) 0 A (d) 1 A

Ans. (c)

Each resistance is converted with two cells combined in opposite direction, so potential drop across each resistor is zero. Hence, the current through each of resistor is zero.

113 In the circuit shown below, the current in the 1Ω resistor is [JEE Main 2015]

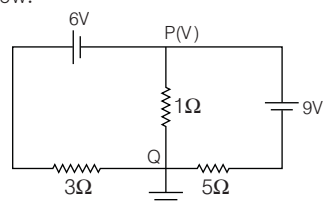


- (a) 1.3 A, from P to Q
(b) 0 A
(c) 0.13 A, from Q to P
(d) 0.13 A, from P to Q

Ans. (c)

Key Idea Connect point Q to ground and apply KCL.

Consider the grounded circuit as shown below.



Applying KCL of point Q we can write

Incoming current at Q = outgoing current from Q

$$\Rightarrow \frac{V+6}{3} + \frac{V}{1} = \frac{9-V}{5}$$

$$\text{or } V \left[\frac{1}{3} + \frac{1}{5} + 1 \right] = \frac{9}{5} - 2$$

$$\text{or } V \left[\frac{5+3+15}{15} \right] = \frac{9-10}{5}$$

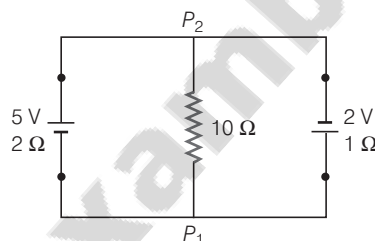
$$\text{or } V \left[\frac{23}{15} \right] = \frac{-1}{5}$$

$$\text{or } V = \frac{-3}{23}$$

$$= -0.13 \text{ V}$$

Thus, current in the 1Ω resistance is 0.13 A, from Q to P.

114 A 5 V battery with internal resistance 2Ω and a 2 V battery with internal resistance 1Ω are connected to a 10Ω resistor as shown in the figure.



The current in the 10Ω resistor is

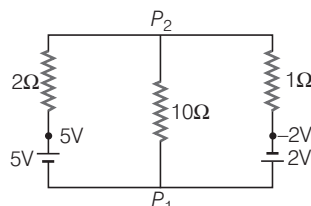
[AIEEE 2008]

- (a) 0.27 A, P_2 to P_1 (b) 0.03 A, P_1 to P_2
(c) 0.03 A, P_2 to P_1 (d) 0.27 A, P_1 to P_2

Ans. (c)

Let potential at P_1 be 0 V and potential at P_2 be V_0 .

Now apply KCL at P_2 ,
 $\Sigma I = 0$



$$\frac{V_0 - 5}{2} + \frac{V_0 - 0}{10} + \frac{V_0 - (-2)}{1} = 0$$

$$\text{or } V_0 = \frac{5}{16}$$

So, current through 10Ω resistor is $\frac{V_0}{10}$ from P_2 to P_1 .

115 The Kirchhoff's first law ($\Sigma i = 0$) and second law ($\Sigma iR = \Sigma E$) where, the symbols have their usual meanings, are respectively based on

[AIEEE 2006]

- (a) conservation of charge, conservation of momentum
(b) conservation of energy, conservation of charge
(c) conservation of momentum, conservation of charge
(d) conservation of charge, conservation of energy

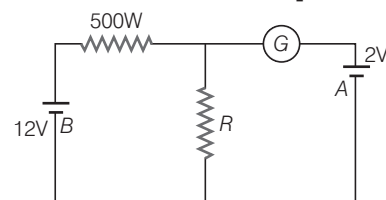
Ans. (d)

Kirchhoff's 1st law or KCL states that the algebraic sum of current meeting at any junction is equal to zero. In other words, we can say that "The sum of all the currents directed towards a junction in a circuit is equal to the sum of all the currents directed away from that junction."

Thus, no charge has been accumulated at any junction i.e., charge is conserved and hence we can say that KCL ($\Sigma i = 0$) is based on conservation of charge.

Kirchhoff's 2nd law or KVL states that algebraic sum of changes in potential around any closed resistor loop must be zero. In other words, "Around any closed loop, voltage drops are equal to voltage rises". No energy is gained or lost in circulating a charge around a loop, thus we can say that KVL is based on conservation of energy.

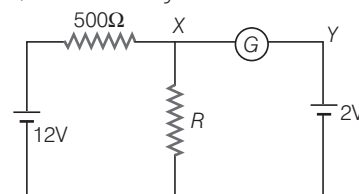
116 In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be [AIEEE 2005]



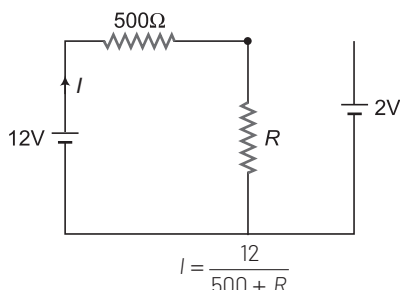
- (a) 200Ω (b) 100Ω (c) 500Ω (d) 1000Ω

Ans. (b)

The galvanometer shows zero deflection i.e., current through XY is zero



As a result potential drop across R is 2 V.
Circuit can be redrawn as



Voltage across R , $V = IR$

$$\Rightarrow 2 = \frac{12}{500 + R} \times R$$

$$\text{or } 1000 + 2R = 12R$$

$$\text{or } R = 100 \Omega$$

- 117** Two sources of equal emf are connected to an external resistance R . The internal resistances of the two sources are R_1 and R_2 , ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then [AIEEE 2005]

$$(a) R = \frac{R_2 \times (R_1 + R_2)}{(R_2 - R_1)}$$

$$(b) R = R_2 - R_1$$

$$(c) R = \frac{R_1 R_2}{(R_1 + R_2)}$$

$$(d) R = \frac{R_1 R_2}{(R_2 - R_1)}$$

Ans. (b)

As R_1 , R_2 and R in series.

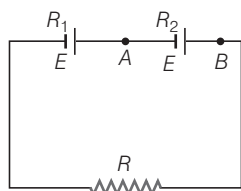
$$R_{eq} = R_1 + R_2 + R$$

$$\therefore \text{Net current, } I = \frac{2E}{R_1 + R_2 + R}$$

According to the question,

$$-(V_A - V_B) = E - IR_2$$

$$\therefore 0 = E - IR_2$$



$$\text{or } E = IR_2 \text{ or } E = \frac{2E}{R_1 + R_2 + R} R_2$$

$$\text{or } R_1 + R_2 + R = 2R_2 \text{ or } R = R_2 - R_1$$

- 118** An energy source will supply a constant current into the load, if its internal resistance is [AIEEE 2005]

- (a) equal to the resistance of the load
(b) very large as compared to the load resistance
(c) zero
(d) non-zero but less than the resistance of the load

Ans. (c)

$$I = \frac{E}{R + r}$$

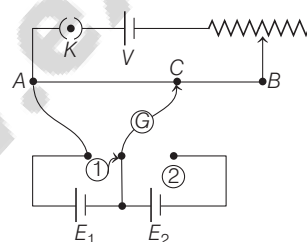
$$I = \frac{E}{R} = \text{constant}$$

where, R = external resistance

r = internal resistance = 0

TOPIC 4 Measuring Instruments

- 119** In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the EMF of two cells, $\frac{\epsilon_1}{\epsilon_2}$ is [2021, 25 July Shift-II]



- (a) $\frac{5}{3}$ (b) $\frac{8}{5}$
(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

Ans. (a)

Given, balancing length for emf (ϵ_1),

$$l_1 = 250 \text{ cm} = 250 \times 10^{-2} \text{ m}$$

and for balancing emf ($\epsilon_1 + \epsilon_2$), length,

$$l_2 = 400 \text{ cm} = 400 \times 10^{-2} \text{ m}$$

As we know that,

$$\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{\epsilon_1}{\epsilon_1 + \epsilon_2} = \frac{250 \times 10^{-2}}{400 \times 10^{-2}}$$

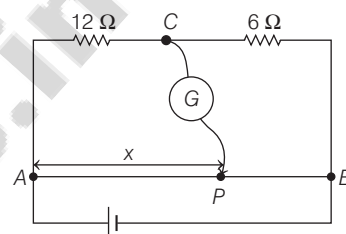
$$\Rightarrow \frac{\epsilon_1}{\epsilon_1 + \epsilon_2} = \frac{250}{400} = \frac{5}{8}$$

$$\Rightarrow 8\epsilon_1 = 5\epsilon_1 + 5\epsilon_2$$

$$\Rightarrow 3\epsilon_1 = 5\epsilon_2$$

$$\Rightarrow \frac{\epsilon_1}{\epsilon_2} = \frac{5}{3}$$

- 120** Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A . The galvanometer shows zero deflection.



The value of x , to the nearest integer, is

[2021, 18 March Shift-II]

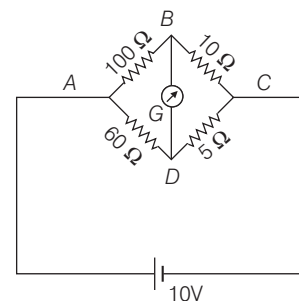
Ans. (48)

At the balanced condition of the Wheatstone bridge,

$$\frac{R}{S} = \frac{L_1}{L - L_1} \Rightarrow \frac{12}{6} = \frac{x}{72 - x} \Rightarrow x = 48 \text{ cm}$$

$\therefore$ The galvanometer jockey is placed at P on AB at a distance of 48 cm from the A .
So, the value of the x to the nearest integer is 48.

- 121** The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD . Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC . [2021, 17 March Shift-II]



- (a) 2.44 μ A (b) 2.44 mA
(c) 4.87 μ A (d) 4.87 mA

Ans. (c)

As, A is directly connected to the positive terminal of the battery, $V_A = 10\text{V}$ and $V_C = 0$

By nodal analysis at B,

$$\frac{V_B - 10}{100} + \frac{V_B - V_D}{15} + \frac{V_B - 0}{10} = 0$$

$$53V_B - 20V_D = 30 \quad \dots (i)$$

By nodal analysis at D,

$$\frac{V_D - 10}{60} + \frac{V_D - V_B}{15} + \frac{V_D - 0}{5} = 0$$

$$-4V_B + 17V_D = 10 \quad \dots (ii)$$

Solving Eqs. (i) and (ii) by substitution method, we get

$$V_D = 0.792\text{V} \Rightarrow V_B = 0.865\text{V}$$

The current through the galvanometer,

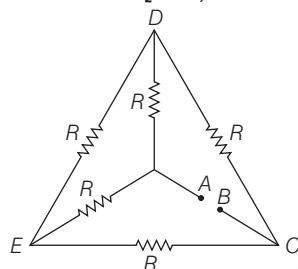
$$I = \frac{V_B - V_D}{R}$$

Substituting the values in the above equation, we get

$$I = \frac{0.865 - 0.792}{15}$$

$$\Rightarrow I = 4.87\text{mA}$$

- 122** Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is

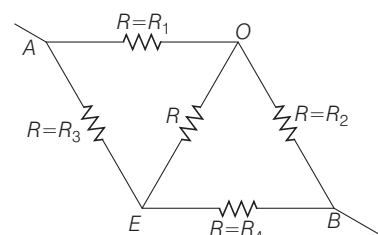
[2021, 26 Feb Shift-I]

- (a) $2R$ (b) $\frac{R}{2}$ (c) $\frac{3R}{2}$ (d) R

Ans. (d)

Given all resistances have same resistance R .

Now, we can redraw the circuit as below



Let resistances be R_1, R_2, R_3 and R_4 .

$$\therefore \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

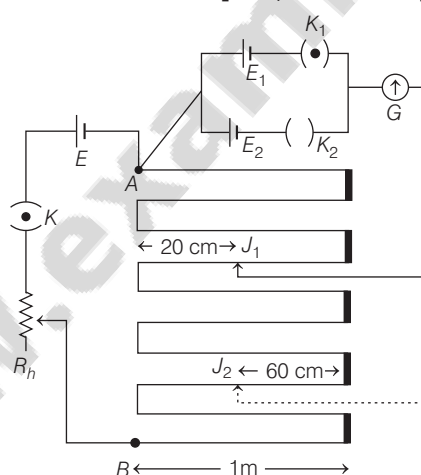
So, circuit will behave as a Wheatstone bridge and no current will flow through middle resistor.

$$\therefore R_{eq} = \frac{(R_1 + R_2)(R_3 + R_4)}{(R_1 + R_2) + (R_3 + R_4)}$$

$$= \frac{(R + R)(R + R)}{(R + R) + (R + R)}$$

$$= R$$

- 123** In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 , so that there is no deflection in the galvanometer. Now, the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2 . The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where $a = \dots\dots\dots$

[2021, 25 Feb Shift-I]**Ans. (1)**

Given, length of AB = 10 m = 1000 cm

$$\text{Length of one arm} = \frac{1000}{10} = 100\text{ cm}$$

For no deflection,

$$\text{In first case, } l_1 = 3 \times 100 + 80 = 380\text{ cm}$$

$$\text{In 2nd case, } l_2 = 7 \times 100 + 60 = 760\text{ cm}$$

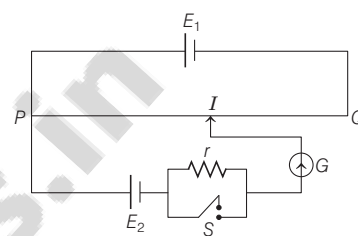
As we know that in balanced potentiometer,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{a}{b} = \frac{380}{760} = \frac{1}{2}$$

$$\therefore a = 1$$

- 124** A potentiometer wire PQ of 1 m length is connected to a standard cell E_1 . Another cell E_2 of emf 1.02 V is connected with a resistance r and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is

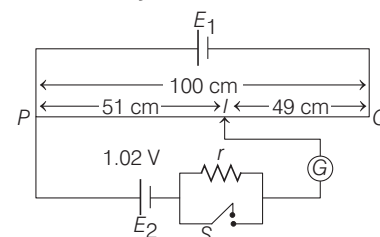
[2020, 2 Sep Shift-II]

- (a) 0.02 V/cm (b) 0.01 V/cm
(c) 0.03 V/cm (d) 0.04 V/cm

Ans. (a)

Resistance r limits current through E_2 when there is no balance situation.

But at balance point no current flows through galvanometer G and E_2 , so r does not affect the position of balance point as shown in figure.



Now, 1.02 V is balanced against 51 cm length, so potential gradient of wire PQ is,

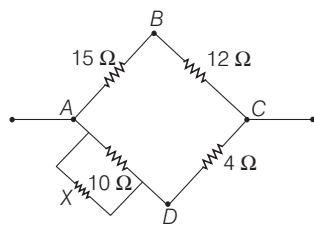
$$\text{Potential gradient} = \frac{\text{Fall of potential}}{\text{unit length}} = \frac{1.02\text{ V}}{51\text{ cm}} = 0.02\text{ V/cm}$$

Hence, correct option is (a).

- 125** Four resistances of 15 Ω , 12 Ω , 4 Ω and 10 Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10 Ω to balance the network is $\dots\dots\dots \Omega$.

[2020, 8 Jan Shift-I]**Ans. (10)**

Cyclic order and resistance $X \Omega$ which is connected to obtain balance condition is as shown in the figure



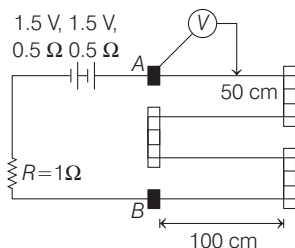
For balance,

$$\frac{15}{12} = \frac{\left(\frac{10X}{10+X}\right)}{4} \Rightarrow \frac{10X}{10+X} = 5$$

$$\Rightarrow X = 10 \Omega$$

- 126** In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01 \Omega/\text{cm}$. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be

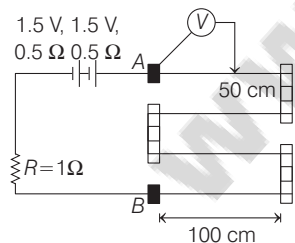
[2019, 8 April Shift-II]



- (a) 0.20 V (b) 0.75 V
(c) 0.25 V (d) 0.50 V

Ans. (c)

In given potentiometer, resistance per unit length is $x = 0.01 \Omega \text{ cm}^{-1}$.



Length of potentiometer wire is
 $L = 400 \text{ cm}$

Net resistance of the wire AB is

$R_{AB} = \text{resistance per unit length} \times \text{length of AB} = 0.01 \times 400$

$$\Rightarrow R_{AB} = 4 \Omega$$

Net internal resistance of the cells connected in series,

$$r = 0.5 + 0.5 = 1 \Omega$$

$\therefore$ Current in given potentiometer circuit is

$$I = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{\text{Net emf}}{r + R + R_{AB}}$$

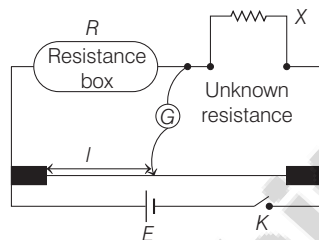
$$= \frac{3}{1 + 1 + 4} = 0.5 \text{ A}$$

Reading of voltmeter when the jockey is at 50 cm (l') from one end A,

$$V = IR = I(xl') = 0.5 \times 0.01 \times 50 = 0.25 \text{ V}$$

- 127** In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure

[2019, 10 April Shift-I]



S. No.	$R(\Omega)$	$l(\text{cm})$
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

- (a) 3 (b) 2 (c) 1 (d) 4

Ans. (d)

Unknown resistance 'X' in meter bridge experiment is given by

$$X = \left(\frac{100 - l}{l} \right) R$$

Case I When $R = 1000 \Omega$ and $l = 60 \text{ cm}$, then

$$X = \frac{(100 - 60)}{60} \times 1000 = \frac{40 \times 1000}{60}$$

$$\Rightarrow X = \frac{2000}{3} \Omega \approx 667 \Omega$$

Case II When $R = 100 \Omega$ and $l = 13 \text{ cm}$, then

$$X = \left(\frac{100 - 13}{13} \right) \times 100 = \frac{100 \times 87}{13}$$

$$= \frac{8700}{13} \Omega \approx 669 \Omega$$

Case III When $R = 10 \Omega$ and $l = 1.5 \text{ cm}$, then

$$X = \left(\frac{100 - 1.5}{1.5} \right) \times 10$$

$$= \frac{98.5}{1.5} \times 10 = \frac{9850}{15} \Omega \approx 656 \Omega$$

Case IV When $R = 1 \Omega$ and $l = 1.0 \text{ cm}$, then

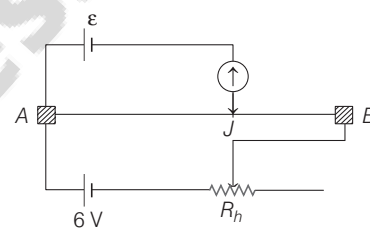
$$X = \left(\frac{100 - 1}{1} \right) \times 1$$

$$\therefore X = 99 \Omega$$

Thus, from the above cases, it can be concluded that, value calculated in case (IV) is inconsistent.

- 128** The resistance of the meter bridge AB in given figure is 4Ω . With a cell of emf $\epsilon = 0.5 \text{ V}$ and rheostat resistance $R_h = 2 \Omega$. The null point is obtained at some point J. When the cell is replaced by another one of emf $\epsilon = \epsilon_2$, the same null point J is found for $R_h = 6 \Omega$. The emf ϵ_2 is

[2019, 11 Jan Shift-I]



- (a) 0.6 V (b) 0.3 V
(c) 0.5 V (d) 0.4 V

Ans. (b)

Let length of null point 'J' be 'x' and length of the potentiometer wire be 'L'.

In first case, current in the circuit

$$I_1 = \frac{6}{4 + 2} = 1 \text{ A}$$

$\therefore$ Potential gradient,

$$= I \times R = \frac{1 \times 4}{L}$$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{1 \times 4}{L} \times x = \epsilon_1$$

$$\text{Given, } \epsilon_1 = 0.5 = \frac{4x}{L}$$

$$\text{or } \frac{x}{L} = \frac{1}{8} \quad \dots(i)$$

In second case, current in the circuit

$$I_2 = \frac{6}{4 + 6} = 0.6 \text{ A}$$

$\therefore$ Potential gradient = $\frac{0.6 \times 4}{L}$

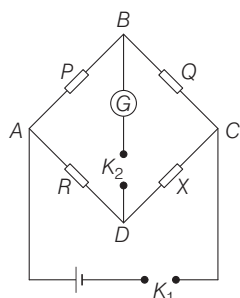
$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{0.6 \times 4}{L} \times x = \epsilon_2$$

$$\Rightarrow \epsilon_2 = \frac{0.6 \times 4}{L} \times \frac{L}{8} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow \epsilon_2 = 0.3 \text{ V}$$

- 129** In a Wheatstone bridge (see figure), resistances P and Q are approximately equal. When $R = 400\Omega$, the bridge is balanced. On interchanging P and Q , the value of R for balance is 405Ω . The value of X is close to [2019, 11 Jan Shift-I]



- (a) 404.5Ω (b) 401.5Ω
(c) 402.5Ω (d) 403.5Ω

Ans. (c)

For a balanced Wheatstone bridge,

$$\frac{P}{R} = \frac{Q}{X}$$

In first case when $R = 400\Omega$, the balancing equation will be

$$\frac{P}{R} = \frac{Q}{X}$$

$$\Rightarrow \frac{P}{400\Omega} = \frac{Q}{X}$$

$$\Rightarrow P = \frac{400 \times Q}{X} \quad \dots(i)$$

In second case, P and Q are interchanged and $R = 405\Omega$

$$\therefore \frac{Q}{R} = \frac{P}{X}$$

$$\Rightarrow \frac{Q}{405} = \frac{P}{X} \quad \dots(ii)$$

Substituting the value of P from Eq. (i) in Eq. (ii), we get

$$\frac{Q}{405} = \frac{Q \times 400}{X^2}$$

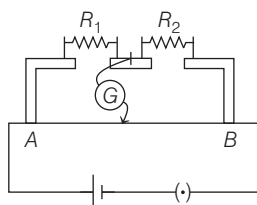
$$\Rightarrow X^2 = 400 \times 405$$

$$\Rightarrow X = \sqrt{400 \times 405} = 402.5$$

The value of X is close to 402.5Ω .

- 130** In the experimental set up of meter bridge shown in the figure, the null point is obtained at a distance of 40 cm from A . If a 10Ω resistor is connected in series with R_1 , the null point shifts by 10 cm .

The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is [2019, 11 Jan Shift-II]



- (a) 60Ω (b) 20Ω (c) 30Ω (d) 40Ω

Ans. (a)

For meter bridge, if balancing length is

$$l\text{ cm, then in first case, } \frac{R_1}{R_2} = \frac{l}{(100-l)}$$

It is given that, $l = 40\text{ cm}$;

$$\text{So; } \frac{R_1}{40} = \frac{R_2}{100-40} \text{ or } \frac{R_1}{R_2} = \frac{2}{3} \dots(i)$$

In second case, $R'_1 = R_1 + 10$, and balancing length is now 50 cm then

$$\frac{R_1 + 10}{50} = \frac{R_2}{(100-50)}$$

$$\text{or } R_1 + 10 = R_2 \dots(ii)$$

Substituting value of R_2 from (ii) to (i) we get,

$$\text{or } \frac{R_1}{10 + R_1} = \frac{2}{3}$$

$$\Rightarrow 3R_1 = 20 + 2R_1 \text{ or } R_1 = 20\Omega$$

$$\Rightarrow R_2 = 30\Omega$$

Let us assume the parallel connected resistance is x .

Then equivalent resistance is $\frac{x(R_1 + 10)}{x + R_1 + 10}$

$$x + R_1 + 10$$

So, this combination should be again equal to R_1 .

$$\frac{(R_1 + 10)x}{R_1 + 10 + x} = R_1 \Rightarrow \frac{30x}{30 + x} = 20$$

$$\text{or } 30x = 600 + 20x \text{ or } x = 60\Omega$$

- 131** An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5Ω . The value of R to give a potential difference of 5 mV across 10 cm of potentiometer wire is [2019, 12 Jan Shift-I]

- (a) 395Ω (b) 495Ω (c) 490Ω (d) 480Ω

Ans. (a)

Given, potential difference of 5 mV is across 10 m length of potentiometer wire. So potential drop per unit length is

$$= \frac{5 \times 10^{-3}}{10 \times 10^{-2}} = 5 \times 10^{-2} \left(\frac{\text{V}}{\text{m}} \right)$$

Hence, potential drop across 1 m length of potentiometer wire is

$$V_{AB} = 5 \times 10^{-2} \left(\frac{\text{V}}{\text{m}} \right) \times 1 = 5 \times 10^{-2}\text{ V}$$

Now, potential drop that must occur across resistance R is

$$V_R = 4 - 5 \times 10^{-2} = \frac{395}{100}\text{ V}$$

$$\text{Now, circuit current is } i = \frac{V}{R_{\text{total}}} = \frac{4}{R + 5}$$

Hence, for resistance R , using $V_R = iR$, we get

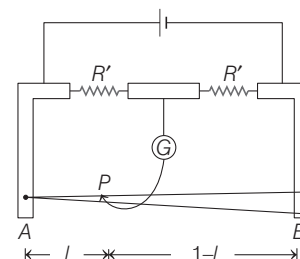
$$\frac{395}{100} = \frac{4}{R + 5} \times R$$

$$395(R + 5) = 400R$$

$$395 \times 5 = (400 - 395)R$$

$$\Rightarrow R = 395\Omega$$

- 132** In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that the variation $\frac{dR}{dl}$ of its resistance R with length l is $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$. Two equal resistance are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P . What is the length AP ? [2019, 12 Jan Shift-I]



- (a) 0.3 m (b) 0.25 m (c) 0.2 m (d) 0.35 m

Ans. (b)

As, galvanometer shows zero deflection. This means, the meter bridge is balanced.

$$\frac{R'}{R_{AP}} = \frac{R'}{R_{PB}} \Rightarrow R_{AP} = R_{PB} \dots(i)$$

Now, for meter bridge wire, $\frac{dR}{dl} = \frac{k}{\sqrt{l}}$

where, 'k' is the constant of proportionality.

$$\Rightarrow dR = \frac{k}{\sqrt{l}} dl$$

Integrating both sides, we get

$$\Rightarrow R = \int \frac{k}{\sqrt{l}} dl$$

$$\text{So, } R_{AP} = \int_0^l \frac{k}{\sqrt{l}} dl = k(2\sqrt{l}) \Big|_0^l = 2k\sqrt{l}$$

$$\text{and } R_{PB} = \int_l^1 \frac{k}{\sqrt{l}} dl = 2k(\sqrt{l}) \Big|_l^1 = 2k(\sqrt{1} - \sqrt{l}) = 2k(1 - \sqrt{l})$$

Substituting values of R_{AP} and R_{PB} in Eq. (i), we get

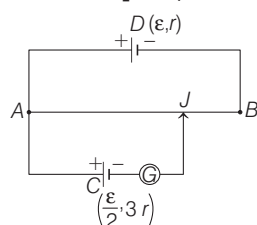
$$\Rightarrow \frac{R_{AP}}{R_{PB}} = \frac{R_{PB}}{R_{AP}} \Rightarrow \frac{2k\sqrt{l}}{2k(1 - \sqrt{l})} = \frac{2k(1 - \sqrt{l})}{2k\sqrt{l}}$$

$$\Rightarrow \sqrt{l} = \frac{1}{2} \text{ or } l = \frac{1}{4} = 0.25 \text{ m}$$

- 133** A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of EMF ϵ and internal resistance r . A cell C having emf $\frac{\epsilon}{2}$

and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is

[2019, 10 Jan Shift-I]



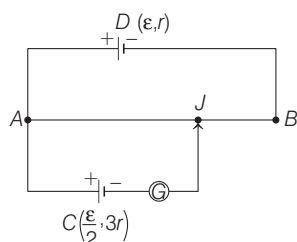
- (a) $\frac{5}{12}L$ (b) $\frac{11}{12}L$
(c) $\frac{13}{24}L$ (d) $\frac{11}{24}L$

Ans. (c)

Given, length of potentiometer wire (AB) = L

Resistance of potentiometer wire (AB) = $12r$

EMF of cell D of potentiometer = ϵ



Internal resistance of cell ' D ' = r

$$\text{EMF of cell 'C'} = \frac{\epsilon}{2}$$

Internal resistance of cell ' C ' = $3r$

Current in potentiometer wire

$$i = \frac{\text{EMF of cell of potentiometer}}{\text{Total resistance of potentiometer circuit}}$$

$$\Rightarrow i = \frac{\epsilon}{r + 12r} = \frac{\epsilon}{13r}$$

Potential drop across the balance length AJ of potentiometer wire is

$$V_{AJ} = i \times R_{AJ}$$

$$\Rightarrow V_{AJ} = i (\text{Resistance per unit length of potentiometer wire} \times \text{length } AJ)$$

$$\Rightarrow V_{AJ} = i \left(\frac{12r}{L} \times x \right)$$

where, x = balance length AJ .

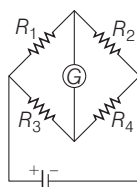
As null point occurs at J so potential drop across balance length AJ = EMF of the cell ' C '.

$$\Rightarrow V_{AJ} = \frac{\epsilon}{2}$$

$$\Rightarrow i \left(\frac{12r}{L} \times x \right) = \frac{\epsilon}{2}$$

$$\Rightarrow \frac{\epsilon}{13r} \times \frac{12r}{L} \times x = \frac{\epsilon}{2} \Rightarrow x = \frac{13}{24}L$$

- 134** The Wheatstone bridge shown in figure here, gets balanced when the carbon resistor is used as R_1 has the color code (orange, red, brown). The resistors R_2 and R_4 are 80Ω and 40Ω , respectively.



Assuming that the color code for the carbon resistors gives their accurate values, the color code for the carbon resistor is used as R_3 would be [2019, 10 Jan Shift-II]

- (a) brown, blue, black
(b) brown, blue, brown
(c) grey, black, brown
(d) red, green, brown

Ans. (b)

The value of R_1 (orange, red, brown)

$$\Rightarrow = 32 \times 10 = 320 \Omega$$

Given, $R_2 = 80 \Omega$ and $R_4 = 40 \Omega$

In balanced Wheatstone bridge condition,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\Rightarrow R_3 = R_4 \times \frac{R_1}{R_2}$$

$$\Rightarrow R_3 = \frac{40 \times 320}{80}$$

$$\text{or } R_3 = 160 \Omega = 16 \times 10^1$$

Comparing the value of R_3 with the colours assigned for the carbon resistor, we get

$$R_3 = 16 \times 10^1$$

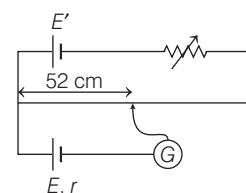
Brown Blue Brown

- 135** In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω , a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. [JEE Main 2018]

- (a) 1Ω (b) 1.5Ω (c) 2Ω (d) 2.5Ω

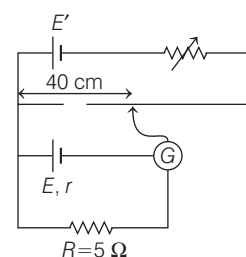
Ans. (b)

With only the cell,



On balancing, $E = 52 \times x$... (i)
where, x is the potential gradient of the wire.

When the cell is shunted,



Similarly, on balancing,

$$V = E - \frac{Er}{(R + r)} = 40 \times x \text{ ... (ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{E}{V} = \frac{1}{1 - \frac{r}{R + r}} = \frac{52}{40}$$

$$\Rightarrow \frac{E}{V} = \frac{R+r}{R} = \frac{52}{40} \Rightarrow \frac{5+r}{5} = \frac{52}{40}$$

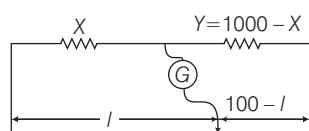
$$\Rightarrow r = \frac{3}{2} \Omega \quad r = 1.5 \Omega$$

- 136** On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 k Ω . How much was the resistance on the left slot before interchanging the resistances?

[JEE Main 2018]

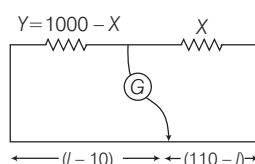
- (a) 990 Ω (b) 505 Ω (c) 550 Ω (d) 910 Ω

Ans. (c)

We have, $X + Y = 1000 \Omega$ 

Initially, $\frac{X}{l} = \frac{1000 - X}{100 - l}$... (i)

When X and Y are interchanged, then



$$\frac{1000 - X}{l - 10} = \frac{X}{100 - (l - 10)}$$

or $\frac{1000 - X}{l - 10} = \frac{X}{110 - l}$... (ii)

From Eqs. (i) and (ii), we get

$$\frac{100 - l}{l} = \frac{l - 10}{110 - l}$$

$$(100 - l)(110 - l) = (l - 10)l$$

$$11000 - 100l - 110l + l^2 = l^2 - 10l$$

$$\Rightarrow 11000 = 200l$$

$$\therefore l = 55 \text{ cm}$$

Substituting the value of l in Eq. (i), we get

$$\frac{X}{55} = \frac{1000 - 55}{100 - 55} \Rightarrow 20X = 11000$$

$$\therefore X = 550 \Omega$$

- 137** Which of the following statements is false? [JEE Main 2017]

- (a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed
(b) A rheostat can be used as a potential divider
(c) Kirchhoff's second law represents energy conservation

- (d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

Ans. (a)

In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

- 138** The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are $4 \times 10^{-7} \Omega\text{-m}$ and $8 \times 10^{-7} \text{m}^2$, respectively. The potential gradient will be equal to

[AIEEE 2011]

- (a) 0.2 V/m
(c) 0.3 V/m

- (b) 1 V/m
(d) 0.1 V/m

Ans. (d)

Potential gradient of a potentiometer,

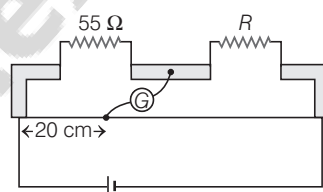
$$K = \frac{V}{l}$$

$$K = \frac{l\rho}{A} = \frac{0.2 \times 4 \times 10^{-7}}{8 \times 10^{-7}}$$

$$= 0.1 \text{ V/m}$$

- 139** Shown in the figure adjacent is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is

[AIEEE 2008]



- (a) 13.75 Ω (b) 220 Ω
(c) 110 Ω (d) 55 Ω

Ans. (b)

When bridge is balanced, ratio of resistance in opposite arms is equal.

$$\frac{P}{Q} = \frac{R}{S}$$

$$\text{Given, } P = 55 \Omega, Q = R$$

$$\text{As } R \propto l_1$$

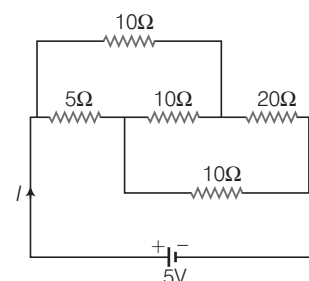
$$R \propto l_1, \text{ where } l_1 = 20 \text{ cm}$$

$$S \propto l_2, \text{ where } l_2 = 100 - 20 = 80 \text{ cm}$$

$$\frac{R}{S} = \frac{l_1}{l_2} = \frac{20}{80}$$

$$\therefore \frac{55}{R} = \frac{20}{80} \text{ or } R = 220 \Omega$$

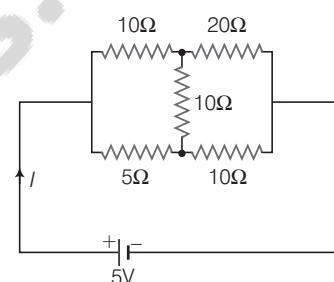
- 140** The current I drawn from the 5 V source will be [AIEEE 2006]



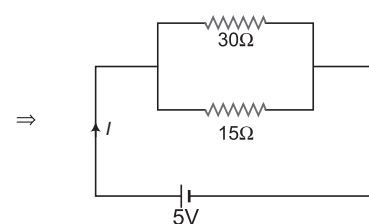
- (a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

Ans. (b)

The given circuit can be redrawn as



Which is a balanced Wheatstone's bridge and hence no current flows in the middle resistor, so equivalent circuit would be as shown below.



$$30 \Omega \parallel 15 \Omega = 10 \Omega$$

$$I = \frac{V}{R} = \frac{5}{10} = 0.5 \text{ A}$$

- 141** In a Wheatstone's bridge, three resistances P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be [AIEEE 2006]

$$(a) \frac{P}{Q} = \frac{2R}{S_1 + S_2} \quad (b) \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

$$(c) \frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2} \quad (d) \frac{P}{Q} = \frac{R}{S_1 + S_2}$$

Ans. (b)

For balanced Wheatstone's bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\text{Here, } S = S_1 \parallel S_2 = \frac{S_1 S_2}{S_1 + S_2}$$

$$\Rightarrow \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

- 142** In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2Ω , the balancing length becomes 120 cm. The internal resistance of the cell is

[AIEEE 2005]

- (a) 1Ω (b) 0.5Ω
(c) 4Ω (d) 2Ω

Ans. (d)

The internal resistance of the cell,

$$r = \left(\frac{l_1 - l_2}{l_2} \right) R = \left(\frac{240 - 120}{120} \right) \times 2 = 2 \Omega$$

- 143** In a meter bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y . If $X < Y$, then

where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y ?

[AIEEE 2004]

- (a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm

Ans. (a)

Meter bridge is an arrangement which works on Wheatstone's principle, so the balancing condition is

$$\frac{R}{S} = \frac{l_1}{l_2}$$

where, $l_2 = 100 - l_1$ **Case I** $R = X, S = Y, l_1 = 20 \text{ cm},$

$$l_2 = 100 - 20 = 80 \text{ cm}$$

$$\therefore \frac{X}{Y} = \frac{20}{80} \quad \dots(i)$$

Case II Let the position of null point be obtained at a distance l from same end.

$$\therefore R = 4X, S = Y,$$

$$l_1 = l, l_2 = 100 - l$$

So, from Eq. (i), we get

$$\frac{4X}{Y} = \frac{l}{100 - l}$$

$$\frac{X}{Y} = \frac{l}{4(100 - l)} \quad \dots(ii)$$

Therefore, from Eqs. (i) and (ii), we get

$$\frac{l}{4(100 - l)} = \frac{20}{80}$$

$$\Rightarrow \frac{l}{4(100 - l)} = \frac{1}{4}$$

$$\text{or } l = 100 - l \text{ or } 2l = 100$$

$$\text{Hence, } l = 50 \text{ cm}$$

- 144** The length of a wire of a potentiometer is 100 cm and the emf of its stand and cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5Ω . If the balance point is obtained at $l = 30 \text{ cm}$ from the positive end, the emf of the battery is

[AIEEE 2003]

$$(a) \frac{30E}{100.5}$$

$$(b) \frac{30E}{100 - 0.5}$$

$$(c) \frac{30(E - 0.5I)}{100}, \text{ where } I \text{ is the current in the potentiometer wire}$$

$$(d) \frac{30E}{100}$$

Ans. (d)

$$\therefore \frac{V}{E} = \frac{l}{L}$$

where, l = balance point distance L = length of potentiometer wire

$$\text{or } V = \frac{l}{L} E$$

$$\text{or } V = \frac{30 \times E}{100} = \frac{30}{100} E$$

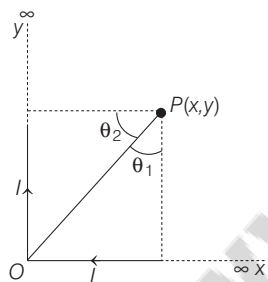
18

Moving Charges and Magnetism

TOPIC 1

Biot Savart's Law and Amperes Circuital Law

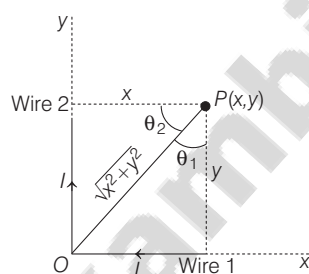
- 01** There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductors is 1 : 1. The magnetic field at point P is [2021, 1 Sep Shift-II]



- (a) $\frac{\mu_0 I}{4\pi xy} [\sqrt{x^2 + y^2} + (x + y)]$
 (b) $\frac{\mu_0 I}{4\pi xy} [\sqrt{x^2 + y^2} - (x + y)]$
 (c) $\frac{\mu_0 I xy}{4\pi} [\sqrt{x^2 + y^2} - (x + y)]$
 (d) $\frac{\mu_0 I xy}{4\pi} [\sqrt{x^2 + y^2} + (x + y)]$

Ans. (a)

The diagram is given below,



The magnetic field due to the wire 1,

$$B_{\text{wire 1}} = \frac{\mu_0 I}{4\pi y} (\sin 90^\circ + \sin \theta_1)$$

$$= \frac{\mu_0 I}{4\pi y} \left(1 + \frac{x}{\sqrt{x^2 + y^2}} \right)$$

The magnetic field due to the wire 2,

$$= \frac{\mu_0 I}{4\pi x} (\sin 90^\circ + \sin \theta_2)$$

$$B_{\text{wire 2}} = \frac{\mu_0 I}{4\pi x} \left(1 + \frac{y}{\sqrt{x^2 + y^2}} \right)$$

Thus, the total magnetic field,

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I}{4\pi} \left(\frac{1}{y} + \frac{x}{y\sqrt{x^2 + y^2}} + \frac{1}{x} + \frac{y}{x\sqrt{x^2 + y^2}} \right)$$

$$= \frac{\mu_0 I}{4\pi xy} [\sqrt{x^2 + y^2} + (x + y)]$$

- 02** A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b , respectively. Find the magnetic field at centre, when a current I passes through coil

[2021, 31 Aug Shift-I]

- (a) $\frac{\mu_0 I N}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 I}{8} \ln\left[\frac{a+b}{a-b}\right]$
 (c) $\frac{\mu_0 I}{4(a-b)} \ln\left[\frac{1}{a} - \frac{1}{b}\right]$ (d) $\frac{\mu_0 I}{8} \ln\left(\frac{a-b}{a+b}\right)$

Ans. (a)

Given, number of turns = N

Inner radius = a , Outer radius = b

Current = I

Consider an element of thickness dr at a distance r from the centre.

Now, number of turns in dr element,

$$dN = \frac{N}{(b-a)} dr$$

Magnetic field due to this element at the

$$\text{centre, } dB = \frac{\mu_0 dN \cdot I}{2r}$$

$$\therefore dB = \frac{1}{2} \frac{\mu_0 I}{r} \frac{N}{(b-a)} dr$$

$\therefore$ Total magnetic field at the centre,

$$B = \int_a^b dB = \int_a^b \frac{\mu_0 I N}{2(b-a)} \frac{dr}{r}$$

$$= \frac{N}{2} \frac{\mu_0 I}{(b-a)} \ln r \Big|_a^b = \frac{N}{2} \frac{\mu_0 I}{(b-a)} \ln\left(\frac{b}{a}\right)$$

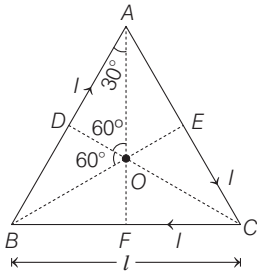
- 03** A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is

(Assume that, the current is flowing in the clockwise direction.) [2021, 31 Aug Shift-II]

- (a) 3×10^{-7} T, outside the plane of triangle
 (b) $2\sqrt{3} \times 10^{-7}$ T, outside the plane of triangle
 (c) $2\sqrt{3} \times 10^{-5}$ T, inside the plane of triangle
 (d) 3×10^{-5} T, inside the plane of triangle

Ans. (d)

Given, electric current, $I = 1.5$ A
 Side of triangle, $l = 9$ cm $= 9 \times 10^{-2}$ m
 Let $OD = OE = OF = r$
 $AD = \frac{l}{2}$



In $\triangle AOD$, $\frac{OD}{AD} = \tan 30^\circ \Rightarrow OD = AD \tan 30^\circ$

$$\therefore r = OD = \frac{l}{2} \times \frac{1}{\sqrt{3}} = \frac{l}{2\sqrt{3}}$$

Magnetic field due to AB at O,

$$B_1 = \frac{\mu_0 I}{4\pi r} [\sin 60^\circ + \sin 60^\circ]$$

$$= \frac{\mu_0 I}{4\pi r} \times \frac{2\sqrt{3}}{2}$$

$$\Rightarrow B_1 = \frac{\mu_0 I \sqrt{3}}{4\pi r} \quad (\text{perpendicular inward})$$

Now, total magnetic field at O due to all sides,

$$B = 3B_1 = \frac{\mu_0 I}{4\pi r} 3\sqrt{3}$$

By substituting the values, we get

$$B = \frac{10^{-7} \times 1.5 \times 3\sqrt{3} \times 2\sqrt{3}}{9 \times 10^{-2}} = 3 \times 10^{-5} \text{ T}$$

(inside the plane of triangle)

- 04** A co-axial cable consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c , respectively. The inner wire carries an electric current i_0 ,

which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) $x < a$ and (ii) $a < x < b$?

[2021, 27 Aug Shift-II]

- (a) $\frac{x^2}{a^2}$ (b) $\frac{a^2}{x^2}$
 (c) $\frac{x^2}{b^2 - a^2}$ (d) $\frac{b^2 - a^2}{x^2}$

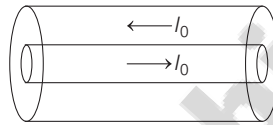
Ans. (a)

Given, radius of inner wire of co-axial cable $= a$

Inner radius of outer shell $= b$

Outer radius of outer shell $= c$

Let, magnetic field at distance $x < a = B_1$



and magnetic field at distance $a < x < b = B_2$

By using Ampere circuital law,

$$\oint \vec{B} \cdot d\vec{l} = n\mu_0 I_{\text{enc}}$$

where, dl = perimeter of small circular element,

n = number of turns,

μ_0 = free space permeability

and I = current.

$$\therefore B_1 \cdot 2\pi x = n\mu_0 I_{\text{enc}}$$

$$\Rightarrow B_1 = \frac{\mu_0 I_0}{2\pi x} \cdot \frac{l_0}{\pi a^2} \cdot \pi x^2$$

$$= \frac{\mu_0 I_0 x}{2\pi a^2} \quad \dots (i)$$

Now, for B_2 ($a < x < b$)

$$B_2 \cdot 2\pi x = \mu_0 I_0$$

$$B_2 = \frac{1}{2\pi} \frac{\mu_0 I_0}{x} \quad \dots (ii)$$

$\therefore$ From Eqs. (i) and (ii) we will get

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I_0 x}{2\pi a^2}}{\frac{1}{2\pi} \frac{\mu_0 I_0}{x}} = \frac{x^2}{a^2}$$

- 05** The fractional change in the magnetic field intensity at a distance r from centre on the axis of current carrying coil of radius a to the magnetic field intensity at the centre of the same coil is (Take, $r < a$) [2021, 26 Aug Shift-II]

- (a) $\frac{3}{2} \frac{a^2}{r^2}$ (b) $\frac{2}{3} \frac{a^2}{r^2}$
 (c) $\frac{2}{3} \frac{r^2}{a^2}$ (d) $\frac{3}{2} \frac{r^2}{a^2}$

Ans. (d)

The magnetic field intensity at a distance r from centre on the axis of a coil of radius a carrying current i is given by

$$B_{\text{axis}} = \frac{\mu_0}{4\pi} \frac{2\pi i a^2}{(a^2 + r^2)^{3/2}} \quad \dots (i)$$

The magnetic field intensity at the centre of the coil is given by

$$B_{\text{centre}} = \frac{\mu_0}{4\pi} \frac{2\pi i}{a} \quad \dots (ii)$$

According to question, the fractional change in the magnetic field intensity is given by

$$\frac{\Delta B}{B} = 1 - \frac{B_{\text{axis}}}{B_{\text{centre}}} \quad \dots (iii)$$

Substituting the values of B_{axis} and B_{centre} from Eqs. (i) and (ii) respectively in Eq. (iii), we get

$$\frac{\Delta B}{B} = 1 - \frac{\frac{\mu_0}{4\pi} \frac{2\pi i a^2}{(a^2 + r^2)^{3/2}}}{\frac{\mu_0}{4\pi} \frac{2\pi i}{a}} = 1 - \frac{a^3}{(a^2 + r^2)^{3/2}}$$

$$= 1 - \frac{a^3}{a^3 \left(1 + \frac{r^2}{a^2}\right)^{3/2}} = 1 - \left(1 + \frac{r^2}{a^2}\right)^{-3/2}$$

Given that, $r < a$, applying Binomial expansion,

$$(1 - x)^{-n} = 1 - x + \frac{2x^2}{2!} - \frac{3 \times 2x^3}{3!} \dots (-1)^n x^n$$

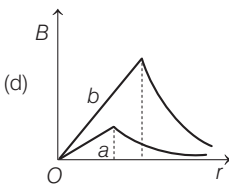
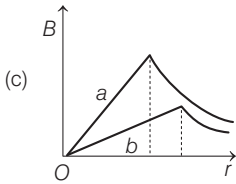
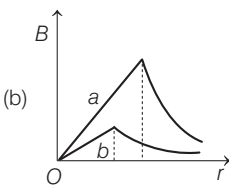
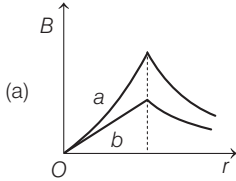
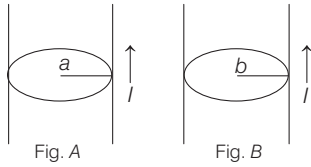
$$= (1 + nx)$$

$$\text{we get } \frac{\Delta B}{B} = 1 - \left(1 - \frac{3}{2} \frac{r^2}{a^2}\right)$$

$$\frac{\Delta B}{B} = 1 - 1 + \frac{3}{2} \frac{r^2}{a^2}$$

$$\frac{\Delta B}{B} = \frac{3}{2} \frac{r^2}{a^2}, \text{ which is the required expression.}$$

- 06** Figures A and B show two long straight wires of circular cross-section (a and b with $a < b$), carrying current I which is uniformly distributed across the cross-section. The magnitude of magnetic field B varies with radius r and can be represented as [2021, 27 July Shift-II]

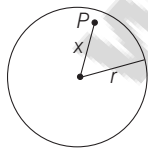


Ans. (c)

We know that magnetic field inside circular cross-section carrying current I (which is uniformly distributed across the cross-section) is given as

$$B = \frac{\mu_0 I x}{2\pi r^2}$$

$$\therefore B \propto x$$



Magnetic field outside the circular cross-section is given as

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow B \propto \frac{1}{r}$$

Hence, graph between B and r showing in option (c) is correct.

07 Match List I with List II.

List I	List II
A. Capacitance, C	I. $M^1 L^1 T^{-3} A^{-1}$
B. Permittivity of free space, ϵ_0	II. $M^{-1} L^{-3} T^4 A^2$
C. Permeability of free space, μ_0	III. $M^{-1} L^{-2} T^4 A^2$
D. Electric field, E	IV. $M^1 L^1 T^{-2} A^{-2}$

Choose the correct answer from the options given below.

[2021, 27 July Shift-II]

	A	B	C	D
(a)	III	II	IV	I
(b)	III	IV	II	I
(c)	IV	II	III	I
(d)	IV	III	II	I

Ans. (a)

For capacitance (C),

$$\therefore \text{Charge, } q = CV$$

$$\Rightarrow [C] = \left[\frac{q}{V} \right] = \frac{[AT]}{[ML^2 T^{-3} A^{-1}]} = [M^{-1} L^{-2} T^4 A^2]$$

For permittivity of free space (ϵ_0),

$\therefore$ According to Coulomb's law

$$F = \frac{kq_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$[\epsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[AT][AT]}{[MLT^{-2}][L^2]}$$

$$\Rightarrow [\epsilon_0] = [M^{-1} L^{-3} T^4 A^2]$$

For permeability of free space (μ_0),

$$\therefore \text{Speed of light, } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow \mu_0 = \frac{1}{\epsilon_0 c^2}$$

$$\Rightarrow [\mu_0] = \frac{1}{[M^{-1} L^{-3} T^4 A^2][L T^{-1}]^2}$$

$$\Rightarrow [\mu_0] = [M^1 L^1 T^{-2} A^{-2}]$$

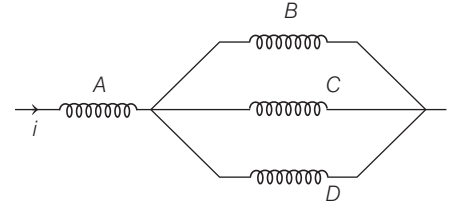
For electric field (E),

$$\therefore [E] = \left[\frac{F}{q} \right] = \left[\frac{MLT^{-2}}{AT} \right]$$

$$\Rightarrow [E] = [M^1 L^1 T^{-3} A^{-1}]$$

08 Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the centre of A is 3T, the field at the centre of C would be (Assume that, the magnetic field is confined with in the volume of respective solenoid)

[2021, 18 March Shift-I]

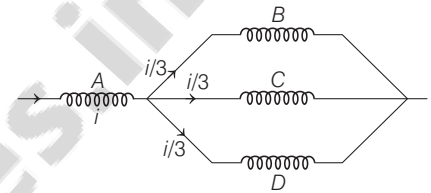


- (a) 12T
(c) 9T
(b) 6T
(d) 1T

Ans. (d)

Given, magnetic field at the centre at A is 3T.

Since, the solenoid B, C and D are connected in parallel arrangement.



We know that,

The magnetic field in the circuit is directly proportional to the current in the circuit.

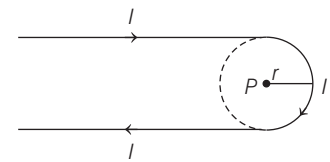
$$\Rightarrow \frac{B_A}{B_C} = \frac{I_A}{I_C} \Rightarrow \frac{3T}{B_C} = \frac{i}{i/3}$$

$$\Rightarrow B_C = 1T$$

So, the field at the centre at C = 1T.

09 A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle?

[2021, 17 March Shift-II]



- (a) $\frac{\mu_0 I}{4\pi r}(2 - \pi)$
(c) $\frac{\mu_0 I}{2\pi r}(2 + \pi)$
(b) $\frac{\mu_0 I}{4\pi r}(2 + \pi)$
(d) $\frac{\mu_0 I}{2\pi r}(2 - \pi)$

Ans. (b)

For the straight wire, the expression of the magnetic field, $B_1 = B_3 = \frac{\mu_0 I}{4\pi r}$

For the semicircular wire, the expression of the magnetic field,

$$B_2 = \frac{\mu_0 I}{4r}$$

Hence, the total magnetic field at the point P , $B = B_1 + B_2 + B_3$

Substituting the values in the above equation, we get

$$B = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r} (1 + 1 + \pi) \Rightarrow B = \frac{\mu_0 I}{4\pi r} (2 + \pi)$$

- 10** Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is [2021, 25 Feb Shift-I]
 (a) 0.2 m (b) 0.1 m (c) 0.15 m (d) 1.0 m

Ans. (b)

Given, $d_1 = 0.05$ m, $d_2 = 0.2$ m

and $B_1 : B_2 = 8 : 1$

Let radius of coil be r .

As we know that, magnetic field at a point on axis of a coil,

$$B = \frac{\mu_0}{2} \cdot \frac{I r^2}{(d^2 + r^2)^{3/2}}$$

$$\therefore \frac{B_1}{B_2} = \frac{r^2 \cdot (d_2^2 + r^2)^{3/2}}{(d_1^2 + r^2)^{3/2} \cdot r^2} = \frac{(d_2^2 + r^2)^{3/2}}{(d_1^2 + r^2)^{3/2}}$$

$$\Rightarrow \frac{8}{1} = \left(\frac{d_2^2 + r^2}{d_1^2 + r^2} \right)^{3/2} \Rightarrow \frac{2^3}{1} = \left(\frac{d_2^2 + r^2}{d_1^2 + r^2} \right)^{3/2}$$

$$\Rightarrow \frac{4}{1} = \frac{d_2^2 + r^2}{d_1^2 + r^2}$$

$$\Rightarrow 4d_1^2 + 4r^2 = d_2^2 + r^2$$

$$\Rightarrow 4(0.05)^2 + 4r^2 = (0.2)^2 + r^2 \Rightarrow 3r^2 = 0.03$$

$$\Rightarrow r = 0.1 \text{ m}$$

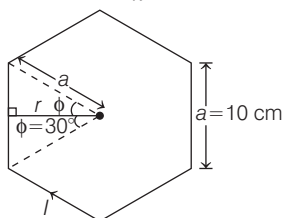
- 11** Magnitude of magnetic field (in SI unit) at the centre of a hexagonal shaped coil of side 10 cm, 50 turns and carrying current I ampere in units of $\frac{\mu_0 I}{\pi}$ is [2020, 3 Sep Shift-I]

- (a) $250\sqrt{3}$ (b) $50\sqrt{3}$
 (c) $500\sqrt{3}$ (d) $5\sqrt{3}$

Ans. (c)

Magnitude of magnetic field due to one side of hexagonal loop at the centre is

$$B_1 = \frac{\mu_0 I}{4\pi r} \cdot 2 \sin \phi$$



Here, $r = 10 \cos 30^\circ$ cm

$$= 10 \times 10^{-2} \times \frac{\sqrt{3}}{2} \text{ m} = 5\sqrt{3} \times 10^{-2} \text{ m}$$

$$\text{So, } B_1 = \frac{\mu_0 I \times 2 \sin 30^\circ}{4\pi (5\sqrt{3} \times 10^{-2})} \quad (\because \phi = 30^\circ)$$

So, magnetic field due to complete 1 turn of loop of six sides is

$$B_2 = 6B_1 = \frac{6 \mu_0 I \times 2 \sin 30^\circ}{4\pi (5\sqrt{3} \times 10^{-2})}$$

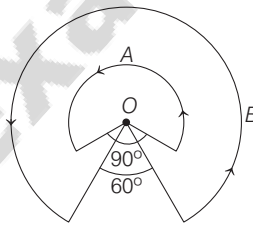
Magnetic field of 50 such turns at centre is

$$B_3 = 50B_2 = \frac{50 \times 6 \times \mu_0 I \times 2 \sin 30^\circ}{4\pi (5\sqrt{3} \times 10^{-2})}$$

$$= 500\sqrt{3} \left(\frac{\mu_0 I}{\pi} \right) \text{ T} = 500\sqrt{3} \text{ in units of } \frac{\mu_0 I}{\pi}$$

Hence, option (c) is correct.

- 12** A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is [2020, 4 Sep Shift-I]



- (a) 4 : 6 (b) 6 : 4
 (c) 6 : 5 (d) 2 : 5

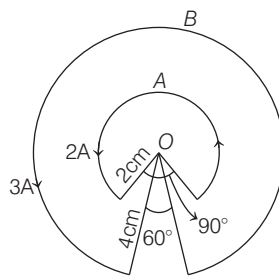
Ans. (c)

For wire A,

$$\theta_A = 360^\circ - 90^\circ = 270^\circ = \frac{3\pi}{2} \text{ rad}$$

$$I_A = 2 \text{ A}$$

$$R_A = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$



For wire B,

$$\theta_B = 360^\circ - 60^\circ = 300^\circ = \frac{5\pi}{3} \text{ rad}$$

$$I_B = 3 \text{ A}$$

$$R_B = 4 \text{ cm} = 4 \times 10^{-2} \text{ m}$$

So, the ratio of magnetic fields due to wires A and B at the common centre O is

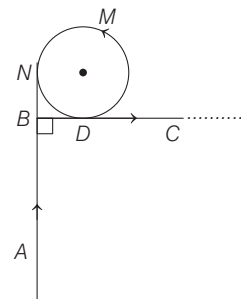
$$\frac{B_A}{B_B} = \frac{\frac{\mu_0 \theta_A I_A}{4\pi R_A}}{\frac{\mu_0 \theta_B I_B}{4\pi R_B}} = \frac{\theta_A I_A R_B}{\theta_B I_B R_A}$$

$$= \frac{\frac{3\pi}{2} \times 2 \times 4 \times 10^{-2}}{\frac{5\pi}{3} \times 3 \times 2 \times 10^{-2}} = \frac{12}{10} = \frac{6}{5}$$

So, $B_A : B_B = 6 : 5$

Hence, correct option is (c).

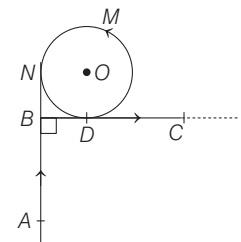
- 13** A very long wire ABDMNDC is shown in figure carrying current I . AB and BC parts are straight, long and at right angle. At D wire forms a circular turn DMND of radius R . AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is [2020, 8 Jan Shift-II]



- (a) $\frac{\mu_0 I}{2\pi R} \left(\pi + \frac{1}{\sqrt{2}} \right)$ (b) $\frac{\mu_0 I}{2R}$
 (c) $\frac{\mu_0 I}{2\pi R} (\pi + 1)$ (d) $\frac{\mu_0 I}{2\pi R} \left(\pi - \frac{1}{\sqrt{2}} \right)$

Ans. (a)

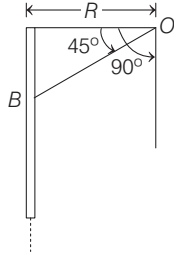
Given arrangement consists of two current carrying wires and one current loop as shown in the figure below.



For wire ABN, magnetic field at O is given by

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R} (\sin\phi_1 + \sin\phi_2)$$

Here, $\phi_1 = +90^\circ$ and $\phi_2 = -45^\circ$.

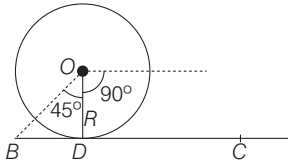


Hence, magnetic field of wire,

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R} [\sin 90^\circ + \sin(-45^\circ)]$$

$$= \frac{\mu_0}{4\pi R} \left(1 - \frac{1}{\sqrt{2}}\right) \otimes$$

Also, field of wire BDC at O,



$$B_2 = \frac{\mu_0 I}{4\pi R} (\sin\phi_1 + \sin\phi_2)$$

Here, $\phi_1 = 45^\circ$ and $\phi_2 = 90^\circ$.

$$\therefore B_2 = \frac{\mu_0 I}{4\pi R} \left(\frac{1}{\sqrt{2}} + 1\right) \otimes$$

Magnetic field of circular coil at O,

$$B_3 = \frac{\mu_0 I}{2R} \odot$$

Hence, net field at O,

$$B_{\text{net}} = B_1 \otimes + B_2 \otimes + B_3 \odot$$

$$= \frac{\mu_0 I}{4\pi R} \left(1 - \frac{1}{\sqrt{2}}\right) - \frac{\mu_0 I}{4\pi R} \left(\frac{1}{\sqrt{2}} + 1\right) - \frac{\mu_0 I}{2R}$$

$$= \frac{\mu_0 I}{2R} \left(\frac{1}{2\pi} - \frac{1}{\sqrt{2} \cdot 2\pi} - \frac{1}{\sqrt{2} \cdot 2\pi} - \frac{1}{2\pi} - 1\right)$$

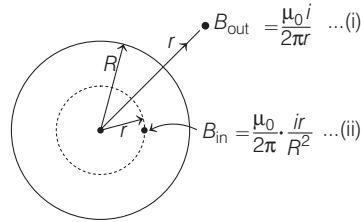
$$= \frac{-\mu_0 I}{2\pi R} \left(\pi + \frac{1}{\sqrt{2}}\right)$$

Negative sign is taken for outward field.

- 14** A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and $2a$ respectively, from the axis of the wire is [2020, 9 Jan Shift-I]
- (a) $\frac{3}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Ans. (c)

By Ampere's circuital rule, magnetic field inside and outside a wire carrying a uniformly distributed current is,



In given case, radius of wire is a . So, magnetic field due to wire at a distance $a/3$ from centre,

$$B_1 = \frac{\mu_0}{2\pi} \times \frac{I \left(\frac{a}{3}\right)}{a^2} = \frac{\mu_0 I}{6\pi a} \quad [\text{from Eq. (ii)}]$$

and magnetic field at a distance $2a$ from centre,

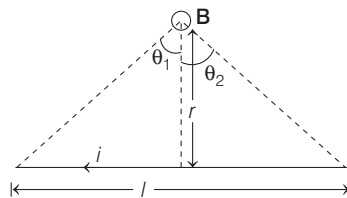
$$B_2 = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a} \quad [\text{from Eq. (i)}]$$

$$\text{Ratio of fields will be } \frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{6\pi a}\right)}{\left(\frac{\mu_0 I}{4\pi a}\right)} = \frac{2}{3}$$

- 15** The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is [Take, $\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$]
- [2019, 10 April Shift-II]
- (a) $9\mu\text{T}$ (b) $1\mu\text{T}$ (c) $3\mu\text{T}$ (d) $18\mu\text{T}$

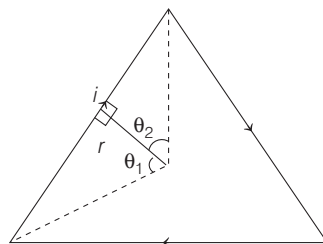
Ans. (d)

For a current carrying wire, from result obtained by Biot-Savart's law, magnetic field at a distance r is given by



$$B = \frac{\mu_0 I}{4\pi r} (\sin\theta_1 + \sin\theta_2)$$

Now, in given case,



Due to symmetry of arrangement, net field at centre of triangle is

$$B_{\text{net}} = \text{Sum of fields of all wires (sides)}$$

$$= 3 \times \frac{\mu_0 I}{4\pi r} (\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = 60^\circ$

$$\therefore \sin\theta_1 = \sin\theta_2 = \frac{\sqrt{3}}{2}, I = 10\text{A}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \text{ NA}^{-2}$$

$$\text{and } r = \frac{1}{3} \times \text{altitude}$$

$$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \times \text{sides length} = \frac{1}{2\sqrt{3}} \times 1$$

$$= \frac{1}{2\sqrt{3}} \text{m}$$

So,

$$B_{\text{net}} = \frac{3 \times 10^{-7} \times 10 \times 2 \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2\sqrt{3}}\right)} = 18 \times 10^{-6} \text{ T}$$

$$\Rightarrow B_{\text{net}} = 18\mu\text{T}$$

- 16** A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of $40\pi \text{ rad s}^{-1}$ about its axis, perpendicular to its plane. If the magnetic field at its centre is $3.8 \times 10^{-9} \text{ T}$, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$).
- [2019, 12 April Shift-I]

$$(a) 2 \times 10^{-6} \text{ C}$$

$$(b) 3 \times 10^{-5} \text{ C}$$

$$(c) 4 \times 10^{-5} \text{ C}$$

$$(d) 7 \times 10^{-6} \text{ C}$$

Ans. (b)

$$\text{Given, } \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2,$$

$$\omega = 40\pi \text{ rad/s}$$

$$B_{\text{at centre}} = 3.8 \times 10^{-9} \text{ T}$$

$$\text{and } R = 10 \text{ cm} = 0.1 \text{ m}$$

Now, we know that, magnetic field at the centre of a current carrying ring is given by

$$B = \frac{\mu_0 I}{2r} \quad \dots(i)$$

Here, I can be determined by flow of charge per rotation, i.e.

$$I = \frac{Q}{T} \quad \dots(ii)$$

Here,

$$T = \frac{2\pi}{\omega}$$

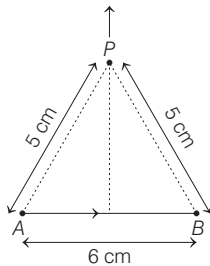
$\Rightarrow$

$$I = \frac{Q\omega}{2\pi} \quad \dots(iii)$$

By putting value of I from Eq. (iii) to Eq. (i), we get

$$\begin{aligned}
 B &= \frac{\mu_0 Q \omega}{2r \times 2\pi} \text{ or } Q = \frac{2Br \times 2\pi}{\mu_0 \omega} \\
 &= \frac{2 \times 3.8 \times 10^{-9} \times 0.1 \times 2\pi}{4\pi \times 10^{-7} \times 40\pi} \\
 &= \frac{2 \times 3.8 \times 0.1}{2 \times 40\pi} \times 10^{-2} \\
 &= 0.003022 \times 10^{-2} \text{ C} \\
 &= 3.022 \times 10^{-5} \text{ C} \\
 \text{or } Q &= 3 \times 10^{-5} \text{ C}
 \end{aligned}$$

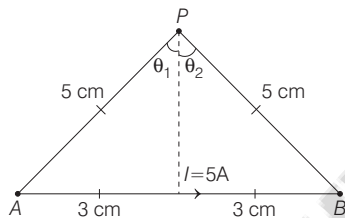
- 17** Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A (See figure). (Take, $\mu_0 = 4\pi \times 10^{-7} \text{ N-A}^{-2}$)
- [2019, 12 April Shift-II]



- (a) $2.0 \times 10^{-5} \text{ T}$ (b) $1.5 \times 10^{-5} \text{ T}$
 (c) $3.0 \times 10^{-5} \text{ T}$ (d) $2.5 \times 10^{-5} \text{ T}$

Ans. (b)

The given figure can be drawn as shown below



By Biot-Savart's law, magnetic field due to a wire segment at point P is

$$B = \frac{\mu_0 I}{4\pi d} (\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = \theta$ (say)

$$\text{Then, } B = \frac{\mu_0 I}{4\pi d} \times 2\sin\theta \quad \dots(i)$$

From given data,

$$I = 5\text{ A,}$$

$$\mu = 4\pi \times 10^{-7} \text{ N-A}^{-2}$$

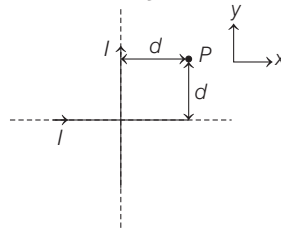
$$d = \sqrt{5^2 - 3^2} = \sqrt{16} = 4 \text{ cm}$$

$$\sin\theta = \frac{3}{5}$$

On substituting these values in Eq. (i), we get

$$\begin{aligned}
 B &= \frac{\mu_0 I}{4\pi d} \times 2\sin\theta = \frac{4\pi \times 10^{-7} \times 5}{4\pi \times 4 \times 10^{-2}} \times 2 \times \left(\frac{3}{5}\right) \\
 &= \frac{5 \times 2 \times 3}{4 \times 5} \times 10^{-5} = 1.5 \times 10^{-5} \text{ T}
 \end{aligned}$$

- 18** Two very long, straight and insulated wires are kept at 90° angle from each other in xy -plane as shown in the figure.



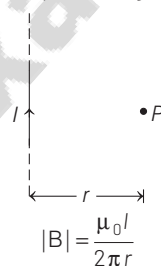
These wires carry currents of equal magnitude I , whose directions are shown in the figure. The net magnetic field at point P will be

[2019, 8 April Shift-II]

- (a) zero (b) $\frac{\mu_0 I}{\pi d} (\hat{x})$
 (c) $-\frac{\mu_0 I}{2\pi d} (\hat{x} + \hat{y})$ (d) $\frac{\mu_0 I}{2\pi d} (\hat{x} + \hat{y})$

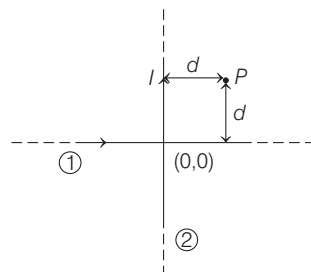
Ans. (a)

Magnetic field due to an infinitely long straight wire at point P is given as



$$|B| = \frac{\mu_0 I}{2\pi r}$$

Thus, in the given situation, magnetic field due to wire 1 at point P is



$$B_1 = \frac{\mu_0 I}{2\pi d}, \otimes$$

Similarly, magnetic field due to wire 2 at point P is

$$B_2 = \frac{\mu_0 I}{2\pi d}, \otimes$$

Resultant field at point P is

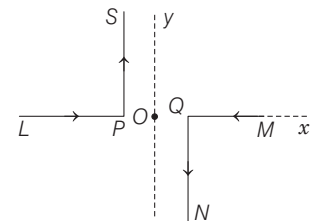
$$B_{\text{net}} = B_1 + B_2$$

Since, $|B_1| = |B_2|$, but they are opposite in direction.

Thus, $B_{\text{net}} = 0$

$\therefore$ Net magnetic field at point P will be zero.

- 19** As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the X -axis, while segments PS and QN are parallel to the Y -axis. If $OP = OQ = 4 \text{ cm}$ and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be (Take, $\mu_0 = 4\pi \times 10^{-7} \text{ N-A}^{-2}$)
- [2019, 12 Jan Shift-I]



- (a) 40 A, perpendicular out of the page
 (b) 20 A, perpendicular into the page
 (c) 20 A, perpendicular out of the page
 (d) 40 A, perpendicular into the page

Ans. (b)

There is no magnetic field along axis of a current-carrying wire.

Also, magnetic field near one of end of an infinitely long wire is $\frac{\mu_0 I}{4\pi r}$ tesla.

Hence, magnetic field due to segments LP and QM at O is zero. Using right hand rule, we can check that magnetic field due to segments PS and QN at O is in same direction perpendicularly into the plane of paper.

$$\text{Hence, } B_O = B_{PS} + B_{QN} = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$

$$\text{So, } I = \frac{2\pi r B_O}{\mu_0}$$

Here, $r = OP = OQ = 4 \text{ cm}$ and $B_O = 10^{-4} \text{ T}$.

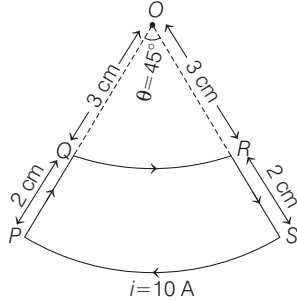
Substituting values, we get

$$\Rightarrow I = \frac{2\pi \times 4 \times 10^{-2} \times 10^{-4}}{4\pi \times 10^{-7}}$$

$\Rightarrow I = 20 \text{ A}$, Also, magnetic field points perpendicularly into the plane of paper.

- 20** A current loop, having two circular arcs joined by two radial lines as shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to

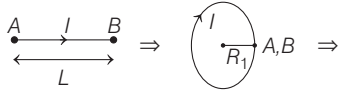
[2019, 9 Jan Shift-I]



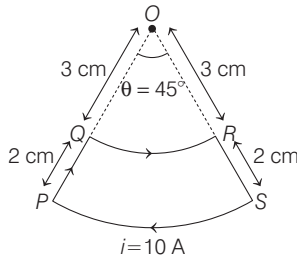
- (a) 1.0×10^{-5} T (b) 1.0×10^{-7} T
(c) 1.5×10^{-7} T (d) 1.5×10^{-5} T

Ans. (a)

Key Idea When a point 'P' lies on the axial position of current-carrying conductor, then magnetic field at P is always zero.



From the given figure as shown below.



The magnetic field at point 'O' due to wires PQ and RS will be zero.
Magnetic field due to arc QR at point 'O' will be

$$B_1 = \frac{\theta}{2\pi} \left(\frac{\mu_0 i}{2a} \right)$$

Here, $\theta = 45^\circ = \frac{\pi}{4}$ rad, $i = 10$ A

and $a = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$

$$\Rightarrow B_1 = \frac{\pi}{2\pi \times 4} \left(\frac{\mu_0 \times 10}{2 \times 3 \times 10^{-2}} \right) = \frac{\mu_0 \times 5}{2 \times 12 \times 10^{-2}} = \frac{5 \times \mu_0 \times 10^2}{24}$$

Direction of field B_1 will be coming out of the plane of figure.

Similarly, field at point 'O' due to arc SP will be

$$B_2 = \frac{\pi}{4} \left(\frac{1}{2\pi} \right) \left[\frac{\mu_0 \times 10}{2 \times (2+3) \times 10^{-2}} \right] = \frac{\mu_0 \times 5}{2 \times 20 \times 10^{-2}} = \frac{\mu_0 \times 10^2}{8}$$

Direction of B_2 is going into the plane of the figure.

$\therefore$ The resultant field at O is

$$B = B_1 - B_2 = \frac{1}{2} \left(\frac{5 \times \mu_0}{12 \times 10^{-2}} - \frac{\mu_0}{4 \times 10^{-2}} \right) = \frac{1}{2} \left(\frac{5 \mu_0 - 3 \mu_0}{12 \times 10^{-2}} \right) = \frac{1}{2} \left(\frac{2 \mu_0}{12 \times 10^{-2}} \right) = \frac{4 \pi \times 10^{-7}}{12 \times 10^{-2}} \approx 1 \times 10^{-5} \text{ T}$$

- 21** One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that

at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$

will be

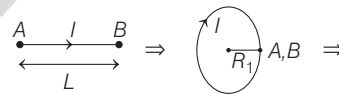
[2019, 9 Jan Shift-I]

- (a) $\frac{1}{N}$ (b) N (c) $\frac{1}{N^2}$ (d) N^2

Ans. (c)

Let consider the length of first wire is L , then according to question, if radius of loop formed is R_1 , then,

For wire 1,

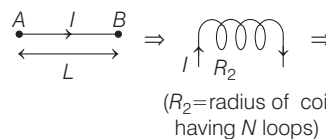


$$L = 2\pi R_1 \Rightarrow R_1 = \frac{L}{2\pi}$$

The magnetic field due to this loop at its centre is

$$B_L = \frac{\mu_0 i}{2R_1} = \frac{\mu_0 i}{2L} \times 2\pi \quad \dots(i)$$

Now, second wire is made into a coil of N -turns.



(R_2 = radius of coil having N loops)

$$\text{Thin, } L = N(2\pi R_2) \Rightarrow R_2 = \frac{L}{2\pi N}$$

The magnetic field due to this circular coil of N -turns is

$$B_C = \left(\frac{\mu_0 i}{2R_2} \right) N = N \cdot \frac{\mu_0 i \cdot (2\pi N)}{2L} \quad \dots(ii)$$

Using Eqs. (i) and (ii), the ratio of $\frac{B_L}{B_C}$ is :

$$\Rightarrow \frac{B_L}{B_C} = \frac{\frac{\mu_0 i}{2R_1}}{N \frac{\mu_0 i}{2R_2}} = \frac{\frac{\mu_0 i}{2L} \cdot (2\pi)}{\frac{\mu_0 i}{2L} \cdot (2\pi) N^2} = \frac{1}{N^2}$$

- 22** Two identical wires A and B, each of length l , carry the same current I . Wire A is bent into a circle of radius R and wire B is bent to form a square of side a . If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

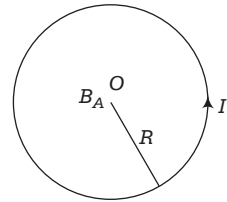
[JEE Main 2016]

- (a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$
(c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

Ans. (d)

Magnetic field in case of circle of radius R , we have

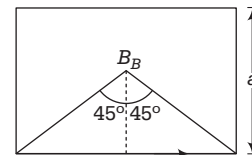
$$B_A = \frac{\mu_0 I}{2R}$$



As, $2\pi R = l$, (where l is length of a wire)

$$R = \frac{l}{2\pi}$$

$$\Rightarrow B_A = \frac{\mu_0 I}{2 \times \frac{l}{2\pi}} = \frac{\mu_0 I \pi}{l} \quad \dots(i)$$



Magnetic field in case of square of side a , we get

$$B_B = 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\left(\frac{a}{2} \right)} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow B_B = \frac{4\mu_0}{\pi a \sqrt{2}} = \frac{\mu_0 2\sqrt{2} I}{a \pi}$$

$$\text{As, } 4a = 1, a = 1/4$$

$$\Rightarrow B_B = \frac{8\sqrt{2}\mu_0 I}{\pi l} \quad \dots (ii)$$

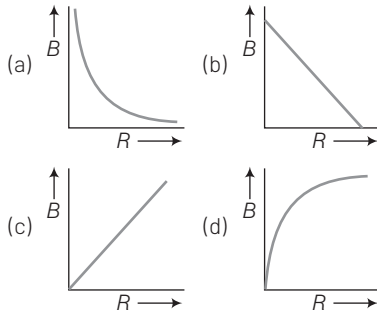
Dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

- 23** A charge Q is uniformly distributed over the surface of non-conducting disc of radius R . The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω . As a result of this rotation, a magnetic field of induction B is obtained at the centre of the disc.

If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by the figure

[AIEEE 2012]



Ans. (a)

Taking an elemental ring of radius r and thickness dr , we obtain magnetic field at the centre of the ring,

$$dB = \frac{\mu_0}{2r} dl = \frac{\mu_0}{2r} \cdot \frac{dq}{T} = \frac{\mu_0}{2r} \cdot \frac{Q}{\pi R^2} \cdot 2\pi r dr$$

$$= \frac{\mu_0 Q \omega}{2\pi R^2} dr$$

So, net magnetic field at the centre of disk,

$$B = \int_0^R dB = \frac{\mu_0 Q \omega}{2\pi R^2} \int_0^R dr = \frac{\mu_0 Q \omega}{2\pi R}$$

i.e., $B \propto \frac{1}{R}$

- 24** A current I flows in an infinitely long wire with cross-section in the form of a semi-circular ring of radius R . The magnitude of the magnetic induction along its axis is

[AIEEE 2011]

- (a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$
(c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

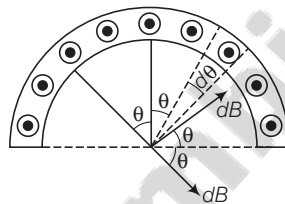
Ans. (d)

Consider the wire to be made up of large number of thin wires of infinite length. Consider such wire of thickness dI subtending an angle $d\theta$ at centre.

Current through this wire,

$$dI = \frac{d\theta}{\pi} I$$

$\therefore$ Magnetic field at centre due to this portion,



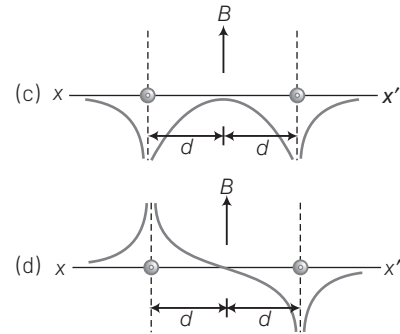
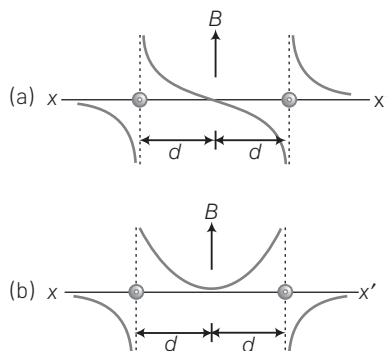
$$dB = \frac{\mu_0}{4\pi} \cdot \frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R} d\theta$$

Net magnetic field at the centre,

$$B = \int_{-\pi/2}^{\pi/2} dB \cos\theta = \frac{\mu_0 I}{2\pi^2 R} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

- 25** Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line xx' is given by

[AIEEE 2010]



Ans. (a)

The magnetic field in between because of each will be in opposite direction.

$$\therefore B_{\text{in between}} = \frac{\mu_0 I}{2\pi x} j - \frac{\mu_0 I}{2\pi(2d-x)} (-j)$$

$$= \frac{\mu_0 I}{2\pi} \left[\frac{1}{x} - \frac{1}{2d-x} \right] (j)$$

At $x=d$, $B_{\text{in between}} = 0$

For $x < d$, $B_{\text{in between}} = (j)$

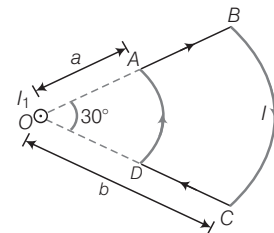
For $x > d$, $B_{\text{in between}} = (-j)$

Towards x , net magnetic field will add up and direction will be $(-j)$.

Towards x' , net magnetic field will add up and direction will be (j) .

- 26** The magnitude of the magnetic field (B) due to loop ABCD at the origin (O) is

[AIEEE 2009]



- (a) zero
(b) $\frac{\mu_0 I(b-a)}{24ab}$
(c) $\frac{\mu_0 I}{4\pi} \left[\frac{b-a}{ab} \right]$
(d) $\frac{\mu_0 I}{4\pi} \left[2(b-a) + \frac{\pi}{3}(a+b) \right]$

Ans. (b)

Net magnetic field due to loop ABCD at O is

$$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$$

$$= 0 + \frac{\mu_0 I}{4\pi a} \times \frac{\pi}{6} + 0 - \frac{\mu_0 I}{\pi b} \times \frac{\pi}{6}$$

$$= \frac{\mu_0 I}{24a} - \frac{\mu_0 I}{24b}$$

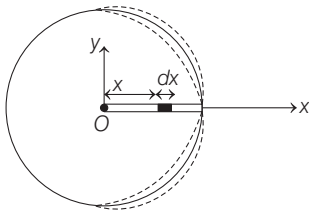
$$= \frac{\mu_0 I}{24ab} (b-a)$$

- 27** Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2 . The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

[AIEEE 2007]

- (a) $\frac{\mu_0}{2\pi} \left(\frac{I_1 + I_2}{d} \right)^{1/2}$ (b) $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$
 (c) $\frac{\mu_0}{2\pi d} (I_1 + I_2)$ (d) $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)$

Ans. (b)



The magnetic field inductions at a point P, at a distance d from O in a direction perpendicular to the plane of the wires due to currents through AOB and COD are perpendicular to each other, is

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \left[\left(\frac{\mu_0}{4\pi} \frac{2I_1}{d} \right)^2 + \left(\frac{\mu_0}{4\pi} \frac{2I_2}{d} \right)^2 \right]^{1/2}$$

$$= \frac{\mu_0}{2\pi d} \sqrt{I_1^2 + I_2^2}$$

- 28** A long straight wire of radius a carries a steady current I . The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

[AIEEE 2007]

- (a) $1/4$ (b) 4
 (c) 1 (d) $1/2$

Ans. (c)

Current density, $J = \frac{I}{\pi a^2}$

From Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot I_{\text{enclosed}}$$

For $r < a$,

$$B \times 2\pi r = \mu_0 \times J \times \pi r^2$$

$$\Rightarrow B = \frac{\mu_0 I}{\pi a^2} \times \frac{r}{2}$$

$$\text{At } r = \frac{a}{2}, \quad B_1 = \frac{\mu_0 I}{4\pi a}$$

$$\text{For } r > a, \quad B \times 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$\text{At } r = 2a, \quad B_2 = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \frac{B_1}{B_2} = 1$$

- 29** A current I flows along the length of an infinitely long, straight, thin walled pipe. Then,

[AIEEE 2007]

- (a) the magnetic field is zero only on the axis of the pipe
 (b) the magnetic field is different at different points inside the pipe
 (c) the magnetic field at any point inside the pipe is zero
 (d) the magnetic field at all points inside the pipe is the same but not zero

Ans. (c)

By using Ampere's circuital law, the magnetic field at any point inside the pipe is zero.

- 30** A long solenoid has 200 turns per cm and carries a current I . The magnetic field at its centre is $6.28 \times 10^{-2} \text{ Wb/m}^2$. Another long solenoid has 100 turns per cm and it carries a current $I/3$. The value of the magnetic field at its centre is

[AIEEE 2006]

- (a) $1.05 \times 10^{-2} \text{ Wb/m}^2$
 (b) $1.05 \times 10^{-5} \text{ Wb/m}^2$
 (c) $1.05 \times 10^{-3} \text{ Wb/m}^2$
 (d) $1.05 \times 10^{-4} \text{ Wb/m}^2$

Ans. (a)

Magnetic field due to a long solenoid is given by

$$B = \mu_0 n I$$

From given data,

$$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times I \quad \dots(i)$$

$$\text{and } B = \mu_0 \times 100 \times 10^2 \times \left(\frac{I}{3} \right) \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$B \approx 1.05 \times 10^{-2} \text{ Wb/m}^2$$

- 31** Two concentric coils each of radius equal to $2\pi \text{ cm}$ are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil respectively. The magnetic induction in Wb/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7} \text{ Wb/Am}$)

[AIEEE 2005]

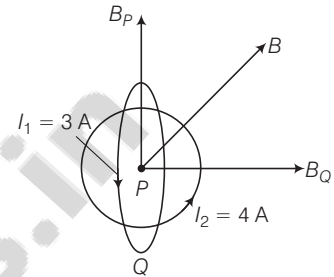
- (a) 12×10^{-5} (b) 10^{-5}
 (c) 5×10^{-5} (d) 7×10^{-5}

Ans. (c)

Magnetic field or magnetic induction at P,

$$B_P = \frac{\mu_0 I_2}{2R} = \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.02\pi}$$

$$= 4 \times 10^{-5} \text{ Wb/m}^2$$



$$\text{and } B_Q = \frac{\mu_0 I_1}{2R} = \frac{4\pi \times 10^{-7} \times 3}{2 \times 0.02\pi}$$

$$= 3 \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore B = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{(4 \times 10^{-5})^2 + (3 \times 10^{-5})^2}$$

$$= 5 \times 10^{-5} \text{ Wb/m}^2$$

- 32** A current I ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is

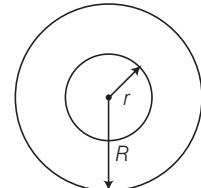
[AIEEE 2004]

- (a) infinite (b) zero
 (c) $\frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$ (d) $\frac{2I}{r}$

Ans. (b)

Let R be the radius of a long thin cylindrical shell.

To calculate the magnetic induction at a distance r , ($r < R$) from the axis of cylinder, a circular shell of radius r is shown.



Since, no current is enclosed in the circle, so from Ampere's circuital law, magnetic induction is zero at every point of circle. Hence, the magnetic induction at any point inside the infinitely long straight thin walled tube (cylindrical) is zero.

- 33** A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B . It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be [AIEEE 2004]

- (a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

Ans. (b)

The magnetic field at the centre of circular coil is

$$B = \frac{\mu_0 I}{2r}$$

where, r = radius of circle = $\frac{l}{2\pi}$

$$[\because l = 2\pi r]$$

$$\therefore B = \frac{\mu_0 I}{2} \times \frac{2\pi}{l} = \frac{\mu_0 I \pi}{l} \quad \dots(i)$$

When wire of length l bends into a circular loops of n turns, then

$$l = n \times 2\pi r' \\ \Rightarrow r' = \frac{l}{n \times 2\pi}$$

Thus, new magnetic field,

$$B' = \frac{\mu_0 n I}{2r'} = \frac{\mu_0 n I}{2} \times \frac{n \times 2\pi}{l} \\ = \frac{\mu_0 \pi}{l} \times n^2 = n^2 B \quad [\text{from Eq. (i)}]$$

- 34** The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is $54 \mu\text{T}$. What will be its value at the centre of the loop? [AIEEE 2004]

- (a) $250 \mu\text{T}$ (b) $150 \mu\text{T}$
(c) $125 \mu\text{T}$ (d) $75 \mu\text{T}$

Ans. (a)

The magnetic field at a point on the axis of a circular loop at a distance x from the centre is

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \quad \dots(i)$$

Given, $B = 54 \mu\text{T}$, $x = 4 \text{ cm}$, $R = 3 \text{ cm}$

Putting the given values in Eq. (i), we get

$$54 = \frac{\mu_0 I \times (3)^2}{2(3^2 + 4^2)^{3/2}} \\ \Rightarrow 54 = \frac{9\mu_0 I}{2(25)^{3/2}} = \frac{9\mu_0 I}{2 \times (5)^3}$$

$$\therefore \mu_0 I = \frac{54 \times 2 \times 125}{9}$$

$$\Rightarrow \mu_0 I = 1500 \mu\text{T-cm} \quad \dots(ii)$$

Now, putting $x = 0$ in Eq. (i), magnetic field at the centre of loop is

$$B = \frac{\mu_0 I R^2}{2R^3} \\ = \frac{\mu_0 I}{2R} = \frac{1500}{2 \times 3} \quad [\text{from Eq. (ii)}] \\ = 250 \mu\text{T}$$

- 35** If in a circular coil A of radius R , current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them will be [AIEEE 2002]

- (a) 1 (b) 2
(c) $1/2$ (d) 4

Ans. (a)

Magnetic field in circular coil A is

$$B_A = \frac{\mu_0 N I}{2R}$$

where, R is radius and I is current flowing in coil.

$$\text{Similarly, } B_B = \frac{\mu_0 N(2I)}{2 \cdot (2R)} = \frac{\mu_0 N I}{2R} \quad \frac{B_A}{B_B} = 1$$

TOPIC 2

Magnetic Force on Charged Particle in Magnetic Field

- 36** Two ions of masses 4 amu and 16 amu have charges $+2e$ and $+3e$, respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same.

Then, [2021, 27 Aug Shift-I]

- (a) lighter ion will be deflected less than heavier ion
(b) lighter ion will be deflected more than heavier ion
(c) both ions will be deflected equally
(d) no ion will be deflected

Ans. (b)

Given, mass of 1st ion, $m_1 = 4 \text{ amu}$

Mass of 2nd ion, $m_2 = 16 \text{ amu}$

Charge on 1st ion, $q_1 = +2e$

Charge on 2nd ion, $q_2 = +3e$

Kinetic energy of both ion is same,

$$KE_1 = KE_2$$

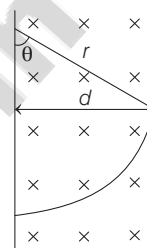
The radius of path traced by a charged particle in magnetic field is given as

$$r = \frac{\sqrt{2mKE}}{qB}$$

$$\Rightarrow r \propto \frac{\sqrt{m}}{q} \\ \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \times \left(\frac{q_2}{q_1} \right) \\ \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{4}{16}} \times \left(\frac{+3e}{+2e} \right) \\ = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$\Rightarrow r_1 = \frac{3}{4} r_2$$

$$\Rightarrow r_1 < r_2$$



Now, the deflection of ion in region with constant magnetic field as shown in figure is denoted by θ .

From diagram,

$$\sin \theta = \frac{d}{r}$$

$$\theta \propto \frac{1}{r} \quad [\because d = \text{constant}]$$

As, $r_1 < r_2$ from above expression $\theta_1 > \theta_2$.

Thus, deflection of lighter ion will be more than the deflection of heavier ion.

- 37** If the maximum value of accelerating potential provided by a radio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is

$$[\text{Given, } m_p = 1.67 \times 10^{-27} \text{ kg, } e = 1.6 \times 10^{-19} \text{ C, } c = 3 \times 10^8 \text{ m/s}]$$

[2021, 26 Aug Shift-II]

Ans. (543)

Given, maximum value of accelerating potential, $V = 12 \text{ kV} = 12000 \text{ V}$,

Speed achieved by a proton cyclotron,

$$v = c/6$$

where, c is the speed of light
 $= 3 \times 10^8 \text{ ms}^{-1}$

Mass of proton, $m_p = 1.67 \times 10^{-27} \text{ kg}$

Charge of proton, $q_p = 1.6 \times 10^{-19} \text{ C}$

According to the question, assume a proton makes

revolutions in the cyclotron to achieve the speed v .

The energy absorbed by proton due to the accelerating potential provided by the radio oscillator

$$= n \times 2 \times q_p \times V$$

This energy absorbed by proton will be equal to the kinetic energy gain by the proton i.e.,

$$2nq_p V = \frac{m_p v^2}{2}$$

Substituting the given values in above expression, we get

$$n \times 2 \times 1.6 \times 10^{-19} \times 12000$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left(\frac{3 \times 10^8}{6} \right)^2$$

$$n \times 3.84 \times 10^{-15} = 2.0875 \times 10^{-12}$$

$$n = 543.62$$

As, n is number of revolutions, so it should be an integer.

$$\therefore n = 543$$

- 38** Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is

[2021, 25 July Shift-II]

- (a) 1 : 4 (b) 4 : 3 (c) 3 : 1 (d) 2 : 3

Ans. (b)

Let masses of ions be m_1 and m_2 respectively, charges be q_1 and q_2 respectively and speed of charges be v_1 and v_2 .

$$\text{Given, } m_1 = m_2 = m$$

$$q_1 : q_2 = 1 : 2$$

$$v_1 : v_2 = 2 : 3$$

As we know that,

$$r = \frac{mv}{qB}$$

$$\therefore \frac{r_1}{r_2} = \frac{m_1 v_1}{q_1 B} \times \frac{q_2 B}{m_2 v_2}$$

$$= \frac{m}{m} \times \frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$$

- 39** A deuteron and an α -particle having equal kinetic energy enter perpendicular into a magnetic field. Let r_d and r_α be their respective radii of circular path. The value of

$$\frac{r_d}{r_\alpha} \text{ is equal to}$$

[2021, 20 July Shift-I]

$$(a) \frac{1}{\sqrt{2}}$$

$$(b) \sqrt{2}$$

$$(c) 1$$

$$(d) 2$$

Ans. (b)

Given, kinetic energy of α -particle (K_α) = kinetic energy of deuteron (K_d)

$$\text{Since, kinetic energy, } K = \frac{1}{2} m v^2$$

$$\Rightarrow m v^2 = 2K$$

$$\Rightarrow v^2 = \frac{2K}{m}$$

We know that,

$$r = \frac{mv}{Bq} \quad \dots(ii)$$

where, r = radius of curvature of path of a charged particle,

m = mass of the charged particle,

q = charge of the particle,

v = velocity of charged particle

and B = magnetic field.

From Eqs. (i) and (ii), we get

$$r = \frac{m \sqrt{\frac{2K}{m}}}{Bq}$$

$$\Rightarrow r = \frac{\sqrt{2Km}}{Bq} \quad \dots(iii)$$

Since, m , K and B are same for both deuteron and

α -particle.

From Eq. (iii), we get

$$r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \frac{r_d}{r_\alpha} = \frac{\sqrt{m_d}}{\sqrt{m_\alpha}} \cdot \frac{q_\alpha}{q_d} = \sqrt{\frac{2}{4}} \left(\frac{2}{1} \right)$$

$$\left[\because m_d = 2m_p \text{ and } m_\alpha = 4m_p \right]$$

$$\left[q_\alpha = 2e \text{ and } q_d = e. \right]$$

$$\frac{r_d}{r_\alpha} = \sqrt{2}$$

- 40** A proton and an α -particle, having kinetic energies K_p and K_α , respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α -particle is 2 : 1. The ratio of $K_p : K_\alpha$ is

[2021, 18 March Shift-II]

$$(a) 1 : 8$$

$$(b) 8 : 1$$

$$(c) 1 : 4$$

$$(d) 4 : 1$$

Ans. (d)

Given, the charge on the proton, $q_p = +e$

The charge on the alpha particles,

$$q_\alpha = +2e$$

The expression of the radius of the orbit,

$$r = \frac{mv}{qB}$$

$$\Rightarrow r = \frac{p}{qB} \quad (\because p = mv)$$

$$\therefore \frac{r_p}{r_\alpha} = \frac{p_p}{q_p} \times \frac{q_\alpha}{p_\alpha}$$

$$\Rightarrow \frac{2}{1} = \frac{+2e}{+e} \times \frac{p_p}{p_\alpha} \left[\because \frac{r_p}{r_\alpha} = \frac{2}{1} \right]$$

$$\Rightarrow \frac{p_p}{p_\alpha} = 1$$

Since, kinetic energy and momentum are related as

$$K = \frac{p^2}{2m}$$

$$\therefore \frac{K_p}{K_\alpha} = \frac{p_p^2 m_\alpha}{m_p p_\alpha^2}$$

$$\Rightarrow \frac{K_p}{K_\alpha} = \frac{m_\alpha}{m_p} \times \frac{4m_p}{m_p} \Rightarrow \frac{K_p}{K_\alpha} = \frac{4}{1}$$

- 41** A charge Q is moving dl distance in the magnetic field $\mathbf{B}$. Find the value of work done by $\mathbf{B}$.

[2021, 16 March Shift-II]

$$(a) 1$$

$$(b) \text{ Infinite}$$

$$(c) \text{ Zero}$$

$$(d) -1$$

Ans. (c)

According to question, a charge Q is moving dl distance in magnetic field B .

As we know that force acting on a point charge by magnetic field is always perpendicular to the velocity of particle $\mathbf{v}$.

$$\therefore \mathbf{F} = Q(\mathbf{v} \times \mathbf{B}) \quad \dots(i)$$

The above, statement means that $\mathbf{F}$ is perpendicular to $\mathbf{v}$.

$$\therefore \text{Work done, } W = \mathbf{F} \cdot d\mathbf{s}$$

$$= Q(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad [\text{using Eq. (i)}]$$

$$= Q(\mathbf{v} \times \mathbf{B}) dl \cos \theta$$

$$= Q(\mathbf{v} \times \mathbf{B}) dl \cos 90^\circ$$

$$[\because \theta = 90^\circ \text{ } \mathbf{F} \perp \mathbf{v} \text{ or } d\mathbf{l}]$$

$$= 0$$

$\therefore$ The work done by the magnetic field will be zero.

- 42** A proton, a deuteron and an α -particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is and their speed is in the ratio.

[2021, 25 Feb Shift-I]

$$(a) 1 : 2 : 4 \text{ and } 2 : 1 : 1$$

$$(b) 2 : 1 : 1 \text{ and } 4 : 2 : 1$$

$$(c) 4 : 2 : 1 \text{ and } 2 : 1 : 1$$

$$(d) 1 : 2 : 4 \text{ and } 1 : 1 : 2$$

Ans. (b)

Let F_p, F_d and F_α be the forces and v_p, v_d and v_α be the velocities of proton, deuteron and α -particle, respectively.

Since, $F = Bqv$

On dividing and multiplying F by m , we get

$$\begin{aligned} F &= Bqv \frac{m}{m} \\ \Rightarrow F &= B \frac{q}{m} p \quad (\because p = mv) \\ \Rightarrow F &\propto q/m \quad (\because p \text{ and } B \text{ are same}) \\ \therefore F_p : F_d : F_\alpha &= \frac{+q}{m} : \frac{+q}{2m} : \frac{+2q}{4m} \\ &= 1 : 1/2 : 1/2 = 2 : 1 : 1 \\ \text{and } F &= Bqv \\ \Rightarrow v &\propto F/q \\ \therefore v_p : v_d : v_\alpha &= \frac{F_p}{q_p} : \frac{F_d}{q_d} : \frac{F_\alpha}{q_\alpha} = \frac{2x}{q} : \frac{x}{q} : \frac{x}{2q} \\ &= 2 : 1 : 1/2 = 4 : 2 : 1 \end{aligned}$$

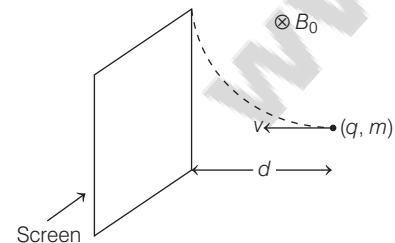
- 43** A particle of charge q and mass m is moving with a velocity $-v\hat{i}$ ($v \neq 0$) towards a large screen placed in the yz -plane at a distance d . If there is a magnetic field $\mathbf{B} = B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is

[2020, 6 Sep Shift-I]

- (a) $\frac{qdB_0}{3m}$ (b) $\frac{2qdB_0}{m}$ (c) $\frac{qdB_0}{m}$ (d) $\frac{qdB_0}{2m}$

Ans. (c)

As we know that, if a charge particle moves in a uniform magnetic field, then its path is always circular. Considering the charge positive, the direction of magnetic force acting on it, has been shown by dotted line.



As, charge particle should not hit the screen this means radius of circular path must be less than screen distance d .

$$\text{i.e. } R \leq d \Rightarrow \frac{mv}{qB_0} \leq d \quad \left[\because R = \frac{mv}{qB_0} \right]$$

$$\text{or } v \leq \frac{qB_0d}{m}$$

$$\text{Maximum value of } v = \frac{qB_0d}{m}$$

Note In this problem, minimum value has been asked which is clearly an error, so read it maximum value.

- 44** A beam of protons with speed $4 \times 10^5 \text{ ms}^{-1}$ enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to

(Take, mass of the proton $= 1.67 \times 10^{-27} \text{ kg}$ and charge of the proton $= 1.69 \times 10^{-19} \text{ C}$)

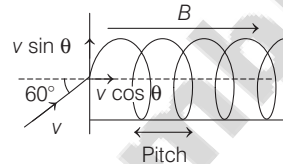
[2020, 2 Sep Shift-I]

- (a) 2 cm (b) 4 cm (c) 5 cm (d) 12 cm

Ans. (b)

Pitch of helical path shown below is given by

$$\text{Pitch} = T \cdot v \cos \theta = \frac{2\pi m}{Bq} \cdot v \cos \theta$$

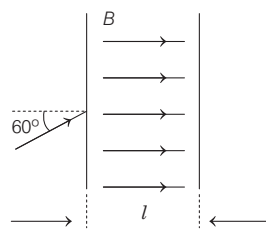


$$\begin{aligned} \Rightarrow \text{Pitch} &= \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 4 \times 10^5 \times \cos 60^\circ}{(1.69 \times 10^{-19}) (0.3)} \\ &= 0.04 \text{ m} = 4 \text{ cm} \end{aligned}$$

Hence, correct option is (b).

- 45** The figure shows a region of length l with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity $4 \times 10^5 \text{ ms}^{-1}$ making an angle 60° with the field. If the proton completes 10 revolutions by the time it cross the region shown, l is close to (Take, mass of proton $= 1.67 \times 10^{-27} \text{ kg}$, charge of the proton $= 1.6 \times 10^{-19} \text{ C}$)

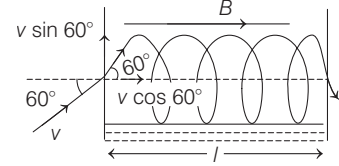
[2020, 2 Sep Shift-II]



- (a) 0.11 m (b) 0.88 m
(c) 0.44 m (d) 0.22 m

Ans. (c)

As $\mathbf{v}$ is not perpendicular to $\mathbf{B}$, so path of particle is helical in magnetic field as shown in figure,



Let T be the time to complete 1 round, then

$$T = 2\pi m / Bq$$

So, total time to cross the length l of region of magnetic field is

$$t = 10T = 10 \times \frac{2\pi m}{Bq}$$

Hence, length of path l is

$$l = \text{speed} \times \text{time}$$

$$= v \cos \theta \times t$$

$$= v \cos 60^\circ \times 10 \times \frac{2\pi m}{Bq}$$

$$\Rightarrow l = \frac{4 \times 10^5 \times \frac{1}{2} \times 10 \times 2\pi \times 1.67 \times 10^{-27}}{0.3 \times 1.6 \times 10^{-19}}$$

$$= 0.437 \approx 0.44 \text{ m}$$

Hence, correct option is (c).

- 46** A charged particle carrying charge $1 \mu\text{C}$ is moving with velocity $(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ ms}^{-1}$. If an external magnetic field of $(5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3} \text{ T}$ exists in the region, where the particle is moving, then the force on the particle is $\mathbf{F} \times 10^{-9} \text{ N}$. The vector $\mathbf{F}$ is

[2020, 3 Sep Shift-I]

$$(a) -0.30\hat{i} + 0.32\hat{j} - 0.09\hat{k}$$

$$(b) -30\hat{i} + 32\hat{j} - 9\hat{k}$$

$$(c) -300\hat{i} + 320\hat{j} - 90\hat{k}$$

$$(d) -3.0\hat{i} + 3.2\hat{j} - 0.9\hat{k}$$

Ans. (b)

Force on a charged particle moving in region of a magnetic field is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

$$\text{Here, } q = 1 \mu\text{C} = 1 \times 10^{-6} \text{ C,}$$

$$\mathbf{v} = (2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ ms}^{-1}$$

$$\text{and } \mathbf{B} = (5\hat{i} + 3\hat{j} - 6\hat{k}) \times 10^{-3} \text{ T}$$

So, force on the particle is

$$\mathbf{F} \times 10^{-9} = 1 \times 10^{-6}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 \times 10^{-3} & 3 \times 10^{-3} & -6 \times 10^{-3} \end{vmatrix}$$

$$= 10^{-9} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{vmatrix}$$

$$\Rightarrow \mathbf{F} = -30\hat{i} + 32\hat{j} - 9\hat{k}$$

Hence, option (b) is correct.

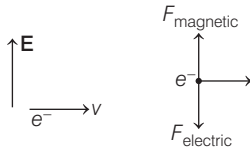
- 47** An electron is moving along +x-direction with a velocity of $6 \times 10^6 \text{ ms}^{-1}$. It enters a region of uniform electric field of 300 V/cm pointing along +y-direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x-direction will be **[2020, 6 Sep Shift-I]**

- (a) $3 \times 10^{-4} \text{ T}$, along +z-direction
 (b) $5 \times 10^{-3} \text{ T}$, along -z-direction
 (c) $5 \times 10^{-3} \text{ T}$, along +z-direction
 (d) $3 \times 10^{-4} \text{ T}$, along -z-direction

Ans. (c)

Given that, $\mathbf{v} = 6 \times 10^6 \hat{i} \text{ m/s}$,

$$\mathbf{E} = 300 \hat{j} \text{ V/cm} = 3 \times 10^4 \hat{j} \text{ V/m}$$



To keep the electron moving along x-direction,

$$\mathbf{F}_{\text{net}} = 0$$

$$\text{or } \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}} = 0$$

$$q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$$

$$\text{or } \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

It is possible when magnetic field $\mathbf{B}$ must be along +z-direction and magnitude is

$$\mathbf{B} = \frac{\mathbf{E}}{\mathbf{v}} = \frac{3 \times 10^4 \hat{j}}{6 \times 10^6 \hat{i}} = 5 \times 10^{-3} \hat{k} \text{ T}$$

- 48** A particle of mass m and charge q has an initial velocity $\mathbf{v} = v_0 \hat{j}$. If an electric field $\mathbf{E} = E_0 \hat{i}$ and magnetic field $\mathbf{B} = B_0 \hat{i}$ act on the particle, its speed will double after a time **[2020, 7 Jan Shift-II]**

- (a) $\frac{3mv_0}{qE_0}$ (b) $\frac{\sqrt{3}mv_0}{qE_0}$
 (c) $\frac{2mv_0}{qE_0}$ (d) $\frac{\sqrt{2}mv_0}{qE_0}$

Ans. (b)

Acceleration produced by electric field,

$$\mathbf{a} = \frac{qE_0}{m} \hat{i}$$

After time t , velocity of the particle,

$$\mathbf{v} = \mathbf{u} + \mathbf{at}$$

$$\Rightarrow \mathbf{v} = v_0 \hat{j} + \left(\frac{qE_0 t}{m} \right) \hat{i} \quad [\because \mathbf{u} = \mathbf{v} = v_0 \hat{j}]$$

So, speed of the particle,

$$|\mathbf{v}| = \sqrt{v_0^2 + \left(\frac{qE_0 t}{m} \right)^2}$$

Now, given $|\mathbf{v}| = 2v_0$ after time t , so

$$2v_0 = \sqrt{v_0^2 + \left(\frac{qE_0 t}{m} \right)^2}$$

$$\Rightarrow 4v_0^2 = v_0^2 + \left(\frac{qE_0 t}{m} \right)^2 \Rightarrow 3v_0^2 = \left(\frac{qE_0 t}{m} \right)^2$$

$$\text{So, time interval, } t = \frac{\sqrt{3}mv_0}{qE_0}$$

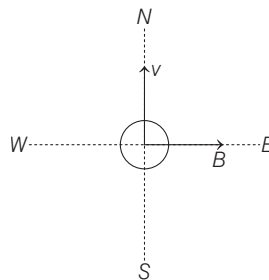
Here, we must note that no change in magnitude of velocity is caused by a perpendicular magnetic field. So, we are not taking effect of magnetic field while calculating change in speed.

- 49** Proton with kinetic energy of 1 MeV moves from south to north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field (rest mass of proton is $1.6 \times 10^{-27} \text{ kg}$) **[2020, 8 Jan Shift-I]**

- (a) 71 mT (b) 0.071 mT
 (c) 0.71 mT (d) 7.1 mT

Ans. (c)

Given, proton is moving from south to north and magnetic field is directed from west to east.



As $\mathbf{v} \perp \mathbf{B}$, force on the charged particle,

$$F = Bqv$$

If m = mass and a = acceleration of the particle, then

$$F = ma$$

$$\text{So, } ma = Bqv \text{ or } B = \frac{ma}{qv} \quad \dots(i)$$

If K = kinetic energy of the particle, then

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2K}{m}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$B = \frac{ma}{q\sqrt{\frac{2K}{m}}} = \frac{m^{3/2} \cdot a}{q\sqrt{2K}} \quad \dots(iii)$$

Here, $m = 1.6 \times 10^{-27} \text{ kg}$, $a = 10^{12} \text{ ms}^{-2}$,
 $q = 1.6 \times 10^{-19} \text{ C}$,
 $K = 1 \text{ MeV} = 1 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$
 $= 1.6 \times 10^{-13} \text{ J}$

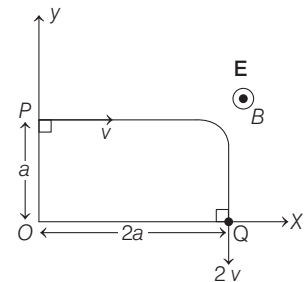
Substituting these given values in Eq. (iii), we get

$$B = \frac{(1.6 \times 10^{-27})^{3/2} \times 10^{12}}{1.6 \times 10^{-19} \times (2 \times 1.6 \times 10^{-13})^{1/2}}$$

$$= \frac{(1.6)^{3/2} \times 10^{-27 \times \frac{3}{2}} \times 10^{12}}{(1.6)^{3/2} \times 2^{1/2} \times 10^{-19} \times 10^{-13/2}}$$

$$= \frac{1}{\sqrt{2}} \times 10^{-3} = 0.71 \text{ mT}$$

- 50** A charged particle of mass m and charge q moving under the influence of uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from point P to Q as shown in the figure. The velocities at P and Q are respectively, $\hat{v}_1$ and $-2\hat{v}_1$. Then, which of the following statements (A,B,C,D) are the correct? (Trajectory shown is schematic and not to scale)



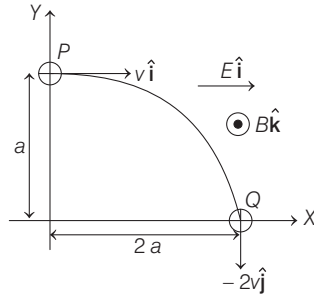
- $E = \frac{3}{4} \left(\frac{mv^2}{qa} \right)$
- Rate of work done by the electric field at P is $\frac{3}{4} \left(\frac{mv^3}{a} \right)$
- Rate of work done by both the fields at Q is zero.
- The difference between the magnitude of angular momentum of the particle at P and Q is $2mav$.

[2020, 9 Jan Shift-I]

- (a) (1), (3), (4) (b) (1), (2), (3)
 (c) (1), (2), (3), (4) (d) (2), (3), (4)

Ans. (b)

Charged particle follows trajectory as shown in the figure.



As magnetic field is perpendicular to the velocity of charged particle, work done by magnetic field is zero.

So, work done by electric field = change in kinetic energy of charged particle.

$$\Rightarrow \mathbf{F} \cdot d\mathbf{s} = \frac{1}{2} m(-2v\mathbf{j})^2 - \frac{1}{2} m(v\mathbf{i})^2$$

$$\Rightarrow (qE\mathbf{i})(2a\mathbf{i} - a\mathbf{j}) = 2mv^2 - \frac{1}{2} mv^2$$

$$\Rightarrow 2qaE = \frac{3}{2} mv^2$$

$$\Rightarrow E = \frac{3}{4} \frac{mv^2}{qa} \quad (\text{statement 1 is correct})$$

Rate of work done

$$= \text{Power} = \mathbf{F} \cdot \mathbf{v} = qE\mathbf{i} \cdot v\mathbf{i}$$

$$= qEv = q \cdot \frac{3}{4} \frac{mv^2}{qa} \cdot v$$

$$= \frac{3}{4} \frac{mv^3}{a} \quad (\text{statement 2 is correct})$$

At point Q, both electric field and magnetic field are perpendicular to velocity of charged particle, so work done by both $\mathbf{E}$ and $\mathbf{B}$ at Q is zero. (statement 3 is correct)

Angular momentum about origin at P = $mva\mathbf{i}$.

Angular momentum about origin at Q = $-4mva\mathbf{j}$.

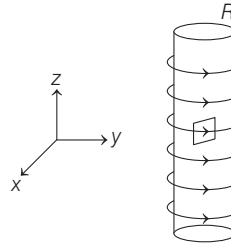
Difference between magnitudes of angular momentum at P and Q,
 $\Delta L = L_Q - L_P = 4mva - mva$
 $= 3mva$ (statement 4 is incorrect)

Thus, option (b) is correct.

- 51** An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I . The electron gun shoots an electron along the radius of the solenoid with speed v . If the electron does not hit the surface of the solenoid, maximum

possible value of v is (all symbols have their standard meaning)

[2020, 9 Jan Shift-II]

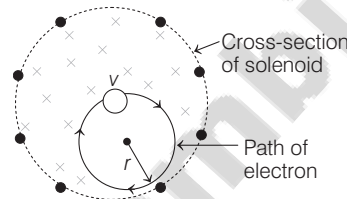


- (a) $\frac{2e\mu_0 nIR}{m}$ (b) $\frac{e\mu_0 nIR}{4m}$
 (c) $\frac{e\mu_0 nIR}{2m}$ (d) $\frac{e\mu_0 nIR}{m}$

Ans. (c)

In given arrangement, electron gun shoots electron perpendicularly to the direction of magnetic field.

Here, we are showing cross-section of solenoid.



As electron is moving at 90° to magnetic field, its path is a circle. Maximum value of radius of path taken by electron, such that it does not collide with solenoid wall is $\frac{R}{2}$.

$$\text{So, } r_{\max} = \frac{R}{2} = \frac{mv_{\max}}{eB}$$

Here,

$$B = \mu_0 nI$$

So,

$$v_{\max} = \frac{ReB}{2m} = \frac{e\mu_0 nIR}{2m}$$

- 52** A proton, an electron and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p , r_e and r_{He} be their respective radii, then

[2019, 10 April Shift-I]

- (a) $r_e < r_p = r_{\text{He}}$ (b) $r_e > r_p = r_{\text{He}}$
 (c) $r_e < r_p < r_{\text{He}}$ (d) $r_e > r_p > r_{\text{He}}$

Ans. (c)

When a moving charged particle is placed in a magnetic field $\mathbf{B}$.

Then, the net magnetic force acting on it is

$$F_m = q(v \times B)$$

$$F_m = qvB \sin \theta$$

Here, $\theta = 90^\circ$

$$F_m = qvB$$

Also, due to this net force, the particle transverses a circular path, whose necessary centripetal force is being provided by F_m , i.e.

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{mv^2}{qvB} = \frac{mv}{qB} \Rightarrow r \propto m$$

So, for electron,

$$r_e = \frac{m_e v}{eB}$$

or $r_e \propto m_e$

For proton,

$$r_p = \frac{m_p v}{eB} \text{ or } r_p \propto m_p$$

For He-particle,

$$r_{\text{He}} = \frac{4m_p v}{2eB} = \frac{2m_p v}{eB}$$

Clearly, $r_{\text{He}} > r_p$ ($\because r_{\text{He}} = 2r_p$)

and we know that, $m_p > m_e$

$$\left[\begin{array}{l} \because m_p \approx 10^{-27} \text{ kg,} \\ m_e \approx 10^{-31} \text{ kg} \end{array} \right]$$

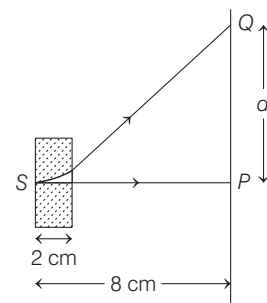
$\therefore r_p > r_e$

$\Rightarrow r_{\text{He}} > r_p > r_e$

- 53** An electron moving along the X-axis with an initial energy of 100 eV, enters a region of magnetic field $\mathbf{B} = (1.5 \times 10^{-3} \text{ T}) \hat{\mathbf{k}}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

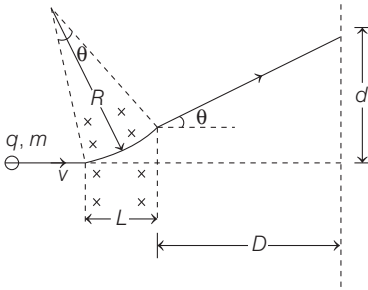
(Take, electron's charge $= 1.6 \times 10^{-19} \text{ C}$, mass of electron $= 9.1 \times 10^{-31} \text{ kg}$)

[2019, 12 April Shift-II]



- (a) 11.65 cm (b) 12.87 cm
 (c) 1.22 cm (d) 2.25 cm

Ans. (*)



When electron enters the region of magnetic field, it experiences a Lorentz force which rotates electron in a circular path of radius R .

So, Lorentz force acts like a centripetal force and we have

$$\frac{mv^2}{R} = Bqv$$

where, m = mass of electron,

q = charge of electron, v = speed of electron,

R = radius of path,

and B = magnetic field intensity.

Radius of path of electron,

$$R = \frac{mv}{Bq}$$

Now, from geometry of given arrangement, comparing values of $\tan\theta$, we have

$$\tan\theta = \frac{L}{R} = \frac{d}{D} \Rightarrow d = \frac{LD}{R} = \frac{BqLD}{mv}$$

$$\Rightarrow d = \frac{BqLD}{\sqrt{2mk}} \quad [\because mv = \sqrt{2mk}]$$

where, k = kinetic energy of electron

Here, $B = 1.5 \times 10^{-3} \text{ T}$,

$q = 1.6 \times 10^{-19} \text{ C}$, $L = 2 \times 10^{-2} \text{ m}$,

$D = 6 \times 10^{-2} \text{ m}$,

$m = 9.1 \times 10^{-31} \text{ kg}$, $k = 100 \times 1.6 \times 10^{-19} \text{ J}$

So,

$$d = \frac{(1.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2}) \times 6 \times 10^{-2}}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})}}$$

$$= \frac{28.8 \times 10^{-26}}{\sqrt{29.12 \times 10^{-48}}} = \frac{28.8 \times 10^{-26}}{5.39 \times 10^{-24}}$$

$$= 5.34 \times 10^{-2} \text{ m}$$

$$= 5.34 \text{ cm}$$

No option is matching.

- 54** A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path,

then the mass of the particle is

(Take, charge of electron

$= 1.6 \times 10^{-19} \text{ C}$) [2019, 9 Jan Shift-II]

(a) $1.6 \times 10^{-19} \text{ kg}$

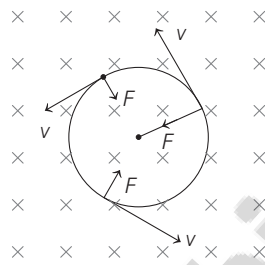
(b) $1.6 \times 10^{-27} \text{ kg}$

(c) $9.1 \times 10^{-31} \text{ kg}$

(d) $2.0 \times 10^{-24} \text{ kg}$

Ans. (d)

According to given condition, when a particle having charge same as electron move in a magnetic field on circular path, then the force always acts towards the centre and perpendicular to the velocity.



Here, $B = 0.5 \text{ T}$

R = radius of circular path = 5 cm

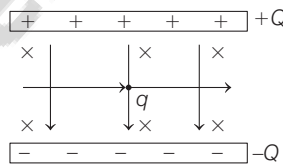
Now, the magnetic force is

$$F_m = q(v \times B)$$

$$= qvB \sin 90^\circ$$

$$F_m = qvB \quad \dots(i)$$

When the electric field applied, then the particle moves in a straight path, then this is the case of velocity selector.



Here, the electric force on charge,

$$F_e = qE \quad \dots(ii)$$

In velocity selector,

$$F_m = F_e$$

$$\Rightarrow qvB = qE \quad \dots(iii)$$

Initially particle moves under the magnetic field. So the radius of circular path taken by the particle is

$$R = \frac{mv}{qB} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$m = \frac{qB^2 R}{E}$$

$$m = \frac{1.6 \times 10^{-19} \times 0.25 \times 0.5 \times 10^{-2}}{10^2}$$

$$m = 2 \times 10^{-24} \text{ kg}$$

- 55** In an experiment, electrons are accelerated, from rest by applying a voltage of 500 V. Calculate the radius of the path, if a magnetic field 100 mT is then applied. (Take, charge of the electron $= 1.6 \times 10^{-19} \text{ C}$ and mass of the electron $= 9.1 \times 10^{-31} \text{ kg}$)

[2019, 11 Jan Shift-I]

(a) $7.5 \times 10^{-2} \text{ m}$ (b) $7.5 \times 10^{-4} \text{ m}$

(c) $7.5 \times 10^{-3} \text{ m}$ (d) 7.5 m

Ans. (b)

During the circular motion of accelerated electron in the presence of magnetic field, its radius is given by

$$r = \frac{mv}{Be} = \frac{\sqrt{2meV}}{eB}$$

where, v is velocity and V is voltage.

After substituting the given values, we get

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}}{1.6 \times 10^{-19} \times 100 \times 10^{-3}}$$

$$= 10 \left[\frac{2 \times 9.1 \times 500}{1.6} \times 10^{-12} \right]^{1/2}$$

$$r = 7.5 \times 10^{-4} \text{ m}$$

- 56** A particle of mass m and charge q is in an electric and magnetic field is given by $\mathbf{E} = 2\hat{i} + 3\hat{j}$, $\mathbf{B} = 4\hat{j} + 6\hat{k}$.

The charged particle is shifted from the origin to the point $P(x=1; y=1)$ along a straight path. The magnitude of the total work done is [2019, 11 Jan Shift-II]

(a) $(0.35)q$ (b) $(0.15)q$ (c) $(2.5)q$ (d) $5q$

Ans. (d)

$$\text{Here, } \mathbf{E} = 2\hat{i} + 3\hat{j}, \mathbf{B} = 4\hat{j} + 6\hat{k},$$

q = charge on a particle.

Initial position, $r_1 = (0,0)$

Final position, $r_2 = (1,1)$

Net force experienced by charge particle inside electromagnetic field is

$$\mathbf{F}_{\text{net}} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = q(2\hat{i} + 3\hat{j})$$

[here, $\mathbf{v} \times \mathbf{B} = 0$]

$$= (2q\hat{i} + 3q\hat{j})$$

$$\therefore dW = \mathbf{F}_{\text{net}} \cdot d\mathbf{r}$$

$$\Rightarrow \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2} (2q\hat{i} + 3q\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

[here $d\mathbf{r} = dx\hat{i} + dy\hat{j}$]

$$\Rightarrow W = 2q \int_0^1 dx + 3q \int_0^1 dy$$

$$\text{or } W = 2q + 3q \text{ or } W = 5q$$

- 57** The region between $y=0$ and $y=d$ contains a magnetic field $\mathbf{B}=B\hat{\mathbf{k}}$. A particle of mass m and charge q enters the region with a velocity $\mathbf{v}=\hat{\mathbf{i}}$. If $d=\frac{mv}{2qB}$ then the

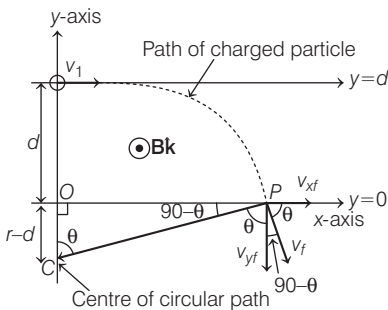
acceleration of the charged particle at the point of its emergence at the other side is

[2019, 11 Jan Shift-II]

- (a) $\frac{qvB}{m} \left(\frac{\sqrt{3}}{2} \hat{\mathbf{i}} + \frac{1}{2} \hat{\mathbf{j}} \right)$ (b) $\frac{qvB}{m} \left(\frac{1}{2} \hat{\mathbf{i}} - \frac{\sqrt{3}}{2} \hat{\mathbf{j}} \right)$
(c) $\frac{qvB}{m} \left(-\frac{\hat{\mathbf{j}} + \hat{\mathbf{i}}}{\sqrt{2}} \right)$ (d) $\frac{qvB}{m} \left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right)$

Ans. (*)

Situation given in question is shown below;



Path taken by particle of charge ' q ' and mass ' m ' is a circle of radius r where,

$$r = \frac{mv}{Bq}$$

Here final velocity

$$\begin{aligned} \mathbf{v}_f &= \mathbf{v}_x \hat{\mathbf{i}} + \mathbf{v}_y (-\hat{\mathbf{j}}) \\ &= v \cos 60^\circ \hat{\mathbf{i}} - v \sin 60^\circ \hat{\mathbf{j}} \\ &= v \left(\frac{1}{2} \hat{\mathbf{i}} - \frac{\sqrt{3}}{2} \hat{\mathbf{j}} \right) \end{aligned}$$

So, change of velocity of charged particle is

$$\begin{aligned} \Delta \mathbf{v} &= \mathbf{v}_f - \mathbf{v}_i \\ &= v \left(\frac{1}{2} \hat{\mathbf{i}} - \frac{\sqrt{3}}{2} \hat{\mathbf{j}} \right) - v \hat{\mathbf{i}} \\ &= -v \left(\frac{1}{2} \hat{\mathbf{i}} + \frac{\sqrt{3}}{2} \hat{\mathbf{j}} \right) \end{aligned}$$

It t = time taken by charged particle to cross region of magnetic field then,

$$\begin{aligned} t &= \frac{\text{distance } OP}{\text{speed in direction } OP} \\ &= \frac{r \times \frac{\sqrt{3}}{2}}{v} = \frac{\frac{mv}{Bq} \times \frac{\sqrt{3}}{2}}{v} = \frac{\sqrt{3}m}{2Bq} \end{aligned}$$

So, acceleration of charged particle at the point its emergence is;

$$\begin{aligned} \text{Acceleration } \mathbf{a} &= \frac{\Delta \mathbf{v}}{\Delta t} = \frac{-v \left(\frac{1}{2} \hat{\mathbf{i}} + \frac{\sqrt{3}}{2} \hat{\mathbf{j}} \right)}{\frac{\sqrt{3}m}{2Bq}} \\ &= \frac{-Bqv}{m} \left(\frac{\hat{\mathbf{i}}}{\sqrt{3}} + \hat{\mathbf{j}} \right) \text{ ms}^{-2} \end{aligned}$$

None of the option given matches with answer.

- 58** A proton and an α -particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V . If a uniform magnetic field B is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be [2019, 12 Jan Shift-I]

- (a) 1 : $\sqrt{2}$ (b) 1 : $\sqrt{3}$
(c) 1 : 3 (d) 1 : 2

Ans. (a)

Radius of path of charged particle q in a uniform magnetic field B of mass ' m ' moving with velocity v is

$$\begin{aligned} r &= \frac{mv}{Bq} = \frac{m \sqrt{(2qV/m)}}{Bq} \\ \Rightarrow r &\propto \frac{\sqrt{m}}{q} \text{ so, required ratio is} \\ \Rightarrow \frac{r_p}{r_\alpha} &= \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}} \\ &= \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \end{aligned}$$

- 59** An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field B . The relation between r_e, r_p, r_α is [JEE Main 2018]

- (a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$
(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

Ans. (b)

From $Bqv = \frac{mv^2}{r}$, we have

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

where, K is the kinetic energy.

As, kinetic energies of particles are same;

$$\begin{aligned} r &\propto \frac{\sqrt{m}}{q} \\ \Rightarrow r_e : r_p : r_\alpha &= \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{4m_p}}{2e} \end{aligned}$$

Clearly, $r_p = r_\alpha$ and r_e is least [$\because m_e < m_p$]
So, $r_p = r_\alpha > r_e$

- 60** Proton, deuteron and alpha particles of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α . Which one of the following relation is correct? [AIEEE 2012]

- (a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p < r_d$
(c) $r_\alpha > r_d > r_p$ (d) $r_\alpha = r_d > r_p$

Ans. (b)

For charged particle in magnetic field, radius

$$\begin{aligned} r &= \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB} \\ [\text{as } mv = p \Rightarrow K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}] \\ p &= \sqrt{2Km} \\ \Rightarrow r &\propto \frac{\sqrt{m}}{q} \text{ or } m_d = 2m_p \text{ and } q_d = q_p; \\ m_\alpha &= 4m_p \text{ and } q_\alpha = 2q_p \\ \Rightarrow r_p : r_d : r_\alpha &= \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{2m_p}}{q_p} : \frac{\sqrt{4m_p}}{2q_p} \\ &= 1 : \sqrt{2} : 1 \Rightarrow r_\alpha = r_p < r_d \end{aligned}$$

- 61** An electric charge $+q$ moves with velocity $\mathbf{v}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+\hat{\mathbf{k}}$, in an electromagnetic field given by $\mathbf{E}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, $\mathbf{B}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$. The y component of the force experienced by $+q$ is [AIEEE 2011]

- (a) $2q$ (b) $11q$ (c) $5q$ (d) $3q$

Ans. (b)

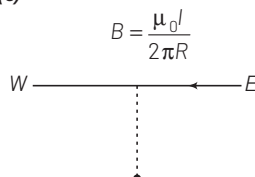
By Lorentz force, net force

$$\begin{aligned} \mathbf{F} &= q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ &= q[(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})] \\ &= q[-10\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + \hat{\mathbf{k}}] \\ \therefore F_y &= 11q \end{aligned}$$

- 62** A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from East to West. The magnetic field directly below it on the ground is (take $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$) [AIEEE 2008]

- (a) 2.5×10^{-7} T, Southward
 (b) 5×10^{-6} T, Northward
 (c) 5×10^{-6} T, Southward
 (d) 2.5×10^{-7} T, Northward

Ans. (c)



Direction is given by right hand palm rule No. 1.

$$B = \frac{2 \times 10^{-7} \times 100}{4} \text{ T, Southward}$$

$$= 5 \times 10^{-6} \text{ T, Southward}$$

- 63** A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\mathbf{E}$ and $\mathbf{B}$ with a velocity $\mathbf{v}$ perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ and comes out without any change in magnitude or direction of $\mathbf{v}$. Then, [AIEEE 2007]

- (a) $\mathbf{v} = \mathbf{E} \times \frac{\mathbf{B}}{B^2}$
 (b) $\mathbf{v} = \mathbf{B} \times \frac{\mathbf{E}}{E^2}$
 (c) $\mathbf{v} = \mathbf{E} \times \frac{\mathbf{B}}{E^2}$
 (d) $\mathbf{v} = \mathbf{B} \times \frac{\mathbf{E}}{E^2}$

Ans. (a)

As $\mathbf{v}$ of charged particle is remaining constant, it means force acting on charged particle is zero.

$$f = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$$

$$q\mathbf{E} - q(\mathbf{v} \times \mathbf{B}) = 0$$

$$\text{So, } q(\mathbf{v} \times \mathbf{B}) = q\mathbf{E}$$

$$\Rightarrow \mathbf{v} \times \mathbf{B} = \mathbf{E}$$

$$\Rightarrow \mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

- 64** A charged particle moves through a magnetic field perpendicular to its direction. Then, [AIEEE 2007]

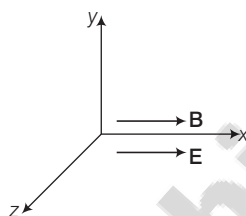
- (a) the momentum changes but the kinetic energy is constant
 (b) both momentum and kinetic energy of the particle are not constant
 (c) both momentum and kinetic energy of the particle are constant
 (d) kinetic energy changes but the momentum is constant

Ans. (a)

In case of motion of a charged particle perpendicular to the motion, i.e., displacement, the work done

$W = \int \mathbf{F} \cdot d\mathbf{s} = \int F ds \cos \theta = 0$ [as $\theta = 90^\circ$]
 and by work-energy theorem, $W = \Delta KE$, the kinetic energy and hence speed v remains constant. But v changes, so momentum changes.

- 65** In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a [AIEEE 2006]



- (a) helix
 (b) straight line
 (c) ellipse
 (d) circle

Ans. (b)

Let $\mathbf{E}$ and $\mathbf{B}$ be along x -axis. When a charged particle is released from rest, it will experience an electric force along the direction of electric field or opposite to the direction of electric field depending on the nature of charge.

Due to this force, it acquires some velocity along x -axis. Due to this motion of charge, magnetic force cannot have non-zero value because angle between $\mathbf{v}$ and $\mathbf{B}$ would be either 0° or 180° .

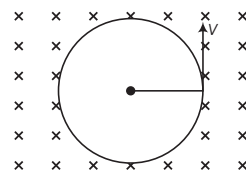
So, only electric force is acting on particle and hence it will move along a straight line.

- 66** A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B . The time taken by the particle to complete one revolution is [AIEEE 2005]

- (a) $\frac{2\pi mq}{B}$ (b) $\frac{2\pi q^2 B}{m}$
 (c) $\frac{2\pi qB}{m}$ (d) $\frac{2\pi m}{qB}$

Ans. (d)

Magnetic force, $F = qvB$ [$\because \theta = 90^\circ$]... (i)



Centripetal force, $F = \frac{mv^2}{r}$... (ii)

From Eqs. (i) and (ii), we have

$$mv^2/r = qvB \text{ or } r = mv/qB$$

The time taken by the particle to complete one revolution,

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$

- 67** A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then [AIEEE 2005]

- (a) its velocity will decrease
 (b) its velocity will increase
 (c) it will turn towards right of direction of motion
 (d) it will turn towards left of direction of motion

Ans. (a)

When $\mathbf{E}$, $\mathbf{v}$ and $\mathbf{B}$ are all along same direction, then magnetic force experienced by electron is zero while electric force is acting opposite to velocity of electron, so velocity of electron will decrease.

$$+ \begin{array}{c} \xrightarrow{\mathbf{B}} \\ \xrightarrow{\mathbf{E}} \\ \xrightarrow{(\mathbf{v})e} \end{array} \left[\begin{array}{l} \because \text{Lorentz force} = 0 \\ \therefore \theta = 0 \\ \Rightarrow qvB \sin \theta = 0 \end{array} \right]$$

- 68** A particle of mass M and charge Q moving with velocity $\mathbf{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B . The work done by the field when the particle completes one full circle is [AIEEE 2003]

- (a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
 (c) $BQ2\pi R$ (d) $BQv2\pi R$

Ans. (b)

When particle describes circular path in a magnetic field, its velocity is always perpendicular to the magnetic force.

$$\text{Power, } P = \mathbf{F} \cdot \mathbf{v} = Fv \cos \theta$$

$$\text{Here, } \theta = 90^\circ \therefore P = 0$$

$$\text{But } P = \frac{W}{t}$$

$$\Rightarrow W = P \cdot t$$

Hence, work done, $W = 0$ [everywhere]

- 69** A particle of charge -16×10^{-18} C moving with velocity 10 ms^{-1} along the X-axis enters a region where a magnetic field of induction B is along the Y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative Z-axis. If the charged particle continues moving along the X-axis, the magnitude of B is [AIEEE 2003]

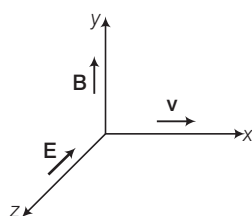
- (a) 10^3 Wb/m^2
 (b) 10^5 Wb/m^2
 (c) 10^{16} Wb/m^2
 (d) 10^{-3} Wb/m^2

Ans. (a)

The force on a particle is

$$\text{So, } \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\text{or } \mathbf{F} = \mathbf{F}_e + \mathbf{F}_m$$



$$\therefore \mathbf{F}_e = q\mathbf{E} = -16 \times 10^{-18} \times 10^4 (-\mathbf{j}) = 16 \times 10^{-14} \mathbf{k}$$

$$\begin{aligned} \text{and } \mathbf{F}_m &= -16 \times 10^{-18} (10 \mathbf{i} \times B \mathbf{j}) \\ &= -16 \times 10^{-17} \times B (+\mathbf{k}) \\ &= -16 \times 10^{-17} B \mathbf{k} \end{aligned}$$

Since, particle will continue to move along +x-axis, so resultant force is equal to zero.

$$\begin{aligned} \Rightarrow 16 \times 10^{-14} &= 16 \times 10^{-17} B \\ \Rightarrow B &= \frac{16 \times 10^{-14}}{16 \times 10^{-17}} = 10^3 \end{aligned}$$

$$\text{or } B = 10^3 \text{ Wb/m}^2$$

- 70** If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then [AIEEE 2002]

- (a) curved path of electron and proton will be same (ignoring the sense of revolution)
 (b) they will move undeflected
 (c) curved path of electron is more curved than that of proton
 (d) path of proton is more curved

Ans. (a)

Since, momenta are same and masses of electron and proton are different, so they will attain different velocities and hence experience different forces. But radius of circular path is depending on momenta, so both will be moving on same trajectory (curved path).

- 71** The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its [AIEEE 2002]

- (a) speed (b) mass
 (c) charge (d) magnetic induction

Ans. (a)

In a circular motion of a uniform magnetic field, the necessary centripetal force to the charged particle is provided by the magnetic force.

$$\text{i.e., } \frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB}$$

Thus, the time period T is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \left(\frac{mv}{qB} \right) = \frac{2\pi m}{qB}$$

So, T is independent of its speed.

TOPIC 3

Force and Torque on Current Carrying Conductor

- 72** A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5} \text{ Nm}$.

The value of x is.....

[2021, 26 Aug Shift-II]

Ans. (3)

Side of equilateral triangular coil,

$$a = 10 \text{ cm} = 0.1 \text{ m}$$

Horizontal magnetic field,

$$B = 20 \text{ mT} = 20 \times 10^{-3} \text{ T}$$

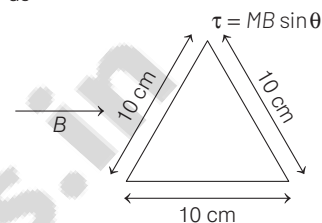
Current in coil,

$$i = 0.2 \text{ A}$$

Torque acting on coil,

$$\tau = \sqrt{x} \times 10^{-5} \text{ N-m}$$

Torque acting on coil can be calculated as



Here, M is pole strength and B is magnetic field.

Magnetic pole strength, $M = iA$

$$\tau = i \left(\frac{\sqrt{3}}{4} a^2 \right) B \sin \theta$$

$$\left[\because A = \frac{\sqrt{3}}{4} a^2 \text{ (for equilateral triangle)} \right]$$

$$= 0.2 \times \frac{\sqrt{3}}{4} \times (0.1)^2 \times 20 \times 10^{-3} \times \sin 90^\circ$$

$$[\because \mathbf{A} \perp \mathbf{B}]$$

$$= \sqrt{3} \times 10^{-5} \text{ N-m}$$

After comparing obtained value with the given value, $x = 3$.

- 73** A uniform conducting wire of length is $24a$, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side a and then in the form of a square of side a . The coil is connected to a voltage source V_0 . The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1:\sqrt{y}$. The value of where y is..... [2021, 27 Aug Shift-I]

Ans. (3)

Length of conducting wire, $l = 24a$

Resistance of wire is R .

Ratio of magnetic moment of coil for equilateral triangle to square is $1:\sqrt{y}$.

Side of equilateral triangle is a and also side of square is a .

Number of turns in equilateral triangle will be

$$n_T = \frac{24a}{3a} = 8 \quad [\because \text{Length of one turn is } 3a]$$

Number of turns in square will be

$$n_s = \frac{24a}{4a} = 6 \quad [\because \text{Length of one turn is } 4a]$$

Magnetic moment of coil is given as

$$M = nIA$$

The ratio of magnetic moment for triangular coil to square coil is given as

$$\frac{M_T}{M_S} = \frac{n_T I A_T}{n_S I A_S} \quad (\because I \text{ is same in both coils})$$

Substituting the values in above expression, we get

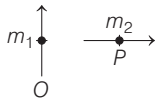
$$\frac{1}{\sqrt{y}} = \frac{\left(8 \times \frac{\sqrt{3}}{4} a^2\right)}{6 \times a^2} = \frac{8\sqrt{3}}{24} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{y} = \sqrt{3} \Rightarrow y = 3$$

Hence, the value of y is 3.

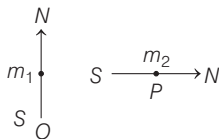
- 74** Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P , respectively. The distance between OP is 1 m. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is $\times 10^{-7} \text{ Nm}$.

[2021, 26 Aug Shift-I]

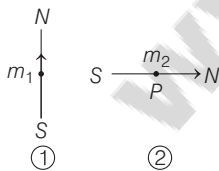


Ans. (1)

In the given figure,



The direction of magnetic moment is from South to North.



When second magnet is kept in the magnetic field produced by first magnet, the torque acting on second magnet is given by

$$\tau = M_2 \times B_1$$

$$|\tau| = M_2 B_1 \sin \theta \quad (\because \theta = 90^\circ)$$

$$|\tau| = M_2 B_1$$

Point P lies on equatorial position, so

$$B_1 = \frac{\mu_0}{4\pi} \frac{M_1}{r^3}$$

$$|\tau| = \left(\frac{\mu_0}{4\pi} \frac{M_1}{r^3} \right) M_2$$

$$M_1 = M_2 = 1 \text{ Am}^2, r = 1 \text{ m} \quad (\text{given})$$

$$|\tau| = \frac{10^{-7} \times 1 \times 1}{(1)^3}$$

$$\Rightarrow |\tau| = 10^{-7} \text{ Nm}$$

$$\Rightarrow |\tau| = 1 \times 10^{-7} \text{ Nm}$$

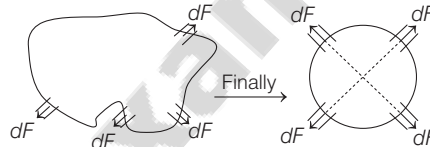
- 75** A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field.

Identify the effect of the field on the wire. [2021, 18 March Shift-I]

- Loop assumes circular shape with its plane normal to the field.
- Loop assumes circular shape with its plane parallel to the field.
- Wire gets stretched to become straight.
- Shape of the loop remains unchanged.

Ans. (a)

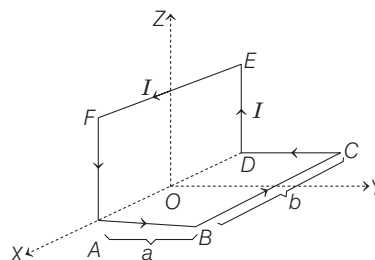
A loop of wire will act like magnetic dipole. In equilibrium, direction of dipole will be parallel to direction of magnetic field and its plane remains normal to the field.



Now on each element of wire force due to magnetic field will be perpendicular to length of element and outward in direction. So, loop will become circular. Hence, the correct option is (a).

- 76** A wire carrying current I is bent in the shape $ABCDEFA$ as shown, where rectangle $ABCD$ and $ADEFA$ are perpendicular to each other. If the sides of the rectangles are of lengths a and b , then the magnitude and direction of magnetic moment of the loop $ABCDEFA$ is

[2020, 2 Sep Shift-II]



$$(a) abI, \text{ along } \left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$$

$$(b) \sqrt{2} abI, \text{ along } \left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}} \right)$$

$$(c) \sqrt{2} abI, \text{ along } \left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \right)$$

$$(d) abI, \text{ along } \left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}} \right)$$

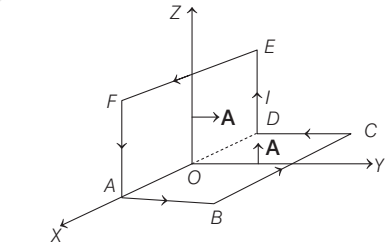
Ans. (b)

Magnetic moment is a vector quantity, $\mathbf{m} = NIA$, with its direction parallel to area vector.

Given rectangular loop can be viewed as a combination of two loops $ABCD$ and $ADEFA$.

$$\text{So, } \mathbf{m}(\text{loop}) = \mathbf{m}(ABCD) + \mathbf{m}(ADEFA)$$

The area vector of $ABCD$ is along Z -axis and that of $ADEFA$ is along Y -axis as shown in figure,



Now,

$$\mathbf{m}(ABCD) = I A \hat{k} = I(ab) \hat{k}$$

$$\text{and } \mathbf{m}(ADEFA) = I A \hat{j} = I(ab) \hat{j}$$

$$\therefore \mathbf{m}(\text{loop}) = Iab\hat{k} + Iab\hat{j} = Iab(\hat{k} + \hat{j})$$

$$= (Iab\sqrt{2}) \left(\frac{\hat{k}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} \right)$$

So, magnitude of $\mathbf{m} = \sqrt{2} Iab$ and direction of $\mathbf{m}$ is along $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$.

Hence, correct option is (b).

- 77** A galvanometer coil has 500 turns and each turn has an average area of $3 \times 10^{-4} \text{ m}^2$. If a torque of 1.5 N-m is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is

[2020, 3 Sep Shift-II]

Ans. (20)

$$\text{Torque, } \tau = BINA \sin \theta$$

$$\text{Here, } \theta = 90^\circ \Rightarrow \sin \theta = 1, \tau = 1.5 \text{ N-m}$$

$$A = 3 \times 10^{-4} \text{ m}^2, N = 500 \text{ and } I = 0.5 \text{ A}$$

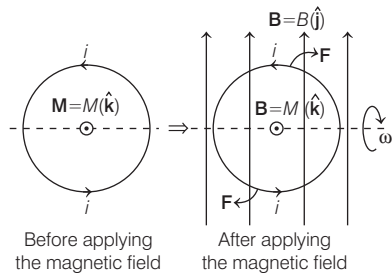
$$\text{So, } B = \frac{\tau}{INA} = \frac{1.5}{0.5 \times 500 \times 3 \times 10^{-4}} = 20 \text{ T}$$

- 78** A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 A m^2 . The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed of the coil, acquired after rotating by 60° , will be

[2020, 4 Sep Shift-II]

- (a) 10 rad s^{-1} (b) $20 \pi \text{ rad s}^{-1}$
(c) $10 \pi \text{ rad s}^{-1}$ (d) 20 rad s^{-1}

Ans. (d)



Given that, $I_{\text{dia}} = 0.8 \text{ kg m}^2$,

$$M = 20 \text{ A m}^2, B = 4 \text{ T}$$

θ = angle between M and B after rotation of 60° by coil $= 90^\circ - 60^\circ = 30^\circ$ and $\omega = ?$

Applying the law of conservation of energy,

$$(TE)_{\text{initial}} = (TE)_{\text{final}} \Rightarrow U_i + K_i = U_f + K_f$$

$$0 + 0 = -MB \cos \theta + \frac{1}{2} I_{\text{dia}} \omega^2$$

$$0 = -20 \times 4 \cos 30^\circ + \frac{1}{2} \times 0.8 \times \omega^2$$

$$0 = -20 \times 4 \times \frac{\sqrt{3}}{2} + 0.4 \omega^2$$

$$0 = -40\sqrt{3} + 0.4 \omega^2$$

$$\Rightarrow 40\sqrt{3} = 0.4 \omega^2 \Rightarrow \omega^2 = \frac{40\sqrt{3}}{0.4} = 100\sqrt{3}$$

$$= 100 \times 1.732 = 173.2$$

$$\omega = \sqrt{173.2} = 13.16 \text{ rad/s}$$

Answer is unavailable in options. So, bonus marks should be provided.

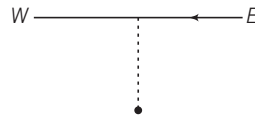
- 79** A square loop of side $2a$ and carrying current I is kept in xy -plane with its centre at origin. A long wire carrying the same current I is placed parallel to the Z -axis and passing through the point $(0, b, 0)$, ($b \gg a$). The magnitude of the torque on the loop about Z -axis is given by

[2020, 5 Sep Shift-I]

- (a) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$ (b) $\frac{2\mu_0 I^2 a^2}{\pi b}$
(c) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ (d) $\frac{\mu_0 I^2 a^2}{2\pi b}$

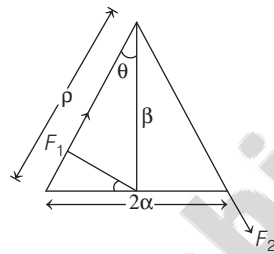
Ans. (b)

The situation given in question is shown in the following figure



$$\text{Force acting towards wire, } F_1 = \frac{\mu_0 I^2}{2\pi r} 2a$$

$$\text{Force acting away from wire, } F_2 = \frac{\mu_0 I^2}{2\pi r} 2a$$



Torque acting on the loop about Z -axis,

$$\tau = F_1 a \cos \theta + F_2 a \cos \theta$$

$$= 2 \left(\frac{\mu_0 I^2 2a}{2\pi r} \right) a \cdot \frac{b}{r} \quad \left(\because \cos \theta = \frac{b}{r} \right)$$

$$= \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)} \quad \left(\because r = \sqrt{a^2 + b^2} \right)$$

$$= \frac{2\mu_0 I^2 a^2 b}{\pi b^2} \quad (\because b \gg a)$$

$$= \frac{2\mu_0 I^2 a^2}{\pi b}$$

Hence, correct option is (b).

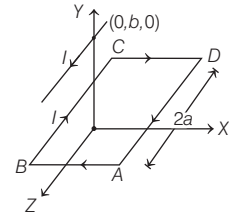
- 80** A square loop of side $2a$ and carrying current I is kept in xz -plane with its centre at origin. A long wire carrying the same current I is placed parallel to Z -axis and passing through point $(0, b, 0)$, ($b \gg a$). The magnitude of torque on the loop about Z -axis will be

[2020, 6 Sep Shift-II]

- (a) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (b) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$
(c) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2 + b^2)}$ (d) $\frac{\mu_0 I^2 a^2}{2\pi b}$

Ans. (a)

According to question, the situation can be drawn as



Force of a current carrying conducting wire, $F = BIl \sin \theta$

So, force acting on wire AB and CD are zero ($\because \theta = 0^\circ$ and $\sin 0^\circ = 0$).

Only sides BC and AD will experience force due to current carrying wire.

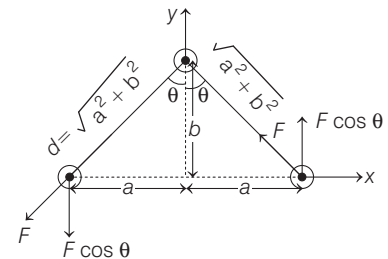
Force on side

$$AD = \text{Force on side } BC = F \text{ (say)}$$

$$\text{Net force on loop, } F = BIl_{\text{loop}} \sin 90^\circ$$

$$= \left(\frac{\mu_0 I}{2\pi d} \right) I(2a)$$

$$F = \frac{\mu_0 I^2 a}{\pi \sqrt{a^2 + b^2}} \quad \because d = \sqrt{a^2 + b^2}$$



Force on AD and BC acts in two different directions as shown in above figure. So, torque is produced in square loop.

$$\therefore \text{Torque on loop} = F \cos \theta \times 2a$$

$$= \frac{\mu_0 I^2 a}{\pi \sqrt{a^2 + b^2}} \times \frac{b}{\sqrt{a^2 + b^2}} \times 2a$$

$$= \frac{2\mu_0 I^2 a^2 b}{\pi(a^2 + b^2)}$$

$$\text{If } b \gg a, \text{ then } a^2 + b^2 \approx b^2$$

$$\therefore \tau = \frac{2\mu_0 I^2 a^2}{\pi b}$$

Hence, correct option is (a).

- 81** A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field B . The magnetic moment of this moving particle

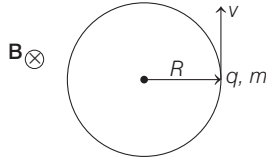
[2020, 6 Sep Shift-II]

$$(a) \frac{mv^2 B}{2B^2} \quad (b) -\frac{mv^2 B}{2\pi B^2}$$

$$(c) -\frac{mv^2 B}{B^2} \quad (d) -\frac{mv^2 B}{2B^2}$$

Ans. (d)

Let us consider the direction of magnetic field be perpendicular to the plane in inward direction as shown below.



Current produced by the motion of charge particle, $i = \frac{q}{T} = \frac{qv}{2\pi R}$

If motion of a charged particle is considered as a current loop, then magnetic moment of charged particle

$$M = iA = \frac{qv}{2\pi R} \times \pi R^2 \Rightarrow M = \frac{1}{2} qvR \quad \dots(i)$$

As we know that, while moving in a circle, necessary centripetal force is provided by magnetic force.

$$\text{i.e., } \frac{mv^2}{R} = qvB \text{ or } R = \frac{mv}{qB}$$

Putting this value in Eq. (i), we get

$$M = \frac{1}{2} qv \left(\frac{mv}{qB} \right) \text{ or } M = \frac{mv^2}{2B}$$

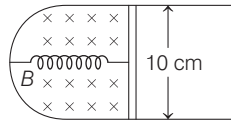
Here the direction of M is determined by right hand thumb rule which is perpendicular to plane but in outward direction, i.e., opposite to that of $\mathbf{B}$.

$$\therefore \mathbf{M} = \frac{mv^2}{2B} (-\hat{\mathbf{B}}) \text{ or } \mathbf{M} = -\frac{mv^2 \mathbf{B}}{2B^2}$$

Hence, correct option is (d).

- 82** A thin strip 10 cm long is on an U-shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N . If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to

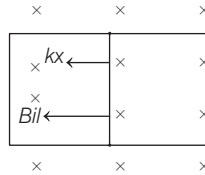
[2019, 8 April Shift-I]



- (a) 1000 (b) 50000
(c) 5000 (d) 10000

Ans. (c)

There are two forces on slider.



Spring force = kx

where, k = spring constant.

As the slider is kept in a uniform magnetic field $B = 0.1 \text{ T}$, hence it will experience a force, i.e.

Magnetic force = Bil

where, l = length of the strip.

Now, using

$$F_{\text{net}} = ma$$

We have,

$$\begin{aligned} (-kx) + (-Bil) &= ma \\ \Rightarrow -kx - Bil - ma &= 0 \\ \Rightarrow -kx - \frac{B^2 l^2}{R} v - ma &= 0 \quad \left[\because i = \frac{Blv}{R} \right] \end{aligned}$$

$$\text{and acceleration, } a = \frac{d^2 x}{dt^2}$$

Hence, the modified equation becomes

$$\Rightarrow \frac{md^2 x}{dt^2} + \frac{B^2 l^2}{R} \left(\frac{dx}{dt} \right) + kx = 0$$

This is the equation of damped simple harmonic motion.

So, amplitude of oscillation varies with time as

$$A = A_0 e^{-\frac{B^2 l^2}{2Rm} t}$$

Now, when amplitude is $\frac{A_0}{e}$, then

$$\frac{A_0}{e} = \frac{A_0}{e^{\frac{B^2 l^2}{2Rm} t}} \quad (\text{as given})$$

$$\Rightarrow \left(\frac{B^2 l^2}{2Rm} \right) t = 1$$

$$\text{or } t = \frac{2Rm}{B^2 l^2}$$

According to the question, magnetic

field $B = 0.1 \text{ T}$, mass of strip

$m = 50 \times 10^{-3} \text{ kg}$,

resistance $R = 10 \Omega$,

$$l = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

$$\therefore t = \frac{2Rm}{B^2 l^2} = \frac{2 \times 10 \times 50 \times 10^{-3}}{(0.1)^2 \times (10 \times 10^{-2})^2}$$

$$= \frac{1}{10^{-4}} = 10000 \text{ s}$$

Given, spring constant, $k = 0.5 \text{ Nm}^{-1}$

Also, time period of oscillation is

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{50 \times 10^{-3}}{0.5}}$$

$$= \frac{2\pi}{\sqrt{10}} \approx 2 \text{ s}$$

So, number of oscillations is

$$N = \frac{t}{T} = \frac{10000}{2} = 5000$$

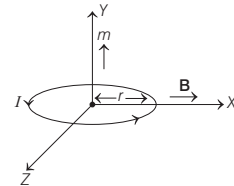
- 83** A circular coil having N turns and radius r carries a current I . It is held in the XZ -plane in a magnetic field $\hat{\mathbf{B}}$. The torque on the coil due to the magnetic field (in N-m) is

[2019, 8 April Shift-I]

- (a) $\frac{Br^2 I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\frac{B\pi r^2 I}{N}$ (d) Zero

Ans. (b)

According to the question, the situation can be drawn as



Let the current I is flowing in anti-clockwise direction, then the magnetic moment of the coil is

$$m = NIA$$

where, N = number of turns in coil

and A = area of each coil = πr^2 .

Its direction is perpendicular to the area of coil and is along Y -axis.

Then, torque on the current coil is

$$\tau = m \times B = mB \sin 90^\circ = NIA B$$

$$= N I \pi r^2 B (\text{N-m})$$

- 84** A rectangular coil (dimension $5 \text{ cm} \times 2.5 \text{ cm}$) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the XZ plane. A magnetic field of 1 T is applied along X -axis. If the coil is tilted through 45° about Z -axis, then the torque on the coil is

[2019, 9 April Shift-I]

- (a) 0.27 N-m (b) 0.38 N-m
(c) 0.42 N-m (d) 0.55 N-m

Ans. (a)

Given,

Area of the rectangular coil,

$$A = 5 \text{ cm} \times 2.5 \text{ cm}$$

$$\Rightarrow A = 12.5 \text{ cm}^2 = 12.5 \times 10^{-4} \text{ m}^2$$

Number of turns, $N = 100$ turns

Current through the coil, $I = 3 \text{ A}$

Magnetic field applied, $B = 1 \text{ T}$

Angle between the magnetic field and area vector of the coil, $\theta = 45^\circ$

As we know that, when a coil is tilted by an

angle θ in the presence of some external magnetic field, then the net torque experienced by the coil is,

$$\tau = \mathbf{M} \times \mathbf{B} = NI(\mathbf{A} \times \mathbf{B})$$

$$= NIAB \sin \theta$$

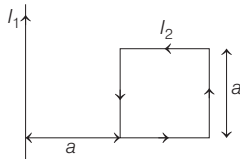
Substituting the given values, we get

$$\tau = 100 \times 3 \times 12.5 \times 10^{-4} \times 1 \times \sin 45^\circ$$

$$\tau = 0.707 \times 100 \times 3 \times 12.5 \times 10^{-4} \text{ N-m}$$

$$= 2.651 \times 10^{-1} \text{ N-m} \approx 0.27 \text{ N-m}$$

- 85** A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be **[2019, 9 April Shift-I]**



- (a) repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$
 (b) attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$
 (c) zero
 (d) repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

Ans. (d)

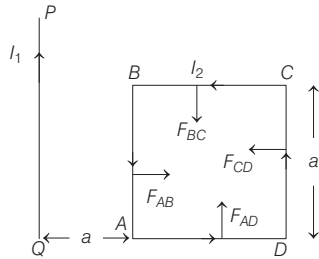
Key Idea Net force experienced by two wires separated by same distance is attractive, if current flow in them in same direction. However, this force is repulsive in nature, if current in them flows in opposite direction.

Force on a wire 1 in which current I_1 is flowing due to another wire 2 which are separated by a distance r is given as

$$F = I_1 (l \times B_2)$$

$$\text{or } F = \frac{\mu_0 I_1 I_2}{2\pi r} l \sin \theta \quad \left[\because B_2 = \frac{\mu_0 I_2}{2\pi r} \right]$$

Thus, the given square loop can be drawn as shown below



$$F_{AB} = \frac{\mu_0 I_1}{2\pi a} \cdot I_2 a \quad (\text{away from wire PQ})$$

$$F_{BC} = F_{AD} = 0 \quad [\because \theta = 0^\circ]$$

$$F_{CD} = \frac{\mu_0 I_1}{2\pi(2a)} \cdot I_2 a$$

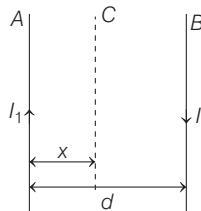
$$= \frac{\mu_0 I_1}{4\pi a} \cdot I_2 a \quad (\text{towards the wire PQ})$$

$$\therefore F_{\text{net}} = F_{AB} - F_{CD}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \quad (\text{away from wire})$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \quad (\text{repulsive in nature})$$

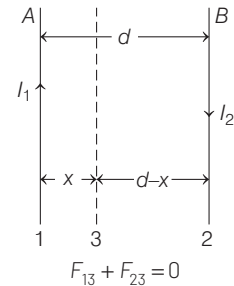
- 86** Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d . A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are **[2019, 10 April Shift-I]**



- (a) $x = \left(\frac{I_2}{I_1 + I_2} \right) d$ and $x = \left(\frac{I_2}{I_1 - I_2} \right) d$
 (b) $x = \left(\frac{I_1}{I_1 - I_2} \right) d$ and $x = \left(\frac{I_2}{I_1 + I_2} \right) d$
 (c) $x = \left(\frac{I_1}{I_1 + I_2} \right) d$ and $x = \left(\frac{I_2}{I_1 - I_2} \right) d$
 (d) $x = \pm \frac{I_1 d}{(I_1 - I_2)}$

Ans. (d)

Net force on the third wire, carrying current I in the following first case is



$$F_{13} + F_{23} = 0$$

Using thumb rule, direction of B at inside region of wires A and B will be same.

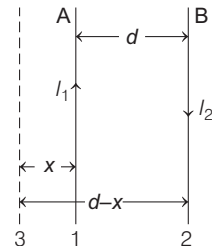
$$\therefore \frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi(d-x)} = 0$$

$$\Rightarrow \frac{I_1}{x} + \frac{I_2}{d-x} = 0 \Rightarrow \frac{I_1}{x} = -\frac{I_2}{d-x}$$

$$\text{or } (x-d)I_1 = xI_2 \Rightarrow x(I_1 - I_2) = dI_1$$

$$\Rightarrow x = \frac{I_1}{(I_1 - I_2)} \cdot d \quad \dots (i)$$

Second case of balanced force can be as shown



Using thumb rule, directions of B at any point on wires A and B will be opposite, so net force,

$$\frac{\mu_0 I_1 I}{2\pi x} - \frac{\mu_0 I_2 I}{2\pi(d+x)} = 0 \text{ or } \frac{I_1}{x} - \frac{I_2}{(d+x)} = 0$$

$$\Rightarrow \frac{I_1}{x} = \frac{I_2}{d+x}$$

$$\Rightarrow (d+x)I_1 = xI_2$$

$$\Rightarrow (I_2 - I_1)x = dI_1$$

$$\Rightarrow x = -\frac{I_1}{(I_1 - I_2)} \cdot d \quad \dots (ii)$$

From Eqs. (i) and (ii), it is clear that

$$x = \pm \frac{I_1}{(I_1 - I_2)} d$$

- 87** A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m . If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop (in A-m) will be **[2019, 10 April Shift-II]**

- (a) $\frac{4m}{\pi}$ (b) $\frac{3m}{\pi}$
 (c) $\frac{2m}{\pi}$ (d) $\frac{m}{\pi}$

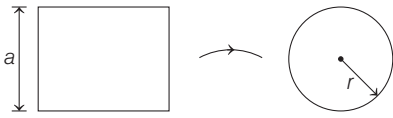
Ans. (a)

Key Idea Magnetic dipole moment of a current carrying loop is
 $m = IA$ (A - m²)

where, I = current in loop and A = area of loop.

Let the given square loop has side a , then its magnetic dipole moment will be
 $m = Ia^2$

When square is converted into a circular loop of radius r ,



Then, wire length will be same in both area,

$$\Rightarrow 4a = 2\pi r \Rightarrow r = \frac{4a}{2\pi} = \frac{2a}{\pi}$$

Hence, area of circular loop formed is,
 $A' = \pi r^2$

$$= \pi \left(\frac{2a}{\pi} \right)^2 = \frac{4a^2}{\pi}$$

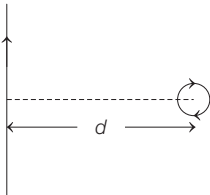
Magnitude of magnetic dipole moment of circular loop will be

$$m' = IA' = I \frac{4a^2}{\pi}$$

Ratio of magnetic dipole moments of both shapes is,

$$\frac{m'}{m} = \frac{I \cdot \frac{4a^2}{\pi}}{Ia^2} = \frac{4}{\pi} \Rightarrow m' = \frac{4m}{\pi} \text{ (A - m)}$$

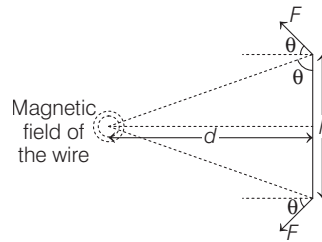
- 88** An infinitely long current-carrying wire and a small current-carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d ($d \gg a$). If the loop applies a force F on the wire, then
 [2019, 9 Jan Shift-I]



- (a) $F \propto \left(\frac{a^2}{d^3} \right)$ (b) $F = 0$
 (c) $F \propto \left(\frac{a}{d} \right)$ (d) $F \propto \left(\frac{a}{d} \right)^2$

Ans. (d)

In the given condition, the current-carrying loop is at a very large distance from the long current-carrying conducting wire. Thus, it can be considered as a dipole (a magnet with north pole facing in upward direction and south in the downward direction). Suppose the effective length of this dipole be l' . Thus, the top view of the condition can be shown in the figure given below.



Now, the net force on the loop (i.e. at the two poles) due to the wire is given as,

$$F_{\text{net}} = 2F \cos \theta = 2mB \cos \theta$$

where, m is the pole strength.

From the figure, we have

$$\cos \theta = \frac{\frac{l'}{2}}{\sqrt{d^2 + \frac{l'^2}{4}}}$$

$$\Rightarrow F_{\text{net}} = \frac{2mBI}{2\sqrt{d^2 + \frac{l'^2}{4}}} \quad \dots(i)$$

Since, the magnetic moment of a loop of radius r is

$$M = IA = I\pi r^2 = ml \quad \dots(ii)$$

and magnetic field due to a straight infinitely long current-carrying conductor at a distance

$$x \text{ is } B = \frac{\mu_0 I'}{2\pi x} \quad \dots(iii)$$

$\therefore$ Using Eqs. (ii) and (iii), rewriting Eq. (i), we get

$$F_{\text{net}} = \frac{I\pi a^2 \mu_0 I'}{2\pi \left(\sqrt{d^2 + \frac{l'^2}{4}} \right)^2} \Rightarrow F_{\text{net}} \propto \frac{a^2}{\left(d^2 + \frac{l'^2}{4} \right)}$$

l' can be neglected as the loop is kept at very large distance.

$$\text{or } F_{\text{net}} \propto \left(\frac{a}{d} \right)^2$$

- 89** An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the

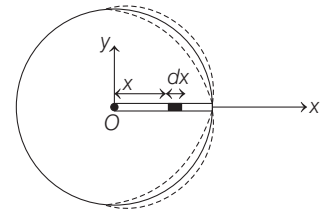
origin ($x=0$) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is
 [2019, 10 Jan Shift-I]

- (a) $n\rho l^3$
 (b) $\pi n\rho l^3$
 (c) $\frac{\pi}{3} n\rho l^3$
 (d) $\frac{\pi}{4} n\rho l^3$

Ans. (d)

Key Idea A rotating charge constitutes a current. Hence, a rotating charged rod behaves like a current carrying coil. If charge q rotates with a frequency n , then equivalent current is $I = qn$ and magnetic moment associated with this current is $M = IA$ where, A = area of coil or area swept by rotating rod.

Let dq be the charge on dx length of rod at a distance x from origin as shown in the figure below.



The magnetic moment dm of this portion dx is given as

$$dm = (dl) A$$

$$dm = n dq A \quad \because I = qn, \therefore dl = n \cdot dq$$

$$= n \rho dx A$$

where, ρ = charge density of rod = $\rho_0 \frac{x}{l}$.

$$\text{So, } dm = \frac{n\rho_0 x dx}{l} \cdot \frac{\pi x^2}{l} = \frac{\pi n\rho_0}{l} \cdot x^3 \cdot dx$$

Total magnetic moment associated with rotating rod is sum of all the magnetic moments of such differentiable elements of rod.

So, magnetic moment associated with complete rod is

$$M = \int_{x=0}^l dm = \int_0^l \frac{\pi n\rho_0}{l} \cdot x^3 \cdot dx$$

$$= \frac{\pi n\rho_0}{l} \cdot \int_0^l x^3 \cdot dx = \frac{\pi n\rho_0}{l} \left[\frac{x^4}{4} \right]_0^l$$

$$= \frac{\pi n\rho_0 l^3}{4}$$

at

$$x = l$$

$$\rho = \rho_0$$

$\therefore$

$$M = \frac{\pi}{4} n\rho l^3$$

- 90** The dipole moment of a circular loop carrying a current I , is m and the magnetic field at the centre of the loop is B_1 . When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2 . The ratio $\frac{B_1}{B_2}$ is

[JEE Main 2018]

- (a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

Ans. (c)

Key Idea As $m = IA$, so to change dipole moment (current is kept constant), we have to change radius of loop.

Initially, $m = I\pi R_1^2$ and $B_1 = \frac{\mu_0 I}{2R_1}$

Finally, $m' = 2m = I\pi R_2^2$

$\Rightarrow 2\pi R_1^2 = \pi R_2^2$ or $R_2 = \sqrt{2}R_1$

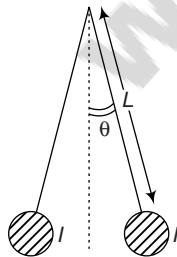
So, $B_2 = \frac{\mu_0 I}{2(R_2)} = \frac{\mu_0 I}{2\sqrt{2}R_1}$

Hence, ratio $\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{2R_1}\right)}{\left(\frac{\mu_0 I}{2\sqrt{2}R_1}\right)} = \sqrt{2}$

$\therefore$ Ratio $\frac{B_1}{B_2} = \sqrt{2}$

- 91** Two long current carrying thin wires, both with current I , are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is (g = gravitational acceleration)

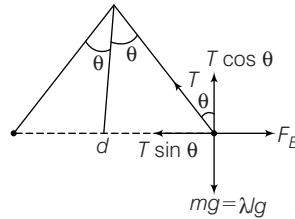
[JEE Main 2015]



- (a) $\sin\theta \sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$ (b) $2\sin\theta \sqrt{\frac{\pi\lambda gL}{\mu_0 \cos\theta}}$
 (c) $2\sqrt{\frac{\pi gL}{\mu_0}} \tan\theta$ (d) $\sqrt{\frac{\pi\lambda gL}{\mu_0}} \tan\theta$

Ans. (b)

Consider free body diagram of the wire. As the wires are in equilibrium, they must carry current in opposite direction.



Here, $F_B = \frac{\mu_0 I^2 l}{2\pi d}$, where l is length of each wire and d is separation between wires.

From figure, $d = 2L \sin\theta$

$T \cos\theta = mg = \lambda l g$
 (in vertical direction)...(i)

$T \sin\theta = F_B = \frac{\mu_0 I^2 l}{4\pi L \sin\theta}$
 (in horizontal direction)...(ii)

From Eqs. (i) and (ii),

$\frac{T \sin\theta}{T \cos\theta} = \frac{\mu_0 I^2 l}{4\pi L \sin\theta \times \lambda l g}$

$\therefore$

$I = \sqrt{\frac{4\pi\lambda L g \sin^2\theta}{\mu_0 \cos\theta}}$
 $= 2\sin\theta \sqrt{\frac{\pi\lambda L g}{\mu_0 \cos\theta}}$

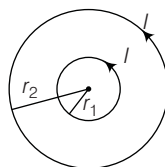
- 92** Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one.

Then, [JEE Main 2015]

- (a) $\mathbf{F}_1 = \mathbf{F}_2 = 0$
 (b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards
 (c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$
 (d) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$

Ans. (a)

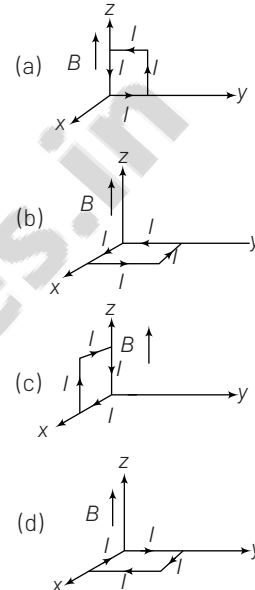
Consider the two coaxial solenoids. Due to one of the solenoids magnetic field at the centre of the other can be assumed to be constant.



Due to symmetry, forces on upper and lower part of a solenoid will be equal and opposite and hence resultant is zero. Therefore, $F_1 = F_2 = 0$

- 93** A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below.

[JEE Main 2015]



If there is a uniform magnetic field of 0.3 T in the positive z -direction in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

[JEE Main 2015]

- (a) (a) and (b) respectively
 (b) (a) and (c) respectively
 (c) (b) and (d) respectively
 (d) (b) and (c) respectively

Ans. (c)

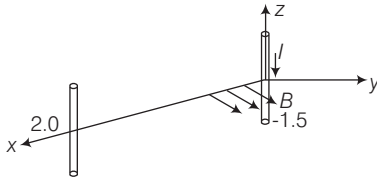
Since B is uniform only torque acts on a current carrying loop.

As, $\tau = \mathbf{M} \times \mathbf{B} \Rightarrow |\tau| = |\mathbf{M}| |\mathbf{B}| \sin\theta$

For orientation shown in (b) $\theta = 0^\circ$, $\tau = 0$ (stable equilibrium) and for (d) $\theta = \pi$, $\tau = 0$ (unstable equilibrium)

- 94** A conductor lies along the z -axis at $-1.5 \leq z \leq 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3} s. Assume parallel motion along the x -axis.

[JEE Main 2014]



- (a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

Ans. (b)

Force exerted on a current carrying conductor,

$$F_{\text{ext}} = B(x) IL$$

$$\text{Average power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \frac{1}{t} \int_0^t F_{\text{ext}} \cdot dx = \frac{1}{t} \int_0^t B(x) IL dx$$

$$= \frac{1}{5 \times 10^{-3}} \int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3 dx$$

$$= 9 [1 - e^{-0.4}]$$

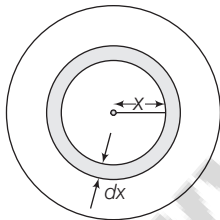
$$= 9 \left[1 - \frac{1}{e^{0.4}} \right] = 2.967 \approx 2.97 \text{ W}$$

- 95** A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed ω . The magnetic moment of the disk is [AIEEE 2011]

- (a) $2\pi R^4 \sigma \omega$ (b) $\pi R^4 \sigma \omega$
(c) $\frac{\pi R^4}{2} \sigma \omega$ (d) $\frac{\pi R^4}{4} \sigma \omega$

Ans. (d)

Let us consider the disc to be made up of large number of concentric elementary rings.



Consider one such ring of radius x and thickness dx . Charge on this elementary ring,

$$dq = \sigma \times 2\pi x dx = 2\pi \sigma x dx$$

Current associated with this elementary ring,

$$dI = \frac{dq}{dt} = dq \times f = \sigma \omega x dx$$

$$[\because f \text{ is frequency and } \omega = 2\pi f]$$

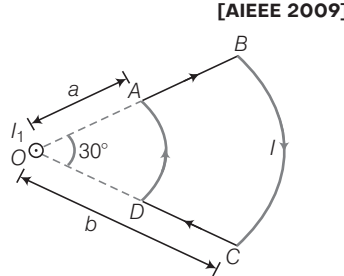
Magnetic moment of this elementary ring,

$$dM = dI \pi x^2 = \pi \sigma \omega x^3 dx$$

$\therefore$ Magnetic moment of the entire disc,

$$M = \int_0^R dM = \pi \sigma \omega \int_0^R x^3 dx = \frac{1}{4} \pi R^4 \sigma \omega$$

- 96** Due to the presence of the current I_1 at the origin in the figure, [AIEEE 2009]



- (a) the forces on AB and DC are zero
(b) the forces on AD and BC are zero
(c) the magnitude of the net force on the loop is given by $\frac{\mu_0 I_1}{4\pi} \left[2(b-a) + \frac{\pi}{3}(a+b) \right]$

- (d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I_1}{24ab}(b-a)$

Ans. (b)

The forces on AD and BC are zero because magnetic field due to a straight wire on AD and BC is parallel to elementary length of the loop.

- 97** Two thin, long, parallel wires, separated by a distance d carry a current of I ampere in the same direction. They will [AIEEE 2005]

- (a) attract each other with a force of

$$\frac{\mu_0 I^2}{(2\pi d)}$$

- (b) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

- (c) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

- (d) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

Ans. (a)

The force per unit length between the two wires is $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2I^2}{d} = \frac{\mu_0 I^2}{2\pi d}$

The force will be attractive as current directions in both are same.

- 98** Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now, the current in one of them is increased to two times and its direction is reversed. The distance is also

increased to $3d$. The new value of the force between them is

- (a) $-2F$ (b) $\frac{F}{3}$ (c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$ [AIEEE 2004]

Ans. (c)

Force acting between two current carrying conductors

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l \quad \dots(i)$$

where d = distance between the conductors

l = length of small conductor

$$\text{Again, } F' = \frac{\mu_0}{2\pi} \frac{(-2I_1 I_2)}{(3d)} \cdot l$$

$$= -\frac{\mu_0}{2\pi} \frac{2I_1 I_2}{3d} \cdot l \quad \dots(ii)$$

Thus, from Eqs. (i) and (ii), we have

$$\frac{F'}{F} = -\frac{2}{3} \Rightarrow F' = -\frac{2}{3} F$$

- 99** A magnetic needle lying parallel to a magnetic field requires W unit of work to turn it through 60° . The torque needed to maintain the needle in this position will be [AIEEE 2003]

- (a) $\sqrt{3}W$ (b) W (c) $(\sqrt{3}/2)W$ (d) $2W$

Ans. (a)

$$W = MB(1 - \cos\theta)$$

$$\Rightarrow W = MB(1 - \cos 60^\circ) \quad [\because \theta = 60^\circ]$$

$$\text{or } W = \frac{MB}{2} \text{ or } MB = 2W$$

Torque, $\tau = MB \sin 60^\circ$

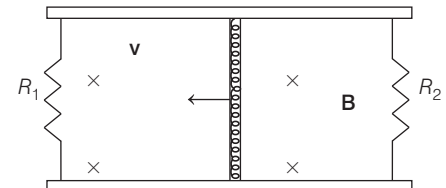
$$= \frac{MB\sqrt{3}}{2} = \frac{2W\sqrt{3}}{2} = W\sqrt{3}$$

- 100** If a current is passed through a spring, then the spring will

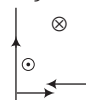
[AIEEE 2002]

- (a) expand (b) compress
(c) remain same (d) None of these

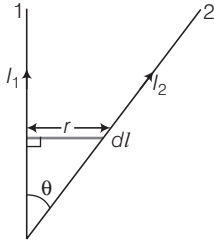
Ans. (b)



Due to flow of current in same direction in two adjacent sides, an attractive magnetic force will be produced due to which spring will get compressed.



- 101** Wires 1 and 2 carrying currents I_1 and I_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1? [AIEEE 2002]



- (a) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \tan \theta$ (b) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \sin \theta$
 (c) $\frac{\mu_0}{2\pi r} I_1 I_2 dl \cos \theta$ (d) $\frac{\mu_0}{4\pi r} I_1 I_2 dl \sin \theta$

Ans. (c)

The component $dl \cos \theta$ of element dl is parallel to the length of the wire 1. Hence, force on this elemental component

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} (dl \cos \theta) \\ = \frac{\mu_0}{2\pi r} I_1 I_2 dl \cos \theta$$

TOPIC 4

Moving Coil Galvanometer

- 102** Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer? [2021, 31 Aug Shift-I]
 (a) 300Ω (b) 344Ω (c) 245Ω (d) 226Ω

Ans. (c)

Given,

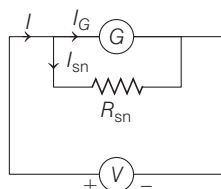
Shunt resistance, $R_{sn} = 5 \Omega$

Let galvanometer resistance = G

Total, current = I

Voltage across galvanometer (V_g) =

Voltage across shunt resistance R_{sn}



$$I_g \times G = I_{sn} \times R_{sn} \\ 0.02I \times G = 0.98I \times 5 \\ \Rightarrow G = \frac{98 \times 5}{2} = 49 \times 5 = 245 \Omega$$

- 103** For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be [2021, 27 Aug Shift-II]

- (a) 1Ω (b) 5Ω
 (c) 4Ω (d) 2Ω

Ans. (d)

Given, number of divisions, $n = 50$

Voltage, $V = 50 \text{ mV} = 50 \times 10^{-3} \text{ V}$

Current sensitivity, $k = \frac{2 \text{ div}}{\text{mA}} = \frac{2 \text{ div}}{10^{-3} \text{ A}}$

$$\therefore \text{Current, } I = \frac{n}{k} = \frac{50 \times 10^{-3}}{2} = 25 \times 10^{-3} \text{ A}$$

By using Ohm's law,

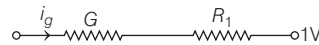
$$\text{Resistance (R)} = \frac{\text{Potential (V)}}{\text{Current (I)}} \\ = \frac{50 \times 10^{-3}}{25 \times 10^{-3}} = 2 \Omega$$

- 104** A galvanometer of resistance G is converted into a voltmeter of range 0-1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0-2V will be [2020, 5 Sep Shift-I]

- (a) $R_1 + G$ (b) R_1
 (c) G (d) $R_1 - G$

Ans. (a)

According to question,

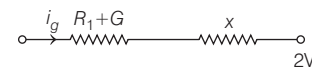


From Ohm's law, $V = IR$

$$\therefore i_g (R_1 + G) = 1 \text{ V} \quad \dots (i)$$

Let x be the required resistance.

$$\text{Now, } i_g (R_1 + G + x) = 2 \text{ V}$$



Using eq. (i) in above equation, we get

$$\frac{1}{(R_1 + G)} (R_1 + G + x) = 2$$

$$R_1 + x + G = 2(R_1 + G) \\ x = R_1 + G$$

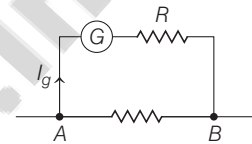
Hence, correct option is (a).

- 104** A galvanometer having a coil resistance 100Ω gives a full scale deflection when a current of 1 mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V? [2020, 8 Jan Shift-II]

- (a) $8.9 \text{ k}\Omega$ (b) $10 \text{ k}\Omega$ (c) $9.9 \text{ k}\Omega$ (d) $7.9 \text{ k}\Omega$

Ans. (c)

Here, I_g = full scale deflection current
 $= 1 \text{ mA} = 1 \times 10^{-3} \text{ A}$



V_{AB} = full scale deflection potential = 10 V

G = Galvanometer resistance = 100Ω

If R = resistance used in series for converting galvanometer into voltmeter.

Then, from above circuit diagram,

$$I_g (G + R) = V \Rightarrow 10^{-3} (100 + R) = 10$$

$$\Rightarrow R = 10000 - 100 = 9900 \Omega = 9.9 \text{ k}\Omega$$

- 106** A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a $5 \text{ k}\Omega$ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to [2019, 9 April Shift-I]

- (a) 40 V (b) 10 V (c) 15 V (d) 20 V

Ans. (d)

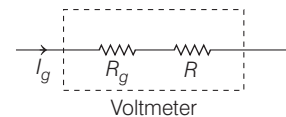
Given, resistance of galvanometer, $R_g = 50 \Omega$

Current, $I_g = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}$

Resistance used in converting a

galvanometer in voltmeter, R

$$= 5 \text{ k}\Omega = 5 \times 10^3 \Omega$$



$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\therefore E = I_g (R + R_g) \\ = 4 \times 10^{-3} \times (5 \times 10^3 + 50) \\ = 5050 \times 4 \times 10^{-3} = 20.2 \text{ V} \approx 20 \text{ V}$$

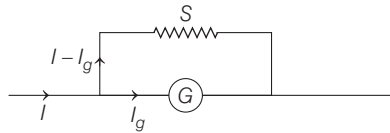
- 107** The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0-0.5 A? [2019, 9 April Shift-II]

- (a) 0.2 ohm (b) 0.5 ohm
(c) 0.002 ohm (d) 0.02 ohm

Ans. (a)

Key Idea An ammeter is a type of galvanometer with a shunt connected in parallel to the galvanometer.

Ammeter circuit is shown in the figure below



$$\text{So, } I_g G = (I - I_g) S$$

$$\text{Here, } I_g = 0.002 \text{ A,}$$

$$I = 0.5 \text{ A,}$$

$$G = 50 \Omega$$

So, shunt resistance required is

$$S = \frac{I_g G}{I - I_g} = \frac{0.002 \times 50}{(0.5 - 0.002)} \approx 0.2 \Omega$$

- 108** A moving coil galvanometer has a coil with 175 turns and area 1 cm^2 . It uses a torsion band of torsion constant 10^{-6} N-m/rad . The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in tesla) is approximately [2019, 9 April Shift-II]

- (a) 10^{-3} (b) 10^{-4}
(c) 10^{-1} (d) 10^{-2}

Ans. (a)

In a moving coil galvanometer in equilibrium, torque on coil due to current is balanced by torque of torsion band.

As, torque on coil,

$$\tau = M \times B = NIAB \sin \alpha$$

where, B = magnetic field strength,

I = current, N = number of turns of coil

Since, plane of the coil is parallel to the field.

$$\therefore \alpha = 90^\circ \Rightarrow \tau = NIBA$$

Torque of torsion band, $T = k\theta$

where, k = torsion constant of torsion band

and θ = deflection of coil in radians or angle of twist of restoring torque.

$$\therefore BINA = k\theta \text{ or } B = \frac{k\theta}{INA} \quad \dots (i)$$

$$\text{Here, } k = 10^{-6} \text{ N-m/rad,}$$

$$I = 1 \times 10^{-3} \text{ A,}$$

$$N = 175,$$

$$A = 1 \text{ cm}^2$$

$$= 1 \times 10^{-4} \text{ m}^2$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ rad}$$

Substituting values in Eq. (i), we get

$$B = \frac{10^{-6} \times 22}{1 \times 10^{-3} \times 175 \times 7 \times 180 \times 10^{-4}} \\ = 0.998 \times 10^{-3} \approx 10^{-3} \text{ T}$$

- 109** A moving coil galvanometer allows a full scale current of 10^{-4} A . A series resistance of $2 \text{ M}\Omega$ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is [2019, 10 April Shift-I]

- (a) 100 Ω (b) 500 Ω
(c) 200 Ω (d) 10 Ω

Ans. (c)

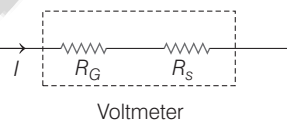
Given data,

$$I = 10^{-4} \text{ A,}$$

$$R_S = 2 \text{ M}\Omega = 2 \times 10^6 \Omega,$$

$$V_{\max} = 5 \text{ V}$$

Let internal resistance of galvanometer is R_G .



Then,

$$I \times R_S + I \times R_G = V_{\max}$$

$$\Rightarrow 2 \times 10^6 \times 10^{-4} + 10^{-4} \times R_G = 5$$

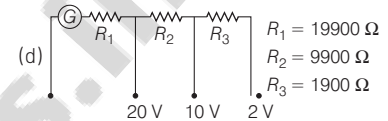
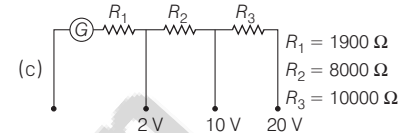
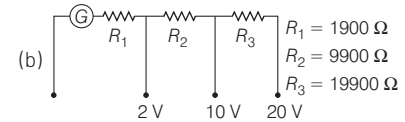
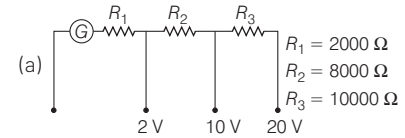
$$\Rightarrow 10^{-4} R_G = 5 - 200 = -195$$

$$\text{or } R_G = -195 \times 10^4 \Omega$$

Resistance cannot be negative.

$\therefore$ No option is correct.

- 110** A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of $20 \mu\text{A/division}$. It is to be converted to a voltmeter with three ranges of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is [2019, 12 April Shift-I]



Ans. (c)

Given, divisions in scale of galvanometer,

$$n = 50$$

Sensitivity of galvanometer,

$$\frac{I_g}{n} = 20 \mu\text{A/division}$$

$\therefore$ Current in galvanometer,

$$I_g = \frac{I}{n} \times n$$

$$= 20 \mu\text{A} \times 50$$

$$\Rightarrow I_g = 1000 \mu\text{A} = 1 \text{ mA}$$

Now, for R_1 , it should be converted into 2V voltmeter.

$$\therefore V_1 = I_g (R_1 + G)$$

$$2 = 10^{-3} (R_1 + 100)$$

$$\Rightarrow 2000 = R_1 + 100$$

$$\therefore R_1 = 1900 \Omega \quad \dots (i)$$

For R_2 , it should be converted into 10V voltmeter.

$$\therefore V_2 = I_g [(R_1 + R_2) + G]$$

$$\Rightarrow 10 = 10^{-3} [(R_1 + R_2) + 100]$$

$$\Rightarrow 10000 = R_1 + R_2 + 100$$

$$= 2000 + R_2$$

$$\therefore R_2 = 8000 \Omega \quad \dots (ii)$$

For R_3 , it should be converted into 20V voltmeter.

$$\therefore V_3 = I_g [(R_1 + R_2 + R_3) + G]$$

$$\Rightarrow 20 = 10^{-3} [1900 + 8000 + R_3 + 100]$$

$$\Rightarrow 20000 = R_3 + 10000$$

$$\therefore R_3 = 10000 \Omega \quad \dots (iii)$$

From Eqs. (i), (ii) and (iii), it is clear that option (c) is correct.

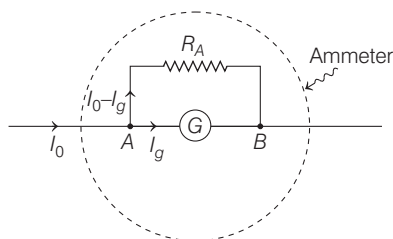
- 111** A moving coil galvanometer, having a resistance G , produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V ($V = GI_0$) by connecting a series resistance R_V to it. Then,

[2019, 12 April Shift-II]

- (a) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g} \right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g} \right)^2$
 (b) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g} \right)^2$
 (c) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g} \right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g} \right)^2$
 (d) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

Ans. (b)

To use galvanometer as an ammeter, a low resistance in parallel is used.

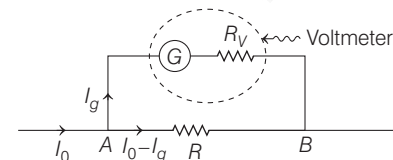


In ammeter, if I_g = full scale deflection current, then equating potential drops across points marked AB, we have

$$\Rightarrow \frac{V_{AB} = I_g G = (I_0 - I_g) R_A}{R_A = \frac{I_g G}{I_0 - I_g}} \quad \dots(i)$$

Here, G = resistance of galvanometer coil.

When a galvanometer is used as a voltmeter, a high resistance (R_V) in series is used.



Equating potential across point AB,

$$\begin{aligned} V_{AB} &= (G + R_V) I_g \\ \text{But } V_{AB} &= I_0 G \quad (\text{given}) \\ \text{So, } I_0 G &= (G + R_V) I_g \end{aligned}$$

$$\Rightarrow R_V = \frac{(I_0 - I_g) G}{I_g} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{R_A}{R_V} = \frac{\left(\frac{I_g G}{(I_0 - I_g) G} \right)}{\frac{(I_0 - I_g) G}{I_g}} = \frac{I_g^2}{(I_0 - I_g)^2}$$

$$\text{and } R_A \times R_V = \frac{I_g G}{(I_0 - I_g)} \times \frac{(I_0 - I_g) G}{I_g} = G^2$$

- 112** A galvanometer having a resistance of 20Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is

[2019, 11 Jan Shift-II]

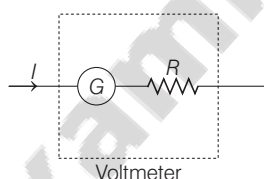
- (a) 100Ω (b) 125Ω (c) 120Ω (d) 80Ω

Ans. (d)

Resistance of galvanometer, $G = 20 \Omega$

Number of divisions on both side = 30

Figure of merit = 0.005 ampere/division



$\therefore$ Full scale deflection current (I_g)

= Number of divisions $\times$ figure of merit

$$\Rightarrow I_g = 30 \times 0.005 = 0.15 \text{ A}$$

Now, for measuring 15V by this galvanometer, let us use a resistance of R ohm in series with the galvanometer.

$\therefore$ Effective resistance of voltmeter, $R_{\text{eff}} = R + 20$

As, maximum potential measured by voltmeter,

$$V = I_g \cdot R_{\text{eff}} \text{ or } 15 = 0.15(R + 20)$$

$$\text{or } R + 20 = 100 \text{ or } R = 80 \Omega$$

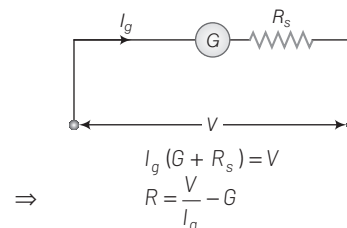
- 113** When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is

[JEE Main 2017]

- (a) $2.045 \times 10^3 \Omega$ (b) $2.535 \times 10^3 \Omega$
 (c) $4.005 \times 10^3 \Omega$ (d) $1.985 \times 10^3 \Omega$

Ans. (d)

Suppose a resistance R_s is connected in series with galvanometer to convert it into voltmeter.



$$\Rightarrow \frac{I_g (G + R_s) = V}{R = \frac{V}{I_g} - G}$$

$$\Rightarrow R = 1985 = 1.985 \text{ k}\Omega$$

$$\text{or } R = 1.985 \times 10^3 \Omega$$

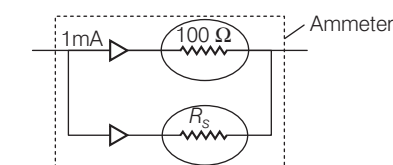
- 114** A galvanometer having a coil resistance of 100Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

[JEE Main 2016]

- (a) 0.01Ω (b) 2Ω
 (c) 0.1Ω (d) 3Ω

Ans. (a)

Maximum voltage that can be applied across the galvanometer coil = $100 \Omega \times 10^{-3} \text{ A} = 0.1 \text{ V}$.



If R_s is the shunt resistance, then

$$\Rightarrow \frac{R_s \times 10 \text{ A} = 0.1 \text{ V}}{R_s = 0.01 \Omega}$$

- 115** A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance in Ohms needed to be connected in series with the coil will be

[AIEEE 2005]

- (a) 10^3 (b) 10^5
 (c) 99995 (d) 9995

Ans. (d)

$$\begin{aligned} \text{Full scale deflection current} &= \frac{150}{10} \text{ mA} \\ &= 15 \text{ mA} \end{aligned}$$

Full scale deflection voltage

$$= \frac{150}{2} \text{ mV} = 75 \text{ mV}$$

Galvanometer resistance,

$$G = \frac{75 \text{ mV}}{15 \text{ mA}} = 5 \Omega$$

Required full scale deflection voltage,

$$V = 1 \times 150 = 150 \text{ V}$$



Let resistance to be connected in series be R .

$$\Rightarrow V = I_g(R + G)$$

$$\therefore 150 = 15 \times 10^{-3}(R + 5)$$

$$\text{or } 10^4 = R + 5$$

$$\text{or } R = 10000 - 5 = 9995$$

116 An ammeter reads upto 1 A. Its internal resistance is 0.81Ω . To

increase the range to 10 A, the value of the required shunt is

[AIEEE 2003]

(a) 0.03Ω

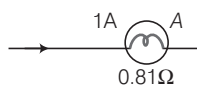
(b) 0.3Ω

(c) 0.9Ω

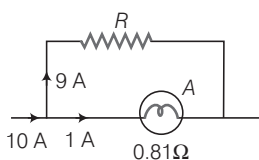
(d) 0.09Ω

Ans. (d)

To increase the range of ammeter, we have to connect a small resistance in parallel (shunt), let its value be R .



Apply KCL at junction to divide the current.



Voltage across R = Voltage across ammeter

i.e.,

$$9R = 0.81 \times 1$$

or

$$R = \frac{0.81}{9} = 0.09 \Omega$$

117 If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a

[AIEEE 2002]

(a) low resistance in parallel

(b) high resistance in parallel

(c) high resistance in series

(d) low resistance in series

Ans. (c)

A voltmeter is a high resistance device and is always connected in parallel with the circuit. While an ammeter is a low resistance device and is always connected in series with the circuit.

So, to use ammeter in place of voltmeter, a high resistance must be connected in series with the ammeter to make its resistance high.

Magnetism and Matter

TOPIC 1

Bar Magnet and Magnetic Dipole

01 Which of the following statements are correct? [2021, 18 March Shift-II]

- A. Electric monopoles do not exist, whereas magnetic monopoles exist.
- B. Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.
- C. Magnetic field lines are completely confined within a toroid.
- D. Magnetic field lines inside a bar magnet are not parallel.
- E. $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below.

- (a) C and E
- (b) B and D
- (c) A and B
- (d) B and C

Ans. (a)

Statement (A) Magnetic monopoles do not exist, they always exist in pairs as South and North poles but electric monopoles exist. So, statement A is incorrect.

Statement (B) In solenoid have a both ends, so magnetic field lines cannot be confined. Inside the solenoids, the magnetic field lines are parallel to the length of the conductor. So, statement B is incorrect.

Statement (C) In toroid has no ends. It is closed loop conductor, so the magnetic field lines are completely confined. So, the statement C is correct.

Statement (D) Inside the bar magnet, the magnetic field lines are parallel. So, the statement D is incorrect.

Statement (E) Magnetic susceptibility of the diamagnetic material is -1 . So, the statement E is correct.

Hence, the correct statements are C and E.

02 A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is

[2020, 4 Sep Shift-I]

- (a) 9.2×10^{-3} J
- (b) 7.2×10^{-2} J
- (c) 6.4×10^{-2} J
- (d) 11.7×10^{-3} J

Ans. (b)

Torque on a bar magnet,

$$\tau = MB \sin \theta$$

$$0.018 = M \times 0.06 \times \sin 30^\circ$$

$$0.018 = M \times 0.06 \times \frac{1}{2}$$

$$M = 0.6 \text{ A-m}^2$$

Now, work done to rotate bar magnet,

$$W = MB [\cos \theta_1 - \cos \theta_2]$$

Here, $\theta_1 = 0^\circ$ (for stable equilibrium position)

and $\theta_2 = 180^\circ$ (for unstable equilibrium position).

$$\text{So, } W = 0.6 \times 0.06 [\cos(0^\circ) - \cos(180^\circ)]$$

$$= 0.036 [(1) - (-1)]$$

$$= 0.036(2) = 0.072 \text{ J}$$

$$= 7.2 \times 10^{-2} \text{ J}$$

Hence, correct option is (b).

03 A small circular loop of conducting wire has radius a and carries current i . It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T . If the mass of the loop is m , then

[2020, 9 Jan Shift-II]

$$(a) T = \sqrt{\frac{\pi m}{iB}}$$

$$(b) T = \sqrt{\frac{2m}{iB}}$$

$$(c) T = \sqrt{\frac{2\pi m}{iB}}$$

$$(d) T = \sqrt{\frac{\pi m}{2iB}}$$

Ans. (c)

Time period of oscillation of a magnetic dipole in a uniform magnetic field,

$$T = 2\pi \sqrt{\frac{I}{\mu B}}$$

where, μ = magnetic dipole.

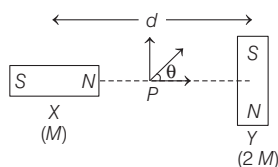
Moment of dipole = Current $\times$ Area = $\pi a^2 i$

I = moment of inertia of a coil = $ma^2/2$

$$\text{So, } T = 2\pi \sqrt{\frac{ma^2/2}{\pi a^2 i B}} = 2\pi \sqrt{\frac{m}{2\pi i B}} = \sqrt{\frac{2\pi m}{iB}}$$

04 Two magnetic dipoles X and Y are placed at a separation d , with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P , at angle $\theta = 45^\circ$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)

[2019, 8 April Shift-II]



- (a) $\left(\frac{\mu_0}{4\pi}\right) \frac{M}{(d/2)^3} \times qv$
 (b) 0
 (c) $\sqrt{2} \left(\frac{\mu_0}{4\pi}\right) \frac{M}{(d/2)^3} \times qv$
 (d) $\left(\frac{\mu_0}{4\pi}\right) \frac{2M}{(d/2)^3} \times qv$

Ans. (b)

Let $2l_1$ and $2l_2$ be the length of dipole X and Y, respectively.

For dipole X, point P lies on its axial line. So, magnetic field strength at P due to X is

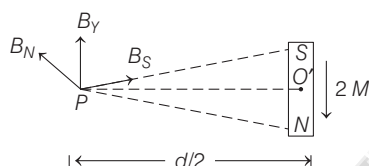
$$B_X = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l_1^2)^{3/2}}, \text{ along } OP$$

Here, $r = \frac{d}{2}$

Also, $d \gg l_1$

$$\Rightarrow |B_X| = \frac{\mu_0}{4\pi} \cdot \frac{2M(d/2)}{(d/2)^4} = \frac{\mu_0}{4\pi} \frac{2M}{(d/2)^3}$$

Similarly, for dipole Y, point P lies on its equatorial line. So, magnetic field strength at P due to Y is



$$B_Y = \frac{\mu_0}{4\pi} \cdot \frac{2M}{(r^2 + l_2^2)^{3/2}},$$

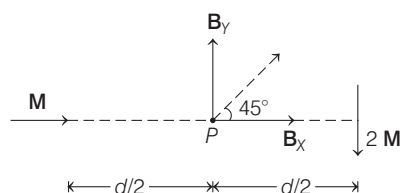
(along a line perpendicular to $O'P$)

Here, $r = \frac{d}{2}$

Also, $d \gg l_2$

$$\Rightarrow |B_Y| = \frac{\mu_0}{4\pi} \frac{2M}{(d/2)^3}$$

Thus, the resultant magnetic field due to X and Y at P is



$$B_{\text{net}} = B_X + B_Y$$

Since, $|B_Y| = |B_X|$

Thus, the resultant magnetic field (B_{net}) at P will be at 45° with the horizontal.

This means, direction of B_{net} and velocity of the charged particle is same.

$\therefore$ Force on the charged particle moving with velocity v in the presence of magnetic field which is

$$B = q(v \times B) = q |v| |B| \sin \theta$$

where, θ is the angle between B and v .

According to the above analysis, we get

$$\theta = 0 \therefore F = 0$$

Thus, magnitude of force on the particle at that instant is zero.

05 A magnet of total magnetic moment $10^{-2} \hat{i} \text{ A-m}^2$ is placed in a time varying magnetic field, $B \hat{i} (\cos \omega t)$, where $B = 1 \text{ T}$ and $\omega = 0.125 \text{ rad/s}$. The work done for reversing the direction of the magnetic moment at $t = 1 \text{ s}$ is

[2019, 10 Jan Shift-I]

- (a) 0.01 J (b) 0.007 J
 (c) 0.014 J (d) 0.028 J

Ans. (c)

Work done in reversing dipole is

$$W = 2MB$$

where, M = magnetic dipole moment

$$= 10^{-2} \text{ A-m}^2$$

and B = external field

$$= B \cos \omega t = 1 \times \cos (0.125 \times 1)$$

$$= \cos(7^\circ) = 0.992$$

Substituting these values, we get

$$W = 2 \times 10^{-2} \times 0.992 = 0.0198 \text{ J}$$

which is nearest to 0.014 J.

06 A magnetic needle of magnetic moment $6.7 \times 10^{-2} \text{ A-m}^2$ and moment of inertia $7.5 \times 10^{-6} \text{ kg m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

[JEE Main 2017]

- (a) 8.89 s (b) 6.98 s
 (c) 8.76 s (d) 6.65 s

Ans. (d)

Time period of oscillation is

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65 \text{ s}$.

07 The magnetic lines of force inside a bar magnet [AIEEE 2013]

- (a) are from North-pole to South-pole of the magnet
 (b) do not exist
 (c) depend upon the area of cross-section of the bar magnet
 (d) are from South-pole to North-pole of the magnet

Ans. (d)

Inside bar magnet, lines of force are from South to North.

08 A magnetic needle is kept in a non-uniform magnetic field. It experiences [AIEEE 2005]

- (a) a torque but not a force
 (b) neither a force nor a torque
 (c) a force and a torque
 (d) a force but not a torque

Ans. (c)

Magnetic needle is placed in non-uniform magnetic field. It experiences force and torque both due to unequal forces acting on poles.

TOPIC 2 Earth Magnetism

09 In a uniform magnetic field, the magnetic needle has a magnetic moment $9.85 \times 10^{-2} \text{ A-m}^2$ and moment of inertia $5 \times 10^{-6} \text{ kg m}^2$. If it performs 10 complete oscillations in 5 s, then the magnitude of the magnetic field is mT. [Take, π^2 as 9.85]

[2021, 27 July Shift-I]

Ans. (8)

Given,

Magnetic moment, $M = 9.85 \times 10^{-2} \text{ A-m}^2$

Moment of inertia, $I = 5 \times 10^{-6} \text{ kg m}^2$

$\therefore$ We know that,

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

where, T = time period of oscillations of a freely suspended magnetic dipole,

I = moment of inertia of the magnet about the axis of angular motion,

M = magnetic dipole moment

and B = magnetic field.

$$\Rightarrow \frac{5}{10} = 2\pi \sqrt{\frac{5 \times 10^{-6}}{9.85 \times 10^{-2} \times B}}$$

$$\Rightarrow B = 8 \times 10^{-3} \text{ T}$$

10 Choose the correct option.

[2021, 22 July Shift-II]

- (a) True dip is not mathematically related to apparent dip.
 (b) True dip is less than apparent dip.
 (c) True dip is always greater than the apparent dip.
 (d) True dip is always equal to apparent dip.

Ans. (b)

We know that, $\tan p' = \frac{\tan p}{\cos \alpha}$

where, p' = apparent dip angle

p = true dip angle

and α = angle made by vertical plane with magnetic meridian.

Since, $\cos \alpha \leq 1$

$\therefore \tan p' \geq \tan p$

$\Rightarrow p' \geq p$

Therefore, true dip is less than or equal to apparent dip.

11 At an angle of 30° to the magnetic meridian, the apparent dip is 45° . Find the true dip.

[2021, 20 July Shift-II]

- (a) $\tan^{-1}(\sqrt{3})$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 (c) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. (d)

Given, $\theta = 30^\circ$

Apparent dip, $\delta' = 45^\circ$

Since, $\tan \delta' = \frac{B_V}{B_H \cos \theta}$

$$\Rightarrow \tan 45^\circ = \frac{B_V}{B_H \cos 30^\circ}$$

$$\Rightarrow 1 = \frac{B_V}{B_H \left(\frac{\sqrt{3}}{2}\right)} \quad \left(\because \cos 30^\circ = \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow 1 = \frac{2B_V}{\sqrt{3}B_H} \Rightarrow \sqrt{3}B_H = 2B_V$$

$$\Rightarrow B_V = \frac{\sqrt{3}}{2} B_H \quad \dots(i)$$

If δ be the true angle of dip, then

$$\tan \delta = \frac{B_V}{B_H}$$

$$\tan \delta = \frac{\frac{\sqrt{3}}{2} B_H}{B_H}$$

$$\Rightarrow \tan \delta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \delta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

[From Eq. (i)]

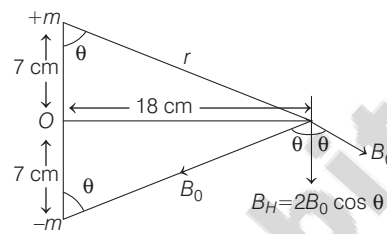
12 A bar magnet of length 14 cm is placed in the magnetic meridian with its North pole pointing towards the geographic North pole. A neutral point is obtained at a distance of 18 cm from the centre of the magnet. If $B_H = 0.4$ G, then the magnetic moment of the magnet is ($1 \text{ G} = 10^{-4} \text{ T}$)

[2021, 16 March Shift-I]

- (a) $2.88 \times 10^3 \text{ J T}^{-1}$ (b) $2.88 \times 10^2 \text{ J T}^{-1}$
 (c) 2.88 J T^{-1} (d) 28.8 J T^{-1}

Ans. (c)

The given situation can be shown as below



From the above figure, it is clear that the neutral point will lie on equatorial plane.

$$B_0 = \frac{\mu_0 m}{4\pi r^2}$$

$$B_H = 2B_0 \cos \theta$$

$$\Rightarrow 0.4 \times 10^{-4} = \frac{2\mu_0 m}{4\pi r^2} \cdot \frac{7 \times 10^{-2}}{r}$$

$$(\because B_H = 0.4 \text{ G} = 0.4 \times 10^{-4} \text{ T})$$

$$[\because \cos \theta = \frac{7}{r} \text{ and } r = (7^2 + 18^2)^{1/2}]$$

$$\Rightarrow 0.4 \times 10^{-4} = 2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4$$

$$\Rightarrow m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14} \quad \dots(i)$$

$\therefore$ Magnetic moment,

$$M = m \times 2l = m \times \frac{14}{100} \quad \dots(ii)$$

From Eqs. (i) & (ii), we get

$$M = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14} \times \frac{14}{100}$$

$$= 4 \times 10^{-4} \times 7203.82 = 2.88 \text{ J/T}$$

13 A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30° . If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio B_1 / B_2 is best given by

[2019, 12 April Shift-I]

- (a) 1.8 T (b) 0.7 T
 (c) 3.6 T (d) 2.2 T

Ans. (b)

Given, at first place, angle of dip, $\theta_1 = 45^\circ$

$$\text{Time period, } T_1 = \frac{60}{30} = 2 \text{ s}$$

At second place, angle of dip, $\theta_2 = 30^\circ$

$$\text{Time period, } T_2 = \frac{60}{40} = \frac{3}{2} \text{ s}$$

Now, at first place,

$$B_{H_1} = B_1 \cos \theta_1 = B_1 \cos 45^\circ = \frac{B_1}{\sqrt{2}} \quad \dots(i)$$

and at second place,

$$B_{H_2} = B_2 \cos \theta_2 = B_2 \cos 30^\circ = \frac{\sqrt{3}}{2} B_2 \quad \dots(ii)$$

Also, time period of a magnetic needle is given by

$$T = 2\pi \sqrt{\frac{I}{MB_H}} \quad \dots(iii)$$

$$\therefore T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \dots(iv)$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{\frac{3}{2}} = \sqrt{\frac{\frac{\sqrt{3}}{2} B_2}{\frac{B_1}{\sqrt{2}}}}$$

$$\text{or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3} \times \sqrt{2} B_2}{2 B_1}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{\sqrt{3} \times \sqrt{2}}{2} \times \frac{9}{16}$$

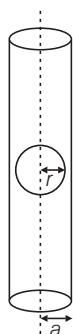
$$\Rightarrow \frac{B_1}{B_2} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{9 \times 1.732}{16 \times 1.414} = \frac{15.588}{22.624}$$

$$\Rightarrow \frac{B_1}{B_2} = 0.689 \approx 0.7 \text{ T}$$

14 A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then

[2019, 10 Jan Shift-II]



- (a) $T_h = 0.5 T_c$ (b) $T_h = T_c$
 (c) $T_h = 2 T_c$ (d) $T_h = 1.5 T_c$

Ans. (b)

The time-period of oscillations made by a magnet of magnetic moment M , moment of inertia I , placed in a magnetic field is given by

$$T = 2\pi \sqrt{\frac{I}{MB}} \quad \dots (i)$$

For the hoop, let us assume its moment of inertia I_h and magnetic moment M_h , then its time period will be

$$T_h = 2\pi \sqrt{\frac{I_h}{M_h B}} \quad \dots (ii)$$

Similarly, for solid cylinder, time period is,

$$T_c = 2\pi \sqrt{\frac{I_c}{M_c B}} \quad \dots (iii)$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{I_h M_c}{M_h I_c}} \quad \dots (iv)$$

Now, it is given that,

$$M_h = 2M_c$$

and we know that, moment of inertia of hoop

$I_h = mR^2$ and moment of inertia of solid cylinder $I_c = \frac{1}{2} mR^2$

Substituting these values in Eq. (iv), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{mR^2 \times M_c}{\frac{1}{2} mR^2 \times 2M_c}} = 1$$

$$\Rightarrow T_h = T_c$$

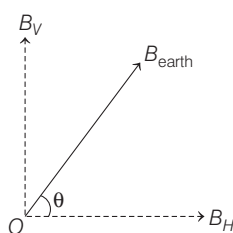
- 15** At some location on earth, the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A-m is suspended from its mid point using

a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is [2019, 10 Jan Shift-II]

- (a) 6.5×10^{-5} N (b) 3.6×10^{-5} N
 (c) 1.3×10^{-5} N (d) 1.8×10^{-5} N

Ans. (a)

Without applied forces, (in equilibrium position) the needle will stay in the resultant magnetic field of earth. Hence, the dip ' θ ' at this place is 45° (given).



We know that, horizontal and vertical components of earth's magnetic field (B_H and B_V) are related as

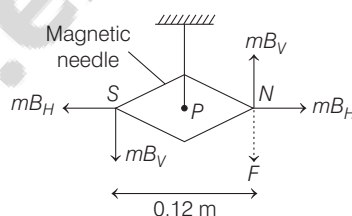
$$\frac{B_V}{B_H} = \tan \theta$$

Here, $\theta = 45^\circ$ and $B_H = 18 \times 10^{-6}$ T

$$\Rightarrow B_V = B_H \tan 45^\circ$$

$$\Rightarrow B_V = B_H = 18 \times 10^{-6} \text{ T} \quad (\because \tan 45^\circ = 1)$$

Now, when the external force F is applied, so as to keep the needle stays in horizontal position is shown below,



Taking torque at point P, we get

$$mB_V \times 2l = Fl$$

$$\therefore F = 2 \times mB_V$$

Substituting the given values, we get

$$= 2 \times 1.8 \times 18 \times 10^{-6}$$

$$= 6.48 \times 10^{-5}$$

$$= 6.5 \times 10^{-5} \text{ N}$$

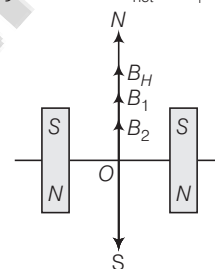
- 16** Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 , respectively. They are placed on a horizontal table parallel to each other with their N poles pointing

towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is $3.6 \times 10^{-5} \text{ Wb/m}^2$) [JEE Main 2013]

- (a) $3.6 \times 10^{-5} \text{ Wb/m}^2$
 (b) $2.56 \times 10^{-4} \text{ Wb/m}^2$
 (c) $3.50 \times 10^{-4} \text{ Wb/m}^2$
 (d) $5.80 \times 10^{-4} \text{ Wb/m}^2$

Ans. (b)

Net magnetic field, $B_{\text{net}} = B_1 + B_2 + B_H$



$$B_{\text{net}} = \frac{\mu_0 (M_1 + M_2)}{4\pi r^3} + B_H$$

$$= \frac{10^{-7} (1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$

- 17** The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be [AIEEE 2004]

- (a) 2 s (b) $\frac{2}{3}$ s
 (c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

Ans. (b)

The time period of oscillations of magnet,

$$T = 2\pi \sqrt{\frac{I}{MH}} \quad \dots (i)$$

where, I = moment of inertia of magnet

$$= \frac{mL^2}{12}$$

[m , being the mass of magnet]

M = Pole strength $\times L$

and H = horizontal component of the earth's magnetic field.

When the three equal parts of magnet are placed on one another with their like poles together, then

$$I' = \frac{1}{12} \left(\frac{m}{3} \right) \times \left(\frac{L}{3} \right)^2 \times 3$$

$$= \frac{1}{12} \frac{mL^2}{9} = \frac{I}{9}$$

and $M' = \text{Pole strength} \times \frac{L}{3} \times 3 = M$

Hence, $T' = 2\pi \sqrt{\left(\frac{I/9}{MB} \right)}$

or $T' = \frac{1}{3} \times T$

or $T' = \frac{2}{3} \text{ s}$

- 18** A thin rectangular magnet suspended freely has a period of oscillation equal to T . Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T' , the ratio T'/T is

[AIEEE 2003]

- (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$
(c) 2 (d) $\frac{1}{4}$

Ans. (b)

When magnet is divided into two equal parts, the magnetic dipole moment

$$M' = \text{Pole strength} \times \frac{L}{2} = \frac{M}{2}$$

[∵ pole strength remains same]

Also, the mass of magnet becomes half i.e.,

$$m' = \frac{m}{2}$$

Moment of inertia of magnet, $I = \frac{mL^2}{12}$

New moment of inertia,

$$I' = \frac{1}{12} \left(\frac{m}{2} \right) \left(\frac{L}{2} \right)^2 = \frac{mL^2}{12 \times 8}$$

∴ $I' = \frac{I}{8}$

Now, $T = 2\pi \sqrt{\left(\frac{I}{MB} \right)}$

$$T' = 2\pi \sqrt{\left(\frac{I'}{M'B} \right)}$$

$$= 2\pi \sqrt{\left(\frac{I/8}{MB/2} \right)}$$

∴ $T' = \frac{T}{2}$

⇒ $\frac{T'}{T} = \frac{1}{2}$

TOPIC 3 Magnetic Materials

- 19** A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3 . If the core material is replaced by another material having relative permeability of 750 with same volume maintaining same current of 0.75 A in the solenoid, the fractional change in the magnetic moment of the core would be approximately $(\chi / 499)$. Find the value of χ .

[2021, 31 Aug Shift-II]

Ans. (250)

Given, number of turns, $n = 1000 \text{ m}^{-1}$

Electric current, $i = 0.75 \text{ A}$

Relative permeability of 1st material

$$\mu_1 = 500$$

Relative permeability of 2nd material

$$\mu_2 = 750$$

we know that $\mu_r = 1 + \chi \Rightarrow \chi = \mu_r - 1$

For 1st and 2nd material,

$$\chi_1 = \mu_1 - 1 = 500 - 1 = 499$$

$$\text{and } \chi_2 = \mu_2 - 1 = 750 - 1 = 749$$

Now, magnetic intensity, $M = ni$

Let M_1 and M_2 be the magnetisation and m_1 and m_2 be the magnetic moments of material 1st and 2nd.

Using, $\frac{l_1}{M} = \chi_1$... (i)

and $\frac{l_2}{M} = \chi_2$... (ii)

Dividing Eqs. (i) and (ii),

$$\frac{l_1}{l_2} = \frac{\chi_1}{\chi_2} \text{ ... (iii)}$$

We also know that,

Magnetisation =

$$\frac{\text{Dipole moment in core material}}{\text{Volume}}$$

i.e. $I = \frac{m}{V}$

∴ $\frac{l_1}{l_2} = \frac{m_1}{m_2}$... (iv)

From Eqs. (iii) and (iv), we get

$$\frac{m_1}{m_2} = \frac{\chi_1}{\chi_2} = \frac{499}{749}$$

Hence, fraction change in magnetic moment,

$$\frac{\Delta m}{m_1} = \frac{m_2 - m_1}{m_1} = \left(\frac{m_2}{m_1} - 1 \right)$$

$$= \left(\frac{749}{499} - 1 \right) = \frac{250}{499}$$

Thus, value of χ is 250.

- 20 Statement I** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 22 July Shift-II]

- (a) Statement I is true but Statement II is false
(b) Both Statement I and Statement II are true
(c) Both Statement I and Statement II are false
(d) Statement I is false but Statement II is true

Ans. (a)

Given, As we know that, ferromagnetism of any object decreases with increase in temperature and above Curie temperature, ferromagnetic material become paramagnetic material.

Hence, statement I is true.

But, in case of ferromagnetic material below Curie temperature, atoms are aligned and parallel. But above Curie temperature atoms losses their ordered magnetic moments.

Hence, statement II is false.

- 21** The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi \times 10^{-7} \text{ H/m}$. Absolute permeability of the material of the rod is

[2021, 20 July Shift-II]

- (a) $4\pi \times 10^{-4} \text{ H/m}$ (b) $2\pi \times 10^{-4} \text{ H/m}$
(c) $3\pi \times 10^{-4} \text{ H/m}$ (d) $\pi \times 10^{-4} \text{ H/m}$

Ans. (b)

Given,

Magnetic susceptibility, $\chi_m = 499$

Permeability in vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

Absolute permeability of the material,

$$\begin{aligned} \mu &= \mu_0 (1 + \chi_m) \\ \Rightarrow \mu &= 4\pi \times 10^{-7} (1 + 499) \\ &= 4\pi \times 10^{-7} \times 500 \\ &= 2\pi \times 10^{-4} \text{ H/m} \end{aligned}$$

- 22** A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is
(Permeability of free space $= 4\pi \times 10^{-7} \text{ H/m}$)

[2021, 17 March Shift-I]

- (a) $\pi \text{ T}$ (b) $2 \times 10^{-3} \pi \text{ T}$
(c) $\frac{\pi}{5} \text{ T}$ (d) $10^{-4} \pi \text{ T}$

Ans. (a)

Given, number of turns per unit length,
 $n = 1000$

Relative magnetic permeability,
 $\mu_r = 500$

Current flowing through solenoid, $I = 5 \text{ A}$

Magnetic permeability of free space,

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

We know that, magnetic field inside the solenoid can be given as

$$B = \mu n I \quad \dots(i)$$

where, μ = permeability of medium.

$$\text{As, } \mu = \mu_r \mu_0 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\begin{aligned} \therefore B &= \mu_r \mu_0 n I \\ &= 500 \times 4\pi \times 10^{-7} \times 1000 \times 5 \\ &= 100\pi \times 10^{-7} \times 10^5 \\ &= \pi \text{ T} \end{aligned}$$

- 23** In a ferromagnetic material, below the Curie temperature, a domain is defined as [2021, 25 Feb Shift-II]

- (a) a macroscopic region with zero magnetisation
(b) a macroscopic region with saturation magnetisation
(c) a macroscopic region with randomly oriented magnetic dipoles
(d) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction

Ans. (b)

Since, in ferromagnetic material, with increase in temperature susceptibility decreases,

$\therefore$ Ferromagnetic material below Curie temperature will show saturation magnetisation.

Hence, option (b) is the correct i.e. domain is defined as a macroscopic region with saturation magnetisation.

- 24** An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be [2020, 5 Sep Shift-II]

- (a) $50 \times 10^2 \text{ Am}^2$ (b) $5 \times 10^2 \text{ Am}^2$
(c) $500 \times 10^2 \text{ Am}^2$ (d) $0.5 \times 10^2 \text{ Am}^2$

Ans. (b)

Magnetic moment of an iron core solenoid, $M = \mu_r N i A$

where, μ_r = relative permeability = 1000,

N = number of turns = 10 turns/cm,

i = current = 0.5 A

and A = area of cross-section.

$$\text{Now, } M = \mu_r N i \frac{V}{l} \quad \left\{ \because A = \frac{V}{l} \right\}$$

Substituting all the given values in above equation, we get

$$\begin{aligned} M &= 1000 \times \frac{10}{10^{-2}} \times 0.5 \times 10^{-3} \text{ V} = 10^{-3} \text{ m}^3 \\ &= 500 \text{ Am}^2 = 5 \times 10^2 \text{ Am}^2 \end{aligned}$$

Hence, correct option is (b).

- 25** A paramagnetic sample shows a net magnetisation of 6 A/m, when it is placed in an external magnetic field of 0.4 T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3 T at a temperature of 24 K, then the magnetisation will be [2020, 4 Sep Shift-II]

- (a) 4 A/m (b) 0.75 A/m
(c) 1 A/m (d) 2.25 A/m

Ans. (b)

If permeability of free space $= \mu_0$

External magnetic field $= B_0$

Magnetisation $= I$

$$\text{Intensity of magnetising field} = H = \frac{B_0}{\mu_0}$$

$$\text{Magnetic susceptibility} = \chi_m = \frac{I}{H}$$

$$\text{Temperature} = T$$

Then, from Curie's law for paramagnetic materials,

$$\chi_m \propto \frac{1}{T} \Rightarrow \frac{I}{H} \propto \frac{1}{T}$$

$$\Rightarrow \frac{I}{(B_0/\mu_0)} \propto \frac{1}{T} \Rightarrow \frac{\mu_0 I}{B_0} \propto \frac{1}{T}$$

$$\Rightarrow I \propto \frac{B_0}{\mu_0 T} \Rightarrow I \propto \frac{B_0}{T} (\because \mu_0 = \text{constant})$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{(B_0)_1}{(B_0)_2} \times \frac{T_2}{T_1}$$

On putting the given values in above equation, we get

$$\frac{6}{I_2} = \frac{0.4}{0.3} \times \frac{24}{4} \Rightarrow \frac{6}{I_2} = \frac{4}{3} \times \frac{6}{1}$$

$$\Rightarrow \frac{6}{I_2} = 8 \Rightarrow I_2 = \frac{6}{8}$$

$$\Rightarrow I_2 = \frac{3}{4} \text{ A/m} \Rightarrow I_2 = 0.75 \text{ A/m}$$

Hence, option (b) is correct.

- 26** Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties, of the following, which property best matches for the type of magnet required?

[2020, 2 Sep Shift-I]

- (a) T: Large retentivity, small coercivity
(b) P: Large retentivity, large coercivity

- (c) P: Small retentivity, large coercivity
(d) T: Large retentivity, large coercivity

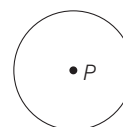
Ans. (b)

Permanent magnets (P) must be able to hold large magnetism for a long time. So, they must have large coercivity and large retentivity.

Electromagnet and transformer cores (T) must be able to lose magnetisation quickly when current in their coils are zero. So, they must have small retentivity.

Hence, correct option is (b).

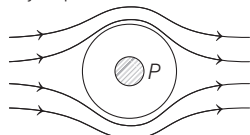
- 27** A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\mathbf{B}$. Then, the field inside the paramagnetic substance is [2020, 3 Sep Shift-II]



- (a) much larger than $|\mathbf{B}|$ and parallel to $\mathbf{B}$
(b) $\mathbf{B}$
(c) zero
(d) much larger than $|\mathbf{B}|$ but opposite to $\mathbf{B}$

Ans. (c)

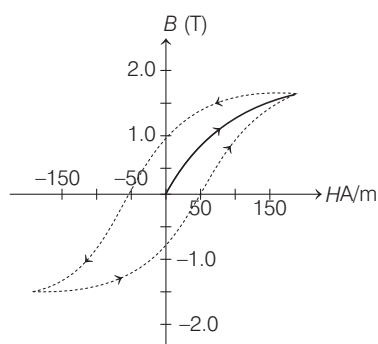
As, the sphere is perfectly diamagnetic, it completely expels field as shown below:



Therefore, there will be no magnetic field inside the cavity of sphere.

So, the paramagnetic sample at the spherical cavity at the centre of sphere remains unaffected by external field and field inside paramagnetic sample is zero. Hence, correct option is (c).

28

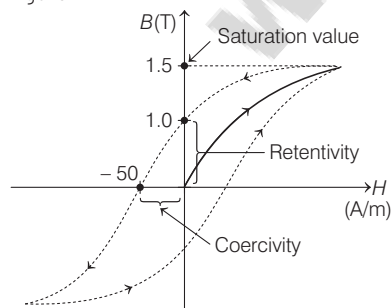


The figure gives experimentally measured B versus H variation in a ferromagnetic material. The retentivity, coercivity and saturation respectively of the material are [2020, 7 Jan Shift-II]

- (a) 1.0 T, 50 A/m and 1.5 T
(b) 150 A/m, 1.0 T and 1.5 T
(c) 1.5 T, 50 A/m and 1.0 T
(d) 1.5 T, 50 A/m and 1.0 T

Ans. (a)

We can obtain the values of retentivity (the remaining value of magnetic field B when $H=0$), coercivity (value of H when $B=0$) and saturation value of induced magnetic field ($B_{\max}$) as shown in the figure.



Clearly, retentivity = 1 T, coercivity = 50 A m⁻¹ and saturation value = 1.5 T

29 A bar magnet is demagnetised by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is [2019, 9 Jan Shift-I]

- (a) 1200 A/m
(b) 285 A/m
(c) 2600 A/m
(d) 520 A/m

Ans. (c)

Coercivity of a bar magnet is the value of magnetic field intensity (H) that is needed to reduce magnetisation to zero. Since, for a solenoid magnetic induction is given as,

$$B = \mu_0 n I \quad \dots(i)$$

where, n is the number of turns (N) per unit, length (l) and I is the current.

$$\text{Also, } B = \mu_0 H \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\mu_0 n I = \mu_0 H \text{ or } H = n I = \frac{N}{l} I$$

Substituting the given values, we get

$$H = \frac{100}{0.2} \times 5.2 = 2600 \text{ A/m}$$

Thus, the value of coercivity of the bar magnet is 2600 A/m.

30 A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is [2019, 11 Jan Shift-II]

- (a) 3.3×10^{-4}
(b) 3.3×10^{-2}
(c) 4.3×10^{-2}
(d) 2.3×10^{-2}

Ans. (a)

Given, side of cube = 1 cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6} \text{ m}^3$

Dipole moment, $M = 20 \times 10^{-6} \text{ J/T}$

Applied magnetic intensity,

$$H = 60 \times 10^3 \text{ A/m}$$

Intensity of magnetisation

$$I = \frac{M}{V} = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ A/m}$$

Now, magnetic susceptibility χ is

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H}$$

$$\Rightarrow \chi = \frac{20}{60 \times 10^3} = \frac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$$

31 A paramagnetic material has 10^{28} atoms/m³. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4} . Its susceptibility at 300 K is [2019, 12 Jan Shift-II]

- (a) 3.726×10^{-4}
(b) 3.672×10^{-4}
(c) 2.672×10^{-4}
(d) 3.267×10^{-4}

Ans. (d)

From Curie's law for paramagnetic substance, we have

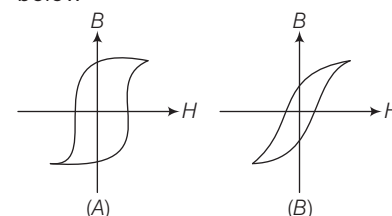
$$\text{Magnetic susceptibility } \chi \propto \frac{1}{T}$$

$$\therefore \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$$

$$\Rightarrow \chi_2 = \frac{\chi_1 \cdot T_1}{T_2}$$

$$\Rightarrow \chi_2 = \frac{2.8 \times 10^{-4} \times 350}{300} = 3.267 \times 10^{-4}$$

32 Hysteresis loops for two magnetic materials A and B are as given below



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

[JEE Main 2016]

- (a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

Ans. (d)

Key Idea Area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation.

For electromagnets and transformers, energy loss should be low.

i.e. thin hysteresis curves.

Also, $|B| \rightarrow 0$ when $H = 0$ and $|H|$ should be small when $B \rightarrow 0$.

- 33** The coercivity of a small magnet where the ferromagnet gets demagnetised is $3 \times 10^3 \text{ Am}^{-1}$. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is [JEE Main 2014]

(a) 30 mA
(b) 60 mA
(c) 3 A
(d) 6 A

Ans. (c)

For solenoid, the magnetic field needed to be magnetised, the magnet is given by

$$B = \mu_0 n I$$

where, n is number of turns per unit length $= \frac{N}{l}$

$$\text{and } N = 100, l = 10 \text{ cm} = \frac{10}{100} \text{ m} = 0.1 \text{ m}$$

$$\Rightarrow 3 \times 10^3 = \frac{100}{0.1} \times I$$

$$\Rightarrow I = 3 \text{ A}$$

- 34** Relative permittivity and permeability of a material are ϵ_r and μ_r , respectively. Which of the following values of these quantities are allowed for a diamagnetic material? [AIEEE 2008]

(a) $\epsilon_r = 0.5, \mu_r = 1.5$
(b) $\epsilon_r = 1.5, \mu_r = 0.5$
(c) $\epsilon_r = 0.5, \mu_r = 0.5$
(d) $\epsilon_r = 1.5, \mu_r = 1.5$

Ans. (b)

For diamagnetic material, $0 < \mu_r < 1$ and for any material, $\epsilon_r > 1$.

- 35** Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will [AIEEE 2006]

(a) attract N_1 and N_2 strongly but repel N_3
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly, but repel N_2 and N_3 weakly
(d) attract all three of them

Ans. (b)

Ferromagnetic substances have strong tendency to get magnetised (induced magnetic moment) in the same direction as that of applied magnetic field, so magnet attracts N_1 strongly.

Paramagnetic substances get weakly magnetised (magnetic moment induced is small) in the same direction as that of applied magnetic field, so magnet attracts N_2 weakly.

Diamagnetic substances also get weakly magnetised when placed in an external magnetic field but in opposite direction and hence N_3 is weakly repelled by magnet.

- 36** The materials suitable for making electro- magnets should have [AIEEE 2004]

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

Ans. (c)

Electromagnets are made of soft iron. The soft iron has high retentivity and low coercivity.

- 37** Curie temperature is the temperature above which [AIEEE 2003]

(a) a ferromagnetic material becomes para-magnetic
(b) a paramagnetic material becomes dia-magnetic
(c) a ferromagnetic material becomes dia-magnetic
(d) a paramagnetic material becomes ferro-magnetic

Ans. (a)

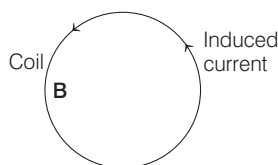
At Curie temperature, a ferromagnetic substance transits into paramagnetic substance.

Electromagnetic Induction

TOPIC 1

Magnetic Flux, Faraday's and Lenz's Laws

- 01** A coil is placed in a magnetic field **B** as shown below.



A current is induced in the coil because **B** is

[2021, 31 Aug Shift-II]

- outward and decreasing with time
- parallel to the plane of coil and decreasing with time
- outward and increasing with time
- parallel to the plane of coil and increasing with time

Ans. (a)

In the given figure, the magnetic field is outward, means *N*-pole is formed on outward face and *S*-pole is formed on inward (back) face.

As the induced current is anti-clockwise which is also making *N*-pole on outward face.

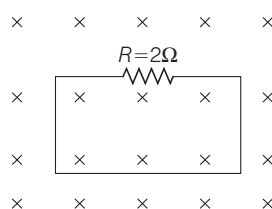
According to the Lenz's law, the induced current always opposes the nature by which it is produced.

Hence, induced current is in the direction, so the strength of *N*-pole is maintained as it is decreasing.

- 02** In the given figure, the magnetic flux through the loop increases according to the relation $\phi_B(t) = 10t^2 + 20t$, where ϕ_B is in milliwebers and t is in seconds.

The magnitude of current through $R = 2\Omega$ resistor at $t = 5$ s is mA.

[2021, 20 July Shift-II]



Ans. (60)

As per question, flux through the loop increases according to

$$\phi_0(t) = 10t^2 + 20t \quad \dots(i)$$

where, ϕ_0 is in milliwebers and t in seconds.

Differentiate Eq. (i) with respect to t , we get

$$\text{Induced emf } |E| = \frac{d\phi}{dt} = (20t + 20) \text{ mV}$$

$$\text{Current, } i = \frac{|E|}{R} \Rightarrow i = \frac{20t + 20}{2}$$

$$\Rightarrow i = (10t + 10) \text{ mA}$$

$$\therefore \text{Current, } = (10 \times 5 + 10) \text{ mA} \quad [\because i(t = 5 \text{ s})]$$

$$= (50 + 10) \text{ mA,}$$

$$i = 60 \text{ mA}$$

- 03** A circular coil of radius 10 cm is placed in a uniform magnetic field of $3.0 \times 10^{-5} \text{ T}$ with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field, so that it undergoes half of rotation in 0.2 s. The maximum value of emf induced (in μV) in the coil will be close to the integer

[2020, 2 Sep Shift-I]

Ans. (15)

Flux linked with the coil is

$$\phi = \mathbf{B} \cdot \mathbf{A} = BA \cos \omega t$$

Magnitude of emf induced in coil

$$= \left| \frac{d}{dt} \phi \right| = |BA\omega \sin \omega t|$$

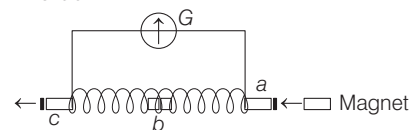
Maximum value of induced emf

$$= BA\omega = \frac{BA2\pi}{T}$$

$$= \frac{3 \times 10^{-5} \times \pi \times (0.1)^2 \times 2\pi}{0.4}$$

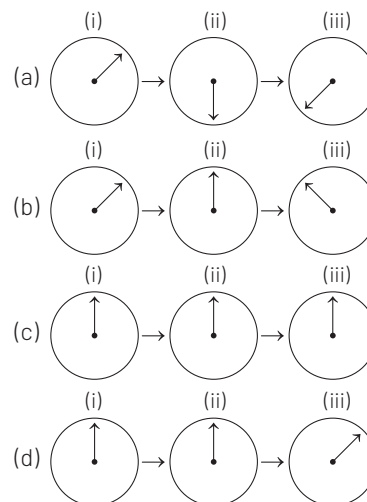
$$\approx 15 \times 10^{-6} = 15 \mu\text{V}$$

- 04** A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer *G* attached across the coil?



Three positions shown describe: (i) the magnet's entry (ii) magnet is completely inside and (iii) magnet's exit.

[2020, 4 Sep Shift-I]



Ans. (b)

At point (i)

The bar magnet enters into the coil, so the magnetic flux passing through the loops will change. In order to oppose that, an emf gets generated and an induced current starts to flow. So, the pointer of galvanometer will deflect in a particular direction.

(We can not be definite about the direction, as which pole of magnet is being entered into the coil is not given.)

At point (ii)

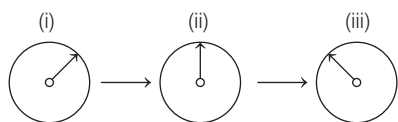
The bar magnet is completely inside the coil, so the flux will not change. No emf will be generated and no current flows. So, the pointer will not deflect towards any side. It will show zero.

At point (iii)

The bar magnet exits from the coil, so again the flux will change in opposite manner to what happened at point (i), so again an opposing emf gets generated and an induced current starts to flow in a direction opposite to that when the magnet was entering.

So in this case, the pointer will deflect in the direction opposite to the case when magnet was entering.

Looking at the options, following cases seems to be possible.



Hence, correct option is (b).

- 05** A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.032 \text{ T s}^{-1}$. The induced current in the loop is close to (Take, resistivity of the metal wire $= 1.23 \times 10^{-8} \Omega \text{ m}$)

[2020, 3 Sep Shift-II]

- (a) 0.61 A (b) 0.43 A
(c) 0.53 A (d) 0.34 A

Ans. (a)

Induced emf in square loop,

$$E = A \left(\frac{dB}{dt} \right) = l^2 \left(\frac{dB}{dt} \right) = \left(\frac{L}{4} \right)^2 \left(\frac{dB}{dt} \right)$$

Here, $L = 30 \text{ cm}$
 $= 30 \times 10^{-2} \text{ m}$

$$\frac{dB}{dt} = 0.032 \text{ T s}^{-1}$$

$$\therefore E = 1.8 \times 10^{-4} \text{ V}$$

$$\text{Induced current} = \frac{E}{R} = \frac{E}{\frac{\rho L}{A}} = \frac{EA}{\rho L}$$

Here, $\rho = 1.23 \times 10^{-8} \Omega \text{ m}$

$$L = 30 \times 10^{-2} \text{ m}$$

$$A = \pi r^2 = \pi (2 \times 10^{-3})^2 = 12.56 \times 10^{-6} \text{ m}^2$$

$$\text{So, } I = \frac{1.8 \times 10^{-4} \times 12.56 \times 10^{-6}}{1.23 \times 10^{-8} \times 30 \times 10^{-2}} = 0.61 \text{ A}$$

Hence, correct option is (a).

- 06** Two concentric circular coils C_1 and C_2 are placed in the xy -plane. C_1 has 500 turns and radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3) \text{ A}$, where t is in second. The emf induced in C_1 (in mV), at the instant $t = 1 \text{ s}$ is $\frac{4}{x}$.

The value of x is

[2020, 5 Sep Shift-I]**Ans. (5)**

Given, $I(t) = 5t^2 - 2t + 3$

$$\frac{dI}{dt} = 10t - 2$$

At $t = 1 \text{ s}$, $\frac{dI}{dt} = 8 \text{ A/s}$

$$\text{Magnetic flux, } \phi = \left(\frac{\mu_0 n_2}{2R} \right) (\pi r^2 n_1)$$

$$\text{Induced emf, } e = \left| \frac{d\phi}{dt} \right| = \left(\frac{\mu_0 n_2}{2R} \right) \pi r^2 n_1 \frac{dI}{dt}$$

Given, $n_1 = 200$, $r = 1 \text{ cm} = 10^{-2} \text{ m}$,
 $n_2 = 500$,

$$R = 20 \text{ cm} = 20 \times 10^{-2} \text{ m} \text{ and}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\therefore e = \frac{4\pi \times 10^{-7} \times 200 \times \pi \times 10^{-4} \times 500}{2 \times 20 \times 10^{-2}} \times 8$$

$$= 8 \times 10^{-4} \text{ V} = 0.8 \text{ mV}$$

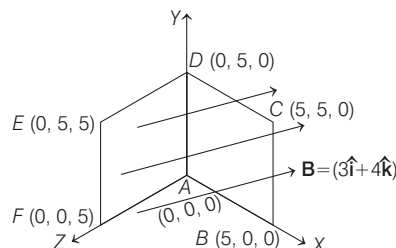
According to question, $0.8 \text{ mV} = \frac{4}{x}$

$$\Rightarrow x = \frac{4}{0.8} = 5$$

- 07** A loop ABCDEFA of straight edges has six corner points $A(0,0,0)$, $B(5,0,0)$, $C(5,5,0)$, $D(0,5,0)$, $E(0,5,5)$ and $F(0,0,5)$. The magnetic field in this region is $\mathbf{B} = (3\hat{i} + 4\hat{k}) \text{ T}$. The quantity of flux through the loop ABCDEFA (in Wb) is

[2020, 7 Jan Shift-I]**Ans. (175)**

If given situation, loop and magnetic field are given as shown in the figure.



Flux through loop ABCDEFA = Flux through part ABCDA + Flux through part ADEFA
 $= \mathbf{B} \cdot \mathbf{A}_1 + \mathbf{B} \cdot \mathbf{A}_2$

Here, from figure,

$$\mathbf{A}_1 = 25\hat{k} \text{ and } \mathbf{A}_2 = 25\hat{i}$$

So, flux associated with complete loop
 $= (3\hat{i} + 4\hat{k}) \cdot 25\hat{k} + (3\hat{i} + 4\hat{k}) \cdot 25\hat{i}$
 $= 100 + 75 = 175 \text{ Wb}$

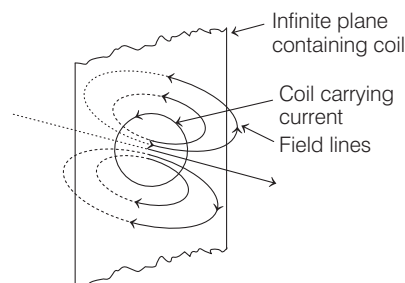
- 08** Consider a circular coil of wire carrying constant current I , forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i . The magnetic flux through the area of the circular coil area is given by ϕ_o . Which of the following option is correct?

[2020, 7 Jan Shift-I]

- (a) $\phi_i > \phi_o$ (b) $\phi_i < \phi_o$
(c) $\phi_i = \phi_o$ (d) $\phi_i = -\phi_o$

Ans. (d)

We are given with following situation,



From the diagram of field lines, we can observe that whatever be the number of field lines emitted from coil, all of them goes back into the infinite plane only. So, magnetic flux emanating from coil is equal and opposite to the flux linked with infinite plane.

So, $\phi_i = -\phi_o$

- 09** A planar loop of wire rotates in a uniform magnetic field. Initially at $t=0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane, then the magnitude of induced emf will be maximum and minimum respectively at [2020, 7 Jan Shift-II]

(a) 2.5 s and 7.5 s (b) 2.5 s and 5.0 s
(c) 5.0 s and 10.0 s (d) 5.0 s and 7.5 s

Ans. (b)

Induced emf in the wire loop,

$$e = BA\omega \sin \omega t$$

Induced emf is maximum when $\sin \omega t = \pm 1$

$$\Rightarrow \omega t = \frac{\pi}{2}$$

$$\Rightarrow \frac{2\pi}{T} \cdot t = \frac{\pi}{2}$$

Here, $T = 10$ s

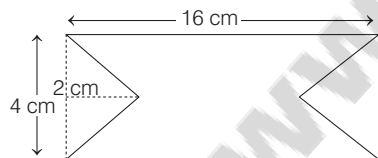
$$\text{So, } t = \frac{10}{4} = 2.5 \text{ s}$$

and induced emf is minimum when $\sin \omega t = 0$

$$\Rightarrow \omega t = \pi \text{ or } \frac{2\pi}{T} \cdot t = \pi$$

$$\Rightarrow t = \frac{10}{2} = 5 \text{ s}$$

- 10** At time $t=0$ magnetic field of 1000 gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 gauss, in the next 5 s, then induced emf in the loop is [2020, 8 Jan Shift-I]



(a) $48 \mu\text{V}$ (b) $28 \mu\text{V}$
(c) $56 \mu\text{V}$ (d) $36 \mu\text{V}$

Ans. (c)

Induced emf in the loop,

$$E = -\frac{\Delta\Phi}{\Delta t} = -\left(\frac{\Delta B}{\Delta t}\right)(A[\because \Phi = BA])$$

$$= -\left(\frac{B_2 - B_1}{\Delta t}\right)A \quad \dots(i)$$

Here, $B_2 = 500 \text{ G} = 500 \times 10^{-4} \text{ T}$,

$B_1 = 1000 \text{ G} = 1000 \times 10^{-4} \text{ T}$,

$\Delta t = 5 \text{ s}$

$\therefore A = \text{area of loop}$
= Area of rectangle – Area of two triangles

$$= \left(16 \times 4 - 2 \times \frac{1}{2} \times 4 \times 2\right) \text{ cm}^2$$

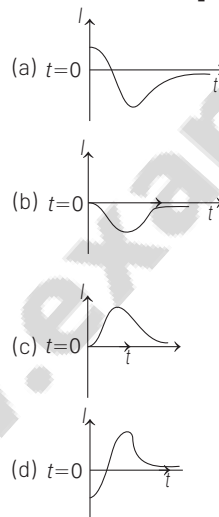
$$= 56 \times 10^{-4} \text{ m}^2$$

Using Eq. (i), we get

$$E = \frac{(1000 - 500) \times 10^{-4} \times 56 \times 10^{-4}}{5}$$

$$= 56 \times 10^{-6} \text{ V} = 56 \mu\text{V}$$

- 11** A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ ($k > 0$), as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by [2019, 9 April Shift-II]



Ans. (d)

Magnetic flux associated with the outer coil is

$$\Phi_{\text{outer}} = \mu_0 \pi N R \cdot I$$

$$= \mu_0 \pi N R (kte^{-\alpha t})$$

$$= Cte^{-\alpha t}$$

where,

$$C = \mu_0 \pi N R k = \text{constant}$$

Induced emf,

$$e = -\frac{d\Phi_{\text{outer}}}{dt} = Ce^{-\alpha t} + (-\alpha C t e^{-\alpha t})$$

$$= Ce^{-\alpha t}(1 - \alpha t)$$

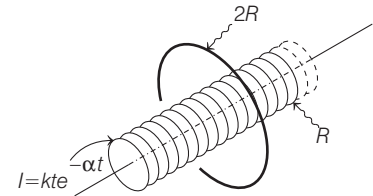
$\therefore$ Induced current, $I = \frac{e}{\text{Resistance}}$

$\Rightarrow$ At $t=0$, $I = -ve$

$\therefore$ The correct graph representing this condition is given in option (d).

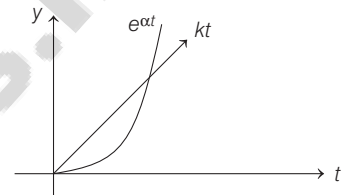
Alternate Solution

Given solenoid is shown below as,



At $t=0$, current in solenoid $= I(t=0) = k(0) e^{-\alpha \cdot 0} = 0$

Graph of $e^{\alpha t}$ and kt versus time can be shown as,



As, $I = \frac{kt}{e^{\alpha t}}$

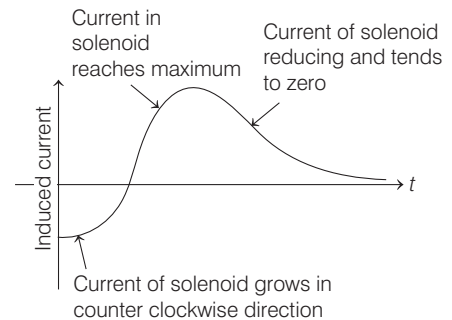
Initially, $kt > e^{\alpha t}$

So, current is increasing in magnitude.

Finally, after a short time $kt < e^{\alpha t}$. So, current is decreasing in magnitude.

But in both cases, it remains positive or counter clockwise. So, current induced is at first anti-clockwise (following Lenz's law) and then it becomes clockwise and finally reduces to zero as $t \rightarrow \infty$.

So, correct graph of induced current is



- 12** A conducting circular loop is made of a thin wire has area $3.5 \times 10^{-3} \text{ m}^2$ and resistance 10Ω . It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4 \text{ T}) \sin(0.5 \pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t=0$ s and $t=10$ ms is close to [2019, 9 Jan Shift-I]
- (a) 6 mC (b) 21 mC
(c) 7 mC (d) 14 mC

Ans. (d)

Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\Delta\phi_B = B A = BA \cos\theta$

where, A is the surface area of the loop and θ is an angle between B and A .

Here, $\theta = 0 \Rightarrow \Delta\phi_B = BA$

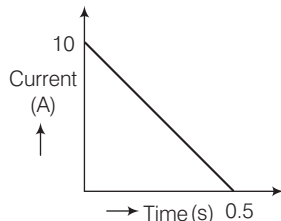
$\therefore$ For the time interval, $t = 0$ ms to $t = 10$ ms

$$Q = \frac{\Delta\phi_B}{R} = \frac{A}{R} (B_{\text{at } 0.01 \text{ s}} - B_{\text{at } 0 \text{ s}})$$

Substituting the given values, we get

$$\begin{aligned} &= \frac{3.5 \times 10^{-3}}{10} [0.4 \sin(0.5\pi) - 0.4 \sin 0] \\ &= 3.5 \times 10^{-4} (0.4 \sin \pi/2) \\ &= 1.4 \times 10^{-4} \text{ C} = 14 \text{ mC} \end{aligned}$$

- 13** In a coil of resistance 100Ω , a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is [JEE Main 2017]



- (a) 225 Wb (b) 250 Wb
(c) 275 Wb (d) 200 Wb

Ans. (b)

Induced constant, $I = \frac{e}{R}$

Here, $e =$ induced emf $= \frac{d\phi}{dt}$

$$I = \frac{I}{R} = \left(\frac{d\phi}{dt} \right) \cdot \frac{1}{R}$$

$$d\phi = IR dt$$

$$\phi = \int IR dt$$

$\therefore$ Here, R is constant

$$\therefore \phi = R \int I dt$$

$$\begin{aligned} \int I \cdot dt &= \text{Area under } I-t \text{ graph} \\ &= \frac{1}{2} \times 10 \times 0.5 = 2.5 \end{aligned}$$

$$\begin{aligned} \therefore \phi &= R \times 2.5 \\ &= 100 \times 2.5 \\ &= 250 \text{ Wb} \end{aligned}$$

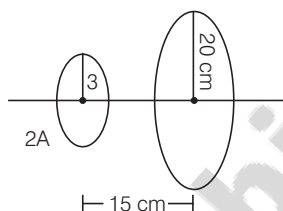
- 14** A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm . The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm . If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is [JEE Main 2013]

- (a) $9.2 \times 10^{-11} \text{ Wb}$ (b) $6 \times 10^{-11} \text{ Wb}$
(c) $3.3 \times 10^{-11} \text{ Wb}$ (d) $6.6 \times 10^{-9} \text{ Wb}$

Ans. (a)

The magnetic field due to the bigger circular loop is

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$



Given, $I = 2 \text{ A}$, $R = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$,

$$\begin{aligned} x &= 15 \text{ cm} \\ &= 15 \times 10^{-2} \text{ m} \end{aligned}$$

$$B = \frac{\mu_0 \times 2 \times (20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]^{3/2}}$$

Flux linked, $\phi = BA$

where, A is area of small circular loop.

$$A = \pi r^2 = \pi \times (0.3 \times 10^{-2})^2$$

$$\therefore \phi = \frac{\mu_0 \times 2 \times (20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]^{3/2}} \times \pi \times (0.3 \times 10^{-2})^2$$

$$\phi = 9.2 \times 10^{-11} \text{ Wb}$$

- 15** A coil is suspended in a uniform magnetic field with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop. But, if an aluminium plate is placed near to the coil, it stops. This is due to [AIEEE 2012]

- (a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

Ans. (d)

According to Lenz's law, electromagnetic induction takes place in the aluminium plate due to which eddy current is developed which oppose the motion or vibrations of coil.

This causes loss in energy which results in damping of oscillatory motion of the coil.

- 16** The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3 \text{ s}$ is [AIEEE 2006]

- (a) -190 V (b) -10 V (c) 10 V (d) 190 V

Ans. (b)

$$\phi = 10t^2 - 50t + 250$$

From Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt}$$

$$\therefore e = -[10 \times 2t - 50]$$

$$\therefore e|_{t=3\text{s}} = -[10 \times 6 - 50] = -10 \text{ V}$$

- 17** A coil having n turns and resistance $R \Omega$ is connected with a galvanometer of resistance $4R \Omega$. This combination is moved for time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is [AIEEE 2004]

- (a) $\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

Ans. (b)

The rate of change of flux or emf induced in the coil is $e = -n \frac{d\phi}{dt}$.

$$\therefore \text{Induced current, } I = \frac{e}{R'} = -\frac{n}{R'} \frac{d\phi}{dt} \dots (i)$$

$$\text{Given, } R' = R + 4R = 5R, d\phi = W_2 - W_1, dt = t$$

[here, W_1 and W_2 are flux associated with one turn]

Putting the given values in Eq. (i), we get

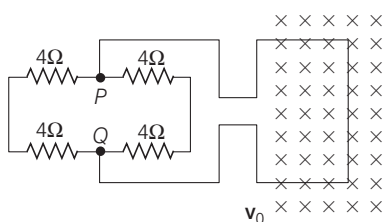
$$I = -\frac{n}{5R} \frac{(W_2 - W_1)}{t}$$

TOPIC 2

Motional EMF and Eddy Current

- 18** A square loop of side 20 cm and resistance 1Ω is moved towards right with a constant speed v_0 . The right arm of the loop is in a uniform

magnetic field of 5 T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4Ω . What should be the value of v_0 , so that a steady current of 2 mA flows in the loop? [2021, 1 Sep Shift-II]



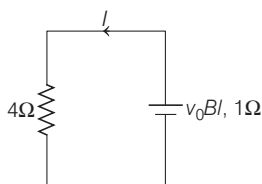
- (a) 1 m/s (b) 1 cm/s
(c) 10^2 m/s (d) 10^{-2} cm/s

Ans. (b)

According to given circuit diagram, equivalent resistance between point P and Q.

$$R_{PQ} = (4 + 4) \parallel (4 + 4) = \frac{8 \times 8}{8 + 8} = 4 \Omega$$

The equivalent circuit can be drawn as,



Equivalent resistance, $R_{eq} = 4 + 1 = 5 \Omega$

Magnetic field, $B = 5 \text{ T}$

The side of the square loop, $l = 20 \text{ cm} = 0.20 \text{ m}$

The steady value of the current, $I = 2 \text{ mA} = 2 \times 10^{-3} \text{ A}$

Induced emf, $e = Bv_0 l$

Induced current, $I = \frac{e}{R_{eq}}$

Substituting the values in the above equation, we get

$$2 \times 10^{-3} = \frac{5 \times v_0 \times 0.2}{5}$$

$$\Rightarrow v_0 = 10^{-2} \text{ m/s} = 1 \text{ cm/s}$$

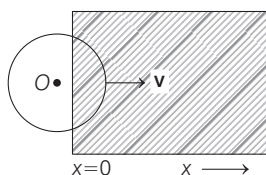
$\therefore$ The value of $v_0 = 1 \text{ cm/s}$ so that a steady current of 2 mA flows in the loop.

- 19** A constant magnetic field of 1 T is applied in the $x > 0$ region. A metallic circular ring of radius 1 m is moving with a constant velocity of 1 m/s along the X-axis. At $t = 0$ s, the centre of O of the ring is at

$x = -1 \text{ m}$. What will be the value of the induced emf in the ring at $t = 1 \text{ s}$?

(Assume the velocity of the ring does not change.)

[2021, 27 Aug Shift-II]



- (a) 1 V (b) $2\pi \text{ V}$
(c) 2 V (d) 0

Ans. (c)

Given, magnetic field, $B = 1 \text{ T}$

Radius of ring, $R = 1 \text{ m}$

Velocity of ring, $v = 1 \text{ m/s}$

Time taken, $t = 1 \text{ s}$

Let emf be ϵ .

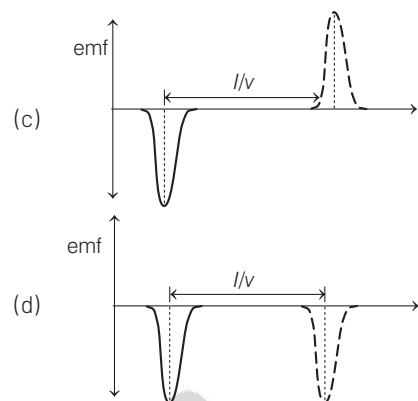
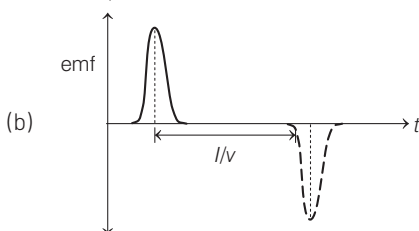
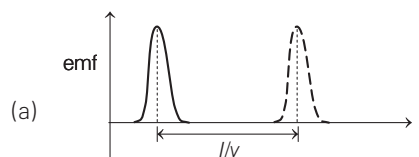
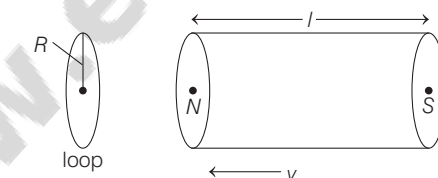
Since, $\epsilon = Blv \sin \theta$

$$\therefore \epsilon = 1 \times 2R \times 1 \times \sin 90^\circ$$

$$= 2R = 2 \text{ V}$$

- 20** A bar magnet is passing through a conducting loop of radius R with velocity v . The radius of the bar magnet is such that it just passes through the loop. The induced emf in the loop can be represented by the approximate curve

[2021, 26 Aug. Shift-I]



Ans. (c)

Given, radius of loop is R and velocity of the bar magnet is v .

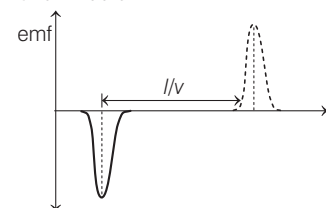
According to Faraday's law of electromagnetic induction, the emf induced in loop is given by

$$e = -\frac{Nd\phi}{dt} \quad \dots(i)$$

As initially the bar magnet is moving towards the coil, so the rate of change of magnetic flux linked with the coil will increase. Thus, initially the emf induced in coil will decrease as per Eq. (i) (due to -ve sign).

When magnet reaches the middle point in the coil, for that moment the emf induced will be equal to zero because in this situation, change in magnetic flux associated with bar magnet becomes zero. Now, after that when magnet moves forward, the magnetic flux again starts changing but the polarity of magnetic flux change will be reversed. So, the emf will now increase in opposite direction.

The graph for change in emf of coil is shown below.



- 21** A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of $3.0 \times 10^{-2} \text{ T}$. The maximum emf induced in the coil will be $\times 10^{-2} \text{ V}$.

(rounded off to the nearest integer.)

[2021, 26 Aug Shift-I]

Ans. (60)

Given, radius of circular coil, $r = 8.0 \text{ cm}$
 $= 0.08 \text{ m}$

Number of turns, $N = 20$

Angular speed of coil, $\omega = 50 \text{ rad s}^{-1}$

Magnetic field, $B = 3.0 \times 10^{-2} \text{ T}$

The maximum emf induced in a coil can be expressed as $\epsilon = \omega NBA$

Here, A is the area of coil.

$$\epsilon = NB\omega(\pi r^2)$$

Substituting the given values, we get

$$\epsilon = 20 \times 3.0 \times 10^{-2} \times 50 \times \pi \times (0.08)^2$$

$$= 60.3 \times 10^{-2} \text{ V} \approx 60 \times 10^{-2} \text{ V}$$

- 22** A circular conducting coil of radius 1 m is being heated by the change of magnetic field $\mathbf{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is $2 \mu\Omega$. The magnetic field is slowly switched off such that its magnitude changes in time as

$$B = \frac{4}{\pi} \times 10^{-3} \left(1 - \frac{t}{100}\right) \text{ T}$$

The energy dissipated by the coil before the magnetic field is switched off completely is $E = \dots \text{ mJ}$.

[2021, 25 July Shift-I]**Ans.**

(80) Given,

Radius of coil, $r = 1 \text{ m}$

Resistance of coil, $R = 2 \mu\Omega = 2 \times 10^{-6} \Omega$

Magnetic field, $B = \frac{4}{\pi} \times 10^{-3} \left(1 - \frac{t}{100}\right) \text{ T}$

As we know that,

$$\text{Induced emf } (\epsilon) = -\frac{d\phi}{dt} = -A \frac{dB}{dt} \quad (\because \phi = \mathbf{B} \cdot \mathbf{A})$$

where, $\frac{d\phi}{dt}$ = rate of change of flux

A = area of coil

$\frac{dB}{dt}$ = rate of change of magnetic field strength

$$\Rightarrow \epsilon = -(\pi r^2) \frac{d}{dt} \left[\frac{4}{\pi} \times 10^{-3} \left(1 - \frac{t}{100}\right) \right]$$

$$= -\pi r^2 \times \frac{4}{\pi} \times 10^{-3} \times \frac{-1}{100}$$

$$= r^2 \times 4 \times 10^{-5} = 1^2 \times 4 \times 10^{-5}$$

$$= 4 \times 10^{-5} \text{ V}$$

If $B = 0$,

$$\text{Then, } \frac{4}{\pi} \times 10^{-3} \left(1 - \frac{t}{100}\right) = 0 \Rightarrow t = 100 \text{ s}$$

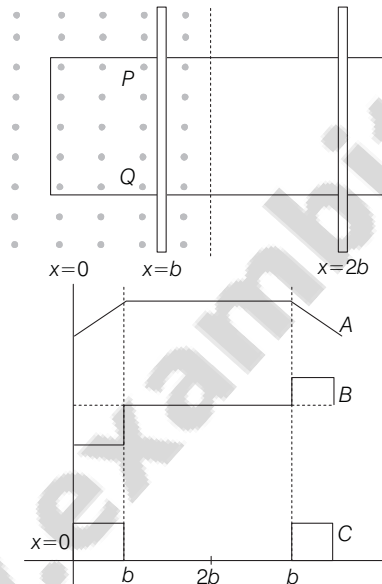
Since, Energy (E) = Power (P) $\times$ Time (t)

$$\text{Also, } P = \frac{\epsilon^2}{R}$$

$$\Rightarrow E = \frac{\epsilon^2}{R} \times t = \frac{(4 \times 10^{-5})^2}{2 \times 10^{-6}} \times 100 = \frac{16 \times 10^2}{2}$$

$$= 8 \times 10^{-2} = 0.08 = 80 \text{ mJ}$$

- 23** The arm PQ of a rectangular conductor is moving from $x=0$ to $x=2b$ outwards and then inwards from $x=2b$ to $x=0$ as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from $x=0$ to $x=b$. Identify the graph showing the variation of different quantities with distance.

[2021, 20 July Shift-I]

- (a) A - flux, B - power dissipated, C - emf
 (b) A - power dissipated, B - flux, C - emf
 (c) A - flux, B - emf, C - power dissipated
 (d) A - emf, B - power dissipated, C - flux

Ans. (c)

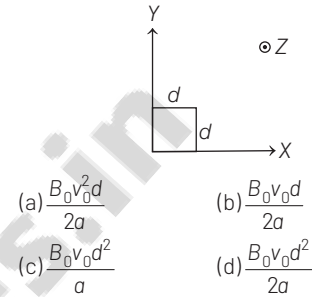
As the rectangular conductor moves in field area, so flux is increasing up to $x=b$, then flux is generated on return journey from $x=b$ to $x=0$. The flux is shown by plot A of the graph.

As, emf, $e = -\frac{d\phi}{dt}$, which is shown by

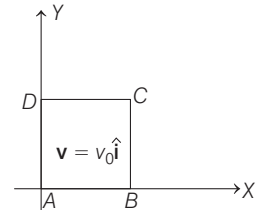
curve B .

and power dissipated, $P = VI$ which is shown by curve C .

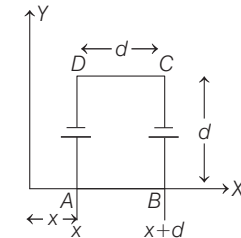
- 24** The magnetic field in a region is given by $\mathbf{B} = B_0 \left(\frac{x}{a}\right) \hat{\mathbf{k}}$. A square loop of side d is placed with its edges along the X and Y -axes. The loop is moved with a constant velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$. The emf induced in the loop is

[2021, 16 March Shift-II]**Ans. (c)**

The given situation can be shown as



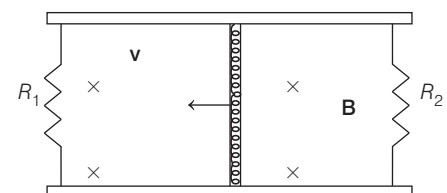
From the above figure, it can be seen that emf induced in side DC and AB will be zero. It is because $\mathbf{v}$ is parallel to length of conductor along X -axis.



$$\therefore \text{Net emf induced} = E_{BC} - E_{AD}$$

$$= \frac{B_0(x+d)v_0 d}{a} - \frac{B_0 x v_0 d}{a} = \frac{B_0 v_0 d^2}{a}$$

- 25** A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\mathbf{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v .

The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is

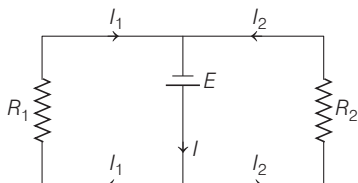
[2021, 16 March Shift-I]

- (a) both I_1 and I_2 are in anti-clockwise direction.
 (b) both I_1 and I_2 are in clockwise direction.
 (c) I_1 is in clockwise direction and I_2 is in anti-clockwise direction.
 (d) I_1 is in anticlockwise direction and I_2 is in clockwise direction.

Ans. (c)

According to Lenz's law, "An induced current flows in a direction in such a way that it always opposes the cause that induced it".

Considering this, it can be concluded that I_1 is in the clockwise direction and I_2 is in the anti-clockwise direction and the effective circuit with the directions of I_1 and I_2 can be shown as follows



- 26** An aeroplane with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of Earth's field at that part is 2.5×10^{-4} Wb/m² and the angle of dip is 60° . The emf induced between the tips of the plane wings will be

[2021, 26 Feb Shift-II]

- (a) 108.25 mV (b) 54.125 mV
 (c) 88.37 mV (d) 62.50 mV

Ans. (a)

Given, length of aeroplane wing, $l = 10$ m
 Speed of aeroplane,

$$v = 180 \text{ km/h} = 50 \text{ ms}^{-1}$$

Magnetic flux density,

$$B = 2.5 \times 10^{-4} \text{ Wb/m}^2$$

Angle of dip, $\theta = 60^\circ$

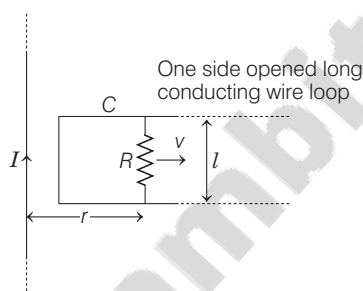
The emf induced between the tips of plane wings,

$$\begin{aligned} \epsilon &= Blv \sin \theta \\ &= 2.5 \times 10^{-4} \times 10 \times 50 \sin 60^\circ \\ &= 2.5 \times 5 \times 10^{-2} \times \sqrt{3}/2 \end{aligned}$$

$$\begin{aligned} &= 6.25 \times 1.732 \times 10^{-2} \\ &= 108.25 \times 10^{-3} \\ &= 108.25 \text{ mV} \end{aligned}$$

- 27** An infinitely long straight wire carrying current I , one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R . It slides to the right with a velocity v . The resistance of the conductor and the self-inductance of the loop are negligible. The induced current in the loop, as a function of separation r between the connector and the straight wire is

[2020, 5 Sep Shift-II]



- (a) $\frac{\mu_0 I l v}{4\pi R r}$ (b) $\frac{\mu_0 I l v}{\pi R r}$
 (c) $\frac{2\mu_0 I l v}{\pi R r}$ (d) $\frac{\mu_0 I l v}{2\pi R r}$

Ans. (d)

Magnetic field due to wire at a distance r is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\text{Induced emf, } \epsilon_{\text{induced}} = Blv = \frac{\mu_0 I l v}{2\pi r}$$

Now, induced current in the loop,

$$I_{\text{induced}} = \frac{\epsilon_{\text{induced}}}{R} = \frac{\mu_0 I l v}{2\pi R r}$$

Hence, correct option is (d).

- 28** A solid metal cube of edge length 2 cm is moving in a positive Y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive Z-direction. The potential difference between the two faces of the cube perpendicular to the X-axis is

[2019, 10 Jan Shift-I]

- (a) 2 mV (b) 12 mV (c) 6 mV (d) 1 mV

Ans. (b)

Potential difference between opposite faces of cube is

$$V = \text{induced emf} = Blv$$

where,

B = magnetic field = 0.1 T,

l = distance between opposite faces of cube

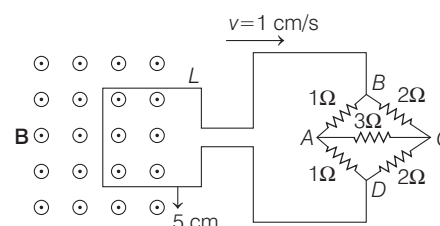
$$= 2 \text{ cm} = 2 \times 10^{-2} \text{ m and}$$

v = speed of cube = 6 ms^{-1} .

$$\text{Hence, } V = 0.1 \times 2 \times 10^{-2} \times 6 = 12 \text{ mV}$$

- 29** The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1} . At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7Ω , the current in the loop at that instant will be close to

[2019, 12 April Shift-I]



- (a) $60 \mu\text{A}$ (b) $170 \mu\text{A}$
 (c) $150 \mu\text{A}$ (d) $115 \mu\text{A}$

Ans. (b)

Induced emf in the conductor of length L moving with velocity of 1 cm/s in the magnetic field of 1T is given by

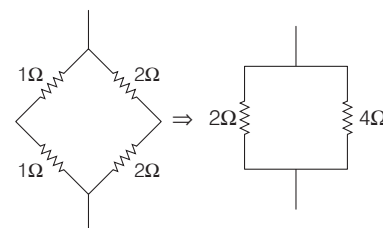
$$V = BLv \quad \dots(i)$$

If equivalent resistance of the circuit is R_{eq} , then current in the loop will be

$$i = \frac{V}{R_{\text{eq}}} = \frac{BLv}{R_{\text{eq}}} \quad \dots(ii)$$

Now, given network is a balanced Wheatstone bridge $\left(\frac{P}{Q} = \frac{R}{S}\right)$.

So, equivalent resistance of the Wheatstone bridge is



$$R_W = \frac{2 \times 4}{2 + 4} = \frac{8}{6} = \frac{4}{3} \Omega$$

Again, resistance of conductor is 1.7Ω .

So, effective resistance will be

$$R_{eq} = \frac{4}{3} + 1.7 = \frac{4}{3} + \frac{17}{10}$$

$$R_{eq} = \frac{40 + 51}{30} = \frac{91}{30} \approx 3 \Omega$$

By putting given values of R_{eq} , B and v in Eq. (ii), we have

$$i = \frac{(1)(5 \times 10^{-2}) \times 10^{-2}}{3}$$

$$[\text{here, } L = 5 \times 10^{-2} \text{ m, } v = 1 \text{ cm/s} = 10^{-2} \text{ m/s}]$$

$$i = \frac{5 \times 10^{-4}}{3} = 1.67 \times 10^{-4} \text{ A}$$

$$i = 167 \mu\text{A} \approx 170 \mu\text{A}$$

- 30** A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field of $0.3 \times 10^{-4} \text{ Wb/m}^2$. The value of the induced emf in wire is

[2019, 12 Jan Shift-II]

- (a) $1.5 \times 10^{-3} \text{ V}$
 (b) $1.1 \times 10^{-3} \text{ V}$
 (c) $0.3 \times 10^{-3} \text{ V}$
 (d) $2.5 \times 10^{-3} \text{ V}$

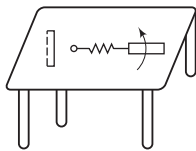
Ans. (a)

Wire falls perpendicularly to horizontal component of earth's magnetic field, so induced electromotive force (ϵ) = Bv

Substituting the given values, we get
 $\epsilon = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3} \text{ V}$

- 31** A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is

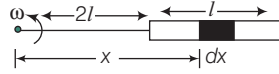
[JEE Main 2013]



- (a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$
 (c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

Ans. (d)

$\therefore$ Induced emf is rate of change of magnetic flux.



- 32** A horizontal straight wire 20 m long extending from East to West is falling with a speed of 5.0 m/s , at right angles to the horizontal component of the earth's magnetic field $0.30 \times 10^{-4} \text{ Wb/m}^2$. The instantaneous value of the emf induced in the wire will be

[AIEEE 2011]

- (a) 6.0 mV (b) 3 mV
 (c) 4.5 mV (d) 1.5 mV

Ans. (b)

Induced emf, $e = B_H l v$
 $= 0.30 \times 10^{-4} \times 20 \times 5.0$
 $= 3 \text{ mV}$

- 33** A boat is moving due East in a region where the earth's magnetic field is $5.0 \times 10^{-5} \text{ NA}^{-1} \text{ m}^{-1}$ due North and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1} , the magnitude of the induced emf in the wire of aerial is

[AIEEE 2011]

- (a) 0.75 mV (b) 0.50 mV
 (c) 0.15 mV (d) 1 mV

Ans. (c)

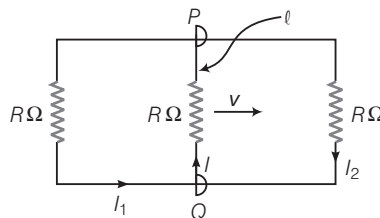
Induced emf,

$$e = B_H l v = 5.0 \times 10^{-5} \times 2 \times 1.50$$

$$= 0.15 \times 10^{-3} \text{ V} = 0.15 \text{ mV}$$

- 34** A rectangular loop has a sliding connector PQ of length l and resistance $R \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are

[AIEEE 2010]



$$(a) I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$$

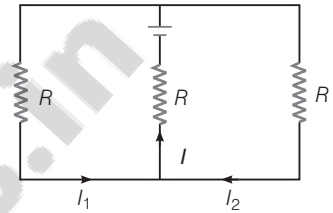
$$(b) I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$$

$$(c) I_1 = I_2 = I = \frac{Blv}{R}$$

$$(d) I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$$

Ans. (b)

A moving conductor is equivalent to a battery of emf $= vBl$ [motion emf]



Equivalent circuit, $I = I_1 + I_2$... (i)

Applying Kirchhoff's law,

$$I_1 R + IR - vBl = 0 \quad \dots (ii)$$

$$I_2 R + IR - vBl = 0 \quad \dots (iii)$$

Adding Eqs. (ii) and (iii), we get

$$I_1 R + IR - vBl + I_2 R + IR - vBl = 0$$

$$\Rightarrow I_1 R + I_2 R + 2IR - 2vBl = 0$$

$$\Rightarrow (I_1 + I_2)R + 2IR - 2vBl = 0$$

$$\Rightarrow 2IR + IR = 2vBl \quad [\text{From Eq. (i)}]$$

$$\Rightarrow I = \frac{2vBl}{3R}$$

Subtracting Eq. (iii) from Eq. (ii), we get

$$I_1 R + IR - vBl = 0$$

$$-I_2 R + IR - vBl = 0$$

$$\frac{I_1 R + IR - vBl}{-I_2 R + IR - vBl} = \frac{0}{0}$$

$$\Rightarrow R(I_1 - I_2) = 0$$

$$\Rightarrow I_1 - I_2 = 0 \quad [\because R \neq 0]$$

$$\Rightarrow I_1 = I_2 \quad \dots (iv)$$

From Eq. (i), we get

$$I = 2I_1 \quad [\because I_1 = I_2]$$

$$\Rightarrow 2I_1 = I$$

$$\Rightarrow I_1 = \frac{I}{2} = \frac{2vBl}{2 \times 3R} = \frac{vBl}{3R}$$

$$\therefore I_1 = I_2 = \frac{vBl}{3R}$$

- 35** In a uniform magnetic field of induction B , a wire in the form of semi-circle of radius r rotates about the diameter of the circle with angular frequency ω . If the total resistance of the circuit is R , the mean power generated per period of rotation is

[AIEEE 2004]

$$(a) \frac{B \pi r^2 \omega}{2R} \quad (b) \frac{(B \pi r^2 \omega)^2}{8R}$$

$$(c) \frac{(B \pi r \omega)^2}{2R} \quad (d) \frac{(B \pi r \omega)^2}{8R}$$

Ans. (b)

The flux associated with coil of area A and magnetic induction B is

$$\phi = BA \cos \theta = \frac{1}{2} B \pi r^2 \cos \omega t \quad \left[\because A = \frac{1}{2} \pi r^2 \right]$$

$$\therefore e_{\text{induced}} = - \frac{d\phi}{dt} = - \frac{d}{dt} \left(\frac{1}{2} B \pi r^2 \cos \omega t \right)$$

$$= \frac{1}{2} B \pi r^2 \omega \sin \omega t$$

$$\therefore \text{Power, } P = \frac{e_{\text{induced}}^2}{R} \quad [\because P = V^2/R]$$

$$= \frac{B^2 \pi^2 r^4 \omega^2 \sin^2 \omega t}{4R}$$

Hence, $P_{\text{mean}} = \langle P \rangle$

$$= \frac{B^2 \pi^2 r^4 \omega^2}{4R} \cdot \frac{1}{2} \quad \left[\because \langle \sin^2 \omega t \rangle = \frac{1}{2} \right]$$

$$= \frac{(B \pi r^2 \omega)^2}{8R}$$

- 36** A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is $0.2 \times 10^{-4} \text{ T}$, then the emf developed between the two ends of the conductor is

[AIEEE 2004]

- (a) $5 \mu\text{V}$ (b) $50 \mu\text{V}$
(c) 5 mV (d) 50 mV

Ans. (b)

The emf induced between ends of conductor



$$e = \frac{1}{2} B \omega L^2 = \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times (1)^2$$

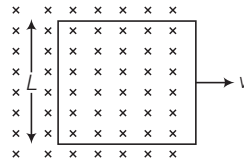
$$= 0.5 \times 10^{-4} \text{ V}$$

$$= 5 \times 10^{-5} \text{ V}$$

$$= 50 \mu\text{V}$$

- 37** A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing

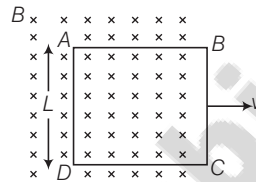
perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is

[AIEEE 2002]

- (a) zero (b) RvB (c) $\frac{vBL}{R}$ (d) vBL

Ans. (d)

As the side BC is outside the field, no emf is induced across BC . Since, AB and CD are not cutting any flux, the emf induced across these two sides will also be zero.



The side AD is cutting the flux and emf induced across this side is BvL with corner A at higher potential.

[according to Lorentz force, force on the charges will be towards A and on $-ve$ charges will be towards D].

TOPIC 3

Inductance (Self & Mutual)

- 38** A small square loop of side a and one turn is placed inside a larger square loop of side b and one turn ($b \gg a$). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side b , then the coefficient of mutual inductance between the two loops is

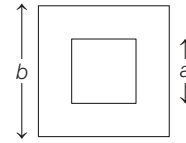
[2021, 31 Aug. Shift-I]

- (a) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$
(b) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a}{b}$
(c) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$
(d) $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a}{b}$

Ans. (a)

Given, side of one square loop = a
Side of 2nd square loop = b

$b \gg a$
Current = i



Since, $\phi = Mi$

$$M = \frac{\phi}{i} = \frac{B \cdot A}{i}$$

where, M = mutual inductance,

ϕ = magnetic flux,

B = magnetic field

and A = area.

Mutual inductance by 4 straight conductor,

$$M = \frac{4B \cdot A}{i}$$

$$M = \frac{4 \left[\frac{\mu_0}{4\pi} \frac{i}{b/2} (\sin \pi/4 + \sin \pi/4) \right] a^2}{i}$$

$$\Rightarrow M = \frac{4}{i} \left(\frac{\mu_0}{4\pi} \frac{2i}{b} (1/\sqrt{2} + 1/\sqrt{2}) a^2 \right)$$

$$\Rightarrow M = 4 \left(\frac{\mu_0}{4\pi} \frac{1}{b} \cdot \frac{2 \times 2}{\sqrt{2}} a^2 \right)$$

$$= \left(\frac{\mu_0}{4\pi} \frac{1}{b} 8\sqrt{2} a^2 \right) = \left(\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b} \right)$$

- 39** An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds.

[2021, 26 Aug. Shift-I]

- a. 0.4 (b) 0.8
c. 0.125 (d) 0.2

Ans. (d)

Magnetic energy is stored in an inductor, whereas energy is dissipated through resistor.

Given, $U_m = 64 \text{ J}$

$$i = 8 \text{ A and } P = 640 \text{ W}$$

As, magnetic energy, $U_m = \frac{1}{2} Li^2$

$$\Rightarrow 64 = \frac{1}{2} \times L \times 64$$

$$\Rightarrow L = 2 \text{ H}$$

Energy dissipated, $P = i^2 R$

$$\Rightarrow 640 = 64 R \Rightarrow R = 10 \Omega$$

Time constant of R - L circuit is given by

$$\tau = \frac{L}{R} \Rightarrow \tau = \frac{2}{10} = \frac{1}{5} = 0.2 \text{ s}$$

- 40** The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R , inductance L is connected to a battery, is

[2021, 18 March Shift-II]

- (a) $\frac{L}{R} \ln 5$ (b) infinite
(c) $\frac{L}{R} \ln 2$ (d) $\frac{L}{R} \ln 10$

Ans. (c)

The expression of the magnetic energy stored in the solenoid, $U = LI^2/2$

The maximum value of the magnetic energy stored in the solenoid,

$$U_0 = LI_0^2/2$$

Given, $U = 25\% \times U_0$

$$\Rightarrow \frac{LI^2}{2} = \frac{1}{4} \times \frac{LI_0^2}{2} \Rightarrow I = \frac{I_0}{2}$$

Therefore, using the formula for the decay current in L - R circuit,

$$I = I_0(1 - e^{-t/\tau}) \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-t/\tau})$$

$$e^{-t/\tau} = 1/2 \Rightarrow t = \tau \ln 2$$

$$\Rightarrow t = \frac{L}{R} \ln 2$$

- 41** A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by $V = 3t$ V (where, t is in second). If the voltage is applied when $t = 0$, then the energy stored in the coil after 4 s is J.

[2021, 25 Feb Shift-I]

Ans. (144)

Given, inductance of coil, $L = 2$ H

Supply voltage, $V = 3t$ V

Let E be the energy stored in the coil.

Since, emf $V = L \cdot \frac{dI}{dt}$

$$3t = L \frac{dI}{dt} \Rightarrow 3t dt = L dI$$

On integrating both sides, we get

$$\frac{3}{2} t^2 = LI$$

At $t = 4$ s,

$$\frac{3}{2} \times 4^2 = LI \Rightarrow \frac{3}{2} \times \frac{16}{L} = I$$

$$\Rightarrow 24/L = I \quad \dots(i)$$

As, $E = 1/2 LI^2$

$$\therefore E = 1/2 L \frac{24^2}{L^2} \quad [\text{From Eq. (i)}]$$

$$= \frac{1}{2} \frac{24^2}{L} = \frac{24^2}{2 \times 2} = 144 \text{ J}$$

- 42** In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be

[2020, 9 Jan Shift-I]

Ans. (10)

From an inductor (choke),

$$E_{\text{induced}} = L \frac{dI}{dt}$$

$$\Rightarrow L = \frac{E \times \Delta t}{\Delta I}$$

Here, $E = 100$ V, $\Delta I = 0.25$ A

and $\Delta t = 0.025$ ms
 $= 0.025 \times 10^{-3}$ s

$$\text{So, } L = \frac{100 \times 0.025 \times 10^{-3}}{0.25} = 100 \times 10^{-4} = 10 \text{ mH}$$

- 43** The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to

[2019, 9 April Shift-I]

- (a) $1/L$ (b) L^2 (c) L (d) $1/L^2$

Ans. (a)

Self inductance L_{sol} of a solenoid is given by

$$L_{\text{sol}} = \mu_0 n^2 \pi r^2 L$$

(Here, $n = N/L$ and L = length of solenoid)

$$\text{or } L_{\text{sol}} = \frac{\mu_0 N^2 \pi r^2}{L}$$

$$\text{Clearly, } L_{\text{sol}} \propto \frac{1}{L}$$

($\because$ All other parameters are fixed)

Note We can determine expression of L as follows

$$\Phi = NBA = L_{\text{sol}} I$$

But for a solenoid, $B = \mu_0 n I$, $A = \pi r^2$

$$\therefore L_{\text{sol}} I = \mu_0 n I \pi r^2 N$$

$$\text{or } L_{\text{sol}} = \mu_0 r^2 \pi r^2 L = \mu_0 \frac{N^2}{L} \pi r^2$$

- 44** Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P , a magnetic flux of 10^{-3} Wb passes through Q . No current is passed through Q . When no current passes through P

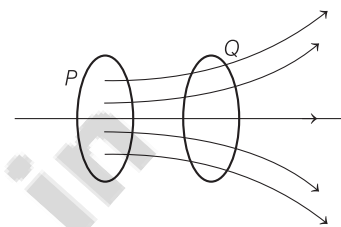
and a current of 2 A passes through Q , the flux through P is

[2019, 9 April Shift-II]

- (a) 6.67×10^{-3} Wb (b) 6.67×10^{-4} Wb
(c) 3.67×10^{-3} Wb (d) 3.67×10^{-4} Wb

Ans. (b)

As, coefficient of mutual induction is same for both coils



$$\therefore \frac{M_{PQ}}{N_P \Phi_{PQ}} = \frac{M_{QP}}{N_Q \Phi_{QP}} \quad \dots(i)$$

Here, $N_P = N_Q = 1$,

$$\Phi_{PQ} = ?, \Phi_{QP} = 10^{-3} \text{ Wb}$$

$$I_Q = 2 \text{ A}, I_P = 3 \text{ A}$$

Substituting values in Eq (i), we get

$$\begin{aligned} \Phi_{PQ} &= \frac{I_Q \cdot \Phi_{QP}}{I_P} = \frac{2}{3} \times 10^{-3} \\ &= 0.667 \times 10^{-3} \\ &= 6.67 \times 10^{-4} \text{ Wb} \end{aligned}$$

- 45** The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy of the inductance is

[2019, 10 Jan Shift-II]

- (a) 437.5 J (b) 740 J
(c) 637.5 J (d) 540 J

Ans. (a)

Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2} LI^2$$

When current is being changed from I_1 to I_2 , change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2} LI_2^2 - \frac{1}{2} LI_1^2 \quad \dots(i)$$

As I_1 and I_2 are given, we need to find value of L .

Now, induced emf in a coil is $\epsilon = L \frac{dI}{dt}$

Here, $\epsilon = 25$ V,

$$dI = I_2 - I_1 = (25 - 10) = 15 \text{ A and } dt = 1 \text{ s}$$

$$\Rightarrow 25 = L \times \frac{15}{1} \text{ or } L = \frac{25}{15} = 5/3 \text{ H}$$

Putting values of L , I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525$$

$$\Delta E = 437.5 \text{ J}$$

- 46** There are two long coaxial solenoids of same length l . The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 , respectively. The ratio of mutual inductance to the self-inductance of the inner coil is

[2019, 11 Jan Shift-I]

- (a) $\frac{n_2 \cdot r_1}{n_1 \cdot r_2}$ (b) $\frac{n_2 \cdot r_2^2}{n_1 \cdot r_1^2}$
(c) $\frac{n_2}{n_1}$ (d) $\frac{n_1}{n_2}$

Ans. (c)

Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2 , respectively is given as,
 $M = \mu_0 n_1 n_2 \pi r_1^2 l$ (for internal coil of radius r_1)

Self inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\therefore \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

- 47** A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil

[2019, 11 Jan Shift-II]

- (a) increases by a factor of 3
(b) decreases by a factor of $9\sqrt{3}$
(c) increases by a factor of 27
(d) decreases by a factor of 9

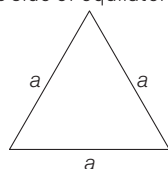
Ans. (c)

Self-inductance of a coil is given by the relation $L = \mu_0 n^2 A \cdot l$

where, n is number of turns per unit length. Shape of the wooden frame is equilateral triangle.

$$\therefore \text{Area of equilateral triangle, } A = \frac{\sqrt{3}}{4} a^2$$

(where, a is side of equilateral triangle)



$\therefore$ Self-inductance,

$$L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2 \right] \cdot l$$

Here, $l = 3a \times N$ (where, N is total turns)

$$\therefore L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2 \right] \times 3aN \text{ or } L \propto a^3$$

When each side of frame is increased by a factor 3 keeping the number of turns per unit length of the frame constant.

Then, $a' = 3a$

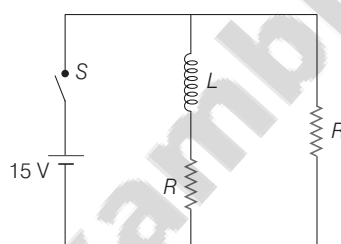
$$\therefore L' \propto (a')^3 \text{ or } L' \propto (3a)^3$$

$$\text{or } L' \propto 27a^3 \text{ or } L' = 27L$$

So, self-inductance will become 27 times.

- 48** In the figure shown, a circuit contains two identical resistors with resistance $R = 5\Omega$ and an inductance with $L = 2 \text{ mH}$. An ideal battery of 15 V is connected in the circuit.

[2019, 12 Jan Shift-I]

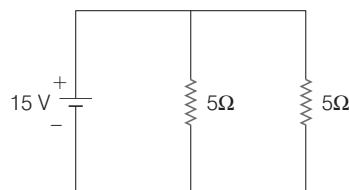


What will be the current through the battery long after the switch is closed?

- (a) 6 A (b) 3 A (c) 5.5 A (d) 7.5 A

Ans. (a)

After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to



$\therefore$ Current in circuit is

$$I = \frac{E}{R_{\text{eq}}} = \frac{15}{\left(\frac{5 \times 5}{5+5} \right)} = \frac{15 \times 2}{5} = 6 \text{ A}$$

- 49** Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon

[AIEEE 2003]

- (a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

Ans. (b)

Mutual inductance M between two coils is given by $M = \mu_0 n_1 n_2 \pi r_1^2 L$ where, n_1, n_2 are number of turns, r_1 is the radius of coil and L is the length. From the above formula, it is clear that mutual inductance depends on distance between the coils and geometry ($\pi r^2 = \text{area}$) of two coils.

- 50** When the current changes from $+2 \text{ A}$ to -2 A in 0.05 s , an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is

[AIEEE 2003]

- (a) 0.2 H (b) 0.4 H
(c) 0.8 H (d) 0.1 H

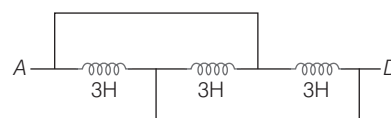
Ans. (d)

$$e = -L \frac{dI}{dt} = -L \frac{(-2-2)}{0.05}$$

$$\Rightarrow 8 = L \frac{(4)}{0.05}$$

$$\therefore L = \frac{8 \times 0.05}{4} = 0.1 \text{ H}$$

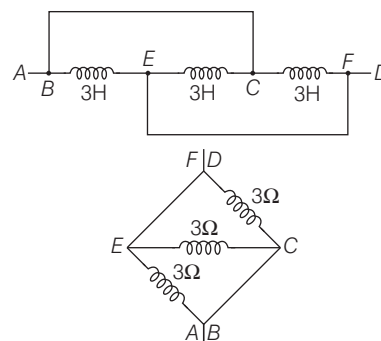
- 51** The inductance between A and D is



[AIEEE 2002]

- (a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

Ans. (d)



Here, inductors are in parallel.

$$\therefore \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

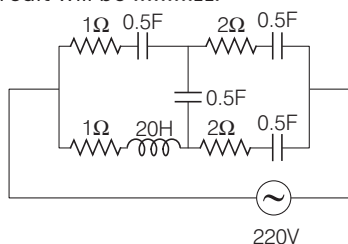
$$\text{or } L = 1 \text{ H}$$

Alternating Current

TOPIC 1

AC Circuits and Power in AC Circuits

- 01** At very high frequencies, the effective impedance of the given circuit will be Ω .



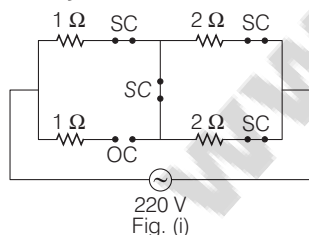
[2021, 31 Aug Shift-II]

Ans. (2)

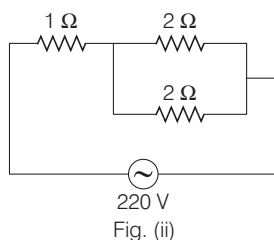
We know that, at very high frequency capacitive reactance becomes negligible i.e. short circuit (SC) and inductive reactance becomes very large i.e. open circuit (OC).

i.e. $X_C \rightarrow 0$ and $X_L \rightarrow \infty$.

Now, the circuit can be rearranged as shown in figure.



Final circuit is



Hence, equivalent resistance,

$$R_{eq} = 1 + \frac{2 \times 2}{2 + 2} = 2\Omega$$

Thus, correct answer is 2.

- 02** An AC circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5}:x$. The value of x is.

[2021, 27 Aug Shift-II]

Ans. (1)

Given, resistance of resistor = R

Inductance, $X_L = 3R$

Capacitance $X_C = 2R$

As we know that,

$$\text{Power factor, } \cos\phi = \frac{R}{Z}$$

where, Z is impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For case I, an inductance connected in series with resistance

$$\begin{aligned} \cos\phi_1 &= \frac{R}{Z_1} = \frac{R}{\sqrt{R^2 + (X_L - 0)^2}} \\ &= \frac{R}{\sqrt{R^2 + (3R)^2}} = \frac{R}{R\sqrt{10}} = \frac{1}{\sqrt{10}} \quad \dots(i) \end{aligned}$$

For case II, A capacitor is also connected in series with resistance

$$\begin{aligned} \therefore \cos\phi_2 &= \frac{R}{Z_2} \\ \Rightarrow \cos\phi_2 &= \frac{R}{\sqrt{R^2 + (3R - 2R)^2}} \\ &= \frac{R}{\sqrt{R^2 + R^2}} = \frac{R}{R\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \dots(ii) \end{aligned}$$

Now, ratio of Eqs. (ii) and (i), we get

$$\frac{\cos\phi_2}{\cos\phi_1} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{10}}} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}:1$$

$$\therefore x = 1$$

- 03** The alternating current is given by

$$i = \left\{ \sqrt{42} \sin\left(\frac{2\pi}{T}t\right) + 10 \right\} \text{ A}$$

The rms value of this current is A.
[2021, 27 Aug Shift-I]

Ans. (11)

Given, equation of alternating current,

$$i = \left[\sqrt{42} \sin\left(\frac{2\pi}{T}t\right) + 10 \right] \text{ A}$$

From given equation, we get

$$i = i_1 + i_2$$

where, $i_1 = \sqrt{42} \sin\left(\frac{2\pi}{T}t\right) \text{ A}$ and $i_2 = 10 \text{ A}$

Now, i_1 is oscillating current, whereas i_2 is direct current and its value does not change with time.

$$(i_1)_{\text{rms}} = \frac{\sqrt{42}}{\sqrt{2}} = \sqrt{21} \text{ A}$$

$$(i_2)_{\text{rms}} = 10 \text{ A}$$

We know that,

$$i_{\text{rms}}^2 = (i_1)_{\text{rms}}^2 + (i_2)_{\text{rms}}^2$$

Substituting the values, we get

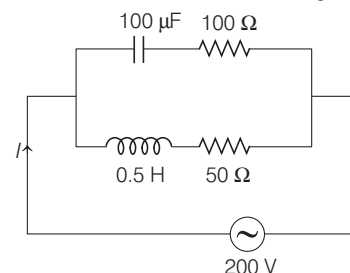
$$i_{\text{rms}} = \sqrt{(\sqrt{21})^2 + 10^2} = \sqrt{121}$$

$$\Rightarrow i_{\text{rms}} = 11 \text{ A}$$

Thus, RMS value of given equation of current is 11 A.

- 04** In the given circuit the AC source has $\omega = 100 \text{ rad s}^{-1}$. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?

[2021, 26 Aug Shift-II]



- (a) 5.9 A (b) 4.24 A
(c) 0.94 A (d) 6 A

Ans. (*)

Given, angular frequency, $\omega = 100 \text{ rad/s}$
Capacitance of capacitor,
 $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$
Inductance of inductor coil, $L = 0.5 \text{ H}$
Resistance in upper branch, $R_1 = 100 \Omega$
Resistance in lower branch, $R_2 = 50 \Omega$
In the given circuit consider current in upper branch be i_1 and current flowing in lower branch be i_2 . The net current flowing in circuit will be I .
Impedance of upper branch can be calculated as

$$\begin{aligned} Z_1 &= \sqrt{X_C^2 + R_1^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R_1^2} \\ &= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2} \\ &= \sqrt{100^2 + 100^2} = 100\sqrt{2} \Omega \end{aligned}$$

Impedance of lower branch can be calculated as

$$\begin{aligned} &= \sqrt{(\omega L)^2 + R_2^2} \\ &= \sqrt{(100 \times 0.5)^2 + 50^2} \\ &= \sqrt{50^2 + 50^2} \\ &= 50\sqrt{2} \Omega \end{aligned}$$

Current flowing in upper branch,

$$i_1 = \frac{V}{Z_1} = \frac{200}{100\sqrt{2}} = \sqrt{2} \text{ A}$$

Phase of current in upper branch,

$$\cos \phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \phi_1 = 45^\circ$$

Thus, in upper branch, current leads voltage by 45° as capacitor is present.

Current flowing in lower branch i_2 is

$$i_2 = \frac{V}{Z_2} = \frac{200}{50\sqrt{2}} = 2\sqrt{2} \text{ A}$$

Phase of current in lower branch is

$$\cos \phi_2 = \frac{R_2}{Z_2} = \frac{50}{50\sqrt{2}} \Rightarrow \phi_2 = 45^\circ$$

Thus, in lower branch current lags voltage by 45° as inductor is present.

Thus, the net current, $I = \sqrt{i_1^2 + i_2^2}$

$$\begin{aligned} I &= \sqrt{(\sqrt{2})^2 + (2\sqrt{2})^2} \\ I &= \sqrt{10} = 3.16 \text{ A} \end{aligned}$$

Thus, no option in the given question is correct.

If $I = i_1 + i_2$ is taken then

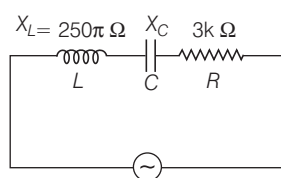
$I = \sqrt{2} + 2\sqrt{2} = 4.24 \text{ A}$ is obtained which is incorrect method of solution.

- 05** A series L - C - R circuit driven by 300 V at a frequency of 50 Hz contains a resistance $R = 3 \text{ k}\Omega$, an inductor of inductive reactance $X_L = 250 \pi \Omega$ and an unknown capacitor. The value of capacitance to maximise the average power should be (Take, $\pi^2 = 10$) [2021, 26 Aug Shift-I]

- (a) $4 \mu\text{F}$ (b) $25 \mu\text{F}$
(c) $400 \mu\text{F}$ (d) $40 \mu\text{F}$

Ans. (a)

The circuit diagram can be drawn as,



Average power of an L - C - R circuit is given by

$$P_{av} = V_{rms} i_{rms} \cos \phi$$

For P_{av} to be maximum, $\cos \phi = 1$... (i)

We know that,

$$\cos \phi = \frac{R}{Z}$$

where, R = resistance

and Z = impedance of L - C - R circuit.

$$\text{Now, } Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\text{As, } \cos \phi = 1 = \frac{R}{Z} \quad [\text{using Eq. (i)}]$$

$$\Rightarrow R = Z$$

$$\Rightarrow R = \sqrt{(X_L - X_C)^2 + R^2}$$

Squaring both sides, we get

$$R^2 = (X_L - X_C)^2 + R^2$$

$$\Rightarrow (X_L - X_C)^2 = 0$$

$$\Rightarrow X_L = X_C$$

$$\text{Since, } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\text{and } X_L = 250 \pi \Omega \quad (\text{given})$$

$$\Rightarrow 250 \pi = \frac{1}{2\pi \times 50 \times C}$$

$$\Rightarrow C = \frac{1}{2\pi^2 \times 50 \times 250}$$

$$\Rightarrow C = 4 \times 10^{-8} \text{ F} = 4 \mu\text{F}$$

- 06** A 100Ω resistance, a $0.1 \mu\text{F}$ capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz. [2021, 27 July Shift-II]

- (a) 0.70 H (b) 70.3 mH
(c) $7.03 \times 10^{-5} \text{ H}$ (d) 70.3 H

Ans. (d)

Given,

Resistance, $R = 100 \Omega$

Capacitance, $C = 0.1 \mu\text{F}$

Inductance, $L = 250 \text{ V}$

Resonant frequency, $f_0 = 60 \text{ Hz}$

$\therefore$ We know that,

$$\text{Resonant frequency, } f_0 = \frac{L}{2\pi\sqrt{LC}}$$

$$\begin{aligned} L &= \frac{1}{4\pi^2 f_0^2 C} \\ &= \frac{1}{4\pi^2 \times (60)^2 \times 0.1 \times 10^{-6}} \\ &= \frac{10^7}{4\pi^2 \times 3600} \\ &= \frac{10^5}{4\pi^2 \times 36} = 70.3 \text{ H} \end{aligned}$$

- 07** A 0.07 H inductor and a 12Ω resistor are connected in series to a 220 V, 50 Hz AC source. The approximate current in the circuit and the phase angle between current and source voltage are, respectively.

[Take, π as $\frac{22}{7}$]

[2021, 27 July Shift-I]

(a) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(b) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(c) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(d) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Ans. (a)

Given,

Inductance, $L = 0.07 \text{ H}$

Resistance, $R = 12 \Omega$

Voltage, $V = 220 \text{ V}$

Frequency, $f = 50 \text{ Hz}$

$\therefore$ We know that,

$$\text{Angular frequency, } \omega = 2\pi f = 2\pi \times 50 = 100 \pi \text{ Hz}$$

$$\therefore \text{Inductive reactance, } X_L = \omega L$$

$$= 100 \pi \times 0.07$$

$$\Rightarrow = 100 \times \frac{22}{7} \times \frac{7}{100} = 22 \Omega$$

$\therefore$ A resistor and an inductor are connected in series,

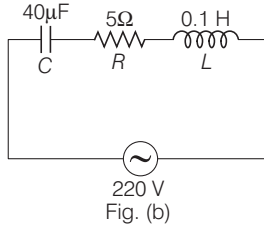
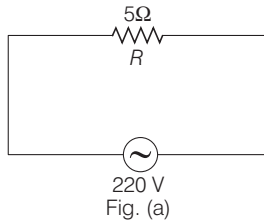
$$\begin{aligned}\therefore \text{Impedance, } Z &= \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 22^2} \\ &= \sqrt{144 + 484} \\ &= \sqrt{628} \approx 25 \Omega \\ \therefore \text{Current, } I &= \frac{V}{Z} = \frac{220}{25} = 8.8 \text{ A} \Rightarrow I = 8.8 \text{ A}\end{aligned}$$

Also, phase angle between current and source voltage can be given as

$$\begin{aligned}\tan \phi &= \left(\frac{X_L}{R} \right) \Rightarrow \tan \phi = \frac{22}{12} \\ \Rightarrow \phi &= \tan^{-1} \left(\frac{11}{6} \right)\end{aligned}$$

- 08** Two circuits are shown in the figures (a) and (b). At a frequency of rad/s, the average power dissipated in one cycle will be same in both the circuits.

[2021, 25 July Shift-II]



Ans. (500)

Given, resistance (R_a) for circuit (a) is 5Ω .

In circuit (b)

Capacitance, $C = 40 \mu\text{F} = 40 \times 10^{-6} \text{ F}$

Resistance, $R = 5 \Omega$

Inductance, $L = 0.1 \text{ H}$

Supply voltage, $V = 220 \text{ V}$

P_a and P_b be the power in circuit (a) and (b)

$$\begin{aligned}\therefore P_a &= \frac{V^2}{R} = \frac{(220)^2}{5} \\ &= \frac{484 \times 100}{5} = 484 \times 20 = 9680 \text{ W}\end{aligned}$$

In R-L-C circuit,

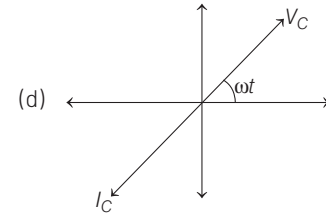
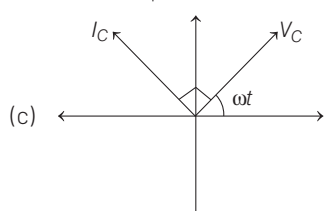
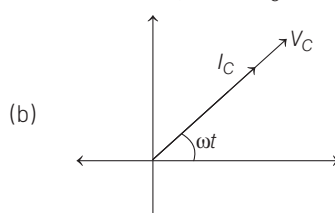
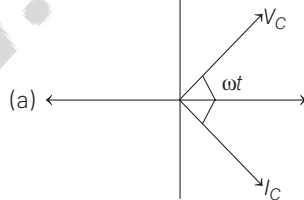
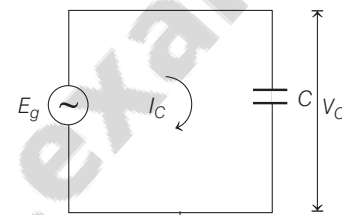
$$P_{av} = \frac{V^2}{Z} \cos \phi$$

where, Z is impedance of circuit and $\cos \phi$ is power factor.

$$\begin{aligned}P_{av} &= \frac{V^2 R}{Z^2} \quad [\because \cos \phi = \frac{R}{Z}] \dots (i) \\ &= \frac{V^2 R}{Z^2}\end{aligned}$$

$$\begin{aligned}\text{and } Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ \Rightarrow Z^2 &= R^2 + (X_L - X_C)^2 \\ \text{Put in Eq. (i), we get} \\ P_{av} &= \frac{(220)^2 \times 5}{5^2 + (X_L - X_C)^2} \\ \Rightarrow 5^2 + (X_L - X_C)^2 &= \frac{(220)^2 \times 5}{484 \times 20} \quad [\because P_{av} = P_a] \\ &= \frac{484 \times 100 \times 5}{484 \times 20} = 25 \\ \Rightarrow (X_L - X_C)^2 &= 0 \\ \Rightarrow X_L &= X_C \\ \Rightarrow \omega L &= \frac{1}{\omega C} \\ \Rightarrow \omega &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 40 \times 10^{-6}}} \\ &= \frac{1}{\sqrt{4 \times 10^{-6}}} = \frac{10^3}{2} \\ &= \frac{1000}{2} = 500 \text{ rad s}^{-1}\end{aligned}$$

- 09** In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0} \sin \omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is [2021, 22 July Shift-II]

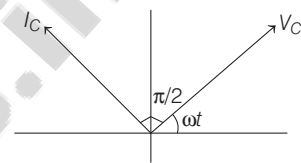


Ans. (c)

Given, input alternating Emf (E_g) = $E_{g_0} \sin \omega t$

As we know that, capacitive current leads voltage by $\frac{\pi}{2}$ rad

$\therefore$ Phasor diagram will be



where, V_C and I_C are voltage and current in capacitive circuit.

- 10** Match List I with List II.

[2021, 22 July Shift-II]

List I	List II
A. $\omega L > \frac{1}{\omega C}$	(i) Current is in phase with EMF
B. $\omega L = \frac{1}{\omega C}$	(ii) Current lags behind the applied EMF
C. $\omega L < \frac{1}{\omega C}$	(iii) Maximum current occurs
D. Resonant frequency	(iv) Current leads the EMF

Choose the correct answer from the options given below.

Codes

A	B	C	D
(a) (ii)	(i)	(iv)	(iii)
(b) (ii)	(i)	(iii)	(iv)
(c) (iii)	(i)	(iv)	(ii)
(d) (iv)	(iii)	(ii)	(i)

Ans. (a)

Given,

$$(A) \quad \omega L > \frac{1}{\omega C}$$

Since, inductive resistance > capacitive resistance, hence Inductive current lag behind applied voltage.

$$(B) \quad \omega L = \frac{1}{\omega C}$$

Since, inductive resistance = capacitive resistance

Hence, this circuit behaves like a L - C - R series resonant circuit. Therefore, $Z = R$ and phase difference $= 0$.

$\therefore$ Circuit will be purely resistive and resistive current remains in phase with voltage.

$$(C) \quad \omega L < \frac{1}{\omega C}$$

Since, inductive resistance $<$ capacitive resistance, hence capacitive current lead with applied voltage.

(D) As we know that, at resonance impedance

$$(Z) = R \text{ (resistance)}$$

$\therefore$ Circuit current will be maximum.

Hence, option (a) is the correct.

- 11** For a series L - C - R circuit with $R = 100 \Omega$, $L = 0.5 \text{ mH}$ and $C = 0.1 \text{ pF}$ connected across 220 V - 50 Hz AC supply, the phase angle between current and supplied voltage and the nature of the circuit is

[2021, 20 July Shift-II]

- (a) 0° , resistive circuit
(b) $\approx 90^\circ$, predominantly inductive circuit
(c) 0° resonance circuit
(d) $\approx 90^\circ$, predominantly capacitive circuit

Ans. (d)

According to question, there is a series L - C - R circuit.

Resistance, $R = 100 \Omega$

Capacitance, $C = 0.1 \text{ pF} = 0.1 \times 10^{-12} \text{ F}$

Inductance, $L = 0.5 \text{ mH} = 0.5 \times 10^{-3} \text{ H}$

Frequency, $f = 50 \text{ Hz}$

Voltage, $V = 220 \text{ V}$

$\therefore$ Inductive reactance, $X_L = \omega L = 2\pi fL$
 $= 2\pi \times 50 \times 0.5 \times 10^{-3}$

$$\Rightarrow X_L = 50\pi \times 10^{-3} \Omega$$

$\therefore$ Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$$\Rightarrow X_C = \frac{1}{2\pi \times 50 \times 10^{-12} \times 0.1}$$

$$\Rightarrow X_C = \frac{1}{100\pi \times \frac{1}{10} \times 10^{-12}}$$

$$\Rightarrow X_C = \frac{10^{11}}{\pi} \Omega$$

On comparing X_L with X_C , we can write

$$X_C \gg X_L$$

and $|X_C - X_L| \gg R$

which means that the circuit is predominantly capacitive circuit.

- 12** A series L - C - R circuit of $R = 5 \Omega$, $L = 20 \text{ mH}$ and $C = 0.5 \mu\text{F}$ is connected across an AC supply of 250 V , having variable frequency. The power dissipated at resonance condition is $\times 10^2 \text{ W}$.

[2021, 20 July Shift-II]

Ans. (125)

Given, resistance, $R = 5 \Omega$

Inductance, $L = 20 \text{ mH}$

Capacitance, $C = 0.5 \mu\text{F}$

Voltage, $V = 250 \text{ V}$

$$I_{\text{rms}} = \frac{V}{Z} = \frac{V}{R} \quad [\text{At resonance, } Z = R]$$

Power dissipated, $P = I_{\text{rms}}^2 R$

$$\begin{aligned} \text{or } P &= \frac{V^2}{R} = \frac{250 \times 250}{5} \\ &= \frac{250 \times 250}{5} = 12500 \text{ J/s} \\ &= 125 \times 10^2 \text{ W} \end{aligned}$$

- 13** AC voltage $V(t) = 20 \sin \omega t$ volt of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2 . The amplitude of the oscillating displacement current for the applied AC voltage is
(Take, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)

[2021, 20 July Shift-I]

- (a) $21.14 \mu\text{A}$ (b) $83.37 \mu\text{A}$
(c) $27.79 \mu\text{A}$ (d) $55.58 \mu\text{A}$

Ans. (c)

Given,

AC voltage, $V(t) = 20 \sin \omega t$ volt.

Frequency, $f = 50 \text{ Hz}$

Separation between the plates, $d = 2 \text{ mm}$
 $= 2 \times 10^{-3} \text{ m}$

Area, $A = 1 \text{ m}^2$

$$\text{As, } C = \frac{\epsilon_0 A}{d}$$

where, ϵ_0 = absolute electrical permittivity of free space
 $= 8.854 \times 10^{-12} \text{ N}^{-1} \text{ kg}^2 \text{ m}^{-2}$

$$\Rightarrow C = \frac{\epsilon_0 \times 1}{2 \times 10^{-3}} \quad \dots(i)$$

$$\text{Capacitive reactance } (X_C) = \frac{1}{\omega C} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$X_C = \frac{2 \times 10^{-3}}{2 \times 50\pi \times \epsilon_0} \quad (\because \omega = 2\pi f)$$

$$\begin{aligned} &= \frac{2 \times 10^{-3}}{25 \times 4\pi \epsilon_0} \\ \Rightarrow X_C &= \frac{2 \times 10^{-3}}{25} \times 9 \times 10^9 \\ \Rightarrow X_C &= \frac{18}{25} \times 10^6 \Omega \end{aligned}$$

By using Ohm's law,

$$\begin{aligned} \text{As, } I_0 &= \frac{V_0}{X_C} = \frac{20 \times 25}{18} \times 10^{-6} \\ &= 27.78 \times 10^{-6} \end{aligned}$$

$$\Rightarrow I_0 = 27.78 \mu\text{A}$$

$\therefore$ The amplitude of the oscillating displacement current for applied AC voltage will be approximately $27.79 \mu\text{A}$.

- 14** In an L - C - R series circuit, an inductor 30 mH and a resistor 1Ω are connected to an AC source of angular frequency 300 rad/s . The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \times 10^{-3} \text{ F}$. Then, the value of x is

[2021, 20 July Shift-I]

Ans. (3)

Given,

Inductance, $L = 30 \text{ mH}$

Resistance, $R = 1 \Omega$

Angular frequency, $\omega = 300 \text{ rad/s}$

We know that in L - C - R circuit,

$$\tan \phi = \frac{X_C - X_L}{R}$$

where, ϕ = phase angle $= 45^\circ$

$$X_C = \text{capacitive reactance} = \frac{1}{\omega C}$$

$$X_L = \text{inductive reactance} = \omega L$$

$$\Rightarrow \tan 45^\circ = \frac{X_C - X_L}{R}$$

$$\Rightarrow X_C - X_L = R \quad [\because \tan 45^\circ = 1]$$

$$\Rightarrow \frac{1}{\omega C} - \omega L = R$$

$$\Rightarrow \frac{1}{\omega C} - 300 \times 30 \times 10^{-3} = 1$$

$$\Rightarrow \frac{1}{\omega C} = 10 \Rightarrow \omega C = \frac{1}{10}$$

$$\Rightarrow C = \frac{1}{10\omega} \Rightarrow C = \frac{1}{10 \times 300}$$

$$\Rightarrow C = \frac{1}{3} \times 10^{-3} \text{ F} \quad \dots(i)$$

According to question, the value of capacitance is $\frac{1}{x} \times 10^{-3} \text{ F}$. So, on comparing it with Eq. (i), we can say $x = 3$.

- 15** In a series L - C - R circuit, the inductive reactance (X_L) is $10\ \Omega$ and the capacitive reactance (X_C) is $4\ \Omega$. The resistance (R) in the circuit is $6\ \Omega$. The power factor of the circuit is [2021, 18 March Shift-II]

(a) $\frac{1}{2}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (c)

Given, inductive reactance, $X_L = 10\ \Omega$

Capacitive reactance, $X_C = 4\ \Omega$

Resistance, $R = 6\ \Omega$

Therefore, power factor = $\cos\theta$

$$= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{6}{\sqrt{6^2 + (10 - 4)^2}} = \frac{1}{\sqrt{2}}$$

- 16** In a series L - C - R resonance circuit, if we change the resistance only, from a lower to higher value, [2021, 18 March Shift-I]

- (a) the bandwidth of resonance circuit will increase
(b) the resonance frequency will increase
(c) the quality factor will increase
(d) the quality factor and the resonance frequency will remain constant

Ans. (a)

Bandwidth of L - C - R series resonance

circuit, $\beta = \frac{R}{L}$

As we increase the value of the resistance from lower to the higher of the circuit, the bandwidth of resonance circuit will also increase.

So, the option (a) is correct.

Resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

Since, the resonance frequency is independent of the resistance.

So, the option (b) is incorrect.

We know that,

Quality factor, $Q = \frac{\omega L}{R}$

The quality factor is inversely proportional to the resistance of the circuit. So, increasing the value of resistance, the quality factor is decreased. So, the option (c) and (d) are incorrect.

- 17** An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is [2021, 18 March Shift-I]

- (a) 2.5 ms (b) 25 ms
(c) 2.5 s (d) 0.25 ms

Ans. (a)

Given, the frequency of the AC source, $f = 50\ \text{Hz}$

Angular frequency of the circuit,

$$\omega = 2\pi f$$

$$\Rightarrow \omega = 2\pi(50) \Rightarrow \omega = 100\pi$$

As we know the general expression of the current in AC circuit,

$$I = I_0 \sin\omega t$$

$$\Rightarrow \frac{I_0}{\sqrt{2}} = I_0 \sin(100\pi t)$$

$$\Rightarrow \sin(100\pi t) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 100\pi t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow 100\pi t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{1}{400}\ \text{sec}$$

$$\Rightarrow t = 2.5 \times 10^{-3}\ \text{s} = 2.5\ \text{ms}$$

- 18** What happens to the inductive reactance and the current in a purely inductive circuit, if the frequency is halved?

[2021, 17 March Shift-II]

- (a) Both inductive reactance and current will be halved.
(b) Inductive reactance will be halved and current will be doubled.
(c) Inductive reactance will be doubled and current will be halved.
(d) Both inductive reactance and current will be doubled.

Ans. (b)

As we know, the inductive reactance is directly proportional to the frequency of the AC circuit

i.e., $X_L = \omega L$

$$\Rightarrow X_L = 2\pi fL \quad (\because \omega = 2\pi f)$$

Here, f is the frequency of the AC circuit,

L is the inductive resistance

and X_L is the inductive reactance.

When the frequency of an AC circuit is halved, then the inductive reactance of the circuit is also halved.

i.e. $X_L' = \frac{X_L}{2}$

Using Ohm's law, $I = \frac{V}{X_L}$

When the frequency is halved, then the current

$$I' = \frac{V}{X_L'} \Rightarrow I' = \frac{V}{X_L/2}$$

$$I' = 2I$$

The current becomes doubled.

- 19** Match List-I with List-II

[2021, 17 March Shift-II]

List-I	List-II
A. Phase difference between current and voltage in a purely resistive AC circuit	1. $\frac{\pi}{2}$; current leads voltage
B. Phase difference between current and voltage in a pure inductive AC circuit	2. zero
C. Phase difference between current and voltage in a pure capacitive AC circuit	3. $\frac{\pi}{2}$; current lags voltage
D. Phase difference between current and voltage in an L - C - R series circuit	4. $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below.

A B C D

(a) 1 3 4 2

(b) 2 4 3 1

(c) 2 3 4 1

(d) 2 3 1 4

Ans. (d)

- A. In a purely resistive AC circuit, the phase difference between the current and voltage is zero.
B. In a purely inductive AC circuit, the current lags the voltage, so the phase difference between the current and voltage is $\pi/2$.
C. In a purely capacitive AC circuit the current leads the voltage, so the phase difference between the current and voltage is $\pi/2$.
D. The phase difference between current & voltage in an L - C - R series circuit is

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

$\therefore$ The correct match is

A-(2), B-(3), C-(1), D-(4).

- 20** An AC current is given by $I = I_1 \sin \omega t + I_2 \cos \omega t$. A hot wire ammeter will give a reading

[2021, 17 March Shift-I]

- (a) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (b) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$
 (c) $\frac{I_1 + I_2}{\sqrt{2}}$ (d) $\frac{I_1 + I_2}{2\sqrt{2}}$

Ans. (b)

Given, $I = I_1 \sin \omega t + I_2 \cos \omega t$... (i)

We know that rms value of current is given by

$$I_{\text{rms}} = \sqrt{\frac{\int_0^T I^2 dt}{T}}$$

$$\Rightarrow I_{\text{rms}} = \sqrt{\frac{\int_0^T (I_1 \sin \omega t + I_2 \cos \omega t)^2 dt}{T}}$$

[using Eq. (i)]

Squaring on both sides of the above equation, we get

$$\Rightarrow (I_{\text{rms}})^2 = \frac{\int_0^T (I_1^2 \sin^2 \omega t + I_2^2 \cos^2 \omega t + 2I_1 I_2 \sin \omega t \cos \omega t) dt}{T}$$

$$\Rightarrow (I_{\text{rms}})^2 = \frac{\int_0^T \left(\frac{I_1^2}{2} + \frac{I_2^2}{2} + 0 \right) dt}{T}$$

$$\Rightarrow I_{\text{rms}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

- 21** A sinusoidal voltage of peak value 250 V is applied to a series L-C-R circuit, in which $R = 8 \Omega$, $L = 24 \text{ mH}$ and $C = 60 \mu\text{F}$. The value of power dissipated at resonant condition is x kW. The value of x to the nearest integer is [2021, 16 March Shift-I]

Ans. (4)

Given, $V_0 = 250 \text{ V}$, $R = 8 \Omega$, $L = 24 \text{ mH}$ and $C = 60 \mu\text{F}$

We know that, at resonance power,

$$P = \frac{V_{\text{rms}}^2}{R}$$

$$\Rightarrow P = \frac{(250/\sqrt{2})^2}{8} \left[\because V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{250}{\sqrt{2}} \text{ V} \right]$$

$$= \frac{(250)^2}{16}$$

$$= \frac{62500}{16} = 3906.25 \text{ W}$$

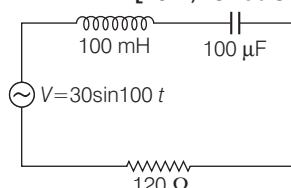
$$\approx 4 \text{ kW}$$

$$x = 4$$

Comparing with the given value in the question i.e., x kW, the value of x = 4.

- 22** Find the peak current and resonant frequency of the following circuit (as shown in figure).

[2021, 26 Feb Shift-II]



- (a) 0.2 A and 50 Hz (b) 0.2 A and 100 Hz
 (c) 2 A and 100 Hz (d) 2 A and 50 Hz

Ans. (a)

Given, inductance, $L = 100 \text{ mH}$
 $= 100 \times 10^{-3} \text{ H}$

Capacitance, $C = 100 \mu\text{F}$
 $= 100 \times 10^{-6} \text{ F}$

Resistance, $R = 120 \Omega$

Maximum voltage, $V_0 = 30 \text{ V}$

Angular frequency, $\omega = 100 \text{ rad s}^{-1}$

Impedance, $Z = \sqrt{(X_L - X_C)^2 + R^2}$

Here, $X_L = \omega L = 100 \times 100 \times 10^{-3} = 10 \Omega$

$\therefore X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100 \Omega$

$$\Rightarrow Z = \sqrt{(X_C - X_L)^2 + R^2}$$

$$= \sqrt{(100 - 10)^2 + (120)^2}$$

$$= \sqrt{90^2 + 120^2} = 153.3 \Omega$$

As we know that, peak current,

$$I_0 = \frac{V_0}{Z} = \frac{30}{153.3} = 0.195 \text{ A} \approx 0.2 \text{ A}$$

Resonance frequency,

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}}$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{10^{-5}}} = \frac{1}{2\pi \times 10^{-2} \sqrt{10^{-1}}}$$

$$= \frac{100}{2\pi} \approx 50 \text{ Hz}$$

- 23** An alternating current is given by the equation $i = i_1 \sin \omega t + i_2 \cos \omega t$. The rms current will be

[2021, 26 Feb Shift-I]

- (a) $\frac{1}{\sqrt{2}} (i_1^2 + i_2^2)^{1/2}$ (b) $\frac{1}{\sqrt{2}} (i_1 + i_2)^2$
 (c) $\frac{1}{2} (i_1^2 + i_2^2)^{1/2}$ (d) $\frac{1}{\sqrt{2}} (i_1 + i_2)$

Ans. (a)

Given, $i = i_1 \sin \omega t + i_2 \cos \omega t$

Let I_{rms} be the rms current.

$$\therefore I_{\text{rms}} = \left(\frac{i_1^2 + i_2^2}{2} \right)^{1/2}$$

$$\Rightarrow I_{\text{rms}} = \frac{1}{\sqrt{2}} (i_1^2 + i_2^2)^{1/2}$$

- 24** In a series L-C-R resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be [2021, 26 Feb Shift-I]

Ans. (400)

Given, initial quality factor (Q_i) = 100

Let initial inductance (X_{Li}) = x

Final inductance (X_{Lf}) = 2x

and initial resistance (R_i) = R

Final resistance (R_f) = $\frac{R}{2}$

Final quality factor = Q_f

Since, $Q_i = \frac{X_L}{R}$

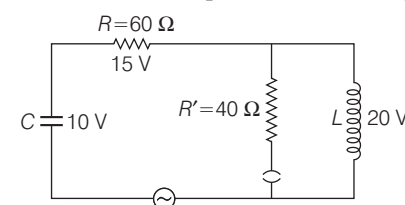
and $Q_f = \frac{2X_L}{R/2}$

$$\Rightarrow Q_f = \frac{4X_L}{R} = 4Q_i = 4 \times 100$$

$$= 400$$

Hence, final quality factor will be 400.

- 25** The angular frequency of alternating current in an L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser. [2021, 25 Feb Shift-I]



- (a) 0.8 H and 150 μF
 (b) 0.8 H and 250 μF
 (c) 1.33 H and 250 μF
 (d) 1.33 H and 150 μF

Ans. (b)

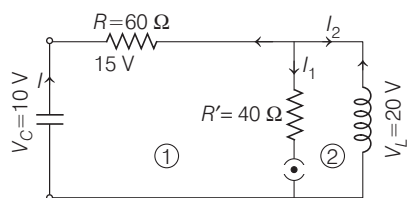
Given, angular frequency,

$\omega = 100 \text{ rad s}^{-1}$

$R = 60 \Omega$, $V_R = 15 \text{ V}$,

$R' = 40 \Omega$, $V_{R'} = V_L = 20 \text{ V}$

and $V_C = 10 \text{ V}$



By using Ohm's law,

$$V = IR \Rightarrow I = V/R$$

$$\Rightarrow I = 15/60 = 1/4 \text{ A} \quad \dots (i)$$

and $I_1 = \frac{V_{R'}}{R'} = 20/40 = 1/2 \text{ A} \quad \dots (ii)$

As, $X_C = \frac{V_C}{I} = \frac{10}{1/4} = 40 \Omega$

and $X_C = \frac{1}{\omega C}$

$$\Rightarrow C = \frac{1}{X_C \omega} = \frac{1}{40 \times 100}$$

$$= 0.25 \times 10^{-3} \text{ F} = 0.25 \text{ mF} = 250 \mu\text{F}$$

By using KCL in loop 2,

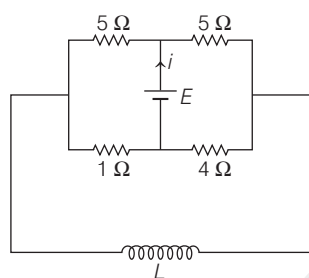
$$I_2 = I - I_1$$

$$= 1/4 - 1/2 = -1/4 \text{ A}$$

$$\therefore X_L = \frac{V_L}{|I_2|} = \frac{20}{1/4} = 80 \Omega \Rightarrow \omega L = 80$$

$$\Rightarrow L = \frac{80}{\omega} = \frac{80}{100} = 0.8 \text{ H}$$

- 26** The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is
[2021, 25 Feb Shift-I]



(a) $\frac{18E}{55}, \frac{5E}{18}$

(b) $\frac{10E}{33}, \frac{5E}{18}$

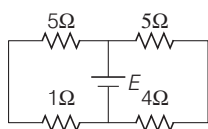
(c) $\frac{5E}{18}, \frac{18E}{55}$

(d) $\frac{5E}{18}, \frac{10E}{33}$

Ans. (d)

As we know that at time $t = 0$, inductor acts as open circuit.

Then, the circuit becomes



Therefore,

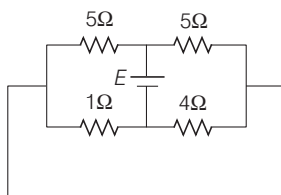
$$R_{eq} = \frac{(5+1)(5+4)}{(5+1)+(5+4)} = \frac{6 \times 9}{6+9} = \frac{54}{15} = \frac{18}{5}$$

By using Ohm's law,

$$V = IR_{eq} \quad [\because V = E]$$

$$\Rightarrow I = \frac{E \times 5}{18} = \frac{5E}{18}$$

At $t = \infty$, inductor will act as short circuit. It is shown below



Therefore,

$$R_{eq} = \frac{5 \times 5}{5+5} + \frac{1 \times 4}{1+4} = \frac{25}{10} + \frac{4}{5} = \frac{5}{2} + \frac{4}{5}$$

$$= \frac{33}{10} \Omega$$

and $I = \frac{E}{R_{eq}} = \frac{10E}{33}$

- 27** A transmitting station releases waves of wavelength 960 m. A capacitor of $2.56 \mu\text{F}$ is used in the resonant circuit. The self-inductance of coil necessary for resonance is $\dots \times 10^{-8} \text{ H}$.
[2021, 25 Feb Shift-I]

Ans. (10)

Given, wavelength of transmission signal,

$$\lambda = 960 \text{ m}$$

Capacitance, $C = 2.56 \mu\text{F}$

$$= 2.56 \times 10^{-6} \text{ F}$$

As we know resonance frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Also, frequency (f) = $\frac{\text{speed } (v)}{\text{wavelength } (\lambda)}$

$$\therefore \frac{v}{\lambda} = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow \sqrt{LC} = \frac{\lambda}{v \times 2\pi}$$

On squaring both sides, we get

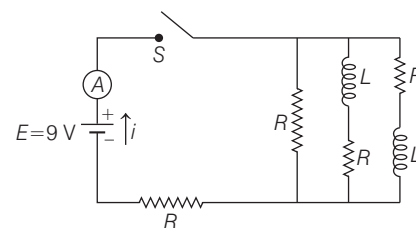
$$\Rightarrow LC = \frac{\lambda^2}{v^2 \times 4\pi^2} \Rightarrow L = \frac{\lambda^2}{v^2 \times 4\pi^2 \times C}$$

$$\Rightarrow L = \frac{(960)^2}{(3 \times 10^8)^2 \times 4\pi^2 \times 2.56 \times 10^{-6}}$$

$$\therefore L = 10 \times 10^{-8} \text{ H}$$

- 28** Figure shows a circuit that contains four identical resistors with resistance $R = 2.0 \Omega$, two identical inductors with inductance $L = 2.0 \text{ mH}$

and an ideal battery with electromotive force $E = 9 \text{ V}$. The current i just after the switch S is closed will be [2021, 24 Feb Shift-II]



(a) 2.25 A

(b) 3.0 A

(c) 3.37 A

(d) 9 A

Ans. (a)

Given, resistance, $R = 2 \Omega$,

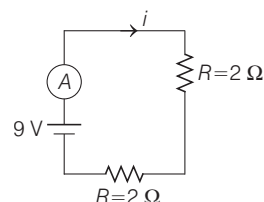
Inductance, $L = 2 \text{ mH}$,

emf, $E = 9 \text{ V}$

and i be the current.

$\therefore$ At $t = 0$ when switch is closed, inductors behave as open circuit.

$\therefore$ Effective circuit will be



By using Ohm's law, $V = i R_{eq}$

$$\Rightarrow i = V/R_{eq}$$

where, R_{eq} is equivalent resistance of series resistors,

i.e., $R_{eq} = R + R = 2R = 2 \times 2 = 4 \Omega$

$$\therefore i = \frac{9}{4} = 2.25 \text{ A}$$

- 29** A resonance circuit having inductance and resistance $2 \times 10^{-4} \text{ H}$ and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is $\dots$.
[Take, $\pi = 3.14$]

[2021, 24 Feb Shift-I]

Ans. (2000)

Given, $L = 2 \times 10^{-4} \text{ H}$, $R = 6.28 \Omega$,

$$f_0 = 10 \text{ MHz} = 10 \times 10^6 \text{ Hz}$$

$$\therefore \text{Quality factor} = \omega_0 \frac{L}{R} = 2\pi f_0 \frac{L}{R}$$

$$= 2\pi \times 10 \times 10^6 \times \frac{2 \times 10^{-4}}{6.28}$$

$$= 2 \times 10^3 = 2000$$

- 30** In an AC-circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is [2020, 2 Sep Shift-II]

(a) $2R^2$ (b) Zero
(c) R (d) $R\sqrt{2}$

Ans. (c)

Given, inductor, capacitor and resistors are connected in series and have same reactance i.e., $X_L = X_C = R$

Since, impedance (Z) = $\sqrt{R^2 + (X_L - X_C)^2}$

$$\therefore Z = \sqrt{R^2 + (R - R)^2} = R$$

- 31** An inductance coil has a reactance of 100Ω . When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45° . The self-inductance of the coil is [2020, 2 Sep Shift-II]

(a) $1.1 \times 10^{-2} \text{ H}$ (b) $1.1 \times 10^{-1} \text{ H}$
(c) $5.5 \times 10^{-5} \text{ H}$ (d) $6.7 \times 10^{-7} \text{ H}$

Ans. (a)

Given, reactance, $X_L = 100 \Omega$

Frequency of AC, $f = 1000 \text{ Hz}$

As voltage leads current by 45° , so there must be some resistance in the coil otherwise

$$\Delta \phi = 90^\circ$$

$$\text{Using, } \tan \phi = \frac{X_L}{R}$$

$$\text{We have, } \tan 45^\circ = \frac{X_L}{R} \text{ or } X_L = R$$

$\therefore$ Reactance of circuit containing resistance and inductance is

$$\sqrt{X_L^2 + R^2} = 100 \Omega$$

$$\Rightarrow \sqrt{X_L^2 + X_L^2} = 100 \Omega$$

$$\text{or } X_L = 50\sqrt{2} \Omega$$

$$\text{But } X_L = L\omega$$

$$\Rightarrow L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{50\sqrt{2}}{2\pi \times 1000}$$

$$\Rightarrow L = 1.125 \times 10^{-2} \text{ H}$$

Hence, correct option is (a).

- 32** A 750 Hz , 20 V (rms) source is connected to a resistance of 100Ω , an inductance of 0.1803 H and a capacitance of $10 \mu\text{F}$ all in series combination. The time in which the resistance (heat capacity $2 \text{ J/}^\circ\text{C}$) will get heated by 10°C is close to. (Assume no loss of heat to the surroundings) [2020, 3 Sep Shift-I]

(a) 418 s (b) 245 s (c) 365 s (d) 348 s

Ans. (d)

Given, frequency, $f = 750 \text{ Hz}$

$$V_{\text{rms}} = 20 \text{ V}, R = 100 \Omega, L = 0.1803 \text{ H}$$

and $C = 10 \mu\text{F}$

So, impedance of L - C - R circuit is

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

$$= \sqrt{(100)^2 + \left(\frac{2\pi \times 750 \times 0.1803 - 1}{2\pi \times 750 \times 10 \times 10^{-6}}\right)^2}$$

$$\approx 834 \Omega$$

Now, power lost by L - C - R circuit, which occurs in resistance is given by

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$\text{As, } \cos \phi = \frac{R}{Z} \text{ and } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$\therefore P = \frac{V_{\text{rms}}^2 \cdot R}{Z^2} = \frac{(20)^2 \times 100}{(834)^2} = 0.0575 \text{ Js}^{-1}$$

Heat developed in resistor,

$$H = Pt = S(\Delta \theta)$$

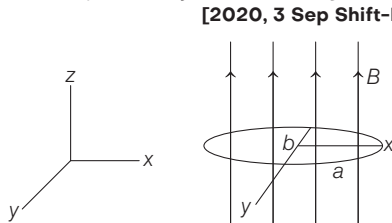
where, t = time, S = heat capacity
 $= 2 \text{ J/}^\circ\text{C}$

$\Delta \theta$ = temperature change = 10°C

$$\Rightarrow t = \frac{S \Delta \theta}{P} = \frac{2 \times 10}{0.0575} = 348 \text{ s}$$

Hence, option (d) is correct.

- 33** An elliptical loop having resistance R , of semi-major axis a and semi-minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the X -axis with angular frequency ω , then the average power loss in the loop due to joule's heating is [2020, 3 Sep Shift-I]



(a) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{2R}$

(b) zero

(c) $\frac{\pi a b B \omega}{R}$

(d) $\frac{\pi^2 a^2 b^2 B^2 \omega^2}{R}$

Ans. (a)

Average power developed which eventually gets lost as heat due to resistance of loop is given by

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{(V_{\text{max}} / \sqrt{2})^2}{R} = \frac{V_{\text{max}}^2}{2R}$$

Also, $V_{\text{max}} = NBA \omega$

In given loop, $N = 1, A = \pi ab$

$$\text{So, } P_{\text{avg}} = \frac{(BA\omega)^2}{2R} = \frac{B^2 \pi^2 a^2 b^2 \omega^2}{2R}$$

Hence, option (a) is correct.

- 34** An AC circuit has $R = 100 \Omega$, $C = 2 \mu\text{F}$ and $L = 80 \text{ mH}$ connected in series. The quality factor of the circuit is [2020, 6 Sep Shift-I]

(a) 2 (b) 0.5 (c) 20 (d) 400

Ans. (a)

Given that, $R = 100 \Omega$, $C = 2 \mu\text{F}$, $L = 80 \text{ mH}$

For a series L - C - R AC circuit,

$$\text{Quality factor, } \phi = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$= \frac{1}{100} \sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$\phi = 2$$

- 35** In a series L - R circuit, power of 400 W is dissipated from a source of 250 V , 50 Hz . The power factor of the circuit is 0.8 . In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R . Taking the value of C as $\left(\frac{n}{3\pi}\right) \mu\text{F}$, then value of n is [2020, 6 Sep Shift-II]

Ans. (400)

Given, for L - R circuit

$$V_{\text{rms}} = 250 \text{ V}, f = 50 \text{ Hz}, P = 400 \text{ W}$$

and $\cos \phi = 0.8$

As, we know, power is given by

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$\Rightarrow P = V_{\text{rms}} \frac{V_{\text{rms}}}{Z} \cos \phi$$

$$\Rightarrow Z = \frac{V_{\text{rms}}^2}{P} \cos \phi$$

$$\Rightarrow Z = \frac{(250)^2}{400} \times 0.8 = 125 \Omega$$

$$\text{Also, } \cos \phi = \frac{R}{Z} \Rightarrow 0.8 = \frac{R}{125} \Rightarrow R = 100 \Omega$$

$$\text{Impedance, } Z = \sqrt{R^2 + X_L^2}$$

$$\Rightarrow 125 = \sqrt{(100)^2 + X_L^2}$$

$$X_L = 75 \Omega$$

Now, to obtain power factor unity, X_C must be equal to X_L .

$$\text{i.e. } X_C = X_L = 75 \Omega$$

$$\Rightarrow \frac{1}{\omega C} = 75 \Rightarrow C = \frac{1}{\omega \times 75} = \frac{1}{2\pi \times 50 \times 75}$$

$$\Rightarrow C = \frac{1}{7500\pi} \text{ F or } C = \frac{400}{3\pi} \mu\text{F}$$

$$\text{Given, } C = \frac{n}{3\pi} \mu\text{F}$$

On comparing the both, we get
 $n = 400$

- 36** An L-C-R circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant b , the correct equivalence would be

[2020, 7 Jan Shift-I]

(a) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

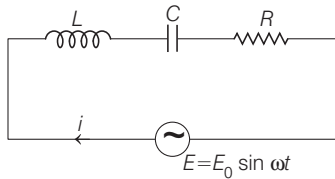
(b) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

(c) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(d) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

Ans. (b)

For an L-C-R circuit,



By KVL, we have

$$-L \frac{di}{dt} - \frac{q}{C} - iR + E = 0$$

Above can be rearranged as

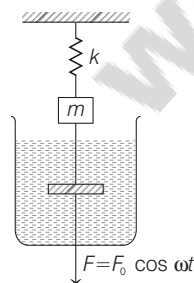
$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E_0 \sin \omega t \quad \dots(i)$$

Now, for a damped harmonic oscillator, we have

$$m\ddot{x} = -kx - b\dot{x} + F_0 \cos \omega t$$

Rearranging above equation, we have

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \cos \omega t \quad \dots(ii)$$



On comparing Eqs. (i) and (ii), we get the following analogy,

$$L \equiv m, R \equiv b \text{ and } \frac{1}{C} \equiv k$$

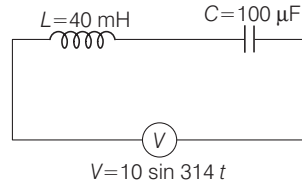
$$\text{or } L \leftrightarrow m, C \leftrightarrow \frac{1}{k} \text{ and } R \leftrightarrow b$$

- 37** In L-C circuit, the inductance $L = 40$ mH and capacitance $C = 100 \mu\text{F}$. If a voltage $V(t) = 10 \sin(314t)$ is applied to the circuit, the current in the circuit is given as

[2020, 9 Jan Shift-II]

- (a) $0.52 \cos 314 t$ (b) $0.52 \sin 314 t$
 (c) $10 \cos 314 t$ (d) $5.2 \cos 314 t$

Ans. (a)



Impedance of given circuit,

$$\begin{aligned} Z &= \sqrt{(X_C - X_L)^2} = X_C - X_L \\ &= \frac{1}{\omega C} - \omega L \\ &= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3} \\ &= 19.28 \Omega \end{aligned}$$

As $X_C > X_L$, circuit is capacitive, hence current in circuit leads emf by $\frac{\pi}{2}$ radians.

Current in circuit is given by

$$\begin{aligned} I &= I_{\max} \sin \left(\omega t + \frac{\pi}{2} \right) \\ &= \frac{V_{\max}}{Z} \cos \omega t \\ &= \frac{10}{19.28} \times \cos 314 t \end{aligned}$$

$$\text{or } I = 0.52 \cos 314 t$$

- 38** An alternating voltage $V(t) = 220 \sin 100\pi t$ volt is applied to a purely resistive load of 50Ω . The time taken for the current to rise from half of the peak value to the peak value is [2019, 8 April Shift-I]

- (a) 5 ms (b) 2.2 ms
 (c) 7.2 ms (d) 3.3 ms

Ans. (d)

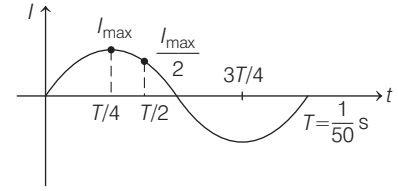
In an AC resistive circuit, current and voltage are in phase.

$$\text{So, } I = \frac{V}{R}$$

$$\Rightarrow I = \frac{220}{50} \sin(100\pi t) \quad \dots(i)$$

$\therefore$ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50} \text{ s}$$



So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200} \text{ s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{\max}}{2} = I_{\max} \sin(100\pi t_2)$$

$$\Rightarrow \sin(100\pi t_2) = \frac{1}{2} \Rightarrow 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120} \text{ s}$

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300} \text{ s} = 3.3 \text{ ms}$$

- 39** A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i . Which of the following circuits will exhibit this?

[2019, 8 April Shift-II]

- (a) RC circuit with $R = 1 \text{ k}\Omega$ and $C = 1 \mu\text{F}$
 (b) RL circuit with $R = 1 \text{ k}\Omega$ and $L = 1 \text{ mH}$
 (c) RC circuit with $R = 1 \text{ k}\Omega$ and $C = 10 \mu\text{F}$
 (d) RL circuit with $R = 1 \text{ k}\Omega$ and $L = 10 \text{ mH}$

Ans. (c)

$$\text{Given, phase difference, } \phi = \frac{\pi}{4}$$

As we know, for R-L or R-C circuit, Capacitive reactance (X_C)

$$\tan \phi = \frac{\text{or inductive reactance } (X_L)}{\text{Resistance } (R)}$$

$$\tan \frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R}$$

$$\Rightarrow R = X_C \text{ or } X_L$$

$$\text{Also, given } e = e_0 \sin(100t)$$

Comparing the above equation with general equation of emf, i.e. $e = e_0 \sin \omega t$, we get

$$\omega = 100 \text{ rad/s} = 10^2 \text{ rad/s}$$

Now, checking option wise,

For R-C circuit, with

$$R = 1 \text{ k}\Omega = 10^3 \Omega \text{ and } C = 1 \mu\text{F} = 10^{-6} \text{ F}$$

$$\text{So, } X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4 \Omega$$

$$\Rightarrow R \neq X_C$$

For R-L circuit, with

$$R = 1k\Omega = 10^3\Omega \text{ and } L = 1mH = 10^{-3}H$$

$$\text{So, } X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\Omega$$

$$\Rightarrow R \neq X_L$$

For R-C circuit, with

$$R = 1k\Omega = 10^3\Omega$$

$$\text{and } C = 10\mu F = 10 \times 10^{-6}F = 10^{-5}F$$

$$\text{So, } X_C = \frac{1}{10^2 \times 10^{-5}} = 10^3\Omega \Rightarrow R = C$$

For R-L circuit, with

$$R = 1k\Omega = 10^3\Omega$$

$$\text{and } L = 10mH = 10 \times 10^{-3}H = 10^{-2}H$$

$$X_L = 10^2 \times 10^{-2} = 1\Omega \Rightarrow R \neq X_L$$

Alternate Solution

$$\text{Since, } \tan \frac{\pi}{4} = 1 = \frac{X_C \text{ or } X_L}{R}$$

$\therefore$ For R-C circuit, we have

$$1 = \frac{1}{C\omega R} \text{ or } \omega = \frac{1}{CR} \quad \dots(i)$$

Similarly, for R-L circuit, we have

$$1 = \frac{\omega L}{R} \Rightarrow \omega = \frac{R}{L} \quad \dots(ii)$$

It is given in the question that,

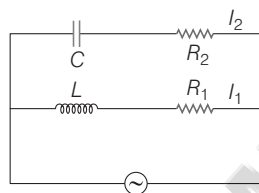
$$\omega = 100 \text{ rad/s}$$

Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c),

$$\omega = \frac{1}{CR} = \frac{1}{10 \times 10^{-6} \times 10^3}$$

$$\text{or } \omega = 100 \text{ rad/s}$$

40



In the above circuit, $C = \frac{\sqrt{3}}{2} \mu F$,

$$R_2 = 20\Omega, L = \frac{\sqrt{3}}{10} \text{ Hand } R_1 = 10\Omega.$$

Current in L-R₁ path is I₁ and in C-R₂ path is I₂. The voltage of AC source is given by $V = 200\sqrt{2} \sin(100t)$ volts. The phase difference between I₁ and I₂ is

- [2019, 12 Jan Shift-II]
 (a) 30° (b) 60°
 (c) 0° (d) 90°

Ans. (a)

Phase difference between I₂ and V, i.e. C-R₂ circuit is given by

$$\tan \phi = \frac{X_C}{R_2} \Rightarrow \tan \phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan \phi = \frac{1}{\frac{\sqrt{3}}{2} \times 10^{-6} \times 100 \times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$ is nearly 90°.

Phase difference between I₁ and V, i.e. in L-R₁ circuit is given by

$$\tan \phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan \phi_2 = -\frac{\frac{\sqrt{3}}{10} \times 100}{10} = -\sqrt{3}$$

$$\text{As, } \tan \phi_2 = -\sqrt{3}$$

$$\therefore \phi_2 = 120^\circ$$

Now, phase difference between I₁ and I₂ is

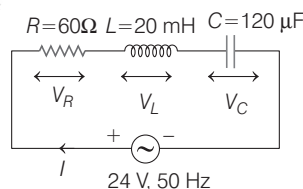
$$\Delta \phi = \phi_2 - \phi_1 = 120^\circ - 90^\circ = 30^\circ$$

41 A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is [2019, 9 Jan Shift-II]

- (a) $3.39 \times 10^3 \text{ J}$ (b) $5.65 \times 10^2 \text{ J}$
 (c) $2.26 \times 10^3 \text{ J}$ (d) $5.17 \times 10^2 \text{ J}$

Ans. (d)

The given series R-L-C circuit is shown in the figure below.



Here,

V_R = potential across resistance (R)
 V_L = potential across inductor (L) and
 V_C = potential across capacitor (C).

Impedance of this series circuit is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \dots(i)$$

$$\therefore X_L = \omega L = (2\pi f)(L) = 2\pi \times 50 \times 20 \times 10^{-3} \Omega$$

$$X_L = 6.28 \Omega \quad \dots(ii)$$

$$\text{and } X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \frac{250}{3\pi} \Omega \quad \dots(iii)$$

$$\text{and } X_L - X_C = \left(6.28 - \frac{250}{3\pi} \right) = -20.23 \Omega \quad \dots(iv)$$

RMS value of current in circuit is,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{\text{rms}} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$$I_{\text{rms}} = 0.379 \text{ A}$$

Therefore, energy dissipated is

$$= I_{\text{rms}}^2 \times R \times t$$

$$E = (0.379)^2 \times 60 \times 60$$

$$\text{or } = 517.10 = 5.17 \times 10^2 \text{ J}$$

42 In an AC circuit, the instantaneous emf and current are given by

$$e = 100 \sin 30t, i = 20 \sin \left(30t - \frac{\pi}{4} \right)$$

In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively [JEE Main 2018]

- (a) 50, 10 (b) $\frac{1000}{\sqrt{2}}$, 10
 (c) $\frac{50}{\sqrt{2}}$, 0 (d) 50, 0

Ans. (b)

Given, $e = 100 \sin 30t$

$$\text{and } i = 20 \sin \left(30t - \frac{\pi}{4} \right)$$

$\therefore$ Average power,

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos \frac{\pi}{4} = \frac{1000}{\sqrt{2}} \text{ watt}$$

Wattless current is,

$$I = I_{\text{rms}} \sin \phi$$

$$= \frac{20}{\sqrt{2}} \times \sin \frac{\pi}{4} = \frac{20}{2} = 10 \text{ A}$$

$$\therefore P_{\text{av}} = \frac{1000}{\sqrt{2}} \text{ watt}$$

$$\text{and } I_{\text{wattless}} = 10 \text{ A}$$

43 For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current

exhibits resonance. The quality factor, Q is given by [JEE Main 2018]

- (a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

Ans. (a)

Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the

selectivity of circuit. This ratio is also called "Quality Factor" or Q -factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

- 44** An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to [JEE Main 2016]

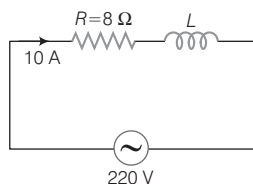
- (a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

Ans. (d)

Given, $I = 10$ A, $V = 80$ V,

$$R = \frac{V}{I} = \frac{80}{10} = 8 \Omega \text{ and } \omega = 50 \text{ Hz}$$

For AC circuit, we have



$$I = \frac{V}{\sqrt{R^2 + X_L^2}} \Rightarrow 10 = \frac{220}{\sqrt{64 + X_L^2}}$$

$$\Rightarrow \sqrt{64 + X_L^2} = 22$$

Squaring on both sides, we get

$$64 + X_L^2 = 484$$

$$\Rightarrow X_L^2 = 484 - 64 = 420$$

$$X_L = \sqrt{420} \Rightarrow 2\pi \times \omega L = \sqrt{420}$$

Series inductor on an arc lamp,

$$L = \frac{\sqrt{420}}{(2\pi \times 50)} = 0.065 \text{ H}$$

- 45** In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right). \text{ The power}$$

consumption in the circuit is given by [AIEEE 2007]

- (a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P = \text{zero}$
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2} E_0 I_0$

Ans. (b)

For given circuit, current is lagging the voltage by $\frac{\pi}{2}$, so circuit is purely

inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

- 46** In a series resonant L - C - R circuit, the voltage across R is 100 V and $R = 1 \text{ k}\Omega$ with $C = 2 \mu\text{F}$. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is [AIEEE 2006]

- (a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

Ans. (c)

At resonance, $\omega L = 1/\omega C$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1 \text{ A}$$

So, voltage across L is given by

$$V_L = I X_L = I \omega L$$

But $\omega L = 1/\omega C$

$$\therefore V_L = \frac{I}{\omega C} = V_C = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250 \text{ V}$$

- 47** A circuit has a resistance of 12Ω and an impedance of 15Ω . The power factor of the circuit will be [AIEEE 2005]

- (a) 0.8 (b) 0.4 (c) 1.25 (d) 0.125

Ans. (a)

$$\text{Power factor} = \cos \phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$$

- 48** The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of [AIEEE 2005]

- (a) $4 \mu\text{F}$ (b) $8 \mu\text{F}$ (c) $1 \mu\text{F}$ (d) $2 \mu\text{F}$

Ans. (c)

Given, $L = 10$ H, $f = 50$ Hz

For maximum power,

$$X_C = X_L \quad [\because \text{resonance condition}]$$

$$\text{or } \frac{1}{\omega C} = \omega L \text{ or } C = \frac{1}{\omega^2 L}$$

$$\therefore C = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

$$\text{or } C = 0.1 \times 10^{-5} \text{ F} = 1 \mu\text{F}$$

- 49** The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit? [AIEEE 2005]

- (a) C alone (b) R, L
(c) L, C (d) L alone

Ans. (c)

- (a) In a circuit having C alone, the voltage lags the current by $\frac{\pi}{2}$.
(b) In circuit containing R and L , the voltage leads the current by $\frac{\pi}{2}$.
(c) In L - C circuit, the phase difference between current and voltage can have any value between 0 to $\frac{\pi}{2}$ depending on the values of L and C .
(d) In a circuit containing L alone, the voltage leads the current by $\pi/2$.

- 50** Alternating current cannot be measured by DC ammeter because [AIEEE 2004]

- (a) AC cannot pass through DC ammeter
(b) AC changes direction
(c) average value of current for complete cycle is zero
(d) DC ammeter will get damaged

Ans. (c)

The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative.

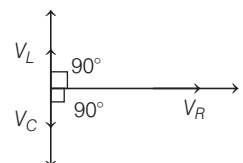
Therefore, for an AC cycle, the net value of current average out to zero. While the DC ammeter, read the average value. Hence, the alternating current cannot be measured by DC ammeter.

- 51** In an L - C - R series AC circuit, the voltage across each of the components L, C and R is 50 V. The voltage across the L - C combination will be [AIEEE 2004]

- (a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) zero

Ans. (d)

In an L - C - R series AC circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90° . $\therefore V = V_L - V_C = 50 - 50 = 0$



Hence, the voltage across L - C combination will be zero.

- 52** In an L - C - R circuit, capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to [AIEEE 2004]
 (a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$

Ans. (c)

In the condition of resonance,

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \quad \dots(i)$$

Since, resonance frequency remains unchanged, so

$$\sqrt{LC} = \text{constant or } LC = \text{constant}$$

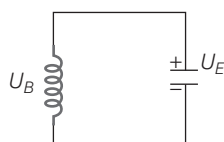
$$\therefore L_1 C_1 = L_2 C_2$$

$$\text{or } L \times C = L_2 \times 2C \text{ or } L_2 = L/2$$

- 53** In an oscillating L - C circuit, the maximum charge on the capacitor is Q . The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is [AIEEE 2003]
 (a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

Ans. (c)

In an L - C circuit, the energy oscillates between inductor (in the magnetic field) and capacitor (in the electric field).



$$U_{E_{\max}} [\text{Maximum energy stored in capacitor}] = \frac{Q^2}{2C}$$

$$U_{B_{\max}} [\text{Maximum energy stored in inductor}] = \frac{Li^2}{2}$$

where, i is the current at this time.

For the given instant, $U_E = U_B$

$$\text{i.e., } \frac{q^2}{2C} = \frac{Li^2}{2} \quad \dots(i)$$

From energy conservation,

$$U_E + U_B = U_{E_{\max}} = U_{B_{\max}}$$

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q^2}{2C}$$

$$\Rightarrow \frac{2q^2}{2C} = \frac{Q^2}{2C} \quad [\text{from Eq. (i)}]$$

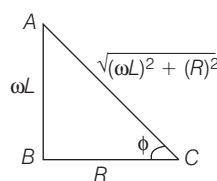
$$\text{or } q = \frac{Q}{\sqrt{2}}$$

- 54** The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is [AIEEE 2002]

$$(a) \frac{R}{\omega L} \quad (b) \frac{R}{(R^2 + \omega^2 L^2)^{1/2}} \\ (c) \frac{\omega L}{R} \quad (d) \frac{R}{(R^2 - \omega^2 L^2)^{1/2}}$$

Ans. (b)

From the relation, $\tan \phi = \frac{\omega L}{R}$



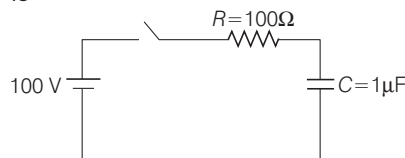
Power factor,

$$\cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} \\ \Rightarrow \cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \\ = \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{1}{\sqrt{1 + \tan^2 \phi}} \\ = \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

TOPIC 2

Growth and Decay of Current

- 55** A capacitor of capacitance $C = 1\mu\text{F}$ is suddenly connected to a battery of 100 V through a resistance $R = 100\Omega$. The time taken for the capacitor to be charged to get 50 V is



[Take, $\ln 2 = 0.69$]

[2021, 27 July Shift-I]

$$(a) 1.44 \times 10^{-4} \text{ s} \quad (b) 3.33 \times 10^{-4} \text{ s} \\ (c) 0.69 \times 10^{-4} \text{ s} \quad (d) 0.30 \times 10^{-4} \text{ s}$$

Ans. (c)

From the given figure in question, it can be seen that a capacitor of capacitance, $C = 1\mu\text{F}$ is connected to a battery of 100 V through a resistance, $R = 100\Omega$.

$\therefore$ We know that in RC -circuit

$$V = V_0 \left(1 - e^{-\left(\frac{t}{RC}\right)} \right)$$

where, V = voltage across the capacitor,

V_0 = supply voltage,

t = elapsed time, since the

application of voltage

and RC = time constant of the RC -charging circuit.

$$\Rightarrow 50 = 100 \left(1 - e^{-\frac{t}{RC}} \right)$$

$$\Rightarrow \frac{1}{2} = 1 - e^{-\frac{t}{RC}}$$

$$\Rightarrow \frac{1}{2} - 1 = -e^{-\frac{t}{RC}}$$

$$\Rightarrow -\frac{1}{2} = -e^{-\frac{t}{RC}}$$

$$\Rightarrow 2 = e^{\frac{t}{RC}} \quad \dots(i)$$

Taking log on both sides of Eq. (i), we get

$$\ln 2 = \ln \left(e^{\frac{t}{RC}} \right)$$

$$\Rightarrow \ln 2 = \frac{t}{RC}$$

$$\Rightarrow t = RC \ln 2 \\ = 100 \times 10^{-6} \times (0.69)$$

$$\Rightarrow t = 0.69 \times 10^{-4} \text{ s}$$

- 56** An inductor of 10 mH is connected to a 20 V battery through a resistor of $10\text{ k}\Omega$ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after

$1\mu\text{s}$ is $\frac{x}{100}\text{ mA}$. Then, x is equal to

..... (Take, $e^{-1} = 0.37$)

[2021, 25 July Shift-I]

Ans. (74)

Given,

inductance of inductor, $L = 10\text{ mH}$
 $= 10 \times 10^{-3} \text{ H}$

Supply voltage, $V = 20\text{ V}$

Resistor of resistance (R) = $10\text{ k}\Omega$
 $= 10 \times 10^3 \Omega$

Time, $t = 1\mu\text{s} = 1 \times 10^{-6} \text{ s}$

As we know that,

In case of circuit having inductor and resistor,

$$I = I_0 e^{-t/\tau} \quad \dots(i)$$

where, I = decayed current,

I_0 = peak current

t = time taken

and $\tau = \text{time constant} = \frac{L}{R}$

Substituting in Eq. (i), we get

$$I = I_0 e^{\frac{-tR}{L}} = \frac{V}{R} e^{\frac{-tR}{L}}$$

$$\Rightarrow I = \frac{20}{10 \times 10^{-3}} e^{\frac{-1 \times 10^{-6} \times 10 \times 10^3}{10 \times 10^{-3}}}$$

$$= 2 \times 10^{-3} e^{-1} = 2e^{-1} \text{ mA}$$

$$= 2 \times 0.37 \text{ mA} = 0.74 \text{ mA}$$

$$= 74/100 \text{ mA}$$

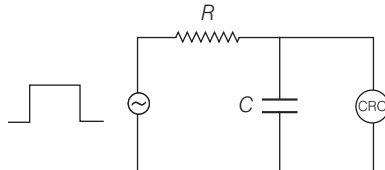
Comparing with given value, i.e.

$$I = \frac{x}{100} \text{ mA, we get}$$

$$x = 74$$

- 57** An R-C circuit as shown in the figure is driven by an AC source generating a square wave. The output wave pattern monitored by CRO would look close to

[2021, 16 March Shift-I]



- (a)
- (b)
- (c)
- (d)

Ans. (c)

When square wave is applied at the input, then

For charging, the capacitor

$$Q_1 = Q(1 - e^{\frac{-t}{RC}})$$

Similarly, for discharging the capacitor,

$$Q_2 = Q_{\max}(e^{\frac{-t}{RC}})$$

In this manner, charging and discharging exponentially with time will keep on happening alternatively. Therefore, the output wave pattern monitored by CRO would look close to



- 58** A series L-R circuit is connected to a battery of emf V . If the circuit is switched ON at $t=0$, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximum value, is

[2020, 4 Sep Shift-II]

- (a) $\frac{L}{R} \ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (b) $\frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$
- (c) $\frac{L}{R} \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (d) $\frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$

Ans. (d)

Let t be the required time at which the energy stored in inductor becomes $\left(\frac{1}{n}\right)$

times of its maximum value.

According to question, for given L-R circuit,

$$U = \frac{1}{n} U_{\max}$$

$$\frac{1}{2} L I^2 = \frac{1}{n} \times \frac{1}{2} L I_{\max}^2$$

$$\frac{1}{2} L \left[I_{\max} \left(1 - e^{\frac{-Rt}{L}} \right) \right]^2 = \frac{1}{n} \times \frac{1}{2} L I_{\max}^2$$

$$\frac{1}{2} L I_{\max}^2 \left(1 - e^{\frac{-Rt}{L}} \right)^2 = \frac{1}{n} \times \frac{1}{2} L I_{\max}^2$$

$$\left(1 - e^{\frac{-Rt}{L}} \right)^2 = \frac{1}{n}$$

$$\Rightarrow 1 - e^{\frac{-Rt}{L}} = \left(\frac{1}{n} \right)^{\frac{1}{2}}$$

$$e^{\frac{-Rt}{L}} = 1 - \frac{1}{\sqrt{n}}$$

$$\Rightarrow e^{\frac{-Rt}{L}} = \frac{\sqrt{n}-1}{\sqrt{n}}$$

Taking natural log on both sides, we get

$$\ln\left(e^{\frac{-Rt}{L}}\right) = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$-\frac{Rt}{L} \ln(e) = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$-\frac{Rt}{L} \times 1 = \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

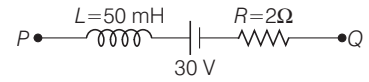
$$\Rightarrow t = \frac{-L}{R} \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$$

$$t = \frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)^{-1}$$

$$\Rightarrow t = \frac{L}{R} \ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$$

Hence, option (d) is correct.

- 59** A part of a complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at the rate of 10^2 A/s . The value of the potential difference $V_P - V_Q$ (in volt) at that instant, is



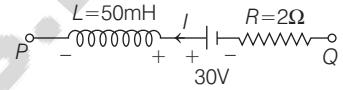
[2020, 6 Sep Shift-I]

Ans. (33)

Given that, at any instant,

$$I = 1 \text{ A}$$

$$\frac{dI}{dt} = -10^2 \text{ A/s (as current is decreasing)}$$



Applying KVL from point P to point Q,

$$V_P + \frac{L dI}{dt} - 30 + IR = V_Q$$

$$V_P - V_Q = 30 - 1 \times 2 - 50 \times 10^{-3} \times (-10^2)$$

$$= 33 \text{ V}$$

- 60** An emf of 20 V is applied at time $t=0$ to a circuit containing in series 10 mH inductor and 5Ω resistor. The ratio of the currents at time $t=\infty$ and at $t=40 \text{ s}$ is close to

(Take, $e^2 = 7.389$) [2020, 7 Jan Shift-II]

- (a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

Ans. (a)

In an L-R circuit, current growth occurs as

$$I = I_0 \left(1 - e^{\frac{-Rt}{L}} \right) \quad \dots(i)$$

where, I = instantaneous current,

I_0 = maximum current

$$= \text{current at } t = \infty = \frac{E}{R}$$

R = resistance of circuit,

L = inductance of circuit

and t = instantaneous time.

Here, $R = 5 \Omega$, $L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$,

$$E = 20 \text{ V}, t = 40 \text{ s}$$

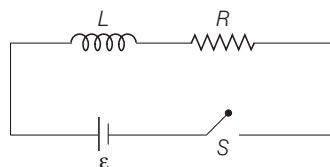
So, substituting these values in Eq. (i), we get

$$\frac{I}{I_0} = \left(1 - e^{\frac{-5}{10 \times 10^{-3}} \times 40} \right) = 1 - e^{-2 \times 10^4}$$

$$\Rightarrow \frac{I_0}{I} = \frac{1}{1 - e^{-2 \times 10^4}} \approx 1$$

$$\therefore \frac{I_0}{I} \approx 1.06 (\text{nearest option})$$

61



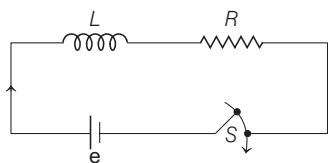
As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at $t = 0$. The total charge that flows from the battery, between $t = 0$ and $t = t_c$ (t_c is the time constant of the circuit) is

[2020, 8 Jan Shift-II]

- (a) $\frac{\varepsilon L}{R^2} \left(1 - \frac{1}{e}\right)$ (b) $\frac{\varepsilon L}{R^2}$
 (c) $\frac{\varepsilon R}{eL^2}$ (d) $\frac{\varepsilon L}{eR^2}$

Ans. (d)

In an L - R circuit as shown in the figure, instantaneous current at time t .



$$i = I \left(1 - e^{-\frac{R}{L}t}\right)$$

where, $\frac{L}{R} = t_c$ = time constant of the circuit.

Now, charge that flows from $t = 0$ to $t = t_c$ from battery,

$$\begin{aligned} q &= \int_0^{t_c} i dt = I \int_0^{t_c} \left(1 - e^{-\frac{R}{L}t}\right) dt \\ &= I \left[t - \frac{1}{-\frac{R}{L}} e^{-\frac{R}{L}t} \right]_0^{t_c} = I \left[t + \frac{L}{R} e^{-\frac{R}{L}t} \right]_0^{t_c} \\ &= I \left[\left(t_c + \frac{L}{R} e^{-\frac{R}{L}t_c} \right) - \left(\frac{L}{R} e^0 \right) \right] \\ &= I \left[\frac{L}{R} + \frac{L}{R} e^{-1} - \frac{L}{R} \right] = I \cdot \frac{L}{R} \cdot e^{-1} \end{aligned}$$

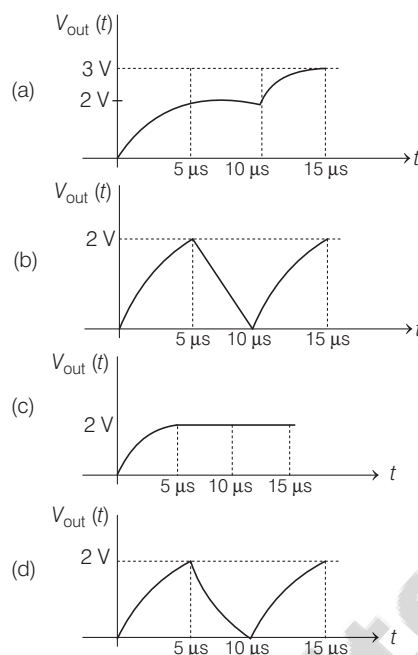
Here, I is maximum current which occurs at $t = \infty$ and its value is,

$$I = \frac{\varepsilon}{R}$$

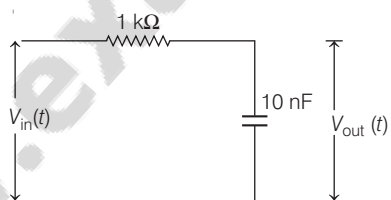
$$\text{So, } q = \frac{\varepsilon}{R} \cdot \frac{L}{R} \cdot \frac{1}{e} = \frac{\varepsilon L}{eR^2}$$

62 For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_{out}(t)$, across the capacitor is correctly depicted by

[2020, 6 Sep Shift-I]


Ans. (a)

The question clearly involves R - C circuit with DC input, which does not remain constant. For first $5\mu\text{s}$, capacitor gets charged as voltage is 5V and for next $5\mu\text{s}$, capacitor gets discharged as voltage is zero. Redrawing the given circuit in simple manner



$$\begin{aligned} \text{Time constant, } \tau &= RC \\ &= (1 \times 10^3) \times (10 \times 10^{-9}) = 10\mu\text{s} \end{aligned}$$

At $t = 5\mu\text{s}$,

Voltage across capacitor,

$$\begin{aligned} V_{out} &= V_m (1 - e^{-t/\tau}) \\ &= 5 \left(1 - e^{-\frac{5\mu\text{s}}{10\mu\text{s}}} \right) \quad [\because V_m = 5\text{V}] \\ &= 5(1 - e^{-0.5}) = 1.96\text{V} \approx 2\text{V} \end{aligned}$$

So, the capacitor is charged to voltage of 2V, i.e. $V_c = 2\text{V}$

For next $5\mu\text{s}$, $V_{in} = 0$, that means capacitor is discharging.

So, output voltage,

$$V_{out} = V_c e^{-t/\tau} = 2e^{-\frac{5\mu\text{s}}{10\mu\text{s}}} = 2e^{-0.5} = 1.21\text{V}$$

Now, for next $5\mu\text{s}$,

$$V_{out} = 5(1 - e^{-t/\tau}) + 1.21e^{-t/\tau}$$

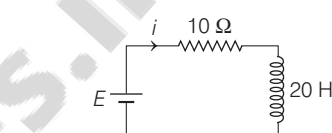
$$= 5 - 3.79e^{-t/\tau}$$

$$= 5 - 3.79e^{-\frac{5\mu\text{s}}{10\mu\text{s}}} = 2.71\text{V} \approx 3\text{V}$$

The given values of output voltage are most closely resembles to that in graph (a).

63 A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is

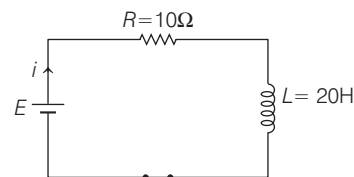
[2019, 8 April Shift-I]



- (a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2} \ln 2$
 (c) $2 \ln 2$ (d) $\ln 2$

Ans. (c)

Given circuit is a series L - R circuit



In an L - R circuit, current increases is

$$i = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2} Li^2$$

where, L = self inductance of the coil and energy dissipated by resistor is

$$U_R = i^2 R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$\begin{aligned} iL \frac{di}{dt} &= i^2 R \Rightarrow \frac{di}{dt} = \frac{R}{L} i \\ \Rightarrow \frac{E}{R} \cdot \frac{R}{L} e^{-\frac{R}{L}t} &= \frac{R}{L} \cdot \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right) \\ \Rightarrow 2e^{-\frac{R}{L}t} &= 1 \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2} \end{aligned}$$

Taking log on both sides, we have

$$\Rightarrow \frac{-R}{L} t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L} t = \ln 2$$

$$\Rightarrow t = \frac{L}{R} \ln 2 = \frac{20}{10} \ln 2 \Rightarrow t = 2 \ln 2$$

- 64** A coil of self inductance 10 mH and resistance 0.1Ω is connected through a switch to a battery of internal resistance 0.9Ω . After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$]

[2019, 10 April Shift-II]

- (a) 0.002 s (b) 0.324 s
(c) 0.103 s (d) 0.016 s

Ans. (d)

Key Idea In an L - R circuit, current during charging of inductor is given by

$$i = i_0 \left(1 - e^{-\frac{R}{L}t}\right)$$

where, i_0 = saturation current.

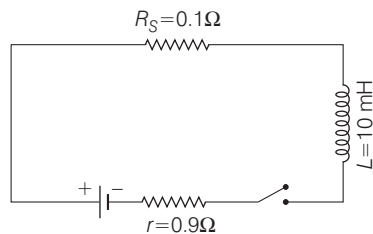
In given circuit,

Inductance of circuit is

$$L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$$

Resistance of circuit is

$$R = (R_s + r) = 0.1 + 0.9 = 1\Omega$$



Now, from

$$i = i_0 \left(1 - e^{-\frac{R}{L}t}\right) \quad \dots(i)$$

Given, $i = 80\%$ of i_0

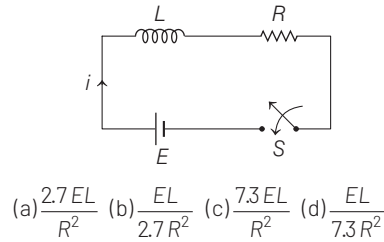
$$\Rightarrow i = \frac{80i_0}{100} = 0.8i_0$$

Substituting the value of i in Eq. (i), we get

$$\begin{aligned} 0.8 &= 1 - e^{-\frac{R}{L}t} \\ \Rightarrow e^{-\frac{R}{L}t} &= 0.2 \Rightarrow e^{\frac{R}{L}t} = 5 \\ \Rightarrow \ln(e)^{\frac{R}{L}t} &= \ln 5 \Rightarrow \frac{R}{L}t = \ln 5 \\ \Rightarrow t &= \frac{L}{R} \cdot \ln 5 = \frac{10 \times 10^{-3}}{1} \times \ln 5 \\ &= 10 \times 10^{-3} \times 1.6 \\ &= 1.6 \times 10^{-2} \text{ s} = 0.016 \text{ s} \end{aligned}$$

- 65** Consider the L - R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is

[2019, 12 April Shift-II]

**Ans. (b)**

In an L - R circuit, current during charging is given by

$$i = i_0 \left(1 - e^{-\frac{R}{L}t}\right)$$

where, $i_0 = \frac{E}{R}$ = saturation current.

$$\text{So, we have } \frac{dq}{dt} = i = i_0 \left(1 - e^{-\frac{R}{L}t}\right)$$

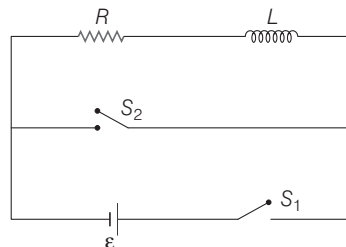
$$\Rightarrow dq = i_0 \left(1 - e^{-\frac{R}{L}t}\right) dt$$

So, charge q that passes through battery from time $t = 0$ to $t = \frac{L}{R}$ is obtained by

integrating the above equation within the specified limits, i.e.

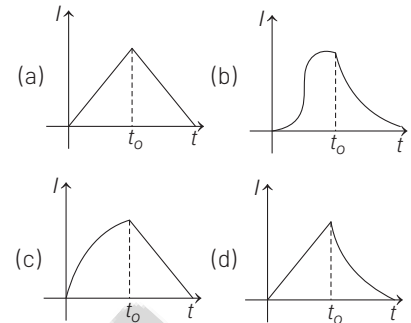
$$\begin{aligned} q &= \int_0^{\frac{L}{R}} dq = \int_0^{\frac{L}{R}} i_0 \left(1 - e^{-\frac{R}{L}t}\right) dt \\ &= i_0 \left[t - \left(-\frac{L}{R}\right) e^{-\frac{R}{L}t} \right]_0^{\frac{L}{R}} \\ &= \frac{E}{R} \left[\left\{ \frac{L}{R} + \frac{L}{R} e^{-1} \right\} - \left\{ 0 + \frac{L}{R} \right\} \right] \\ &= \frac{E}{R} \times \frac{L}{R} = \frac{EL}{R^2} \\ \Rightarrow q &= \frac{EL}{2.7 R^2} \quad [\because e \approx 2.72] \end{aligned}$$

- 66** In the circuit shown,



The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the

current I as a function of time ' t ' is given by



[2019, 11 Jan Shift-I]

Ans. (b)

Initially in the given RL circuit with a source, when S_1 is closed and S_2 is open at $t \leq t_0$.

$$I_1 = \frac{V}{R} \left[1 - \exp\left(-\frac{R}{L}t\right) \right]$$

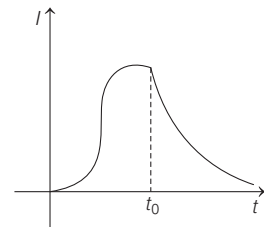
In this case, inductor L is charging.

When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.

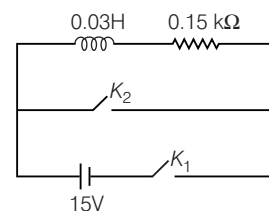
In this case ($t > t_0$),

$$I_2 = \frac{V}{R} \exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of I with t approximately is shown below



- 67** An inductor ($L = 0.03 \text{ H}$) and a resistor ($R = 0.15 \text{ k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1 \text{ ms}$, the current in the circuit will be ($e^5 \approx 150$) [JEE Main 2015]



- (a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67 mA

Ans. (d)

Key Idea After long time inductor behaves as short-circuit.

At $t = 0$, the inductor behaves as short-circuited. The current

$$I_0 = \frac{E_0}{R} = \frac{15\text{V}}{0.15\text{k}\Omega} = 100\text{ mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0 e^{-\frac{tR}{L}} = (100\text{ mA}) e^{-\frac{-t \times 15000}{3}}$$

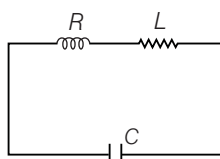
At $t = 1\text{ms}$

$$I = (100\text{ mA}) e^{-\frac{1 \times 10^{-3} \times 15 \times 10^3}{3}}$$

$$I = (100\text{ mA}) e^{-5} = 0.6737\text{ mA}$$

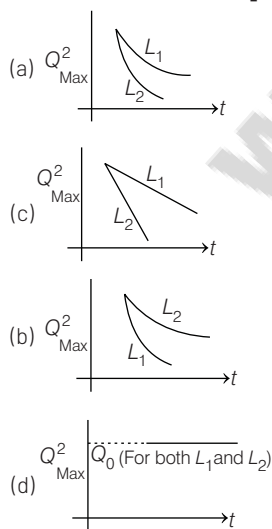
or $I = 0.67\text{ mA}$

- 68** An LCR circuit is equivalent to a damped pendulum. In an LCR circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below.

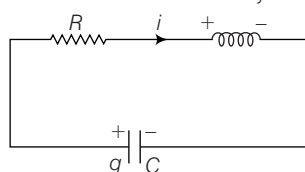


If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L , then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

[JEE Main 2015]


Ans. (a)

Consider the LCR circuit at any time t



Now, applying KVL

$$\text{We have } \frac{q}{C} - iR - \frac{L di}{dt} = 0$$

As current is decreasing with time we can write $i = -\frac{dq}{dt}$

$$\Rightarrow \frac{q}{C} + \frac{dq}{dt} R + \frac{L d^2 q}{dt^2} = 0$$

$$\text{or } \frac{d^2 q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = 0$$

This equation is equivalent to that of a damped oscillator

Thus, we can write the solution as

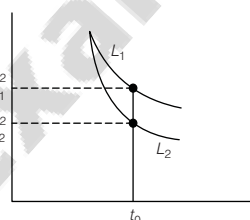
$$Q_{\text{max}}(t) = Q_0 \cdot e^{-Rt/2L}$$

$$\text{or } Q_{\text{max}}^2 = Q_0^2 e^{-\frac{Rt}{L}}$$

As $L_1 > L_2$ damping is faster for L_2

Aliter Inductance is inertia of circuit. It means inductance opposes the flow of charge, more inductance means decay of charge is slow.

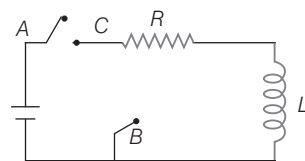
In option (a), in a given time t , $Q_1^2 > Q_2^2$.



So, $L_1 > L_2$. Hence, option (a) is correct.

- 69** In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to

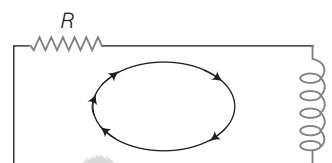
[JEE Main 2014]



- (a) $\frac{e}{1-e}$ (b) 1
(c) -1 (d) $\frac{1-e}{e}$

Ans. (c)

After connecting C to B hanging the switch, the circuit will act like L-R discharging circuit.



Applying Kirchhoff's loop equation,

$$V_R + V_L = 0 \Rightarrow V_R = -V_L$$

$$\therefore \frac{V_R}{V_L} = -1$$

$$e = \int Bv dx \quad [\because v = \omega \times]$$

$$\Rightarrow e = \int_{2l}^{3l} (\omega x) \cdot B \cdot dx$$

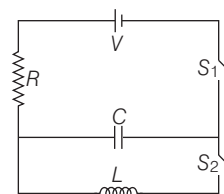
$$\text{Using } \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow e = B\omega \left[\frac{x^2}{2} \right]_{2l}^{3l}$$

$$e = B\omega \frac{[(3l)^2 - (2l)^2]}{2} = \frac{5Bl^2\omega}{2}$$

- 70** In an L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 is closed and S_2 kept open (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct?

[JEE Main 2013]



- (a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau$, $q = CV/2$
(c) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(d) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

Ans. (c)

For charging capacitor, q is given as

$$q = q_0(1 - e^{-t/\tau}) = CV(1 - e^{-t/\tau})$$

At $t = 2\tau$,

$$q = CV(1 - e^{-\frac{2t}{\tau}})$$

$$\Rightarrow q = CV(1 - e^{-2})$$

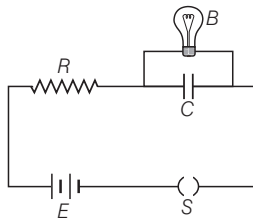
- 71** A resistor R and $2\mu\text{F}$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed (take $\log_{10} 2.5 = 0.4$)

[AIEEE 2011]

- (a) $1.7 \times 10^5 \Omega$ (b) $2.7 \times 10^6 \Omega$
 (c) $3.3 \times 10^7 \Omega$ (d) $1.3 \times 10^4 \Omega$

Ans. (b)

Neon bulb is filled with gas, so its resistance is infinite, hence no current flows through it.



Now, $V_c = E(1 - e^{-t/RC})$
 $\Rightarrow 120 = 200(1 - e^{-t/RC})$
 $\Rightarrow e^{-t/RC} = \frac{2}{5} \Rightarrow t = RC \ln 2.5$
 $\Rightarrow R = \frac{t}{C \ln 2.5} = \frac{t}{2.303 C \log 2.5}$
 $= 2.7 \times 10^6 \Omega$

- 72** Let C be the capacitance of a capacitor discharging through a resistor R . Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio t_1/t_2 will be [AIEEE 2010]
- (a) 1 (b) 1/2 (c) 1/4 (d) 2

Ans. (c)

Energy stored in capacitor,

$$U = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2C} (q_0 e^{-t/\tau})^2$$

$$= \frac{q_0^2}{2C} e^{-2t/\tau} \text{ [where, } \tau = CR]$$

$$U = U_i e^{-2t/\tau}, \frac{1}{2} U_i = U_i e^{-2t_1/\tau}$$

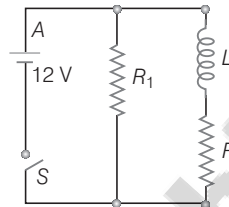
$$\frac{1}{2} = e^{-2t_1/\tau} \Rightarrow t_1 = \frac{\tau}{2} \ln 2$$

Now, $q = q_0 e^{-t/\tau}, \frac{1}{4} q_0 = q_0 e^{-t_2/\tau}$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$$\therefore \frac{t_1}{t_2} = \frac{1}{4}$$

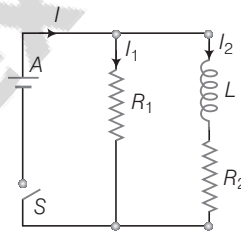
- 73** An inductor of inductance $L = 400 \text{ mH}$ and resistors of resistances $R_1 = 4 \Omega$ and $R_2 = 2 \Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is [AIEEE 2009]



- (a) $6e^{-5t} \text{ V}$ (b) $\frac{12}{t} e^{-3t} \text{ V}$
 (c) $6(1 - e^{-t/0.2}) \text{ V}$ (d) $12e^{-5t} \text{ V}$

Ans. (d)

$$I_1 = \frac{E}{R_1} = \frac{12}{4} = 3 \text{ A}$$



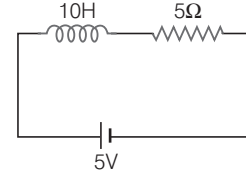
$\therefore$ Potential drop $= E - I_2 R_2$
 $I_2 = I_0(1 - e^{-t/t_c})$
 [current as a function of time]
 $\Rightarrow I_0 = \frac{E}{R_2} = \frac{12}{2} = 6 \text{ A}$
 and $t_c = \frac{L}{R_2} = \frac{400 \times 10^{-3}}{2} = 0.2$
 $I_2 = 6(1 - e^{-t})$
 Potential drop across L
 $L = E - R_2 I_2 = 12 - 2 \times 6(1 - e^{-5t})$
 $= 12e^{-5t}$

- 74** An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. After 2 s, the

connection is made, the current flowing (in ampere) in the circuit is
 (a) $(1 - e)$ (b) e [AIEEE 2007]
 (c) e^{-1} (d) $(1 - e^{-1})$

Ans. (d)

Rise of current in L - R circuit is given by

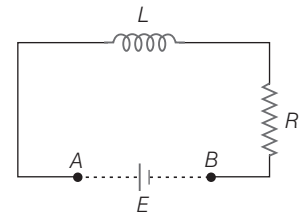


$$I = I_0(1 - e^{-t/\tau})$$

where, $I_0 = \frac{E}{R} = \frac{5}{5} = 1 \text{ A}$
 Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2 \text{ s}$

After 2 s i.e., at $t = 2 \text{ s}$,
 Rise of current, $I = (1 - e^{-1}) \text{ A}$

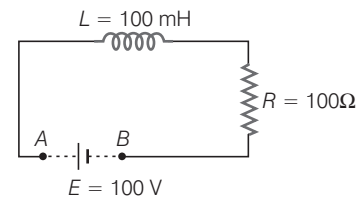
- 75** An inductor ($L = 100 \text{ mH}$), a resistor ($R = 100 \Omega$) and a battery ($E = 100 \text{ V}$) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 millisecond after the short circuit is [AIEEE 2006]



- (a) $1/e \text{ A}$ (b) $e \text{ A}$ (c) 0.1 A (d) 1 A

Ans. (a)

This is a combined example of growth and decay of current in an L - R circuit.



The current through circuit just before shorting the battery,

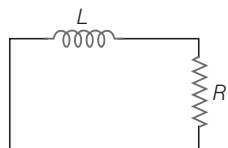
$$I_0 = \frac{E}{R} = \frac{100}{100} = 1 \text{ A}$$

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit, according to the equation,

$$I = I_0 e^{-t/\tau}$$

where, $\tau = L/R$



$$I = 1 \times e^{-(1 \times 10^{-3}) / (100 \times 10^{-3} / 100)} = \left(\frac{1}{e}\right) \text{ A}$$

$$[\because t = 1 \text{ millisecond} = 1 \times 10^{-3} \text{ s} \text{ and } L = 100 \times 10^{-3} \text{ H}]$$

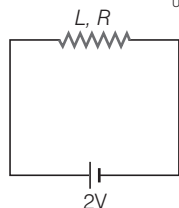
- 76** A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in [AIEEE 2005]

- (a) 0.05 s (b) 0.1 s
(c) 0.15 s (d) 0.3 s

Ans. (b)

The current at any instant is given by

$$I = I_0 (1 - e^{-Rt/L})$$



$$\Rightarrow \frac{I_0}{2} = I_0 (1 - e^{-Rt/L})$$

$$\text{or } \frac{1}{2} = (1 - e^{-Rt/L})$$

$$\text{or } e^{-Rt/L} = \frac{1}{2} \text{ or } \frac{Rt}{L} = \ln 2$$

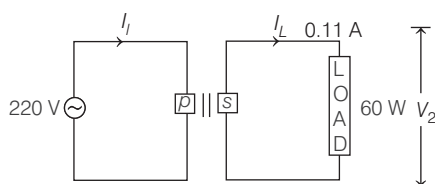
$$\therefore t = \frac{L}{R} \ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693$$

$$= 150 \times 0.693 \times 10^{-3} = 0.10395 \text{ s} \approx 0.1 \text{ s}$$

TOPIC 3

AC Generator and Transformer

- 77** For the given circuit, comment on the type of transformer used. [2021, 16 March Shift-II]



- (a) Auxilliary transformer
(b) Auto transformer
(c) Step-up transformer
(d) Step down transformer

Ans. (c)

$$\text{Voltage across secondary coil, } V_2 = \frac{\text{Power across load}}{\text{Current passing through load}}$$

$$\Rightarrow V_2 = \frac{P}{I_L} = \frac{60}{0.11} \Rightarrow V_2 = 545.45 \text{ V}$$

$$\text{Voltage across primary coil, } V_1 = 220 \text{ V}$$

$$\Rightarrow V_2 > V_1$$

It means that step-up transformer is used.

- 78** An electrical power line, having a total resistance of 2Ω , delivers 1 kW at 220 V. The efficiency of the transmission line is approximately [2020, 5 Sep Shift-I]

- (a) 91% (b) 85%
(c) 96% (d) 72%

Ans. (c)

$$\text{Given, } P = 1 \text{ kW} = 1000 \text{ W}$$

$$V = 220 \text{ V, } R = 2 \Omega$$

$$\text{Current, } I = \frac{P}{V} = \frac{1000}{220} \text{ A}$$

$$\text{Power loss, } P_{\text{loss}} = I^2 R = \left(\frac{1000}{220}\right)^2 \times 2$$

$$= 41.32 \text{ W}$$

Now, the efficiency of transmission line,

$$\eta = \left(\frac{P}{P + P_{\text{loss}}}\right) \times 100$$

$$= \left(\frac{1000}{1000 + 41.32}\right) \times 100$$

$$\eta = 96.03\% \approx 96\%$$

Hence, correct option is (c).

- 79** A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are [2019, 10 April Shift-I]
- (a) 440 V and 5 A (b) 220 V and 20 A
(c) 220 V and 10 A (d) 440 V and 20 A

Ans. (a)

Given,

$$\text{Number of turns in primary, } N_1 = 300$$

$$\text{Number of turns in secondary, } N_2 = 150$$

$$\text{Output power, } P_2 = 2.2 \text{ kW} = 2.2 \times 10^3 \text{ W}$$

$$\text{Current in secondary coil, } I_2 = 10 \text{ A}$$

$$\text{Output power, } P_2 = I_2 V_2$$

$$\Rightarrow V_2 = \frac{P_2}{I_2} = \frac{2.2 \times 10^3}{10} = 220 \text{ V} \quad \dots (i)$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}}$$

$$= \frac{V_1}{V_2} \Rightarrow V_1 = \left(\frac{N_1}{N_2}\right) V_2$$

$$\Rightarrow V_1 = \left(\frac{300}{150}\right) \times (220 \text{ V}) \quad [\text{using Eq. (i)}]$$

$$V_1 = 440 \text{ V} \quad \dots (ii)$$

$$\text{Again, } \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\Rightarrow I_1 = \left(\frac{V_2}{V_1}\right) I_2 = \frac{220}{440} \times 10$$

[using Eqs. (i) and (ii)]

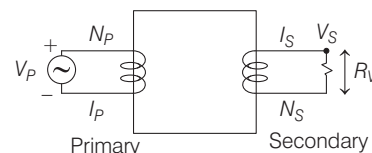
$$\Rightarrow I_1 = 5 \text{ A}$$

- 80** A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5 A and its efficiency is 90%, the output current would be [2019, 9 Jan Shift-II]

- (a) 45 A (b) 50 A
(c) 25 A (d) 35 A

Ans. (a)

For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_s (currents) respectively as shown below.



$$\text{Here, input voltage, } V_p = 2300 \text{ V}$$

$$\text{Number of turns in primary coil, } N_p = 4000$$

$$\text{Output voltage, } V_s = 230 \text{ V}$$

$$\text{Output power, } P_s = V_s \cdot I_s$$

$$\text{Input power, } P_p = V_p \cdot I_p$$

$\therefore$ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$\Rightarrow \eta = \frac{V_s \cdot I_s}{V_p \cdot I_p} \times 100$$

$$\Rightarrow \eta = \frac{(230)(I_s)}{(2300)(5)} \times 100$$

$$90 = \frac{230 I_s}{(2300) \times 5} \times 100$$

$$\Rightarrow I_s = 45 \text{ A}$$

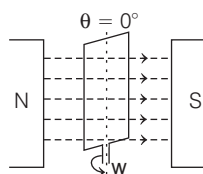
- 81** In an AC generator, a coil with N turns, all of the same area A and total resistance R , rotates with frequency ω in a magnetic field B . The maximum value of emf generated in the coil is [AIEEE 2006]

- (a) $NABR\omega$
 (b) NAB
 (c) $NABR$
 (d) $NAB\omega$

Ans. (d)

The emf generated would be maximum when flux (cutting) would be maximum i.e., angle between area vector of coil and magnetic field is 0° . The emf

generated is given by (as a function of time)



$$e = NBA\omega \cos \omega t$$

$$\Rightarrow e_{\max} = NAB\omega$$

$$[\because \cos \omega t = \cos \theta = 1 \Rightarrow \theta = 0^\circ]$$

- 82** The core of any transformer is laminated so as to [AIEEE 2003]

- (a) reduce the energy loss due to eddy currents
 (b) make it light weight
 (c) make it robust and strong
 (d) increase the secondary voltage

Ans. (a)

The core of transformer is laminated to reduce energy loss due to eddy currents

because induction is reduced by laminating.

- 83** In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is [AIEEE 2002]

- (a) 4 A
 (b) 2 A
 (c) 6 A
 (d) 10 A

Ans. (b)

Given, $I_p = 4 \text{ A}$, $N_p = 140$ and $N_s = 280$

From the formula,

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$\text{or } \frac{4}{I_s} = \frac{280}{140}$$

or

So,

$$I_s = 2 \text{ A}$$

Electromagnetic Waves

TOPIC 1

Displacement Current and Properties of EM Waves

- 01** Electric field of plane electromagnetic wave propagating through a non-magnetic medium is given by $E = 20 \cos(2 \times 10^{10} t - 200x)$ V/m. The dielectric constant of the medium is equal to

(Take, $\mu_r = 1$) [2021, 1 Sep. Shift-II]
 (a) 9 (b) 2
 (c) $\frac{1}{3}$ (d) 3

Ans. (a)

Given, electric field,

$$E = 20 \cos(2 \times 10^{10} t - 200x) \text{ V/m}$$

Comparing with the standard equation,

$$E = E_0 \cos(\omega t - kx) \text{ V/m, we get}$$

Wave constant, $k = 200$

Angular frequency, $\omega = 2 \times 10^{10} \text{ rad/s}$

Speed of the wave, $v = \frac{\omega}{k} = \frac{2 \times 10^{10}}{200}$

$$= 10^8 \text{ m/s}$$

Refractive index, $\mu = \frac{c}{v} = \frac{3 \times 10^8}{10^8} = 3$

As we know the relation between the refractive index and dielectric constant,

$$\mu = \sqrt{\epsilon_r \mu_r}$$

Substituting the value in the above equations, we get

$$3 = \sqrt{\epsilon_r (1)}$$

$$\epsilon_r = 9$$

Thus, the dielectric constant of the medium is 9.

- 02** A light beam is described by $E = 800 \sin\left(\omega t - \frac{x}{c}\right)$. An electron is

allowed to move normal to the propagation of light beam with a speed $3 \times 10^7 \text{ ms}^{-1}$. What is the maximum magnetic force exerted on the electron?

[2021, 26 Aug Shift-II]

- (a) $1.28 \times 10^{-17} \text{ N}$
 (b) $1.28 \times 10^{-18} \text{ N}$
 (c) $12.8 \times 10^{-17} \text{ N}$
 (d) $12.8 \times 10^{-18} \text{ N}$

Ans. (a, d)

Equation for light beam is given as

$$E = 800 \sin\left(\omega t - \frac{x}{c}\right)$$

Here, c is speed of light.

Comparing with standard equation, we get

$$E_0 = 800 \text{ Vm}^{-1}$$

As we know, the magnitude of magnetic field is given as

$$E_0 = B_0 c$$

$$\Rightarrow 800 = B_0 \times 3 \times 10^8$$

$$\Rightarrow B_0 = \frac{800}{3 \times 10^8} \text{ T}$$

Maximum magnetic force exerted on electron will be

$$F_{\max} = evB_0$$

$$= (1.6 \times 10^{-19}) \times (3 \times 10^7) \times \frac{800}{3 \times 10^8}$$

(Given, $v = 3 \times 10^7 \text{ ms}^{-1}$)

$$F_{\max} = 12.8 \times 10^{-18} \text{ N}$$

$$\text{or } = 1.28 \times 10^{-17}$$

- 03** The magnetic field vector of an electromagnetic wave is given by

$$\mathbf{B} = B_0 \frac{\hat{i} + \hat{j}}{\sqrt{2}} \cos(kz - \omega t) \text{ T where } \hat{i}, \hat{j}$$

represents unit vector along X and Y-axis respectively. At $t = 0$, two electric charges q_1 of $4\pi \text{ C}$ and q_2

of $2\pi \text{ C}$ located at $\left(0, 0, \frac{\pi}{k}\right)$ and

$\left(0, 0, \frac{3\pi}{k}\right)$ respectively, have the

same velocity of $0.5c \hat{i}$. (where, c is the velocity of light). The ratio of the force acting on charge q_1 to q_2 is

[2021, 31 Aug. Shift-I]

(a) $2\sqrt{2} : 1$ (b) $1 : \sqrt{2}$ (c) $2 : 1$ (d) $\sqrt{2} : 1$

Ans.

(c) The equation of magnetic field vector of an electromagnetic field is

$$\mathbf{B} = \frac{B_0}{\sqrt{2}} (\hat{i} + \hat{j}) \cos(kz - \omega t) \text{ T}$$

Electric charges

$$q_1 \text{ at } P\left(0, 0, \frac{\pi}{k}\right) = 4\pi \text{ C}$$

$$q_2 \text{ at } Q\left(0, 0, \frac{3\pi}{k}\right) = 2\pi \text{ C}$$

At $t = 0$ and $P\left(0, 0, \frac{\pi}{k}\right)$, where $z = \frac{\pi}{k}$,

Magnetic field vector,

$$\mathbf{B} = \frac{B_0}{\sqrt{2}} (\hat{i} + \hat{j}) \cos\left(k \frac{\pi}{k} - 0\right) \text{ T}$$

$$= \frac{B_0}{\sqrt{2}} (\hat{i} + \hat{j}) \cos \pi \text{ T} = -\frac{B_0}{\sqrt{2}} (\hat{i} + \hat{j}) \text{ T}$$

and at $t = 0$ and $Q\left(0, 0, \frac{3\pi}{k}\right)$, where $z = \frac{3\pi}{k}$.

Magnetic field vector,

$$\mathbf{B} = \frac{B_0}{\sqrt{2}}(\hat{i} + \hat{j}) \cos\left(k\frac{3\pi}{k} - 0\right) \\ = -\frac{B_0}{\sqrt{2}}(\hat{i} + \hat{j})$$

[$\because \cos \pi = \cos 3\pi = -1$]

Velocity of charges q_1 and q_2 , $\mathbf{v} = 0.5c\hat{i}$

We know that, force on charge in magnetic field is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

$$\therefore \text{For } q_1, \mathbf{F}_1 = 4\pi \left[0.5c\hat{i} \times \left\{ -\frac{B_0}{\sqrt{2}}(\hat{i} + \hat{j}) \right\} \right] \text{ N} \\ \mathbf{F}_1 = -\frac{2\pi c B_0}{\sqrt{2}} \hat{k} \text{ N}$$

Similarly, for q_2

$$\Rightarrow \mathbf{F}_2 = 2\pi \left[0.5c\hat{i} \times \left\{ -\frac{B_0}{\sqrt{2}}(\hat{i} + \hat{j}) \right\} \right] \text{ N} \\ \Rightarrow \mathbf{F}_2 = -\frac{\pi c B_0}{\sqrt{2}} \hat{k} \text{ N}$$

Hence, ratio of magnitudes of $\mathbf{F}_1$ to $\mathbf{F}_2$,

$$\frac{F_1}{F_2} = \frac{\frac{2\pi c B_0}{\sqrt{2}}}{\frac{\pi c B_0}{\sqrt{2}}} = \frac{2}{1} \Rightarrow F_1 : F_2 = 2 : 1$$

- 04** The electric field in an electromagnetic wave is given by $E = (50 \text{ NC}^{-1}) \sin \omega \left(t - \frac{x}{c} \right)$.

The energy contained in a cylinder of volume V is $5.5 \times 10^{-12} \text{ J}$. The value of V is cm^3 .
(Given $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$). [2021, 31 Aug. Shift-I]

Ans. (500)

Given, electric field in electromagnetic wave,

$$E = 50 \sin \omega \left(t - \frac{x}{c} \right)$$

Energy contained in cylinder,

$$U = 5.5 \times 10^{-12} \text{ J}$$

Let, volume = V

and permittivity in free space,

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

As we know that,

$$\frac{\text{Energy (U)}}{\text{Volume (V)}} = \frac{1}{2} \epsilon_0 E_0^2$$

$$\therefore V = \frac{2U}{\epsilon_0 E_0^2} = \frac{2 \times 5.5 \times 10^{-12}}{8.854 \times 10^{-12} \times (50)^2} \\ = \frac{11}{8.854 \times 50^2} = 4.97 \times 10^{-4} \\ = 497 \times 10^{-6} \\ = 500 \text{ cm}^3$$

- 05** A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8} \text{ T}$. The value of x is.

[2021, 27 Aug. Shift-II]

Ans. (2)

Given, frequency of electromagnetic wave (f)

$$= 30 \text{ MHz} \\ = 30 \times 10^6 \text{ Hz}$$

Electric field, $E = 6 \text{ V/m}$

Magnetic field, $B = x \times 10^{-8} \text{ T}$

Speed of light in air, $c = 3 \times 10^8 \text{ ms}^{-1}$

We know that, $B = \frac{E}{c}$

$$\Rightarrow B = \frac{6}{3 \times 10^8}$$

$$\Rightarrow B = 2 \times 10^{-8} \text{ T}$$

$$\therefore x = 2$$

- 06** Electric field in a plane electromagnetic wave is given by $E = 50 \sin(500x - 10 \times 10^{10} t) \text{ V/m}$.

The velocity of electromagnetic wave in this medium is

(Given, c = speed of light in vacuum)

[2021, 27 Aug. Shift-I]

- (a) $\frac{3}{2}c$ (b) c
(c) $\frac{2}{3}c$ (d) $\frac{c}{2}$

Ans. (c)

Given, equation of electromagnetic wave, $E = 50 \sin(500x - 10 \times 10^{10} t) \text{ V/m}$

The standard equation of electromagnetic wave,

$$E = E_0 \sin(kx - \omega t) \text{ V/m}$$

Comparing with standard equation, we get

$$E_0 = 50 \text{ V/m}, \omega = 10^{10} \text{ rad s}^{-1}, \\ k = 500 \text{ rad m}^{-1}$$

Speed of wave in medium will be

$$v = \frac{\omega}{k} \\ = \frac{10 \times 10^{10}}{500} \\ = 2 \times 10^8 \text{ ms}^{-1} \\ = \frac{2}{3} \times 3 \times 10^8 \text{ m/s} \\ = \frac{2}{3} c$$

- 07** The electric field in a plane electromagnetic wave is given by

$$\mathbf{E} = 200 \cos \left[\left(\frac{0.5 \times 10^3}{m} \right) x - \left(1.5 \times 10^{11} \frac{\text{rad}}{\text{s}} \times t \right) \right] \frac{\text{V}}{\text{m}} \hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2 . If the radiation pressure exerted by the EM wave on the surface during a 10 min exposure is $\frac{x}{10^9}$. Find the

value of x .

[2021, 26 Aug. Shift-I]

Ans. (354)

The electric field in a plane electromagnetic wave is given by

$$\mathbf{E} = 200 \cos \left[\left(\frac{0.5 \times 10^3}{m} \right) x - \left(1.5 \times 10^{11} \frac{\text{rad}}{\text{s}} \times t \right) \right] \frac{\text{V}}{\text{m}} \hat{j}$$

$$\therefore E_0 = 200 \text{ V/m}$$

Intensity provided by electric field is given by

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

The radiation pressure exerted by electromagnetic wave on surface is given by

$$P = \frac{2I}{c} = \frac{2 \times \frac{1}{2} \epsilon_0 E_0^2 c}{c} = \epsilon_0 E_0^2 \\ \Rightarrow P = \epsilon_0 E_0^2 \\ = 8.85 \times 10^{-12} \times (200)^2 \\ = \frac{354}{10^9}$$

According to question,

$$\frac{x}{10^9} = \frac{354}{10^9}$$

$$\therefore x = 354$$

- 08** The relative permittivity of distilled water is 81. The velocity of light in it will be

(Take, $\mu_r = 1$)

[2021, 27 July Shift-I]

- (a) $4.33 \times 10^7 \text{ m/s}$ (b) $2.33 \times 10^7 \text{ m/s}$
(c) $3.33 \times 10^7 \text{ m/s}$ (d) $5.33 \times 10^7 \text{ m/s}$

Ans. (c)

Given, relative permittivity of distilled water = 81

$$\therefore \text{We know that, } v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$$

where, v = speed of light in medium,
 c = speed of light in air,
 ϵ_r = relative permittivity
 and μ_r = relative permeability.
 $\Rightarrow v = \frac{3 \times 10^8}{\sqrt{1 \times 81}} \quad [\because \mu_r = 1]$
 $= \frac{3 \times 10^8}{9} = \frac{10^8}{3} = 3.33 \times 10^7 \text{ ms}^{-1}$

- 09** A linearly polarised electromagnetic wave in vacuum is $E = 3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$ is incident normally on a perfectly reflecting wall at $z = a$. Choose the correct option.
 (a) The wavelength is 5.4 m
 (b) The frequency of electromagnetic wave is $54 \times 10^4 \text{ Hz}$.
 (c) The transmitted wave will be $3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$
 (d) The reflected wave will be $3.1 \cos[(1.8)z + (5.4 \times 10^6)t] \hat{i} \text{ N/C}$

Ans. (d)

Given,

Linearly polarised electromagnetic wave,

$$E = 3.1 \cos[(1.8)z - (5.4 \times 10^6)t] \hat{i} \text{ N/C}$$

The given wave is travelling along +ve X-axis.

Angular frequency, $\omega = 5.4 \times 10^6 \text{ rads}^{-1}$

Wave constant, $k = 1.8 \text{ m}^{-1}$

As, we know that,

If incident wave is $x = A \sin(\omega t - kx)$

Then, for normal incidence, reflected wave $x_r = A \sin(\omega t + kx)$

Hence, reflected wave will be

$$E_r = 3.1 \cos[(1.8)z + (5.4 \times 10^6)t] \hat{i} \text{ N/C}.$$

- 10** Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point?
 $(\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^{-2} \text{ N}^{-1} \text{ m}^{-2})$

[2021, 22 July Shift-II]

- (a) $2.77 \times 10^{-8} \text{ T}$ (b) $1.96 \times 10^{-8} \text{ T}$
 (c) 8.31 T (d) 5.88 T

Ans. (a)

Given, intensity of sunlight,

$$I = 0.092 \text{ W/m}^2$$

As we know that,

$$I = \frac{\epsilon_0 c}{2} E_0^2$$

$$E = \sqrt{\frac{2I}{\epsilon_0 c}}$$

The peak value of electric field (E_0) and magnetic field (B_0) are related as

$$\begin{aligned} \text{Since, } B_0 &= \frac{E_0}{c} = \frac{1}{c} \sqrt{\frac{2I}{\epsilon_0 c}} \\ &= \frac{1}{3 \times 10^8} \sqrt{\frac{2 \times 0.092}{8.85 \times 10^{-12} \times 3 \times 10^8}} \\ &= \frac{1}{3 \times 10^8} \sqrt{0.0069 \times 10^4} \\ &= \frac{1}{3 \times 10^8} \times \sqrt{69} = 2.77 \times 10^{-8} \text{ T} \end{aligned}$$

- 11** In an electromagnetic wave, the electric field vector and magnetic field vector are given as $\mathbf{E} = E_0 \hat{i}$ and $\mathbf{B} = B_0 \hat{k}$, respectively. The direction of propagation of electromagnetic wave is along [2021, 20 July Shift-II]
 (a) $\hat{k}$ (b) $\hat{j}$
 (c) $(-\hat{k})$ (d) $(-\hat{j})$

Ans. (d)

In an electromagnetic wave, the electric field vector and magnetic field vector are given as

$$\mathbf{E} = E_0 \hat{i} \quad \dots(i)$$

$$\text{and } \mathbf{B} = B_0 \hat{k} \quad \dots(ii)$$

$\therefore$ Direction of electromagnetic wave is given by direction of $\mathbf{E} \times \mathbf{B}$ because it is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ vectors.

$$\begin{aligned} \text{So, unit vector in direction of } \mathbf{E} \times \mathbf{B} \text{ will be} \\ = \frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{E} \times \mathbf{B}|} \quad \dots(iii) \end{aligned}$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{E_0 \hat{i} \times B_0 \hat{k}}{E_0 B_0 \sin 90^\circ} = \hat{i} \times \hat{k} = -\hat{j}$$

- 12** A plane electromagnetic wave propagating along y-direction can have the following pair of electric field ($\mathbf{E}$) and magnetic field ($\mathbf{B}$) components. [2021, 18 March Shift-II]

- (a) E_y, B_y or E_z, B_z (b) E_y, B_x or E_x, B_y
 (c) E_x, B_z or E_z, B_x (d) E_x, B_y or E_y, B_x

Ans. (c)

Electric field, magnetic field and the direction of wave propagation are mutually perpendicular to each other.

For electromagnetic waves,

$$\mathbf{E} \times \mathbf{B} = \mathbf{c}$$

The direction of wave propagation is in along + y-direction.

Therefore, the possible direction of electric field and magnetic field are (E_x, B_z) and (E_z, B_x) .

- 13** A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\mathbf{B} = 2.0 \times 10^{-8} \hat{k} \text{ T}$ (where, $\hat{k}$ is unit vector along z-direction). What is $\mathbf{E}$ at this point? [2021, 18 March Shift-I]
 (a) $0.6 \hat{j} \text{ V/m}$ (b) $6.0 \hat{k} \text{ V/m}$
 (c) $6.0 \hat{j} \text{ V/m}$ (d) $0.6 \hat{k} \text{ V/m}$

Ans. (c)

Given, $f = 100 \text{ MHz}$

$$\mathbf{B} = (2.0 \times 10^{-8}) \hat{k} \text{ T}$$

The speed of the light in free space,

$$\mathbf{c} = (3 \times 10^8) \hat{i} \text{ m/s}$$

(the direction of wave propagates in x-direction)

Magnitude of electric field,

$$E = Bc = 2 \times 10^{-8} \times 3 \times 10^8$$

$$\Rightarrow E = 6 \text{ V/m}$$

If $\hat{n}$ be the direction of $\mathbf{E}$, then $\hat{i} = \hat{n} \times \hat{k}$

$$\Rightarrow \hat{n} = \hat{j}$$

Hence, the electric field at this point is $6\hat{j} \text{ V/m}$.

- 14** The electric field intensity produced by the radiation coming from a 100 W bulb at a distance of 3 m is E . The electric field intensity produced by the radiation coming from 60 W at the same distance is $\sqrt{\frac{x}{5}} E$, where the value of x is

[2021, 17 March Shift-II]

Ans. (3)

Intensity of the electromagnetic radiation is given as

$$\begin{aligned} I &= \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}} \\ &= U_{\text{avg}} c \\ &= \frac{1}{2} c \epsilon_0 E_0^2 \end{aligned}$$

Here,

U_{avg} = average energy density,

c = speed of radiation in air (or vacuum),

ϵ_0 = permittivity of the free space

and E_0 = peak value of the electric field.

$$\Rightarrow P \propto E_0^2$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{E_1^2}{E_2^2}$$

$$\Rightarrow \frac{E_1}{E_2} = \sqrt{\frac{P_1}{P_2}}$$

Substituting the values in the above equation, we get

$$\Rightarrow \frac{E_1}{E_2} = \sqrt{\frac{60}{100}}$$

$$\Rightarrow E_1 = \sqrt{\frac{3}{5}} E_2$$

$$= \sqrt{\frac{3}{5}} E \quad (\because E_2 = E)$$

Comparing with $\sqrt{\frac{x}{5}} E$, we get

$$x = 3$$

15 Seawater at a frequency $f = 9 \times 10^2$

Hz, has permittivity $\epsilon = 80\epsilon_0$ and resistivity $r = 0.25 \Omega\text{-m}$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi ft)$. Then, the conduction current density becomes 10^x times the displacement current density after time $t = \frac{1}{800}$ s. The value of x is

.....

(Take, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N}\cdot\text{m}^2\text{C}^{-2}$)

[2021, 17 March Shift-II]

Ans. (6)

As we know, conduction current,

$$I_c = \frac{[V_0 \sin(2\pi ft)]}{\rho \frac{d}{A}}$$

$$\Rightarrow I_c = \frac{A[V_0 \sin(2\pi ft)]}{\rho d} \quad \dots (i)$$

As we know the displacement current equation,

$$I_d = \frac{\epsilon_0 \epsilon_r A}{d} \frac{d}{dt} [V_0 \sin(2\pi ft)]$$

$$I_d = \frac{\epsilon_0 \epsilon_r A}{d} V_0 (2\pi f) [\cos(2\pi ft)] \quad \dots (ii)$$

Divide Eq. (ii) by Eq. (i), we get

$$\frac{I_d}{I_c} = \epsilon_0 \epsilon_r \times \rho \times 2\pi f \cot(2\pi ft)$$

Substituting the values in the above equation, we get

$$\frac{I_d}{I_c} = \frac{1}{4\pi \times 9 \times 10^9} \times (80) \times 2\pi \times 900 (0.25)$$

$$\cot\left(2\pi(900)\frac{1}{800}\right)$$

$$\frac{I_d}{I_c} = \frac{1}{10^6}$$

Comparing with 10^x , we get $x = 6$.

16 A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\mathbf{B} = 8.0 \times 10^{-8} \hat{\mathbf{z}} \text{ T}$. The value of electric field at this point is (speed of light $= 3 \times 10^8 \text{ ms}^{-1}$; $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are unit vectors along x, y and z-direction).

[2021, 16 March Shift-I]

- (a) $-24\hat{\mathbf{x}} \text{ V/m}$ (b) $2.6\hat{\mathbf{x}} \text{ V/m}$
(c) $24\hat{\mathbf{x}} \text{ V/m}$ (d) $-2.6\hat{\mathbf{x}} \text{ V/m}$

Ans. (a)

Given, frequency, $f = 500 \text{ MHz} = 5 \times 10^8 \text{ Hz}$

$$\mathbf{B} = 8.0 \times 10^{-8} \hat{\mathbf{z}} \text{ T}$$

$\therefore$ Magnitude of peak value of magnetic field is given by

$$B_0 = 8 \times 10^{-8} \text{ T}$$

We know that, $\frac{E_0}{B_0} = c$

where, E_0 is the magnitude of peak value of electric field and c is the speed of electromagnetic wave in air (or vacuum).

$$\Rightarrow E_0 = cB_0$$

$$= 3 \times 10^8 \times 8 \times 10^{-8} = 24 \text{ V/m}$$

Since, the direction of propagation of electromagnetic wave is perpendicular to the direction of E and B both.

$\therefore$ Direction of propagation is given by $\hat{\mathbf{E}} \times \hat{\mathbf{B}}$.

As, the wave is travelling in y-direction, and the magnetic field is in z-direction.

$$\Rightarrow \hat{\mathbf{E}} \times \hat{\mathbf{z}} = \hat{\mathbf{y}}$$

$$\hat{\mathbf{E}} = -\hat{\mathbf{x}}$$

$\therefore$ The value of electric field will be $-24\hat{\mathbf{x}} \text{ V/m}$.

17 For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is [2021, 16 March Shift-I]

- (a) $U_e = U_m$ (b) $U_e > U_m$
(c) $U_e < U_m$ (d) $U_e \neq U_m$

Ans. (a)

We know that,

$$U_e = \frac{1}{2} \epsilon_0 E^2 \quad \dots (i)$$

where, U_e = average energy density due to electric field,

ϵ_0 = electrical permittivity of free space and E = electric field.

$$\text{and } U_m = \frac{B^2}{2\mu_0} \quad \dots (ii)$$

where, U_m = average energy density due to magnetic field, μ_0 = magnetic permeability of free space and B = magnetic field.

$$\text{Also, } E = cB \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right)$$

$$\Rightarrow E = \frac{B}{\sqrt{\mu_0 \epsilon_0}}$$

Put this value in Eq. (i), we get

$$U_e = \frac{1}{2} \epsilon_0 \times \frac{1}{\mu_0 \epsilon_0} B^2$$

$$\Rightarrow U_e = \frac{B^2}{2\mu_0}$$

$$\Rightarrow U_e = U_m \quad [\text{using Eq. (ii)}]$$

18 A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P , placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is

(Rounded-off to the nearest integer)

[Take, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$,

$c = 3 \times 10^8 \text{ ms}^{-1}$] [2021, 26 Feb Shift-I]

Ans. (137)

Given, power of bulb, $P = 1000 \text{ W}$

Distance $d = 2 \text{ m}$

Efficiency of bulb = 1.25%,

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{N}^{-1} \text{m}^{-2},$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \text{Intensity, } (I) = \frac{\text{Power}(P)}{\text{Area}(A)} = \frac{1}{2} \epsilon_0 E^2 c$$

$$\Rightarrow \frac{1.25 \times 1000}{100 \times 4\pi(2)^2} = \frac{1}{2} \times 8.854 \times 10^{-12}$$

$$\times E^2 \times 3 \times 10^8$$

$$\Rightarrow E = \sqrt{\frac{12.5}{16\pi} \times \frac{2}{8.854 \times 10^{-4} \times 3}}$$

$$= 13.7 \text{ NC}^{-1} = 137 \times 10^{-1} \text{ NC}^{-1} \text{ or } \text{Vm}^{-1}$$

$$\therefore x = 137$$

19 The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is

$$\frac{x}{10} \sqrt{\frac{\mu_0 c}{\pi}} \frac{\text{V}}{\text{m}}$$

The efficiency of the bulb is 10% and it is a point source.

The value of x is

[2021, 25 Feb Shift-II]

Ans. (4)

Given, radius of disc, $R = 20 \text{ cm}$

$$= 20 \times 10^{-2} \text{ m}$$

Mass of disc, $m = 10 \text{ kg}$

Angular velocity, $\omega = 600 \text{ rpm}$

$$\omega = 600 \times \frac{2\pi}{60} = 20\pi \text{ rads}^{-1}$$

Time taken, $t = 10 \text{ s}$

Let torque be τ ,

As we know that,

$$\tau = I\alpha \quad \dots(i)$$

where, I is moment of inertia of disc

about centre of mass $= \frac{1}{2}MR^2$

and α is angular acceleration.

$$\therefore \alpha = \frac{\omega}{t} = \frac{20\pi}{10} = 2\pi \text{ rads}^{-2}$$

$\therefore$ Put value of I in Eq. (i), we get

$$\begin{aligned} \tau &= \frac{1}{2}MR^2\alpha = \frac{1}{2} \times 10 \times \left(\frac{20}{100}\right)^2 \times 2\pi \\ &= \frac{1}{2} \times 10 \times \frac{4}{100} \times 2\pi \\ &= 0.4\pi = 4\pi \times 10^{-1} \text{ N-m} \end{aligned}$$

- 20** An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be $\times 10^{-2} \text{ cm}$.

[2021, 24 Feb Shift-II]

Ans. (667)

Given, frequency of wave,

$$f = 3 \text{ GHz} = 3 \times 10^9 \text{ Hz}$$

Relative permittivity, $\epsilon_r = 2.25$

Since, $f = c/\lambda$

$$\Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

$\therefore \lambda_m$ (wavelength of wave in a medium) $= \lambda/\mu$

and as we know that, $\mu = \sqrt{\mu_r \epsilon_r}$

As, dielectric is non-magnetic, $\mu_r = 1$

$$\Rightarrow \mu = \sqrt{2.25} = 1.5$$

$$\begin{aligned} \Rightarrow \lambda_m &= \frac{0.1}{1.5} = \frac{1}{15} = 0.0667 \text{ m} \\ &= 667 \text{ cm} = 667 \times 10^{-2} \text{ cm} \end{aligned}$$

- 21** An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is $\times 10^7 \text{ m/s}$.

[2021, 24 Feb 2021 Shift-I]

Ans. (15)

Given, $\mu_r = \epsilon_r = 2$

where, μ_r is relative permeability,

ϵ_r is relative permittivity.

Speed of electromagnetic wave v is given by

$$v = \frac{c}{n}$$

where, n = refractive index

$$= \sqrt{\mu_r \epsilon_r} = \sqrt{4} = 2$$

$$\Rightarrow v = \frac{3 \times 10^8}{2}$$

$$= 15 \times 10^7 \text{ m/s}$$

$$\therefore x \times 10^7 = 15 \times 10^7$$

$$\Rightarrow x = 15$$

- 22** A plane electromagnetic wave, has frequency of $2.0 \times 10^{10} \text{ Hz}$ and its energy density is $1.02 \times 10^{-8} \text{ J/m}^3$ in vacuum. The amplitude of the magnetic field of the wave is close to (Take, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$ and speed of light $= 3 \times 10^8 \text{ ms}^{-1}$)

[2020, 2 Sep Shift-I]

- (a) 190 nT (b) 160 nT
(c) 180 nT (d) 150 nT

Ans. (b)

Energy density of magnetic field of an electromagnetic wave is given by

$$u_B = \frac{1}{2} \left(\frac{B_0^2}{\mu_0} \right)$$

Here, $\mu_0 = 4\pi \times 10^{-7} \text{ units}$,

and $u_B = 1.02 \times 10^{-8} \text{ Jm}^{-3}$

So, amplitude of magnetic field,

$$\begin{aligned} B_0 &= \sqrt{2\mu_0 u_B} \\ &= \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}} \\ &= 16 \times 10^{-8} \text{ T} \\ &= 160 \text{ nT} \end{aligned}$$

Hence, correct option is (b).

- 23** In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i} - 2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave?

[2020, 2 Sep Shift-II]

- (a) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$
(b) $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$
(c) $\frac{1}{\sqrt{5}}(\hat{i} + 2\hat{j})$
(d) $\frac{1}{\sqrt{5}}(2\hat{i} + \hat{j})$

Ans. (a)

Direction of propagation of an electromagnetic

wave is given by $\hat{E} \times \hat{B}$.

$\therefore$ A unit vector in the direction of propagation

$$\begin{aligned} &= \frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{E} \times \mathbf{B}|} = \frac{\hat{k} \times (2\hat{i} - 2\hat{j})}{|\mathbf{E} \times \mathbf{B}|} \\ &\left[\begin{aligned} \therefore \mathbf{E} \times \mathbf{B} &= \hat{k} \times (2\hat{i} - 2\hat{j}) \\ &= 2(\hat{k} \times \hat{i}) - 2(\hat{k} \times \hat{j}) \\ &= 2(\hat{j}) - 2(-\hat{i}) \\ &= 2\hat{j} + 2\hat{i} \end{aligned} \right] \\ &\therefore |\mathbf{E} \times \mathbf{B}| = \sqrt{2^2 + 2^2} = 2\sqrt{2} \\ &= \frac{2\hat{j} + 2\hat{i}}{2\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} \end{aligned}$$

Hence, correct option is (a).

- 24** The magnetic field of a plane electromagnetic wave is

$$\mathbf{B} = 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{i} \text{ T,}$$

where $c = 3 \times 10^8 \text{ ms}^{-1}$ is the speed of light. The corresponding electric field is

[2020, 3 Sep Shift-I]

- (a) $\mathbf{E} = 9 \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$
(b) $\mathbf{E} = -10^{-6} \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$
(c) $\mathbf{E} = 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$
(d) $\mathbf{E} = -9 \sin[200\pi(y + ct)] \hat{k} \text{ V/m}$

Ans. (d)

We have the following facts :

An electromagnetic wave propagates in direction of vector $\mathbf{E} \times \mathbf{B}$.

Phase gives velocity of wave.

Electric field vector of EM wave is given by

$$\mathbf{E} = c\mathbf{B}$$

Here, $\mathbf{B} = B_0 \sin[200\pi(y + ct)] \hat{i}$

$$= 3 \times 10^{-8} \sin[200\pi(y + ct)] \hat{i}$$

$$\begin{aligned} \Rightarrow E_0 &= cB_0 = 3 \times 10^8 \times 3 \times 10^{-8} \\ &= 9 \text{ V/m} \end{aligned}$$

Phase $= y + ct = \text{constant}$

$$\Rightarrow \frac{dy}{dt} = -c$$

i.e., wave propagates in negative y -direction.

$\Rightarrow \mathbf{E} \times \mathbf{B}$ is parallel to $-\hat{j}$

$$\Rightarrow \hat{E} \times \hat{i} = \lambda(-\hat{j})$$

So, $\hat{E} = -\hat{k}$

Hence, electric field vector is given by

$$\begin{aligned} \mathbf{E} &= E_0 \sin(ky + \omega t)(-\hat{k}) \\ &= -9 \sin[200\pi(y + ct)] \hat{k} \text{ V/m} \end{aligned}$$

Hence, option (d) is correct.

- 25** The electric field of a plane electromagnetic wave propagating along the x-direction in vacuum is $\mathbf{E} = E_0 \hat{j} \cos(\omega t - kx)$. The magnetic field $\mathbf{B}$, at the moment $t = 0$ is

[2020, 3 Sep Shift-II]

(a) $\mathbf{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{j}$

(b) $\mathbf{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{j}$

(c) $\mathbf{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}} \cos(kx) \hat{k}$

(d) $\mathbf{B} = E_0 \sqrt{\mu_0 \epsilon_0} \cos(kx) \hat{k}$

Ans. (d)

Given, $\mathbf{E} = E_0 \hat{j} \cos(\omega t - kx)$

This equation represents electric field vector of a wave which travels in positive x-direction, i.e. along $\hat{i}$.

As, $\mathbf{E} \times \mathbf{B}$ = direction of wave propagation, we have

$$\hat{j} \times \mathbf{B} = \hat{i}$$

$\therefore \mathbf{B}$ is along $\hat{k}$

Also, $\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

so $B_0 = E_0 (\sqrt{\mu_0 \epsilon_0})$

So, $\mathbf{B} = B_0 \hat{k} \cos(\omega t - kx)$

or $\mathbf{B} = E_0 (\sqrt{\mu_0 \epsilon_0}) \hat{k} \cos kx$

Hence, correct option is (d).

- 26** The electric field of a plane electromagnetic wave is given by $\mathbf{E} = E_0 (\hat{x} + \hat{y}) \sin(kz - \omega t)$

Its magnetic field will be given by

[2020, 4 Sep Shift-II]

(a) $\frac{E_0}{c} (\hat{x} + \hat{y}) \sin(kz - \omega t)$

(b) $\frac{E_0}{c} (-\hat{x} + \hat{y}) \sin(kz - \omega t)$

(c) $\frac{E_0}{c} (\hat{x} - \hat{y}) \sin(kz - \omega t)$

(d) $\frac{E_0}{c} (\hat{x} - \hat{y}) \cos(kz - \omega t)$

Ans. (b)

Electric field vector of the given EM wave is

$$\mathbf{E} = E_0 (\hat{x} + \hat{y}) \sin(kz - \omega t)$$

$\therefore$ Amplitude of electric field $= E_0$

and unit vector of electric field,

$$\hat{\mathbf{E}} = \frac{\hat{x} + \hat{y}}{\sqrt{1^2 + 1^2}} = \frac{\hat{x} + \hat{y}}{\sqrt{2}}$$

The coefficient of k in the given equation is z . So, wave travels along positive z -direction. So, unit vector into the direction of propagation of EM wave $\hat{\mathbf{c}} = \hat{\mathbf{z}}$.

Now, for magnetic field vector of the given EM wave, we can conclude

Amplitude of magnetic field, $B_0 = \frac{E_0}{c}$

and unit vector of magnetic field $= \hat{\mathbf{B}}$

So, the expected equation of magnetic field vector will be

$$\begin{aligned} \mathbf{B} &= B_0 (\hat{\mathbf{B}}) \sin(kz - \omega t) \\ &= \frac{E_0}{c} (\hat{\mathbf{B}}) \sin(kz - \omega t) \quad \dots(i) \end{aligned}$$

Also,

$$\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{c}} = \hat{\mathbf{z}} \quad \dots(ii)$$

Now, let's check the given alternatives

(a) $\mathbf{B} = \frac{E_0}{c} (\hat{x} + \hat{y}) \sin(kz - \omega t)$

Here, $\hat{\mathbf{B}} = \frac{\hat{x} + \hat{y}}{\sqrt{1^2 + 1^2}} = \frac{\hat{x} + \hat{y}}{\sqrt{2}}$

So, $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \left(\frac{\hat{x} + \hat{y}}{\sqrt{2}} \right) \times \left(\frac{\hat{x} + \hat{y}}{\sqrt{2}} \right)$

$$= \frac{1}{2} [(\hat{x} \times \hat{x}) + (\hat{x} \times \hat{y}) + (\hat{y} \times \hat{x}) + (\hat{y} \times \hat{y})]$$

$$= \frac{1}{2} [(0) + (\hat{z}) + (-\hat{z}) + (0)]$$

$$= \frac{1}{2} [\hat{z} - \hat{z}] = 0$$

which is impossible for an EM wave.

So, this option is wrong.

(b) $\mathbf{B} = \frac{E_0}{c} (-\hat{x} + \hat{y}) \sin(kz - \omega t)$

Here, $\hat{\mathbf{B}} = \frac{-\hat{x} + \hat{y}}{\sqrt{1^2 + 1^2}} = \frac{-\hat{x} + \hat{y}}{\sqrt{2}}$

So, $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \left(\frac{\hat{x} + \hat{y}}{\sqrt{2}} \right) \times \left(\frac{-\hat{x} + \hat{y}}{\sqrt{2}} \right)$

$$= \frac{1}{2} [(-\hat{x} \times \hat{x}) + (\hat{x} \times \hat{y}) - (\hat{y} \times \hat{x}) + (\hat{y} \times \hat{y})]$$

$$= \frac{1}{2} [-(0) + (\hat{z}) - (-\hat{z}) + (0)]$$

$$= \frac{1}{2} [\hat{z} + \hat{z}] = \frac{1}{2} (2\hat{z}) = \hat{z}$$

which satisfies for eq. (ii).

So, this option is correct.

(c) $\mathbf{B} = \frac{E_0}{c} (\hat{x} - \hat{y}) \sin(kz - \omega t)$

Here, $\hat{\mathbf{B}} = \frac{\hat{x} - \hat{y}}{\sqrt{1^2 + 1^2}} = \frac{\hat{x} - \hat{y}}{\sqrt{2}}$

So, $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \left(\frac{\hat{x} + \hat{y}}{\sqrt{2}} \right) \times \left(\frac{\hat{x} - \hat{y}}{\sqrt{2}} \right)$

$$= \frac{1}{2} [(\hat{x} \times \hat{x}) - (\hat{x} \times \hat{y}) + (\hat{y} \times \hat{x}) - (\hat{y} \times \hat{y})]$$

$$= \frac{1}{2} [(0) - (\hat{z}) + (-\hat{z}) - (0)]$$

$$= \frac{1}{2} [-\hat{z} - \hat{z}]$$

$$= \frac{1}{2} (-2\hat{z}) = -\hat{z}$$

which doesn't satisfy for eq. (ii).

So, this option is wrong.

(d) $\mathbf{B} = \frac{E_0}{c} (\hat{x} - \hat{y}) \cos(kz - \omega t)$

This option is wrong, as the function in eq. (i) is a "sine" function, not a "cosine" function.

Hence, option (b) is correct.

- 27** An electron is constrained to move along the Y-axis with a speed of $0.1c$ (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\mathbf{E} = 30 \hat{j} \sin(1.5 \times 10^7 t - 5 \times 10^{-2} x) \text{ V/m}$.

The maximum magnetic force experienced by the electron will be (Given, $c = 3 \times 10^8 \text{ ms}^{-1}$ and electron charge $= 1.6 \times 10^{-19} \text{ C}$)

[2020, 5 Sep Shift-I]

(a) $2.4 \times 10^{-18} \text{ N}$

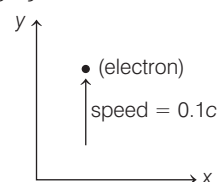
(b) $4.8 \times 10^{-19} \text{ N}$

(c) $1.6 \times 10^{-19} \text{ N}$

(d) $3.2 \times 10^{-18} \text{ N}$

Ans. (b)

The moving electron is shown in the following figure.



In electromagnetic wave,

$$B_{\max} = \frac{E_{\max}}{c}$$

We know that, magnetic force on a moving charge (electron) is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Here, angle between $\mathbf{v}$ and $\mathbf{B}$ is 90° .

So, $F_{\max} = qvB_{\max} \sin\theta$

$$= qvB_{\max} \sin 90^\circ$$

$$= \frac{qvE_{\max}}{c}$$

Given, $c = 3 \times 10^8 \text{ m/s}$,

$$v = 0.1c$$

Charge on electron, $q = 1.6 \times 10^{-19} \text{ C}$

Substituting these values in above equation, we get

$$\begin{aligned} F_{\max} &= \frac{1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 30}{3 \times 10^8} \\ &= 4.8 \times 10^{-19} \text{ N} \end{aligned}$$

Hence, correct option is (b).

- 28** Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right) \text{ W/m}^2$. The rms electric field (in V/m) associated with this source is close to the nearest integer is
- (Take, $\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2 \text{ Nm}^{-2}$ and $c = 3 \times 10^8 \text{ ms}^{-1}$)

[2020, 6 Sep Shift-I]

Ans. (194)

Given, intensity, $I = \left(\frac{315}{\pi}\right) \text{ W/m}^2$

Absolute permittivity,

$$\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2 \text{ Nm}^{-2}$$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

As, intensity, $I = \frac{1}{2} \epsilon_0 E_0^2 c$

$$\text{or } E_0 = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2 \times \left(\frac{315}{\pi}\right)}{8.86 \times 10^{-12} \times 3 \times 10^8}}$$

$$E_0 \approx 275 \text{ V/m}$$

$$\therefore E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{275}{\sqrt{2}} \approx 194 \text{ V/m}$$

- 29** For a plane electromagnetic wave, the magnetic field at a point x and time t is

$$\mathbf{B}(x, t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}] \text{ T}$$

The instantaneous electric field $\mathbf{E}$ corresponding to $\mathbf{B}$ is (Given, speed of light, $c = 3 \times 10^8 \text{ m/s}$)

[JEE Main 2020, 6 Sep Shift-II]

- (a) $\mathbf{E}(x, t) = [-36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{j}}] \frac{\text{V}}{\text{m}}$
- (b) $\mathbf{E}(x, t) = [36 \sin(1 \times 10^3 x + 0.5 \times 10^{11} t) \hat{\mathbf{j}}] \frac{\text{V}}{\text{m}}$
- (c) $\mathbf{E}(x, t) = [36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}] \frac{\text{V}}{\text{m}}$
- (d) $\mathbf{E}(x, t) = [36 \sin(1 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{j}}] \frac{\text{V}}{\text{m}}$

Ans. (a)

Given that,

$$\mathbf{B}(x, t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{k}}] \text{ T}$$

Speed of light, $c = 3 \times 10^8 \text{ m/s}$

Maximum value of electric field,

$$E_0 = (\text{maximum value of } B_0) \times (\text{speed of light})$$

$$= B_0 \times c = 1.2 \times 10^{-7} \times 3 \times 10^8 = 36 \text{ V/m}$$

Since, wave is propagating in negative x -direction and magnetic field is in z -direction. So, electric field vector must be in negative y -direction as direction of wave propagation is in the direction of $\mathbf{E} \times \mathbf{B}$.

So, electric field at a point x and time t is given by

$$\mathbf{E}(x, t) = -36 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{\mathbf{j}} \text{ V/m}$$

Here, phase of $\mathbf{E}$ will be same that of $\mathbf{B}$.

Hence, correct option is (a).

- 30** If the magnetic field in a plane electromagnetic wave is given by $\mathbf{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{j}} \text{ T}$, then what will be expression for electric field? [2020, 7 Jan Shift-I]

- (a) $\mathbf{E} = 60 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{k}} \text{ V/m}$
- (b) $\mathbf{E} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{i}} \text{ V/m}$
- (c) $\mathbf{E} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{j}} \text{ V/m}$
- (d) $\mathbf{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{k}} \text{ V/m}$

Ans. (d)

An electromagnetic wave is a travelling wave in which electric vector $\mathbf{E}$ is perpendicular to magnetic vector $\mathbf{B}$ and both of these vectors are also perpendicular to the direction of propagation of electromagnetic wave.

So, it is given that

$$\mathbf{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{j}} \text{ T}$$

$$= B_0 \sin(kx + \omega t) \hat{\mathbf{j}} \text{ T}$$

Here, $k = 1.6 \times 10^3 \text{ units}$

and $\omega = 48 \times 10^{10} \text{ units}$

Then, it can be implied that electromagnetic wave is travelling along $-X$ -axis (this is obtained from phase $kx + \omega t$) and its magnetic vector $\mathbf{B}$ is directed along $+Y$ -axis (this is indicated by unit vector $\hat{\mathbf{j}}$).

Hence, electric field vector is along $+Z$ -axis and its magnitude is given by

$$E_0 = c \cdot B_0 = 3 \times 10^8 \times 3 \times 10^{-8} = 9 \text{ V/m}$$

$$\therefore \text{Expression for } \mathbf{E} = 9 \sin(kx + \omega t) \hat{\mathbf{k}}$$

$$= 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{\mathbf{k}} \text{ V/m}$$

- 31** The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0 \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \cos(kz + \omega t)$$

At $t = 0$, a positively charged particle is at the point

$$(x, y, z) = \left(0, 0, \frac{\pi}{k}\right).$$

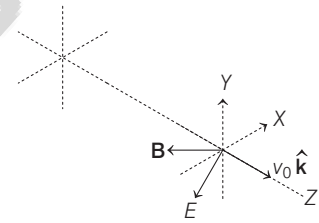
If its instantaneous velocity at ($t = 0$) is $v_0 \hat{\mathbf{k}}$, the force acting on it due to the wave is

[2020, 7 Jan Shift-II]

(a) zero

(b) antiparallel to $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ (c) parallel to $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ (d) parallel to $\hat{\mathbf{k}}$ **Ans. (b)**

We are given with following situation



$$\text{Given, } \mathbf{E} = E_0 \left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} \right) \cdot \cos(kz + \omega t)$$

So, electromagnetic wave is propagating along Z -axis and its electric vector $\mathbf{E}$ is perpendicular to Z -axis; and is along angle bisector of IIIrd quadrant of xy -plane as shown in the figure. So, magnetic vector $\mathbf{B}$ is along angle bisector of IInd quadrant of xy -plane and is perpendicular to both $\mathbf{E}$ and $\mathbf{v}$ vectors.

Now, force due to electric field is along $\mathbf{E}$ vector and force due to magnetic field is obtained by Fleming's left hand rule.

Thus, direction of magnetic force is also along $\mathbf{E}$. So, force on charged particle will

be along $\mathbf{E}$ or along $-\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)$ vector.

Hence, force on charged particle is

antiparallel to $\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)$ vector.

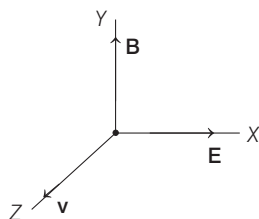
- 32** A plane electromagnetic wave of frequency 25 GHz is propagating in vacuum along the z -direction. At a particular point in space and time, the magnetic field is given by $\mathbf{B} = 5 \times 10^{-8} \hat{\mathbf{j}} \text{ T}$. The corresponding electric field $\mathbf{E}$ is (speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$) [2020, 8 Jan Shift-II]

- (a) $-1.66 \times 10^{-16} \hat{i} \text{ V/m}$
 (b) $1.66 \times 10^{-16} \hat{i} \text{ V/m}$
 (c) $-15 \hat{i} \text{ V/m}$
 (d) $15 \hat{i} \text{ V/m}$

Ans. (d)

Given,

$$\mathbf{B} = 5 \times 10^{-8} \hat{j} \text{ and } \mathbf{v} = 3 \times 10^8 \hat{k}$$



Using $\mathbf{E} = \mathbf{B} \times \mathbf{v}$, we have

$$\mathbf{E} = (5 \times 10^{-8} \hat{j}) \times (3 \times 10^8 \hat{k}) = 15 \hat{i} \text{ Vm}^{-1}$$

- 33** A plane electromagnetic wave is propagating along the direction $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ with its polarisation along the direction $\hat{k}$. The correct form of the magnetic field of the wave would be (here B_0 is an appropriate constant) [2020, 9 Jan Shift-II]

- (a) $B_0 \hat{k} \cos\left(\omega t - k \frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$
 (b) $B_0 \frac{\hat{j} - \hat{i}}{\sqrt{2}} \cos\left(\omega t + k \frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$
 (c) $B_0 \frac{\hat{i} - \hat{j}}{\sqrt{2}} \cos\left(\omega t - k \frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$
 (d) $B_0 \frac{\hat{i} + \hat{j}}{\sqrt{2}} \cos\left(\omega t - k \frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

Ans. (c)

Given, direction of propagation of electromagnetic wave is along $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ also

direction of its electric field vector $\mathbf{E}$ (or direction of its polarisation) is $\hat{k}$.

As, direction of propagation of wave is along $\mathbf{E} \times \mathbf{B}$, so direction of $\mathbf{B}$ is such that

$$\hat{k} \times \mathbf{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

From options,

$$\hat{k} \times \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right) = \frac{\hat{k} \times \hat{i} - \hat{k} \times \hat{j}}{\sqrt{2}} = \frac{\hat{j} - (-\hat{i})}{\sqrt{2}} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$$

So, correct expression of $\mathbf{B}$ is,

$$\mathbf{B} = B_0 \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right) \cos\left(\omega t - k \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right)\right)$$

- 34** A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6 \text{ Vm}^{-1}$ along y-direction. Its corresponding magnetic field component, $\mathbf{B}$ would be

- (a) $2 \times 10^{-8} \text{ T}$ along z-direction
 (b) $6 \times 10^{-8} \text{ T}$ along x-direction
 (c) $6 \times 10^{-8} \text{ T}$ along z-direction
 (d) $2 \times 10^{-8} \text{ T}$ along y-direction

[2019, 8 April Shift-I]

Ans. (a)

Key Idea For an electromagnetic wave, ratio of magnitudes of electric and magnetic field is

$$\frac{E}{B} = c$$

where, c is the speed of electromagnetic wave in vacuum.

$$\text{Given, } E = 6 \text{ V/m, } c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\text{So, } B = \frac{E}{c} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8} \text{ T}$$

Also, direction of propagation of electromagnetic wave is given by

$$\hat{n} = \mathbf{E} \times \mathbf{B}$$

Here, $\hat{n} = \hat{i}$ and $\mathbf{E} = \text{Unit vector of electric field } (\hat{j})$

$\mathbf{B} = \text{unit vector of magnetic field.}$

$$\Rightarrow \hat{i} = \hat{j} \times \mathbf{B}$$

$$\therefore \mathbf{B} = \hat{k}$$

Hence, magnetic field components

$$\mathbf{B} = 2 \times 10^{-8} \hat{k} \text{ T} = 2 \times 10^{-8} \text{ T}$$

(along z-direction)

- 35** The magnetic field of an electromagnetic wave is given by $\mathbf{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t) (2\hat{i} + \hat{j}) \text{ Wbm}^{-2}$

The associated electric field will be [2019, 8 April Shift-II]

- (a) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t) (-2\hat{j} + \hat{i}) \text{ Vm}^{-1}$
 (b) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t) (2\hat{j} + \hat{i}) \text{ Vm}^{-1}$
 (c) $\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t) (\hat{i} - 2\hat{j}) \text{ Vm}^{-1}$

$$(d) \mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t) (-\hat{i} + 2\hat{j}) \text{ Vm}^{-1}$$

Ans. (c)

(c) Given,

$$\mathbf{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t) (2\hat{i} + \hat{j}) \text{ Wbm}^{-2}$$

From the given equation, it can be said that the electromagnetic wave is propagating negative z-direction, i.e. $-\hat{k}$.

Equation of associated electric field will be

$$\mathbf{E} = (|\mathbf{B}|c) \cos(kz + \omega t) \cdot \hat{n}$$

where, $\hat{n}$ = a vector perpendicular to $\mathbf{B}$.

So,

$$|\mathbf{E}| = |\mathbf{B}|c = 1.6 \times 10^{-6} \times 3 \times 10^8 = 4.8 \times 10^2 \text{ V/m}$$

Since, we know that for an electromagnetic wave, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular to each other.

So, $\mathbf{E} \cdot \mathbf{B} = 0$

From the given options, when $\hat{n} = \hat{i} - 2\hat{j}$

$$\mathbf{E} \cdot \mathbf{B} = (2\hat{i} + \hat{j}) \cdot (\hat{i} - 2\hat{j}) = 0$$

Also, when $\hat{n} = -\hat{i} + 2\hat{j}$

$$\mathbf{E} \cdot \mathbf{B} = (2\hat{i} + \hat{j}) \cdot (-\hat{i} + 2\hat{j}) = 0$$

But, we also know that the direction of propagation of electromagnetic wave is perpendicular to both $\mathbf{E}$ and $\mathbf{B}$, i.e. it is in the direction of $\mathbf{E} \times \mathbf{B}$.

Again, when $\hat{n} = \hat{i} - 2\hat{j}$

$$\mathbf{E} \times \mathbf{B} = (2\hat{i} + \hat{j}) \times (\hat{i} - 2\hat{j}) = -\hat{k}$$

and when $\hat{n} = -\hat{i} + 2\hat{j}$

$$\mathbf{E} \times \mathbf{B} = (2\hat{i} + \hat{j}) \times (-\hat{i} + 2\hat{j}) = \hat{k}$$

But, it is been given in the question that the direction of propagation of wave is in $-\hat{k}$.

Thus, associated electric field will be

$$\mathbf{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t) (\hat{i} - 2\hat{j}) \text{ Vm}^{-1}$$

- 36** 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3 \times 10^8 \text{ m/s}$) [2019, 9 April Shift-II]

- (a) $20 \times 10^{-8} \text{ N}$
 (b) $35 \times 10^{-8} \text{ N}$
 (c) $15 \times 10^{-8} \text{ N}$
 (d) $10 \times 10^{-8} \text{ N}$

Ans. (a)

Radiation pressure or momentum imparted per second per unit area when light falls is

$$p = \begin{cases} \frac{2I}{c} & ; \text{ for reflection of radiation} \\ \frac{I}{c} & ; \text{ for absorption of radiation} \end{cases}$$

where, I is the intensity of the light.

In given case, there is 25% reflection and 75% absorption, so radiation pressure

= force per unit area

$$\begin{aligned} &= \frac{25}{100} \times \frac{2I}{c} + \frac{75}{100} \times \frac{I}{c} \\ &= \frac{1}{2} \times \frac{I}{c} + \frac{3}{4} \times \frac{I}{c} \\ &= \frac{5}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{50}{3 \times 10^8} \\ &= 20.83 \times 10^{-8} \text{ N} \\ &\approx 20 \times 10^{-8} \text{ N} \end{aligned}$$

- 37** Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2} . If the surface has an area of 25 cm^2 , the momentum transferred to the surface in 40 min time duration will be **[2019, 10 April Shift-II]**

- (a) $3.5 \times 10^{-6} \text{ N-s}$ (b) $6.3 \times 10^{-4} \text{ N-s}$
(c) $1.4 \times 10^{-6} \text{ N-s}$ (d) $5.0 \times 10^{-3} \text{ N-s}$

Ans. (d)

Radiation pressure over an absorbing surface is,

$$p = \frac{I}{c}$$

where, I = intensity or energy flux
and c = speed of light.

If A = area of surface, then force due to radiation on the surface is

$$F = p \times A = \frac{IA}{c}$$

If force F acts for a duration of Δt seconds, then momentum transferred to the surface is

$$\Delta p = F \times \Delta t = \frac{IA}{c} \times \Delta t$$

Here, $I = 25 \text{ W cm}^{-2}$, $A = 25 \text{ cm}^2$,
 $c = 3 \times 10^8 \text{ ms}^{-1}$,

$$\Delta t = 40 \text{ min} = 2400 \text{ s}$$

So, momentum transferred to the surface,

$$\begin{aligned} \Delta p &= \frac{25 \times 25 \times 2400}{3 \times 10^8} \\ &= 5 \times 10^{-3} \text{ N-s} \end{aligned}$$

- 38** The magnetic field of a plane electromagnetic wave is given by

$$\mathbf{B} = B_0 [\cos(kz - \omega t)]\hat{i} + B_1 \cos(kz + \omega t)\hat{j}$$

where, $B_0 = 3 \times 10^{-5} \text{ T}$ and

$B_1 = 2 \times 10^{-6} \text{ T}$. The rms value of the force experienced by a stationary charge $Q = 10^{-4} \text{ C}$ at $z = 0$ is closest to **[2019, 9 April Shift-I]**

- (a) 0.1 N (b) $3 \times 10^{-2} \text{ N}$
(c) 0.6 N (d) 0.9 N

Ans. (c)

Given, magnetic field of an electromagnetic wave is

$$\mathbf{B} = B_0 [\cos(kz - \omega t)]\hat{i} + B_1 [\cos(kz + \omega t)]\hat{j}$$

Here, $B_0 = 3 \times 10^{-5} \text{ T}$ and $B_1 = 2 \times 10^{-6} \text{ T}$

Also, stationary charge, $Q = 10^{-4} \text{ C}$ at $z = 0$

As charge is released from the rest at $z = 0$, in this condition.

Maximum electric field, $E_0 = cB_0$ and $E_1 = cB_1$

$$\text{So, } E_0 = c \times 3 \times 10^{-5}$$

$$\text{and } E_1 = c \times 2 \times 10^{-6}$$

Now, the direction of electric field of an electromagnetic wave is perpendicular to $\mathbf{B}$ and to the direction of propagation of wave ($\mathbf{E} \times \mathbf{B}$) which is $\hat{k}$.

So, for E_0 ,

$$\begin{aligned} E_0 \times B_0 &= \hat{k} \\ \Rightarrow E_0 \times \hat{i} &= \hat{k} \\ \Rightarrow E_0 &= -\hat{j} \end{aligned}$$

Similarly, for E_1 $E_1 \times B_1 = \hat{k}$

$$\begin{aligned} \Rightarrow E_1 \times \hat{j} &= \hat{k} \\ \Rightarrow E_1 &= \hat{i} \end{aligned}$$

$$\begin{aligned} \therefore E_0 &= c \times 3 \times 10^{-5} (-\hat{j}) \text{ NC}^{-1} \\ E &= c \times 2 \times 10^{-6} (\hat{i}) \text{ NC}^{-1} \end{aligned}$$

$\therefore$ Maximum force experienced by stationary charge is

$$\begin{aligned} F_{\max} &= QE = Q(E_0 + E_1) \\ &= Q \times c [-3 \times 10^{-5} \hat{j} + 2 \times 10^{-6} \hat{i}] \\ \Rightarrow |F_{\max}| &= 10^{-4} \times 3 \times 10^8 \\ &\quad \times \sqrt{(3 \times 10^{-5})^2 + (2 \times 10^{-6})^2} \\ &= 3 \times 10^4 \times 10^{-6} \sqrt{900 + 4} \\ &= 3 \times 10^{-2} \times \sqrt{904} \approx 0.9 \text{ N} \end{aligned}$$

$\therefore$ rms value of experienced force is

$$\begin{aligned} F_{\text{rms}} &= \frac{F_{\max}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.707 \times 0.9 \\ &= 0.6363 \text{ N} \approx 0.6 \text{ N} \end{aligned}$$

- 39** The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0 \hat{i} \cos(kz) \cos(\omega t)$$

The corresponding magnetic field $\mathbf{B}$ is then given by

[2019, 10 April Shift-I]

- (a) $\mathbf{B} = \frac{E_0}{c} \hat{j} \sin(kz) \sin(\omega t)$
(b) $\mathbf{B} = \frac{E_0}{c} \hat{j} \sin(kz) \cos(\omega t)$
(c) $\mathbf{B} = \frac{E_0}{c} \hat{k} \sin(kz) \cos(\omega t)$
(d) $\mathbf{B} = \frac{E_0}{c} \hat{j} \cos(kz) \sin(\omega t)$

Ans. (a)

Key Idea For an electromagnetic wave, its electric field vector ($\mathbf{E}$) and magnetic field vector ($\mathbf{B}$) is mutually perpendicular to each other and also to its direction of propagation.

We know that, $\mathbf{E} \times \mathbf{B}$ represents direction of propagation of an electromagnetic wave

$$\Rightarrow (\mathbf{E} \times \mathbf{B}) \parallel \mathbf{v}$$

$\therefore$ From the given electric field, we can state that direction of propagation is along Z -axis and direction of $\mathbf{E}$ is along X -axis.

Thus, from the above discussion, direction of $\mathbf{B}$ must be Y -axis.

From Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\text{Here, } \frac{\partial \mathbf{E}}{\partial z} = -\frac{\partial \mathbf{B}}{\partial t} \quad \dots (i)$$

$$\text{and } B_0 = E_0 / c \quad \dots (ii)$$

Given, $\mathbf{E} = E_0 \hat{i} \cos kz \cos \omega t$

$$\Rightarrow \frac{-\partial \mathbf{E}}{\partial z} = k E_0 \sin kz \cos \omega t$$

$\therefore$ Using Eq. (i), we get

$$\frac{\partial \mathbf{B}}{\partial t} = k E_0 \sin kz \cos \omega t$$

Integrating both sides of the above equation w.r.t. t , we get

$$\Rightarrow \mathbf{B} = \frac{k}{\omega} E_0 \sin kz \sin \omega t$$

$$= \frac{E_0}{c} \sin kz \sin \omega t$$

$$\Rightarrow \mathbf{B} = \frac{E_0}{c} \sin(kz) \sin(\omega t) \hat{j}$$

- 40** An electromagnetic wave is represented by the electric field $\mathbf{E} = E_0 \hat{n} \sin[\omega t + (6y - 8z)]$. Taking

unit vectors in x, y and z- directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation $\hat{s}$, is

[2019, 12 April Shift-I]

$$\begin{aligned} \text{(a) } \hat{s} &= \frac{3\hat{i} - 4\hat{j}}{5} & \text{(b) } \hat{s} &= \frac{-4\hat{k} + 3\hat{j}}{5} \\ \text{(c) } \hat{s} &= \left(\frac{-3\hat{j} + 4\hat{k}}{5} \right) & \text{(d) } \hat{s} &= \frac{4\hat{j} - 3\hat{k}}{5} \end{aligned}$$

Ans. (c)

Key Idea Standard expression of electromagnetic wave is given by

$$\mathbf{E} = E_0 \hat{n} [\sin(\omega t - \mathbf{k} \cdot \mathbf{r})] \quad \dots(i)$$

Here, $\mathbf{k}$ is the propagation vector.

Direction of propagation in this case is $\hat{k}$.

Given expression of electromagnetic wave,

$$\mathbf{E} = E_0 \hat{n} \sin[\omega t + (6y - 8z)]$$

$$\Rightarrow \mathbf{E} = E_0 \hat{n} \sin[\omega t - (8z - 6y)] \quad \dots(ii)$$

Comparing Eq. (ii) with Eq. (i), we get

$$\mathbf{k} \cdot \mathbf{r} = 8z - 6y \quad \dots(iii)$$

$$\text{Here, } \mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{and } \mathbf{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$$

$$\therefore \mathbf{k} \cdot \mathbf{r} = xk_x + yk_y + zk_z \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$xk_x = \text{zero} \Rightarrow k_x = 0$$

$$yk_y = -6y \Rightarrow k_y = -6$$

$$zk_z = 8z \Rightarrow k_z = 8$$

$$\text{Hence, } \mathbf{k} = -6\hat{j} + 8\hat{k}$$

So, direction of propagation,

$$\begin{aligned} \hat{s} &= \hat{k} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{-6\hat{j} + 8\hat{k}}{\sqrt{6^2 + 8^2}} \\ &= \frac{-6\hat{j} + 8\hat{k}}{10} = \frac{-3\hat{j} + 4\hat{k}}{5} \end{aligned}$$

- 41** A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave? [2019, 12 April Shift-II]

- (a) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t) \hat{i}$
 (b) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t) \hat{i}$
 (c) $\mathbf{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \hat{k}$
 (d) $\mathbf{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t) \hat{j}$

Ans. (b)

In an electromagnetic wave, magnetic field and electric field are perpendicular to each other and both are also

perpendicular to the direction of propagation of wave.

Now, given direction of propagation is along z-direction. So, magnetic field is in either x or y direction. Also, angular wave number for wave is

$$\begin{aligned} k &= \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c} \\ &= \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8} \\ &\approx 0.5 \times 10^3 \text{ m}^{-1} \end{aligned}$$

and angular frequency ω for wave is

$$\begin{aligned} \omega &= 2\pi\nu = 2\pi \times 23.9 \times 10^9 \text{ Hz} \\ &= 1.5 \times 10^{11} \text{ Hz} \end{aligned}$$

Magnitude of magnetic field is

$$\begin{aligned} B_0 &= \frac{E_0}{c} = \frac{60}{3 \times 10^8} \\ &= 2 \times 10^{-7} \text{ T} \end{aligned}$$

As the general equation of magnetic field of an electromagnetic wave propagating in +z-direction is given as,

$$\mathbf{B} = B_0 \sin(kz - \omega t) \hat{i} \text{ or } \hat{j}$$

Thus, substituting the values of B_0 , k and ω , we get

$$\Rightarrow \mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t) \hat{i} \text{ or } \hat{j}$$

- 42** A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\mathbf{E} = 6.3 \hat{j}$ V/m. The corresponding magnetic field $\mathbf{B}$, at that point will be [2019, 9 Jan Shift-I]

- (a) $18.9 \times 10^{-8} \hat{k}$ T (b) $6.3 \times 10^{-8} \hat{k}$ T
 (c) $18.9 \times 10^{-8} \hat{k}$ T (d) $2.1 \times 10^{-8} \hat{k}$ T

Ans. (d)

Since, the magnitude of electric field vector and magnetic field vector of electromagnetic wave is related as,

$$\frac{|\mathbf{E}|}{|\mathbf{B}|} = c$$

where, c is the speed of light in vacuum.

Here, $|\mathbf{E}| = 6.3$ and $c = 3 \times 10^8$ m/s.

$$\begin{aligned} \Rightarrow |\mathbf{B}| &= \frac{|\mathbf{E}|}{c} = \frac{6.3}{3 \times 10^8} \\ &= 2.1 \times 10^{-8} \end{aligned}$$

Now, as we know for a EM wave, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other as well as to direction of propagation of wave ($\mathbf{E} \times \mathbf{B}$).

As here, direction of $\mathbf{E}$ is +y-direction i.e. $\hat{j}$ and direction of propagation of wave is in +x-direction i.e., $\hat{i}$

$\therefore$ If $\mathbf{B}$ has to be perpendicular to $\mathbf{E}$ and direction of propagation, then it must in +z-direction i.e., $\hat{k}$.

Thus, magnetic field, $\mathbf{B} = 2.1 \times 10^{-8} \hat{k}$ T.

- 43** In free space, the energy of electromagnetic wave in electric field is U_E and in magnetic field is U_B . Then [2019, 9 Jan Shift-II]

- (a) $U_E = U_B$ (b) $U_E > U_B$
 (c) $U_E < U_B$ (d) $U_E = \frac{U_B}{2}$

Ans. (a)

Energy density of an electromagnetic wave in electric field,

$$U_E = \frac{1}{2} \epsilon_0 E^2 \quad \dots(i)$$

Energy density of an electromagnetic wave in magnetic field,

$$U_B = \frac{B^2}{2\mu_0} \quad \dots(ii)$$

where, E = electric field,

B = magnetic field,

ϵ_0 = permittivity of medium and

μ_0 = magnetic permeability of medium.

From the theory of electro-magnetic waves, the relation between μ_0 and ϵ_0 is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \dots(iii)$$

where, c = velocity of light

$$= 3 \times 10^8 \text{ m/s}$$

$$\text{and } \frac{E}{B} = c \quad \dots(iv)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{U_E}{U_B} = \frac{\frac{1}{2} \epsilon_0 E^2}{\frac{1}{2} B^2 \times \frac{1}{\mu_0}} = \frac{\mu_0 \epsilon_0 E^2}{B^2} \quad \dots(v)$$

Using Eqs. (iii), (iv) and (v), we get

$$\frac{U_E}{U_B} = \frac{c^2}{c^2} = 1$$

Therefore, $U_E = U_B$

- 44** If the magnetic field of a plane electromagnetic wave is given by $\mathbf{B} = 100 \times 10^{-6} \sin \left[2\pi \times 2 \times 10^{15} \left(t - \frac{x}{c} \right) \right]$

then the maximum electric field associated with it is (Take, the speed of light = 3×10^8 m/s)

[2019, 10 Jan Shift-I]

- (a) 6×10^4 N/C (b) 4×10^4 N/C
 (c) 3×10^4 N/C (d) 4.5×10^4 N/C

Ans. (c)

Given, instantaneous value of magnetic field

$$B = 100 \times 10^{-6} \sin \left[2\pi \times 2 \times 10^{15} \left(t - \frac{x}{c} \right) \right]$$

and speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$

For an electromagnetic wave,

$$E_{\max} = B_{\max} \times c$$

where, $E_{\max}$ = maximum value of the electric field.

We get,

$$E_{\max} = 100 \times 10^{-6} \times 3 \times 10^8 \\ = 3 \times 10^4 \frac{\text{N}}{\text{C}}$$

- 45** The electric field of a plane polarised electromagnetic wave in free space at time $t=0$ is given by an expression.

$$\mathbf{E}(x, y) = 10\hat{j} \cos[(6x + 8z)]$$

The magnetic field $\mathbf{B}(x, z, t)$ is given by (where, c is the velocity of light) **[2019, 10 Jan Shift-II]**

- (a) $\frac{1}{c}(6\hat{k} - 8\hat{i}) \cos[(6x + 8z + 10ct)]$
 (b) $\frac{1}{c}(6\hat{k} - 8\hat{i}) \cos[(6x + 8z - 10ct)]$
 (c) $\frac{1}{c}(6\hat{k} + 8\hat{i}) \cos[(6x - 8z + 10ct)]$
 (d) $\frac{1}{c}(6\hat{k} + 8\hat{i}) \cos[(6x + 8z - 10ct)]$

Ans. (b)

We are given electric field as

$$\mathbf{E} = 10\hat{j} \cos(6x + 8z) \quad \dots (i)$$

where, phase angle is independent of time,

i.e., phase angle at $t=0$ is $\phi = 6x + 8z$.

Phase angle for $\mathbf{B}$ will also be $6x + 8z$ because for an electromagnetic wave $\mathbf{E}$ and $\mathbf{B}$ oscillate in same phase.

Thus, direction of wave propagation

$$= \frac{6\hat{i} + 8\hat{k}}{\sqrt{6^2 + 8^2}} = \frac{6\hat{i} + 8\hat{k}}{10} \quad \dots (ii)$$

Let magnetic field vector,

$\mathbf{B} = a\hat{i} + b\hat{j} + d\hat{k}$, then direction of wave propagation is given by

$$\frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{E}'||\mathbf{B}|} = \frac{10\hat{j} \times (a\hat{i} + b\hat{j} + d\hat{k})}{10 \times (a^2 + b^2 + d^2)^{1/2}} \\ = \frac{-10a\hat{k} + 10d\hat{i}}{10(a^2 + b^2 + d^2)^{1/2}} \quad \dots (iii)$$

As, $|\mathbf{B}| = \frac{|\mathbf{E}|}{c} = \frac{10}{c}$

We get, $|\mathbf{B}| = \sqrt{a^2 + b^2 + d^2} = 10/c$

By putting this value in Eqs. (iii) and (ii), we get direction of propagation

$$\Rightarrow \frac{c10(d\hat{i} - a\hat{k})}{10 \times 10} = \frac{6\hat{i} + 8\hat{k}}{10}$$

$$\Rightarrow \begin{aligned} d &= 6/c \\ \text{and } a &= -8/c \end{aligned}$$

$$\text{Hence, } \mathbf{B} = \frac{6\hat{k}}{c} - \frac{8\hat{i}}{c} = \frac{1}{c}(6\hat{k} - 8\hat{i})$$

As the general equation of magnetic field of an EM wave propagating in positive y -direction is given as,

$$B = B_0 \cos(Ry - \omega t)$$

$$\therefore \mathbf{B} = \frac{1}{c}(6\hat{k} - 8\hat{i}) \cos(6x + 8z - 10ct)$$

Alternate Solution Given, electric field is $\mathbf{E}(x, y)$, i.e. electric field is in xy -plane which is given as

$$\therefore \mathbf{E} = 10\hat{j} \cos(6x + 8z)$$

Since, the magnetic field given is $\mathbf{B}(x, z, t)$, this means $\mathbf{B}$ is in xz -plane.

$\therefore$ Propagation of wave is in y -direction.

[$\therefore$ for an electromagnetic wave, $\mathbf{E} \perp \mathbf{B} \perp$ propagation direction]

As Poynting vector suggests that $\mathbf{E} \times \mathbf{B}$ is parallel to $(6\hat{i} + 8\hat{k})$.

$$\text{Let } \mathbf{B} = (x\hat{i} + z\hat{k}),$$

$$\text{then } \mathbf{E} \times \mathbf{B} = \hat{j} \times (x\hat{i} + z\hat{k})$$

$$= 6\hat{i} + 8\hat{k}$$

$$\text{or } -x\hat{k} + z\hat{i} = 6\hat{i} + 8\hat{k}$$

$$\text{or } x = -8 \text{ and } z = 6$$

$$\therefore \mathbf{B} = \frac{1}{c}(6\hat{k} - 8\hat{i}) \cos(6x + 8z - 10ct)$$

$$\therefore \frac{|\mathbf{E}|}{|\mathbf{B}|} = c$$

- 46** An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index ' n ' without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by

[2019, 11 Jan Shift-I]

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

Ans. (d)

In the free space, the speed of electromagnetic wave is given as,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0} \quad \dots (i)$$

where, E_0 and B_0 are the amplitudes of varying electric and magnetic fields, respectively.

Now, when it enters in a medium of refractive index ' n ', its speed is given as,

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{K \epsilon_0 \mu}} = \frac{E}{B} \quad \dots (ii)$$

where, K is dielectric strength of the medium.

Using Eqs. (i) and (ii), we get

$$\frac{v}{c} = \frac{1}{\sqrt{K}} \quad \dots (iii)$$

($\therefore$ For a transparent medium, $\mu_0 \approx \mu$)

Also, refractive index of medium is ' n ' and is given as

$$\frac{c}{v} = n$$

$$\text{or } \frac{v}{c} = \frac{1}{n} \quad \dots (iv)$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \text{ or } K = n^2 \quad \dots (v)$$

The intensity of an EM wave is given as,

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

and in the medium, it is given as

$$I' = \frac{1}{2} K \epsilon_0 E'^2 v$$

It is given that, $I = I'$

$$\Rightarrow \frac{1}{2} \epsilon_0 E_0^2 c = \frac{1}{2} K \epsilon_0 E'^2 v$$

$$\text{or } \left(\frac{E_0}{E'}\right)^2 = \frac{Kv}{c} \quad \dots (vi)$$

From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E'}\right)^2 = \frac{n^2}{n} = n \text{ or } E_0/E' = \sqrt{n}$$

$$\text{Similarly, } \frac{1}{2} \frac{B_0^2}{\mu_0} c = \frac{1}{2} \frac{B'^2}{\mu_0} v$$

$$\Rightarrow \frac{B_0}{B'} = \frac{1}{\sqrt{n}}$$

Alternate Solution

We know that,

$$\left(\frac{E_0}{B_0}\right)_{\text{air/vacuum}} = c$$

$$\text{and } \left(\frac{E}{B}\right)_{\text{medium}} = v$$

$$\text{Also, } n = \frac{c}{v} \\ \frac{E_0/B_0}{E/B} = \frac{c}{v} = n$$

$$\Rightarrow \frac{E_0/E}{B_0/B} = n$$

This is possible only if $\frac{E_0}{E} = \sqrt{n}$ and

$$\frac{B_0}{B} = \frac{1}{\sqrt{n}}$$

- 47** A 27 mW laser beam has a cross-sectional area of 10 mm^2 . The magnitude of the maximum electric field in this electromagnetic wave is given by [Take, permittivity of space, $\epsilon_0 = 9 \times 10^{-12} \text{ SI units}$ and speed of light, $c = 3 \times 10^8 \text{ m/s}$]

[2019, 11 Jan Shift-II]

- (a) 1 kV/m (b) 0.7 kV/m
(c) 2 kV/m (d) 1.4 kV/m

Ans. (d)

Given,

Power of laser beam

$$(P) = 27 \text{ mW} = 27 \times 10^{-3} \text{ W}$$

Area of cross-section (A)

$$= 10 \text{ m}^2 = 10 \times 10^{-6} \text{ m}^2$$

Permittivity of free space (ϵ_0) = 9×10^{-12} SI unitSpeed of light (c) = $3 \times 10^8 \text{ m/s}$

Intensity of electromagnetic wave is given by the relation

$$I = \frac{1}{2} n c \epsilon_0 E^2$$

where, n is refractive index, for air $n = 1$.

$$\therefore I = \frac{1}{2} c \epsilon_0 E^2 \quad \dots(i)$$

$$\text{Also, } I = \frac{P}{A} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} c \epsilon_0 E^2 = \frac{P}{A}$$

$$\text{or } E^2 = \frac{2P}{Ac \epsilon_0}$$

$$\text{or } E = \sqrt{\frac{2 \times 27 \times 10^{-3}}{10 \times 10^{-6} \times 3 \times 10^8 \times 9 \times 10^{-12}}} \\ \approx 1.4 \times 10^3 \text{ V/m} \\ = 1.4 \text{ kV/m}$$

- 48** A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propagating in the glass medium will be

[2019, 12 Jan Shift-I]

- (a) 30 V/m (b) 6 V/m
(c) 10 V/m (d) 24 V/m

Ans. (d)Energy of a light wave $\propto$ Intensity of the light waveSince, intensity = $\epsilon v A^2$

where, ϵ is the permittivity of the medium in which light is travelling with velocity v and A is its amplitude.

Since, only 4% of the energy of the light gets reflected.

$\therefore$ 96% of the energy of the light is transmitted.

$$E_{\text{transmitted}} (E_t) = 96\% \text{ of } E_{\text{incident}} (E_i)$$

$$\epsilon_0 \epsilon_r v A_t^2 = \frac{96}{100} \times \epsilon_0 \times c \times A_i^2$$

$$A_t^2 = \frac{96}{100} \cdot \frac{\epsilon_0}{\epsilon_r} \cdot \frac{c}{v} A_i^2$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v^2}{c^2} \cdot \frac{c}{v} A_i^2 \left[\because \sqrt{\frac{\epsilon_0}{\epsilon_r}} = \frac{v}{c} \right]$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v}{c} \cdot A_i^2 \\ = \frac{96}{100} \times \frac{1}{3/2} \times 30^2$$

$$\left[\because \mu = \frac{c}{v} = 1.5 \right]$$

$$A_t = \sqrt{\frac{64}{100} \times 30^2}$$

$$A_t = 24 \text{ V/m}$$

- 49** An EM wave from air enters a medium. The electric fields are $E_1 = E_{01} \hat{x} \cos \left[2\pi v \left(\frac{z}{c} - t \right) \right]$ in air and

$E_2 = E_{02} \hat{x} \cos [k(2z - ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic.

If ϵ_{r1} and ϵ_{r2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

[JEE Main 2018]

- (a) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 4$ (b) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 2$
(c) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{4}$ (d) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{2}$

Ans. (c)

Key Idea Speed of progressive wave is given by, $v = \frac{\omega}{k}$

As electric field in air is,

$$E_1 = E_{01} \hat{x} \cos \left(\frac{2\pi v z}{c} - 2\pi v t \right)$$

$$\therefore \text{Speed in air} = \frac{2\pi v}{\left(\frac{2\pi v}{c} \right)} = c$$

$$\text{Also, } c = \frac{1}{\sqrt{\mu_0 \epsilon_{r1} \epsilon_0}} \quad \dots(i)$$

In medium,

$$E_2 = E_{02} \times \cos(2kz - kct)$$

$$\therefore \text{Speed in medium} = \frac{kc}{2k} = \frac{c}{2}$$

$$\text{Also, } \frac{c}{2} = \frac{1}{\sqrt{\mu_0 \epsilon_{r2} \epsilon_0}} \quad \dots(ii)$$

As medium is non-magnetic medium,

$$\mu_{\text{medium}} = \mu_{\text{air}}$$

On dividing Eq. (i) by Eq. (ii), we have

$$2 = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \Rightarrow \frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{4}$$

- 50** During the propagation of electromagnetic waves in a medium, [JEE Main 2014]

- (a) electric energy density is double of the magnetic energy density
(b) electric energy density is half of the magnetic energy density
(c) electric energy density is equal to the magnetic energy density
(d) Both electric and magnetic energy densities are zero

Ans. (c)

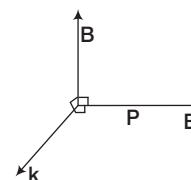
Both the energy densities are equal i.e., is equally divided between electric and magnetic field.

- 51** An electromagnetic wave in vacuum has the electric and magnetic fields E and B , which are always perpendicular to each other. The direction of polarisation is given by X and that of wave propagation by k . Then, [AIEEE 2012]

- (a) $X \parallel B$ and $k \parallel B \times E$
(b) $X \parallel E$ and $k \parallel E \times B$
(c) $X \parallel B$ and $k \parallel E \times B$
(d) $X \parallel E$ and $k \parallel B \times E$

Ans. (b)

In electromagnetic wave, the direction of propagation of wave, electric field and magnetic field are mutually perpendicular, i.e., wave propagates perpendicular to E and B or along $E \times B$. While polarisation of wave takes place parallel to electric field vector.



- 52** The rms value of the electric field of the light coming from the sun is 720 NC^{-1} . The average total energy density of the electromagnetic wave is [AIEEE 2006]

- (a) $4.58 \times 10^{-6} \text{ Jm}^{-3}$
 (b) $6.37 \times 10^{-9} \text{ Jm}^{-3}$
 (c) $81.35 \times 10^{-12} \text{ Jm}^{-3}$
 (d) $3.3 \times 10^{-3} \text{ Jm}^{-3}$

Ans. (a)

$$\begin{aligned} \text{Total average energy} &= \epsilon_0 E_{\text{rms}}^2 \\ &= 8.85 \times 10^{-12} \times (720)^2 \\ &= 4.58 \times 10^{-6} \text{ Jm}^{-3} \end{aligned}$$

- 53** An electromagnetic wave of frequency $\nu = 3.0 \text{ MHz}$ passes from vacuum into a dielectric medium with permittivity $\epsilon = 4.0$. Then, [AIEEE 2004]

- (a) wavelength is doubled and the frequency remains unchanged
 (b) wavelength is doubled and frequency becomes half
 (c) wavelength is halved and frequency remains unchanged
 (d) wavelength and frequency both remain unchanged

Ans. (c)

In vacuum, $\epsilon_0 = 1$

In medium, $\epsilon = 4$

$$\therefore \text{Refractive index, } \mu = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\frac{4}{1}} = 2$$

$$\text{Wavelength, } \lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$$

$$\text{and wave velocity, } v = \frac{c}{\mu} = \frac{c}{2} \quad \left[\because \mu = \frac{c}{v} \right]$$

Hence, it is clear that wavelength and velocity will become half but frequency remains unchanged when the wave is passing through any medium.

- 54** A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is [AIEEE 2004]

- (a) $\frac{E}{c}$ (b) $\frac{2E}{c}$
 (c) Ec (d) $\frac{E}{c^2}$

Ans. (b)

Initial momentum of surface, $p_i = E/c$ where, c = velocity of light (constant). Since, the surface is perfectly reflecting, so the same momentum will be reflected completely. Final momentum,

$$p_f = E/c \quad [\text{negative value}]$$

$\therefore$ Change in momentum,

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

- 55** Electromagnetic waves are transverse in nature is evident by [AIEEE 2002]

- (a) polarisation (b) interference
 (c) reflection (d) diffraction

Ans. (a)

Polarisation, using polariser and analyser, we get change in intensity of light at different rotations, hence we conclude that light waves are transverse in nature.

TOPIC 2 EM Spectrum

- 56** Match List-I with List-II.

List-I	List-II
A. Source of microwave frequency	1. Radioactive decay of nucleus
B. Source of infrared frequency	2. Magnetron
C. Source of gamma rays	3. Inner shell electrons
D. Source of X-rays	4. Vibration of atoms and molecules
	5. LASER
	6. R-C circuit

Choose the correct answer from the options given below.

[2021, 24 Feb Shift-II]

- A B C D
 (a) 6 4 1 5
 (b) 6 5 1 4
 (c) 2 4 6 3
 (d) 2 4 1 3

Ans. (d)

As we know that;

- A. Source of microwave frequency is magnetron.
 B. Source of infrared frequency is vibration of atoms and molecules.
 C. Source of gamma ray is radioactive decay of nucleus.

D. Source of X-ray is transition of electron in inner shells.

$\therefore$ The correct match is $A \rightarrow (2)$, $B \rightarrow (4)$, $C \rightarrow (1)$, $D \rightarrow (3)$.

- 57** Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum [2020, 4 Sep Shift-I]

- (a) $\lambda_{\text{X-rays}} < \lambda_{\text{microwaves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$
 (b) $\lambda_{\text{visible}} > \lambda_{\text{X-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{microwaves}}$
 (c) $\lambda_{\text{radio waves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X-rays}}$
 (d) $\lambda_{\text{visible}} < \lambda_{\text{microwaves}} < \lambda_{\text{radio waves}} < \lambda_{\text{X-rays}}$

Ans. (c)

$$\lambda_{\text{X-rays}} = 0.01 \text{ \AA} \text{ to } 100 \text{ \AA}$$

$$\lambda_{\text{visible}} = 3800 \text{ \AA} \text{ to } 7000 \text{ \AA}$$

$$\lambda_{\text{microwaves}} = 10^7 \text{ \AA} \text{ to } 3 \times 10^9 \text{ \AA}$$

$$\lambda_{\text{radio waves}} = 10^7 \text{ \AA} \text{ to } 10^{15} \text{ \AA}$$

So, it is clear from the values that

$$\lambda_{\text{radio waves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X-rays}}$$

So, correct option is (c).

- 58** The correct match between the entries in table are

[2020, 5 Sep Shift-II]

Column I (Radiation)	Column II (Wavelength)
(A) Microwaves	(I) 100 m
(B) γ -rays	(II) 10^{-15} m
(C) AM radiowaves	(III) 10^{-10} m
(D) X-rays	(IV) 10^{-3} m

- A B C D
 (a) II I IV III
 (b) I III IV II
 (c) III II I IV
 (d) IV II I III

Ans. (d)

Energy is inversely proportional to wavelength, i.e., $E = \frac{hc}{\lambda} \Rightarrow E \propto \frac{1}{\lambda}$

Energies of given radiations have the following order:

$$E_{\gamma\text{-rays}} > E_{\text{X-rays}} > E_{\text{microwaves}} > E_{\text{AM radiowaves}}$$

$$\therefore \lambda_{\gamma\text{-rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{microwaves}} < \lambda_{\text{AM radiowaves}}$$

Thus, microwaves $\rightarrow 10^{-3} \text{ m}$

$$\gamma\text{-rays} \rightarrow 10^{-15} \text{ m}$$

AM radiowaves $\rightarrow 100 \text{ m}$

X-rays $\rightarrow 10^{-10} \text{ m}$

$\therefore (A) \rightarrow (IV), (B) \rightarrow (II), (C) \rightarrow (I), (D) \rightarrow (III).$

Hence, correct option is (d).

- 59** Arrange the following electromagnetic radiations per quantum in the order of increasing energy.

A. Blue light B. Yellow light
C. X-ray (d) Radio wave

[JEE Main 2016]

- (a) D, B, A, C
(b) A, B, D, C
(c) C, A, B, D
(d) B, A, D, C

Ans. (a)

As, we know energy liberated, $E = \frac{hc}{\lambda}$

i.e. $E \propto \frac{1}{\lambda}$

So, lesser the wavelength, than greater will be energy liberated by electromagnetic radiations per quantum.

As, order of wavelength is given by

X-ray, VIBGYOR, Radio waves

(C) (A) (B) (D)

$\therefore$ Order of electromagnetic radiations per quantum.

$\Rightarrow D < B < A < C$

- 60** Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists.

[JEE Main 2014]

List I	List II
A. Infrared waves	1. To treat muscular strain
B. Radio waves	2. For broadcasting
C. X-rays	3. To detect fracture of bones
D. Ultraviolet	5. Absorbed by the ozone layer of the atmosphere

Codes

- | | | | | |
|-----|---|---|---|---|
| | A | B | C | D |
| (a) | 4 | 3 | 2 | 1 |
| (b) | 1 | 2 | 4 | 3 |
| (c) | 3 | 2 | 1 | 4 |
| (d) | 1 | 2 | 3 | 4 |

Ans. (d)

- (a) Infrared rays are used to treat muscular strain.
(b) Radiowaves are used for broadcasting purposes.
(c) X-rays are used to detect fracture of bones.
(d) Ultraviolet rays are absorbed by ozone.

- 61** Which of the following are not electromagnetic waves?

[AIEEE 2002]

- (a) cosmic rays (b) γ -rays
(c) β -rays (d) X-rays

Ans. (c)

Cosmic rays, γ -rays and X-rays are part of electromagnetic spectrum, while β -rays are emitted by radioactive elements. Hence, β -rays is not electromagnetic waves.

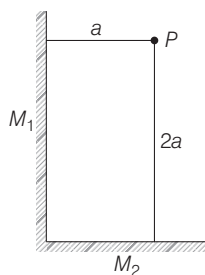
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Ray Optics and Optical Instruments

TOPIC 1

Reflection of Light

- 01** Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source P is placed at a and $2a$ meter away from M_1 and M_2 , respectively. The shortest distance between the images thus formed is (Take $\sqrt{5} = 2.3$) [2021, 31 Aug Shift-I]



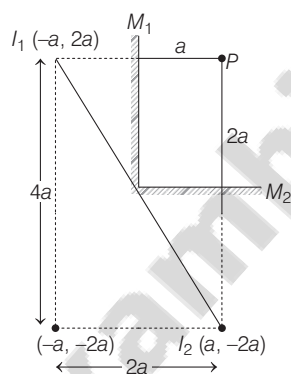
- (a) $3a$
 (b) $4.6a$
 (c) $2.3a$
 (d) $2\sqrt{10}a$

Ans. (b)

According to the given figure, we have two plane mirror, placed at 90° to each other.

Since, image formed by plane mirror are virtual, erect, same size and at same distance behind mirror.

Let I_1 is the image of P due to mirror M_1 and I_2 is the image of P due to mirror M_2



Therefore, the shortest distance between images i.e., between I_1 and I_2

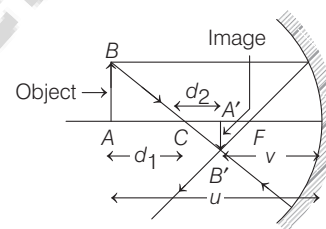
$$\begin{aligned} I_1 I_2 &= \sqrt{(4a)^2 + (2a)^2} \\ &= \sqrt{16a^2 + 4a^2} \\ &= a\sqrt{20} = a2\sqrt{5} \\ &= a \times 2 \times 2.3 = 4.6a \end{aligned}$$

- 02** An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C , the radius of curvature of this mirror is [2021, 27 Aug Shift-I]

- (a) $\frac{2d_1 d_2}{d_1 - d_2}$ (b) $\frac{2d_1 d_2}{d_1 + d_2}$
 (c) $\frac{d_1 d_2}{d_1 + d_2}$ (d) $\frac{d_1 d_2}{d_1 - d_2}$

Ans. (a)

The given situation is shown in the following ray diagram.



In concave mirror, when object is placed beyond C the image is formed between C and focus of mirror.

Consider radius of curvature of mirror is R .

Object distance, $u = -(R + d_1)$

Image distance, $v = -(R - d_2)$

Focal length of mirror, $f = \frac{-R}{2}$

Now, from mirror formula, we have

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{u+v}{uv}$$

$$\Rightarrow uv = f(u+v)$$

Substituting the values in above expression, we get

$$[-(R + d_1)] [-(R - d_2)] = -\frac{R}{2}(R + d_1 + R - d_2)$$

$$\begin{aligned} \Rightarrow 2(R^2 - d_2 R + d_1 R - d_1 d_2) \\ = 2R^2 + d_1 R - d_2 R = 2d_1 d_2 \end{aligned}$$

$$\Rightarrow d_1 R - d_2 R = 2d_1 d_2$$

$$\Rightarrow R = \frac{2d_1 d_2}{d_1 - d_2}$$

Thus, radius of curvature of mirror is

$$\frac{2d_1 d_2}{d_1 - d_2}$$

- 03** Car B overtakes another car A at a relative speed of 40 ms^{-1} . How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

[2021, 26 Aug Shift-I]

- (a) 4 ms^{-1} (b) 0.2 ms^{-1}
(c) 40 ms^{-1} (d) 0.1 ms^{-1}

Ans. (d)

According to the question,

Velocity of car B w.r.t. mirror of car A,

$$v_{BM} = 40 \text{ m/s}$$

Distance of car B from mirror of car A,

$$u = 1.9 \text{ m} = 190 \text{ cm}$$

Focal length of mirror, $f = 10 \text{ cm}$

We have to find the velocity of image of car B in the mirror, i.e. v_{IM}

We know that,

$$v_{IM} = -m^2 v_{BM} \quad \dots(i)$$

where, m is the magnification produced by mirror of car A,

$$\Rightarrow m = \frac{f}{f - u} = \frac{10}{10 - (-190)} \\ = \frac{10}{200} = \frac{1}{20}$$

Substituting the values in Eq. (i), we get

$$v_{IM} = -\left(\frac{1}{20}\right)^2 \times 40 \\ = -\frac{1}{400} \times 40 = -0.1 \text{ m/s}$$

Here, negative sign shows that, the speed of image w.r.t. mirror of car A is in opposite direction.

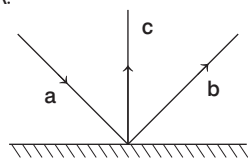
- 04** The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors **a**, **b** and **c**, respectively. Then, choose the correct relation for these vectors.

[2021, 26 Feb Shift-II]

- (a) $\mathbf{b} = \mathbf{a} + 2\mathbf{c}$ (b) $\mathbf{b} = 2\mathbf{a} + \mathbf{c}$
(c) $\mathbf{b} = \mathbf{a} - 2(\mathbf{a} \cdot \mathbf{c})\mathbf{c}$ (d) $\mathbf{b} = \mathbf{a} - \mathbf{c}$

Ans. (c)

Taking component of **a**, **b** and **c** along $\hat{i}$, $\hat{j}$ and $\hat{k}$.



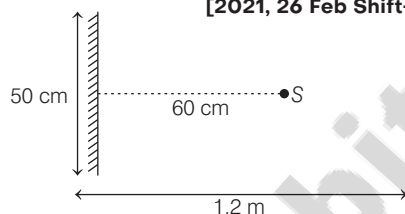
$$\therefore \mathbf{a} = \sin\theta \hat{i} - \cos\theta \hat{j} \\ \mathbf{b} = \sin\theta \hat{i} + \cos\theta \hat{j} \\ \mathbf{c} = \hat{j}$$

Now, on solving option (c), we get

$$\mathbf{a} - 2(\mathbf{a} \cdot \mathbf{c})\mathbf{c} = (\sin\theta \hat{i} - \cos\theta \hat{j}) \\ - 2[(\sin\theta \hat{i} - \cos\theta \hat{j}) \cdot \hat{j}]\hat{j} \\ = \sin\theta \hat{i} + \cos\theta \hat{j} = \mathbf{b}$$

- 05** A point source of light S, placed at a distance 60 cm in front of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points, where he can see the image of the light source in the mirror is cm.

[2021, 26 Feb Shift-II]



Ans. (150)

Given, length of mirror, $m = 50 \text{ cm}$

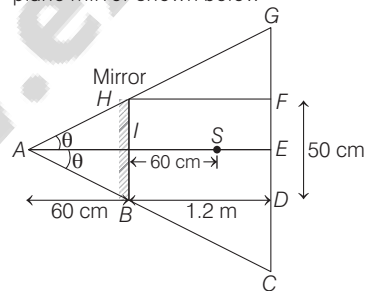
$$= 50 \times 10^{-2} \text{ m}$$

Distance of source from mirror, $d = 60 \text{ cm}$

$$= 60 \times 10^{-2} \text{ m}$$

Distance of man from mirror, $d_m = 1.2 \text{ m}$

By using the concept of ray diagram of plane mirror shown below



Now, using the concept of similar triangle,

$$\triangle HAI \sim \triangle GAE \text{ and } \triangle BAI \sim \triangle CAE$$

$$\therefore \frac{AI}{AE} = \frac{HI}{EG}$$

$$\Rightarrow \frac{0.60}{1.8} = \frac{0.25}{EG} \quad (\because AI = IS)$$

$$\Rightarrow EG = 0.25 \times \frac{1.8}{0.6} = 0.25 \times 3 = 0.75 \text{ m}$$

$$\text{As, } CG = 2EG$$

$$\Rightarrow CG = 0.75 \times 2 = 1.50 \text{ m}$$

Hence, distance between the extreme points, where he can see image of light source in mirror is 150 cm.

- 06** A short straight object of height 100 cm lies before the central axis of a spherical mirror, whose focal length has absolute value $f = 40 \text{ cm}$. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information.

[2021, 26 Feb Shift-I]

- (a) Image is real, same side of concave mirror
(b) Image is virtual, opposite side of concave mirror
(c) Image is real, same side of convex mirror
(d) Image is virtual, opposite side of convex mirror

Ans. (d)

Given, height of object, $h_o = 100 \text{ cm}$

Focal length of mirror, $f = 40 \text{ cm}$

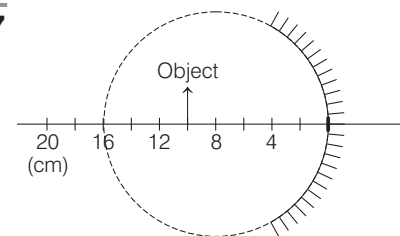
Height of image, $h_i = 25 \text{ cm}$

Nature of image is erect means virtual.

As, h_i is less than h_o , so mirror used is convex mirror.

Hence, (d) is correct option, i.e. image is virtual, opposite and mirror is convex.

07



A spherical mirror is obtained as shown in the above figure from a hollow glass sphere. If an object is in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)

[2020, 2 Sep Shift-I]

- (a) Erect, virtual and unmagnified
(b) Inverted, real and magnified
(c) Erect, virtual and magnified
(d) Inverted, real and unmagnified

Ans. (d)

Radius of curvature of given concave mirror $R = -8 \text{ cm}$

$$\text{So, focal length, } f = \frac{R}{2} = -\frac{8}{2} = -4 \text{ cm}$$

Object distance, $u = -10 \text{ cm}$

Using mirror formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{4} - \frac{1}{-10} = -\frac{3}{20}$$

$$\Rightarrow v = -6.67 \text{ cm (real)}$$

Also, magnification,

$$m = -\frac{v}{u} = -\frac{(-6.67)}{(-10)} = -0.667$$

As m is negative and less than one, so image is inverted and diminished or unmagnified. Hence, correct option is (d).

- 08** When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cms^{-1} , the speed (in cms^{-1}) with which image moves at that instant is

[2020, 3 Sep Shift-II]

Ans. (1)

Given, object distance, $u = -30 \text{ cm}$,

Image distance, $v = -10 \text{ cm}$

and speed of object, $v_o = 9 \text{ cms}^{-1}$

Let v_i be the speed of image.

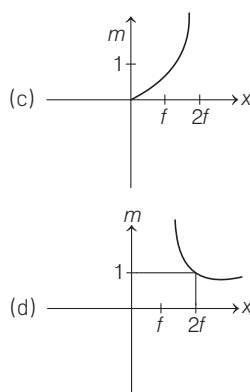
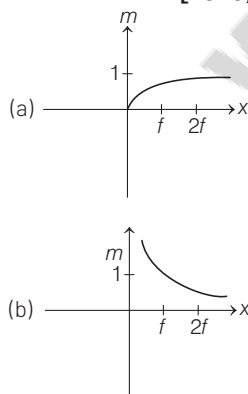
Magnification of concave mirror,

$$m = -\frac{v}{u} = \frac{-10}{30} = -\frac{1}{3}$$

$$\text{Now, } v_i = v_o(m^2) = 9 \times \left(\frac{-1}{3}\right)^2 = 1 \text{ cms}^{-1}$$

- 09** An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by (graphs are drawn schematically and are not to scale)

[2020, 8 Jan Shift-II]

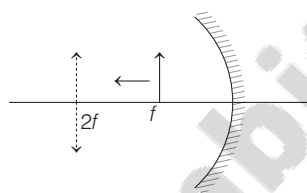


Ans. (d)

In case of a concave mirror, when object is at f , image is formed at infinity.

So, magnification when object is at f ,

$$m = \frac{-v}{u} = \infty$$



Also, if an object is at $2f$ from mirror, then the image is also formed at $2f$. So, magnification,

$$m = \frac{-v}{u} = \frac{-(-2f)}{(-2f)} = -1$$

or $|m| = 1$

So, these values are correctly depicted in option (d) only.

- 10** A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is

[2019, 9 April Shift-I]

- (a) 0.16 m (b) 1.60 m
(c) 0.32 m (d) 0.24 m

Ans. (c)

Given, focal length of concave mirror,

$$f = -0.4 \text{ m}$$

Magnification = 5

We know that, magnification produced by a mirror, $m = -\frac{\text{image distance}}{\text{object distance}}$

$$\Rightarrow \frac{v}{u} = -5 \text{ or } v = -5u$$

$$\text{Using mirror formula, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Substituting the given values in the above equation, we get

$$\frac{1}{-5u} + \frac{1}{u} = -\frac{1}{0.4} \Rightarrow \frac{4}{5u} = -\frac{1}{0.4}$$

$$\Rightarrow u = -\frac{1.6}{5} = -0.32 \text{ m}$$

Alternate Solution

Magnification produced by a mirror can also be given as

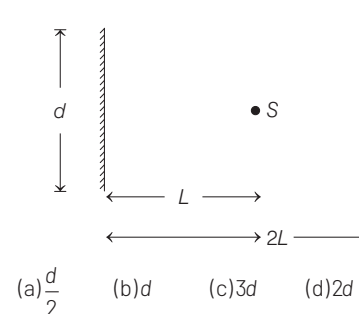
$$m = \frac{f}{f - u}$$

Substituting the given values, we get

$$5 = \frac{-0.4}{-0.4 - u} \text{ or } u = -0.32 \text{ m}$$

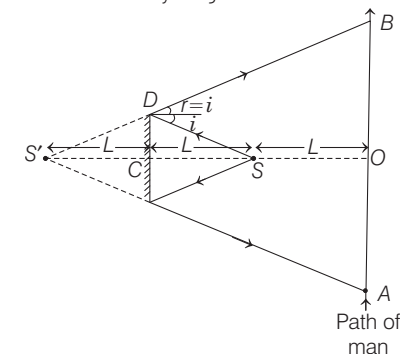
- 11** A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below.

The distance over which the man can see the image of the light source in the mirror is [2019, 12 Jan Shift-I]



Ans. (c)

Light from mirror is reflected in a straight line and it appears to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below.



From ray diagram in similar triangles

$\Delta S'CD$ and $\Delta S'OB$, we have $\frac{S'C}{S'O} = \frac{CD}{OB}$

$$\text{So, } OB = \frac{CD \times S'O}{S'C} = \frac{\frac{d}{2} \times 3L}{L} = \frac{3d}{2}$$

$$\text{Also, } OA = \frac{3d}{2}$$

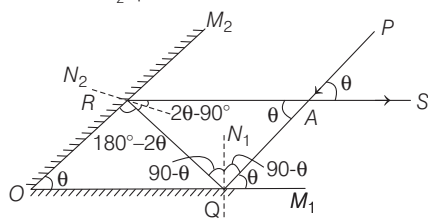
So, distance over which man can see the image S' is $\frac{3d}{2} + \frac{3d}{2} = 3d$

- 12** The plane mirrors (M_1 and M_2) are inclined to each other such that a ray of light incident on mirror M_1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1 . The angle between the two mirrors is **[2019, 9 Jan Shift-II]**

(a) 45° (b) 75° (c) 90° (d) 60°

Ans. (d)

The given condition is shown in the figure given below, where two plane mirrors inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from second mirror (M_2) parallel to the first mirror.



where, PQ = incident ray parallel to the mirror M_2 , QR = reflected ray from the mirror M_1 ,

RS = reflected ray from the mirror M_2 which is parallel to the M_1 and θ = angle between M_1 and M_2 .

According to geometry,

$$\angle PAS = \angle PQM_1 = \theta \text{ (angle on same line)}$$

$$\angle AQN_1 = \text{angle of incident} = 90^\circ - \theta$$

$$\angle N_1QR = \text{angle of reflection} = (90^\circ - \theta).$$

Therefore, for triangle ΔORQ , (according to geometry)

$$\angle \theta + \angle \theta + \angle ORQ = 180^\circ$$

$$\angle ORQ = 180^\circ - 2\theta \quad \dots(i)$$

For normal N_2 , angle of incident = angle of reflection

$$= 2\theta - 90^\circ \quad \dots(ii)$$

Therefore, for the triangle ΔRAQ

$$\Rightarrow 4\theta - 180^\circ + 180^\circ - 2\theta + \theta = 180^\circ$$

$$3\theta = 180^\circ$$

$$\theta = 60^\circ$$

- 13** A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE 2011]**

(a) $\frac{1}{15}$ m/s (b) 10 m/s
(c) 15 m/s (d) $\frac{1}{10}$ m/s

Ans. (a)

$$\text{For the mirror, } \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Differentiate this equation w.r.t. t , we get

$$-\frac{1}{u^2} \frac{du}{dt} - \frac{1}{v^2} \frac{dv}{dt} = 0$$

$$\Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2} \left(\frac{du}{dt} \right)$$

$$\text{But } \frac{v}{u} = \frac{f}{u-f}$$

$$\therefore \frac{dv}{dt} = -\left(\frac{f}{u-f} \right)^2 \left(\frac{du}{dt} \right) \\ = -\left(\frac{0.2}{-2.8-0.2} \right)^2 \times 15 = \frac{1}{15} \text{ ms}^{-1}$$

- 14** To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 60° (b) 90°
(c) 120° (d) 30°

Ans. (b)

$$\text{Number of images, } n = \frac{360^\circ}{\theta} - 1$$

where, θ is angle between mirrors.

$$\therefore 3 = \frac{360^\circ}{\theta} - 1$$

$$\text{or } \theta = 90^\circ$$

- 15** If two mirrors are kept at 60° to each other, then the number of images formed by them is

[AIEEE 2002]

(a) 5 (b) 6
(c) 7 (d) 8

Ans. (a)

$$\text{Number of images, } n = \frac{360^\circ}{\theta} - 1$$

where, θ is angle between mirrors.

$$\text{Thus, } \theta = 60^\circ \quad [\text{given}]$$

$$\text{So, number of images, } n = \frac{360^\circ}{60^\circ} - 1 = 5$$

TOPIC 2

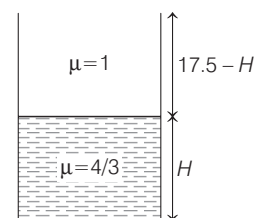
Refraction, TIR and Prism

- 16** A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water ($\mu = 4/3$) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled? **[2021, 1 Sep Shift-II]**

(a) 11.7 cm (b) 10 cm
(c) 7.5 cm (d) 8.75 cm

Ans. (b)

Let us draw the diagram of glass tumbler.



Consider the actual height of the tumbler be H .

The refractive index of the water, $\mu = 4/3$

The refractive index of the air, $\mu = 1$

As we know that,

$$\mu_{\text{water}} = \frac{H_{\text{real}}}{H_{\text{apparent}}}$$

$$\frac{4}{3} = \frac{H}{H_{\text{apparent}}}$$

$$\Rightarrow H_{\text{apparent}} = \frac{3H}{4}$$

The height of air observed by observer = $17.5 - H$

According to the above figure,

$$\frac{3H}{4} = 17.5 - H$$

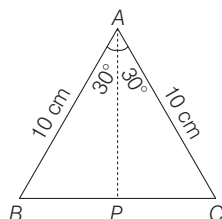
$$\Rightarrow H = 10.11 \text{ cm} \approx 10 \text{ cm}$$

The actual height of the water in tumbler is 10 cm.

- 17** Cross-section view of a prism is the equilateral triangle ABC in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (mid-point of BC) to A is $\dots \times 10^{-10}$ s.

(Given, speed of light in vacuum
 $= 3 \times 10^8$ m/s and $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

[2021, 31 Aug Shift-II]



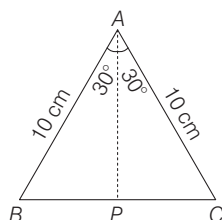
Ans. (5)

Given, the base of a prism is equilateral triangle of side 10 cm as shown in figure.

At minimum deviation,

Angle of incidence = Angle of prism

i.e. $i = A = 60^\circ$



Angle of refraction, $r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

Let μ be the refractive index.

Then, by Snell's law of refraction,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Now, in $\triangle ABP$, $AP = AB \cos 30^\circ$

$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

$$= 5\sqrt{3} \times 10^{-2} \text{ m}$$

i.e. optical distance travelled by light along AP, $d = \mu \times AP = \sqrt{3} \times 5\sqrt{3} \times 10^{-2}$
 $= 15 \times 10^{-2} \text{ m}$

Now, the time taken by light ($c = 3 \times 10^8$ m/s) to travel from P to A,

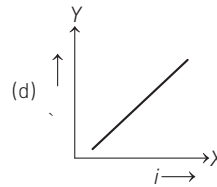
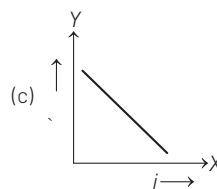
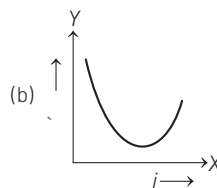
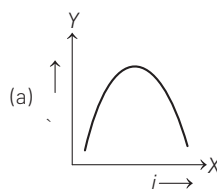
$$t = \frac{d}{c} = \frac{15 \times 10^{-2}}{3 \times 10^8} \text{ s}$$

$$= 5 \times 10^{-10} \text{ s}$$

Thus, correct answer is 5.

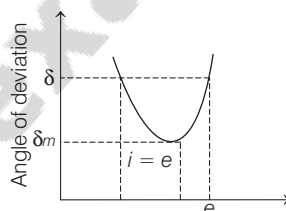
- 18** The expected graphical representation of the variation of angle of deviation δ with angle of incidence i in a prism is

[2021, 27 July Shift-II]



Ans. (b)

If the angle of incidence is increased gradually, then the angle of deviation first decreases, attains a minimum value (δ_m) and then again starts increasing.



Hence, the correct option is (b).

- 19** A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ , the wavelength of light, according to the relation

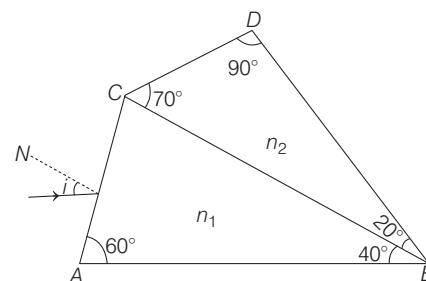
$$n_1 = 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2}$$

$$\text{and } n_2 = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$$

The wavelength for which rays incident at any angle on the interface BC pass through without

bending at that interface will be nm.

[2021, 27 July Shift-I]



Ans.

(600) $\because$ For no deviation, $n_1 = n_2$

$$\Rightarrow 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2} = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$$

$$\Rightarrow 0.25 = \frac{9 \times 10^{-14}}{\lambda^2}$$

$$\Rightarrow \lambda^2 = \frac{9 \times 10^{-14}}{0.25}$$

$$\Rightarrow \lambda = \frac{3}{5} \times 10^{-6}$$

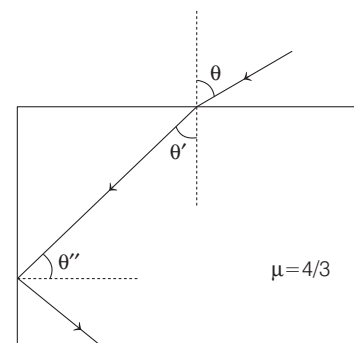
$$\Rightarrow \lambda = 6 \times 10^{-7} \text{ m}$$

$$\Rightarrow \lambda = 600 \text{ nm}$$

- 20** A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The

light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to

[2021, 25 July Shift-II]



(a) $\sin^{-1} \frac{\sqrt{7}}{3}$

(b) $\sin^{-1} \frac{\sqrt{5}}{4}$

(c) $\sin^{-1} \frac{\sqrt{7}}{4}$

(d) $\sin^{-1} \frac{\sqrt{5}}{3}$

Ans. (a)

Given, refractive index, $\mu = \frac{4}{3}$

By using Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin \theta}{\sin \theta'} \quad \dots(i)$$

From diagram,

$$\theta'' = 90^\circ - \theta'$$

For total internal reflection,

$$\frac{1}{\mu} = \sin \theta''$$

$$\Rightarrow \sin(90^\circ - \theta') = \frac{3}{4} \Rightarrow \cos \theta' = \frac{3}{4}$$

Since, $\sin \theta' = \sqrt{1 - \cos^2 \theta'}$

$$\Rightarrow \sin \theta' = \sqrt{1 - \left(\frac{3}{4}\right)^2} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Putting this value in Eq. (i), we get

$$\frac{4}{3} = \frac{\sin \theta \times 4}{\sqrt{7}}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{7}}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$$

The maximum value of θ is $\sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$.

- 21** A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option. [2021, 25 July Shift-I]

- (a) Angle of refraction is 24.41°
 (b) Angle of refraction is 30°
 (c) Refraction is not possible
 (d) Angle of refraction is 53.4°

Ans. (c)

Given, wavelength of laser light (λ) = 630 nm

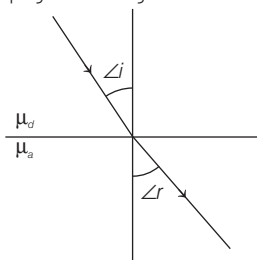
$$= 630 \times 10^{-9} \text{ m}$$

Angle of incidence (i) = 30°

Refractive index of diamond, $\mu_d = 2.42$

Refractive index of air, $\mu_r = 1$

The propagation of light can be shown as



By using Snell's law,

$$\frac{\mu_d}{\mu_r} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \sin r = \frac{\mu_d}{\mu_r} \sin i$$

$$= 2.42 \times \sin 30^\circ$$

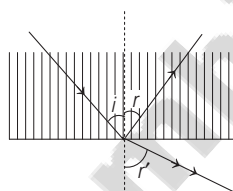
$$= 2.42 \times \frac{1}{2} = 1.21 > 1$$

But the maximum value of $\sin \theta$ is 1.

$\therefore$ In above case, refraction is not possible.

- 22** A ray of light passes from a denser medium to a rarer medium at an angle of incidence i . The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r' . The critical angle is given by

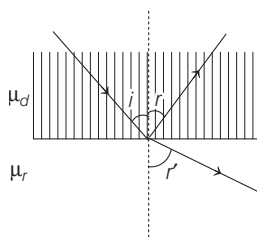
[2021, 22 July Shift-II]



- (a) $\sin^{-1}(\cot r)$
 (b) $\tan^{-1}(\sin i)$
 (c) $\sin^{-1}(\tan r')$
 (d) $\sin^{-1}(\tan r)$

Ans. (d)

If refractive index of denser medium and rarer medium be μ_d and μ_r respectively, then



By using Snell's law,

$$\mu_{rd} = \frac{\mu_r}{\mu_d} = \frac{\sin i}{\sin r'}$$

$$\Rightarrow \sin i = \frac{\mu_r}{\mu_d} \sin r' \quad \dots(i)$$

According to given situation,

$$r + r' = 90^\circ$$

$$\Rightarrow r' = 90^\circ - r \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\sin i = \frac{\mu_r}{\mu_d} \sin(90^\circ - r)$$

$$= \frac{\mu_r}{\mu_d} \cos r$$

By using law of reflection $\angle i = \angle r$, therefore,

$$\frac{\mu_r}{\mu_d} = \frac{\sin r}{\cos r} = \tan r \quad \dots(ii)$$

For critical angle, $\frac{\mu_r}{\mu_d} = \frac{\sin C}{\sin 90^\circ}$

$$\therefore \sin 90^\circ = 1$$

$$\therefore \frac{\mu_r}{\mu_d} = \sin C$$

Put in Eq. (ii), we get

$$\Rightarrow \sin C = \tan r$$

$$\Rightarrow C = \sin^{-1}(\tan r)$$

- 23** A ray of light passing through a prism ($\mu = \sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is (in degrees). [2021, 22 July Shift-II]

Ans. (60)

Given, let refractive index of prism, $\mu = \sqrt{3}$

In the case of minimum deviation,

$$i = 2r = A \quad \dots(i)$$

As we know that,

$$i + e = A + \delta_m \quad [\because i = e]$$

$$2i = A + \delta_m$$

$$2A = A + \delta_m$$

$$\Rightarrow A = \delta_m$$

By using prism formula,

$$\mu = \frac{\sin\left(A + \frac{\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\sin\left(A + \frac{A}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}}$$

$$\Rightarrow \sqrt{3} \sin\frac{A}{2} = \sin A = \sin 2\frac{A}{2}$$

$$= 2 \sin\frac{A}{2} \cos\frac{A}{2}$$

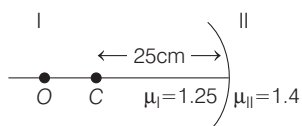
$$\Rightarrow \frac{\sqrt{3}}{2} = \cos\frac{A}{2}$$

$$\Rightarrow \cos 30^\circ = \cos\frac{A}{2}$$

$$\therefore \frac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$$

- 24** Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is

[2021, 20 July Shift-I]



- (a) 55.44 cm (b) 9.52 cm
(c) 18.23 cm (d) 37.58 cm

Ans. (d)

As we know that, the equation of refraction at spherical surface is

$$\frac{\mu_{II}}{v} - \frac{\mu_I}{u} = \frac{\mu_{II} - \mu_I}{R}$$

where, μ_{II} = refractive index of region II = 1.4

μ_I = refractive index of region I = 1.25

R = radius of curvature = -25 cm

u = object distance = -40 cm

and v = image distance.

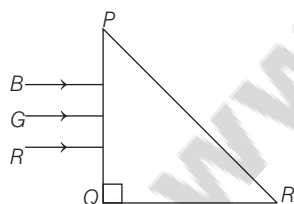
$$\Rightarrow \frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{-25}$$

$$\Rightarrow \frac{1.4}{v} = \frac{-0.15}{25} - \frac{1.25}{40}$$

$$\Rightarrow v = -37.58 \text{ cm}$$

where, negative sign indicate real image.

- 25** Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure



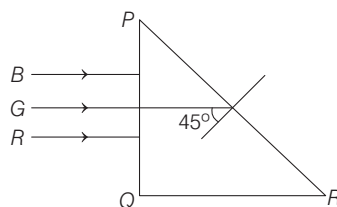
The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49, respectively. The colour of the ray(s) emerging out of the face PR is

[2021, 18 March Shift-II]

- (a) green (b) red
(c) blue and green (d) blue

Ans. (b)

From the given figure,



We know that,

$$\mu = \frac{1}{\sin i_c}$$

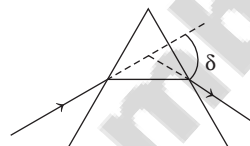
Here, i_c is the critical angle of incidence and μ is the refractive index.

$$\mu = \frac{1}{\sin 45^\circ} \Rightarrow \mu = 1.414$$

The rays will emerge out when angle of incidence is less than the angle of critical angle of glass-air interface PR.

As $\mu_R < \mu$ while μ_G and $\mu_B > \mu$, so only red colour will be transmitted through face PR while green and blue rays will suffer total internal reflection.

- 26** The angle of deviation through a prism is minimum when



- A. incident ray and emergent ray are symmetric to the prism
B. the refracted ray inside the prism becomes parallel to its base
C. angle of incidence is equal to that of the angle of emergence
D. angle of emergence is double the angle of incidence

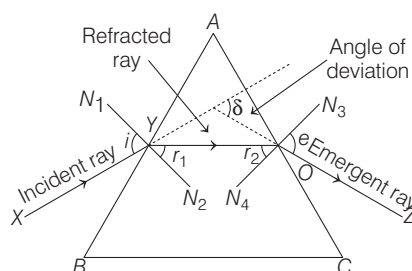
Choose the correct answer from the options given below.

[2021, 16 March Shift-I]

- (a) Statements (A), (B) and (C) are true.
(b) Only statement (D) is true.
(c) Only statements (A) and (B) are true.
(d) Statements (B) and (C) are true.

Ans. (a)

The propagation of light ray from a prism is shown below



From the above figure, we can say that minimum value of angle of deviation can only be achieved when

- A. incident ray and emergent ray are symmetric to the prism.
B. the refracted ray inside the prism becomes parallel to its base.
C. angle of incidence is equal to angle of emergence.

$\therefore$ Statement (A), (B) and (C) are true.

Note Refracted ray inside the prism is parallel to the base only for equilateral and isosceles prism.

- 27** The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of x is

[2021, 17 March Shift-II]

Ans. (30)

Given,

Image distance, $v = 10$ m

Object distance, $u = \left(\frac{3}{2}\right) \times 10 = 15$ m

and $n_2 = \frac{3}{2} n_1$ ($\because n \propto \frac{1}{\lambda}$)

Using the lens Maker's formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \left(\frac{n_2 - n_1}{R} \right)$$

Substituting the values in the above equation, we get

$$\frac{\frac{3}{2} n_1}{10} - \frac{n_1}{-15} = \left(\frac{\frac{3}{2} n_1 - n_1}{R} \right) \Rightarrow R = \frac{30}{13} \text{ m}$$

The radius of the curved surface is $30/13$ m. Comparing with $x/13$, we get

$$x = 30$$

- 28** Red light differs from blue light as they have [2021, 16 March Shift-II]

- (a) different frequencies and different wavelengths
(b) different frequencies and same wavelengths

- (c) same frequencies and same wavelengths
 (d) same frequencies and different wavelengths

Ans. (a)

Since, $\lambda v = c = \text{constant}$

where, λ = wavelength of light

and v = frequency of light.

Red light and blue light have different wavelengths and different frequencies but same speed.

- 29** A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6, respectively. The refracting angles for crown glass prism will be $^\circ$ (in degree). (Round off to the nearest integer)

[2021, 16 March Shift-II]

Ans. (12)

Given,

Dispersive power of crown glass,

$$\omega_1 = 0.02$$

Dispersive power of flint glass, $\omega_2 = 0.03$

Refractive index of yellow light,

for crown glass, $\mu_1 = 1.5$ and

for flint glass, $\mu_2 = 1.6$

This is a case of achromatic combination.

$$\therefore \theta_{\text{net}} = 0$$

$$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$$

$$\Rightarrow \omega_1 \delta_1 = \omega_2 \delta_2 \quad [\because \theta = \omega \delta] \quad \dots (i)$$

$$\text{and} \quad \delta_{\text{net}} = \delta_1 - \delta_2 = 2^\circ \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\delta_1 - \frac{\omega_1 \delta_1}{\omega_2} = 2^\circ$$

$$\Rightarrow \delta_1 \left(1 - \frac{\omega_1}{\omega_2} \right) = 2^\circ \Rightarrow \delta_1 \left(1 - \frac{0.02}{0.03} \right) = 2^\circ$$

$$\Rightarrow \delta_1 = 6^\circ$$

$$\text{Also, } \delta_1 = (\mu_1 - 1) A_1$$

$$\Rightarrow 6^\circ = (1.5 - 1) A_1$$

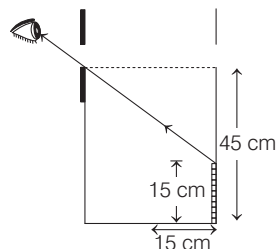
$$\Rightarrow A_1 = 12^\circ$$

$\therefore$ The refracting angle for crown glass prism will be 12° .

- 30** An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure).

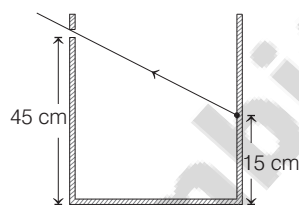
The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm, the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where N is an integer, the value of N is

[2020, 3 Sep Shift-I]

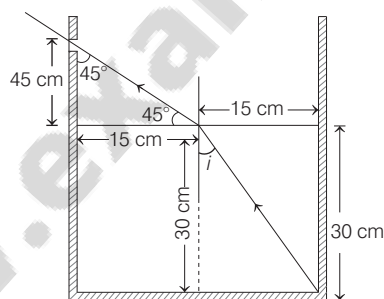


Ans. (158)

Initially, when the jar is empty.



Finally, when the jar is filled with liquid.



If i be the angle of incidence, then

$$\tan i = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \sin i = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{5}}$$

According to law of refraction,

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow \mu \left(\frac{1}{\sqrt{5}} \right) = 1 \times \sin 45^\circ$$

$$\Rightarrow \mu = \frac{\sqrt{5}}{\sqrt{2}} = 1.581$$

Also, given that,

$$\frac{N}{100} = \mu$$

$$\Rightarrow N = 100\mu = 100 \times 1.581 = 158.1$$

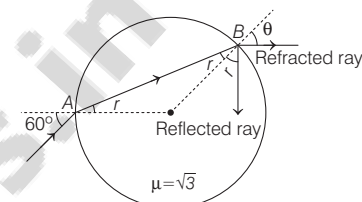
So, nearest integer value of N is 158.

- 31** A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60° . The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degree) between the reflected and refracted rays at this surface is

[2020, 2 Sep Shift-II]

Ans. (90)

We are given following situation as shown in figure,



Using Snell's law at point A,

$$n_1 \sin i = n_2 \sin r$$

$$1 \times \sin 60^\circ = \sqrt{3} \times \sin r$$

$$\Rightarrow \sin r = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2} \Rightarrow r = 30^\circ$$

Now, at point B, angle of incidence $= r = 30^\circ$

Again, using Snell's law at point B,

$$\sqrt{3} \sin 30^\circ = 1 \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^\circ$$

And, angle of reflection, $r = 30^\circ$

$\therefore$ Angle between reflected ray and refracted ray at point B $= 180^\circ - \theta - r$
 $= 180^\circ - 60^\circ - 30^\circ = 90^\circ$

- 32** A prism of angle $A = 1^\circ$ has a refractive index $\mu = 1.5$. A good estimate for the minimum angle of deviation (in degree) is close to $N/10$. Value of N is

[2020, 5 Sep Shift-II]

Ans. (5)

Deviation for small-angled prism is given by

$$\delta = (\mu - 1) A$$

Given, $A = 1^\circ$, $\mu = 1.5$

Substituting these values in above equation, we get

$$\delta = (1.5 - 1) 1 = 0.5$$

According to question, $\delta = \frac{N}{10}$

$$\Rightarrow 0.5 = \frac{N}{10} \Rightarrow N = 5$$

Hence, the value of N is 5.

- 33** The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be

[2020, 8 Jan Shift-I]

- (a) 45° (b) 60° (c) 15° (d) 30°

Ans. (d)

Speed of light in a medium is given by

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon_0\mu_0}} \times \frac{1}{\sqrt{\epsilon_r\mu_r}}$$

$$= \frac{c}{\sqrt{\epsilon_r\mu_r}}$$

Here, $\epsilon_r = 3$ and $\mu_r = \frac{4}{3}$

$$\therefore \frac{v}{c} = \frac{1}{\sqrt{\epsilon_r\mu_r}} = \frac{1}{\sqrt{3 \times \frac{4}{3}}} = \frac{1}{2}$$

So, refractive index of medium w.r.t. vacuum or air,

$$n = \frac{c}{v} = 2$$

Hence, critical angle,

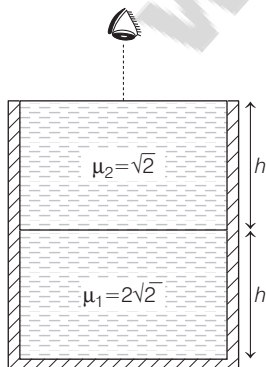
$$i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

$$= \sin^{-1}\frac{1}{2} = 30^\circ$$

- 34** A vessel of depth $2h$ is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be

[2020, 9 Jan Shift-I]

- (a) $\frac{h}{\sqrt{2}}$ (b) $\frac{3}{4}h\sqrt{2}$
(c) $\frac{h}{3\sqrt{2}}$ (d) $\frac{h}{2(\sqrt{2}+1)}$

Ans. (b)

When a vessel contains immiscible liquids, then apparent depth will be

$$d_{\text{apparent}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \dots$$

In given case,

$$d_{\text{apparent}} = \frac{h}{\mu_1} + \frac{h}{\mu_2}$$

$$= \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}$$

$$\text{or } d_{\text{apparent}} = \frac{3}{4}h\sqrt{2}$$

- 35** There is a small source of light at some depth below the surface of water (refractive index $= \frac{4}{3}$) in a

tank of large cross-sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly)

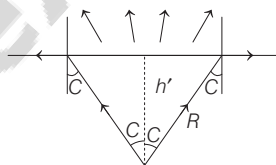
[Use the fact that surface area of a spherical cap of height h and radius of curvature R is $2\pi Rh$]

[2020, 9 Jan Shift-II]

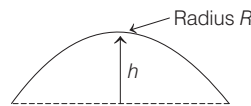
- (a) 34% (b) 50% (c) 17% (d) 21%

Ans. (c)

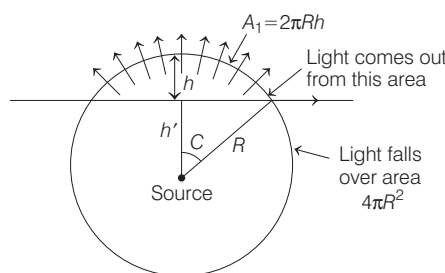
Due to total internal reflection, light comes out from a cone of angle $2C$ as shown in the figure (C = angle of critical incidence).



Now, it is given that surface area of spherical cap of height h and radius R is $2\pi Rh$.



Here, we are given with following situation



So, percentage of light that emerges out of surface is given by,

$$\% \text{ of light coming out} = \frac{\text{Area through which light comes out}}{\text{Total area over which light falls}} \times 100$$

$$= \frac{2\pi Rh}{4\pi R^2} \times 100 = \frac{1}{2} \times \left(\frac{R-h'}{R}\right) \times 100$$

$$\text{Now, } \frac{h'}{R} = \frac{\text{base}}{\text{hypotenuse}} = \cos C$$

$$\text{As, } \sin C = \frac{1}{n} = \frac{1}{\frac{4}{3}} = \frac{3}{4} \Rightarrow \cos C = \frac{\sqrt{7}}{4} = \frac{h'}{R}$$

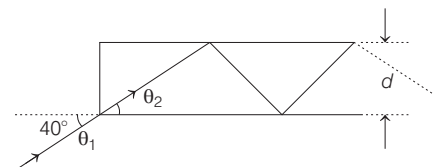
$\therefore$ % of light coming out

$$= \frac{1}{2} \times \left(\frac{R-h'}{R}\right) \times 100 = \frac{1}{2} \times \left(1 - \frac{h'}{R}\right) \times 100$$

$$= \frac{1}{2} \times \frac{\sqrt{7}}{4} \times 100 = 16.9\% \approx 17\%$$

Nearest answer is 17 %.

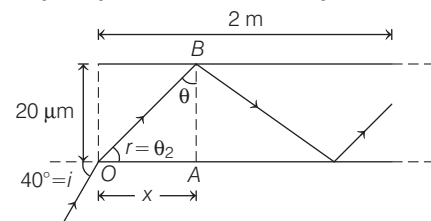
- 36** In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20\mu\text{m}$. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40^\circ$, the number of reflections it makes before emerging from the other end is close to (refractive index of fibre is 1.31 and $\sin 40^\circ = 0.64$) [2019, 8 April Shift-I]



- (a) 55000 (b) 66000
(c) 45000 (d) 57000

Ans. (d)

Total internal reflection occurs through given glass rod as shown in figure.



From Snell's law, $n_1 \sin i = n_2 \sin r$

where, $n_1 = 1$, $n_2 = 1.31$ and $i = 40^\circ$

So, we get

$$1 \sin 40^\circ = 1.31 \sin r$$

$$\Rightarrow \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$$

So, $r = 30^\circ$

From $\triangle OAB$, $\theta = 90^\circ - r = 60^\circ$

$$\text{Now, } \tan \theta = \frac{x}{20 \mu\text{m}}$$

$$\Rightarrow x = 20\sqrt{3} \mu\text{m} \quad [\because \tan 60^\circ = \sqrt{3}]$$

One reflection occurs in $20\sqrt{3} \mu\text{m}$.

$\therefore$ Total number of reflections occurring in 2 m

$$= n = \frac{2\text{m}}{20\sqrt{3} \mu\text{m}} = \frac{2}{20\sqrt{3} \times 10^{-6}}$$

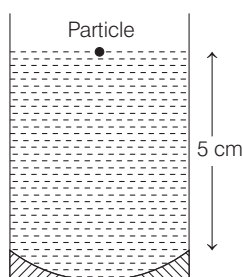
$$= 57735 \text{ reflections}$$

$$\approx 57000 \text{ reflections}$$

- 37** A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to

[Refractive index of water = 1.33]

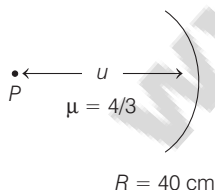
[2019, 12 April Shift-I]



- (a) 6.7 cm (b) 13.4 cm
(c) 8.8 cm (d) 11.7 cm

Ans. (c)

In the given case, $u = -5 \text{ cm}$



$$\text{Focal length, } f = \frac{-R}{2} = \frac{-40}{2} = -20 \text{ cm}$$

Now, using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{5} = +\frac{3}{20}$$

$$\Rightarrow v = +\frac{20}{3} \text{ cm}$$

For the light getting refracted at water surface, this image will act as an object.

So, distance of object,

$$d = 5 \text{ cm} + \frac{20}{3} \text{ cm} = \frac{35}{3} \text{ cm}$$

(below the surface). Let's assume final image at distance d' after refraction.

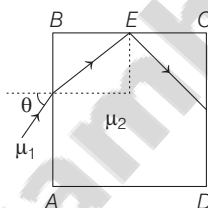
$$\frac{d'}{d} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow d' = d \left(\frac{\mu_2}{\mu_1} \right) = \left(\frac{35}{3} \text{ cm} \right) \left(\frac{1}{\frac{4}{3}} \right)$$

$$= \frac{35}{3} \times \frac{3}{4} \text{ cm} = \frac{35}{4} \text{ cm}$$

$$= 8.75 \text{ cm} \approx 8.8 \text{ cm}$$

- 38** A transparent cube of side d , made of a material of refractive index μ_2 , is immersed in a liquid of refractive index μ_1 ($\mu_1 < \mu_2$). A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC . [2019, 12 April Shift-II]



Then, θ must satisfy

$$(a) \theta < \sin^{-1} \frac{\mu_1}{\mu_2} \quad (b) \theta > \sin^{-1} \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$$

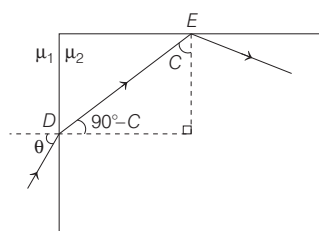
$$(c) \theta < \sin^{-1} \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1} \quad (d) \theta > \sin^{-1} \frac{\mu_1}{\mu_2}$$

Ans. (c)

Key Idea The critical angle is defined as the angle of incidence that provides an angle of refraction of 90° .

$$\text{So, } \theta_c = \sin^{-1} \frac{\mu_2}{\mu_1}$$

For total internal reflection, angle of incidence (i) at medium interface must be greater than critical angle (C).



$$\text{where, } \sin C = \frac{\mu_1}{\mu_2} \quad \dots(i)$$

Now, in given arrangement, at point D ,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad (\text{Snell's law})$$

$$\Rightarrow \frac{\sin \theta}{\sin(90^\circ - C)} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\sin \theta}{\cos C} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \sin \theta = \frac{\mu_2}{\mu_1} \cdot \cos C = \frac{\mu_2}{\mu_1} \sqrt{1 - \sin^2 C}$$

[from Eq. (i)]

$$= \frac{\mu_2}{\mu_1} \sqrt{1 - \frac{\mu_1^2}{\mu_2^2}} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$$

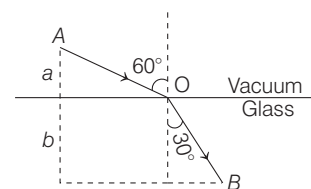
$$\Rightarrow \theta = \sin^{-1} \sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1 \right)}$$

For TIR at E , $i > C$

$$\Rightarrow \theta < \sin^{-1} \sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1 \right)}$$

- 39** A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure.

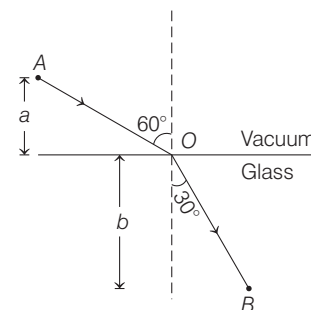
The optical path length of light ray from A to B is [2019, 10 April Shift-I]



- (a) $\frac{2\sqrt{3}}{a} + 2b$ (b) $2a + \frac{2b}{3}$
(c) $2a + 2b$ (d) $2a + \frac{2b}{\sqrt{3}}$

Ans. (c)

From the figure,



$$\cos 60^\circ = \frac{a}{AO}$$

$$\Rightarrow AO = \frac{a}{\cos 60^\circ} = 2a \quad \dots(i)$$

$$\text{and } \cos 30^\circ = \frac{b}{BO}$$

$$\text{or } BO = \frac{b}{\cos 30^\circ} = \frac{2}{\sqrt{3}} b \quad \dots (ii)$$

$$\text{Optical path length of light ray} \\ = AO + \mu (BO) \quad \dots (iii)$$

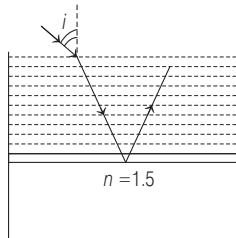
Here, μ can be determined using Snell's law, i.e.

$$\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \dots (iv)$$

Substituting the values from Eqs. (i), (ii) and (iv) in Eq. (iii), we get

$$\therefore \text{Optical path} = 2a + (\sqrt{3} \times \frac{2}{\sqrt{3}}b) \\ = 2a + 2b$$

- 40** Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index μ . A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.



For this to happen, the minimum value of μ is [2019, 9 Jan Shift-I]

- (a) $\frac{3}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) $\sqrt{\frac{5}{3}}$

Ans. (a)

Key Idea When a beam of unpolarised light is reflected from a transparent medium of refractive index μ , then the reflected light is completely plane polarised at a certain angle of incidence i_B , which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised. So, $i_C > i_B$ where, i_C is the critical angle.

$$\Rightarrow \sin i_C < \sin i_B \quad \dots (i)$$

From Brewster's law, we know that

$$\tan i_B = \mu_g = \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{1.5}{\mu} \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

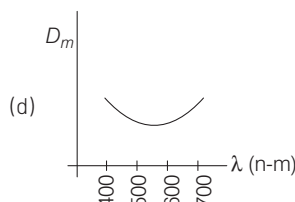
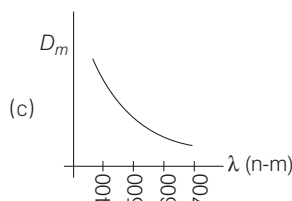
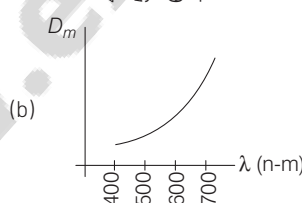
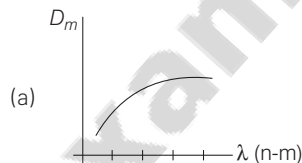
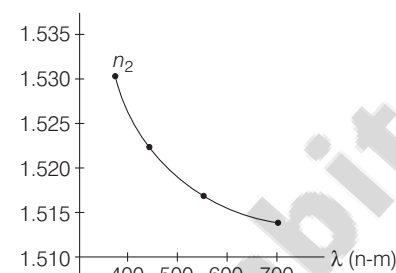
$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$$\Rightarrow \sqrt{(1.5)^2 + \mu^2} < 1.5 \mu$$

$$\mu^2 + (1.5)^2 < (1.5\mu)^2 \text{ or } \mu < \frac{3}{\sqrt{5}}$$

$\therefore$ The minimum value of μ should be $\frac{3}{\sqrt{5}}$.

- 41** The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation ? [2019, 11 Jan Shift-I]



Ans. (c)

For a crown glass thin prism i.e., prism with small angle, the angle of minimum deviation is given as,

$$D_m = (n - 1) A$$

where, A is prism angle and n is refractive index.

$$\Rightarrow D_m \propto n \quad \dots (i)$$

Since from the given graph, the value of n decrease with the increase in λ . Thus, from relation (i), we can say that, D_m will also decrease with the increase in λ .

$\therefore$ Hence, option (c) is correct.

- 42** A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is [2019, 11 Jan Shift-II]
(a) 45° (b) 90° (c) 60° (d) 30°

Ans. (c)

Given, refractive index of material of prism $n = \sqrt{3}$, prism angle $A = 60^\circ$

Method 1

$$\text{Using prism formula, } n = \frac{\sin\left(\frac{A + \delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\sin\left(\frac{60 + \delta}{2}\right)}{\sin 30^\circ}$$

$$\Rightarrow \sin\left(\frac{60 + \delta}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\left(\frac{60 + \delta}{2}\right) = \sin 60^\circ$$

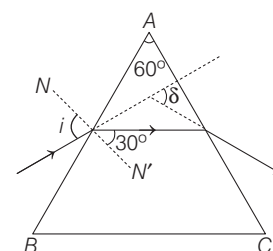
$$\text{or } \frac{60 + \delta}{2} = 60$$

or angle of minimum deviation $\delta = 60^\circ$

$$\text{Incident angle, } i = \frac{60 + \delta}{2} = 60^\circ$$

Method 2 For minimum deviation, ray should pass symmetrically (i.e. parallel to the base of the equilateral prism)

$\Rightarrow$ From geometry of given figure, we have,
 $r = 30^\circ$



Using Snell's law, $n = \frac{\sin i}{\sin r}$

$$\sin i = n \sin r = \sqrt{3} \sin 30^\circ$$

$$\Rightarrow \sin i = \frac{\sqrt{3}}{2} \text{ or } i = 60^\circ$$

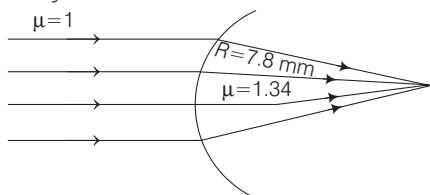
- 43** The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

[2019, 10 Jan Shift-II]

- (a) 4.0 cm (b) 2 cm
(c) 3.1 cm (d) 1 cm

Ans. (c)

The given condition is shown in the figure below



where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \quad \dots (i)$$

Given, $\mu_r = 1.34$, $\mu_i = 1$, $u = \infty$ (ve) and $R = 7.8$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8} \text{ or } \frac{1.34}{v} = \frac{0.34}{7.8}$$

$$\Rightarrow v = \frac{1.34 \times 7.8}{0.34} \text{ mm}$$

$$\Rightarrow v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$$\Rightarrow v = 31.2 \text{ mm or } 3.12 \text{ cm}$$

- 44** In an experiment for determination of refractive index of glass of a prism by i - δ plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79° . In that

case, which of the following is closest to the maximum possible value of the refractive index?

[JEE Main 2016]

- (a) 1.5 (b) 1.6
(c) 1.7 (d) 1.8

Ans. (a)

Key Idea If μ is refractive index of material of prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin(A + \delta_m)/2}{\sin A/2} \quad \dots (i)$$

where, A is angle of prism and δ_m is minimum deviation through prism.

Given, $i = 35^\circ$, $\delta = 40^\circ$, $e = 79^\circ$.

So, angle of deviation by a glass prism,

$$\delta = i + e - A \Rightarrow 40^\circ = 35^\circ + 79^\circ - A$$

i.e. Angle of prism $\Rightarrow A = 74^\circ$.

Such that, $r_1 + r_2 = A = 74^\circ$.

Let us put $\mu = 1.5$ in eq. (i), we get

$$1.5 = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin A/2}$$

$$\Rightarrow 1.5 = \frac{\sin\left(\frac{74^\circ + \delta_{\min}}{2}\right)}{\sin 37^\circ}$$

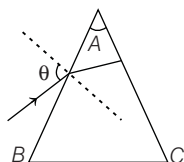
$$\Rightarrow 0.9 = \sin\left(37^\circ + \frac{\delta_{\min}}{2}\right) \quad (\because \sin 37^\circ \approx 0.6)$$

$$\sin 64^\circ = \sin\left(37^\circ + \frac{\delta_{\min}}{2}\right) \quad (\because \sin 64^\circ = 0.9)$$

$$37^\circ + \frac{\delta_{\min}}{2} = 64^\circ \Rightarrow \delta_{\min} \approx 54^\circ$$

This angle is greater than the 40° deviation angle already given. For greater μ , deviation will be even higher. Hence, μ of the given prism should be lesser than 1.5. Hence, the closest option will be 1.5.

- 45** Monochromatic light is incident on a glass prism of angle A . If the refractive index of the material of the prism is μ , a ray incident at an angle θ , on the face AB would get transmitted through the face AC of the prism provided [JEE Main 2015]



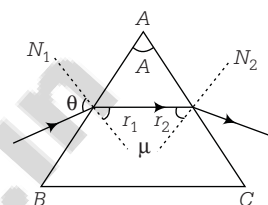
- (a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(b) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

- (c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Ans. (a)

Key Idea The ray will get transmitted through face AC if $i_{AC} < i_c$

Consider the ray diagram as shown below.



A ray of light incident on face AB at an angle θ .

r_1 = Angle of refraction on face AB

r_2 = Angle of incidence at face AC

For transmission of light through face AC

$$i_{AC} < i_c \text{ or } A - r_1 < i_c$$

$$\text{or } \sin(A - r_1) < \sin i_c \text{ or } \sin(A - r_1) < \frac{1}{\mu}$$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\text{or } \sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Now, applying Snell's law at the face AB

$$1 \times \sin \theta = \mu \sin r_1 \text{ or } \sin r_1 = \frac{\sin \theta}{\mu}$$

$$\Rightarrow \frac{\sin \theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\text{or } \theta > \sin^{-1}\left[\mu \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right]$$

- 46** A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement. [JEE Main 2014]

- (a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water at various angles to the normal

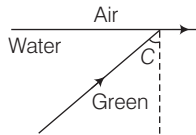
Key Idea For total internal reflection of light take place, then following conditions must be obeyed.

- The ray must travel from denser to rarer medium.
- Angle of incidence (θ) must be greater than or equal to critical angle (C) i.e., $C = \sin^{-1} \left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} \right]$

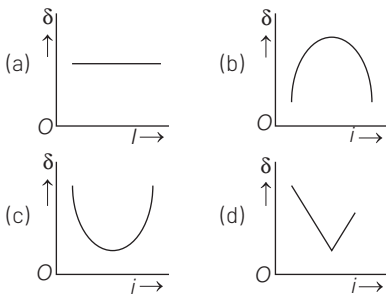
Ans. (b)

Here, $\sin C = \frac{1}{n_{\text{water}}}$ and $n_{\text{water}} = a + \frac{b}{\lambda^2}$

If frequency is less, then λ is greater and hence RI $n_{\text{(water)}}$ is less and therefore critical angle increases. So, they do not suffer reflection and come out at angle less than 90° of the air medium.

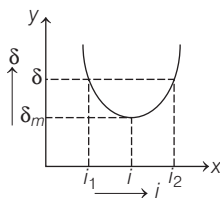


- 47** The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by [JEE Main 2013]



Ans. (c)

We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on x-axis) and δ (on y-axis), we get a curve as shown in figure.



Clearly, if angle of incidence is gradually increased from a small value, the angle of deviation first decreases, becomes

minimum for a particular angle of incidence and then begins to increase.

- 48** A beaker contains water upto a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2 . The apparent shift in the position of the bottom of the beaker when viewed from above is [AIEEE 2011]

- $\left(1 - \frac{1}{\mu_1}\right)h_2 + \left(1 - \frac{1}{\mu_2}\right)h_1$
- $\left(1 + \frac{1}{\mu_1}\right)h_1 + \left(1 + \frac{1}{\mu_2}\right)h_2$
- $\left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$
- $\left(1 + \frac{1}{\mu_1}\right)h_2 - \left(1 + \frac{1}{\mu_2}\right)h_1$

Ans. (c)

We know, apparent shift, $\Delta h = \left(1 - \frac{1}{\mu}\right)h$

$\therefore$ Apparent shift produced by water,

$$\Delta h_1 = \left(1 - \frac{1}{\mu_1}\right)h_1$$

and apparent shift produced by kerosene,

$$\Delta h_2 = \left(1 - \frac{1}{\mu_2}\right)h_2$$

$$\therefore \Delta h = \Delta h_1 + \Delta h_2 = \left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$$

- 49** Let the zx -plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is [AIEEE 2011]

- 45°
- 60°
- 75°
- 30°

Ans. (a)

As refractive index for $z > 0$ and $z \leq 0$ is different, xy -plane should be boundary between two media.

Angle of incidence,

$$\cos i = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \frac{1}{2}$$

$$\therefore i = 60^\circ$$

$$\text{From Snell's law, } \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\sin r = \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60^\circ = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{2} = 45^\circ$$

$$\Rightarrow r = 45^\circ$$

- 50** As the beam enters the medium, it will [AIEEE 2010]

- diverge
- converge
- diverge near the axis and converge near the periphery
- travel as a cylindrical beam

Ans. (b)

As intensity is maximum at axis. Therefore, μ will be maximum and speed will be minimum on the axis of the beam. Hence, beam will converge.

- 51** The speed of light in the medium is [AIEEE 2010]

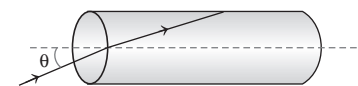
- minimum on the axis of the beam
- the same everywhere in the beam
- directly proportional to the intensity I
- maximum on the axis of the beam

Ans. (c)

The speed of light in the medium is directly proportional to the intensity I .

- 52** A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is

surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is [AIEEE 2009]

- $\sin^{-1} \left(\frac{1}{2} \right)$
- $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$
- $\sin^{-1} \left(\frac{2}{\sqrt{3}} \right)$
- $\sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$

Ans. (d)

$$\sin C = \frac{\sqrt{3}}{2} \quad \dots(i)$$

$$\sin r = \sin(90^\circ - C) = \cos C = \frac{1}{2}$$

$$\frac{\sin \theta}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\sin \theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

- 53** The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviations for red and blue light respectively in a prism of this glass. Then, [AIEEE 2006]

- (a) $D_1 < D_2$
 (b) $D_1 = D_2$
 (c) can be less than or greater than D_2 depending upon the angle of prism
 (d) $D_1 > D_2$

Ans. (a)

$$D = (\mu - 1) A$$

For blue light, μ is greater than that for red light, so $D_2 > D_1$.

Aliter

$$D_1 = (1.520 - 1) A$$

$$D_2 = (1.525 - 1) A$$

$$\Rightarrow \frac{D_1}{D_2} < 1$$

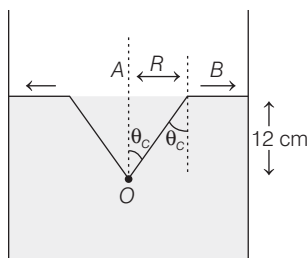
$$\Rightarrow D_2 > D_1$$

- 54** A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is $4/3$ and the fish is 12 cm below the water surface, the radius of this circle (in cm) is [AIEEE 2005]

- (a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$
 (c) $36\sqrt{5}$ (d) $4\sqrt{5}$

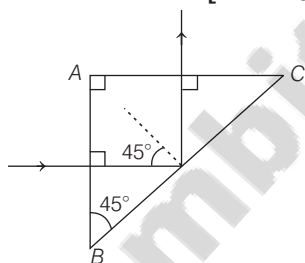
Ans. (b)

The situation is shown in figure.



$$\begin{aligned} \sin \theta_c &= \frac{1}{\mu} \Rightarrow \tan \theta_c = \frac{AB}{OA} \\ \therefore AB &= OA \tan \theta_c \\ &= OA \frac{\sin \theta_c}{\cos \theta_c} = \frac{\sin \theta_c}{\sqrt{1 - \sin^2 \theta_c}} \times OA \\ &= \frac{1/\mu}{\sqrt{1 - 1/\mu^2}} \times OA = \frac{OA}{\sqrt{\mu^2 - 1}} \\ \text{or } AB &= \frac{OA}{\sqrt{\mu^2 - 1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{36}{\sqrt{7}} \end{aligned}$$

- 55** A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45° , we conclude that for the refractive index n as [AIEEE 2004]



- (a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
 (c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

Ans. (b)

For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.

$$\text{i.e., } C < i$$

$$\text{or } C < 45^\circ \quad [\because \angle i = 45^\circ]$$

$$\text{But } n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$$

$$\text{or } \sin^{-1}\left(\frac{1}{n}\right) < 45^\circ \text{ or } \frac{1}{n} < \sin 45^\circ$$

$$\text{or } n > \frac{1}{\sin 45^\circ} \text{ or } n > \frac{1}{(1/\sqrt{2})} \text{ or } n > \sqrt{2}$$

- 56** Which of the following is used in optical fibres? [AIEEE 2002]

- (a) Total internal reflection
 (b) Scattering
 (c) Diffraction
 (d) Refraction

Ans. (a)

Optical fibres work on the principle of total internal reflection.

TOPIC 3

Lenses

- 57** An object is placed at the focus of concave lens having focal length f . What is the magnification and distance of the image from the optical centre of the lens?

[2021, 31 Aug Shift-I]

- (a) 1, ∞ (b) Very high, ∞
 (c) $\frac{1}{2}$, $\frac{f}{2}$ (d) $\frac{1}{4}$, $\frac{f}{4}$

Ans. (c)

Given, focal length of concave lens, $f' = -f$

Object distance from lens, $u = -f$

Image distance from lens = v

Magnification = m

By using lens formula,

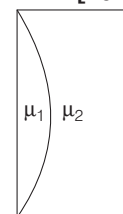
$$\begin{aligned} \frac{1}{f'} &= \frac{1}{v} - \frac{1}{u} \\ \Rightarrow \frac{1}{v} &= \frac{1}{f'} + \frac{1}{u} \\ &= -\frac{1}{f} - \frac{1}{f} = -2/f \end{aligned}$$

$$\Rightarrow v = -f/2$$

$$\therefore |v| = \frac{f}{2}$$

$$\therefore m = \frac{v}{u} = \frac{-f/2}{-f} = \frac{1}{2}$$

- 58** Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses. [2021, 27 Aug Shift-I]



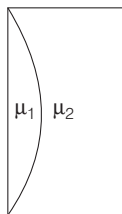
- (a) $\frac{1}{\mu_2 - \mu_1}$ (b) $\mu_1 - \mu_2$
 (c) $\frac{1}{\mu_1 - \mu_2}$ (d) $\mu_2 - \mu_1$

Ans. (b)

According to given diagram, Refractive indices are μ_1 and μ_2 .

Radius of two lenses are $R_1 = R_2 = R$

By using lens Maker's formula,



$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

where, f = focal length.

Now, for plano-convex lens $R_1 = \infty$ and $R_2 = -R$

$$\frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{\infty} - \frac{1}{-R} \right)$$

$$\Rightarrow \frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{R} \right) \quad \dots(i)$$

For plano-concave lens, $R_1 = -R$ and $R_2 = \infty$

$$\frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{-R} - \frac{1}{\infty} \right)$$

$$\frac{1}{f_2} = (\mu_2 - 1) \left(-\frac{1}{R} \right) \quad \dots(ii)$$

Therefore, combined focal length is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, f = net focal length of combination of lenses.

$$= (\mu_1 - 1) \left(\frac{1}{R} \right) + (\mu_2 - 1) \left(-\frac{1}{R} \right)$$

$$= \frac{\mu_1}{R} - \frac{1}{R} - \frac{\mu_2}{R} + \frac{1}{R}$$

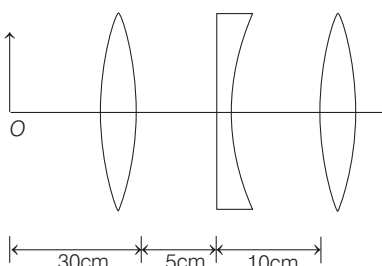
$$\frac{1}{f} = \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \frac{R}{f} = \mu_1 - \mu_2$$

- 59** Find the distance of the image from object O , formed by the combination of lenses in the figure.

[2021, 27 Aug Shift-II]

$f = +10\text{ cm}$ $f = -10\text{ cm}$ $f = +30\text{ cm}$



(a) 75 cm

(c) 20 cm

(b) 10 cm

(d) infinity

Ans. (a)

Given, focal length of first lens,

$f_1 = +10\text{ cm}$

Focal length of second lens,

$f_2 = -10\text{ cm}$

Focal length of third lens, $f_3 = +30\text{ cm}$

Object distance from first lens,

$u_1 = -30\text{ cm}$

Using lens formula for first lens, we get

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\Rightarrow \frac{1}{-10} = \frac{1}{v_1} - \frac{1}{-30}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{-10} - \frac{1}{-30} = \frac{2}{-30}$$

$$\Rightarrow v_1 = 15\text{ cm}$$

Thus, the image formed by first lens is 15 cm to the right of first lens.

The object distance for second lens will be

$$u_2 = 15\text{ cm} - 5\text{ cm}$$

$$= 10\text{ cm}$$

Using lens formula for first lens, we get

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\Rightarrow \frac{1}{-10} = \frac{1}{v_2} - \frac{1}{10}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{-10} + \frac{1}{10} = 0$$

$$\Rightarrow v_2 = \infty$$

Thus, the object distance for third lens, $u_3 = \infty$.

Using lens formula for third lens, we get

$$\frac{1}{f_3} = \frac{1}{v_3} - \frac{1}{u_3}$$

$$\Rightarrow \frac{1}{30} = \frac{1}{v_3} - \frac{1}{\infty}$$

$$\Rightarrow \frac{1}{v_3} = \frac{1}{30}$$

$$v_3 = 30\text{ cm}$$

The final image will form 30 cm from the third lens.

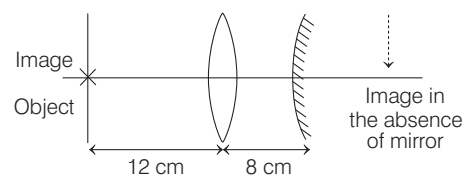
The distance of final image from object will be

$$30 + 10 + 5 + 30 = 75\text{ cm}.$$

Thus, the distance of final image formed from object O is 75 cm.

- 60** An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm

as shown in the figure. Image of object coincides with the object.

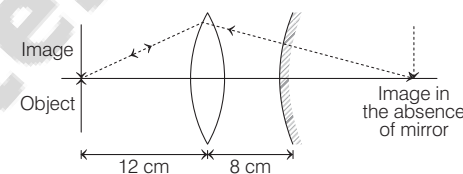


When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be cm.

[2021, 26 Aug Shift-II]

Ans. (50)

It is given that image coincides with the object. In the arrangement given, the image will only coincide with the object when the ray on mirror falls directly perpendicular on the surface of it.



The light will converge at centre of curvature of mirror and due to that after removing the mirror the light will converge at same position.

The focal length of mirror, $f = 15\text{ cm}$. So, radius of curvature of mirror, $2f = 30\text{ cm}$

The image distance from the object after convex mirror is removed will be calculated as, $12\text{ cm} + 8\text{ cm} + 30\text{ cm} = 50\text{ cm}$.

Thus, the image will be formed at a distance of 50 cm from object.

- 61** The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is $2 \times 10^8\text{ ms}^{-1}$, then the focal length of the lens is [2021, 17 March Shift-I]

(a) 0.30 cm

(b) 15 cm

(c) 1.5 cm

(d) 30 cm

Ans. (d)

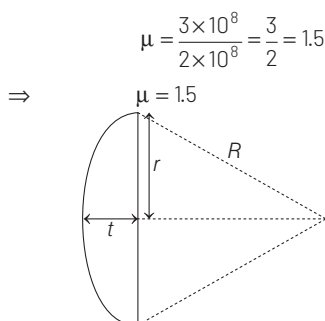
Given, thickness at the centre of plano-convex lens, $t = 3\text{ mm} = 3 \times 10^{-3}\text{ m}$
Diameter of plano-convex lens, $d = 6\text{ cm}$
 $\therefore$ Radius of plano-convex lens, $r = 3\text{ cm} = 3 \times 10^{-2}\text{ m}$

Speed of light in lens material,

$$v = 2 \times 10^8\text{ ms}^{-1}$$

Refractive index

$$= \frac{\text{Speed of light in air}}{\text{Speed of light in medium}}$$



We know that, $R^2 = (R-t)^2 + r^2$
where, R is the radius of curvature of plano convex lens.

$$\Rightarrow R^2 = R^2 + t^2 - 2Rt + r^2$$

$$\Rightarrow 2Rt = r^2 + t^2$$

As, t is small, then t^2 will be very very small, so it can be neglected.

$$\Rightarrow 2Rt = r^2$$

$$\Rightarrow R = r^2 / 2t \quad \dots(i)$$

From lens Maker's formula, we have

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Here $R_1 = R$ & $R_2 = \infty$

$$\Rightarrow \frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

$$\Rightarrow \frac{1}{f} = \frac{\mu - 1}{R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{f} = \frac{\mu - 1(2t)}{r^2}$$

$$\Rightarrow f = \frac{r^2}{(\mu - 1)2t}$$

Putting the given values in above equation, we get

$$f = \frac{(3 \times 10^{-2})^2}{(1.5 - 1)2 \times 3 \times 10^{-3}} \Rightarrow f = 0.3 \text{ m}$$

$$f = 30 \text{ cm}$$

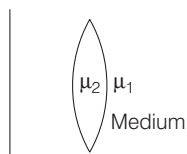
- 62** The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively)

[2021, 16 March Shift-II]

- (a) 1 (b) Infinite
(c) $\frac{R_1 R_2}{R_1 - R_2}$ (d) Zero

Ans. (b)

Consider a convex lens of refractive index μ_2 and μ_1 is the refractive index of the medium in which it is placed.



$$\Rightarrow \mu_2 = \mu_1 \quad \dots(i)$$

According to the lens Maker's formula,

$$\frac{1}{f} = \left[\frac{\mu_1}{\mu_2} - 1 \right] \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \quad \dots(ii)$$

where, f is the focal length of the lens, R_1 and R_2 are the radii of curvature of respective faces of lens.

From Eqs. (i) and (ii), we can write

$$\frac{1}{f} = [1 - 1] \left[\frac{1}{R_1} - \frac{1}{R_2} \right] = 0$$

$$\Rightarrow \frac{1}{f} = 0 \Rightarrow f = \text{Infinite}$$

- 63** The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is cm.

[2021, 25 Feb Shift-I]

Ans. (15)

Let v be the position of image, h_i be the height of image and h_o be the height of object.

$$\text{Given, } h_i = h_o$$

$$\text{Since, magnification, } m = h_i / h_o = h_i / h_o \quad \dots(i)$$

By using lens formula,

$$1/f = 1/v - 1/u \Rightarrow 1/v = 1/f + 1/u$$

$$\Rightarrow v = \frac{fu}{u + f}$$

$$\therefore m = \frac{v}{u} = \frac{f}{f + u}$$

Now, from Eq. (i), m can be ± 1 .

$$\text{For, } m = +1 = \frac{f}{-10 + f} \quad \dots(ii)$$

For $m = -1$

$$= \frac{f}{-20 + f} \quad \dots(iii)$$

On dividing Eq. (iii) by Eq. (ii), we get

$$-1 = \frac{-10 + f}{-20 + f}$$

$$\Rightarrow 20 - f = -10 + f$$

$$\Rightarrow 30 = 2f$$

$$\Rightarrow f = 15 \text{ cm}$$

- 64** A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane

mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is [2020, 6 Sep Shift-I]

- (a) 2.6 m from the mirror, real
(b) 1 m from the mirror, virtual
(c) 1 m from the mirror, real
(d) 2.6 m from the mirror, virtual

Ans. (a)

Given that, for convex lens,

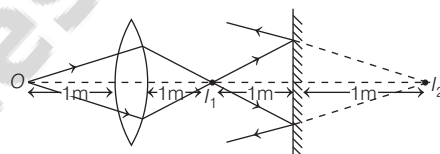
$$u_1 = -1 \text{ m, } f = +0.5 \text{ m}$$

Applying lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$$\frac{1}{v_1} - \frac{1}{-1} = \frac{1}{0.5}$$

$$\Rightarrow v_1 = +1 \text{ m}$$



The image I_1 will behave as an object for plane mirror. So, image will be formed 1 m behind the mirror as shown in the above figure.

Now, the virtual image I_2 will act as an object at a distance of 3m from lens.

For final image I_3 ,

$$u = -3 \text{ m,}$$

$$f = +0.5 \text{ m}$$

Again, applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-3} = \frac{1}{0.5}$$

$$\text{or } v = \frac{3}{5} = +0.6 \text{ m}$$

So, total distance of final image from mirror = $2 + 0.6 = 2.6 \text{ m}$. The image will be real (but in answer it was given virtual).

- 65** The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right) \text{ D}$, where N is an integer.

Then, the value of N is

[2020, 4 Sep Shift-II]

Ans. (476)

Using displacement method, the focal length of a lens is given by

$$f = \frac{D^2 - d^2}{4D} \quad \dots(i)$$

Given that, distance between object and screen, $D = 100$ cm

Displacement travelled by object, $d = 40$ cm

Substituting these values in eq. (i), we get

$$\begin{aligned} f &= \frac{(100)^2 - (40)^2}{4(100)} = \frac{10000 - 1600}{400} \\ &= \frac{8400}{400} = 21 \text{ cm} = 0.21 \text{ m} \end{aligned}$$

Now, optical power of lens is

$$\begin{aligned} P &= \frac{1}{f} = \frac{1}{0.21 \text{ m}} = \frac{100}{21} \\ &= \frac{100}{21} \text{ m}^{-1} = \frac{100}{21} \text{ D} = 4.76 \text{ D} = \left(\frac{476}{100}\right) \text{ D} \end{aligned}$$

Given that,

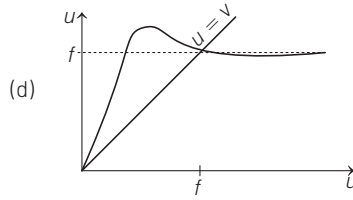
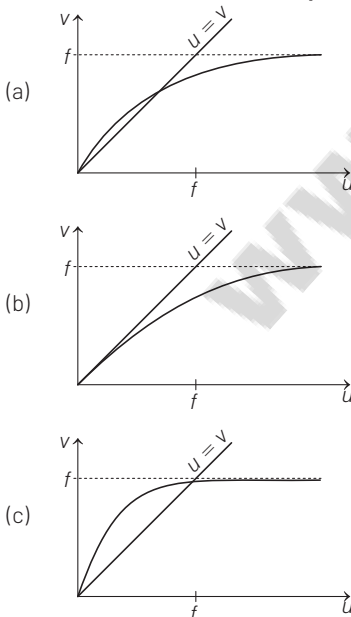
$$P = \left(\frac{N}{100}\right) \text{ D}$$

On comparing both the equations, we get

$$N = 476$$

- 66** For a concave lens of focal length f , the relation between object and image distances u and v , respectively, from its pole can best be represented by ($u = v$ is the reference line)

[2020, 5 Sep Shift-I]

**Ans. (b)**

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow v = \frac{uf}{u+f}$$

Now, considering two cases:

Case I If $v = u$,

$$\Rightarrow u = \frac{uf}{u+f} \Rightarrow u+f = u \Rightarrow u = 0$$

So, the curve will pass through origin.

Case II If $u = \infty \Rightarrow v = f$

So, as u tends towards infinity, v tends towards focus and all other distances will be less than focal length f and it will never intersect $u = v$ line.

Hence, correct option is (b).

- 67** A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power $1.5P$ is

[2020, 6 Sep Shift-II]

- (a) $2R$ (b) $\frac{R}{2}$
(c) $\frac{3R}{2}$ (d) $\frac{R}{3}$

Ans. (d)

For double convex lens,

$$R_1 = R_2 = R \text{ (given)}$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right)$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{2}{R} \right)$$

$$\text{Power, } P = \frac{1}{f} = \frac{2(\mu - 1)}{R} \quad \dots(i)$$

For plano-convex lens,

$$R_1 = R', R_2 = \infty$$

$$\frac{1}{f'} = (\mu - 1) \left(\frac{1}{R'} - \frac{1}{\infty} \right)$$

$$\frac{1}{f'} = \frac{\mu - 1}{R'}$$

$$\text{Power, } P' = \frac{1}{f'} = \frac{(\mu - 1)}{R'}$$

$$\text{Given, } P' = 1.5P \Rightarrow 1.5P = \frac{\mu - 1}{R'} \quad \dots(ii)$$

Dividing Eqs. (i) by (ii), we get

$$\frac{P}{1.5P} = \frac{\frac{2(\mu - 1)}{R}}{\frac{(\mu - 1)}{R'}} \Rightarrow \frac{2}{3} = \frac{2R'}{R} \text{ or } R' = \frac{R}{3}$$

Hence, correct option is (d).

- 68** A thin lens made of glass (refractive index = 1.5) of focal length $f = 16$ cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1 , then the ratio f_1 / f is closest to the integer

[2020, 7 Jan Shift-II]

- (a) 1 (b) 5 (c) 9 (d) 17

Ans. (c)

Focal length of lens of refractive index n_2 placed in a medium of refractive index n_1 is

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

For a glass-lens in air, focal length,

$$\frac{1}{f_{\text{air}}} = \frac{1}{f} = \left(\frac{n_g}{n_a} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

When lens is dipped in a liquid, its focal length,

$$\frac{1}{f_{\text{liquid}}} = \frac{1}{f_1} = \left(\frac{n_g}{n_l} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{f_1}{f} = \frac{\left(\frac{n_g}{n_a} - 1 \right)}{\left(\frac{n_g}{n_l} - 1 \right)}$$

As $f = 16$ cm, $n_g = 1.5$, $n_l = 1.42$, we have

$$\begin{aligned} \frac{f_1}{f} &= \frac{\left(\frac{1.5}{1} - 1 \right)}{\left(\frac{1.5}{1.42} - 1 \right)} \Rightarrow \frac{f_1}{f} = \frac{\left(\frac{3}{2} - 1 \right)}{\left(\frac{150}{142} - 1 \right)} = \left(\frac{1}{\frac{2}{4}} \right) \\ &= \frac{71}{8} \approx 8.9 \end{aligned}$$

So, $\frac{f_1}{f}$ is closest to integer 9.

- 69** A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is

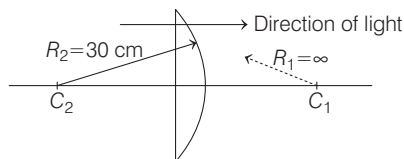
[2020, 8 Jan Shift-I]

Ans. (60)

Focal length of lens by lens maker's formula,

$$\frac{1}{f} = (n_{21} - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

For a plano-convex lens,



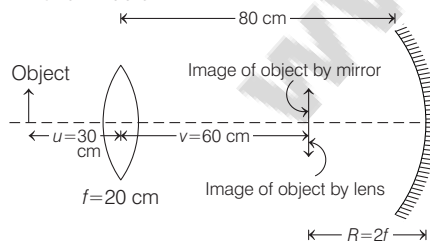
$$\begin{aligned} R_1 &= \infty, R_2 = -30 \text{ cm}, n_{21} = 1.5 \\ \therefore \frac{1}{f} &= (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{-30} \right) \\ \Rightarrow \frac{1}{f} &= \frac{1}{2} \times \frac{1}{30} \\ \Rightarrow f &= 60 \text{ cm} \end{aligned}$$

- 70** A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **[2019, 8 April Shift-II]**

- (a) 25 cm (b) 20 cm
(c) 10 cm (d) 30 cm

Ans. (c)

The given situation can be drawn as shown below



For lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow v = 60 \text{ cm}$$

So, this image is at a distance of $80 - 60 = 20 \text{ cm}$ from the mirror.

As, the image formed by the mirror coincides with image formed by the lens.

This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, i.e. at $2f$.

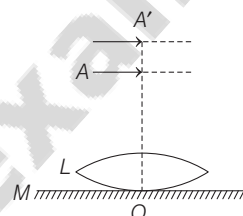
So, radius of curvature of mirror is $R = 20 \text{ cm}$

$$\therefore \text{Focal length of mirror, } f = \frac{R}{2} = 10 \text{ cm}$$

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

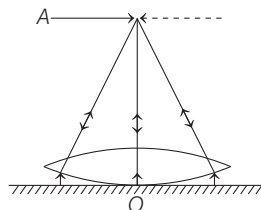
- 71** A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M . When a pin is placed at A , such that $OA = 18 \text{ cm}$, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A' , such that $OA' = 27 \text{ cm}$, to get its inverted real image at A' itself. The value of μ_l will be **[2019, 9 April Shift-II]**



- (a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

Ans. (c)

Light from plane mirror is reflected back on its path, so that image of A coincides with A itself.



This would happen when rays refracted by the convex lens falls normally on the plane mirror, i.e. the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens is,

$$f_1 = 18 \text{ cm}$$

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27}$$

$$\Rightarrow f_2 = -54 \text{ cm}$$

For convex lens by lens maker's formula, we have

$$\begin{aligned} \frac{1}{f} &= (\mu - 1) \left(\frac{2}{R} \right) \\ \Rightarrow \frac{1}{18} &= 0.5 \times \frac{2}{R} \Rightarrow R = 18 \text{ cm} \end{aligned}$$

and for plano-convex liquid lens, we have

$$\begin{aligned} \frac{1}{f} &= (\mu_l - 1) \left(\frac{-1}{R} \right) \\ \Rightarrow -\frac{1}{54} &= (\mu_l - 1) \left(\frac{-1}{18} \right) \\ \Rightarrow \mu_l &= 1 + \frac{1}{3} = \frac{4}{3} \end{aligned}$$

- 72** A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is **[2019, 9 April Shift-II]**
- (a) 5 : 3 (b) 2 : 1
(c) 4 : 3 (d) 3 : 1

Ans. (d)

Since, image formed by a convex lens can be real or virtual in nature.

Thus, $m = +2$ or -2 .

First let us take image to be real in nature,

then $m = -2 = \frac{v}{u}$, where v is image distance and u is object distance.

$$\Rightarrow v = -2x_1 \quad [\text{Taking } u = x_1]$$

Now, by using lens equation,

$$\begin{aligned} \frac{1}{v} - \frac{1}{u} &= \frac{1}{f} \\ \Rightarrow \frac{1}{-2x_1} - \frac{1}{x_1} &= \frac{1}{20} \Rightarrow \frac{-3}{2x_1} = \frac{1}{20} \\ \Rightarrow x_1 &= -30 \text{ cm} \end{aligned}$$

Now, let us take image to be virtual in nature, then

$$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2 \quad [\text{Taking } u = x_2]$$

Again, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$$

$$\Rightarrow \frac{-1}{2x_2} = \frac{1}{20}$$

$$\Rightarrow x_2 = -10 \text{ cm}$$

So, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{-30}{-10} = 3:1$$

Alternate Solution

Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u} \right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f+x_1}$$

$$\Rightarrow -2f + (-2)x_1 = f \text{ or } x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image)

$$+2 = \frac{f}{f+x_2} \Rightarrow 2f + 2x_2 = f$$

$$\text{or } x_2 = \frac{-f}{2}$$

Now, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{-\frac{3f}{2}}{-\frac{f}{2}} = \frac{3}{1}$$

- 73** An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be

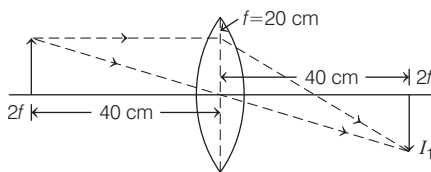
[2019, 8 April Shift-I]

- 20 cm from the convergent mirror, same size as the object
- 40 cm from the convergent mirror, same size as the object
- 40 cm from the convergent lens, twice the size of the object
- 20 cm from the convergent mirror, twice size of the object

Ans. (*)

In given system of lens and mirror, position of object O in front of lens is at a distance $2f$.

i.e. $u = 2f = 40 \text{ cm}$

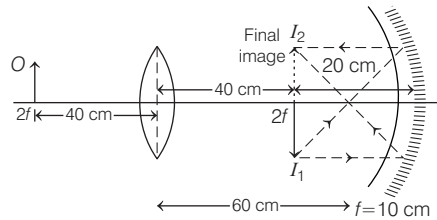


So, image (I_1) formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40 \text{ cm}$, (behind lens) magnification,

$$m_1 = \frac{v}{u} = \frac{40}{40} = 1$$

Thus, size of image is same as that of object.

This image (I_1) acts like a real object for mirror.



As object distance for mirror is

$$u = C = 2f = -20 \text{ cm}$$

where, C = centre of curvature.

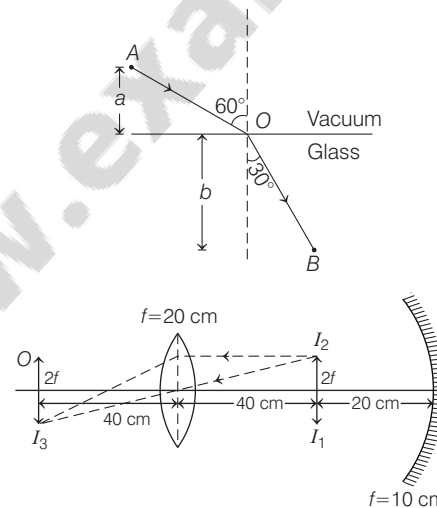
So, image (I_2) formed by mirror is at $2f$.

$\therefore$ For mirror $v = 2f = 2(-10) = -20 \text{ cm}$

$$\text{Magnification, } m_2 = -\frac{v}{u} = -\frac{(-20)}{(-20)} = -1$$

Thus, image size is same as that of object.

The image I_2 formed by the mirror will act like an object for lens.



As the object is at $2f$ distance from lens, so image (I_3) will be formed at a distance $2f$ or 40 cm. Thus, magnification,

$$m_3 = \frac{v}{u} = \frac{40}{40} = 1$$

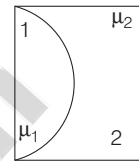
So, final magnification,

$$m = m_1 \times m_2 \times m_3 = -1$$

Hence, final image (I_3) is real, inverted of same size as that of object and coinciding with object.

- 74** One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2 , then the focal length of the combination is

[2019, 10 April Shift-I]



- $\frac{2R}{\mu_1 - \mu_2}$
- $\frac{R}{2 - (\mu_1 - \mu_2)}$
- $\frac{R}{2(\mu_1 - \mu_2)}$
- $\frac{R}{\mu_1 - \mu_2}$

Ans. (d)

Key Idea Focal length of two lenses in contact is given as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, f_1 and f_2 are the focal lengths of the respective lenses.

Focal length of a lens is given as

$$\frac{1}{f} = (\mu_1 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$\therefore$ Focal length of plano-convex lens, i.e. lens 1,

$$\Rightarrow \frac{1}{f_1} = \frac{\mu_1 - 1}{R} \quad \left(\begin{array}{c} 1 \\ \mu_1 \end{array} \right) \quad \dots (i)$$

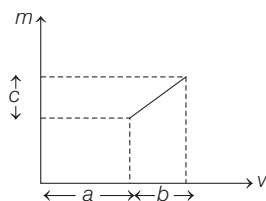
Similarly, focal length of plano-concave lens, i.e. lens 2,

$$\Rightarrow \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R} \quad \left(\begin{array}{c} \mu_2 \\ 2 \end{array} \right) \quad \dots (ii)$$

From Eqs. (i) and (ii), net focal length is

$$\Rightarrow \frac{1}{f} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R}$$

- 75** The graph shows how the magnification m produced by a thin lens varies with image distance v . What is the focal length of the lens used? [2019, 10 April Shift-II]



- (a) $\frac{b^2c}{a}$ (b) $\frac{b^2}{ac}$
(c) $\frac{a}{c}$ (d) $\frac{b}{c}$

Ans. (d)

First we find relation of magnification m and focal length f . By lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow 1 - \frac{v}{u} = \frac{v}{f}$$

$$\Rightarrow 1 - m = \frac{v}{f} \quad \left[\because \frac{v}{u} = m \right]$$

$$\Rightarrow m = 1 - \frac{v}{f} \quad \dots(i)$$

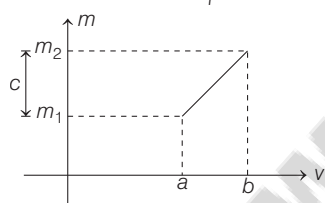
Now, from given graph and from Eq. (i),

At $v = a$, magnification is

$$m_1 = 1 - \frac{a}{f} \quad \dots(ii)$$

At $v = a + b$, magnification is

$$m_2 = 1 - \frac{a+b}{f} \quad \dots(iii)$$



From graph, we can also say that

$$m_2 - m_1 = c \quad \dots(iv)$$

So, from Eqs. (ii), (iii) and (iv), we have

$$\left(1 - \frac{a+b}{f}\right) - \left(1 - \frac{a}{f}\right) = c$$

$$\Rightarrow \frac{a - (a+b)}{f} = c$$

$$\Rightarrow f = -\frac{b}{c} \text{ or } |f| = \frac{b}{c}$$

- 76** A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block

(refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

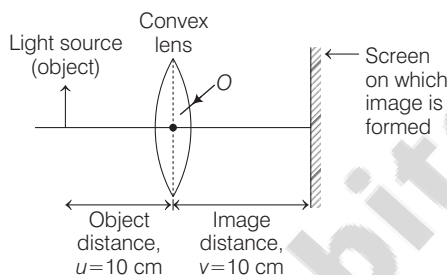
To get the sharp image again, the screen is shifted by a distance d .

Then, d is [2019, 9 Jan Shift-I]

- (a) 0
(b) 1.1 cm away from the lens
(c) 0.55 cm away from the lens
(d) 0.55 cm towards the lens

Ans. (c)

Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,



Since, $u = v$ which can only be possible in the situation when the object is placed at $2f$ of the lens.

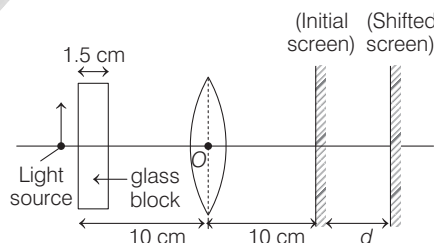
So, that the image can form at $2f$ only on the other side of the lens.

Thus, the distance from the optical centre (O) of the lens and $2f$ is $2 \times$ focal length (f) = object distance

$$\Rightarrow 2 \times f = 10$$

$$\text{or } f = 5 \text{ cm}$$

Now, when a glass block is placed in contact with the light source i.e., object, then the situation is shown in the figure given below



Then due to the block, the position of the object in front of the lens would now be shifted due to refraction of the light source rays through the block.

The shift in the position of the object is given as

$$x = \left(1 - \frac{1}{\mu}\right)t$$

where, μ is the refractive index of the block and t is its thickness.

$$\Rightarrow x = \left(1 - \frac{1}{1.5}\right)1.5$$

$$= \left(1 - \frac{2}{3}\right)\frac{3}{2}$$

$$= \frac{1}{3} \times \frac{3}{2} = \frac{1}{2} = 0.5 \text{ cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$$u' = 10 - 0.5 = 9.5 \text{ cm}$$

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} \quad (\text{using lens formula})$$

Substituting the values, we get

$$\frac{1}{v'} = \frac{1}{5} + \left(\frac{1}{-9.5}\right) = \frac{+9.5 - 5}{47.5} = \frac{4.5}{47.5}$$

$$\text{or } v' = 10.55 \text{ cm}$$

$$\therefore \text{The value of } d = v' - v = 10.55 - 10$$

$$= 0.55 \text{ cm, away from the lens}$$

Focal length in the above question can be calculated by using lens formula i.e.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

- 77** A plano-convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano-concave lens of refractive index μ_2 and focal length f_2 . If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as [2019, 10 Jan Shift-I]

- (a) $3\mu_2 - 2\mu_1 = 1$ (b) $2\mu_2 - \mu_1 = 1$
(c) $2\mu_1 - \mu_2 = 1$ (d) $\mu_1 + \mu_2 = 3$

Ans. (c)

Given, $f_1 = 2f_2$

$$\Rightarrow \frac{1}{|f_1|} = \frac{1}{|2f_2|} \quad \dots(i)$$

Using lens Maker's formula, we get

$$\frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{R'_1} - \frac{1}{R'_2} \right)$$

$$\Rightarrow \left| (\mu_1 - 1) \left(\frac{1}{\infty} - \frac{1}{-R} \right) \right|$$

$$= \left| \frac{1}{2} (\mu_2 - 1) \left(\frac{1}{-R} - \frac{1}{\infty} \right) \right| \quad [\because \text{using Eq. (i)}]$$

$$\Rightarrow \frac{\mu_1 - 1}{R} = \frac{\mu_2 - 1}{2R}$$

$$\Rightarrow 2\mu_1 - \mu_2 = 1$$

- 78** An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be

[2019, 11 Jan Shift-I]

- (a) 3.22×10^{-3} m/s towards the lens
 (b) 0.92×10^{-3} m/s away from the lens
 (c) 2.26×10^{-3} m/s away from the lens
 (d) 1.16×10^{-3} m/s towards the lens

Ans. (d)

Lens formula is given as

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \dots (i)$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \frac{uf}{u+f} = v \quad \dots (ii)$$

Now, by differentiating Eq. (i), we get

$$0 = -\frac{1}{v^2} \cdot \frac{dv}{dt} + \frac{1}{u^2} \cdot \frac{du}{dt}$$

[$\because f$ (focal length of a lens is constant)]

$$\text{or } \frac{dv}{dt} = \frac{v^2}{u^2} \frac{du}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \left(\frac{f}{u+f} \right)^2 \frac{du}{dt} \quad [\text{using Eq. (ii)}]$$

Given, $f = 0.3$ m, $u = -20$ m, $du/dt = 5$ m/s

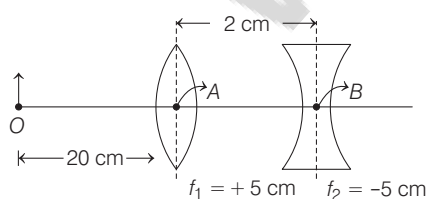
$$\therefore \frac{dv}{dt} = \left(\frac{0.3}{0.3-20} \right)^2 \times 5 = \left(\frac{3}{197} \right)^2 \times 5$$

$$= 1.16 \times 10^{-3} \text{ m/s}$$

Thus, the image is moved with a speed of 1.16×10^{-3} m/s towards the lens.

- 79** What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths)

[2019, 12 Jan Shift-I]



- (a) $\frac{20}{3}$ cm from point B at right, real
 (b) 70 cm from point B at right, real
 (c) 40 cm from point B at right, real
 (d) 70 cm from point B at left, virtual

Ans. (b)

For lens A, object distance, $u = -20$ cm

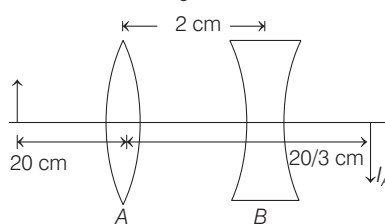
Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\text{We have, } \frac{1}{v} = \frac{1}{5} - \frac{1}{20}$$

$$\frac{1}{v} = \frac{20-5}{20 \times 5} = \frac{15}{100}$$

$$v = \frac{20}{3} \text{ cm}$$



For lens B, image of A is object for B.

$$\therefore u = \frac{20}{3} - 2 = +\frac{14}{3} \text{ cm}$$

$$f = -5 \text{ cm}$$

Now, from lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{-5} + \frac{3}{14}$$

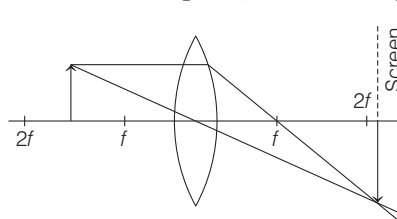
$$\frac{1}{v} = \frac{15-14}{5 \times 14}$$

$$\Rightarrow v = 70 \text{ cm}$$

Hence, image is on right of lens B and is real in nature.

- 80** Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?

[2019, 12 Jan Shift-II]



- (a) No change
 (b) Magnified image
 (c) Image disappears
 (d) Erect real image

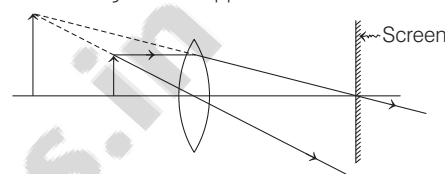
Ans. (c)

When given set up is immersed in water, focal length of lens increases,

$$\left\{ \begin{aligned} \frac{f_{\text{liquid}}}{f_{\text{air}}} &= \frac{n_{ga} - 1}{n_{gl} - 1} = \frac{n_{ga} - 1}{\left(\frac{n_{ga}}{n_{la}} - 1 \right)} \\ \text{Note } \left\{ \begin{aligned} \text{Now, } n_{ga} &= \frac{3}{2} \text{ and } n_{la} = \frac{4}{3} \\ \therefore f_{\text{liquid}} &= f_{\text{air}} \left(\frac{\frac{1}{2}}{\frac{2}{1}} \right) = 4 f_{\text{air}} \end{aligned} \right. \end{aligned} \right.$$

As focal length increases there is no focussing of image on screen

$\therefore$ image will disappear



Actually a virtual image is formed on same side of object.

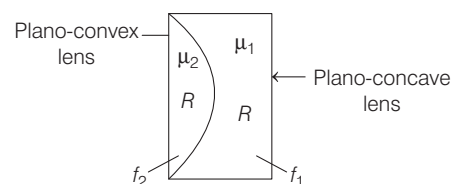
- 81** A plano-convex lens (focal length f_2 , refractive index μ_2 , radius of curvature R) fits exactly into a plano-concave lens (focal length f_1 , refractive index μ_1 , radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be

[2019, 12 Jan Shift-II]

- (a) $f_1 - f_2$ (b) $\frac{R}{\mu_2 - \mu_1}$
 (c) $f_1 + f_2$ (d) $\frac{2f_1 f_2}{f_1 + f_2}$

Ans. (b)

Given combination is as shown below



As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{-R} - \frac{1}{\infty} \right)$$

Here,

$$= \frac{(1 - \mu_1)}{R}$$

$$\text{and } \frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{\infty} - \frac{1}{(-R)} \right) = \frac{(\mu_2 - 1)}{R}$$

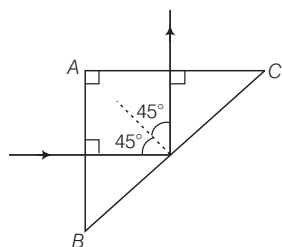
$$\therefore \frac{1}{f_{eq}} = \left(\frac{\mu_2 - 1}{R} \right) + \left(\frac{1 - \mu_1}{R} \right)$$

$$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$$

$$\text{So, } f_{eq} = \frac{R}{\mu_2 - \mu_1}$$

- 82** A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

[JEE Main 2017]

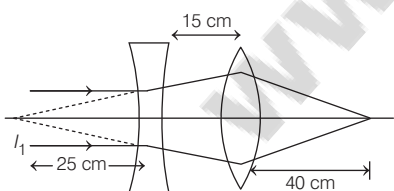


- (a) virtual and at a distance of 40 cm from convergent lens
 (b) real and at a distance of 40 cm from the divergent lens
 (c) real and at a distance of 6 cm from the convergent lens
 (d) real and at a distance of 40 cm from convergent lens

Ans. (d)

Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.



Now, the image (I_1) will work as object for converging lens.

For converging lens, distance of object u (i.e. distance of I_1) = $-(25 + 15) = -40$ cm

$$f = 20 \text{ cm}$$

$\therefore$ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{90} \Rightarrow v = 40 \text{ cm}$$

v is positive so image will be real and will form at right side of converging lens at 40 cm.

- 83** A thin convex lens made from crown glass ($\mu = \frac{3}{2}$) has focal

length f . When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$. It has

the focal lengths f_1 and f_2 , respectively. The correct relation between the focal lengths is

- (a) $f_1 = f_2 < f$ [JEE Main 2014]
 (b) $f_1 > f$ and f_2 becomes negative
 (c) $f_2 > f$ and f_1 becomes negative
 (d) f_1 and f_2 both become negative

Key Idea It is based on lens maker's formula

$$\text{i.e., } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Ans. (b)

According to lens maker's formula, when the lens is in the air

$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$$

$$\text{Here, } \frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}$$

In case of liquid, where refractive indices are $4/3$ and $5/3$, we get

Focal length in first liquid,

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1 \right) \frac{1}{x}$$

$\Rightarrow F_1$ is positive.

Nature of lens is not change.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f} \Rightarrow f_1 = 4f$$

Focal length in second liquid,

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

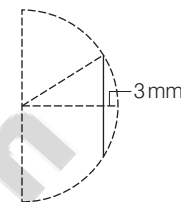
$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1 \right) \left(\frac{1}{x} \right)$$

$\Rightarrow f_2$ is negative.

Nature of lens change i.e., convex behave as concave.

- 84** Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is [JEE Main 2013]

- (a) 15 cm (b) 20 cm
 (c) 30 cm (d) 10 cm

Ans. (c)

$$n = \frac{3}{2} \quad \left[\because n = \frac{c}{v} \right]$$

$$3^2 + (R - 3\text{mm})^2 = R^2$$

$$\Rightarrow 3^2 + R^2 - 2R(3\text{mm}) + (3\text{mm})^2 = R^2$$

$$\Rightarrow R \approx 15 \text{ cm}$$

$$\therefore \frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{15} \right)$$

$$\left[\because \frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \right]$$

$$\text{and } R_1 = \infty, R_2 = -15$$

$$\Rightarrow f = 30 \text{ cm}$$

- 85** An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? [AIEEE 2012]

- (a) 7.2 m (b) 2.4 m
 (c) 3.2 m (d) 5.6 m

Ans. (d)

Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu} \right) t = \left(1 - \frac{1}{1.5} \right) \times 1 \text{ cm} = \frac{1}{3} \text{ cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240}$$

or

$$\frac{1}{f} = \frac{20 + 1}{240}$$

$$\Rightarrow f = \frac{240}{21} \text{ cm}$$

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3} \right) \text{ cm} = \frac{35}{3} \text{ cm}$

such that the glass plate will shift the image on the film.

$$\begin{aligned} \text{As } \frac{1}{f} &= \frac{1}{v} - \frac{1}{u} \\ \Rightarrow \frac{1}{u} &= \frac{1}{v} - \frac{1}{f} = \frac{3}{35} - \frac{21}{240} \\ &= \frac{48 \times 3 - 7 \times 21}{1680} = -\frac{1}{560} \\ \Rightarrow u &= -5.6 \text{ m} \end{aligned}$$

- 86** When monochromatic red light is used instead of blue light in a convex lens, its focal length will [AIEEE 2011]

- (a) not depend on colour of light
(b) increase
(c) decrease
(d) remain same

Ans. (b)

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Also, by Cauchy's formula,

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots \Rightarrow \mu \propto \frac{1}{\lambda}$$

As $\lambda_{\text{blue}} < \lambda_{\text{red}} \Rightarrow \mu_{\text{blue}} > \mu_{\text{red}}$

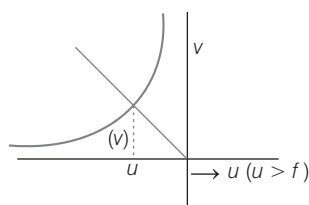
Hence, $f_{\text{red}} > f_{\text{blue}}$

- 87** In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v , from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x -axis meets the experimental curve at P . The coordinates of P will be [AIEEE 2009]

- (a) $(2f, 2f)$ (b) $\left(\frac{f}{2}, \frac{f}{2}\right)$
(c) (f, f) (d) $(4f, 4f)$

Ans. (a)

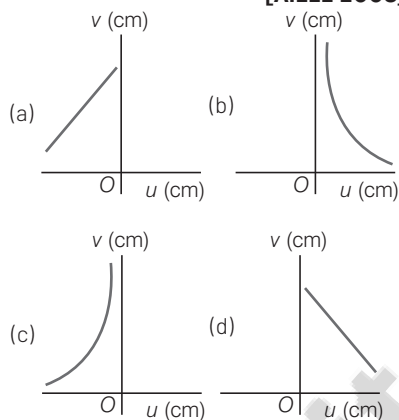
It is possible when object kept at centre of curvature.



For $u = v$, $u = 2f$, $v = 2f$

- 88** A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

[AIEEE 2008]



Ans. (c)

Experimental observation at $v = \infty$, $u = -f$

At $u = \infty$, $v = f$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \text{constant}$$

So, (c) is the correct graph.

- 89** Two lenses of power -15D and $+5\text{D}$ are in contact with each other. The focal length of the combination is [AIEEE 2007]

- (a) -20 cm (b) -10 cm
(c) $+20\text{ cm}$ (d) $+10\text{ cm}$

Ans. (b)

Power of a lens is reciprocal of its focal length. Power of combined lens is

$$\begin{aligned} P &= P_1 + P_2 \\ &= -15 + 5 \\ &= -10 \text{ D} \end{aligned}$$

$$\therefore f = \frac{1}{P} = \frac{100}{-10} \text{ cm}$$

or $f = -10\text{ cm}$

- 90** A thin glass (refractive index 1.5) lens has optical power of -5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be [AIEEE 2005]

- (a) 1 D (b) -1 D
(c) 25 D (d) -25 D

Ans. (a)

$$\frac{1}{f_a} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

$$= (1.5 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{and } \frac{1}{f_m} = \left(\frac{\mu_g - \mu_m}{\mu_m} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

$$\frac{1}{f_m} = \left(\frac{1.5 - 1}{1.6} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{Thus, } \frac{f_m}{f_a} = \frac{(1.5 - 1)}{\left(\frac{1.5 - 1}{1.6} \right)} = -8$$

$$\text{or } f_m = -8 \times f_a = -8 \times \frac{-1}{5} = 1.6 \text{ m}$$

$$\left[\because f_a = \frac{1}{P} = -\frac{1}{5} \text{ m} \right]$$

$$\therefore P_m = \frac{\mu}{f_m} = \frac{1.6}{1.6} = 1 \text{ D}$$

- 91** A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object [AIEEE 2004]

- (a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

Ans. (a)

A plano-convex lens behaves as a concave mirror, if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens be F .

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering,

f_m = focal length of spherical mirror.

$$\frac{1}{F} = \frac{2}{f} + \frac{1}{R} \quad [\because R = 2f_m] \quad \dots(i)$$

$$\text{Now, } \frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

Here, $R_1 = \infty$, $R_2 = 30\text{ cm}$

$$\therefore \frac{1}{f} = (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{30} \right)$$

$$\text{or } \frac{1}{f} = -\frac{0.5}{30} = -\frac{1}{60} \text{ or } f = -60 \text{ cm}$$

Hence, from Eq. (i), we get

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10 \text{ cm}$$

Again given that,

Size of object = Size of image

i.e., $O = I$

$$\therefore m = -\frac{v}{u} = \frac{l}{O} \Rightarrow \frac{v}{u} = -1$$

or $v = -u$

Thus, from lens formula,

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$$\therefore u = -20 \text{ cm}$$

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

TOPIC 4

Optical Instruments

- 92** An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification 6, gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to cm.

[2021, 20 July Shift-I]

Ans. (25)

Given, magnification, $M=6$

Since we know that magnifying power of a simple microscope is given by

$$M = 1 + \frac{D}{f_0}$$

where, D = least distance of distinct vision = 25 cm

and f_0 = focal length of objective lens.

$$\Rightarrow 6 = 1 + \frac{D}{f_0}$$

$$\Rightarrow 6 = 1 + \frac{25}{f_0}$$

$$\Rightarrow 5 = \frac{25}{f_0} \Rightarrow f_0 = 5 \text{ cm}$$

For compound microscope, magnifying power is given by

$$M = \frac{l \cdot D}{f_0 f_e} = 2M_{\text{simple microscope}}$$

where, f_0 and f_e are the focal lengths of the objective lens and eye piece respectively and l = length of the given tube = 0.6 m

$$\Rightarrow 12 = \frac{60 \times 25}{5 \cdot f_e}$$

[$\because$ magnification is doubled]

$$\Rightarrow f_e = 25 \text{ cm}$$

This is the required focal length of eyepiece.

- 93** Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as

[2021, 18 March Shift-I]

- (a) astigmatism
(b) myopia with astigmatism
(c) presbyopia with astigmatism
(d) myopia and hypermetropia

Ans. (b)

A friend is not seen clearly the distant object, then its diagnosis is myopia because in myopia the distant object is blurry and it also appear non-uniform and distorted images of the object, then its diagnosis is astigmatism also.

- 94** Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) For a simple microscope, the angular size of the object equals the angular size of the image.

Reason (R) Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence, it subtends a large angle.

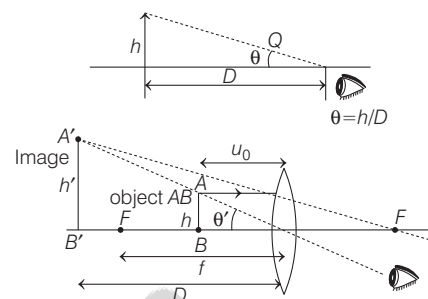
In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 26 Feb Shift-II]

- (a) A is true but R is false.
(b) Both A and R are true but R is not the correct explanation of A.
(c) Both A and R are true and R is the correct explanation of A.
(d) A is false but R is true.

Ans. (c)

The formation of image with simple microscope is shown below.



$$\text{Here, } \theta' = \frac{h}{u_0} = \frac{h'}{D} = \frac{h'}{25}$$

where, $D = 25 \text{ cm}$ (least distance of distinct vision)

Here, θ' is same for both object and image, hence Assertion is true.

$$\text{Magnification, } m = \frac{\theta'}{\theta} = \frac{D}{u_0}$$

Hence, if $u_0 < D$ (25 cm), hence the value of θ' will obtain large.

So, option (c) is the correct.

- 95** In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is

[2020, 4 Sep Shift-I]

Ans. (6)

When the final image is formed at least distance of distinct vision (i.e., at 25 cm) from eye-piece of a compound microscope, then the magnification is given by

$$m = -\frac{v_0}{u_0} \left(1 + \frac{D}{f_e} \right)$$

$$\Rightarrow m = \frac{-l}{f_0} \left(1 + \frac{D}{f_e} \right)$$

$$\Rightarrow -100 = -\frac{20}{1} \left(1 + \frac{25}{f_e} \right)$$

$$\Rightarrow 5 = 1 + \frac{25}{f_e}$$

$$4 = \frac{25}{f_e}$$

$$\Rightarrow f_e = \frac{25}{4} = 6.25 \approx 6 \text{ cm}$$

- 96** A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is

[2020, 5 Sep Shift-I]

Ans. (50)

Strain on the eye is minimum so the intermediate image is formed at focal length of eye-piece.

For objective, $v = 5$ cm, $f = 1$ cm

$\therefore$ Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{5} - \frac{1}{u} = \frac{1}{1}$$

$$\Rightarrow u = \frac{-5}{4} \text{ cm}$$

$$\Rightarrow |u| = \frac{5}{4} \text{ cm}$$

$$\Rightarrow \frac{n}{40} = \frac{5}{4}$$

$$\Rightarrow n = \frac{5}{4} \times 40$$

$$\Rightarrow n = 50$$

- 97** If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eyepiece should be close to

[2020, 7 Jan Shift-I]

- (a) 22 mm (b) 2 mm
(c) 12 mm (d) 33 mm

Ans. (a)

In normal setting, magnification obtained by a microscope is given by

$$m = \frac{L \times D}{f_o \times f_e}$$

where,

L = tube length = 150 mm,

D = distance of distinct vision = 25 cm = 250 mm,

f_o = focal length of objective = 5 mm,

f_e = focal length of eyepiece

and m = magnification = 375.

So, we have

$$375 = \frac{150 \times 250}{5 \times f_e}$$

$$\text{or } f_e = 20 \text{ mm}$$

So, focal length of eyepiece required is closest to 22 mm.

- 98** The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eyepiece? [2020, 8 Jan Shift-I]

- (a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

Ans. (a)

For a telescope in normal setting,

$$f_o + f_e = L$$

(length of the tube of telescope)

$$\text{and } \frac{f_o}{f_e} = m \text{ (magnification)}$$

where, f_o and f_e is the focal length of the objective and eyepiece, respectively.

According to the given values in the question, we have

$$f_o + f_e = 60 \text{ cm and } \frac{f_o}{f_e} = 5$$

$$\Rightarrow f_o = 5 f_e$$

Solving the above equations, we have

$$f_e = 10 \text{ cm}$$

- 99** An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

[JEE Main 2016]

- (a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

Ans. (c)

Height of image depends upon the magnifying power to see a 20 times taller object, as the angular magnification should be 20 and we observe angular magnification. Option (c) would not be very correct as the telescope can be adjusted to form the image anywhere between infinity and least distance for distinct vision.

Suppose that the image is formed at infinity. Then, the observer will have to focus the eyes at infinity to observe the image. Hence, it is incorrect to say that the image will be appear nearer to the observer.

- 100** The image formed by an objective of a compound microscope is

[AIIEE 2003]

- (a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

Ans. (c)

Objective of compound microscope is a convex lens. Convex lens form real and enlarged image when an object is placed between its focus and lens.

24

Wave Optics

TOPIC 1

Huygens Principle and Interference of Light

- 01** The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is $x:4$ where x is

[2021, 1 Sep Shift-II]

Ans. (1)

Given, $b_1 = 3b_2$

Here, b_1 = width of the one of the two slit and b_2 = width of the other slit.

As we know that,

Intensity, $I \propto (\text{Amplitude})^2$

$$\Rightarrow \frac{I_1}{I_2} = \left(\frac{b_1}{b_2}\right)^2 \Rightarrow \frac{I_1}{I_2} = \left(\frac{3b_2}{b_2}\right)^2$$

$$I_1 = 9I_2$$

As we know, the ratio of the minimum intensity to the maximum intensity in the interference pattern,

$$\frac{I_{\min}}{I_{\max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2$$

Substituting the values in the above equations, we get

$$\frac{I_{\min}}{I_{\max}} = \left(\frac{\sqrt{9I_2} - \sqrt{I_2}}{\sqrt{9I_2} + \sqrt{I_2}}\right)^2 = \left(\frac{3-1}{3+1}\right)^2$$

$$\frac{I_{\min}}{I_{\max}} = \frac{1}{4}$$

Comparing with, $\frac{I_{\min}}{I_{\max}} = \frac{x}{4}$

The value of $x = 1$.

- 02** In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright is 2.4 cm. The frequency of light used is $\times 10^{14}$ Hz. [2021, 31 Aug Shift-II]

Ans. (5)

Given, slit separation, $d = 0.3 \text{ mm}$
 $= 0.3 \times 10^{-3} \text{ m}$

Distance between slit and screen,
 $D = 1.5 \text{ m}$

Distance between 4th bright fringes
 $= 2Y_4 = 2.4 \text{ cm}$

Position of 4th fringe,

$$Y_4 = \frac{2.4}{2} = 1.2 \text{ cm} = 1.2 \times 10^{-2} \text{ m}$$

Using equation, $Y_n = \frac{n\lambda D}{d}$

For $n = 4$

$$1.2 \times 10^{-2} = \frac{4 \times \lambda \times 1.5}{0.3 \times 10^{-3}}$$

$\therefore$ Wavelength, $\lambda = 6 \times 10^{-7} \text{ m}$

Using $c = \lambda f$

$$\Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6 \times 10^{-7}} = 5 \times 10^{14}$$

Hz

Hence, the frequency of light is
 $5 \times 10^{14} \text{ Hz}$.

So, correct answer is 5.

- 03** The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima?

[2021, 27 Aug Shift-II]

- (a) I_0 (b) $2I_0$ (c) $5I_0$ (d) $4I_0$

Ans. (d)

Given, intensity of two light waves

$$I_1 = I_2 = I_0$$

$$I_{\min} = 0$$

Since, $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$\text{and } I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$= (\sqrt{I_0} + \sqrt{I_0})^2 = (2\sqrt{I_0})^2 = 4I_0$$

- 04** White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and violet light is nm. [2021, 26 Aug Shift-I]

Ans. (300)

In a Young's double slit experiment for interference pattern, the position of bright fringe is given by

$$y_n = \frac{n\lambda D}{d}$$

Here, $D = 1.5 \text{ m}$,

$$d = 0.3 \times 10^{-3} \text{ m}$$

and $n = 1$

$$\lambda = \frac{y_n d}{D}$$

For first violet, $y_n = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$\therefore \lambda_{\text{violet}} = 2 \times 10^{-3} \frac{d}{D}$$

For first red $y_n = 3.5 \text{ mm}$

$$= 3.5 \times 10^{-3} \text{ m}$$

$$\lambda_{\text{red}} = 3.5 \times 10^{-3} \frac{d}{D}$$

The difference in wavelengths of red and violet light is

$$\begin{aligned}
 \Delta\lambda &= \lambda_{\text{red}} - \lambda_{\text{violet}} \\
 &= 3.5 \times 10^{-3} \frac{d}{D} - 2 \times 10^{-3} \frac{d}{D} \\
 &= \frac{d}{D} (1.5) \times 10^{-3} \\
 &= \frac{0.3 \times 10^{-3}}{1.5} \times 1.5 \times 10^{-3} \\
 &= 0.3 \times 10^{-6} \\
 &= 0.3 \times 10^{-6} \times 10^3 \times 10^{-3} \\
 &= 300 \times 10^{-9} \\
 &= 300 \text{ nm}
 \end{aligned}$$

- 05** In Young's double slit experiment, if the source of light changes from orange to blue, then

[2021, 27 July Shift-I]

- the central bright fringe will become a dark fringe
- the distance between consecutive fringes will decrease
- the distance between consecutive fringes will increase
- the intensity of the minima will increase

Ans. (b)

We know that in Young's double slit experiment, fringe width, $\beta = \frac{D\lambda}{d}$

$$\begin{aligned}
 \beta &\propto \lambda \\
 \text{Since, } \lambda_{\text{blue}} &< \lambda_{\text{orange}} \\
 \text{Thus, } \beta_{\text{blue}} &< \beta_{\text{orange}}
 \end{aligned}$$

$\therefore$ In Young's double slit experiment, if the source of light changes from orange to blue, then distance between the consecutive fringes will decrease.

- 06** In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin \omega t$, where d_0 , ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as

[2021, 25 July Shift-I]

- $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$
- $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$
- $\frac{\lambda D}{d_0^2} a_0$
- $\frac{\lambda D}{d_0 + a_0}$

Ans. (b)

Given, separation between slits,

$$d(t) = d_0 + a_0 \sin \omega t$$

As we know that,

$$\text{Fringe width } (\beta) = \frac{\lambda D}{d}$$

where, λ = wavelength,

D = distance between slit and screen

and d = distance between slits.

$$\Rightarrow \beta = \frac{\lambda D}{d_0 + a_0 \sin \omega t}$$

Now, maximum fringe width

$$(\beta_{\text{max}}) = \frac{\lambda D}{d_0 - a_0} \quad (\text{for } \sin \omega t = -1)$$

and minimum fringe width

$$(\beta_{\text{min}}) = \frac{\lambda D}{d_0 + a_0} \quad (\text{for } \sin \omega t = +1)$$

$$\begin{aligned}
 \therefore \beta_{\text{max}} - \beta_{\text{min}} &= \lambda D \left[\frac{1}{d_0 - a_0} - \frac{1}{d_0 + a_0} \right] \\
 &= \lambda D \left[\frac{d_0 + a_0 - d_0 + a_0}{(d_0 - a_0)(d_0 + a_0)} \right] \\
 &= \lambda D \left[\frac{2a_0}{d_0^2 - a_0^2} \right]
 \end{aligned}$$

- 07** In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is

[2021, 18 March Shift-I]

- $1178 \times 10^{-9} \text{ m}$
- $1178 \times 10^{-6} \text{ m}$
- $1178 \times 10^{-12} \text{ m}$
- $5890 \times 10^{-7} \text{ m}$

Ans. (b)

Given,

The distance of sources from the screen, $D = 0.5 \text{ m}$

The distance between the slits,

$$d = 0.5 \text{ mm} = 0.0005 \text{ m}$$

The wavelength of the monochromatic light,

$$\lambda = 5890 \text{ Å} = 5890 \times 10^{-10} \text{ m}$$

The distance between the first and the third bright fringe formed,

$$Y = 2\beta$$

$$\text{As we know, } \beta = \frac{\lambda D}{d}$$

Substituting the values in the above equation,

$$\begin{aligned}
 Y &= 2 \left(\frac{\lambda D}{d} \right) \\
 &= 2 \left(\frac{5890 \times 10^{-10} \times 0.5}{0.0005} \right) \\
 &= 1178 \times 10^{-6} \text{ m}
 \end{aligned}$$

- 08** A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10

m away. The wavelength of light used is $x \text{ nm}$. The value of x to the nearest integer is

[2021, 16 March Shift-I]

Ans. (600)

Given, Slit width, $d = 1 \text{ mm} = 10^{-3} \text{ m}$

Fringe width, $\beta = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

Distance between the screen and the slit, $D = 10 \text{ m}$

Wavelength of light used $\lambda = ?$

$$\begin{aligned}
 \text{As, } \beta &= \frac{D\lambda}{d} \Rightarrow \lambda = \frac{\beta d}{D} \\
 &= \frac{6 \times 10^{-3} \times 10^{-3}}{10} = 6 \times 10^{-6} \times 10^{-1} \\
 &= 6 \times 10^{-7} \text{ m} = 600 \times 10^{-9} \text{ m} \\
 &= 600 \text{ nm}
 \end{aligned}$$

Comparing with the given value in the question i.e., $x \text{ nm}$, the value of $x = 600$.

- 09** In a Young's double slit experiment, two slits are separated by 2 mm and the screen is placed one metre away. When a light of wavelength 500 nm is used, the fringe separation will be

[2021, 26 Feb Shift-I]

- 0.25 mm
- 0.50 mm
- 0.75 mm
- 1 mm

Ans. (a)

Given, in YDSE, the separation between two slits, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

Distance between slit and screen, $D = 1 \text{ m}$

Wavelength of light, $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$

Let B will be the fringe width

$$\begin{aligned}
 \therefore B &= \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1}{2 \times 10^{-3}} = 250 \times 10^{-6} \\
 &= 0.25 \text{ mm}
 \end{aligned}$$

- 10** Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$ will be

[2021, 25 Feb Shift-I]

- $\frac{2\sqrt{2x}}{x+1}$
- $\frac{\sqrt{2x}}{2x+1}$
- $\frac{\sqrt{2x}}{x+1}$
- $\frac{2\sqrt{2x}}{2x+1}$

Ans. (d)

$$\text{Given, } \frac{I_1}{I_2} = 2x \Rightarrow I_1 = 2I_2 x$$

As we know,

$$\begin{aligned}
 I_{\text{max}} &= (\sqrt{I_1} + \sqrt{I_2})^2 \\
 \text{and } I_{\text{min}} &= (\sqrt{I_1} - \sqrt{I_2})^2
 \end{aligned}$$

$$\therefore \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

(using dividendo rule)

$$= \frac{2\sqrt{2I_2 \times I_2}}{2I_2 + I_2} = \frac{2\sqrt{2}x}{2x + 1}$$

- 11** If the source of light used in a Young's double slit experiment is changed from red to violet, then
[2021, 24 Feb Shift-II]

- (a) the consecutive fringe lines will come closer
(b) the central bright fringe will become a dark fringe
(c) the fringes will become brighter
(d) the intensity of minima will increase

Ans. (a)

According to Young's double slit experiment,

The distance of n th bright fringe from the centre,

$$y_n = \frac{n\lambda D}{d}$$

Since, $\lambda_{\text{violet}} < \lambda_{\text{red}}$

$\therefore y_{\text{violet}} < y_{\text{red}}$

$\therefore$ Consecutive fringe lines will come closer.

- 12** The focal length f is related to the radius of curvature r of the spherical convex mirror by
[2021, 24 Feb Shift-I]

- (a) $f = r$ (b) $f = -r$
(c) $f = -\frac{r}{2}$ (d) $f = +\frac{r}{2}$

Ans. (d)

For convex mirror, the focal length (f) and radius of curvature (r) are related as

$$f = +\frac{r}{2}$$

- 13** In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.
[2021, 24 Feb Shift-I]

- (a) 4 : 1 (b) 2 : 1
(c) 1 : 4 (d) 3 : 1

Ans. (a)

Given, amplitude $\propto$ width of slit

$$\Rightarrow A_2 = 3A_1$$

We know that,

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$\therefore$ Intensity, $I \propto A^2$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$= \left(\frac{A_1 + 3A_1}{A_1 - 3A_1} \right)^2 = \left(\frac{4A_1}{-2A_1} \right)^2 = \frac{4}{1}$$

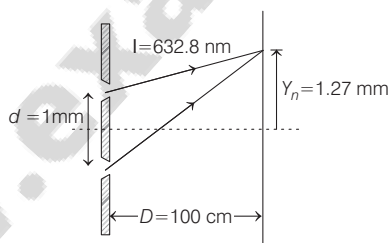
$$\therefore I_{\max} : I_{\min} = 4 : 1$$

- 14** Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source ($\lambda = 632.8$ nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to
[2020, 2 Sep Shift-I]

- (a) 2 nm (b) 1.27 μ m
(c) 2.87 nm (d) 2.05 μ m

Ans. (b)

Given, setup of Young's double slit experiment is as shown



Distance of a bright fringe from central bright fringe is

$$y_n = \frac{n\lambda D}{d}$$

where, $n\lambda$ = path difference between rays from

$$\text{lower and upper slit} = \frac{y_n d}{D}$$

$$= \frac{1.27 \times 10^{-3} \times 10^{-3}}{100 \times 10^{-2}}$$

$$= 1.27 \times 10^{-6} \text{ m} = 1.27 \mu\text{m}$$

Hence, correct option is (b).

- 15** In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed

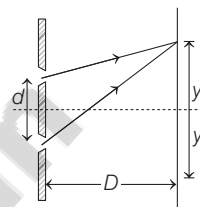
to 400 nm, the number of fringes observed in the same segment of the screen would be

[2020, 2 Sep Shift-II]

- (a) 24 (b) 30
(c) 18 (d) 28

Ans. (d)

In YDSE, let segment length is $2y$ as shown in figure.



$$\text{Then, } 2y = 2 \times N \times \frac{\lambda D}{d}$$

where, N = number of fringes in segment length $2y$.

$$\Rightarrow N\lambda = \frac{yd}{D}$$

So, for same segment length $N\lambda$ is constant.

$$\Rightarrow N_1 \lambda_1 = N_2 \lambda_2$$

$$\text{Given, } N_1 = 16, \lambda_1 = 700 \text{ nm}$$

$$N_2 = ?, \lambda_2 = 400 \text{ nm}$$

$$\Rightarrow N_2 = \frac{N_1 \lambda_1}{\lambda_2} = 16 \times \frac{700}{400} = 28$$

$\therefore$ 28 fringes are observed in same segment length.

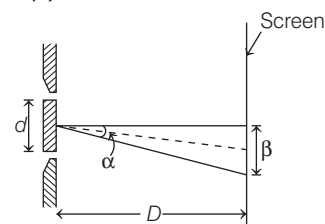
Hence, correct option is (d).

- 16** In Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to

[2020, 3 Sep Shift-I]

- (a) 0.17° (b) 0.57°
(c) 1.7° (d) 0.07°

Ans. (b)



Angular width of central maxima in YDSE is given by,

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d} \text{ (in radian)}$$

$$= \frac{\lambda}{d} \times \frac{180}{\pi} \text{ (in degree)}$$

Here, $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$
and $d = 0.05 \text{ mm} = 0.05 \times 10^{-3} \text{ m}$.

Hence, angular width is

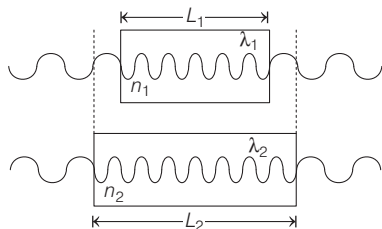
$$\alpha = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}} \times \frac{180}{\pi} = 0.57^\circ$$

Hence, option (b) is correct.

- 17** Two light waves having the same wavelength λ in vacuum are in phase initially. Then, the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2 . After this, the phase difference between the two waves is [2020, 3 Sep Shift-II]

(a) $\frac{2\pi}{\lambda}(n_1 L_1 - n_2 L_2)$ (b) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_1}{n_2}\right)$
(c) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$ (d) $\frac{2\pi}{\lambda}(n_2 L_1 - n_1 L_2)$

Ans. (a)



Let λ_1 be the wavelength inside medium of refractive index n_1 and it travels a length L_1 . Let λ_2 , n_2 and L_2 be the parameters for second wave.

So, $\frac{\lambda_1}{v} = \frac{\lambda}{c} \Rightarrow \lambda_1 = \frac{\lambda}{n_1}$ and $\lambda_2 = \frac{\lambda}{n_2}$

Let there are N_1 number of wavelengths in medium 1 of length L_1 , then

$$N_1 = \frac{L_1}{\lambda_1} = \frac{L_1 n_1}{\lambda} \text{ and } N_2 = \frac{L_2}{\lambda_2} = \frac{L_2 n_2}{\lambda}$$

So, difference in number of waves, ΔN

$$= N_1 - N_2 = \frac{L_1 n_1}{\lambda} - \frac{L_2 n_2}{\lambda} = \frac{L_1 n_1 - L_2 n_2}{\lambda}$$

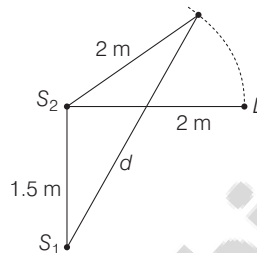
As each wavelength is equivalent to a phase of 2π , we can write

$$\Delta\phi = 2\pi\Delta N = 2\pi(N_1 - N_2)$$

$$= 2\pi \frac{L_1 n_1 - L_2 n_2}{\lambda}$$

Hence, correct option is (a).

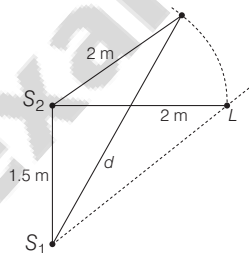
- 18** Two coherent sources of sound S_1 and S_2 , produce sound waves of the same wavelength $\lambda = 1 \text{ m}$, in phase. S_1 and S_2 are placed 1.5 m apart (see figure). A listener, located at L , directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2 . The listener moves away from S_1 , keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1 . Then, d is [2020, 5 Sep Shift-II]



- (a) 12 m (b) 5 m (c) 2 m (d) 3 m

Ans. (d)

In the given figure, join the points S_1 and L .



From figure, $S_2L = 2 \text{ m}$

$$S_1L = \sqrt{(2)^2 + (1.5)^2}$$

$$= \sqrt{4 + 2.25} = 2.5 \text{ m}$$

Initial path difference where minima is observed,

$$\Delta x_1 = S_1L - S_2L = 2.5 - 2 = 0.5 \text{ m}$$

Path difference at new position, where maxima is observed,

$$\Delta x_2 = d - 2$$

Path difference between adjacent maxima and minima is $\frac{\lambda}{2}$.

$$\text{i.e. } \Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

$$\Rightarrow d - 2 - 0.5 = \frac{1}{2} \quad (\because \lambda = 1 \text{ m})$$

$$\Rightarrow d = 3 \text{ m}$$

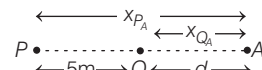
Hence, correct option is (d).

- 19** In the given figure, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m . The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90° . A , B and C are three distinct points of observation, each equidistant from the mid-point of PQ . The intensities of radiation at A , B and C will be in the ratio [2020, 6 Sep Shift-I]

- (a) $0 : 1 : 4$ (b) $2 : 1 : 0$
(c) $0 : 1 : 2$ (d) $4 : 1 : 0$

Ans. (b)

For point A



Path difference, $\Delta x = x_{PA} - x_{QA} = 5 \text{ m}$

Phase difference due to path difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{20} \times 5 \Rightarrow \Delta\phi = \frac{\pi}{2}$$

Also, phase of P is ahead of Q by 90° or $\frac{\pi}{2}$.

So, total phase difference at point A ,

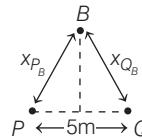
$$\Delta\phi = \frac{\pi}{2} - \frac{\pi}{2} = 0^\circ$$

$\therefore$ Intensity at point A ,

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$$

$$= I_0 + I_0 + 2I_0 \cos 0^\circ = 4I_0$$

For point B



$$\Delta x = x_{PB} - x_{QB} = 0$$

$$\therefore \Delta\phi = 0$$

(due to path difference)

So, total phase difference at point B ,

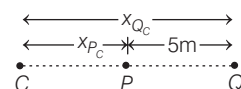
$$\Delta\phi = \frac{\pi}{2} + 0 = 90^\circ$$

$\therefore$ Intensity at point B ,

$$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\phi$$

$$= I_0 + I_0 + 2I_0 \cos 90^\circ = 2I_0$$

For point C



$$\Delta x = x_{0C} - x_{PC} = 5 \text{ m}$$

$\Delta\phi$ due to path difference,

$$\Delta\phi = \pi/2$$

Total phase difference at point C,

$$\Delta\phi = \frac{\pi}{2} + \frac{\pi}{2} = \pi = 180^\circ$$

($\therefore$ phase of P is ahead of Q by 90°)

$\therefore$ Intensity at point C,

$$I_C = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\Delta\phi$$

$$= I_0 + I_0 + 2I_0 \cos 180^\circ = 0$$

Ratio of intensities,

$$I_A : I_B : I_C = 4I_0 : 2I_0 : 0$$

$$I_A : I_B : I_C = 2 : 1 : 0$$

- 20** A Young's double slit experiment is performed using monochromatic light of wavelength λ . The intensity of light at a point on the screen, where the path difference is λ , is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is

[2020, 6 Sep Shift-II]

Ans. (9)

For path difference of λ , i.e. $\Delta x = \lambda$
 Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\Delta\phi$
 As, light is monochromatic, so

$$I_1 = I_2 = I_0 \text{ (say)}$$

and $I = K$ (given)

$$\Rightarrow K = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos 2\pi$$

$$K = 4I_0 \text{ or } I_0 = \frac{K}{4}$$

For path difference of $\frac{\lambda}{6}$, i.e. $\Delta x = \frac{\lambda}{6}$

$$\text{Phase difference, } \Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$$

So, intensity, $I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\Delta\phi$

$$= I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \frac{\pi}{3}$$

$$= \frac{K}{4} + \frac{K}{4} + 2 \times \frac{K}{4} \times \frac{1}{2} = \frac{3K}{4} = \frac{9K}{12}$$

$$\text{Given, } I' = \frac{nK}{12}$$

On comparing the both, we get
 $n = 9$

- 21** In a Young's double slit experiment, the separation between the slits is 0.15 mm . In the experiment, a

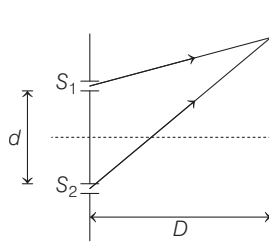
source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is

[2020, 7 Jan Shift-II]

- (a) 4.9 mm (b) 3.9 mm
 (c) 6.9 mm (d) 5.9 mm

Ans. (d)

In YDSE, separation between successive bright fringes is width of one dark fringe.



As, fringe width in YDSE,

$$\beta = \frac{\lambda D}{d} \quad \dots (i)$$

Here, $\lambda = 589 \text{ nm} = 589 \times 10^{-9} \text{ m}$,

$$D = 1.5 \text{ m}, d = 0.15 \text{ mm} = 0.15 \times 10^{-3} \text{ m}$$

Substituting the given values in Eq. (i), we get

$$\beta = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}} = 589 \times 10^{-5} \text{ m}$$

$$= 5.89 \text{ mm or } 5.9 \text{ mm}$$

- 22** In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$ th of a

wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is

[2020, 8 Jan Shift-II]

- (a) 0.568 (b) 0.853 (c) 0.760 (d) 0.672

Ans. (b)

In YDSE, intensity at a point is given by

$$I_{\text{net}} = 2I_0(1 + \cos\phi) \quad \dots (i)$$

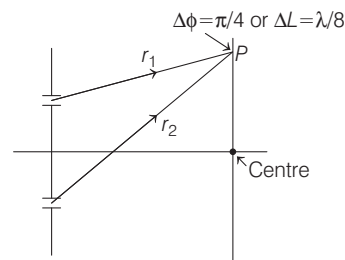
where, ϕ = phase difference between interfering waves.

$$\text{Here, } \phi = \frac{1}{8} \lambda = \frac{1}{8} \times 2\pi$$

(a path difference of one wavelength is equivalent to a phase difference of 2π radians)

$$\Rightarrow \phi = \frac{\pi}{4} \Rightarrow \cos\phi = \frac{1}{\sqrt{2}} \approx 0.7$$

So, I_{net} at given point P is



Also, intensity maxima at centre of a bright fringe is obtained by substituting

$$\phi = 0, 2\pi, \dots, n\pi \text{ in Eq. (i).}$$

So, $I_{\text{net}} = I_2 = 2I_0(1 + \cos 0^\circ) = 4I_0$

Hence, required ratio is

$$\frac{I_1}{I_2} = \frac{2 \times 1.7 I_0}{4 I_0}$$

$$= \frac{1.7}{2} = 0.853$$

- 23** In a Young's double slit experiment, 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then, the value of λ is (in nm)

[2020, 9 Jan Shift-II]

Ans. (750)

Given, in a YDSE,

Fringe width of 15 fringes for wavelength of light of 500 nm

Fringe width of 10 fringes for wavelength of light of λ_{nm}

$$\Rightarrow \frac{15 \times 500 \times D}{d} = \frac{10 \times \lambda \times D}{d}$$

$$\Rightarrow \lambda = \frac{15 \times 500}{10} = 750 \text{ nm}$$

- 24** In an interference experiment, the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of

maximum and minimum intensities of fringes will be

- (a) 2 (b) 18
 (c) 4 (d) 9

[2019, 8 April Shift-I]

Ans. (c)

In Young's double slit experiment, ratio of maxima and minima intensity is given by

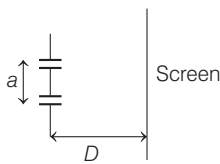
$$\frac{I_{\text{max}}}{I_{\text{min}}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} \right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1} \right)^2$$

As, intensity (I) $\propto$ [amplitude (a)]²

$$\therefore \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\text{So, } \frac{I_{\max}}{I_{\min}} = \left(\frac{\frac{1}{3} + 1}{\frac{1}{3} - 1}\right)^2 = 4 : 1$$

- 25** The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ , t will be **[2019, 9 April Shift-I]**



- (a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{2D\lambda}{a(\mu-1)}$
(c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{nD\lambda}{a(\mu-1)}$

Ans. (*)

Path difference introduced by a slab of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

Position of the fringe is

$$x = \frac{\Delta D}{d} = \frac{(\mu - 1)t D}{d}$$

Also, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

According to the question,

$$\Rightarrow \frac{n\lambda D}{d} = (\mu - 1)t \frac{D}{d}$$

$$\Rightarrow n\lambda = (\mu - 1)t$$

$$\text{or } t = \frac{n\lambda}{(\mu - 1)}$$

Alternate Solution

$$\text{Path difference, } \Delta = \frac{xd}{D} = (\mu - 1)t$$

$$\text{or } t = \frac{xd}{(\mu - 1)D}$$

$$\text{Also, } \beta = \frac{\lambda D}{d} \Rightarrow \frac{d}{D} = \frac{\lambda}{\beta}$$

$$\therefore t = \frac{x\lambda}{(\mu - 1)\beta} = \frac{n\lambda D \times \lambda}{d(\mu - 1)\beta}$$

$$[\because x = n\beta]$$

$$\Rightarrow t = \frac{n\lambda}{(\mu - 1)}$$

So, no option is correct.

- 26** In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be **[2019, 10 April Shift-II]**

- (a) 4 : 1 (b) 25 : 9
(c) 9 : 1 (d) $(\sqrt{3} + 1)^4 : 16$

Ans. (c)

Key Idea Amplitude of light is directly proportional to area through which light is passing.

For same length of slits,

$$\text{amplitude} \propto (\text{width})^{1/2}$$

$$\text{Also, intensity} \propto (\text{amplitude})^2$$

In YDSE, ratio of intensities of maxima and minima is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

where, I_1 and I_2 are the intensities obtained from two slits, respectively.

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

where, a_1 and a_2 are light amplitudes from slits 1 and 2, respectively.

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2}$$

where, W_1 and W_2 are the widths of slits, respectively.

$$\text{Here, } \left(\frac{W_1}{W_2}\right) = \left(\frac{a_1}{a_2}\right)^2 = \frac{4}{1} \Rightarrow \sqrt{\frac{W_1}{W_2}} = \frac{2}{1}$$

So, we have

$$\begin{aligned} \frac{I_{\max}}{I_{\min}} &= \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2} = \left(\frac{\sqrt{\frac{W_1}{W_2}} + 1}{\sqrt{\frac{W_1}{W_2}} - 1}\right)^2 \\ &= \left(\frac{2 + 1}{2 - 1}\right)^2 = 9 : 1 \end{aligned}$$

- 27** In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the

maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used)

[2019, 12 April Shift-I]

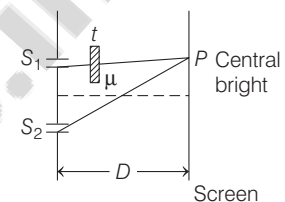
- (a) $\frac{2\lambda}{(\mu-1)}$ (b) $\frac{\lambda}{2(\mu-1)}$
(c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{(2\mu-1)}$

Ans. (c)

As we know,

Path difference introduced by thin film,

$$\Delta = (\mu - 1)t \quad \dots(i)$$



and if fringe pattern shifts by one fringes width, then path difference,

$$\Delta = 1 \times \lambda = \lambda \quad \dots(ii)$$

So, from Eqs. (i) and (ii), we get

$$\begin{aligned} (\mu - 1)t &= \lambda \\ \Rightarrow t &= \frac{\lambda}{\mu - 1} \end{aligned}$$

Alternate Solution

Path difference introduced by the thin film of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

$\therefore$ Position of the fringe is

$$x = \frac{\Delta D}{d} = \frac{(\mu - 1)t D}{d} \quad \dots(i)$$

Fringe width of one fringe is given by

$$\beta = \frac{\lambda D}{d} \quad \dots(ii)$$

Given that $x = \beta$, so from Eqs. (i) and (ii), we get

$$\Rightarrow \frac{(\mu - 1)t D}{d} = \frac{\lambda D}{d}$$

$$\Rightarrow (\mu - 1)t = \lambda \text{ or } t = \frac{\lambda}{(\mu - 1)}$$

- 28** Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **[2019, 9 Jan Shift-I]**

- (a) 16 : 9 (b) 5 : 3
(c) 25 : 9 (d) 4 : 1

Ans. (c)

Let the intensity of two coherent sources be I_1 and I_2 , respectively.

It is given that,

$$\frac{\text{maximum intensity}}{\text{minimum intensity}} = \frac{I_{\max}}{I_{\min}} = \frac{16}{1}$$

Since, we know,

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \text{ and } I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\therefore \text{We can write, } \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{4}{1} \Rightarrow \sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow 5\sqrt{I_2} = 3\sqrt{I_1}, \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

$$\text{Squaring both the sides, we get } \frac{I_1}{I_2} = \frac{25}{9}$$

- 29** In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \leq \theta \leq 30^\circ$ is

[2019, 9 Jan Shift-II]

- (a) 320 (b) 321
(c) 640 (d) 641

Ans. (d)

In Young's double slit experiment, the condition of bright fringe and dark fringe are,

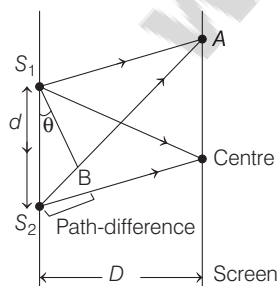
for bright fringes (maxima), path difference = $n\lambda$

$$d \sin \theta = n\lambda$$

for dark fringes (minima),

$$\text{path-difference} = (2n-1) \frac{\lambda}{2}$$

$$d \sin \theta = (2n-1) \frac{\lambda}{2}$$



For the given question, distance between slits (d) = 0.320 mm
Wavelength of light used (λ) = 500 nm

Angular range for bright fringe

$$(\theta) = -30^\circ \text{ to } 30^\circ$$

Hence, for bright-fringe,

$$n\lambda = d \sin \theta$$

$$n = \frac{d \sin \theta}{\lambda}$$

$$= \frac{0.320 \times 10^{-3} \times \sin 30^\circ}{500 \times 10^{-9}}$$

$$n_{\max} = 320$$

$\therefore$ Total number of maxima between the two lines are

$$n = (n_{\max} \times 2) + 1$$

Here, $n = (320 \times 2) + 1$

$$n = 641$$

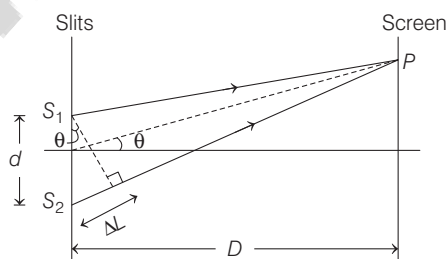
- 30** In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$

rad by using light of wavelength λ_1 . When the light of the wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are [2019, 10 Jan Shift-I]

- (a) 380 nm, 525 nm
(b) 400 nm, 500 nm
(c) 380 nm, 500 nm
(d) 625 nm, 500 nm

Ans. (d)

Key Idea In a YDSE, path difference between 2 rays, reaching at some common point P located at angular position θ as shown in the figure below is



$$\Delta L = d \sin \theta$$

For small value of θ ; $\sin \theta \approx \theta$

So, path difference = $\Delta L = d\theta$

For a bright fringe at same angular position θ , both of the rays from slits S_1 and S_2 are in phase.

Hence, path difference is an integral multiple of wavelength of light used.

$$\text{i.e., } \Delta L = n\lambda$$

$$\text{or } d\theta = n\lambda$$

$$\Rightarrow \lambda = \frac{d\theta}{n}$$

$$\text{Here, } \theta = \frac{1}{40} \text{ rad, } d = 0.1 \text{ mm}$$

$$\text{Hence, } \lambda = \frac{0.1}{40n} \text{ mm}$$

$$= \frac{0.1 \times 10^{-3} \text{ m}}{40n}$$

$$= \frac{0.1 \times 10^{-3} \times 10^9}{40n} \text{ nm}$$

$$\Rightarrow \lambda = \frac{2500}{n} \text{ nm}$$

So, with light of wavelength λ_1 we have

$$\lambda_1 = \frac{2500}{n_1} \text{ (nm)}$$

and with light of wavelength λ_2 , we have

$$\lambda_2 = \frac{2500}{n_2} \text{ (nm)}$$

Now, choosing different integral values for n_1 and n_2 , (i.e., $n_1, n_2 = 1, 2, 3, \dots$ etc) we find that for

$$n_1 = 4,$$

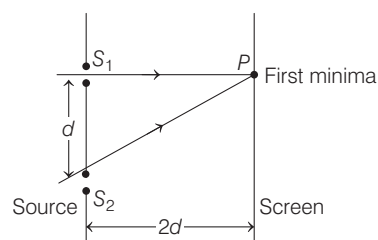
$$\lambda_1 = \frac{2500}{4} = 625 \text{ nm}$$

and for $n_2 = 5,$

$$\lambda_2 = \frac{2500}{5} = 500 \text{ nm}$$

These values lie in given interval 500 nm to 625 nm.

- 31** Consider a Young's double slit experiment as shown in figure.

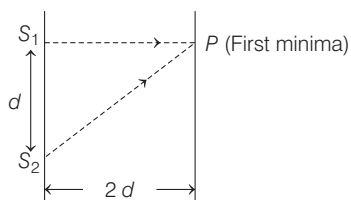


What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? [2019, 10 Jan Shift-II]

- (a) $\frac{\lambda}{2(5-\sqrt{2})}$ (b) $\frac{\lambda}{(5-\sqrt{2})}$
(c) $\frac{\lambda}{2(\sqrt{5}-2)}$ (d) $\frac{\lambda}{(\sqrt{5}-2)}$

Ans. (c)

In the given case, figure for first minima will be as shown below



We know that condition for minima in Young's double slit experiment is path difference,

$$\Delta x = (2n - 1) \lambda / 2$$

For first minima, $n = 1$

$$\Rightarrow \Delta x = \lambda / 2 \quad \dots (i)$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point 'P' will be

$$\Delta x = S_2P - S_1P \quad \dots (ii)$$

From triangle S_1S_2P ,

$$S_1P = 2d \quad \dots (iii)$$

$$\text{and } (S_2P)^2 = (S_1S_2)^2 + (S_1P)^2 \\ = d^2 + (2d)^2$$

$$\Rightarrow (S_2P)^2 = 5d^2 \text{ or } S_2P = \sqrt{5}d \quad \dots (iv)$$

Substituting the values from Eqs. (iii)

and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}d - 2d \quad \dots (v)$$

From Eqs. (i) and (v), we get

$$\sqrt{5}d - 2d = \lambda / 2 \Rightarrow d = \frac{\lambda}{2(\sqrt{5} - 2)}$$

- 32** In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\frac{1}{8}$ th of

wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to

[2019, 11 Jan Shift-I]

- (a) 0.80 (b) 0.74 (c) 0.94 (d) 0.85

Ans. (d)

Let intensity of each wave is I_0 .

Then, intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda / 8$

$$\therefore \text{Phase difference, } \phi = \Delta x \times \frac{2\pi}{\lambda}$$

$$\Rightarrow \phi = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} \text{ or } \phi = \pi / 4$$

Intensity of light at this point,

$$I' = I_0 + I_0 + 2I_0 \cos(\pi / 4) \\ = 2I_0 + \sqrt{2}I_0 = 3.41I_0$$

$$\text{Now, } \frac{I'}{4I_0} = \frac{3.41}{4} = 0.85$$

- 33** In a double-slit experiment, green light ($5303 \text{ \AA}$) falls on a double slit having a separation of $19.44 \mu\text{-m}$ and a width of $4.05 \mu\text{-m}$. The number of bright fringes between the first and the second diffraction minima is [2019, 11 Jan Shift-II]

- (a) 5 (b) 10 (c) 9 (d) 4

Ans. (a)

Here, wavelength of light used (λ) = $5303 \text{ \AA}$.

Distance between two slit (d) = $19.44 \mu\text{m}$

Width of single slit (a) = $4.05 \mu\text{m}$

Here, angular width between first and second diffraction minima $\theta = \frac{\lambda}{a}$

and angular width of a fringe due to double slit is $\theta' = \frac{\lambda}{d}$

$\therefore$ Number of fringes between first and

$$\text{second diffraction minima, } n = \frac{\theta}{\theta'} = \frac{\frac{\lambda}{a}}{\frac{\lambda}{d}}$$

$$= \frac{d}{a} = \frac{19.44}{4.05} = 4.81$$

or $n \approx 5$

$\therefore$ 5 interfering bright fringes lie between first and second diffraction minima.

- 34** In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm , is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is

[JEE Main 2017]

- (a) 7.8 mm (b) 9.75 mm
(c) 15.6 mm (d) 1.56 mm

Ans. (a)

Let n_1 th fringe formed due to first wavelength and n_2 th fringe formed due to second wavelength coincide i.e. their distance from common central maxima will be same

$$\text{i.e. } y_{n_1} = y_{n_2} \\ \Rightarrow \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \\ \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{4}{5}$$

Hence, distance of the point of coincidence from the central maxima is

$$y = \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \\ = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} \\ = 7.8 \text{ mm}$$

- 35** On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam [JEE Main 2015]

- (a) becomes narrower
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards

Ans. (d)

According to Snell's law,

$$\mu \sin \theta = \text{constant}$$

$$\therefore \sin \theta \propto \frac{1}{\mu}$$

As μ increases, θ decreases.

Hence, beam will bend upward.

- 36** In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m is the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ , is given by [AIEEE 2012]

- (a) $\frac{I_m}{9} (4 + 5 \cos \phi)$
(b) $\frac{I_m}{3} \left(1 + 2 \cos^2 \frac{\phi}{2} \right)$
(c) $\frac{I_m}{5} \left(1 + 4 \cos^2 \frac{\phi}{2} \right)$
(d) $\frac{I_m}{9} \left(1 + 8 \cos^2 \frac{\phi}{2} \right)$

Ans. (d)

Given, $a_1 = 2a_2$

$$\Rightarrow I_1 = 4I_2 = 4I_0 \quad [\because I \propto a^2]$$

$$\therefore I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \\ = (3\sqrt{I_2})^2 = 9I_2 = 9I_0$$

$$\text{Now, } I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \\ = 4I_0 + I_0 + 2\sqrt{4I_0 \cdot I_0} \cos \phi \\ = 5I_0 + 4I_0 \cos \phi$$

$$\begin{aligned}
 &= \frac{I_m}{9} (5 + 4 \cos \phi) \quad \left[\because I_0 = \frac{I_m}{9} \right] \\
 &= \frac{I_m}{9} [1 + 4(1 + \cos \phi)] \\
 &= \frac{I_m}{9} (1 + 8 \cos^2 \phi/2) \\
 &\quad \left[\because \cos \phi = 2 \cos^2 \frac{\phi}{2} - 1 \right]
 \end{aligned}$$

- 37** At two points P and Q on screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$,

respectively. The ratio of intensities at P and Q will be

[AIEEE 2011]

- (a) 3:2 (b) 2:1 (c) $\sqrt{2}$:1 (d) 4:1

Ans. (b)

Let I_0 be intensity of light emitted from the source, then

Resultant intensity

$$\begin{aligned}
 I &= 4I_0 \cos^2 \phi/2 \\
 \therefore I_1 &= 4I_0 \cos^2 0/2 = 4I_0 \quad [\because \phi = 0] \\
 \text{Now, } \Delta x &= \lambda/4 \\
 \therefore \phi &= \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \Rightarrow \phi = \frac{\pi}{2} \\
 \text{and } I_2 &= 4I_0 \cos^2 \frac{\pi}{4} = 2I_0 \quad \left[\because \phi = \frac{2\pi}{\lambda} \Delta x \right] \\
 \therefore I_1 : I_2 &= 2:1
 \end{aligned}$$

- 38** In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ . In another experiment with the same arrangement, the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2 , then the ratio $\frac{I_1}{I_2}$ is

[AIEEE 2011]

- (a) 4 (b) 2 (c) 1 (d) 0.5

Ans. (b)

For coherent sources,

$$I_1 = 4I_0 \cos^2 \frac{\phi}{2} = 4I_0$$

For incoherent sources,

$$\begin{aligned}
 I_2 &= I_0 + I_0 = 2I_0 \\
 \therefore \frac{I_1}{I_2} &= 2
 \end{aligned}$$

- 39** This question has a paragraph followed by two statements, Statement I and Statement II. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement I When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π .

Statement II The centre of the interference pattern is dark.

[AIEEE 2011]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

Ans. (b)

Both statements are correct but Statement II does not explain Statement I.

- 40** The initial shape of the wavefront of the beam is [AIEEE 2010]

- (a) convex
(b) concave
(c) convex near the axis and concave near the periphery
(d) planar

Ans. (d)

For a parallel cylindrical beam, wavefront will be planar.

- 41** A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third

bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is

[AIEEE 2009]

- (a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm

Ans. (c)

$$\begin{aligned}
 3\lambda_1 &= 4\lambda_2 \quad \left[\because \frac{3D\lambda_1}{d} = 4 \frac{D\lambda_2}{d} \right] \\
 \Rightarrow \lambda_2 &= \frac{3}{4} \lambda_1 = \frac{3}{4} \times 590 \\
 &= \frac{1770}{4} = 442.5 \text{ nm}
 \end{aligned}$$

- 42** In a Young's double slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I . If I_0 denotes the maximum intensity, then I/I_0 is equal to [AIEEE 2007]

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Ans. (d)

Phase difference = $\frac{2\pi}{\lambda} \times$ Path difference

$$\text{i.e., } \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$$

$$\text{As, } I = I_{\max} \cos^2 \left(\frac{\phi}{2} \right) \text{ or } \frac{I}{I_{\max}} = \cos^2 \left(\frac{\phi}{2} \right)$$

$$\text{or } \frac{I}{I_0} = \cos^2 \left(\frac{\pi}{6} \right) = \frac{3}{4} \quad [\because I_{\max} = I_0]$$

- 43** A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is [AIEEE 2005]
(a) hyperbola (b) circle
(c) straight line (d) parabola

Ans. (d)

The shape of the interference fringes formed on a screen is parabola.

- 44** The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is [AIEEE 2004]
(a) infinite (b) five
(c) three (d) zero

Ans. (b)

For possible interference maxima on the screen, the condition is

$$d \sin \theta = n\lambda \quad \dots(i)$$

Given, $d = \text{slit-width} = 2\lambda$

$$2\lambda \sin \theta = n\lambda$$

$$\text{or } 2 \sin \theta = n$$

The maximum value of $\sin \theta$ is 1, hence

$$n = 2 \times 1 = 2$$

Thus, Eq. (i) must be satisfied by 5 integer values i.e., -2, -1, 0, 1, 2. Hence, the maximum number of possible interference maxima is 5.

- 45** To demonstrate the phenomenon of interference, we require two sources which emit radiations of **[AIEEE 2003]**

- (a) nearly the same frequency
- (b) the same frequency
- (c) different wavelength
- (d) the same frequency and having a definite phase relationship

Ans. (d)

Sustained interference is possible with coherent sources only i.e., sources having same frequency and constant or zero phase difference.

TOPIC 2

Diffraction and Polarisation of Light

- 46** A source of light is placed in front of a screen. Intensity of light on the screen is I . Two polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is $I/2$. P_2 should be rotated by an angle of (degrees), so that the intensity of light on the screen becomes $3I/8$.

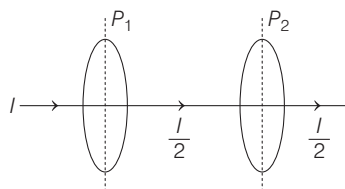
[2021, 26 Aug Shift-II]**Ans. (30)**

Given, intensity of light falling on screen is I .

Intensity of light when P_1 and P_2 placed between screen and source is $I/2$.

As per Malus's law, the intensity of transmitted light when passing through a pair of polariser and analyser having angle ϕ between their axis is given as

$$I = I_0 \cos^2 \phi$$



Initially let us consider the angle between analyser and polariser is 0° .

Then, the intensity of transmitted light is $I/2$. Thus, intensity of light become $I/2$ after transmitting through 1st polariser as shown in the figure.

If we rotate the polariser P_2 by angle ϕ , then the resulting intensity of light on screen will be $3I/8$.

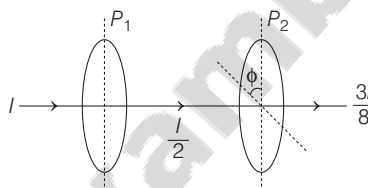
Now, for second condition, apply Malus's law,

$$\frac{3I}{8} = \frac{I}{2} \cos^2 \phi$$

$$\cos^2 \phi = \frac{3}{4}$$

$$\Rightarrow \cos \phi = \frac{\sqrt{3}}{2}$$

$$\phi = 30^\circ$$



Thus, P_2 should be rotated by 30° to get intensity of light $3I/8$ on screen.

- 47** With what speed should a galaxy move outward with respect to Earth, so that the sodium-D line at wavelength $5890 \text{ \AA}$ is observed at $5896 \text{ \AA}$? **[2021, 20 July Shift-II]**

- (a) 306 km/s
- (b) 322 km/s
- (c) 296 km/s
- (d) 336 km/s

Ans. (a)

Given, observed wavelength, $\lambda_o = 5896 \text{ \AA}$

Actual wavelength, $\lambda_1 = 5890 \text{ \AA}$

$\therefore$ Change in wavelength,

$$\Delta \lambda = \lambda_o - \lambda_1 = 5896 - 5890 = 6 \text{ \AA}$$

Speed of galaxy with respect to Earth is given as

$$\begin{aligned} v &= c \times \frac{\Delta \lambda}{\lambda_1} \\ &= 3 \times 10^8 \times \frac{6}{5890} \\ &= 305.6 \times 10^3 \text{ m/s} \\ &\approx 306 \text{ km/s} \end{aligned}$$

- 48** A galaxy is moving away from the Earth at a speed of 286 kms^{-1} . The shift in the wavelength of a red line at 630 nm is $x \times 10^{-10} \text{ m}$. The value of x to the nearest integer, is

.....

(Take the value of speed of light c , as $3 \times 10^8 \text{ ms}^{-1}$)

[2021, 18 March Shift-II]**Ans. (6)**

The shift in the wavelength of a red line,

$$\begin{aligned} v &= c \frac{\Delta \lambda}{\lambda} \Rightarrow \Delta \lambda = \frac{v}{c} \times \lambda \\ &= \frac{286 \times 10^3}{3 \times 10^8} \times 630 \times 10^{-9} \\ &= 6 \times 10^{-10} \text{ m} \end{aligned}$$

Hence, the value of the x to the nearest integer is 6.

- 49** Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter $0.1 \mu\text{m}$. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that

[2021, 25 Feb Shift-II]

- (a) its size increases and intensity increases
- (b) its size increases, but intensity decreases
- (c) its size decreases, but intensity increases
- (d) its size decreases and intensity decreases

Ans. (c)

Given, diameter of pinhole, $a = 0.1 \mu\text{m}$
 $= 0.1 \times 10^{-6} \text{ m}$

$\therefore$ Path difference

$$(\Delta x) = a \sin \phi = n\lambda \quad \dots(i)$$

where, ϕ is the phase difference and λ be the wavelength.

$$\text{As, } I = 4I_0 \cos^2 \phi$$

$$\text{and } \sin \phi = \frac{n\lambda}{a} \quad [\text{from Eq. (i)}]$$

If a increases $\leftrightarrow \sin \phi$ or ϕ decreases

As ϕ decreases $\leftrightarrow \cos \phi$ increases

$\therefore$ Intensity increases.

Hence, on decreasing diameter of pinhole, the size of diffraction pattern decreases and intensity increases.

- 50** An unpolarised light beam is incident on the polariser of a polarisation experiment and the intensity of light beam emerging

from the analyser is measured as 100 lumens. Now, if the analyser is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be lumens.

[2021, 24 Feb Shift-I]

Ans. (75)

Given, $I_0 = 100$ lumens

When analyser is rotated through an angle θ , the intensity of light will become

$$I = I_0 \cos^2 \theta = 100 \times \cos^2 30^\circ \\ = 100 \times \left(\frac{\sqrt{3}}{2} \right)^2 = 75 \text{ lumens}$$

- 51** A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 W m^{-2} falls normally on a polariser, which rotates about its axis with an angular speed of 31.4 rad/s . The energy of light passing through the polariser per revolution is close to (Take, cross-sectional area $= 3 \times 10^{-4} \text{ m}^2$) [2020, 4 Sep Shift-I]

- (a) $1.5 \times 10^{-4} \text{ J}$ (b) $5.0 \times 10^{-4} \text{ J}$
(c) $1.0 \times 10^{-4} \text{ J}$ (d) $1.0 \times 10^{-5} \text{ J}$

Ans. (c)

Intensity falling on polariser, $I_0 = 3.3 \text{ W/m}^2$

Area of polariser, $A = 3 \times 10^{-4} \text{ m}^2$

Angular speed of rotation of polariser,
 $\omega = 31.4 \text{ rad/s}$

Now, average energy is given by

$$\langle E \rangle = \langle I A t \rangle$$

$$\langle E \rangle = \langle I_0 \cos^2 \theta \times A \times t \rangle$$

$$\langle E \rangle = I_0 A t \langle \cos^2 \theta \rangle$$

$$\langle E \rangle = I_0 A t \left(\frac{1}{2} \right) \quad \left[\because \langle \cos^2 \theta \rangle = \frac{1}{2} \right]$$

Now, the calculation is about one revolution, then

Time taken = Time period

$$t = T = \frac{2\pi}{\omega} = \frac{2\pi}{31.4} = \frac{2\pi}{10\pi} = \frac{1}{5} \text{ s}$$

$$\text{Now, } \langle E \rangle = I_0 A t \left(\frac{1}{2} \right)$$

Substituting all the values in it, we get

$$\langle E \rangle = 3.3 \times 3 \times 10^{-4} \times \frac{1}{5} \times \left(\frac{1}{2} \right)$$

$$= 0.99 \times 10^{-4} \text{ J}$$

$$= 1 \times 10^{-4} \text{ J}$$

Hence, correct option is (c).

- 52** Orange light of wavelength $6000 \times 10^{-10} \text{ m}$ illuminates a single slit of width $0.6 \times 10^{-4} \text{ m}$. The maximum possible number of diffraction minima produced on both sides of the central maximum is [2020, 4 Sep Shift-II]

Ans. (198)

In single slit diffraction, if width of slit is a , wavelength of light is λ and angular position on screen is θ , then for n th minima, the condition is

$$a \sin(\theta) = n\lambda \Rightarrow \sin(\theta) = \frac{n\lambda}{a} \quad \dots(i)$$

$$\because \sin(\theta) < 1 \Rightarrow \frac{n\lambda}{a} < 1 \Rightarrow n < \frac{a}{\lambda}$$

$$\Rightarrow n < \frac{0.6 \times 10^{-4}}{6000 \times 10^{-10}} \Rightarrow n < \frac{10^{-4}}{10^4 \times 10^{-10}}$$

$$\Rightarrow n < 10^2 \Rightarrow n < 100$$

So, maximum possible number of minima on one side of central maximum = 99.

Therefore, maximum possible number of minima on both sides of central maximum = $2 \times 99 = 198$

- 53** A polariser-analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity.

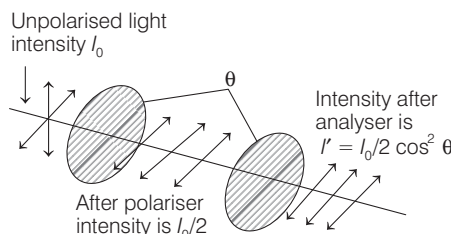
Assuming that the polariser-analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is

[2020, 7 Jan Shift-I]

- (a) 71.6° (b) 90°
(c) 45° (d) 18.4°

Ans. (d)

A polariser-analyser set is as shown below.



By Malus' law,

Output intensity obtained from analyser,

$$I' = \frac{I_0}{2} \cos^2 \theta$$

where, θ = angle between polariser and analyser.

Now, given $I' = 10\%$ of initial intensity

$$\Rightarrow I' = \frac{1}{10} I_0 = \frac{I_0}{2} \cos^2 \theta$$

$$\Rightarrow \cos^2 \theta = \frac{1}{5}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \theta = \cos^{-1} \frac{1}{\sqrt{5}} \approx 63^\circ$$

Now, output intensity is zero when angle between polariser and analyser is 90° .

So, analyser must be rotated by $90^\circ - 63^\circ = 27^\circ$ to reduce output intensity to zero.

From given options, we can see that output intensity is least when analyser rotation is 18.4° .

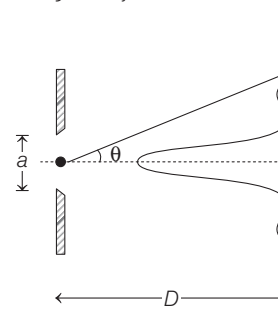
- 54** Visible light of wavelength $6000 \times 10^{-8} \text{ cm}$ falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1 , then θ_1 is close to

[2020, 7 Jan Shift-I]

- (a) 25° (b) 30°
(c) 20° (d) 45°

Ans. (a)

In case of a single slit diffraction pattern as shown below, angular position of n th minima is given by



angular position of n th minima is given by

$$\sin \theta_n = \frac{n\lambda}{D} \quad \dots(i)$$

Given, $\theta_2 = 60^\circ$ and $\lambda = 6000 \times 10^{-8} \text{ cm}$

$$\text{As, } \sin \theta_2 = \frac{2\lambda}{D}$$

$$\text{We have, } \sin 60^\circ = \frac{2\lambda}{D}$$

$$\text{or } \frac{\lambda}{D} = \frac{\sin 60^\circ}{2}$$

$$= 0.43 \approx \sin 25^\circ$$

[Note $\sin 30^\circ = 0.5$, $\sin 25^\circ = 0.422$ and $\sin 20^\circ = 0.34$]

Now, if θ_1 = angular position of first minima, then

$$\sin \theta_1 = \frac{\lambda}{D} \quad [\text{from Eq. (i), } n = 1]$$

$$\Rightarrow \sin \theta_1 = 0.43 \approx \sin 25^\circ \text{ or } \theta_1 \approx 25^\circ$$

- 55** The aperture of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of $5500 \text{ \AA}$, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to

[2020, 9 Jan Shift-I]

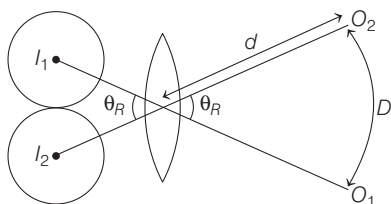
- (a) 600 m (b) 20 m (c) 60 m (d) 200 m

Ans. (c)

From Rayleigh's criteria, two objects will be just resolved when their angular separation,

$$\theta_R = 1.22 \frac{\lambda}{a}$$

where, λ = wavelength and a = aperture size.



Two objects are just resolved when their central maximas does not overlap.

From diagram, distance between objects in object plane,

$$D = \theta_R \cdot d = 1.22 \frac{\lambda d}{a}$$

Here, $a = 5 \text{ m}$, $d = 4 \times 10^5 \text{ km} = 4 \times 10^8 \text{ m}$,

$$\lambda = 5500 \text{ \AA} = 5500 \times 10^{-10} \text{ m}$$

$$\text{So, } D = \frac{(1.22 \times 5500 \times 10^{-10} \times 4 \times 10^8)}{5} \approx 54 \text{ m}$$

From options, minimum separation distance is 60 m.

- 56** Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to

[2019, 9 April Shift-II]

- (a) $3.0 \times 10^{-7} \text{ rad}$ (b) $2.0 \times 10^{-7} \text{ rad}$
(c) $1.5 \times 10^{-7} \text{ rad}$ (d) $4.5 \times 10^{-7} \text{ rad}$

Ans. (a)

Limit of resolution for a telescope from Rayleigh's criteria is

$$\theta_R = \frac{1.22 \lambda}{D}$$

Here, $D = 250 \text{ cm} = 250 \times 10^{-2} \text{ m}$

and $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$

So, limit of resolution is

$$\theta_R = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}} = 2.93 \times 10^{-7} \text{ rad} \approx 3 \times 10^{-7} \text{ rad}$$

- 57** The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength $5000 \text{ \AA}$ is used, the minimum separation between two points, to be seen as distinct, will be

[2019, 12 April Shift-I]

- (a) $0.24 \mu\text{m}$ (b) $0.38 \mu\text{m}$
(c) $0.12 \mu\text{m}$ (d) $0.48 \mu\text{m}$

Ans. (a)

Key Idea Numerical Aperture (NA) of the microscope is given by

$$\text{NA} = \frac{0.61 \lambda}{d}$$

Here, d = minimum separation between two points to be seen as distinct and λ = wavelength of light.

Given, $\lambda = 5000 \text{ \AA} = 5000 \times 10^{-10} \text{ m}$ and

$$\text{NA} = 1.25$$

$$\text{Now, } d = \frac{0.61 \lambda}{\text{NA}} = \frac{0.61 \times 5000 \times 10^{-10}}{1.25}$$

$$\text{or } d = \frac{3.05}{1.25} \times 10^{-7} \text{ m}$$

$$= 2.4 \times 10^{-7} \text{ m}$$

$$\text{or } d = 0.24 \mu\text{m}$$

- 58** Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

[2019, 8 April Shift-II]

- (a) $610 \times 10^{-9} \text{ rad}$ (b) $305 \times 10^{-9} \text{ rad}$
(c) $457.5 \times 10^{-9} \text{ rad}$ (d) $152.5 \times 10^{-9} \text{ rad}$

Ans. (b)

For a telescope, limit of resolution is

$$\text{given by } \Delta\theta = \frac{1.22 \lambda}{D}$$

Here, $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$,

$$D = 200 \text{ cm}$$

$$= 200 \times 10^{-2} \text{ m}$$

So, limit of resolution is

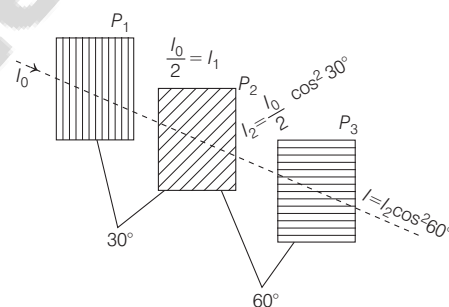
$$\Delta\theta = \frac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}} = 305 \times 10^{-9} \text{ rad}$$

- 59** A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1 . The pass axis of P_2 is inclined at 60° to the pass axis of P_3 . When a beam of unpolarised light of intensity I_0 is incident on P_1 , the intensity of light transmitted by the three polarisers is I . The ratio (I_0 / I) equals (nearly)

[2019, 12 April Shift-II]

- (a) 5.33 (b) 16.00
(c) 10.67 (d) 1.80

Ans. (c)



When unpolarised light pass through polaroid P_1 , intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it pass through polariser P_2 . So, intensity I_2 transmitted is obtained by Malus law.

$$\Rightarrow I_2 = I_1 \cos^2 \theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60° , so angle between pass axis of P_1 and P_2 is $(90^\circ - 60^\circ) = 30^\circ$. So,

$$I_2 = \frac{I_0}{2} \cos^2 30^\circ = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3 , intensity I transmitted is again obtained by Malus law.

$$\text{So, } I = I_2 \cos^2 60^\circ = \left(\frac{3}{8} I_0\right) \cos^2 60^\circ$$

$$= \frac{3}{8} I_0 \times \left(\frac{1}{2}\right)^2 = \frac{3}{32} I_0$$

$$\text{So, ratio } \frac{I_0}{I} = \frac{32}{3} = 10.67$$

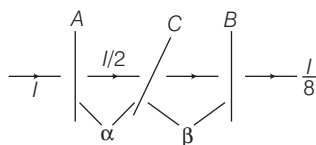
- 60** Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$.

Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polariser A and C is

[JEE Main 2018]

- (a) 0° (b) 30°
(c) 45° (d) 60°

Ans. (c)



Using Malus's law, intensity available after C = $\frac{I}{2} \times \cos^2 \alpha$

and intensity available after B = $\frac{I}{2} \cos^2 \alpha \times \cos^2 \beta = \frac{I}{8}$ (given)

So, $\frac{I}{2} \times \cos^2 \alpha \cdot \cos^2 \beta = \frac{I}{8}$

$\Rightarrow \cos^2 \alpha \cdot \cos^2 \beta = \frac{1}{4}$

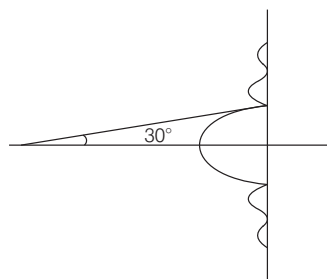
This is satisfied with $\alpha = 45^\circ$ and $\beta = 45^\circ$.
So, angle between A and C is 45° .

- 61** The angular width of the central maximum in a single slit diffraction pattern is 60° . The width of the slit is $1 \mu\text{m}$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.) [JEE Main 2018]

- (a) $25 \mu\text{m}$
(b) $50 \mu\text{m}$
(c) $75 \mu\text{m}$
(d) $100 \mu\text{m}$

Ans. (a)

Angular width of diffraction pattern = 60°



For first minima,

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2}$$

[here, $a = 10^{-6} \text{ m}$, $\theta = 30^\circ$]

$$\Rightarrow \lambda = 10^{-6} \times \sin 30^\circ$$

$$\Rightarrow \lambda = \frac{10^{-6}}{2} \text{ m}$$

Now, in case of interference caused by bringing second slit,

$$\therefore \text{Fringe width, } \beta = \frac{\lambda D}{d}$$

$$\left[\text{here, } \lambda = \frac{10^{-6}}{2} \text{ m}, \beta = 1 \text{ cm} = \frac{1}{100} \text{ m}, \right. \\ \left. d = ? \text{ and } D = 50 \text{ cm} = \frac{50}{100} \text{ m} \right]$$

$$\text{So, } d = \frac{\lambda D}{\beta} = \frac{10^{-6} \times 50}{2 \times \frac{1}{100} \times 100} \\ = 25 \times 10^{-6} \text{ m}$$

$$\text{or } d = 25 \mu\text{m}$$

- 62** The box of a pin hole camera, of length L , has a hole of radius a . It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{\min}$) when [JEE Main 2016]

$$(a) a = \frac{\lambda^2}{L} \text{ and } b_{\min} = \left(\frac{2\lambda^2}{L} \right)$$

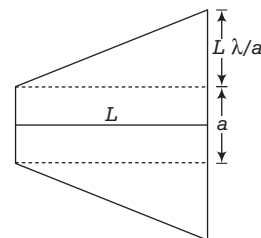
$$(b) a = \sqrt{\lambda L} \text{ and } b_{\min} = \left(\frac{2\lambda^2}{L} \right)$$

$$(c) a = \sqrt{\lambda L} \text{ and } b_{\min} = \sqrt{4\lambda L}$$

$$(d) a = \frac{\lambda^2}{L} \text{ and } b_{\min} = \sqrt{4\lambda L}$$

Ans. (c)

In diffraction, first minima, we have ,
 $\sin \theta = \frac{\lambda}{a}$



$$\text{So, size of a spot } b = 2a + \frac{2L\lambda}{a} \quad \dots(i)$$

Then, minimum size of a spot, we get

$$\frac{\partial b}{\partial a} = 0 \Rightarrow 1 - \frac{L\lambda}{a^2} = 0$$

$$\Rightarrow a = \sqrt{\lambda L} \quad \dots(ii)$$

So, $b_{\min} = 2\sqrt{\lambda L} + 2\sqrt{\lambda L}$

[by substituting the value of a from Eq. (ii) in Eq. (i)]

$$= 4\sqrt{\lambda L}$$

So, the radius of the spot,

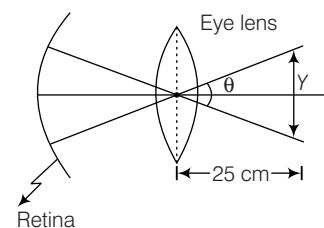
$$b_{\min} = \frac{4}{2} \sqrt{\lambda L} = \sqrt{4\lambda L}$$

- 63** Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm , the minimum separation between two objects that human eye can resolve at 500 nm wavelength is [JEE Main 2015]

- (a) $1 \mu\text{m}$ (b) $30 \mu\text{m}$
(c) $100 \mu\text{m}$ (d) $300 \mu\text{m}$

Ans. (b)

We can write resolving angle of necked eye as



$$\theta = 1.22 \frac{\lambda}{D}$$

where, D is the diameter of eye lens.

$$\text{or } \frac{Y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}} \\ Y = 30 \times 10^{-6} \text{ m} = 30 \mu\text{m}$$

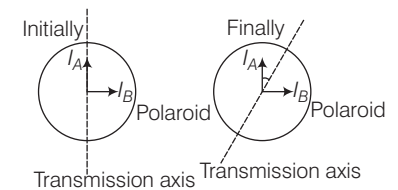
- 64** Two beams, A and B, of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position, when the beam A has maximum intensity

(and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A/I_B equals [JEE Main 2014]

- (a) 3 (b) $3/2$ (c) 1 (d) $1/3$

Ans. (d)

By law of Malus i.e., $I = I_0 \cos^2 \theta$



Now, $I_{A'} = I_A \cos^2 30^\circ$, $I_{B'} = I_B \cos^2 60^\circ$
As, $I_{A'} = I_{B'}$

$$I_A \cos^2 30^\circ = I_B \cos^2 60^\circ$$

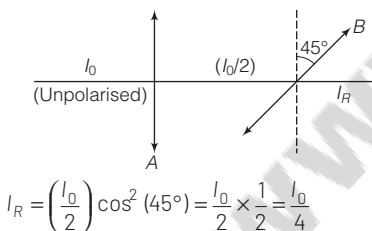
$$\Rightarrow I_A \frac{3}{4} = I_B \frac{1}{4} \Rightarrow \frac{I_A}{I_B} = \frac{1}{3}$$

- 65** A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is [JEE Main 2013]

- (a) I_0 (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{4}$ (d) $\frac{I_0}{8}$

Ans. (c)

Relation between intensities is



$$I_R = \left(\frac{I_0}{2}\right) \cos^2 (45^\circ) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

- 66 Statement I** On viewing the clear, blue portion of the sky through a Calcite crystal, the intensity of transmitted light varies as the crystal is rotated.

Statement II The light coming from the sky is polarised due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. [AIEEE 2011]

- (a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. (c)

Scattering for blue light is largest and it is polarised by scattering.

Also, for polarised light,
 $I = I_0 \cos^2 \theta$

- 67** Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]

[AIEEE 2005]

- (a) 5 m (b) 1 m (c) 6 m (d) 3 m

Ans. (a)

We know $\frac{y}{D} \geq 1.22 \frac{\lambda}{d} \Rightarrow D \leq \frac{yd}{1.22 \lambda}$

$$\therefore D = \frac{10^{-3} \times 3 \times 10^{-3}}{1.22 \times 5 \times 10^{-7}} = 5 \text{ m} \Rightarrow D_{\max} = 5 \text{ m}$$

- 68** When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is [AIEEE 2005]

- (a) $\frac{1}{2} I_0$ (b) $\frac{1}{4} I_0$ (c) zero (d) I_0

Ans. (a)

$$I = I_0 \cos^2 \theta$$

Intensity of polarised light = $I_0/2$

$$\therefore \text{Intensity of untransmitted light} \\ = I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

- 69** The angle of incidence at which reflected light is totally polarised for reflection from air to glass (refractive index n), is [AIEEE 2004]

- (a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$
(c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$

Ans. (d)

The particular angle of incidence for which reflected light is totally polarised for reflection from air to glass, is called the angle of polarisation (i_p) or Brewster's law.

Accordingly, $n = \tan i_p$

$$\Rightarrow i_p = \tan^{-1}(n)$$

where, n is refractive index of glass.

- 70** Wavelength of light used in an optical instrument are $\lambda_1 = 4000 \text{ \AA}$ and $\lambda_2 = 5000 \text{ \AA}$, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is [AIEEE 2002]

- (a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4

Ans. (d)

Resolving power of an optical instrument is inversely proportional to λ i.e.,

$$\text{RP} \propto \frac{1}{\lambda}$$

$$\therefore \frac{\text{Resolving power at } \lambda_1}{\text{Resolving power at } \lambda_2} = \frac{\lambda_2}{\lambda_1}$$

$$= \frac{5000}{4000} = 5:4$$

- 71** An astronomical telescope has a large aperture to [AIEEE 2002]

- (a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion

Ans. (b)

Resolving power of telescope,

$$\text{RP} = \frac{d}{1.22 \lambda}$$

25

Dual Nature of Radiation and Matter

TOPIC 1

Photoelectric Effect

- 01** A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm? **[2021, 27 Aug Shift-II]**

- (a) 0.96 V (b) 1.25 V
(c) 0.24 V (d) 1.5 V

Ans. (b)

In, case I given, incident wavelength,
 $\lambda_1 = 670.5 \text{ nm}$

Stopping potential, $V_1 = 0.48 \text{ V}$

In case II Incident wavelength,
 $\lambda_2 = 474.6 \text{ nm}$

Stopping potential is V_2 .

By using Einstein photoelectric equation, $eV_1 = \frac{hc}{\lambda_1} - \phi_0$

where, h is Planck's constant
 $= 6.63 \times 10^{-34} \text{ Js}$

c is speed of light $= 3 \times 10^8 \text{ ms}^{-1}$

ϕ_0 is work function

$$\therefore eV_1 = \frac{hc}{\lambda_1} - \phi_0$$

$$\text{and similarly, } eV_2 = \frac{hc}{\lambda_2} - \phi_0$$

$$\Rightarrow \phi_0 = \frac{hc}{\lambda_2} - eV_2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$$

$$\begin{aligned} \Rightarrow eV_1 - eV_2 &= hc \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \\ \Rightarrow V_1 - V_2 &= \frac{hc}{e} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \\ \Rightarrow V_2 &= V_1 + \frac{hc}{e} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) \\ &= 0.48 + \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \left(\frac{1}{474.6} - \frac{1}{670.5} \right) \\ &= 0.48 + 12.4 \times 10^{-7+9} \\ &\quad (2.11 \times 10^{-3} - 1.49 \times 10^{-3}) \\ &= 0.48 + 12.4 \times 10^{-2-3} \times 0.62 \\ &= 0.48 + 0.77 = 1.25 \text{ V} \end{aligned}$$

- 02** In a photoelectric experiment, increasing the intensity of incident light **[2021, 27 Aug Shift-I]**

- (a) increases the number of photons incident and also increases the KE of the ejected electrons.
(b) increases the frequency of photons incident and increases the KE of the ejected electrons.
(c) increases the frequency of photons incident and the KE of the ejected electrons remains unchanged.
(d) increases the number of photons incident and the KE of the ejected electrons remains unchanged.

Ans. (d)

As per Einstein's theory of photoelectric effect when a photon strikes the metal surface it ejects an electron. If we increase the intensity of light, the number of photons incident on surface will increase and thus, the number of

electrons ejected will increase proportionally and the photocurrent will also increase.

Kinetic energy of ejected electrons is given as

$$\frac{1}{2} m_e v_e^2 = eV_0$$

Here, V_0 is stopping potential.

The stopping potential of surface is constant and it will not increase with increase in intensity of light, so the kinetic energy of ejected electrons will not increase.

Thus, increasing the intensity of light will increase the number of photons incident and the kinetic energy will remain unchanged.

- 03** In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work-function $\phi = 2.5 \text{ eV}$. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. ($h = 6.63 \times 10^{-34} \text{ Js}$, and $c = 3 \times 10^8 \text{ ms}^{-1}$) **[2021, 26 Aug Shift-I]**

- (a) 1.3 V (b) 1.1 V (c) 1.9 V (d) 0.6 V

Ans. (a)

By Einstein's photoelectric equation, we have

$$hf = \phi + (KE)_{\max}$$

where, f is the frequency, ϕ is the work-function of metal and $(KE)_{\max}$ is the maximum kinetic energy of emitted photoelectron.

$$\Rightarrow \frac{hc}{\lambda} = \phi + eV_0$$

$$\Rightarrow eV_0 = \frac{hc}{\lambda} - \phi$$

Here, $\lambda = 280 \text{ nm}$ and $\phi = 2.5 \text{ eV}$

$$\Rightarrow e(V_0)_1 = \frac{1240}{280} - 2.5$$

$$\Rightarrow e(V_0)_1 = 1.93 \text{ eV}$$

$$\Rightarrow (V_0)_1 = 1.93 \text{ V}$$

Similarly, stopping potential for the light of wavelength of 400 nm ,

$$e(V_0)_2 = \frac{1240}{400} - 2.5$$

$$\Rightarrow (V_0)_2 = 0.6 \text{ V}$$

$\therefore$ Change in stopping potential,

$$\begin{aligned}\Delta V &= (V_0)_1 - (V_0)_2 \\ &= 1.93 - 0.6 \\ &= 1.33 \approx 1.3 \text{ V}\end{aligned}$$

- 04** A light beam of wavelength 500 nm is incident on a metal having work-function of 1.25 eV , placed in a magnetic field of intensity B . The electrons emitted perpendicular to the magnetic field B , with maximum kinetic energy are bent into circular arc of radius 30 cm . The value of B is $\times 10^{-7} \text{ T}$.
(Take, $hc = 20 \times 10^{-26} \text{ J-m}$, mass of electron $= 9 \times 10^{-31} \text{ kg}$)

[2021, 25 July Shift-II]

Ans. (125)

Given, wavelength of light,

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

Work-function, $\phi_0 = 1.25 \text{ eV}$

Radius of arc, $r = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$

Let magnetic field be (B)

$$hc = 20 \times 10^{-26} \text{ J-m}$$

Mass of electron, $m = 9 \times 10^{-31} \text{ kg}$

According to Einstein's photoelectric equation,

$$\begin{aligned}\text{KE} &= E_i - \phi_0 = \frac{hc}{\lambda} - \phi_0 \\ &= \frac{20 \times 10^{-26}}{500 \times 10^{-9} \times 1.6 \times 10^{-19}} - 1.25 \\ &= 0.025 \times 10^2 - 1.25 \\ &= 2.5 - 1.25 = 1.25 \text{ eV}\end{aligned}$$

$$\therefore \text{KE} = \frac{p^2}{2m}$$

where, p is momentum.

$$\begin{aligned}\Rightarrow p &= \sqrt{2m\text{KE}} \\ &= \sqrt{2 \times 9 \times 10^{-31} \times 1.25 \times 1.6 \times 10^{-19}} \\ &= 6 \times 10^{-25} \text{ kgms}^{-1}\end{aligned}$$

$$\text{Now, } r = \frac{mv}{qB}$$

Here r is radius, mv is momentum and B is magnetic field.

$$\begin{aligned}\Rightarrow B &= \frac{p}{qr} \\ &= \frac{6 \times 10^{-25}}{1.6 \times 10^{-19} \times 30 \times 10^{-2}} \\ &= 0.125 \times 10^{-4} = 125 \times 10^{-7} \text{ T}\end{aligned}$$

- 05** When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V . If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V . The threshold wavelength of the metal is

[2021, 25 July Shift-II]

- (a) 2λ (b) 4λ (c) 8λ (d) 6λ

Ans. (b)

Given,

In Case I

Wavelength of light $= \lambda$

Stopping potential of ejected electron,

$$V_{01} = 4.8 \text{ V}$$

In Case 2

Let wavelength of light be $\lambda_2 = 2\lambda$

Stopping potential, $V_{02} = 1.6 \text{ V}$

By using equation for photoelectric effect,

$$E = h\nu - h\nu_0$$

$$E = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$E = hc \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right]$$

$$\Rightarrow V = \frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \left[\because E = \frac{V}{e} \right]$$

For case I

$$4.8 = \frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(i)$$

For case II

$$\text{and } 1.6 = \frac{hc}{e} \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{4.8}{1.6} = \frac{\frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}{\frac{hc}{e} \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right)}$$

$$\Rightarrow 3 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right)}$$

$$\begin{aligned}\Rightarrow \frac{3}{2\lambda} - \frac{3}{\lambda_0} &= \frac{1}{\lambda} - \frac{1}{\lambda_0} \\ \Rightarrow \frac{3}{2\lambda} - \frac{1}{\lambda} &= \frac{3}{\lambda_0} - \frac{1}{\lambda_0} \\ \Rightarrow \frac{1}{2\lambda} &= \frac{2}{\lambda_0} \\ \Rightarrow \lambda_0 &= 4\lambda\end{aligned}$$

- 06** A certain metallic surface is illuminated by monochromatic radiation of wavelength λ . The stopping potential for photoelectric current for this radiation is $3V_0$.

If the same surface is illuminated with a radiation of wavelength 2λ , the stopping potential is V_0 . The threshold wavelength of this surface for photoelectric effect is

..... λ . [2021, 20 July Shift-II]

Ans. (4)

We know that, according to Einstein's photoelectric equation,

$$\text{KE} = \frac{hc}{\lambda} - \phi$$

$$\text{or } eV_0 = \frac{hc}{\lambda} - \phi \quad \dots(i)$$

As per question, when the metallic surface is illuminated by monochromatic radiation of wavelength λ , the stopping potential is $3V_0$.

So, Eq. (i) becomes

$$\Rightarrow e(3V_0) = \frac{hc}{\lambda} - \phi \quad \dots(ii)$$

and when the same surface is illuminated with a radiation of wavelength 2λ , stopping potential becomes V_0 .

$$\Rightarrow eV_0 = \frac{hc}{2\lambda} - \phi \quad \dots(iii)$$

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{e(3V_0)}{eV_0} = \frac{\frac{hc}{\lambda} - \phi}{\frac{hc}{2\lambda} - \phi}$$

On solving, we will get

$$\phi = \frac{hc}{4\lambda} \quad \dots(iv)$$

$\therefore$ We know that

$$\phi = \frac{hc}{\lambda_t} \quad \dots(v)$$

where, λ_t is the threshold wavelength.

$\therefore$ From Eqs (iv) and (v), we get

$$\lambda_t = 4\lambda$$

So, the threshold wavelength of this surface for photoelectric effect is 4λ .

- 07** Two identical photocathodes receive the light of frequencies f_1 and f_2 , respectively. If the velocities of the photoelectrons coming out are v_1 and v_2 respectively, then

[2021, 17 March Shift-II]

$$(a) v_1^2 - v_2^2 = \frac{2h}{m} [f_1 - f_2]$$

$$(b) v_1^2 + v_2^2 = \frac{2h}{m} [f_1 + f_2]$$

$$(c) v_1 - v_2 = \left[\frac{2h}{m} (f_1 + f_2) \right]^{\frac{1}{2}}$$

$$(d) v_1 - v_2 = \left[\frac{2h}{m} (f_1 - f_2) \right]^{\frac{1}{2}}$$

Ans. (a)

We know that, the photoelectric effect equation,

$$KE = hf - \phi$$

Here, KE = kinetic energy of the electrons

h = Planck's constant

f = frequency of the light

and ϕ = work-function of the photocathodes.

For two identical photocathodes,

$$\frac{1}{2} mv_1^2 = hf_1 - \phi \quad \dots (i)$$

$$\frac{1}{2} mv_2^2 = hf_2 - \phi \quad \dots (ii)$$

Since, the material of photocathodes is same, so the value of the work-function ϕ is same.

Subtracting Eq. (ii) from Eq. (i) we get

$$\frac{1}{2} mv_1^2 - \frac{1}{2} mv_2^2 = hf_1 - hf_2$$

$$v_1^2 - v_2^2 = \frac{2h}{m} (f_1 - f_2)$$

- 08** The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation [2021, 16 March Shift-I]

- (a) phase (b) intensity
(c) amplitude (d) frequency

Ans. (d)

The stopping potential V_0 is related to the frequency of incident radiations by following relation

$$eV_0 = h\nu - \phi \quad (\phi = \text{work-function})$$

The stopping potential in the context of photoelectric effect depends only on the frequency ν of incident electromagnetic radiation.

- 09** Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is $x : y$. The value of x is [2021, 26 Feb Shift-II]

Ans. (1)

Given, first incident energy, $E_1 = 2(\text{work function } \phi_0)$ Second incident energy, $E_2 = 10\phi_0$

If v_1 and v_2 be the respective maximum velocities in two cases, then by using the concept of photoelectric effect,

$$KE = E_i - \phi_0$$

$$\therefore \frac{1}{2} mv_1^2 = 2\phi_0 - \phi_0 = \phi_0 \quad \dots (i)$$

$$\text{and } \frac{1}{2} mv_2^2 = 10\phi_0 - \phi_0 = 9\phi_0 \quad \dots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\left(\frac{v_1}{v_2} \right)^2 = \frac{1}{9}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{1}{3} = \frac{x}{y}$$

$$\therefore x = 1$$

- 10** The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is [2021, 25 Feb Shift-II]

- (a) 309 nm (b) 329 nm
(c) 382 nm (d) 400 nm

Ans. (c)

Given, stopping potential (V_1) = 0.710 V
Incident wavelength of electrons (λ_1) = 491 nm

$$= 491 \times 10^{-9} \text{ m}$$

Let λ_2 will be the new incident wavelength of electrons.

Stopping potential (V_2) = 1.43 V

$$\text{As, energy } (E) = \frac{1240}{\lambda_1} = \phi_0 + eV$$

where, ϕ_0 is work-function and V is applied potential

$$\therefore E_1 = \frac{1240}{491} = \phi_0 + 0.71 \quad \dots (i)$$

$$\text{and } E_2 = \frac{1240}{\lambda_2} = \phi_0 + 1.43 \quad \dots (ii)$$

Now, subtracting Eqs. (i) from (ii), we get

$$E_2 - E_1 = 1240 \left(\frac{1}{\lambda_2} - \frac{1}{491} \right) = 0.72$$

$$\Rightarrow \frac{1}{\lambda_2} = 0.00058 + 0.00204 = 0.00262$$

$$\therefore \lambda_2 = 381.7 \text{ nm}$$

$$\lambda_2 \approx 382 \text{ nm}$$

- 11** When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V . When the same surface is illuminated with radiation of wavelength 3λ , the stopping potential is $\frac{V}{4}$. If the threshold

wavelength for the metallic surface is $n\lambda$, then value of n will be [2020, 2 Sep Shift-I]

Ans. (9)

With radiation wavelength λ and work function ϕ_0 , stopping potential is V .

$$\Rightarrow \frac{hc}{\lambda} - \phi_0 = eV \quad \dots (i)$$

When wavelength is 3λ , stopping potential is $\frac{V}{4}$.

$$\Rightarrow \frac{hc}{3\lambda} - \phi_0 = \frac{eV}{4} \quad \dots (ii)$$

Now, multiply Eq. (ii) by 3 and subtracting Eq (i) from it, we get

$$\Rightarrow \left(\frac{hc}{\lambda} - 3\phi_0 \right) - \left(\frac{hc}{\lambda} - \phi_0 \right) = \frac{3eV}{4} - eV$$

$$\Rightarrow 2\phi_0 = \frac{1}{4} eV \Rightarrow eV = 8\phi_0$$

Putting this value in Eq. (i), we get

$$\frac{hc}{\lambda} - \phi_0 = 8\phi_0 \Rightarrow \frac{hc}{\lambda} = 9\phi_0$$

$$\Rightarrow \frac{hc}{\lambda} = 9 \frac{hc}{\lambda_0}$$

$$\left\{ \text{As } \phi_0 = \frac{hc}{\lambda_0}, \text{ where } \lambda_0 = \text{threshold wavelength} \right\}$$

$$\Rightarrow \lambda_0 = 9\lambda \therefore n = 9$$

- 12** When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to [2020, 3 Sep Shift-I]

- (a) 0.81 eV (b) 1.02 eV
(c) 0.52 eV (d) 0.61 eV

Ans. (d)

Maximum kinetic energy of emitted electrons is given by,

$$K_{\max} = \frac{hc}{\lambda} - \phi_0$$

where, $hc \approx 1240 \text{ eV} \cdot \text{nm}$ and $\phi_0 =$ work function.

Here, as per question

$$K = \frac{1240}{500} - \phi_0 \quad \dots(i)$$

$$\text{and } 3K = \frac{1240}{200} - \phi_0 \quad \dots(ii)$$

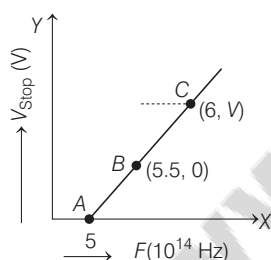
From Eqs. (i) and (ii), we get

$$\begin{aligned} 3\left(\frac{1240}{500} - \phi_0\right) &= \frac{1240}{200} - \phi_0 \\ \Rightarrow 2\phi_0 &= \frac{3 \times 1240}{500} - \frac{1240}{200} \\ \Rightarrow 2\phi_0 &= \frac{1240}{100} \left(\frac{3}{5} - \frac{1}{2}\right) = \frac{1240}{100} \left(\frac{1}{10}\right) \\ \Rightarrow \phi_0 &\approx \frac{1240}{2 \times 1000} \approx 0.62 \text{ eV} \end{aligned}$$

Hence, option (d) is correct.

- 13** The following figure shows few data points in a photoelectric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is (Take, Planck's constant, $h = 6.62 \times 10^{-34} \text{ J-s}$)

[2020, 4 Sep Shift-I]



- (a) 1.93 eV (b) 2.59 eV
(c) 2.27 eV (d) 2.10 eV

Ans. (c)

From Einstein's photoelectric equation,

$$\phi = \phi_0 + eV_0$$

[here, $V_0 =$ stopping potential]

$$\Rightarrow hf = \phi_0 + eV_0 \quad \dots(i)$$

From graph, take point B

$$(5.5 \times 10^{14} \text{ Hz}, 0 \text{ V})$$

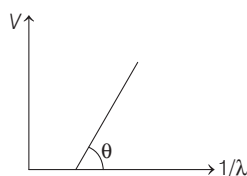
On satisfying eq. (i) with point B, we get

$$\begin{aligned} 6.62 \times 10^{-34} \times 5.5 \times 10^{14} &= \phi_0 + 1.6 \times 10^{-19} \times 0 \\ 36.41 \times 10^{-20} &= \phi_0 + 0 \end{aligned}$$

$$\begin{aligned} \phi_0 &= 36.41 \times 10^{-20} \text{ J} \\ &= \frac{36.41 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 22.75 \times 10^{-1} \text{ eV} \\ &= 2.275 \text{ eV} \approx 2.27 \text{ eV} \end{aligned}$$

Hence, correct option is (c).

- 14** In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength $(1/\lambda)$ obtained is shown in the figure.



As the intensity of incident radiation is increased,

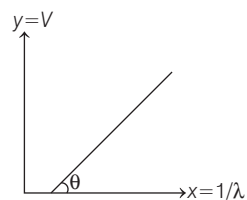
[2020, 4 Sep Shift-II]

- (a) graph does not change
(b) straight line shifts to left
(c) slope of the straight line get more steep
(d) straight line shifts to right

Ans. (a)

From photoelectric equation,

$$\begin{aligned} \phi &= \phi_0 + eV \Rightarrow \frac{hc}{\lambda} = \phi_0 + eV \\ eV &= \frac{hc}{\lambda} - \phi_0 \Rightarrow V = \frac{hc}{\lambda e} - \frac{\phi_0}{e} \\ V &= \left(\frac{hc}{e}\right) \left(\frac{1}{\lambda}\right) - \frac{\phi_0}{e} \end{aligned}$$



On comparing the above equation with equation of straight line, i.e., $y = mx + c$, we get

$$\text{Slope, } m = hc/e$$

$$\text{y-intercept } c = -\frac{\phi_0}{e}$$

Now, if we change intensity of incident radiation, then h , c , e and ϕ_0 will not change. Therefore, neither slope of the graph, nor the y-intercept cut by graph will change on changing the intensity of incident radiation.

So, graph will not change.

- 15** The surface of a metal is illuminated alternately with photons of energies $E_1 = 4 \text{ eV}$ and $E_2 = 2.5 \text{ eV}$ respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal (in eV) is

[2020, 5 Sep Shift-II]

Ans. (2)

Let the work function of the metal be ϕ .

$$KE = \frac{hc}{\lambda} - \phi$$

$$\text{For case I, } \frac{1}{2}mv_1^2 = 4 - \phi \quad \dots(i)$$

$$\text{For case II, } \frac{1}{2}mv_2^2 = 2.5 - \phi \quad \dots(ii)$$

$$\text{Given, } v_1/v_2 = 2$$

From eqs. (i) and (ii), we get

$$\frac{v_1^2}{v_2^2} = \frac{4 - \phi}{2.5 - \phi} \Rightarrow (2)^2 = \frac{4 - \phi}{2.5 - \phi}$$

$$\Rightarrow 4(2.5 - \phi) = 4 - \phi \Rightarrow 10 - 4\phi = 4 - \phi$$

$$\Rightarrow 3\phi = 6 \Rightarrow \phi = \frac{6}{3} = 2 \text{ eV}$$

So, the work function of the metal is 2 eV.

- 16** Radiation with wavelength $6561 \text{ \AA}$ falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of $3 \times 10^{-4} \text{ T}$. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to

[2020, 9 Jan Shift-I]

- (a) 0.8 eV (b) 1.1 eV
(c) 1.8 eV (d) 1.6 eV

Ans.(b)

Radius of largest circular path of electron in perpendicular magnetic field,

$$r = \frac{mv_{\max}}{Be}$$

$\therefore$ Velocity of most energetic electron emitted from metal surface,

$$v_{\max} = \frac{Be r}{m}$$

Kinetic energy with which electron is emitted,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m \cdot \frac{B^2 e^2 r^2}{m^2}$$

$$\Rightarrow K_{\max} = \frac{1}{2} \left(\frac{B^2 e^2 r^2}{m} \right) \text{ joule}$$

Kinetic energy of emitted electron (in electron volts),

$$K_{\max}(\text{eV}) = \frac{K_{\max}(\text{in joule})}{e}$$

$$\therefore K_{\max}(\text{eV}) = \frac{1}{2} \left(\frac{B^2 e r^2}{m} \right)$$

Here given,

$$B = 3 \times 10^{-4} \text{ T}, \quad e = 1.6 \times 10^{-19} \text{ C},$$

$$r = 10 \text{ mm} = 10^{-2} \text{ m and}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

So,

$$K_{\max} = \frac{(3 \times 10^{-4})^2 \times (1.6 \times 10^{-19}) \times (10^{-2})^2}{2 \times (9.1 \times 10^{-31})}$$

$$= \frac{9 \times 1.6}{2 \times 9.1} \times 10^{31-31} \approx 0.8 \text{ eV}$$

$$\therefore \text{Energy of incident photon} = hf = \frac{hc}{\lambda}$$

$$= \frac{12400 \text{ eV} \cdot \text{\AA}}{6561 \text{\AA}} \approx 1.9 \text{ eV}$$

(Note $hc = 1240 \text{ eV} \cdot \text{nm} = 12400 \text{ eV} \cdot \text{\AA}$)

So, work function of metal,

$$\phi_0 = \text{incident energy} - \text{kinetic energy of emitted electron}$$

$$= 1.9 - 0.8 = 1.1 \text{ eV}$$

- 17** The electric field of light wave is given as

$$\mathbf{E} = 10^{-3} \cos \left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t \right) \hat{x} \text{ NC}^{-1}.$$

This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is

[2019, 9 April Shift-I]

$$\text{Given, } E(\text{in eV}) = \frac{12375}{\lambda(\text{in \AA})}$$

- (a) 0.48 V (b) 0.72 V
(c) 2.0 V (d) 2.48 V

Ans. (a)

Given,

$$E = 10^{-3} \cos \left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t \right) \hat{x} \text{ NC}^{-1}$$

By comparing it with the general equation of electric field of light, i.e.

$$E = E_0 \cos(kx - \omega t) \hat{x}, \text{ we get}$$

$$k = \frac{2\pi}{5 \times 10^{-7}} = 2\pi / \lambda$$

(from definition, $k = 2\pi / \lambda$)

$$\Rightarrow \lambda = 5 \times 10^{-7} \text{ m} = 5000 \text{\AA} \quad \dots(i)$$

Or

The value of λ can also be calculated as, after comparing the given equation of E with standard equation, we get

$$\omega = 6 \times 10^{14} \times 2\pi$$

$$\Rightarrow v = 6 \times 10^{14} \quad [\because 2\pi v = \omega]$$

As, $c = v\lambda$

$$\Rightarrow \lambda = \frac{c}{v} = \frac{3 \times 10^8}{6 \times 10^{14}}$$

$$= 5 \times 10^{-7} \text{ m}$$

$$= 5000 \text{\AA}$$

According to Einstein's equation for photoelectric effect, i.e.

$$\frac{hc}{\lambda} - \phi = (KE)_{\max} = eV_0 \quad \dots(ii)$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375 \text{ eV}}{\lambda} \quad [\text{given}]$$

$$\text{or } \frac{hc}{\lambda} = \frac{12375}{5000} \text{ eV} \quad [\text{using Eq. (i)}] \dots(iii)$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000} \text{ eV} - 2 \text{ eV} = eV_0$$

$$\Rightarrow 2.475 \text{ eV} - 2 \text{ eV} = eV_0$$

$$\text{or } V_0 = 2475 \text{ V} - 2 \text{ V}$$

$$= 0.475 \text{ V}$$

$$\Rightarrow V_0 \approx 0.48 \text{ V}$$

- 18** In a photoelectric effect experiment, the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be

$$\text{Given, } E(\text{in eV}) = \frac{1237}{\lambda(\text{in nm})}$$

[2019, 10 April Shift-I]

- (a) 15.1 eV
(b) 3.0 eV
(c) 1.5 eV
(d) 4.5 eV

Ans. (c)

Given, threshold wavelength, $\lambda_0 = 380 \text{ nm}$

Wavelength of incident light, $\lambda = 260 \text{ nm}$

Using Einstein's relation of photoelectric effect,

$$(KE)_{\max} = eV_0 = h\nu - h\nu_0 \quad \dots(i)$$

$$\text{But } h\nu = E = \frac{1237}{\lambda(\text{nm})} \text{ eV} \quad (\text{Given})$$

$$\therefore E_0 = \frac{1237}{\lambda_0(\text{nm})} \text{ eV} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$(KE)_{\max} = E - E_0 = \left(\frac{1237}{\lambda} - \frac{1237}{\lambda_0} \right) \text{ eV}$$

$$= 1237 \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right] \text{ eV} (\lambda \text{ in nm}) \quad \dots(iii)$$

By putting values of λ and λ_0 in Eq. (iii), we get

$$(KE)_{\max} = 1237 \left(\frac{1}{260} - \frac{1}{380} \right) \text{ eV}$$

$$= 1237 \times \left[\frac{380 - 260}{380 \times 260} \right] \text{ eV}$$

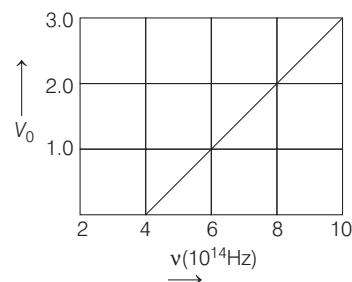
$$\Rightarrow (KE)_{\max} = 1.5 \text{ eV}$$

- 19** The stopping potential V_0 (in volt)

as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be

(Take, Planck's constant $(h) = 6.63 \times 10^{-34} \text{ J-s}$, electron charge, $e = 1.6 \times 10^{-19} \text{ C}$)

[2019, 12 April Shift-I]



- (a) 1.82 eV (b) 1.66 eV
(c) 1.95 eV (d) 2.12 eV

Ans. (b)

Given, Planck's constant,

$$h = 6.63 \times 10^{-34} \text{ J-s}, \quad e = 1.6 \times 10^{-19} \text{ C}$$

and there is a graph between stopping potential and frequency.

We need to determine work function W .

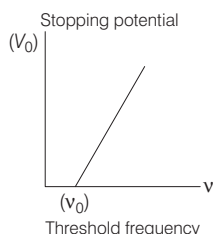
Using Einstein's relation of photoelectric effect,

$$(KE)_{\max} = eV_0 = h\nu - h\nu_0 = h\nu - W$$

$$[\because W = h\nu_0]$$

$$\text{or } V_0 = \frac{h}{e} \nu - \frac{W}{e}$$

From graph at $V_0 = 0$ and $\nu = 4 \times 10^{14}$ Hz



$$\therefore 0 = \frac{6.63 \times 10^{-34}}{e} \times 4 \times 10^{14} - \frac{W}{e}$$

$$\Rightarrow \frac{W}{e} = \frac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{e} \text{ J}$$

or $W = 6.63 \times 4 \times 10^{-20} \text{ J}$

or $W = \frac{6.63 \times 4 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 1.657 \text{ eV}$

$$\therefore W = 1.66 \text{ eV}$$

Alternate Solution

From graph, threshold frequency,

$$\nu_0 = 4 \times 10^{14} \text{ Hz (where, } V_0 = 0)$$

$\therefore$ Work function, $W = h\nu_0$

$$\Rightarrow W = 6.63 \times 10^{-34} \times 4 \times 10^{14} \text{ J}$$

$$\Rightarrow W = \frac{6.63 \times 4 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.657 \text{ eV} \approx 1.66 \text{ eV}$$

- 20** Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350 \text{ nm}$ and then by light of wavelength $\lambda_2 = 540 \text{ nm}$. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

$$(\text{energy of photon} = \frac{1240}{\lambda(\text{nm})} \text{ eV})$$

[2019, 9 Jan Shift-I]

- (a) 5.6 (b) 2.5
(c) 1.8 (d) 1.4

Ans. (c)

Let maximum speed of photo electrons in first case is v_1 and maximum speed of photo electrons in second case is v_2

Assumption I if we assume difference in maximum speed in two cases is 2 then $v_1 = v$ and $v_2 = 3v$

According to Einstein's photo electron equation

Energy of incident photon = work function + KE

$$\text{i.e. } \frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$$

Where $hc = 1240 \text{ eV}$, λ is wavelength of light incident, ϕ_0 is work function and v is speed of photo electrons.

When $\lambda_1 = 350 \text{ nm}$

$$\therefore \frac{hc}{350} = \phi_0 + \frac{1}{2}mv^2$$

$$\text{or } \frac{hc}{350} - \phi_0 = \frac{1}{2}mv^2 \quad \dots(i)$$

when $\lambda_2 = 540 \text{ nm}$

$$\therefore \frac{hc}{540} = \phi_0 + \frac{1}{2}m(3v)^2$$

$$\therefore \frac{hc}{540} - \phi_0 = \left(\frac{1}{2}mv^2\right) \times 9 \quad \dots(ii)$$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{350} - \phi_0}{\frac{hc}{540} - \phi_0} = \frac{\frac{1}{2}mv^2}{\left(\frac{1}{2}mv^2\right) \times 9} = \frac{1}{9}$$

$$\text{or } 9\left(\frac{hc}{350} - \phi_0\right) = \frac{hc}{540} - \phi_0$$

$$\text{or } 8\phi_0 = hc \left[\frac{9}{350} - \frac{1}{540} \right]$$

$$\text{or } \phi_0 = \frac{1}{8} \times 1240 \left[\frac{9 \times 540 - 350}{350 \times 540} \right]$$

$$= 3.7 \text{ eV.}$$

No option given is correct.

Alternate Solution

Assumption II If we assume velocity of one is twice in factor with second then.

Let $v_1 = 2v$ and $v_2 = v$

We know that from Einstein's photoelectric equation, energy of incident radiation = work function + KE

$$\text{or } \frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$$

Let when $\lambda_1 = 350 \text{ nm}$ then $v_1 = 2v$

and when $\lambda_2 = 540 \text{ nm}$ then $v_2 = v$

$\therefore$ Above Eq. becomes

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v)^2$$

$$\text{or } \frac{hc}{\lambda_1} - \phi = \frac{1}{2}m \times 4v^2 \quad \dots(i)$$

$$\text{and } \frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

$$\text{or } \frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2 \quad \dots(ii)$$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{\frac{1}{2}m \times 4v^2}{\frac{1}{2}mv^2} = 4$$

$$\text{or } \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\text{or } \phi = \frac{1}{3}hc \left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1} \right)$$

$$= \frac{1}{3} \times 1240 \left(\frac{4 \times 350 - 540}{350 \times 540} \right)$$

$$\text{or } \phi = 1.8 \text{ eV}$$

According to the assumption II, correct option is (c).

- 21** The magnetic field associated with a light wave is given at the origin, by $B = B_0 [\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$.

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons?

(Take, $c = 3 \times 10^8 \text{ ms}^{-1}$ and $h = 6.6 \times 10^{-34} \text{ J-s}$)

[2019, 9 Jan Shift-II]

- (a) 7.72 eV (b) 6.82 eV
(c) 8.52 eV (d) 12.5 eV

Ans. (a)

According to question, the wave equation of the magnetic field which produce photoelectric effect

$$B = B_0 [(\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct))]$$

Here, the photoelectric effect produced by the angular frequency (ω) = $6.28 \times 10^7 \text{ rad/s}$

$$\Rightarrow \omega = 6.28 \times 10^7 \times 3 \times 10^8$$

$$\omega = 2\pi \times 10^7 \times 3 \times 10^8 \text{ rad/s} \quad \dots(i)$$

Using Eqs. (i)

$$h\nu = \frac{h\omega}{2\pi} = \frac{h \times 2\pi \times 10^7 \times 3 \times 10^8}{2\pi} = 124 \text{ eV}$$

Therefore, according to Einstein equation for photoelectric effect

$$E = h\nu = \phi + KE_{\max}$$

$$\Rightarrow KE_{\max} = E - \phi$$

(where, ϕ = work-function = 4.7 eV)

$$KE_{\max} = 124 - 4.7$$

$$= 7.7 \text{ eV}$$

$$\text{or } KE_{\max} = 7.7 \text{ eV}$$

- 22** In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\frac{hc}{e} = 1240 \text{ nm} \cdot \text{V}\right)$

[2019, 11 Jan Shift-II]

(a) 0.5 V (b) 2.0 V (c) 1.5 V (d) 1.0 V

Ans. (d)Given, $\lambda_1 = 300 \text{ nm}$; $\lambda_2 = 400 \text{ nm}$

$$\frac{hc}{e} = 1240 \text{ nm}$$

Using Einstein equation for photoelectric effect,

$$E = h\nu = \phi + eV_0 \quad \dots(i)$$

(here, ϕ is work function of the metal and V_0 is stopping potential)For λ_1 wavelength's wave,

$$E_1 = h\nu_1 = \phi + eV_{01}$$

$$\text{or } \frac{hc}{\lambda_1} = \phi + eV_{01} \quad \dots(ii)$$

$$\text{Similarly, } \frac{hc}{\lambda_2} = \phi + eV_{02} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$hc \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] = e(V_{01} - V_{02})$$

$$\text{or } \frac{hc}{e} \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] = \Delta V$$

By using given values,

$$\Delta V = 1240 \left[\frac{1}{300} - \frac{1}{400} \right] \frac{\text{nm} \cdot \text{V}}{\text{nm}}$$

$$= 1240 \times \frac{1}{1200} \text{ V}$$

$$\Rightarrow \Delta V = 1.03 \text{ V} \approx 1 \text{ V}$$

- 23** When a certain photosensitive surface is illuminated with monochromatic light of frequency ν , the stopping potential for the photocurrent is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

[2019, 12 Jan Shift-II]

(a) $\frac{4}{3}\nu$ (b) 2ν (c) $\frac{3\nu}{2}$ (d) $\frac{5\nu}{3}$ **Ans. (c)**Relation between stopping potential and incident light's frequency is $eV_0 = hf - \phi_0$ where, V_0 is the stopping potential and ϕ_0 is the work function of the photosensitive surface.

So, from given data, we have

$$-e \frac{V_0}{2} = h\nu - \phi_0 \quad \dots(i)$$

$$\text{and } -eV_0 = \frac{h\nu}{2} - \phi_0 \quad \dots(ii)$$

Subtracting Eqs. (i) from (ii), we have

$$-eV_0 - \left(-\frac{eV_0}{2}\right) = \frac{h\nu}{2} - h\nu$$

$$\Rightarrow -\frac{eV_0}{2} = -\frac{h\nu}{2}$$

$$\Rightarrow eV_0 = h\nu$$

Substituting this in Eq. (i), we get

$$-\frac{eV_0}{2} = eV_0 - \phi_0$$

$$\Rightarrow -\left(\frac{3}{2}eV_0\right) = -\phi_0$$

$$\text{or } \frac{3}{2}h\nu = \phi_0$$

If threshold frequency is ν_0 , then

$$h\nu_0 = \frac{3}{2}h\nu \Rightarrow \nu_0 = \frac{3}{2}\nu$$

- 24** Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed v . If the wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

[JEE Main 2016]

$$(a) > v \left(\frac{4}{3}\right)^{1/2} \quad (b) < v \left(\frac{4}{3}\right)^{1/2}$$

$$(c) = v \left(\frac{4}{3}\right)^{1/2} \quad (d) = v \left(\frac{3}{4}\right)^{1/2}$$

Ans. (c)

Key Idea According to the law of conservation of energy, i.e. Energy of a photon ($h\nu$) = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2}mv_{\text{max}}^2\right)$

According to Einstein's photoelectric emission of light,

$$E = (KE)_{\text{max}} + \phi$$

$$\text{As, } \frac{hc}{\lambda} = (KE)_{\text{max}} + \phi$$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\Rightarrow \frac{4hc}{3\lambda} = \left(\frac{4}{3}(KE)_{\text{max}} + \frac{\phi}{3}\right) + \phi$$

For fastest emitted electron,

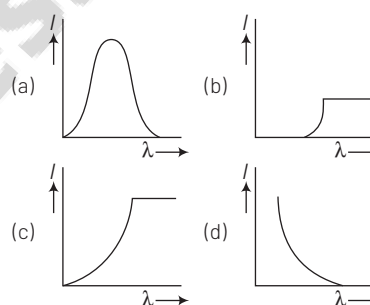
$$(KE)_{\text{max}} = \frac{1}{2}mv'^2 + \phi$$

$$\Rightarrow \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

$$\text{i.e. } v' > v \left(\frac{4}{3}\right)^{1/2}$$

- 25** The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of photocell varies as follows

[JEE Main 2013]

**Ans. (d)**As λ is increased, there will be a value of λ above which photoelectron will be cease to come out, so photocurrent will becomes zero.

- 26 Statement I** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max} . When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement II Photoelectrons are emitted with speeds ranging from zero to a maximum value, because of the range of frequencies present in the incident light. [AIEEE 2010]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

Ans. (d)

Since, the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays

$$KE_{\text{photoelectron}} = h\nu - \phi$$

Stopping potential is to stop the fastest photoelectron

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

So, KE_{max} and V_0 both increases. But KE ranges from zero to KE_{max} , because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

- 27** This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 , respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement II The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. [AIEEE 2011]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
 (b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
 (c) Statement I is false, Statement II is true
 (d) Statement I is true, Statement II is false

Ans. (c)

$$K_{\text{max}} = eV_0 = h(\nu - \nu_0)$$

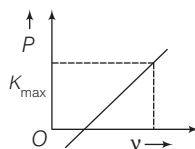
$$\text{If } \nu' = 2\nu$$

$$\therefore K'_{\text{max}} = eV'_0 = h(2\nu - \nu_0) = 2K_{\text{max}} + h\nu_0$$

$$\therefore K'_{\text{max}} > 2K_{\text{max}} \text{ and } V'_0 > 2V_0$$

Aliter

From graph, we can predict that K_{max} and ν do not correspond linear graph from origin.



- 28** The surface a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is ($hc = 1240 \text{ eV}\cdot\text{nm}$)

[AIEEE 2009]

- (a) 3.09 eV (b) 1.42 eV
 (c) 151 eV (d) 1.68 eV

Ans. (b)

$$\therefore KE_{\text{max}} = eV_0$$

$$\Rightarrow \frac{1}{2}mv^2 = eV_0 = 1.68 \text{ eV}$$

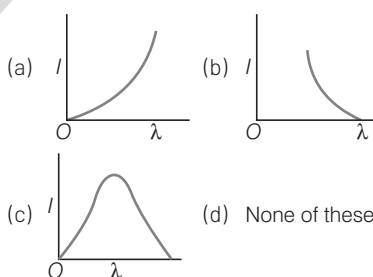
$$\Rightarrow h\nu = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{400 \text{ nm}} = 3.1 \text{ eV}$$

$$\Rightarrow 3.1 \text{ eV} = W_0 + 1.6 \text{ eV}$$

[from Einstein equation, $E = W_0 + K_{\text{max}}$]

$$W_0 = 1.42 \text{ eV}$$

- 29** The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows [AIEEE 2006]

**Ans. (d)**

According to the photoelectric effect in a photocell, if a light of wavelength λ is incident on a cathode, then electrons are emitted, which constitute the photoelectric current.

Photocell is based on the principle of photoelectric effect. As the wavelength of light changes, there is no change in

number of electrons emitted and hence, no change in current (plate current of photocell). Thus, the two wavelength of incident light and plate current are independent to each other.

Plate current depends on intensity of light used.

Hence option (d) is true.

- 30** A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed 1/2 m away, the number of electrons emitted by photocathode would [AIEEE 2004]

- (a) decrease by a factor of 4
 (b) increase by a factor of 4
 (c) decrease by a factor of 2
 (d) increase by a factor of 2

Ans. (b)

We know intensity depends on source distance.

$$\therefore \frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 \quad \left[\text{as } I \propto \frac{1}{r^2}\right]$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{(1)^2}{(1/2)^2} \Rightarrow I_2 = 4I_1$$

Now, since number of electrons emitted per second is directly proportional to intensity, so number of electrons emitted by photocathode would increase by a factor of 4.

- 31** The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in [AIEEE 2006]

- (a) ultra-violet region (b) infrared region
 (c) visible region (d) X-ray region

Ans. (a)

$$h\nu_0 = 6.2 \text{ eV}, eV_0 = 5 \text{ eV}$$

From Einstein's photoelectric equation

$$h\nu = h\nu_0 + eV_0$$

$$= 6.2 + 5 = 11.2 \text{ eV}$$

$$\Rightarrow \frac{hc}{\lambda} = 11.2 \text{ eV} \text{ or } \lambda = \frac{hc}{11.2} = 1108.9 \text{ \AA}$$

Which belongs to ultra-violet region.

- 32** The time taken by a photoelectron to come out after the photon strikes is approximately [AIEEE 2006]

- (a) 10^{-4} s (b) 10^{-10} s
 (c) 10^{-16} s (d) 10^{-1} s

Ans. (b)

The photoelectric effect is an instantaneous phenomenon (experimentally proved). It takes approximate time of the order of 10^{-10} s.

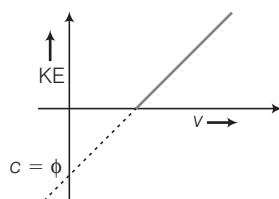
- 33** According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal Versus the frequency of the incident radiation gives a straight line whose slope [AIEEE 2004]

- (a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depend both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation

Ans. (d)

Einstein's photoelectric equation is

$$KE_{\max} = h\nu - \phi \quad \dots(i)$$



The equation of line is

$$y = mx + c \quad \dots(ii)$$

Comparing above two equations, we get

$$m = h, \quad c = -\phi$$

Hence, slope of graph is equal to Planck's constant (non-variable) and does not depend on intensity of radiation. But interception 'c' depends on work function ϕ .

- 34** The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately [AIEEE 2004]

- (a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

Ans. (c)

$$\begin{aligned} \frac{hc}{\lambda_0} = \phi_0 &\Rightarrow \lambda_{\max} = \frac{hc}{\phi_0} \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}} = 310 \text{ nm} \end{aligned}$$

- 35** Two identical, photocathodes receive light of frequencies f_1 and f_2 . If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2 , then [AIEEE 2003]

- (a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2) \right]^{1/2}$
(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$
(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2) \right]^{1/2}$

Ans. (a)

We know, Einstein's equations

$$\begin{array}{c} hf = hf_0 + \frac{1}{2}mv^2 \\ \downarrow \quad \quad \downarrow \quad \quad \downarrow \\ \text{Energy of incident light} \quad \text{Work function} \quad \text{Kinetic energy} \end{array}$$

$$\begin{aligned} \Rightarrow v_1^2 &= \frac{2hf_1}{m} - \frac{2hf_0}{m} \\ \text{and } v_2^2 &= \frac{2hf_2}{m} - \frac{2hf_0}{m} \\ \therefore v_1^2 - v_2^2 &= \frac{2h}{m}[f_1 - f_2] \end{aligned}$$

- 36** Sodium and copper have work functions 2.3 eV and 4.5 eV, respectively. Then, the ratio of the wavelengths is nearest to [AIEEE 2002]

- (a) 1 : 2
(b) 4 : 1
(c) 2 : 1
(d) 1 : 4

Ans. (c)

$$\text{Work function, } W = \frac{hc}{\lambda}$$

[here, they are interested in asking threshold wavelength]

where, h = Planck's constant,

c = velocity of light.

$$\text{Therefore, } \frac{W_{\text{Na}}}{W_{\text{Cu}}} = \frac{\lambda_{\text{Cu}}}{\lambda_{\text{Na}}}$$

$$\begin{aligned} \text{or } \frac{\lambda_{\text{Na}}}{\lambda_{\text{Cu}}} &= \frac{W_{\text{Cu}}}{W_{\text{Na}}} \\ &= \frac{4.5}{2.3} = 2 \text{ (nearly)} \end{aligned}$$

TOPIC 2

Particle Nature of Light-The photon

- 37** If 2.5×10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 min of time span, the energy flux of light just before it falls on the surface is W/cm^2 .

(Round off to the nearest integer. Assume complete absorption and normal incidence conditions are there.) [2021, 17 March Shift-I]

Ans. (25)

Given, average force, $F = 2.5 \times 10^{-6}$ N

Area, $A = 30 \text{ cm}^2$

Time, $t = 40 \text{ min}$

We know that, $F = \frac{IA}{c}$

where, I = energy flux of light

and c = speed of light in air.

$$\begin{aligned} \Rightarrow I &= \frac{Fc}{A} = \frac{2.5 \times 10^{-6} \times 3 \times 10^8}{30} \\ &= 25 \text{ W/cm}^2 \end{aligned}$$

- 38** Given below are two statements :

Statement I Two photons having equal linear momenta have equal wavelengths.

Statement II If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below. [2021, 24 Feb Shift-I]

- (a) Both Statement I and Statement II are true.
(b) Both Statement I and Statement II are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

Ans. (c)

$$\text{As we know, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

If linear momenta of two photons are equal, then their wavelengths is also equal.

Also, if the wavelength is decreased, then the momentum and energy of photon will increase.

Hence, option(c) is correct.

- 39** Two sources of light emit X-rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is

[2020, 3 Sep Shift-II]

- (a) $\frac{1}{250}$ (b) 500
(c) 250 (d) $\frac{1}{500}$

Ans. (b)

$$\begin{aligned} \text{Power of source} &= \frac{\text{Energy emitted}}{\text{Time}} \\ &= \frac{\text{Number of photons} \times \text{Energy of 1 photon}}{\text{Time}} \\ &= \text{Number density of photons} \times \text{Energy of 1 photon} \\ &= n \cdot \frac{hc}{\lambda} \end{aligned}$$

$$\text{Now, } P_{X\text{-rays}} = P_{\text{visible light}}$$

$$\begin{aligned} \Rightarrow n_X \left(\frac{hc}{\lambda_X} \right) &= n_V \left(\frac{hc}{\lambda_V} \right) \\ \Rightarrow \frac{n_V}{n_X} &= \frac{\lambda_V}{\lambda_X} = \frac{500}{1} = 500 \end{aligned}$$

Hence, correct option is (b).

- 40** A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} \text{ W/cm}^2$ is comprised of wavelength $\lambda = 310 \text{ nm}$. It falls normally on a metal (work function $\phi = 2 \text{ eV}$) of surface area of 1 cm^2 . If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x . ($hc = 1240 \text{ eVnm}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$), then x is

[2020, 7 Jan Shift-I]

Ans. (11)

Given, intensity of falling radiation,

$$I = 6.4 \times 10^{-5} \text{ Wcm}^{-2}$$

Wavelength of radiation, $\lambda = 310 \text{ nm}$

Work function of metal, $\phi = 2 \text{ eV}$

Area of metal surface, $A = 1 \text{ cm}^2$

Energy of one photon of radiation,

$$E_1 = \frac{hc}{\lambda} = \frac{1240 \text{ eVnm}}{\lambda (\text{nm})}$$

$$\begin{aligned} \text{or } E_1 &= \frac{1240}{310} \text{ eV} = \frac{1240}{310} \times 1.6 \times 10^{-19} \text{ J} \\ &= 6.4 \times 10^{-19} \text{ J} \end{aligned}$$

So, photon flux of radiation falling over surface = number of photons falling over surface per second

$$= \frac{\text{Total incident energy}}{\text{Energy of one photon}} = \frac{I \times A}{E_1}$$

$$= \frac{6.4 \times 10^{-5} \times 1}{6.4 \times 10^{-19}} = 10^{14} \text{ photons/second}$$

As only one in 10^3 photons are able to emit an electron, number of ejected electrons in 1 second

$$= \frac{10^{14}}{10^3} = 10^{11}$$

$$\Rightarrow x = 11$$

- 41** A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is

[Given, Planck's constant $h = 6.6 \times 10^{-34} \text{ Js}$, speed of light $c = 3.0 \times 10^8 \text{ m/s}$]

[2019, 10 April Shift-2]

- (a) 1×10^{16} (b) 5×10^{15}
(c) 1.5×10^{16} (d) 2×10^{16}

Ans. (b)

Power of laser is given as, $P = \frac{\text{Energy}}{\text{Time}}$

$$\begin{aligned} \text{Number of photons emitted} \times \text{Energy of one photon} \\ = \frac{\text{Energy}}{\text{Time}} \end{aligned}$$

$$\Rightarrow P = \frac{NE}{t} = \left(\frac{N}{t} \right) \cdot E$$

So, number of photons emitted per second

$$\begin{aligned} &= \frac{N}{t} = \frac{P}{E} \\ &= \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \quad \left[\because E = h\nu = \frac{hc}{\lambda} \right] \end{aligned}$$

Here, $h = 6.6 \times 10^{-34} \text{ J-s}$, $\lambda = 500 \text{ nm}$

$$= 500 \times 10^{-9} \text{ m}; c = 3 \times 10^8 \text{ ms}^{-1}$$

$$P = 2 \text{ mW} = 2 \times 10^{-3} \text{ W}$$

$$\begin{aligned} \therefore \frac{N}{t} &= \frac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 5.56 \times 10^{15} \\ &\approx 5 \times 10^{15} \text{ photons per second} \end{aligned}$$

- 42** A metal plate of area $1 \times 10^{-4} \text{ m}^2$ is illuminated by a radiation of intensity 16 W/m^2 . The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively will be (Take, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$) [2019, 10 Jan Shift-II]
- (a) 10^{11} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{10} and 5 eV (d) 10^{14} and 10 eV

Ans. (a)

We know that, intensity of a radiation I with energy ' E ' incident on a plate per second per unit area is given as

$$\Rightarrow I = \frac{dE}{dA \times dt} \Rightarrow \frac{dE}{dt} = IdA \text{ or } IA$$

i.e., energy incident per unit time = IA

Substituting the given values, we get

$$\frac{dE}{dt} = 16 \times 10^{-3} \times 1 \times 10^{-4}$$

$$\frac{dE}{dt} = 16 \times 10^{-7} \text{ W} \quad \dots(i)$$

Using Einstein's photoelectric equation, we can find kinetic energy of the incident radiation as

$$E = \frac{1}{2}mv^2 + \phi$$

(Here, ϕ is work function of metal)

$$\text{or } E = KE + \phi$$

$$KE = E - \phi = 10 \text{ eV} - 5 \text{ eV}$$

$$\Rightarrow KE = 5 \text{ eV} \quad \dots(ii)$$

Now, energy per unit time for incident photons will be

$$\therefore E = Nh\nu$$

$$\therefore \frac{dE}{dt} = h\nu \frac{dN}{dt} \text{ or } h\nu \dot{N} \quad \dots(iii)$$

From Eqs. (i) and (iii), we get

$$h\nu \dot{N} = 16 \times 10^{-7} \text{ or } E \dot{N} = 16 \times 10^{-7}$$

But $E = 10 \text{ eV}$, so

$$\dot{N} (10 \times 1.6 \times 10^{-19}) = 16 \times 10^{-7}$$

$$\Rightarrow \dot{N} = 10^{12}$$

$\therefore$ Only 10% of incident photons emit electrons.

So, emitted electrons per second are

$$\frac{10}{100} \times 10^{12} = 10^{11}$$

- 43** In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

[2019, 12 Jan Shift-II]

- (a) 250 nm (b) 2020 nm
(c) 1700 nm (d) 220 nm

Ans. (a)

Minimum wavelength occurs when mercury atom deexcites from highest energy level.

∴ Maximum possible energy absorbed by mercury atom = $\Delta E = 5.6 - 0.7 = 4.9$ eV

Wavelength of photon emitted in deexcitation is

$$\lambda = \frac{hc}{E} \approx \frac{1240 \text{ eVnm}}{4.9 \text{ eV}} \approx 250 \text{ nm}$$

Note Frank-Hertz experiment was the first electrical measurement to show quantum nature of atoms. In a vacuum tube energetic electrons are passed through thin mercury vapour film. It was discovered that when an electron collided with a mercury atom, it loses only a specific quantity (4.9 eV) of its kinetic energy. This experiment shows existence of quantum energy levels.

- 44** If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called

[AIEEE 2010]

- (a) X-rays (b) ultraviolet rays
(c) microwaves (d) γ -rays

Ans. (a)

As power = Energy $\times$ time
or $4 \times 10^3 = hf \times 10^{20} \left[\because \frac{\text{energy}}{\text{time}} = \text{power} \right]$

$$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

$$f = 6.64 \times 10^{16} \text{ Hz}$$

The obtained frequency lies in the band of X-rays.

- 45** Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is

[AIEEE 2007]

- (a) $\frac{\nu}{c}$ (b) $h\nu c$
(c) $\frac{h\nu}{c^2}$ (d) $\frac{h\nu}{c}$

Ans. (d)

The momentum of the photon,

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

TOPIC 3 Wave Nature of Light

- 46** The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is

$$[m_e = \text{mass of electron} = 9 \times 10^{-31} \text{ kg}]$$

$$h = \text{Planck's constant} = 6.6 \times 10^{-34} \text{ Js}$$

$$K_B = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

[2021, 1 Sep Shift-II]

- (a) 6.26 nm (b) 8.46 nm
(c) 2.26 nm (d) 3.25 nm

Ans. (a)

Given, Planck's constant, $h = 6.6 \times 10^{-34} \text{ Js}$
Boltzmann constant, $K_B = 1.38 \times 10^{-23} \text{ J/K}$
Mass of an electron, $m_e = 9 \times 10^{-31} \text{ kg}$
Temperature of an ideal gas, $T = 300 \text{ K}$
As we know that, de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \quad \dots(i)$$

Here, E is the kinetic energy.

$$E = \frac{3K_B T}{2}$$

Substituting value of E in Eq. (i), we get,

$$\lambda = \frac{h}{\sqrt{3mK_B T}}$$

Substituting the given values in the above equation, we get

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} = 6.26 \text{ nm}$$

∴ The corresponding de-Broglie wavelength of an electron approximately at 300 K is 6.26 nm.

- 47** Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to

[2021, 31 Aug Shift-II]

$$(a) \left(\frac{m_p}{m_e} \right)^{\frac{3}{2}} \quad (b) \sqrt{\frac{m_e}{m_p}} \\ (c) \sqrt{\frac{m_p}{m_e}} \quad (d) \frac{m_p}{m_e}$$

Ans. (c)

At same temperature kinetic energy of proton and electron are same

$$KE_p = KE_e \Rightarrow \frac{p_p^2}{2m_p} = \frac{p_e^2}{2m_e}$$

$$\frac{p_p^2}{p_e^2} = \frac{m_p}{m_e} \quad \dots(i)$$

Using uncertainty in determining the position, $\Delta x \approx \frac{h}{\Delta p}$

$$\therefore \text{Ratio, } \frac{\Delta x_e}{\Delta x_p} = \frac{\Delta p_p}{\Delta p_e} = \sqrt{\frac{m_p}{m_e}} \quad [\text{from Eq. (i)}]$$

- 48** A moving proton and electron have the same de-Broglie wavelength. If k and p denote the KE and momentum, respectively. Then, choose the correct option.

[2021, 31 Aug Shift-I]

- (a) $K_p < K_e$ and $p_p = p_e$
(b) $K_p = K_e$ and $p_p = p_e$
(c) $K_p < K_e$ and $p_p < p_e$
(d) $K_p > K_e$ and $p_p = p_e$

Ans. (a)

Let de-Broglie wavelength of proton and electron are λ_p and λ_e .

Kinetic energy of electron and proton are K_e and K_p .

Momentum of electron and proton are p_e and p_p .

According to question, $\lambda_p = \lambda_e = \lambda$

By using de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

where, m is mass and h is Planck's constant.

$$\therefore p = h/\lambda \quad \dots(i)$$

From Eq. (i)

$$p_e = p_p \text{ as } \lambda_e = \lambda_p$$

$$\text{Again, } K = \frac{h^2}{2m\lambda} \quad \dots(ii)$$

From Eq. (ii)

$$\therefore \frac{K_e}{K_p} = \frac{m_p}{m_e} \times \frac{\lambda_p}{\lambda_e}$$

$$\Rightarrow \frac{K_e}{K_p} = \frac{m_p}{m_e} \quad [\because \lambda_e = \lambda_p]$$

$$\therefore m_p > m_e \\ K_e > K_p$$

- 49** The de-Broglie wavelength of a particle having kinetic energy E is λ . How much extra energy must be given to this particle, so that the de-Broglie wavelength reduces to 75% of the initial value?

[2021, 26 Aug Shift-II]

- (a) $\frac{1}{9}E$ (b) $\frac{7}{9}E$ (c) E (d) $\frac{16}{9}E$

Ans. (b)

Given, initial kinetic energy, $E_1 = E$

Initial de-Broglie wavelength, $\lambda_1 = \lambda$

Consider the wavelength of particle changes to 75% of λ after providing energy ΔE to the particle.

Hence, final wavelength of particle, $\lambda_2 = 0.75\lambda$

Final energy, $E_2 = E + \Delta E$

Relationship between de-Broglie wavelength and energy of particle is given as

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Here, h is Planck's constant and m is mass of particle which is also constant term.

Therefore, we get

$$\lambda \propto \frac{1}{\sqrt{E}}$$

Thus, we can write the relationship as

$$\begin{aligned} \frac{\lambda_1}{\lambda_2} &= \frac{\sqrt{E_2}}{\sqrt{E_1}} \\ \Rightarrow \frac{\lambda}{0.75\lambda} &= \frac{\sqrt{E + \Delta E}}{\sqrt{E}} \\ \Rightarrow \frac{4}{3} &= \frac{\sqrt{E + \Delta E}}{\sqrt{E}} \end{aligned}$$

Squaring both sides of above equation, we get

$$\begin{aligned} \frac{16}{9} &= \frac{E + \Delta E}{E} \\ \Rightarrow 16E &= 9E + 9\Delta E \\ \Rightarrow \Delta E &= \frac{7}{9}E \end{aligned}$$

- 50** A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^6 m/s and a photon of a radiation of linear momentum 10^{-27} kg-m/s travels in vacuum. The wavelength of photon is times the wavelength of the particle.

[2021, 27 July Shift-I]

Ans.

(910) Given, $m = 9.1 \times 10^{-31}$ kg
Speed, $v = 10^6$ ms $^{-1}$

For photon,

Momentum, $p = 10^{-27}$ kg ms $^{-1}$

$\therefore$ We know that, de-Broglie wavelength can be given as $\lambda = \frac{h}{mv}$

$\therefore$ For particle,

$$\lambda_{pa} = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6} \dots(i)$$

and for photon,

$$\lambda_{ph} = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{10^{-27}} \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \frac{\lambda_{ph}}{\lambda_{pa}} = \frac{\frac{6.6 \times 10^{-34}}{10^{-27}}}{\frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^6}}$$

$$\Rightarrow \frac{\lambda_{ph}}{\lambda_{pa}} = 910 \Rightarrow \lambda_{ph} = 910(\lambda_{pa})$$

- 51** An electron moving with speed v and a photon moving with speed c , have same de-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is

[2021, 25 July Shift-II]

- (a) $\frac{3c}{v}$ (b) $\frac{v}{3c}$
(c) $\frac{v}{2c}$ (d) $\frac{2c}{v}$

Ans. (c)

Given, speed of electron and photon be v and c , respectively.

Let λ_e and λ_p be the de-Broglie wavelength of electron and proton respectively and $\lambda_e = \lambda_p$ (i)

As we know that, de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{mc} \dots(ii)$$

where, h = Planck's constant

p = momentum and c = speed of photon.

Kinetic energy of photon,

$$E_p = \frac{hc}{\lambda_p}$$

Using Eq. (ii)

$$E_p = \frac{hcm_p c}{h} = m_p c^2 \dots(iii)$$

$$\text{Similarly, } E_e = \frac{1}{2} m_e v^2 \dots(iv)$$

Dividing Eqs. (iv) by (iii), we get

$$\frac{E_e}{E_p} = \frac{\frac{1}{2} m_e v^2}{m_p c^2} \dots(v)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow p_e = p_p \Rightarrow m_e v = m_p c \Rightarrow \frac{m_e}{m_p} = \frac{c}{v}$$

Putting this value in Eq. (v), we get

$$\frac{E_e}{E_p} = \frac{1}{2} \frac{c}{v} \cdot \frac{v^2}{c^2} = \frac{v}{2c}$$

- 52** An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is

[2021, 22 July Shift-II]

- (a) $\frac{m_p}{m_e}$ (b) 1 (c) $\sqrt{\frac{m_p}{m_e}}$ (d) $\frac{m_e}{m_p}$

Ans. (c)

de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\text{Hence, } \frac{\lambda_e}{\lambda_p} = \frac{\sqrt{m_p q_p}}{\sqrt{m_e q_e}} = \sqrt{\frac{m_p}{m_e}} [\because q_p = q_e]$$

- 53** An electron having de-Broglie wavelength λ is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is

[2021, 20 July Shift-II]

- (a) Zero (b) $\frac{2m^2 c^2 \lambda^2}{h^2}$
(c) $\frac{2mc\lambda^2}{h}$ (d) $\frac{hc}{mc}$

Ans. (c)

Since, we know that de-Broglie wavelength of an electron can be given as

$$\lambda = \frac{h}{mv} \dots(i)$$

Also, we know that,

$$\text{Kinetic energy, } K = \frac{p^2}{2m} \dots(ii)$$

where, p is the linear momentum.

$\therefore$ Linear momentum,

$$p = \frac{h}{\lambda} \dots(iii)$$

From Eq. (ii) and Eq. (iii), we get

$$K = \frac{h^2}{2\lambda^2 m}$$

$$\Rightarrow hv = \frac{h^2}{2\lambda^2 m} \quad \left\{ \because K = hv = \frac{hc}{\lambda} \right\}$$

$$\Rightarrow \frac{hc}{\lambda_c} = \frac{h^2}{2\lambda^2 m}$$

where, λ_c = cut off wavelength.

$$\Rightarrow \lambda_c = \frac{2m\lambda^2 c}{h}$$

- 54** The speed of electrons in a scanning electron microscope is $1 \times 10^7 \text{ ms}^{-1}$. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of **[2021, 18 March Shift-II]**

- (a) 1837 (b) $\frac{1}{1837}$
(c) $\sqrt{1837}$ (d) $\frac{1}{\sqrt{1837}}$

Ans. (a)

According to de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

Since, resolving power,

$$\begin{aligned} \text{RP} &\propto \frac{1}{\lambda} \\ \Rightarrow \text{RP} &\propto \frac{mv}{h} \\ \Rightarrow \text{RP} &\propto m \quad (\text{speed} = \text{constant}) \\ \Rightarrow \frac{\text{RP}_p}{\text{RP}_e} &= \frac{m_p}{m_e} \\ \Rightarrow \frac{\text{RP}_p}{\text{RP}_e} &= 1837 \quad (\text{given, } m_p = 1837 m_e) \end{aligned}$$

Hence, the resolving power of scanning proton microscope will be changed by factor of 1837.

- 55** A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is **[2021, 18 March Shift-I]**

- (a) $\frac{1}{16}$ times the mass of electron
(b) 8 times the mass of electron
(c) 16 times the mass of electron
(d) $\frac{1}{8}$ times the mass of electron

Ans. (d)

$$\text{Given, } \frac{\lambda_p}{\lambda_e} = \frac{2}{1}, \frac{v_e}{v_p} = \frac{1}{4}$$

We know that, de-Broglie wavelength,

$$\begin{aligned} \lambda &= \frac{h}{mv} \\ \Rightarrow \frac{\lambda_p}{\lambda_e} &= \frac{m_e v_e}{m_p v_p} \Rightarrow \frac{2}{1} = \frac{m_e}{m_p} \left(\frac{1}{4} \right) \\ \Rightarrow m_p &= \frac{1}{8} m_e \end{aligned}$$

- 56** An electron of mass m and a photon have same energy E . The ratio of wavelength of electron to that of photon is (c being the velocity of light) **[2021, 17 March Shift-I]**

- (a) $\frac{1}{c} \left(\frac{2m}{E} \right)^{1/2}$ (b) $\frac{1}{c} \left(\frac{E}{2m} \right)^{1/2}$
(c) $\left(\frac{E}{2m} \right)^{1/2}$ (d) $c(2mE)^{1/2}$

Ans. (b)

We know that

$$\lambda = \frac{h}{p} \quad \dots(i)$$

where, λ = wavelength,
 h = Planck's constant
and p = linear momentum

$$\text{Also, kinetic energy (KE)} = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2mKE} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{\sqrt{2mKE}} \quad \text{or} \quad \lambda = \frac{h}{\sqrt{2mE}} \quad [\because KE = E]$$

$$\text{Wavelength of electron, } \lambda_e = \frac{h}{\sqrt{2mE}}$$

$$\text{and wavelength of photon, } \lambda_p = \frac{hc}{E}$$

[$\because$ Both electron and photon have same energy.]

$$\begin{aligned} \Rightarrow \frac{\lambda_e}{\lambda_p} &= \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}} \\ \Rightarrow \frac{\lambda_e}{\lambda_p} &= \frac{1}{c} \left(\frac{E}{2m} \right)^{1/2} \end{aligned}$$

- 57** The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths? ($m_p = 1.00727 \text{ u}$, $m_e = 0.00055 \text{ u}$) **[2021, 16 March Shift-II]**

- (a) 1860 : 1 (b) $(1860)^2$: 1
(c) 41.4 : 1 (d) 43 : 1

Ans. (d)

Given, mass of proton, $m_p = 1.00727 \text{ u}$

Mass of electron, $m_e = 0.00055 \text{ u}$

Potential, $V = 100 \text{ V}$

As, de-Broglie wavelength can be given as

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2mqV}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}} \\ \Rightarrow \frac{\lambda_1}{\lambda_2} &= \sqrt{\frac{m_2}{m_1}} \\ \Rightarrow \frac{\lambda_e}{\lambda_p} &= \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.00727 \text{ u}}{0.00055 \text{ u}}} \\ \frac{\lambda_e}{\lambda_p} &= \sqrt{1831.4} = 42.79 \\ \text{or } \frac{\lambda_e}{\lambda_p} &\approx \frac{43}{1} \\ \text{or } \lambda_e : \lambda_p &= 43 : 1 \end{aligned}$$

- 58** Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A An electron microscope can achieve better resolving power than an optical microscope.

Reason R The de-Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below. **[2021, 26 Feb Shift-I]**

- (a) A is true but R is false.
(b) Both A and R are true and R is the correct explanation of A.
(c) Both A and R are true but R is not the correct explanation of A.
(d) A is false but R is true.

Ans. (b)

As we know that,

$$\text{Resolving power of microscope} = \frac{1}{\Delta\theta} = \frac{2u \sin\theta}{\lambda}$$

$$\text{i.e. resolving power} \propto \frac{1}{\lambda}$$

and since, wavelength of electron emitted (λ_e) < wavelength of visible light (λ_v)

$\therefore$ Resolving power of electron microscope > Resolving power of optical microscope.

Hence, option (b) is the correct.

- 59** The wavelength of an X-ray beam is $10 \text{ \AA}$. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3} h \text{ kg}$.

The value of x is

(h = Planck's constant)

[2021, 25 Feb Shift-II]

Ans. (10)

Given, wavelength of X-rays,

$$\lambda = 10 \text{ \AA} = 10 \times 10^{-10} \text{ m}$$

Speed of light in free space,

$$c = 3 \times 10^8 \text{ m/s}$$

Since, energy

$$(E) = \frac{hc}{\lambda} = mc^2 \quad \dots (i)$$

$$\Rightarrow m = \frac{h}{c\lambda} = \frac{h}{3 \times 10^8 \times 10 \times 10^{-10}}$$

$$= \frac{h}{3 \times 10^{-1}} = \frac{10h}{3} \text{ kg}$$

$$\therefore x = 10$$

- 60** An electron of mass m_e and a proton of mass $m_p = 1836 m_e$ are moving with the same speed. The ratio of their de-Broglie wavelength $\frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}}$ will be

[2021, 25 Feb Shift-II]

- (a) 1
(c) $\frac{1}{1836}$
- (b) 1836
(d) 918

Ans. (b)

Given, mass of proton (m_p) is 1836 times the mass of electron (m_e) and velocity of proton (v_p) is equal to velocity of electron (v_e).

$$\text{As, wavelength } (\lambda) = \frac{h}{p} = \frac{h}{mv}$$

where, h is Planck's constant and p is momentum.

$$\therefore \lambda \propto \frac{1}{m}$$

$$\Rightarrow \frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = \frac{1836 m_e}{m_e} = 1836$$

- 61** An α -particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de-Broglie wavelengths are λ_α and λ_p , respectively.

The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is

[2021, 25 Feb Shift-I]

- (a) 3.8
(c) 7.8
- (b) 8
(d) 2.8

Ans. (d)

de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p \cdot 2e}{m_p \cdot e}}$$

$$= \frac{2\sqrt{2}}{1} = 2 \times 1.414 = 2.8$$

- 62** The de-Broglie wavelength of a proton and α -particle are equal. The ratio of their velocities is

[2021, 24 Feb Shift-II]

- (a) 4 : 3
(c) 4 : 2
- (b) 4 : 1
(d) 1 : 4

Ans. (b)

Let $\lambda_p, \lambda_\alpha, m_p, m_\alpha, v_p, v_\alpha, p_p$ and p_α be the wavelength, mass, velocity and momentum of proton and α -particle, respectively.

$$\text{Given, } \lambda_p = \lambda_\alpha$$

As we know that,

$$\lambda = h/p$$

$$\therefore \frac{h}{p_p} = \frac{h}{p_\alpha}$$

$$\Rightarrow p_p = p_\alpha$$

$$\Rightarrow m_p v_p = m_\alpha v_\alpha$$

$$\Rightarrow m_p v_p = 4m_p v_\alpha \quad (\because m_\alpha = 4m_p)$$

$$\Rightarrow \frac{v_p}{v_\alpha} = \frac{4}{1} \text{ or } 4 : 1$$

- 63** An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be

[2021, 24 Feb Shift-II]

- (a) 10^{-3} nm
(c) 10^{-2} nm
- (b) 10^{-1} nm
(d) 10^{-4} nm

Ans. (a)

Given,

$$V = 1.24 \text{ million volt}$$

$$= 1.24 \times 10^6 \text{ volt}$$

Since, energy (E) = eV

$$\text{where, } e \text{ is the charge of electron}$$

$$= 1.6 \times 10^{-19} \text{ C}$$

$$\therefore E = 1.6 \times 10^{-19} \times 1.24 \times 10^6 \quad \dots (i)$$

As we know that,

Energy of photon,

$$E = \frac{hc}{\lambda} \quad \dots (ii)$$

Here, Planck's constant,

$$h = 6.67 \times 10^{-34} \text{ J-s,}$$

c = speed of light in free space,
 $c = 3 \times 10^8 \text{ ms}^{-1}$

Equating Eqs. (i) and (ii), we get

$$1.6 \times 10^{-19} \times 1.24 \times 10^6$$

$$= \frac{6.67 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\Rightarrow \lambda = \frac{20.01 \times 10^{-13}}{1.6 \times 1.24} = 10.09 \times 10^{-13}$$

$$= 1.009 \times 10^{-12} \approx 10^{-3} \times 10^{-9}$$

$$= 10^{-3} \text{ nm}$$

- 64** A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4} . The mass of the particle is close to

[2020, 2 Sep Shift-II]

- (a) $4.8 \times 10^{-27} \text{ kg}$
(c) $12 \times 10^{-28} \text{ kg}$
- (b) $9.1 \times 10^{-31} \text{ kg}$
(d) $9.7 \times 10^{-28} \text{ kg}$

Ans. (d)

de-Broglie wavelength of a particle is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{Given, } \frac{\lambda_{\text{particle}}}{\lambda_{\text{electron}}} = 1.878 \times 10^{-4}$$

$$\Rightarrow \frac{m_e v_e}{m_p v_p} = 1.878 \times 10^{-4}$$

$$\Rightarrow \frac{m_e \times v_e}{m_p \times 5v_e} = 1.878 \times 10^{-4} \quad (\because v_p = 5v_e)$$

$$\Rightarrow m_p = \frac{m_e}{5 \times 1.878 \times 10^{-4}}$$

$$= \frac{9.1 \times 10^{-31}}{5 \times 1.878 \times 10^{-4}}$$

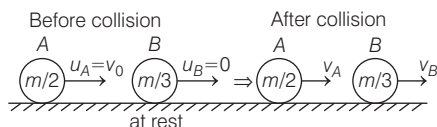
$$= 9.7 \times 10^{-28} \text{ kg}$$

Hence, correct option is (d).

- 65** Particle A of mass $m_A = m/2$ moving along the X-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = m/3$. If both particles move along the X-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength λ_0 before collision is

[2020, 4 Sep Shift-I]

- (a) $\Delta\lambda = \frac{5}{2}\lambda_0$
(c) $\Delta\lambda = 2\lambda_0$
- (b) $\Delta\lambda = 4\lambda_0$
(d) $\Delta\lambda = \frac{3}{2}\lambda_0$

Ans. (b)

Using law of conservation of linear momentum, we have

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$\frac{m}{2} \times v_0 + \frac{m}{3} \times 0 = \frac{m}{2} \times v_A + \frac{m}{3} \times v_B$$

$$\frac{v_0}{2} = \frac{v_A}{2} + \frac{v_B}{3} \Rightarrow \frac{v_0}{2} = \frac{3v_A + 2v_B}{6}$$

$$3v_A + 2v_B = 3v_0 \quad \dots (i)$$

By the definition of coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - u_B} \Rightarrow 1 = \frac{v_B - v_A}{v_0 - 0}$$

($\because$ for elastic collision, $e = 1$)

$$\Rightarrow -v_A + v_B = v_0$$

$$\Rightarrow -3v_A + 3v_B = 3v_0 \quad \dots (ii)$$

On adding eqs. (i) and (ii), we get

$$\Rightarrow 5v_B = 6v_0 \Rightarrow v_B = \frac{6v_0}{5}$$

Substituting this value in eq. (i), we get

$$3v_A + 2\left(\frac{6v_0}{5}\right) = 3v_0$$

$$\Rightarrow 3v_A = 3v_0 - \frac{12}{5}v_0$$

$$3v_A = \frac{3}{5}v_0 \Rightarrow v_A = \frac{v_0}{5}$$

Now, de-Broglie wavelength of particle A before collision,

$$(\lambda_A)_i = \frac{h}{(p_A)_i}$$

$$\lambda_0 = \frac{h}{m_A u_A} \quad [(\lambda_A)_i = \lambda_0 \text{ (given)}]$$

$$= \frac{h}{\frac{m}{2} \times v_0} = \frac{2h}{mv_0} \quad \dots (iii)$$

And, de-Broglie wavelength of particle A after collision,

$$(\lambda_A)_f = \frac{h}{(p_A)_f} = \frac{h}{m_A v_A}$$

$$= \frac{h}{\frac{m}{2} \times \frac{v_0}{5}} = \frac{10h}{mv_0} = 5 \times \frac{2h}{mv_0} = 5\lambda_0$$

So, change in de-Broglie wavelength of particle A is

$$(\Delta\lambda)_A = (\lambda_A)_f - (\lambda_A)_i$$

$$= 5\lambda_0 - \lambda_0 = 4\lambda_0$$

Hence, correct option is (b).

- 66** Assuming the nitrogen molecule is moving with rms velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to
(Given, weight of N_2 molecule = 4.64×10^{-26} kg, Boltzmann constant = 1.38×10^{-23} J/K and Planck's constant = 6.63×10^{-34} J-s)

[2020, 6 Sep Shift-II]

- (a) 0.24 Å (b) 0.20 Å
(c) 0.34 Å (d) 0.44 Å

Ans. (a)

Given, $T = 400$ K

Mass of single molecule of N_2 , $m = 4.64 \times 10^{-26}$ kg

Boltzmann constant, $k = 1.38 \times 10^{-23}$ J/K

Planck's constant, $h = 6.63 \times 10^{-34}$ J-s

The de-Broglie wavelength of a gas molecule moving with rms velocity at T temperature is given by

$$\lambda = \frac{h}{\sqrt{3kTm}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.38 \times 10^{-23} \times 400 \times 4.64 \times 10^{-26}}}$$

$$\lambda = 0.24 \text{ Å}$$

Hence, correct option is (a).

- 67** A beam of electrons of energy E scatters from a target having atomic spacing of 1 Å. The first maximum intensity occurs at $\theta = 60^\circ$. Then, E (in eV) is

(Given, Planck's constant,

$h = 6.64 \times 10^{-34}$ Js,

$1 \text{ eV} = 1.6 \times 10^{-19}$ J and

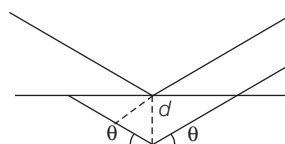
electron mass, $m = 9.1 \times 10^{-31}$ kg.)

[2020, 5 Sep Shift-I]

Ans. (50)

From Bragg's law,

$$2d \sin \theta = n\lambda$$



For first maxima, $n = 1$

$$\Rightarrow 2d \sin \theta = \lambda$$

$$\Rightarrow 2d \sin \theta = \frac{h}{\sqrt{2Em}}$$

$$\left\{ \begin{array}{l} \because \frac{1}{2}mv^2 = E \\ p = \sqrt{2Em} \\ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \end{array} \right\}$$

$$\Rightarrow 2Em = \frac{h^2}{4d^2 \sin^2 \theta} \Rightarrow E = \frac{h^2}{8md^2 \sin^2 \theta}$$

Given, $d = 1 \text{ Å} = 10^{-10} \text{ m}$, $\theta = 60^\circ$

$$h = 6.64 \times 10^{-34} \text{ Js}, m = 9.1 \times 10^{-31} \text{ kg}$$

Substituting these values in above equation, we get

$$E = \frac{(6.64 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2 \sin^2 60^\circ}$$

$$= \frac{44.0896 \times 10^{-68}}{8 \times 9.1 \times 10^{-51} \times \left(\frac{3}{4}\right)}$$

$$= 0.8075 \times 10^{-17} \text{ J}$$

$$= \frac{0.8075 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 50.47 \text{ eV} \approx 50 \text{ eV}$$

- 68** An electron, a doubly ionised helium ion He^{++} and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths λ_e , $\lambda_{\text{He}^{++}}$ and λ_p is

[2020, 6 Sep Shift-I]

- (a) $\lambda_e > \lambda_{\text{He}^{++}} > \lambda_p$ (b) $\lambda_e < \lambda_{\text{He}^{++}} = \lambda_p$
(c) $\lambda_e > \lambda_p > \lambda_{\text{He}^{++}}$ (d) $\lambda_e < \lambda_p < \lambda_{\text{He}^{++}}$

Ans. (c)

de-Broglie wavelength of a particle having kinetic energy, is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(\text{KE})}}$$

For same kinetic energy, $\lambda \propto \frac{1}{\sqrt{m}}$

$$\because m_{\text{He}^{++}} > m_p > m_e \Rightarrow \lambda_{\text{He}^{++}} < \lambda_p < \lambda_e$$

- 69** An electron (of mass m) and a photon have the same energy E in the range of a few electron volt. The ratio of the de Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

[2020, 7 Jan Shift-II]

- (a) $\left(\frac{E}{2m}\right)^{1/2}$ (b) $\frac{1}{c} \left(\frac{2E}{m}\right)^{1/2}$
(c) $c(2mE)^{1/2}$ (d) $\frac{1}{c} \left(\frac{E}{2m}\right)^{1/2}$

Ans. (d)

Here, energy of electron = E

$$\Rightarrow \frac{1}{2}mv^2 = E \Rightarrow m^2v^2 = 2mE$$

$$\Rightarrow p^2 = 2mE$$

or momentum of electron, $p = \sqrt{2mE}$

So, de Broglie wavelength associated with electron, $\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

Now, photon energy = E

$$\Rightarrow hf = E \Rightarrow \frac{hc}{\lambda_p} = E$$

or wavelength of photon, $\lambda_p = hc/E$

$$\text{Hence, ratio } \frac{\lambda_e}{\lambda_p} = \frac{\left(\frac{h}{\sqrt{2mE}}\right)}{\left(\frac{hc}{E}\right)} = \frac{1}{c} \left(\frac{E}{2m}\right)^{1/2}$$

- 70** When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A . The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is

[2020, 8 Jan Shift-I]

- (a) 4 eV
(b) 2 eV
(c) 1.5 eV
(d) 3 eV

Ans. (a)

Kinetic energy T of electron and its de Broglie wavelength λ are related as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{m^2 v^2}} = \frac{h}{\sqrt{2m \times \frac{1}{2} m v^2}} = \frac{h}{\sqrt{2mT}}$$

and given, $\lambda_B = 2\lambda_A$

$$\Rightarrow \frac{h}{\sqrt{2mT_B}} = 2 \cdot \frac{h}{\sqrt{2mT_A}} \Rightarrow T_A = 4T_B$$

Also given,

$$T_B = T_A - 1.5 \Rightarrow T_B = 4T_B - 1.5$$

$$\Rightarrow 3T_B = 1.5 \Rightarrow T_B = 0.5 \text{ eV}$$

Now, for metal B,

incident energy = kinetic energy of electron + work function

$$\Rightarrow 4.5 = 0.5 + \phi_0 \Rightarrow \phi_0 = 4 \text{ eV}$$

- 71** An electron (mass m) with initial velocity $\mathbf{v} = v_0 \hat{i} + v_0 \hat{j}$ is in an electric field $\mathbf{E} = -E_0 \hat{k}$. If λ_0 is initial de-Broglie wavelength of electron, then its de Broglie wavelength at time t is given by

[2020, 8 Jan Shift-II]

(a) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$ (b) $\frac{\lambda_0}{\sqrt{2 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$

(c) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{2m^2 v_0^2}}}$ (d) $\frac{\lambda_0 \sqrt{2}}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$

Ans. (c)

Initial velocity of electron, $\mathbf{v} = v_0 \hat{i} + v_0 \hat{j}$

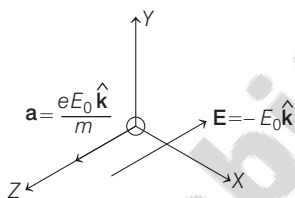
$$\Rightarrow |\mathbf{u}| = \sqrt{v_0^2 + v_0^2} = v_0 \sqrt{2}$$

Initial de Broglie wavelength associated with electron,

$$\lambda_0 = \frac{h}{p} = \frac{h}{mv_0 \sqrt{2}} \quad \dots(i)$$

After t seconds velocity of electron,

$$\mathbf{v} = \mathbf{u} + \mathbf{at}$$



$$\text{or } \mathbf{v} = (v_0 \hat{i} + v_0 \hat{j}) + \frac{eE_0}{m} \cdot t \cdot \hat{k}$$

Note that electron is accelerated opposite to the field.

Magnitude of velocity after time t ,

$$|\mathbf{v}| = \sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0 t}{m}\right)^2}$$

$$= \sqrt{2v_0^2 + \frac{e^2 E_0^2 t^2}{m^2}} = v_0 \sqrt{2 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}$$

$\therefore$ de Broglie wavelength of electron at time t ,

$$\lambda = \frac{h}{p} = \frac{h}{m|\mathbf{v}|} \Rightarrow \lambda = \frac{h}{mv_0 \sqrt{2 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$$

$$= \frac{\lambda_0 \sqrt{2}}{\sqrt{2 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}} = \frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{2m^2 v_0^2}}}$$

- 72** A particle moving with kinetic energy E has de Broglie wavelength λ . If energy ΔE is added to its energy, the wavelength become $\lambda/2$. Value of ΔE is

[2020, 9 Jan Shift-I]

- (a) $2E$
(b) $4E$
(c) $3E$
(d) E

Ans. (c)

de-Broglie wavelength of a moving particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

where, K = kinetic energy of the particle.

Now, in given condition,

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \dots(i)$$

$$\text{and } \frac{\lambda}{2} = \frac{h}{\sqrt{2m(E + \Delta E)}} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$2 = \frac{\sqrt{E + \Delta E}}{\sqrt{E}} \Rightarrow \Delta E = 3E$$

- 73** An electron of mass m and magnitude of charge $|e|$ initially at rest gets accelerated by a constant electric field $\mathbf{E}$. The rate of change of de Broglie wavelength of this electron at time t ignoring relativistic effects is

[2020, 9 Jan Shift-II]

- (a) $\frac{h}{|e|\mathbf{E}\sqrt{t}}$ (b) $-\frac{h}{|e|\mathbf{E}t}$
- (c) $\frac{-h}{|e|\mathbf{E}t^2}$ (d) $\frac{|e|\mathbf{E}t}{h}$

Ans. (c)

Initial acceleration of electron,

$$a = \frac{F}{m} = \frac{|e|\mathbf{E}}{m}$$

Velocity of electron after time t ,

$$\mathbf{v} = \mathbf{u} + \mathbf{at} = \frac{|e|\mathbf{E}}{m} \cdot t$$

Momentum of electron, $m\mathbf{v} = |e|\mathbf{E} \cdot t$

de Broglie wavelength λ associated with

$$\text{electron, } \lambda = \frac{h}{p} = \frac{h}{|e|\mathbf{E} \cdot t}$$

Rate of change of de Broglie wavelength

$$\text{with time, } \frac{d\lambda}{dt} = \frac{-h}{|e|\mathbf{E}t^2}$$

Here, negative sign shows wavelength decreases with time.

- 74** Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2 , respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by

[2019, 8 April Shift-I]

$$(a) \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \quad (b) \lambda = \sqrt{\lambda_1 \lambda_2}$$

$$(c) \lambda = \frac{\lambda_1 + \lambda_2}{2} \quad (d) \frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

Ans. (a)

Given, de-Broglie wavelengths for particles are λ_1 and λ_2 .

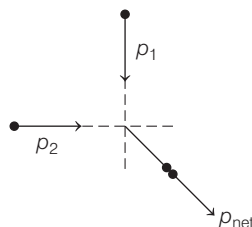
$$\text{So, } \lambda_1 = \frac{h}{p_1} \text{ and } \lambda_2 = \frac{h}{p_2}$$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.



So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{\text{net}} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2 \cos 90^\circ}$$

$$= \sqrt{p_1^2 + p_2^2}$$

$$\text{Since, } p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

$$\text{So, } p_{\text{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \quad \dots(i)$$

Let the de-Broglie wavelength after the collision is λ_{net} , then

$$p_{\text{net}} = \frac{h}{\lambda_{\text{net}}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{\text{net}}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \Rightarrow \frac{1}{\lambda_{\text{net}}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

- 75** A particle P is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y , respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of P is

[2019, 9 April Shift-II]

$$(a) \lambda_x - \lambda_y \quad (b) \frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}$$

$$(c) \frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y} \quad (d) \lambda_x + \lambda_y$$

Ans. (b)

Initially,

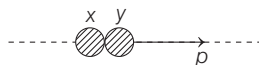


We have, de-Broglie wavelengths associated with particles are

$$\lambda_x = \frac{h}{p_x} \text{ and } \lambda_y = \frac{h}{p_y}$$

$$\Rightarrow p_x = \frac{h}{\lambda_x} \text{ and } p_y = \frac{h}{\lambda_y}$$

Finally, particles collided to form a single particle.



As we know that, linear momentum is conserved in collision.

$$\text{So, } p_p = |p_x - p_y|$$

$$\Rightarrow p_p = \left| \frac{h}{\lambda_x} - \frac{h}{\lambda_y} \right|$$

So, de-Broglie wavelength of combined particle is

$$\lambda_p = \frac{h}{|p_p|} = \frac{h}{\left| \frac{h}{\lambda_x} - \frac{h}{\lambda_y} \right|} = \frac{h}{\left| \frac{h\lambda_y - h\lambda_x}{\lambda_x \lambda_y} \right|}$$

$$= \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$$

- 76** A particle A of mass ' m ' and charge ' q ' is accelerated by a potential difference of 50 V. Another particle B of mass ' $4m$ ' and charge ' q ' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to

[2019, 12 Jan Shift-I]

- (a) 4.47 (b) 10.00
(c) 0.07 (d) 14.14

Ans. (d)

de-Broglie wavelength associated with a moving charged particle of charge q is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where, V = accelerating potential.

Ratio of de-Broglie wavelength for particle A and B is,

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{m_B q_B V_B}}{\sqrt{m_A q_A V_A}} = \sqrt{\frac{m_B}{m_A}} \cdot \sqrt{\frac{q_B}{q_A}} \cdot \sqrt{\frac{V_B}{V_A}}$$

Substituting the given values, we get

$$= \sqrt{\frac{4m}{m}} \cdot \sqrt{\frac{q}{q}} \cdot \sqrt{\frac{2500}{50}} = 2 \times 1 \times 5 \times 1.414$$

$$= 14.14$$

- 77** In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to

[2019, 10 Jan Shift-I]

- (a) 500 keV (b) 1 keV
(c) 100 keV (d) 25 keV

Ans. (d)

Given, resolution achieved in electron microscope is of the order of wavelength.

So, to resolve 7.5×10^{-12} m separation wavelength associated with electrons is

$$\lambda = 7.5 \times 10^{-12} \text{ m}$$

$\therefore$ Momentum of electrons required is

$$p = h/\lambda$$

or kinetic energy of electron must be

$$KE = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$$

Substituting the given values, we get

$$= \frac{\left(\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}} \right)^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.5 \times 10^{-12})^2} \text{ eV}$$

$$\times (1.6 \times 10^{-19})$$

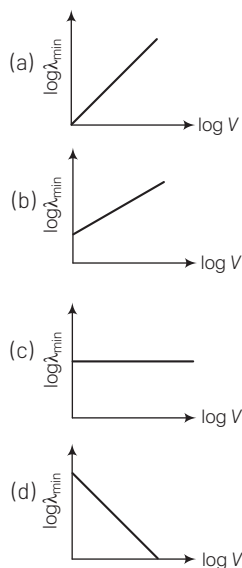
$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$$= 26593.4 \approx 26.6 \times 10^3 \text{ eV} \approx 26 \text{ keV}$$

which is nearest to 25 keV.

- 78** An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{\text{min}}$ with $\log V$ is correctly represented in

[JEE Main 2017]

**Ans. (d)**

$$\lambda_{\min} = hc / eV$$

$$\log(\lambda_{\min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

- 79** A particle A of mass m and initial velocity v collides with a particle B of mass $m/2$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is [JEE Main 2017]

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$
 (c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

Ans. (a)

For elastic collision,

$$p_{\text{before collision}} = p_{\text{after collision}}$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \dots(i)$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$$\therefore 1 = \frac{v_B - v_A}{v} \Rightarrow v = v_B - v_A \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$v_A = \frac{v}{3} \quad \text{and} \quad v_B = \frac{4v}{3}$$

$$\text{Hence, } \frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{h}{mv_A}\right)}{\left(\frac{h}{\frac{m}{2} \cdot v_B}\right)} = \frac{v_B}{2v_A} = \frac{4/3}{2/3} = 2$$

- 80** Match List I (fundamental experiment) with List II (its conclusion) and select the correct option from the choices given below the list. [JEE Main 2015]

List I	List II
A. Franck-Hertz experiment	1. Particle nature of light
B. Photo-electric experiment	2. Discrete energy levels of atom
C. Davisson-Germer experiment	3. Wave nature of electron
	4. Structure of atom

A	B	C	A	B	C
(a) 1	4	3	(b) 2	4	3
(c) 2	1	3	(d) 4	3	2

Ans. (c)

- (A) Franck-Hertz experiments is associated with discrete energy levels of atom.
 (B) Photo-electric experiment is associated with particle nature of light.
 (C) Davisson-Germer experiment is associated with wave nature of electron.

- 81** This question has Statement I and Statement II. Of the four choices given the statements, choose the one that describes the two statements. [AIEEE 2012]

Statement I Davisson-Germer experiment established the wave nature of electrons.

Statement II If electrons have wave nature, they can interfere and show diffraction.

- (a) Statement I is false, Statement II is true
 (b) Statement I is true, Statement II is false
 (c) Statement I is true, Statement II is true; Statement I is the correct explanation of Statement II
 (d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. (c)

Davisson and Germer experimentally established wave nature of electron by observing diffraction pattern while bombarding electrons on Ni-crystal.

- 82** After absorbing a slowly moving neutron of mass m_N (momentum ~ 0), a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1$, ($6m_1 = M + m_N$), respectively. If the de-Broglie wavelength of the nucleus with mass m_1 is λ , then de-Broglie wavelength of the other nucleus will be [AIEEE 2011]
 (a) 25λ (b) 5λ (c) $\lambda/5$ (d) λ

Ans. (d)

$$\text{de-Broglie wavelength, } \lambda = \frac{h}{mv} = \frac{h}{p}$$

where, p = momentum

By conservation of momentum,

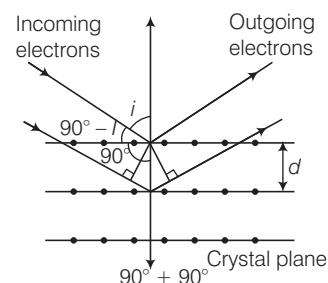
$$p_1 + p_2 = 0 \text{ or } p_1 = p_2 \Rightarrow \frac{h}{p_1} = \frac{h}{p_2}$$

$$\therefore \lambda_1 = \lambda_2 = \lambda$$

$$\left(\because \lambda = \frac{h}{p} \right)$$

Directions Q. Nos. 83 and 85 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure). [AIEEE 2008]



- 83** Electrons accelerated by potential V are diffracted from a crystal. If $d = 1\text{\AA}$ and $i = 30^\circ$, V should be about ($h = 6.6 \times 10^{-34} \text{ J-s}$, $m_e = 9.1 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$) [AIEEE 2008]
 (a) 2000 V (b) 50 V
 (c) 500 V (d) 1000 V

Ans. (b)

For constructive interference,

$$2d \cos i = n\lambda = \frac{h}{\sqrt{2meV}}$$

On substituting values, we get

$$V \approx 50 \text{ V}$$

- 84** If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de-Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)

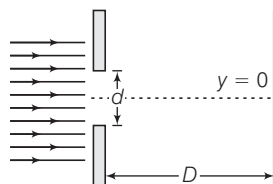
[AIEEE 2008]

- (a) $d \sin i = n\lambda_{dB}$
 (b) $2d \cos i = n\lambda_{dB}$
 (c) $2d \sin i = n\lambda_{dB}$
 (d) $d \cos i = n\lambda_{dB}$

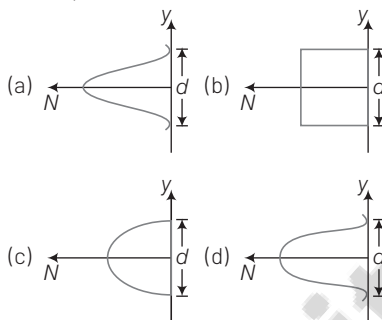
Ans. (b)Expression is given by $2d \cos i = n\lambda_{dB}$.

- 85** In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de-Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure).

[AIEEE 2008]



Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y=0$ corresponds to the middle of the slit)?

**Ans. (d)**

As diffraction pattern has to be wider than slit width, so (d) is the correct option.

- 86** If the kinetic energy of a free electron doubles, its de-Broglie wavelength changes by the factor

[AIEEE 2005]

- (a) $\frac{1}{2}$ (b) 2
 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Ans. (c)We know that, $\lambda = \frac{h}{mv}$

$$\text{and } K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m}$$

$$\Rightarrow mv = \sqrt{2mK}$$

$$\text{Thus, } \lambda = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{K}}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{\sqrt{K_1}}{\sqrt{K_2}} = \frac{\sqrt{K_1}}{\sqrt{2K_1}} \quad [\because K_2 = 2K_1]$$

$$\text{or } \frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}} \quad \text{or } \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

- 87** Formation of covalent bonds in compounds exhibits [AIEEE 2002]

- (a) wave nature of electron
 (b) particle nature of electron
 (c) both wave and particle nature of electron
 (d) None of the above

Ans. (a)

Formation of covalent bonds due to the wave nature of particles is done in compounds.

26

Atoms

TOPIC 1

Alpha-Particle Scattering and Rutherford Nuclear Model of Atom

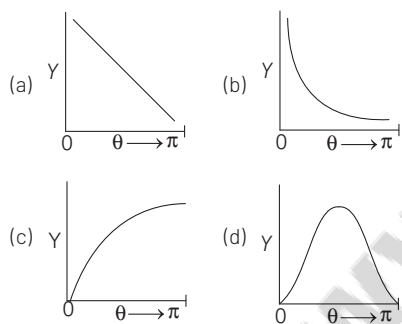
- 01** The graph which depicts the results of Rutherford's gold foil experiment with α -particles is

θ : Scattering angle

Y : Number of scattered α -particles detected

(plots are schematic and not to scale)

[JEE Main 2020, 8 Jan Shift-I]

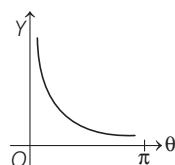


Ans. (b)

In Rutherford's experiment, number of particles scattered at large angles is very less and most of the particles are scattered at small angles.

Hence, graph of Y = number of α -particles and

θ = scattering angle is as shown in the figure below.



- 02** An alpha nucleus of energy $(1/2)mv^2$ bombards a heavy nuclear target of charge Ze . Then, the distance of closest approach for the alpha nucleus will be proportional to [AIEEE 2006]
- (a) v^2 (b) $1/m$ (c) $1/v^4$ (d) $1/Ze$

Ans. (d)

Since, nuclear target is heavy, it can be assumed safely that it will remain stationary and will not move due to the Coulombic interaction force.

At distance of closest approach, relative velocity of two particles is v .

Here, target is considered as stationary, so α -particle comes to rest instantaneously at distance of closest approach. Let required distance be r , then from work-energy theorem,

$$0 - \frac{mv^2}{2} = -\frac{1}{4\pi\epsilon_0} \frac{Ze \times 2e}{r}$$

$$\Rightarrow r \propto \frac{1}{m} \text{ or } r \propto \frac{1}{v^2} \text{ or } r \propto Ze^2$$

TOPIC 2

Bohr Model and Hydrogen Spectra

- 03** x different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number $n=6$? The value of x is [2021, 27 Aug Shift-II]

Ans. (15)

Given, principal quantum number, $n=6$

Number of wavelength, $N = \frac{n(n-1)}{2}$

$$= \frac{6(6-1)}{2} = 3(5) = 15$$

$\therefore x = 15$

- 04** A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atom in the first excited state and a photon is released. Find the frequency of the emitted photon. ($h = 6.6 \times 10^{-34}$ Js)

[2021, 31 Aug Shift-II]

- (a) 1.45×10^{16} MHz (b) 0.19×10^{15} MHz
(c) 1.45×10^9 MHz (d) 9.0×10^{27} MHz

Ans. (c)

Given that, energy of free electron,

$$E_1 = 2.6 \text{ eV}$$

We know that, the energy of H-atom in its first excited state ($n=2$),

$$E_2 = \frac{-13.6}{2^2} = -\frac{13.6}{4} \text{ eV}$$

Now, the energy of emitted photons will be the difference of these two energies E_1 and E_2 .

$$\Delta E = E_1 - E_2$$

$$\therefore h\nu = \left[2.6 - \left(-\frac{13.6}{4} \right) \right] \text{ eV}$$

$$\Rightarrow \nu = \frac{(10.4 + 13.6) \times 1.6 \times 10^{-19}}{4 \times 6.6 \times 10^{-34}}$$

$$= 1.45 \times 10^9 \text{ MHz}$$

- 05** The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work-function of the metal is (Take, mass of electron = 9.1×10^{-31} kg)

[2021, 20 July Shift-I]

- (a) 1.36 eV (b) 1.88 eV
(c) 0.16 eV (d) 0.82 eV

Ans. (d)

Energy of photon can be given as

$$E_p = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{eV}$$

where, n_1 = lower energy level
and n_2 = higher energy level.

As per question, $n_1 = 2$, $n_2 = 3$

$$\begin{aligned} \therefore E_p &= 13.6 \left[\frac{1}{(2)^2} - \frac{1}{(3)^2} \right] \\ &= 13.6 \left[\frac{1}{4} - \frac{1}{9} \right] = 13.6 \left[\frac{9-4}{36} \right] = 1.89 \text{ eV} \end{aligned}$$

We know that work-function is the minimum energy required to eject photoelectrons from metal surface.

For gold plate, it will be

$$\phi = E_p - KE_{\max} \quad \dots(i)$$

[Given, $B = 5 \times 10^{-4} \text{ T}$, $r = 7 \text{ mm} = 7 \times 10^{-3} \text{ m}$,
 $q = 1.6 \times 10^{-19} \text{ C}$ and $m = 9.1 \times 10^{-31} \text{ kg}$]

Therefore, velocity of photoelectrons will be

$$\begin{aligned} v &= \frac{Bqr}{m} = \frac{5 \times 10^{-4} \times 1.6 \times 10^{-19} \times 7 \times 10^{-3}}{9.1 \times 10^{-31}} \\ &= 6.15 \times 10^5 \text{ ms}^{-1} \end{aligned}$$

Kinetic energy will be

$$\begin{aligned} \therefore KE &= \frac{1}{2} mv^2 \\ &= \frac{1 \times 9.1 \times 10^{-31} \times (6.15 \times 10^5)^2}{2 \times 1.6 \times 10^{-19}} \text{ eV} \\ &= 1.075 \text{ eV} \end{aligned}$$

Now, putting the values, in Eq. (i), we get

$$\begin{aligned} \phi &= (1.89 - 1.075) \text{ eV} \\ &= 0.82 \text{ eV} \end{aligned}$$

06 A particular hydrogen like ion emits radiation of frequency 2.92×10^{15}

Hz when it makes transition from $n=3$ to $n=1$. The frequency in Hz of radiation emitted in transition from $n=2$ to $n=1$ will be

[2021, 26 Aug Shift-I]

- (a) 0.44×10^{15} (b) 6.57×10^{15}
(c) 4.38×10^{15} (d) 2.46×10^{15}

Ans. (d)

The frequency of the emitted radiation by a particular hydrogen like ion when it makes transition from $n_1 = n_{\text{initial}}$ to $n_2 = n_{\text{final}}$ is given by

$$f = RZ^2 c \left(\frac{1}{n_2} - \frac{1}{n_1} \right)$$

where, c = velocity of lightand R = Rydberg's constant.For H-atom, $Z = 1$ So, for transition from $n=3$ to $n=1$,

$$f_1 = Rc \left(1 - \frac{1}{3^2} \right) \quad \dots(i)$$

Similarly for transition from $n=2$ to $n=1$,

$$f_2 = Rc \left(1 - \frac{1}{2^2} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_1}{f_2} = \frac{Rc \left(1 - \frac{1}{9} \right)}{Rc \left(1 - \frac{1}{4} \right)} = \frac{\frac{8}{9}}{\frac{3}{4}} = \frac{32}{27}$$

Here, $f_1 = 2.92 \times 10^{15} \text{ Hz}$

$$\Rightarrow \frac{2.92 \times 10^{15}}{f_2} = \frac{32}{27}$$

$$\begin{aligned} \Rightarrow f_2 &= \frac{2.92 \times 10^{15} \times 27}{32} \\ &= 2.46 \times 10^{15} \text{ Hz} \end{aligned}$$

07 The K_α X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atom with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be keV.

(Round off to the nearest integer)

[Take, $h = 4.14 \times 10^{-15} \text{ eV-s}$,

$$c = 3 \times 10^8 \text{ ms}^{-1}]$$

[2021, 27 July Shift-II]**Ans. (10)**

As per question, K_α X-ray of molybdenum, has wavelength 0.071 nm.

Since, we know that,

$$K_\alpha = \frac{hc}{\lambda} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{0.071 \times 10^{-9}} = 17.5 \text{ keV}$$

$\therefore$ Energy difference between L and K will be,

$$\begin{aligned} E_K - E_L &= K_\alpha \\ \Rightarrow E_K - E_L &= 17.49 \\ \Rightarrow E_L &= E_K - 17.49 \\ \Rightarrow E_L &= 27.5 - 17.49 \\ \Rightarrow E_L &\approx 10 \text{ keV} \end{aligned}$$

$\therefore$ The energy of molybdenum atom when an L-electron is knocked out will be 10 keV.

08 An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3 eV. The proton captures the electrons and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive

metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron? **[2021, 27 July Shift-II]**

- (a) 7.61 eV
(b) 1.41 eV
(c) 3.3 eV
(d) No photoelectron would be emitted

Ans. (b)Threshold wavelength, $\lambda_0 = 4000 \text{ Å}$

Initial energy of electron = 3 eV

After this, the proton captures the electron and forms a hydrogen atom in second excited state. This means

$$\begin{aligned} \text{final energy of electron} &= \frac{-13.6}{n^2} = \frac{-13.6}{(3)^2} \text{ eV} \\ &= -1.51 \text{ eV} \end{aligned}$$

[For second excited state, $n=3$] $\therefore$ Loss of energy is emitted as photon,

$$\text{Photon energy, } \Delta E = (3 - (-1.51)) \text{ eV} = \frac{hc}{\lambda}$$

$$\Rightarrow \frac{hc}{\lambda} = 4.51 \text{ eV} \quad \dots(i)$$

 $\therefore$ According to photoelectric equation,

$$KE_{\max} = \frac{hc}{\lambda} - \phi \quad \dots(ii)$$

 $\therefore$ From Eqs. (i) and (ii), we get

$$\begin{aligned} \Rightarrow KE_{\max} &= 4.51 - \left(\frac{hc}{\lambda_0} \right) \\ &= 4.51 - \frac{12400 \text{ eVÅ}}{4000 \text{ Å}} \end{aligned}$$

On solving, we will get

$$KE_{\max} = 1.41 \text{ eV}$$

09 Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionisation potential of this hydrogen atom will be

[2021, 18 March Shift-I]

- (a) 13.6 eV (b) 2815.2 eV
(c) 331.2 eV (d) 27.2 eV

Ans. (b)

As we know that, the ionisation potential of the hydrogen,

$$IP_H = 13.6 \text{ eV}$$

We know that,

$$IP = \frac{Z^2 mq^4}{8n^2 h^2 \epsilon_0^2}$$

$$\Rightarrow IP \propto mq^4 \Rightarrow \frac{IP_\mu}{IP_H} = \frac{m_\mu q_\mu^4}{m_e q_e^4}$$

$$\Rightarrow \frac{IP}{IP_{\mu}} = \frac{m_{\mu}}{m_e} \quad [\because q_{\mu} = q_e]$$

$$IP_{\mu} = \frac{m_{\mu}}{m_e} \times IP_H = \frac{207 m_e}{m_e} \times 13.6$$

$$= 2815.2 \text{ eV}$$

- 10** The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region? [2021, 17 March Shift-II]

(a) Brackett series (b) Paschen series
(c) Lyman series (d) Balmer series

Ans. (d)

When an electron jumps from the higher energy level to $n=2$ orbit, Balmer series of the line spectrum is obtained.

The Balmer series of the hydrogen atom lies in the visible region.

However, Brackett and Paschen series of hydrogen atom lies in the infrared region and Lyman series of hydrogen atom lies in the ultraviolet region.

- 11** A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantisation conditions are applied, radii of possible orbitals r_n vary with $n^{1/\alpha}$, where α is

[2021, 17 March Shift-II]

Ans. (3)

As we know, the force on the particle is given as

$$F = -\frac{dU(r)}{dr}$$

Given, $U(r) = U_0 r^4$

$$\Rightarrow F = -\frac{d(U_0 r^4)}{dr} \Rightarrow F = -4U_0 r^3$$

As we know, the force on the particle moving in a circular orbit of radius r will be centripetal force

$$\left| \frac{mv^2}{r} \right| = 4U_0 r^3$$

$$\Rightarrow \frac{v^2 \propto r^4}{v \propto r^2} \quad \dots(i)$$

Hence, the velocity of the particle is square of the radius of the orbit.

Using the Bohr's quantisation condition, which states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$, here, h is

Planck's constant.

$$\text{i.e., } n \frac{h}{2\pi} = mvr \quad [\text{From Eq. (i)}]$$

$$\Rightarrow r^3 \propto n \Rightarrow r \propto n^{1/3}$$

Comparing the above relation with $n^{1/\alpha}$, we get

$$\alpha = 3$$

- 12** Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom? [2021, 17 March Shift-I]

(a) 1 (b) 6 (c) 4 (d) 8

Ans. (b)

Since we know that, energy of hydrogen atom is

$$E = -13.6 \frac{Z^2}{n^2}$$

where, Z is the atomic number and n is the energy state.

$\therefore$ For hydrogen atom, $Z = 1$ and $n = 1$

$$\Rightarrow E = \frac{-13.6(1)^2}{1^2}$$

$$\Rightarrow E = -13.6 \text{ eV} \quad \dots(i)$$

For carbon atom, $Z = 6$, $n = ?$

$$\Rightarrow E = \frac{-13.6(6)^2}{n^2} \quad \dots(ii)$$

According to question, carbon has same energy as the ground state energy of hydrogen atom.

$\therefore$ From Eqs. (i) and (ii), we get

$$-13.6 = \frac{-13.6(6)^2}{n^2} \Rightarrow n^2 = 6^2$$

$$\Rightarrow n = 6$$

- 13** If an electron is moving in the n th orbit of the hydrogen atom, then its velocity v_n for the n th orbit is given as [2021, 17 March Shift-I]

(a) $v_n \propto n$ (b) $v_n \propto \frac{1}{n}$
(c) $v_n \propto n^2$ (d) $v_n \propto \frac{1}{n^2}$

Ans. (b)

As we know that, the velocity of electron in the n th shell of hydrogen atom is given by

$$v_n = \frac{Ze^2}{2\epsilon_0 nh}$$

where, Z = atomic number,

n = order of orbit,

e = charge on an electron,

h = Planck's constant

and ϵ_0 = absolute permittivity of free space

$$\Rightarrow v_n \propto \frac{1}{n}$$

- 14** The first three spectral lines of H-atom in the Balmer series are given $\lambda_1, \lambda_2, \lambda_3$ considering the Bohr atomic model, the wavelengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a

factor of approximately $x \times 10^{-1}$. The value of x to the nearest integer, is

[2021, 16 March Shift-I]

Ans. (15)

Since, we know that,

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(i)$$

where, λ = wavelength of light emitted,

R = Rydberg's constant,

Z = atomic number,

n_1 = principal quantum number of lower energy level

and n_2 = principal quantum number of higher energy level.

Therefore, for 1st spectral line of Balmer series, $n_1 = 2$ and $n_2 = 3$

$$\frac{1}{\lambda_1} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \frac{1}{\lambda_1} = RZ^2 \left(\frac{5}{36} \right) \quad \dots(ii)$$

Similarly, for 3rd spectral line, $n_1 = 2$ and $n_2 = 5$

$$\frac{1}{\lambda_3} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{5^2} \right)$$

$$\Rightarrow \frac{1}{\lambda_3} = RZ^2 \left(\frac{21}{100} \right) \quad \dots(iii)$$

Now, dividing Eq. (iii) by Eq. (ii), we get

$$\frac{\frac{1}{\lambda_3}}{\frac{1}{\lambda_1}} = \frac{RZ^2 \left(\frac{21}{100} \right)}{RZ^2 \left(\frac{5}{36} \right)} \Rightarrow \frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_3} = 1.512 = 15.12 \times 10^{-1}$$

Comparing with the given value in the question i.e., $x \times 10^{-1}$, the value of $x = 15$.

- 15** The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state will be [2021, 26 Feb Shift-II]

(a) 4.17 m/s (b) 2.19 m/s
(c) 3.25 m/s (d) 4.34 m/s

Ans. (a)

As we know, mass of photon, $m = 1.6 \times 10^{-27} \text{ kg}$

Using de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \dots (i)$$

where, h is Planck's constant

$$= 6.63 \times 10^{-34} \text{ J-s}$$

and p is momentum of photon.

By using Rydberg wavelength equation,

$$\frac{1}{\lambda} = R \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$$

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{25} \right]$$

$$\Rightarrow \frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{24}{25} \right]$$

$$\Rightarrow \lambda = \frac{25}{1.09 \times 10^7 \times 24}$$

Substituting values in Eq. (i), we get

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34} \times 1.09 \times 10^7 \times 24}{1.6 \times 10^{-27} \times 25} \\ = \frac{6.63 \times 1.09 \times 24}{1.6 \times 2.5} = 4.17 \text{ ms}^{-1}$$

- 16** If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is

[2021, 26 Feb Shift-I]

- (a) 1 : 9 (b) 7 : 108
(c) 7 : 135 (d) 1 : 3

Ans. (c)

By using Rydberg's formula,

$$\frac{1}{\lambda} = R \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$$

where, R is Rydberg constant.

For wavelength of third member of Lyman series $n_2 = 1$ and $n_1 = 4$.

$$\therefore \frac{1}{\lambda_1} = R \left[\frac{1}{1^2} - \frac{1}{4^2} \right] = R \left[\frac{1}{1} - \frac{1}{16} \right] \quad \dots (i)$$

For wavelength of first member of Paschen series, $n_2 = 3$ and $n_1 = 4$

$$\therefore \frac{1}{\lambda_2} = R \left[\frac{1}{3^2} - \frac{1}{4^2} \right] = R \left[\frac{1}{9} - \frac{1}{16} \right] \quad \dots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{\left[\frac{1}{9} - \frac{1}{16} \right]}{\left[\frac{1}{1} - \frac{1}{16} \right]} = \frac{7}{9 \times 15} = \frac{7}{135}$$

- 17** The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from $n=2$ to $n=1$ state is

[2021, 25 Feb Shift-II]

- (a) 121.8 nm (b) 194.8 nm
(c) 490.7 nm (d) 913.3 nm

Ans. (a)

Given, electron is moving from $n=2$ to $n=1$.

From Bohr's hydrogen spectrum (Rydberg formula)

$$\frac{1}{\lambda} = R \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$$

where, λ = wavelength,

$R = 1.097 \times 10^7 \text{ m}^{-1}$ (Rydberg's constant)

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{1} - \frac{1}{4} \right] = \frac{3}{4} \times 1.097 \times 10^7$$

$$\lambda = \frac{4}{3 \times 1.097 \times 10^7} = 1.215 \times 10^{-7}$$

$$= 121.5 \times 10^{-9} \\ \approx 121.8 \text{ nm}$$

- 18** According to Bohr atom model, in which of the following transitions will the frequency be maximum?

[2021, 24 Feb Shift-II]

- (a) $n=4$ to $n=3$
(b) $n=2$ to $n=1$
(c) $n=5$ to $n=4$
(d) $n=3$ to $n=2$

Ans. (b)

Let n_f, n_i be the final and initial orbit.

$$\text{As we know that, } \frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{n_f^2} - \frac{1}{n_i^2} \right]$$

Now, checking for each option, we get

$$(a) \frac{1}{\lambda} \propto \left[\frac{1}{3^2} - \frac{1}{4^2} \right] = \left[\frac{1}{9} - \frac{1}{16} \right] = 0.05 \quad \dots (i)$$

$$(b) \frac{1}{\lambda} \propto \left[\frac{1}{1^2} - \frac{1}{4^2} \right] = 0.75 \quad \dots (ii)$$

$$(c) \frac{1}{\lambda} \propto \left[\frac{1}{16} - \frac{1}{25} \right] = 0.0225 \quad \dots (iii)$$

$$(d) \frac{1}{\lambda} \propto \left[\frac{1}{4} - \frac{1}{9} \right] = 0.14 \quad \dots (iv)$$

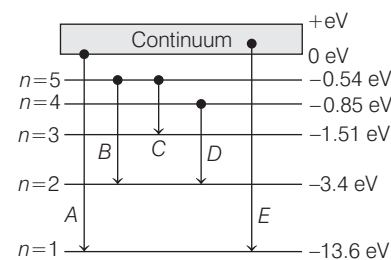
The option (b) has highest value.

$$\text{Since, frequency, } f = \frac{c}{\lambda} \Rightarrow f \propto \frac{1}{\lambda}$$

$\therefore$ Frequency will be maximum for transition $n=2$ to $n=1$.

- 19** In the given figure, the energy levels of hydrogen atom have been shown alongwith some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent

[2021, 24 Feb Shift-I]



- (a) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.
(b) The ionisation potential of hydrogen, second member of Balmer series and third member of Paschen series.
(c) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.
(d) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

Ans. (d)

In transition A, hydrogen atom comes from higher energy state $n = \infty$ to lower energy state $n = 1$. Hence, transition A represents series limit of Lyman series. In transition B, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 2$. Hence, transition B represents 3rd line of Balmer series.

In transition C, hydrogen atom comes from higher energy state $n = 5$ to lower energy state $n = 3$. Hence, transition C represents 2nd line of Paschen series. Hence, option (d) is correct.

- 20** In a hydrogen atom, electron makes a transition from $(n+1)$ th level to the n th level. If $n \gg 1$, the frequency of radiation emitted is proportional to

[2020, 2 Sep Shift-II]

- (a) $\frac{1}{n}$ (b) $\frac{1}{n^3}$
(c) $\frac{1}{n^2}$ (d) $\frac{1}{n^4}$

Ans. (b)

Frequency of emitted radiation during transition from $(n+1)$ th level to the n th level in hydrogen atom is

$$f = \frac{\Delta E}{h} \quad \{\because \Delta E = hf\} \\ = \frac{1}{h} [E_{n+1} - E_n] = \frac{1}{h} \left(\frac{-13.6}{(n+1)^2} - \left(\frac{-13.6}{n^2} \right) \right)$$

$$\begin{aligned}
 &= \frac{1}{h} \times 13.6 \times \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right) \\
 &= \frac{13.6}{h} \left\{ \frac{2n+1}{n^2(n+1)^2} \right\} \\
 \Rightarrow f &= \frac{13.6}{h} \left\{ \frac{n \left(2 + \frac{1}{n} \right)}{n^4 \left(1 + \frac{1}{n^2} \right)^2} \right\} \quad \dots(i)
 \end{aligned}$$

If $n \gg 1$, $\frac{1}{n}$ and $\frac{1}{n^2} \approx 0$

So, neglecting $\frac{1}{n}$ and $\frac{1}{n^2}$ in Eq (i), we have

$$\begin{aligned}
 f &= \frac{13.6}{h} \times \left(\frac{2n}{n^4} \right) = \frac{2 \times 13.6}{h} \times \frac{1}{n^3} \\
 \Rightarrow f &\propto \frac{1}{n^3}
 \end{aligned}$$

Hence, correct option is (b).

- 21** Hydrogen ion and singly ionised helium atom are accelerated from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to

[2020, 3 Sep Shift-II]

(a) 1 : 2 (b) 10 : 7 (c) 2 : 1 (d) 5 : 7

Ans. (c)

When a charged particle of charge q and mass m is accelerated through a potential difference of V volts, then by energy conservation,

$$qV = \frac{1}{2}mv^2$$

where, v = velocity gained by charged particle.

$$\text{So, } v = \sqrt{\frac{2qV}{m}}$$

$$\text{Hence, } \frac{v_H}{v_{He}} = \frac{\text{velocity of H-ion}}{\text{velocity of He-ion}}$$

$$\begin{aligned}
 &= \frac{\sqrt{\frac{e}{m}}}{\sqrt{\frac{e}{4m}}} = \sqrt{4} = 2 \\
 &\quad \sqrt{\frac{e}{4m}}
 \end{aligned}$$

$$v_H : v_{He} = 2 : 1$$

Hence, correct option is (c).

- 22** In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is $304 \text{ \AA}$. The corresponding difference for the Paschen series (in $\text{\AA}$) is

[2020, 4 Sep Shift-I]

Ans. (10553)

$$\begin{aligned}
 \text{As, } \frac{1}{\lambda} &= R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \\
 \Rightarrow \lambda &= \frac{1}{R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)} \quad \dots(i)
 \end{aligned}$$

For Lyman series

Largest wavelength ($n_i = 2$ and $n_f = 1$)

$$\lambda_1 = \frac{1}{R \left(\frac{1}{1^2} - \frac{1}{2^2} \right)} = \frac{1}{R \left(\frac{3}{4} \right)} = \frac{4}{3R}$$

Shortest wavelength ($n_i = \infty$ and $n_f = 1$)

$$\lambda_2 = \frac{1}{R \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right)} = \frac{1}{R(1)} = \frac{1}{R}$$

Now, it is given that the difference between the largest and the shortest wavelengths of the Lyman series is $304 \text{ \AA}$.

$$\begin{aligned}
 \text{So, } \Delta\lambda &= \lambda_1 - \lambda_2 \\
 304 &= \frac{4}{3R} - \frac{1}{R} \Rightarrow 304 = \frac{1}{3R} \\
 R &= \frac{1}{3 \times 304} \Rightarrow R = \frac{1}{912} \quad \dots(ii)
 \end{aligned}$$

For Paschen series

Largest wavelength ($n_i = 4$ and $n_f = 3$)

$$\lambda_3 = \frac{1}{R \left(\frac{1}{3^2} - \frac{1}{4^2} \right)} = \frac{1}{R \left(\frac{7}{144} \right)} = \frac{144}{7R}$$

Shortest wavelength ($n_i = \infty$ and $n_f = 3$)

$$\lambda_4 = \frac{1}{R \left(\frac{1}{3^2} - \frac{1}{\infty^2} \right)} = \frac{1}{R \left(\frac{1}{9} \right)} = \frac{9}{R}$$

Now, the difference between the largest and the shortest wavelengths of the Paschen series,

$$\begin{aligned}
 \Delta\lambda &= \lambda_3 - \lambda_4 \\
 &= \frac{144}{7R} - \frac{9}{R} = \frac{144 - 63}{7R} = \frac{81}{7R} = \frac{81}{7 \left(\frac{1}{912} \right)} \\
 &= \frac{81 \times 912}{7} = 10553.14 \text{ \AA} \approx 10553 \text{ \AA} \quad [\text{From eq. (ii)}]
 \end{aligned}$$

- 23** A particle of mass $200 \text{ MeV}/c^2$ collides with a hydrogen atom at rest. Soon after the collision, the particle comes to rest and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $N/4$. The value of N is
- (Given, the mass of the hydrogen atom to be $1 \text{ GeV}/c^2$)

[2020, 5 Sep Shift-I]

Ans. (51)

Here, mass of hydrogen atom is 5 times mass of colliding particle.

Using law of conservation of linear momentum,

$$mv + 0 = 0 + 5mv' \Rightarrow v' = \frac{v}{5}$$

When the atom goes to its first excited state,

Energy dissipated

$$\begin{aligned}
 &= \left[\frac{-13.6}{(2)^2} \right] \text{eV} - \left[\frac{-13.6}{(1)^2} \right] \text{eV} \\
 &= -3.4 + 13.6 = 10.2 \text{ eV} \quad \dots(i)
 \end{aligned}$$

Loss of kinetic energy

$$\begin{aligned}
 &= \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2 \\
 &= \frac{1}{2}mv^2 \left(1 - \frac{1}{5} \right) = \frac{4}{5} \left(\frac{mv^2}{2} \right) = \frac{4}{5} K \quad \dots(ii)
 \end{aligned}$$

From eqs. (i) and (ii), we get

$$\begin{aligned}
 \frac{4}{5} K &= 10.2 \text{ eV} \\
 \Rightarrow K &= \frac{10.2 \times 5}{4} \text{ eV} = \frac{51.0}{4} \text{ eV} \\
 \Rightarrow \frac{N}{4} &= \frac{51}{4} \text{ eV} \Rightarrow N = 51
 \end{aligned}$$

- 24** The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16} \text{ s}$. The frequency of revolution of the electron in its first excited state (in Hz) is

[2020, 7 Jan Shift-I]

(a) 1.6×10^{14} (b) 5.6×10^{12}
(c) 6.2×10^{15} (d) 7.8×10^{14}

Ans. (d)

Time period of revolution of an electron in its n th state (or n th orbit) of H-atom,

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi r_1 \cdot n^2}{(v_1/n)}$$

where, r_1 and v_1 are radius and velocity of electron in ground state.

$$\text{So, } T_n = \frac{2\pi r_1 \cdot n^3}{v_1} \text{ or } T_n = T_1 \cdot n^3$$

where, T_1 = time period of revolution in ground state.

Now, in first excited state $n = 2$, so time period,

$$T_2 = T_1 \times 2^3 = 8 \times 1.6 \times 10^{-16} \text{ s}$$

$$\text{As, frequency} = \frac{1}{\text{time period}}$$

So, frequency of revolution of electron in first excited state,

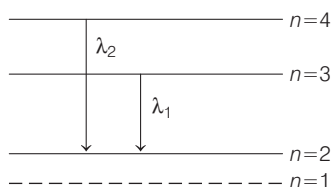
$$f_2 = \frac{1}{T_2} = \frac{1}{8 \times 1.6 \times 10^{-16}} = 7.8 \times 10^{14} \text{ Hz}$$

- 25** The first member of the Balmer series of hydrogen atom has a wavelength of 6561 Å. The wavelength of the second member of the Balmer series (in nm) is

..... [2020, 8 Jan Shift-II]

Ans. (486)

Balmer series of H-atom spectrum occurs when an excited electron jumps to second level or $n=2$ state.



Emitted wavelengths are given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where, $n=3, 4, 5 \dots \infty$.

First member is obtained in transition $n=3$ to $n=2$.

$$\Rightarrow \frac{1}{\lambda_1} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \quad \dots(i)$$

Second member is obtained in transition $n=4$ to $n=2$.

$$\Rightarrow \frac{1}{\lambda_2} = R \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\begin{aligned} \frac{\frac{1}{\lambda_1}}{\frac{1}{\lambda_2}} &= \frac{\left(\frac{1}{2^2} - \frac{1}{3^2} \right)}{\left(\frac{1}{2^2} - \frac{1}{4^2} \right)} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{4} - \frac{1}{9} \right)}{\left(\frac{1}{4} - \frac{1}{16} \right)} \\ &= \frac{5}{36} \times \frac{64}{12} \\ \Rightarrow \frac{\lambda_2}{\lambda_1} &= \frac{5}{9} \times \frac{4}{3} = \frac{20}{27} \Rightarrow \lambda_2 = \lambda_1 \times \frac{20}{27} \\ &= 6561 \times \frac{20}{27} = 4860 \text{ Å} = 486 \text{ nm} \end{aligned}$$

- 26** The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ?

[2020, 9 Jan Shift-II]

- (a) 8.6 nm (b) 24.2 nm
(c) 11.4 nm (d) 35.8 nm

Ans. (c)

As, ionisation energy of ground state = -(ground state energy)

$$\Rightarrow 9 \times 13.6 = -E_1 \quad (\because 1 \text{ Rydberg} = 13.6 \text{ eV})$$

$$\text{or } E_1 = -9 \times 13.6 \text{ eV}$$

and energy of n th state for the given atom will be

$$E_n = \frac{E_1}{n^2} = \frac{-9 \times 13.6}{n^2} \text{ eV}$$

In transition from second excited state ($n=3$) to ground state ($n=1$), energy released will be

$$\begin{aligned} \Delta E &= E(n=3) - E(n=1) \\ &= \left(\frac{-9 \times 13.6}{3^2} \right) - \left(\frac{-9 \times 13.6}{1^2} \right) \end{aligned}$$

$$\begin{aligned} \text{or } \Delta E &= 9 \times 13.6 \left(1 - \frac{1}{9} \right) \\ &= 8 \times 13.6 \text{ eV} \end{aligned}$$

So, wavelength of radiation emitted will be

$$\begin{aligned} \lambda &= \frac{hc}{\Delta E} = \frac{1240 \text{ (eV} \cdot \text{nm)}}{\Delta E \text{ (eV)}} \\ \Rightarrow \lambda &= \frac{1240}{8 \times 13.6} \\ &= 11.39 \text{ nm} \\ &\approx 11.4 \text{ nm} \end{aligned}$$

- 27** In Li^{++} , electron in first Bohr orbit is excited to a level by a radiation of wavelength λ . When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ?

[Take, $h = 6.63 \times 10^{-34} \text{ Js}$;

$$c = 3 \times 10^8 \text{ ms}^{-1}]$$

[2019, 10 April Shift-II]

- (a) 9.4 nm (b) 12.3 nm
(c) 10.8 nm (d) 11.4 nm

Ans. (c)

Number of spectral lines produced as an excited electron falls to ground state ($n=1$) is,

$$N = \frac{n(n-1)}{2}$$

In given case, $N=6$

$$\therefore 6 = \frac{n(n-1)}{2}$$

$$\Rightarrow n=4$$

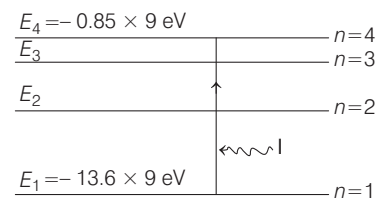
So, Li^{++} electron is in its 3rd excited state.

Now, using the expression of energy of an electron in n th energy level,

$$E_n = -\frac{13.6Z^2}{n^2} \text{ eV}$$

where, Z is the atomic number.

$\therefore$ Energy levels of Li^{++} electron are as shown



So, energy absorbed by electron from incident photon of wavelength λ is

$$\begin{aligned} \Delta E &= \frac{hc}{\lambda} \\ \Rightarrow (13.6 \times 9 - 0.85 \times 9) &= \frac{hc}{\lambda} \\ \Rightarrow \lambda &= \frac{hc}{9(13.6 - 0.85)} \\ \Rightarrow \lambda &= \frac{1240 \text{ eV} \cdot \text{nm}}{9 \times 12.75 \text{ eV}} = 10.8 \text{ nm} \end{aligned}$$

- 28** Radiation coming from transitions $n=2$ to $n=1$ of hydrogen atoms fall on He^+ ions in $n=1$ and $n=2$ states. The possible transition of helium ions as they absorb energy from the radiation is

[2019, 8 April Shift-I]

- (a) $n=2 \rightarrow n=3$ (b) $n=1 \rightarrow n=4$
(c) $n=2 \rightarrow n=5$ (d) $n=2 \rightarrow n=4$

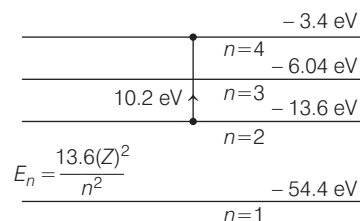
Ans. (d)

De-excitation energy of hydrogen electron in transition $n=2$ to $n=1$ is

$$\begin{aligned} E &= 13.6 \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ eV} \\ &= 13.6 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \\ &= 10.2 \text{ eV} \end{aligned}$$

Now, energy levels of helium ion's (He^+) electron are

(For helium, $Z=2$)



So, a photon of energy 10.2 eV can cause a transition $n=2$ to $n=4$ in a He^+ ion.

Alternate Solution

For He^+ ion, when in $n=1$ state,

$$10.2 = 13.6 \times 2^2 \left(\frac{1}{1^2} - \frac{1}{n^2} \right) \Rightarrow n=1$$

Thus, no transition takes place.

Similarly, when in $n=2$ state,

$$10.2 = 13.6 \times 2^2 \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \Rightarrow n=4$$

- 29** Taking the wavelength of first Balmer line in hydrogen spectrum ($n=3$ to $n=2$) as 660 nm, the wavelength of the 2nd Balmer line ($n=4$ to $n=2$) will be

[2019, 9 April Shift-I]

- (a) 889.2 nm (b) 388.9 nm
(c) 642.7 nm (d) 488.9 nm

Ans. (d)

Expression for the energy of the hydrogenic electron states for atoms of atomic number Z is given by

$$E = h\nu = \frac{Z^2 me^4}{8h^2 E_0^2} \left[\frac{1}{m^2} - \frac{1}{n^2} \right] \quad (\text{Here, } m < n)$$

$$\text{or } \frac{hc}{\lambda} = \frac{Z^2 me^4}{8h^2 E_0^2} \left[\frac{1}{m^2} - \frac{1}{n^2} \right]$$

$$\Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2} \right) Z^2$$

For first case, $\lambda = 660 \text{ nm}$, $m=2$ and $n=3$

$$\therefore \frac{1}{660 \text{ nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2} \right] Z^2$$

$$\Rightarrow \frac{1}{660 \text{ nm}} \propto \left(\frac{1}{4} - \frac{1}{9} \right) Z^2 \text{ or } \frac{5}{36} Z^2 \quad \dots(i)$$

For second case, transition is from $n=4$ to $n=2$, i.e. $m=2$ and $n=4$

$$\therefore \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2} \right) Z^2$$

$$\Rightarrow \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16} \right) Z^2$$

$$\text{or } \frac{1}{\lambda} \propto \frac{3}{16} Z^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\begin{aligned} \frac{\lambda}{660 \text{ nm}} &= \frac{5}{36} \times \frac{16}{3} \\ \Rightarrow \lambda &= \frac{80}{108} \times 660 \text{ nm} \\ &= 488.9 \text{ nm} \end{aligned}$$

- 30** A He^+ ion is in its first excited state. Its ionisation energy is

[2019, 9 April Shift-II]

- (a) 54.40 eV (b) 13.6 eV
(c) 48.36 eV (d) 6.04 eV

Ans. (b)

Energy of a hydrogen atom like ion by Bohr's

$$\text{model is } E_n = -13.6 \frac{Z^2}{n^2}$$

where, Z = atomic number

and n = principal quantum number.

For a He^+ ion in first excited state,

$$n=2, Z=2$$

$$\therefore E_2 = -13.6 \times \frac{4}{4} = -13.6 \text{ eV}$$

- 31** An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n corresponding to its initial excited state is [for photon of wavelength

$$\lambda, \text{ energy } E = \frac{1240 \text{ eV}}{\lambda (\text{in nm})}]$$

[2019, 12 April Shift-I]

- (a) $n=4$ (b) $n=5$
(c) $n=7$ (d) $n=6$

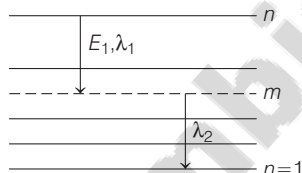
Ans. (b)

Change in energy in transition from n to m stage is given by ($n > m$),

$$E_n = -\frac{E_0 Z^2}{n^2}$$

Here, $Z=2$

$$\Delta E_n = +13.6 \times 4 \left[\frac{1}{m^2} - \frac{1}{n^2} \right] = \frac{hc}{\lambda} \quad \dots(i)$$



Let it start from n to m and then m to ground.

So, in first case,

$$13.6 \times 4 \times \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = \frac{hc}{108.5 \text{ nm}} \quad \dots(ii)$$

and in second case,

$$\begin{aligned} 13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2} \right) &= \frac{hc}{30.4 \text{ nm}} \\ \Rightarrow \left(1 - \frac{1}{n^2} \right) &= \frac{1240 \text{ eV}}{30.4 \times 13.6 \times 4} \\ &\quad \left(\text{Given, } E = \frac{1240 \text{ eV}}{\lambda (\text{in nm})} \right) \end{aligned}$$

$$\Rightarrow \left(1 - \frac{1}{n^2} \right) = 0.74980 \approx 0.75$$

$$\text{or } \frac{1}{n^2} = 1 - 0.75 = 0.25$$

$$\Rightarrow \frac{1}{n^2} = \frac{1}{0.25} = 4$$

Hence, $m=2$

So, by putting the value of m in Eq. (ii), we get

$$13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2} \right) = \frac{1240 \text{ eV}}{108.5}$$

$$\Rightarrow \left(\frac{1}{4} - \frac{1}{n^2} \right) = \frac{1240}{108.5 \times 13.6 \times 4}$$

$$\Rightarrow \frac{1}{4} - \frac{1}{n^2} = 0.21$$

$$\text{or } 1/n^2 = 0.25 - 0.21 = 0.04$$

$$n^2 = \frac{1}{0.04} = 25 \Rightarrow n^2 = 25 \Rightarrow n = 5$$

- 32** The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths λ_1 / λ_2 of the photons emitted in this process is

[2019, 12 April Shift-II]

- (a) 20/7 (b) 27/5 (c) 7/5 (d) 9/7

Ans. (a)

Wavelength λ of emitted photon as an electron transits from an initial energy level n_i to some final energy level n_f is given by Balmer's formula,

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where, R = Rydberg constant.

In transition from $n=4$ to $n=3$, we have

$$\frac{1}{\lambda_1} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = R \left(\frac{7}{9 \times 16} \right) \quad \dots(i)$$

In transition from $n=3$ to $n=2$, we have

$$\frac{1}{\lambda_2} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = R \left(\frac{5}{9 \times 4} \right) \quad \dots(ii)$$

So, from Eqs. (i) and (ii), the ratio of $\frac{\lambda_1}{\lambda_2}$ is

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{9 \times 16}{7R} \right)}{\left(\frac{9 \times 4}{5R} \right)} = \frac{20}{7}$$

- 33** Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is

[2019, 12 April Shift-II]

- (a) 3.5 Å (b) 6.6 Å (c) 12.9 Å (d) 9.7 Å

Ans. (d)

By Bohr's IInd postulate, for revolving electron,

$$\text{Angular momentum} = \frac{nh}{2\pi} \Rightarrow mvr_n = \frac{nh}{2\pi}$$

$$\Rightarrow \text{Momentum of electron, } p = mv = \frac{nh}{2\pi r_n}$$

de-Broglie wavelength associated with electron is

$$\lambda_n = h/p = 2\pi r_n / n$$

Given, $n=3, r_n = 4.65 \text{ Å}$

$$\therefore \lambda_n = \frac{(2 \times \pi \times 4.65)}{3} \approx 9.7 \text{ Å}$$

- 34** A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius a_0 will be
(Take $hc = 12500 \text{ eV-Å}$)

[2019, 11 Jan Shift-I]

- (a) $4a_0$ (b) $9a_0$ (c) $16a_0$ (d) $25a_0$

Ans. (c)

We know that net change in energy of a photon in a transition with wavelength λ is $\Delta E = hc/\lambda$.

Here, $hc = 12500 \text{ eV-Å}$ and $\lambda = 980 \text{ Å}$
 $\Delta E = 12500/980 = 12.76 \text{ eV}$

$$\Rightarrow E_n - E_1 = 12.76 \text{ eV}$$

Since, the energy associated with an electron in n^{th} Bohr's orbit is given as,

$$E_n = \frac{-13.6}{n^2} \text{ eV} \quad \dots(i)$$

$$\Rightarrow E_n = E_1 + 12.76 \text{ eV} \quad \dots(ii)$$

$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of n^{th} orbit, $r_n = n^2 a_0$

$$\Rightarrow r_n = 16a_0$$

- 35** In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ . If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be

[2019, 11 Jan Shift-II]

- (a) $\frac{27}{20} \lambda$ (b) $\frac{25}{16} \lambda$ (c) $\frac{20}{27} \lambda$ (d) $\frac{16}{25} \lambda$

Ans. (c)

For hydrogen or hydrogen like atoms, we know that

$$\frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \quad \dots(i)$$

where, R is Rydberg constant and Z is atomic number. When electron jumps from M-shell to the L-shell, then

$$n_1 = 2 \quad (\text{for L-shell})$$

$$n_2 = 3 \quad (\text{for M-shell})$$

$\therefore$ Eq (i) becomes

$$\frac{1}{\lambda} = RZ^2 \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = \frac{5}{36} RZ^2 \quad \dots(ii)$$

Now, electron jumps from N-shell to the L-shell, for this

$$n_1 = 2 \quad (\text{for L-shell})$$

$n_2 = 4$ (for N-shell)
 $\therefore$ Eq (i) becomes

$$\frac{1}{\lambda'} = RZ^2 \left[\frac{1}{2^2} - \frac{1}{4^2} \right] = \frac{3}{16} RZ^2 \quad \dots(iii)$$

Now, we divide Eq (ii) by Eq (iii)

$$\frac{\lambda'}{\lambda} = \left(\frac{5}{36} RZ^2 \right) \div \left(\frac{3}{16} RZ^2 \right) = \frac{20}{27} \text{ or } \lambda' = \frac{20}{27} \lambda$$

- 36** A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2} kr^2$. If Bohr's

quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as

[2019, 12 Jan 2019, Shift-I]

(a) $r_n \propto n$, $E_n \propto n$ (b) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

(c) $r_n \propto \sqrt{n}$, $E_n \propto n$ (d) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$

Ans. (c)

As, for conservative fields $F = -\left(\frac{dU}{dr}\right)$

$\therefore$ Magnitude of force on particle is

$$\Rightarrow F = \frac{dU}{dr} = \frac{d}{dr} \left(\frac{1}{2} kr^2 \right) \Rightarrow F = kr$$

This force is acting like centripetal force.

$$\therefore \frac{mv^2}{r} = kr \quad \dots(i)$$

So, for n^{th} orbit,

$$\Rightarrow m^2 v_n^2 = mkr_n^2$$

$$\Rightarrow \frac{n^2 h^2}{4\pi^2 r^2} = mkr_n^2 \quad \left[\because v_n = \frac{nh}{2\pi mr} \right]$$

$$\text{Therefore, } r_n^4 \propto n^2 \Rightarrow r_n^2 \propto n$$

$$\text{So, } r_n \propto \sqrt{n}$$

Energy of particle is

$$E_n = PE + KE = \frac{1}{2} kr_n^2 + \frac{1}{2} mv_n^2$$

$$= \frac{1}{2} kr_n^2 + \frac{1}{2} kr_n^2 \quad [\text{using Eq. (i)}]$$

$$= kr_n^2$$

$$\text{So, energy, } E_n \propto r_n^2 \Rightarrow E_n \propto n$$

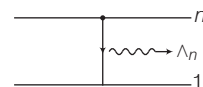
- 37** An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de-Broglie wavelength of the electron in the n^{th} state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n , (A, B are constants) [JEE Main 2018]

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n^2$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

Ans. (a)

If wavelength of emitted photon in de-excitation is Λ_n ;



Then, $\frac{hc}{\Lambda_n} = E_n - E_g$

$$\frac{hc}{\Lambda_n} = \frac{p_n^2}{2m} - \frac{p_g^2}{2m} \quad \left[\because E = \frac{p^2}{2m} \right]$$

As energies are negative, we get

$$\frac{hc}{\Lambda_n} = \frac{p_g^2}{2m} - \frac{p_n^2}{2m}$$

$$= \frac{p_g^2}{2m} \left(1 - \left(\frac{p_n}{p_g} \right)^2 \right) = \frac{h^2}{2m\lambda_g^2} \left(1 - \frac{\lambda_g^2}{\lambda_n^2} \right)$$

$$[\because p \propto \lambda^{-1}, p = \frac{h}{\lambda}]$$

$$\Rightarrow \Lambda_n = \frac{2m\lambda_g^2 c}{h} \left(1 - \frac{\lambda_g^2}{\lambda_n^2} \right)^{-1}$$

$$\Rightarrow \Lambda_n = \frac{2m\lambda_g^2 c}{h} \left(1 + \frac{\lambda_g^2}{\lambda_n^2} \right)$$

$$[\because (1-x)^{-n} = 1 + nx]$$

$$\Rightarrow \Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

where, $A = \left[\frac{2mc\lambda_g^2}{h} \right]$ and $B = \left[\frac{2mc\lambda_g^4}{h} \right]$

are constants.

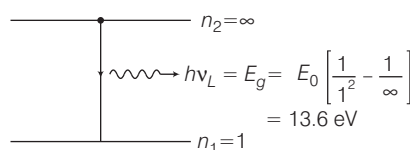
- 38** If the series limit frequency of the Lyman series is ν_L , then the series limit frequency of the Pfund series is [JEE Main 2018]

(a) $25 \nu_L$ (b) $16 \nu_L$

(c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{25}$

Ans. (d)

Key Idea Series limit occurs in the transition $n_2 = \infty$ to $n_1 = 1$ in Lyman series and $n_2 = \infty$ to $n_1 = 5$ in pfund series. For Lyman series,



$$h\nu_L = 13.6 \quad \dots(i)$$

In pfund series

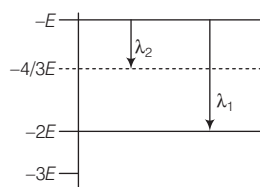
$$hv_p = E_0 \left[\frac{1}{5^2} - \frac{1}{\infty} \right] = \frac{13.6}{5^2}$$

$$hv_p = \frac{13.6}{5^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$25hv_p = hv_L \Rightarrow v_p = \frac{v_L}{25}$$

- 39** Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by [JEE Main 2017]



- (a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$
(c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

Ans. (c)

We have, $\lambda = \frac{hc}{\Delta E}$

$\therefore$ So, ratio of wave lengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc / \Delta E_1}{hc / \Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

- 40** As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion [JEE Main 2015]

- (a) its kinetic energy increases but potential energy and total energy decrease
(b) kinetic energy, potential energy and total energy decrease
(c) kinetic energy decreases, potential energy increases but total energy remains same
(d) kinetic energy and total energy decrease but potential energy increases

Ans. (a)

As we know that kinetic energy of an electron is

$$KE \propto \left(\frac{Z}{n}\right)^2$$

When the electron makes transition from an excited state to the ground state, then n decreases and KE increases. We know that PE is lowest for ground state. As $TE = -KE$, TE also decreases.

- 41** The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T.

If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to [JEE Main 2014]

- (a) 1.8 eV
(b) 1.1 eV
(c) 0.8 eV
(d) 1.6 eV

Ans. (b)

Key Idea The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency) is focus on a metal surface, then some electrons are emitted from the metal with substantial initial speed.

When an electron moves in a circular path, then

$$\text{Radius, } r = \frac{mv}{eB} \Rightarrow \frac{r^2 e^2 B^2}{2} = \frac{m^2 v^2}{2}$$

$$KE_{\max} = \frac{(mv)^2}{2m}$$

$$\Rightarrow \frac{r^2 e^2 B^2}{2m} = (KE)_{\max}$$

Work function of the metal (W),

$$\text{i.e., } W = hv - KE_{\max}$$

$$1.89 - \phi = \frac{r^2 e^2 B^2}{2m} \cdot \frac{1}{2} \text{ eV} = \frac{r^2 e^2 B^2}{2m} \text{ eV}$$

$[hv \rightarrow 1.89 \text{ eV, for the transition on from third to second orbit of H-atom}]$

$$= \frac{100 \times 10^{-6} \times 1.6 \times 10^{-19} \times 9 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}}$$

$$\phi = 1.89 - \frac{1.6 \times 9}{2 \times 9.1}$$

$$= 1.89 - 0.79 = 1.1 \text{ eV}$$

- 42** Hydrogen (${}_1\text{H}^1$), deuterium (${}_1\text{H}^2$), singly ionised helium (${}_2\text{He}^4$)⁺ and doubly ionised lithium (${}_3\text{Li}^8$)⁺⁺ all have one electron around the nucleus. Consider an electron transition from $n=2$ to $n=1$.

If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively for four elements, then approximately which one of the following is correct?

[JEE Main 2014]

- (a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

Ans. (c)

For hydrogen atom, we get

$$\frac{1}{\lambda} = R Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\Rightarrow \frac{1}{\lambda_1} = R(1)^2 \left(\frac{3}{4} \right)$$

$$\Rightarrow \frac{1}{\lambda_2} = R(1)^2 \left(\frac{3}{4} \right)$$

$$\Rightarrow \frac{1}{\lambda_3} = R(2)^2 \left(\frac{3}{4} \right)$$

$$\frac{1}{\lambda_4} = R(3)^2 \left(\frac{3}{4} \right)$$

$$\Rightarrow \frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

- 43** In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to [JEE Main 2013]

- (a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$
(c) $\frac{1}{n^3/2}$ (d) $\frac{1}{n^3}$

Ans. (d)

$$\Delta E = hv$$

where, ΔE is energy of radiation, h is Planck's constant and v is frequency.

$$\Rightarrow v = \frac{\Delta E}{h} = k \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right]$$

$$= \frac{k2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

- 44** Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be [JEE Main 2013]

- (a) 2 (b) 3 (c) 5 (d) 6

Ans. (d)

In emission spectrum, number of bright lines is given by

$$\frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

- 45** A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r . If we calculate its rotational energy by applying Bohr's rule of angular momentum quantisation, its energy will be given by (n is an integer)

[AIEEE 2012]

- (a) $\frac{(m_1 + m_2)^2 n^2 h^2}{2 m_1 m_2 r^2}$
 (b) $\frac{n^2 h^2}{2(m_1 + m_2) r^2}$
 (c) $\frac{2 n^2 h^2}{(m_1 + m_2) r^2}$
 (d) $\frac{(m_1 + m_2) n^2 h^2}{2 m_1 m_2 r^2}$

Ans. (d)

Rotational kinetic energy of the two body system rotating about their centre of mass is

$$\text{RKE} = \frac{1}{2} \mu \omega^2 r^2$$

where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

and angular momentum, $L = \mu \omega r^2 = \frac{nh}{2\pi}$

$$\begin{aligned} \therefore \text{RKE} &= \frac{1}{2} \mu \omega^2 r^2 = \frac{1}{2} \mu \cdot \left(\frac{nh}{2\pi \mu r^2} \right)^2 r^2 \\ &= \frac{n^2 h^2}{8\pi^2 \mu r^2} = \frac{n^2 h^2}{2\mu r^2} \\ &= \frac{(m_1 + m_2) n^2 h^2}{2 m_1 m_2 r^2} \end{aligned}$$

- 46** The transition from the state $n=4$ to $n=3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from **[AIEEE 2009]**

- (a) $2 \rightarrow 1$ (b) $3 \rightarrow 2$
 (c) $4 \rightarrow 2$ (d) $5 \rightarrow 4$

Ans. (d)

IR corresponds to least value of

$$\left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

i.e., from Paschen, Brackett and Pfund series. Thus, the transition corresponds to $5 \rightarrow 4$.

- 47** Suppose an electron is attracted towards the origin by a force k/r , where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n th orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n . Then, which of the following is true?

[AIEEE 2008]

- (a) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$
 (b) T_n is independent of n , $r_n \propto n$
 (c) $T_n \propto \frac{1}{n}$, $r_n \propto n$
 (d) $T_n \propto \frac{1}{n}$, $r_n \propto n^2$

Ans. (b)

$$\frac{mv^2}{r_n} = \frac{k}{r_n} \quad [\text{given}]$$

$$mvr_n = \frac{nh}{2\pi} \quad [\text{from Bohr's theory}]$$

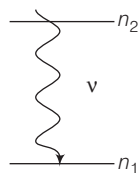
On solving, $r_n \propto n$ and T_n is independent of n .

- 48** Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[AIEEE 2007]**

- (a) $n=2$ to $n=6$
 (b) $n=6$ to $n=2$
 (c) $n=2$ to $n=1$
 (d) $n=1$ to $n=2$

Ans. (c)

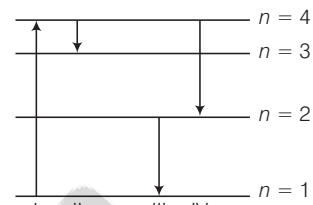
Emission spectrum would rise when electron makes a jump from higher energy level to lower energy level.



Frequency of emitted photon is proportional to change in energy of two energy levels, i.e.,

$$\nu = RcZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

- 49** The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?

[AIEEE 2005]

- (a) III (b) IV (c) I (d) II

Ans. (a)

$$E = Rhc \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\begin{aligned} E_{(4 \rightarrow 3)} &= Rhc \left[\frac{1}{3^2} - \frac{1}{4^2} \right] \\ &= Rhc \left[\frac{7}{9 \times 16} \right] = 0.05 Rhc \end{aligned}$$

$$\begin{aligned} E_{(4 \rightarrow 2)} &= Rhc \left[\frac{1}{2^2} - \frac{1}{4^2} \right] \\ &= Rhc \left[\frac{3}{16} \right] = 0.2 Rhc \end{aligned}$$

$$\begin{aligned} E_{(2 \rightarrow 1)} &= Rhc \left[\frac{1}{1^2} - \frac{1}{2^2} \right] \\ &= Rhc \left[\frac{3}{4} \right] = 0.75 Rhc \end{aligned}$$

$$\begin{aligned} E_{(1 \rightarrow 3)} &= Rhc \left[\frac{1}{3^2} - \frac{1}{1^2} \right] \\ &= -\frac{8}{9} Rhc = -0.9 Rhc \end{aligned}$$

Thus, III transition gives most energy. I transition represents the absorption of energy.

- 50** An α -particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of **[AIEEE 2004]**

- (a) $1 \text{ \AA}$ (b) 10^{-10} cm
 (c) 10^{-12} cm (d) 10^{-15} cm

Ans. (c)

According to law of conservation of energy,

Kinetic energy of α -particle = Potential energy of α -particle at distance of closest approach

$$\begin{aligned}
 \text{i.e., } \frac{1}{2}mv^2 &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \\
 \therefore 5 \text{ MeV} &= \frac{9 \times 10^9 \times (2e) \times (92e)}{r} \\
 &\quad \left[\because \frac{1}{2}mv^2 = 5 \text{ MeV, given} \right] \\
 \Rightarrow r &= \frac{9 \times 10^9 \times 2 \times 92 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}} \\
 \text{or } r &= 5.3 \times 10^{-14} \text{ m} \\
 &\approx 10^{-12} \text{ cm}
 \end{aligned}$$

51 The manifestation of band structure in solids is due to

[AIEEE 2004]

- (a) Heisenberg's uncertainty principle
- (b) Pauli's exclusion principle
- (c) Bohr's correspondence principle
- (d) Boltzmann's law

Ans. (b)

According to Pauli's exclusion principle, the electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell is found.

Hence, on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

52 Which of the following atoms has the lowest ionisation potential?

[AIEEE 2003]

- (a) ${}^{14}_7\text{N}$ (b) ${}^{133}_{55}\text{Cs}$
- (c) ${}^{40}_{18}\text{Ar}$ (d) ${}^{16}_8\text{O}$

Ans. (b)

According to problem,

Since, ionisation potential is minimum energy required to eject the electron from the outermost orbit. As ${}^{133}_{55}\text{Cs}$ has larger size among the four atoms given, thus electrons present in the outermost orbit will be away from the nucleus and the electrostatic force experienced by electrons due to nucleus will be minimum.

Therefore, the energy required to liberate electrons from outer orbit will be minimum in case of ${}^{133}_{55}\text{Cs}$.

53 If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is

[AIEEE 2003]

- (a) 30.6 eV (b) 13.6 eV
- (c) 3.4 eV (d) 122.4 eV

Ans. (a)

$$E = -Z^2 \frac{13.6}{n^2} \text{ eV}$$

For first excited state,

$$E_2 = -3^2 \times \frac{13.6}{4}$$

[$\because n=2$, for 1st excited state]

$$= -30.6 \text{ eV}$$

Ionisation energy for first excited state of Li^{2+} is 30.6 eV.

54 If 13.6 eV energy is required to ionise the hydrogen atom, then the energy required to remove an electron from $n=2$ is

[AIEEE 2002]

- (a) 10.2 eV (b) zero
- (c) 3.4 eV (d) 6.8 eV

Ans. (c)

Energy required to remove an electron from n th orbit is

$$E_n = -\frac{13.6}{n^2}$$

Here, $n=2$

$$\text{Therefore, } E_2 = -\frac{13.6}{2^2} = -3.4 \text{ V}$$

27

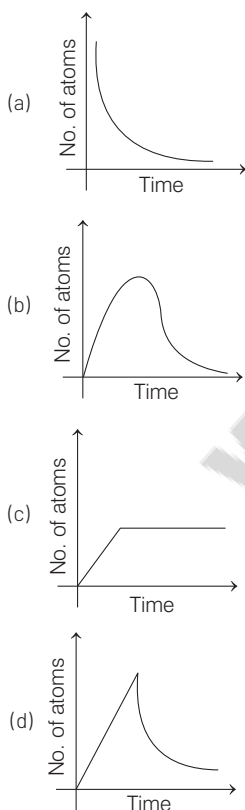
Nuclei

TOPIC 1

Nucleus and Radioactivity

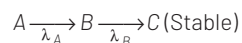
- 01** A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B versus time is (Assume that at $t = 0$, there are no B atoms in the sample)

[2021, 31 Aug Shift-I]



Ans. (b)

According to given question, the decay is as shown below



Let N_0 be the original number of atoms in the radioactive nucleus A.

Initially at $t = 0$,

Number of atoms of B in the sample is zero.

According to radioactive decay law, i.e.

$$N = N_0 e^{-\lambda t} \quad \dots(i)$$

With the increase in time, the number of atoms of B will start increasing and reaches a maximum value.

After reaching the maximum value, B will start decaying into C.

Since, number of atoms varies with time following an exponential law i.e. Eq. (i).

So, the graph between number of atoms and t will be exponential in nature. Thus, the correct option is (b).

- 02** There are 10^{10} radioactive nuclei in a given radioactive element. Its half-life time is 1min. How many nuclei will remain after 30 s?

$$(\sqrt{2} = 1.414)$$

[2021, 27 Aug Shift-I]

$$(a) 2 \times 10^{10}$$

$$(b) 7 \times 10^9$$

$$(c) 10^5$$

$$(d) 4 \times 10^{10}$$

Ans. (b)

Initial number of radioactive nuclei, $N_0 = 10^{10}$

Half-life, $t_{1/2} = 1 \text{ min} = 60 \text{ s}$

Time, $t = 30 \text{ s}$

We know that, number of nuclei remaining in radioactive decay after time t is given by

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Substituting the values in above relation, we get

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{30/60} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow N = \frac{1}{\sqrt{2}} \times 10^{10} = 0.707 \times 10^{10}$$

$$\Rightarrow N \approx 7 \times 10^9$$

Thus, the radioactive nuclei remaining after 30 s is 7×10^9 .

- 03** At time $t = 0$, a material is

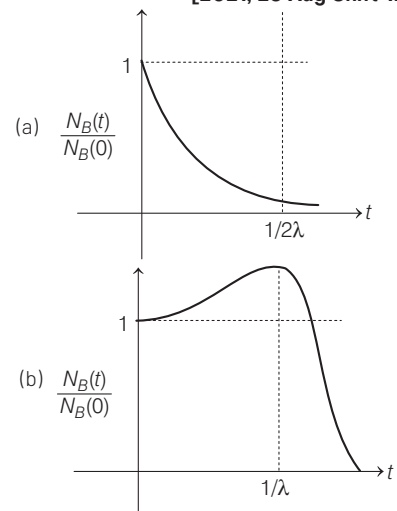
composed of two radioactive atoms A and B, where

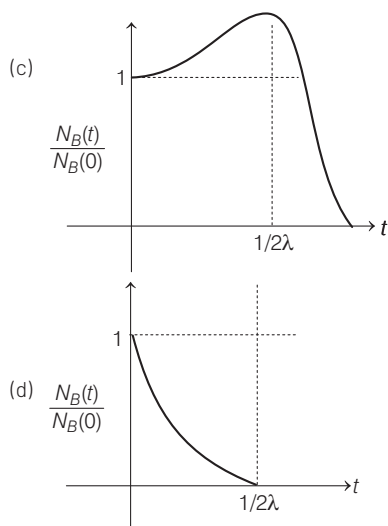
$N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ . However, A disintegrates to B

and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t) / N_B(0)$ with respect to time t ?

$N_A(0)$ = Number of A atoms at $t = 0$
 $N_B(0)$ = Number of B atoms at $t = 0$

[2021, 26 Aug Shift-II]



**Ans. (c)**

Given, at time $t = 0$, $N_A(0) = 2N_B(0)$
 Decay constant is same for both radioactive atoms as λ .

For $A \rightarrow B$, $\frac{dN_B(t)}{dt} = \lambda N_A(t) - \lambda N_B(t)$

Substituting $N_A(0)e^{-\lambda t}$ for $N_A(t)$ in above expression, we get

$$\begin{aligned} \frac{dN_B(t)}{dt} &= \lambda N_A(0)e^{-\lambda t} - \lambda N_B(t) \\ &= 2\lambda N_B(0)e^{-\lambda t} - \lambda N_B(t) \\ \Rightarrow \frac{dN_B(t)}{dt} + \lambda N_B(t) &= 2\lambda N_B(0)e^{-\lambda t} \end{aligned}$$

Multiplying both sides by $e^{\lambda t}$, we get

$$\begin{aligned} e^{\lambda t} \left[\frac{dN_B(t)}{dt} + \lambda N_B(t) \right] &= 2\lambda N_B(0)e^{-\lambda t} \times e^{\lambda t} \\ \Rightarrow \frac{d}{dt} [N_B(t)e^{\lambda t}] &= 2\lambda N_B(0) \end{aligned}$$

Integrating both sides,

$$[N_B(t)e^{\lambda t}] = 2\lambda N_B(0)t + C \quad \dots (i)$$

Putting $t = 0$ in above expression,

$$\begin{aligned} [N_B(0)e^{\lambda \times 0}] &= 2\lambda N_B(0) \times 0 + C \\ \Rightarrow C &= N_B(0) \end{aligned}$$

Putting value of C in Eq. (i)

$$\begin{aligned} [N_B(t)e^{\lambda t}] &= 2\lambda N_B(0)t + N_B(0) \quad \dots (ii) \\ \Rightarrow N_B(t) &= N_B(0)[1 + 2\lambda t]e^{-\lambda t} \\ \Rightarrow \frac{N_B(t)}{N_B(0)} &= [1 + 2\lambda t]e^{-\lambda t} \end{aligned}$$

$$\text{As, } N_B(t) = N_B(0)[1 + 2\lambda t]e^{-\lambda t}$$

$$\Rightarrow N_B(t) = C[1 + 2\lambda t]e^{-\lambda t} \quad \dots (iii)$$

The maximum value of $N_B(t)$ is obtained at

$$\frac{dN_B(t)}{dt} = 0$$

From Eq. (ii),

$$\frac{dN_B(t)}{dt} = -\lambda C[1 + 2\lambda t]e^{-\lambda t} + 2C\lambda e^{-\lambda t} = 0$$

Solving the above expression,

$$t = \frac{1}{2\lambda} \text{ s}$$

Thus, maximum value of function will be

$$\text{at } t = \frac{1}{2\lambda}$$

Hence, graph (c) is correct option.

04 Consider the following statements.
[2021, 27 July Shift-II]

- Atoms of each element emit characteristics spectrum.
 - According to Bohr's postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.
 - The density of nuclear matter depends on the size of the nucleus.
 - A free neutron is stable but a free proton decay is possible.
 - Radioactivity is an indication of the instability of nuclei.
- Choose the correct answer from the options given below.

(a) A, B, C, D and E (b) A, B and E

(c) B and D (d) A, C and E

Ans. (b)

Statement A is true, because atom of each element emits characteristics spectrum.

Statement B is true,

$\therefore$ According to Bohr's postulates,

$mvr = \frac{nh}{2\pi}$ and due to this reason, the

electron resides into orbits of specific radius called stationary orbits.

Statement C is false, because density of nucleus is constant.

Statement D is false because a free neutron is unstable which decays into proton, electron and anti-neutrino.

Statement E is true because unstable nucleus shows radioactivity. So, we can say that radioactivity is the instability of nuclei.

05 A radioactive sample has an average life of 30 ms and is decaying. A capacitor of capacitance $200 \mu\text{F}$ is first charged and later connected with resistor R . If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time, then the value of R should be Ω .

[2021, 27 July Shift-I]

Ans. (150)

We know that,

$$q = q_0 e^{-\frac{t}{RC}} \quad \dots (i)$$

where, RC is the time constant of RC - circuit.

At time t , activity A is given by

$$A = A_0 e^{-\lambda t} \quad \dots (ii)$$

where, A_0 = initial activity

and λ = decay constant.

On dividing Eqs. (i) and (ii), we get

$$\begin{aligned} \frac{q}{A} &= \frac{q_0 e^{-\frac{t}{RC}}}{A_0 e^{-\lambda t}} \\ \Rightarrow 1 &= \frac{e^{-\frac{t}{RC}}}{e^{-\lambda t}} \\ \Rightarrow e^{-\lambda t} &= e^{-\frac{t}{RC}} \end{aligned}$$

Taking log on both sides of above equation, we get

$$\ln(e^{-\lambda t}) = \ln\left(e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow -\lambda t = -\frac{t}{RC}$$

$$\Rightarrow \lambda = \frac{1}{RC}$$

$$\Rightarrow R = \frac{1}{\lambda C}$$

$$\Rightarrow R = \frac{30 \times 10^{-3}}{200 \times 10^{-6}}$$

$$\Rightarrow R = 150 \Omega$$

06 If f denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at $t = 0$ (N_0), then for a collection of radioactive nuclei, the rate of change of f with respect to time is given as **[2021, 27 July Shift-I]**

$[\lambda$ is the radioactive decay constant]

- $-\lambda(1 - e^{-\lambda t})$
- $\lambda(1 - e^{-\lambda t})$
- $\lambda e^{-\lambda t}$
- $-\lambda e^{-\lambda t}$

Ans. (c)

According to law of radioactive decay,

$$N = N_0 e^{-\lambda t} \quad \dots (i)$$

where, N = number of undecayed radioactive atoms,

N_0 = original number of radioactive atoms,

t = elapsed time

and λ = decay constant.

According to question,

$$N_d = N_0 - N \quad \dots (ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\begin{aligned}
 N_d &= N_0 - N_0 e^{-\lambda t} \\
 \Rightarrow N_d &= N_0(1 - e^{-\lambda t}) \\
 \Rightarrow \frac{N_d}{N_0} &= 1 - e^{-\lambda t} \\
 \Rightarrow f &= 1 - e^{-\lambda t} \quad \dots(iii)
 \end{aligned}$$

On differentiating Eq. (iii), we get

$$\frac{df}{dt} = \lambda e^{-\lambda t}$$

- 07** The nuclear activity of a radioactive element becomes (1/8)th of its initial value in 30 yr. The half-life of radioactive element is yr.

[2021, 25 July Shift-II]

Ans. (10)

Given, let initial amount of nuclear element = A_0

After time (t) = 30 years

Final amount of nuclear element (A) = $\frac{A_0}{8}$

Let, half-life be $t_{1/2}$.

As we know that,

$$A = A_0 e^{-\lambda t}$$

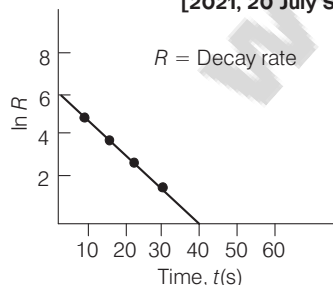
$$\frac{A_0}{8} = A_0 e^{-\lambda \cdot 30} \quad \dots(i)$$

On taking natural log on both side

$$\begin{aligned}
 \ln \frac{1}{8} &= -30\lambda \ln e \Rightarrow \ln \left(\frac{1}{8} \right) = -30\lambda \\
 \ln(2^{-3}) &= -30\lambda \Rightarrow -3\ln 2 = -30\lambda \\
 \Rightarrow \lambda &= \frac{\ln 2}{10} = 0.0693 \text{ and } t_{1/2} = \frac{0.693}{\lambda} \\
 \therefore t_{1/2} &= \frac{0.693}{0.0693} = 10 \text{ yr}
 \end{aligned}$$

- 08** For a certain radioactive process, the graph between $\ln R$ and t (sec) is obtained as shown in the figure. Then, the value of half-life for the unknown radioactive material is approximately

[2021, 20 July Shift-II]



- (a) 9.15 s (b) 6.93 s (c) 2.62 s (d) 4.62 s

Ans. (d)

According to law of radioactive decay,

$$R = R_0 e^{-\lambda t} \quad \dots(i)$$

where, R is remaining amount, R_0 is initial amount and λ is decay constant.

Taking natural log on both sides of Eq. (i), we get

$$\begin{aligned}
 \ln R &= \ln R_0 + \ln e^{-\lambda t} \\
 \Rightarrow \ln R &= \ln R_0 - \lambda t \quad \dots(ii)
 \end{aligned}$$

So, for a given graph between $\ln R$ and t , $-\lambda$ is slope.

According to above graph given in question

at $t = 0$, $\ln R = 6$

$\therefore$ Eq. (ii) becomes

$$\ln R_0 = 6 \quad \dots(iii)$$

and at $t = 40$ s, $\ln R = 0$

$\therefore$ Eq. (ii) becomes,

$$\lambda = \frac{\ln R_0}{t} = \frac{\ln R_0}{40} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\lambda = \frac{6}{40} \quad \dots(v)$$

$\therefore$ We know that,

$$\text{Half-life, } t_{1/2} = \frac{0.693}{\lambda} \quad \dots(vi)$$

$\therefore$ From Eq. (v) and (vi), we get

$$t_{1/2} = \frac{0.693 \times 40}{6} \Rightarrow t_{1/2} = 4.62 \text{ s}$$

- 09** A radioactive substance decays to (1/16)th of its initial activity in 80 days. The half-life of the radioactive substance expressed in days is [2021, 20 July Shift-II]

Ans. (20)

According to question, a radioactive substance decays to $\left(\frac{1}{16}\right)$ th of its initial activity in 80 days.

If N_0 is the number of atoms present initially, then in 80 days, the substance decays to $\frac{1}{16}$ th of N_0 as follows

$$N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16}$$

$$\begin{aligned}
 \Rightarrow 4 \times t_{1/2} &= 80 \text{ days} \Rightarrow t_{1/2} = \frac{80}{4} \\
 \Rightarrow t_{1/2} &= 20 \text{ days}
 \end{aligned}$$

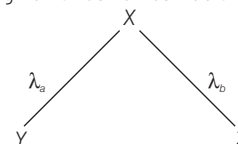
- 10** A radioactive material decays by simultaneous emission of two particles with half-lives of 1400 yr and 700 yr, respectively. What will be the time after the which one-third of the material remains? [Take, $\ln 3 = 1.1$]

[2021, 20 July Shift-I]

- (a) 1110 yr (b) 700 yr
(c) 340 yr (d) 740 yr

Ans. (d)

The given situation can be shown as



Here, radioactive material X is decayed into two particles Y and Z with their respective decay constant, λ_a and λ_b . It means that

$$\therefore \lambda = \frac{\ln 2}{t_{1/2}}$$

where, $t_{1/2}(a) = 700$ yr

and $t_{1/2}(b) = 1400$ yr

$$\lambda_a = \frac{\ln 2}{700} \text{ yr}^{-1} \text{ and } \lambda_b = \frac{\ln 2}{1400} \text{ yr}^{-1}$$

$$\begin{aligned}
 \therefore \lambda_{\text{total}} &= \lambda_a + \lambda_b \\
 &= \left(\frac{\ln 2}{700} + \frac{\ln 2}{1400} \right) \text{ yr}^{-1} \\
 &= \ln 2 \left(\frac{1}{700} + \frac{1}{1400} \right) \text{ yr}^{-1} \\
 &= \left(\frac{3 \ln 2}{1400} \right) \text{ yr}^{-1}
 \end{aligned}$$

Suppose the initial number of radioactive nuclei was N_0 .

$$\therefore N = N_0 e^{-\lambda t}$$

where, N = number of nuclei present at time = t

and N_0 = number of nuclei present at time = 0

$$\Rightarrow \frac{N_0}{3} = N_0 e^{-\lambda t} \text{ or } \frac{N_0}{3} = N_0 e^{-\lambda_{\text{total}} t}$$

$$\Rightarrow \frac{1}{3} = e^{-\lambda_{\text{total}} t}$$

Taking log on both the sides of above equation, we get

$$\ln \left(\frac{1}{3} \right) = \ln(e^{-\lambda_{\text{total}} t})$$

$$\Rightarrow \ln \left(\frac{1}{3} \right) = -\lambda_{\text{total}} t$$

$$\Rightarrow 1.1 = \frac{3 \times 0.693}{1400} \times t$$

$$\Rightarrow t \approx 740 \text{ yr}$$

- 11** The decay of a proton to neutron is

[2021, 18 March Shift-II]

- (a) not possible as proton mass is less than the neutron mass
(b) possible only inside the nucleus
(c) not possible but neutron to proton conversion is possible
(d) always possible as it is associated only with β^+ decay

Ans. (b)

The decay of a proton to neutron is possible only inside the nucleus because the mass of proton is less than the mass of neutron. This mass difference is compensated with the binding energy of nucleus. This decay is not possible outside the nucleus because particle cannot be decayed into the greater mass.

- 12** A radioactive sample disintegrates via two independent decay processes having half-lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$, respectively. The effective half-life $T_{1/2}$ of the nuclei is

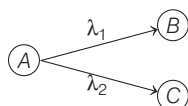
[2021, 18 March Shift-I]

$$(a) T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}} \quad (b) T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$$

$$(c) T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}} \quad (d) \text{None of these}$$

Ans. (c)

The radioactive sample disintegrates via two independent decays as shown in the figure,



Here, λ is the net disintegrate constant.

λ_1 is the first decay constant, and λ_2 is the second decay constant.

From the above figure,

$$\lambda = \lambda_1 + \lambda_2 \quad \dots (i)$$

As, we know that,

$$\text{Half-life, } T_{1/2} = \frac{\ln(2)}{\lambda} \Rightarrow \lambda = \frac{\ln(2)}{T_{1/2}}$$

Substituting in Eq. (i), we get

$$\frac{\ln(2)}{T_{1/2}} = \frac{\ln(2)}{T_{1/2}^{(1)}} + \frac{\ln(2)}{T_{1/2}^{(2)}}$$

$$\Rightarrow T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$$

- 13** Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500 g, decay constant = 20 g/s, then how much time is required for the amplitude of the system to drop to half of its initial value? ($\ln 2 = 0.693$)

[2021, 16 March Shift-II]

- (a) 34.65 s (b) 17.32 s
(c) 0.034 s (d) 15.01 s

Ans. (a)

Given,

Mass, $m = 500$ g

Decay constant, $\lambda = 20$ g/s

For damped oscillation,

$$A = A_0 e^{-bt/2m} \quad \dots (i)$$

Here, b = damped constant = decay constant

As per question, the amplitude of the system is dropped to half of its initial value.

So, Eq. (i) becomes

$$\Rightarrow \frac{A_0}{2} = A_0 e^{-bt/2m} \Rightarrow \frac{1}{2} = e^{-bt/2m}$$

$$\Rightarrow 2 = e^{bt/2m}$$

Taking log on both sides of above equation, we get

$$\log 2 = \log(e^{bt/2m})$$

$$\Rightarrow \log 2 = \frac{bt}{2m} \Rightarrow t = \frac{2m}{b} \log 2$$

$$= \frac{2 \times 500 \times 0.693}{20} \quad [\because \log 2 = 0.693]$$

$$\Rightarrow t = 34.65 \text{ s}$$

- 14** Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 min.

[2021, 16 March Shift-II]

- (a) 60 min (b) 20 min
(c) 40 min (d) 13 min

Ans. (b)

As we know that,

$$N = N_0 e^{-\lambda t}$$

where,

N = number of radioactive atoms left undecayed,

N_0 = original number of radioactive atoms,

t = elapsed time

and λ = decay constant.

$$\Rightarrow N_1 = N_0 e^{-\lambda t_1} \quad \dots (i)$$

$$\text{and } N_2 = N_0 e^{-\lambda t_2} \quad \dots (ii)$$

Consider Eq. (i), $N_1 = N_0 e^{-\lambda t_1}$

$$\Rightarrow \frac{N_1}{N_0} = e^{-\lambda t_1} \Rightarrow 0.67 = e^{-\lambda t_1}$$

$$\Rightarrow \log(0.67) = \log(e^{-\lambda t_1})$$

$$\Rightarrow \log 0.67 = -\lambda t_1 \quad \dots (iii)$$

Consider Eq. (ii), $N_2 = N_0 e^{-\lambda t_2}$

$$\Rightarrow \frac{N_2}{N_0} = e^{-\lambda t_2} \Rightarrow 0.33 = e^{-\lambda t_2}$$

$$\Rightarrow \log(0.33) = \log(e^{-\lambda t_2})$$

$$\Rightarrow \log(0.33) = -\lambda t_2 \quad \dots (iv)$$

Subtracting Eq. (iv) from Eq. (iii), we get

$$\log 0.67 - \log 0.33 = -\lambda t_1 + \lambda t_2$$

$$\Rightarrow \log\left(\frac{0.67}{0.33}\right) = \lambda(t_2 - t_1)$$

$$\Rightarrow \lambda(t_2 - t_1) = \log 2$$

$$\Rightarrow t_2 - t_1 = \frac{\log 2}{\lambda} = t_{1/2}$$

$$\therefore \text{Half-life} = t_{1/2} = 20 \text{ min}$$

- 15** The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is ($N_A = 6 \times 10^{23} / \text{mol}$)

[2021, 16 March Shift-II]

- (a) 240 Ci (b) 357 Ci
(c) 535 Ci (d) 252 Ci

Ans. (b)

Given,

Half-life of Au^{198} , $T_{1/2} = 2.7$ days

Amount of Au^{198} , $m = 1.5 \text{ mg} = 1.5 \times 10^{-3} \text{ g}$

Atomic weight of Au^{198} , $A = 198 \text{ g mol}^{-1}$

We know that activity, $A_0 = \lambda N_0$

$$\text{where, } \lambda = \text{decay constant} = \frac{\ln 2}{T_{1/2}}$$

$$N_0 = \text{initial number of atoms} = \frac{m}{A} N_A$$

$$\Rightarrow A_0 = \lambda \cdot \frac{m}{A} N_A = \frac{\ln 2}{T_{1/2}} \times \frac{1.5 \times 10^{-3}}{198} \times 6.023 \times 10^{23}$$

$$\Rightarrow A_0 = \frac{\ln 2 \times 1.5 \times 10^{-3} \times 6 \times 10^{23}}{2.7 \times 3600 \times 24 \times 198 \times 3.7 \times 10^{10}} \text{ Ci}$$

$$[\because 1 \text{ curie (Ci)} = 3.7 \times 10^{10} \text{ Bq}]$$

$$\Rightarrow A_0 = 3.65 \times 10^2 \text{ Ci} = 365 \text{ Ci}$$

The answer is close to 357 Ci.

- 16** A radioactive sample is undergoing α -decay. At any time t_1 , its activity is A and another time t_2 , the activity is $\frac{A}{5}$. What is the average

life time for the sample?

[2021, 26 Feb Shift-II]

- (a) $\frac{\ln 5}{t_2 - t_1}$ (b) $\frac{t_1 - t_2}{\ln 5}$
(c) $\frac{t_2 - t_1}{\ln 5}$ (d) $\frac{\ln(t_2 + t_1)}{2}$

Ans. (c)

Given, activity of radioactive element at time t_1 ,

$$A_1 = A$$

and at time t_2 , $A_2 = A/5$

As we know that,

activity at any time (A) = $A_0 e^{-\lambda t}$

where A_0 is activity at time $t = 0$.

$$\therefore A = A_0 e^{-\lambda t_1} \quad \dots (i)$$

$$\text{and } A/5 = A_0 e^{-\lambda t_2} \quad \dots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{A}{A/5} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}}$$

$$\Rightarrow 5 = e^{-\lambda t_1 + \lambda t_2}$$

Taking log on both sides, we get

$$\ln 5 = \lambda(t_2 - t_1) \ln e$$

$$\Rightarrow \ln 5 = \lambda(t_2 - t_1) \quad [\because \ln e = 1]$$

$$\Rightarrow \lambda = \ln 5 / (t_2 - t_1)$$

Also, average life, $\tau = \frac{1}{\lambda} \Rightarrow \tau = \frac{t_2 - t_1}{\ln 5}$

- 17** Two radioactive substances X and Y originally have N_1 and N_2 nuclei, respectively. Half-life of X is half of the half-life of Y. After three half-lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be

equal to **[2021, 25 Feb Shift-I]**

- (a) $\frac{1}{8}$ (b) $\frac{3}{1}$
(c) $\frac{8}{1}$ (d) $\frac{1}{3}$

Ans. (c)

Given, initial amount of X and Y be N_1 and N_2 .

Let half-life of X be t_x and y be t_y .

According to question, $t_x = \frac{t_y}{2}$

$$\Rightarrow t_x = t \text{ and } t_y = 2t$$

After 3 half-lives of Y,

$$3t_y = 6t$$

As we know that,

$$N = N_0 e^{-\lambda t}$$

where, N is the number of nuclei left undecayed.

$$\text{and } t_{1/2} = \frac{0.693}{\lambda}$$

$$\text{or } \lambda = \frac{0.693}{t_{1/2}}$$

$$\Rightarrow \lambda_1 = \frac{0.693}{t} \text{ and } \lambda_2 = \frac{0.693}{2t}$$

Since, after 3 half-lives of Y number of nuclei of both become equal.

$$\therefore N_1 e^{-\lambda_1 6t} = N_2 e^{-\lambda_2 6t}$$

$$\Rightarrow N_1 / N_2 = e^{6t(-\lambda_2 + \lambda_1)}$$

$$\Rightarrow N_1 / N_2 = e^{6t \left(\frac{0.693}{t} - \frac{0.693}{2t} \right)}$$

$$= e^{0.693 \left(\frac{6t}{t} - \frac{6t}{2t} \right)} = e^{0.693 \times 3}$$

$$= 7.9 \approx 8$$

$$\frac{N_1}{N_2} = \frac{8}{1}$$

- 18** In a radioactive material, fraction of active material remaining after time t is $9/16$. The fraction that was remaining after time $t/2$ is

[2020, 3 Sep Shift-I]

- (a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{3}{4}$ (d) $\frac{7}{8}$

Ans. (c)

By radioactive disintegration (Rutherford and Soddy) law, amount of active (undecayed) radioactive material remained in a sample after time t is given by,

$$M(t) = M_0 e^{-\lambda t}$$

where, M_0 = initial sample of mass taken at $t = 0$ and λ = disintegration constant. Fraction of active material remained after time t is

$$\frac{M(t)}{M_0} = e^{-\lambda t} \quad \dots(i)$$

$$\text{Given that, after time } t, \frac{M(t)}{M_0} = \frac{9}{16}$$

$$\text{So, } e^{-\lambda t} = \frac{9}{16} \quad [\text{from Eq. (i)}]$$

Similarly, fraction of active material remained after time $t/2$ is

$$\frac{M\left(\frac{t}{2}\right)}{M_0} = e^{-\lambda \frac{t}{2}} = (e^{-\lambda t})^{\frac{1}{2}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

Hence, option (c) is correct.

- 19** The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15}) A^{1/3}$ m. It

follows that the mass density of a nucleus is of the order of

$$(M_{\text{proton}} \cong M_{\text{neutron}} \approx 1.67 \times 10^{-27} \text{ kg})$$

[2020, 3 Sep Shift-II]

- (a) $10^{17} \text{ kg m}^{-3}$ (b) $10^{24} \text{ kg m}^{-3}$
(c) $10^{10} \text{ kg m}^{-3}$ (d) 10^3 kg m^{-3}

Ans. (a)

$$\text{Mass of nucleus} = \text{Mass number} \times \text{Mass of a nucleon}$$

$$= A \times 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Volume of nucleus} = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi \times (1.3 \times 10^{-15} \times A^{1/3})^3$$

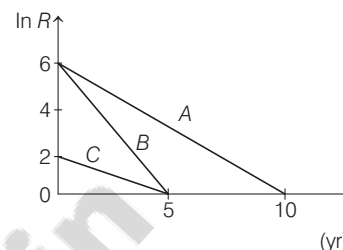
$$= \frac{4}{3} \pi \times (1.3)^3 \times 10^{-45} \times A$$

$$\therefore \text{Density of nucleus} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3} \pi \times (1.3)^3 \times 10^{-45} \times A} \approx 10^{17} \text{ kg m}^{-3}$$

Hence, correct option is (a).

- 20** Activities of three radioactive substances A, B and C are represented by the curves A, B and C in the figure. Then, their half-lives $T_{1/2}(A):T_{1/2}(B):T_{1/2}(C)$ are in the ratio **[2020, 5 Sep Shift-I]**



- (a) 4 : 3 : 1 (b) 3 : 2 : 1
(c) 2 : 1 : 1 (d) 2 : 1 : 3

Ans. (d)

According to law of disintegration for radioactive substances,

$$R = R_0 e^{-\lambda t} \Rightarrow \ln R = -\lambda t + \ln R_0$$

Slope of $\ln R$ versus t graph will give λ .

Also, decay constant,

$$\lambda = \frac{\ln 2}{T_{1/2}} \Rightarrow T_{1/2} = \frac{1}{\lambda} \ln 2$$

$\therefore$ Slope of curve A

$$= \lambda_A = \frac{6}{10} \Rightarrow T_{1/2}(A) = \frac{10}{6} \ln 2$$

Similarly, slope of curve

$$B = \lambda_B = \frac{6}{5} \Rightarrow T_{1/2}(B) = \frac{5}{6} \ln 2$$

and slope of curve

$$C = \lambda_C = \frac{2}{5} \Rightarrow T_{1/2}(C) = \frac{5}{2} \ln 2$$

$$\text{So, } T_{1/2}(A) : T_{1/2}(B) : T_{1/2}(C) = \frac{10}{6} : \frac{5}{6} : \frac{5}{2}$$

$$= 20 : 10 : 30 = 2 : 1 : 3$$

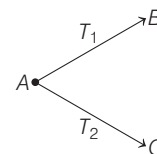
Hence, correct option is (d).

- 21** A radioactive nucleus decays by two different processes. The half-life for the first process is 10 s and that for the second is 100 s. The effective half-life of the nucleus is close to **[2020, 5 Sep Shift-II]**

- (a) 9 s (b) 6 s
(c) 55 s (d) 12 s

Ans. (a)

If a radioactive sample undergoes two different processes simultaneously, then



$$\frac{dN}{dT} = -\lambda_1 N - \lambda_2 N \Rightarrow \frac{dN}{dT} = -(\lambda_1 + \lambda_2)N$$

$$\frac{dN}{dT} = -\lambda_{\text{eff}} N$$

Here, $\lambda_{\text{eff}} = \lambda_1 + \lambda_2$

Let T_{eff} be the effective half-life of the nucleus.

$$\Rightarrow \frac{\ln 2}{T_{\text{eff}}} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2}$$

$$\frac{1}{T_{\text{eff}}} = \frac{1}{T_1} + \frac{1}{T_2} = \frac{1}{10} + \frac{1}{100} = \frac{11}{100}$$

$$\Rightarrow T_{\text{eff}} = \frac{100}{11} = 9.09 \text{ s} \approx 9 \text{ s}$$

Hence, correct option is (a).

- 22** The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 min. Its half-life is close to

[2020, 7 Jan Shift-II]

- (a) 62 min (b) 66 min
(c) 72 min (d) 52 min

Ans. (a)

Activity of a sample after time t is related to initial activity as

$$A = A_0 \left(\frac{1}{2} \right)^n \quad \dots (i)$$

where, A = present activity
and A_0 = initial activity.

$$\text{Also, } n = t / T_{1/2} \quad \dots (ii)$$

where, t = time interval

and $T_{1/2}$ = half-life time.

Here, $A = 500 \text{ s}^{-1}$, $A_0 = 700 \text{ s}^{-1}$,

$t = 30 \text{ min}$

So, from Eq. (i), we get

$$500 = 700 \left(\frac{1}{2} \right)^n \Rightarrow 2^n = 1.4 \approx \sqrt{2}$$

$$\text{So, } n = \frac{1}{2}$$

Now, from Eq. (ii), we get

$$\frac{t}{T_{1/2}} = \frac{1}{2} \Rightarrow T_{1/2} = 2 \times t = 60 \text{ min}$$

which is close to 62 min.

- 23** The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to

[2019, 8 April Shift-II]

- (a) 5 (b) 2
(c) 0.1 (d) 1

Ans. (d)

Mass density of nuclear matter is a constant quantity for all elements. It does not depend on element's mass number or atomic radius.

$\therefore$ The ratio of mass densities of ^{40}Ca and ^{16}O is 1 : 1.

- 24** Two radioactive materials A and B have decay constants 10λ and λ , respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time

[2019, 10 April Shift-I]

- (a) $\frac{1}{11\lambda}$ (b) $\frac{11}{10\lambda}$
(c) $\frac{1}{9\lambda}$ (d) $\frac{1}{10\lambda}$

Ans. (c)

Given, $\lambda_A = 10\lambda$ and $\lambda_B = \lambda$

Number of nuclei (at any instant) present in material is

$$N = N_0 e^{-\lambda t}$$

So, for materials A and B, we can write

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t} \quad \dots (i)$$

$$\text{Given, } \frac{N_A}{N_B} = \frac{1}{e} \quad \dots (ii)$$

Equating Eqs. (i) and (ii), we get

$$\frac{1}{e} = e^{-(\lambda_A - \lambda_B)t} \Rightarrow e^{-1} = e^{-(\lambda_A - \lambda_B)t}$$

Comparing the power of 'e' on both sides, we get

$$\text{or } (\lambda_A - \lambda_B)t = 1 \Rightarrow t = \frac{1}{\lambda_A - \lambda_B}$$

By putting values of λ_A and λ_B in the above equation, we get

$$t = \frac{1}{10\lambda - \lambda} \Rightarrow t = \frac{1}{9\lambda}$$

- 25** Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be

[2019, 12 April Shift-II]

- (a) 3 : 8 (b) 1 : 8
(c) 8 : 1 (d) 9 : 8

Ans. (d)

For substance A, half-life is 10 min, so it decays as

$$N_{0A} \xrightarrow{10\text{min}} \frac{N_{0A}}{2} \xrightarrow{10\text{min}} \frac{N_{0A}}{4} \xrightarrow{10\text{min}} \frac{N_{0A}}{8}$$

(initial number of nuclei at $t = 0$) (Active nuclei remained after 10 min)

$$\xrightarrow{10\text{min}} \frac{N_{0A}}{16} \xrightarrow{10\text{min}} \frac{N_{0A}}{32} \xrightarrow{10\text{min}} \frac{N_{0A}}{64}$$

(number of active nuclei remained after 60 min)

$\therefore$ For substance A, number of nuclei decayed in 60 min is

$$N_{0A} - \frac{N_{0A}}{64} = \frac{63N_{0A}}{64}$$

Similarly, for substance B, half-life is 20 min, so its decay scheme is

$$N_{0B} \xrightarrow{20\text{min}} \frac{N_{0B}}{2} \xrightarrow{20\text{min}} \frac{N_{0B}}{4} \xrightarrow{20\text{min}} \frac{N_{0B}}{8}$$

So, number of nuclei of B decayed in 60 min is

$$N_{0B} - \frac{N_{0B}}{8} = \frac{7N_{0B}}{8}$$

Hence, ratio of decayed nuclei of A and B in 60 min is

$$\frac{\frac{63}{64}N_{0A}}{\frac{7}{8}N_{0B}} = \frac{9}{8} \quad [\because N_{0A} = N_{0B}]$$

Alternate Solution

Number of active nuclei remained after 60 min can also be calculated as

$$N' = \frac{N}{2^{T/T_{1/2}}}$$

where, $T = 60 \text{ min}$

So, for nuclei A,

$$N'_{0A} = \frac{N_{0A}}{2^6} = \frac{N_{0A}}{64}$$

Similarly, for nuclei B,

$$N'_{0B} = \frac{N_{0B}}{2^3} = \frac{N_{0B}}{8}$$

- 26** Two radioactive substances A and B have decay constants 5λ and λ , respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$

will be

[2019, 10 April Shift-II]

- (a) $2/\lambda$ (b) $1/2\lambda$
(c) $1/4\lambda$ (d) $1/\lambda$

Ans. (b)

Number of active nuclei remained after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$ and λ = decay constant.

Here, at $t = 0$,

Number of nuclei in sample A and B are equal, i.e. $N_{0A} = N_{0B} = N_0$

Also, $\lambda_A = 5\lambda$ and $\lambda_B = \lambda$

So, after time t , number of active nuclei of A and B are

$$N_A = N_0 e^{-5\lambda t} \text{ and } N_B = N_0 e^{-\lambda t}$$

If $\frac{N_A}{N_B} = \frac{1}{e^2}$, then $\frac{N_A}{N_B} = \frac{N_0 e^{-5\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e^2}$
 $\Rightarrow e^{(\lambda - 5\lambda)t} = e^{-2}$

Comparing the power of e on both sides, we get

$$4\lambda t = 2 \Rightarrow t = \frac{1}{2\lambda}$$

- 27** A sample of radioactive material A, that has an activity of 10 mCi ($1 \text{ Ci} = 3.7 \times 10^{10}$ decays/s) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would, then be respectively [2019, 9 Jan Shift-I]

- (a) 20 days and 10 days
 (b) 5 days and 10 days
 (c) 10 days and 40 days
 (d) 20 days and 5 days

Ans. (d)

Activity of a radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant and N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = 10 \text{ mCi}$$

For substance B,

$$R_B = \lambda_B N_B = 20 \text{ mCi} \quad \dots(i)$$

As given in the question,

$$N_A = 2N_B \Rightarrow R_A = \lambda_A (2N_B) = 10 \text{ mCi} \quad \dots(ii)$$

$\therefore$ Dividing Eq. (ii) and Eq. (i), we get

$$\frac{R_A}{R_B} = \frac{\lambda_A (2N_B)}{\lambda_B (N_B)} = \frac{10}{20} \text{ or } \frac{\lambda_A}{\lambda_B} = \frac{1}{4} \quad \dots(iii)$$

As, half-life of a radioactive material is given as

$$T_{1/2} = \frac{0.693}{\lambda}$$

$\therefore$ For material A and B, we can write

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{\frac{0.693}{\lambda_A}}{\frac{0.693}{\lambda_B}} = \frac{\lambda_B}{\lambda_A}$$

Using Eq. (iii), we get, $\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{4}{1}$

Hence, from the given options, only option (d) satisfies this ratio.

Therefore, $(T_{1/2})_A = 20$ days and $(T_{1/2})_B = 5$ days

- 28** In given time $t = 0$, Activity of two radioactive substances A and B are equal. After time t , the ratio of their activities $\frac{R_B}{R_A}$ decreases according to e^{-3t} . If the half life of A is $\ln 2$, the half-life of B will be

[2019, 9 Jan Shift-II]

- (a) $4\ln 2$ (b) $\frac{\ln 2}{4}$
 (c) $\frac{\ln 2}{2}$ (d) $2\ln 2$

Ans. (b)

Activity of radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = \lambda_A N_{0A} \text{ (initially } N_A = N_{0A})$$

For substance B,

$$R_B = \lambda_B N_B = \lambda_B N_{0B} \text{ (initially } N_B = N_{0B})$$

At $t = 0$, activity is equal, therefore

$$\lambda_A N_{0A} = \lambda_B N_{0B} \quad \dots(i)$$

The half-life is given by

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

So, for substance A,

$$(T_{1/2})_A = \frac{\ln 2}{\lambda_A} \Rightarrow \ln 2 = \frac{\ln 2}{\lambda_A} \quad \dots(ii)$$

$$\lambda_A = 1$$

According to the given question,

$$\text{at time } t, \frac{R_B}{R_A} = e^{-3t} \quad \dots(iii)$$

Using Eqs. (i), (ii) and (iii), we get

$$\begin{aligned} \frac{R_B}{R_A} &= e^{-3t} = \frac{\lambda_B N_{0B} e^{-\lambda_B t}}{\lambda_A N_{0A} e^{-\lambda_A t}} \\ \Rightarrow e^{-3t} &= e^{(\lambda_A - \lambda_B)t} \Rightarrow -3 = \lambda_A - \lambda_B \\ \Rightarrow \lambda_B &= \lambda_A + 3 \Rightarrow \lambda_B = 1 + 3 = 4 \quad \dots(iv) \end{aligned}$$

The half-life of substance B is

$$(T_{1/2})_B = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$$

- 29** Using a nuclear counter, the count rate of emitted particles from a radioactive source is measured. At $t = 0$, it was 1600 counts per second and $t = 8$ s, it was 100 counts per second. The count rate observed as counts per second at $t = 6$ s is close to

[2019, 10 Jan Shift-I]

- (a) 400 (b) 200
 (c) 150 (d) 360

Ans. (b)

Here given, at $t = 0$, count rate or initial activity is $A_0 = 1600 \text{ s}^{-1}$.

At $t = 8$ s, count rate or activity is $A = 100 \text{ s}^{-1}$

So, decay scheme for given sample is

$$1600 \xrightarrow{T_{1/2}} 800 \xrightarrow{T_{1/2}} 400 \xrightarrow{T_{1/2}} 200 \xrightarrow{T_{1/2}} 100$$

So, $8 \text{ s} = 4T_{1/2}$
 where, $T_{1/2}$ = Half-life time.

$$\Rightarrow T_{1/2} = 2 \text{ s}$$

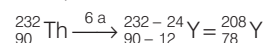
$\therefore$ From above decay scheme, we see that activity after 6 s is 200 counts per second.

- 30** In a radioactive decay chain, the initial nucleus is ${}_{90}^{232}\text{Th}$. At the end, there are 6 α -particles and 4 β -particles which are emitted. If the end nucleus is ${}_Z^AX$, A and Z are given by [2019, 12 Jan Shift-II]
 (a) $A = 202$; $Z = 80$ (b) $A = 208$; $Z = 82$
 (c) $A = 200$; $Z = 81$ (d) $A = 208$; $Z = 80$

Ans. (b)

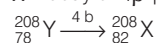
An α -particle decay (${}_2^4\text{He}$) reduces, mass number by 4 and atomic number by 2.

$\therefore$ Decay of 6 α -particles results



A β -decay does not produce any change in mass number but it increases atomic number by 1.

$\therefore$ Decay of 4 β -particles results,



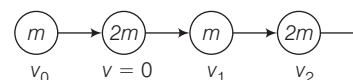
$\therefore$ In the end nucleus $A = 208$, $Z = 82$

- 31** It is found that, if a neutron suffers an elastic collision with deuterium at rest, fractional loss of its energy is P_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c . The values of P_d and P_c are respectively [JEE Main 2018]

- (a) (.89, .28) (b) (.28, .89)
 (c) (0, 0) (d) (0, 1)

Ans. (a)

Neutron-Deuterium collision;



Momentum conservation gives;

$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_0 = v_1 + 2v_2 \quad \dots(i)$$

Collision given is elastic.

So, coefficient of restitution, $e = 1$

$$\therefore e = 1 = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$\Rightarrow 1 = \frac{v_2 - v_1}{v_0 - 0} \Rightarrow v_0 = v_2 - v_1 \quad \dots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$2v_0 = 3v_2 \Rightarrow \frac{2v_0}{3} = v_2$$

So, from Eq. (i), we get

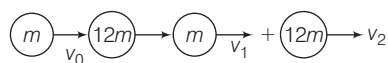
$$v_1 = v_0 - 2v_2 = v_0 - \frac{4v_0}{3} \Rightarrow v_1 = -\frac{v_0}{3}$$

Fractional loss of energy of neutron

$$= \left(\frac{-K_f + K_i}{K_i} \right)_{\text{for neutron}}$$

$$= \frac{-\frac{1}{2}mv_1^2 + \frac{1}{2}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{-\frac{v_0^2}{9} + v_0^2}{v_0^2}$$

$$= \left(-\frac{1}{9} + 1 \right) = \frac{8}{9} = 0.8\bar{8} = 0.89$$



Similarly, for neutron-carbon atom collision;

Momentum conservation gives;

$$v_0 = v_1 + 12v_2 \text{ and } e = 1$$

$$\Rightarrow v_0 = v_2 - v_1$$

$$\text{So, } v_1 = \frac{11}{13}v_0$$

$$\therefore \text{Loss of energy} = \left(-\frac{121}{169} + 1 \right) = 0.28$$

So, $P_d = 0.89$ and $P_c = 0.28$

- 32** A radioactive nucleus A with a half-life T , decays into a nucleus B. At $t=0$, there is no nucleus B. After sometime t , the ratio of the number of B to that of A is 0.3.

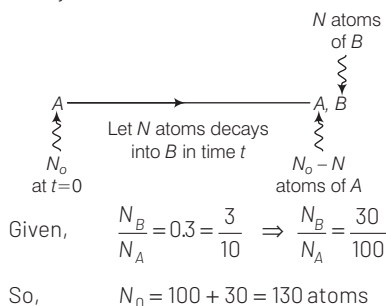
Then, t is given by [JEE Main 2017]

$$(a) t = T \frac{\log 1.3}{\log_e 2} \quad (b) t = T \log 1.3$$

$$(c) t = \frac{T}{\log 1.3} \quad (d) t = \frac{T \log_e 2}{2 \log 1.3}$$

Ans. (a)

Decay scheme is,



$$\text{By using } N = N_0 e^{-\lambda t}$$

$$\text{We have, } 100 = 130 e^{-\lambda t}$$

$$\Rightarrow \frac{1}{1.3} = e^{-\lambda t} \Rightarrow \log 1.3 = \lambda t$$

$$\text{If } T \text{ is half-life, then } \lambda = \frac{\log_e 2}{T}$$

$$\Rightarrow \log 1.3 = \frac{\log_e 2}{T} \cdot t \Rightarrow t = \frac{T \cdot \log (1.3)}{\log_e 2}$$

- 33** Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be [JEE Main 2016]

- (a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 5 : 4

Ans. (d)

Given, 80 min = 4 half-lives of A = 2 half-lives of B.

Let the initial number of nuclei in each sample be N .

For radioactive element A,

$$N_A \text{ after 80 min} = \frac{N}{2^4}$$

$$\Rightarrow \text{Number of A nuclides decayed}$$

$$= N - \frac{N}{16} = \frac{15}{16}N$$

For radioactive element B,

$$N_B \text{ after 80 min} = \frac{N}{2^2} \Rightarrow \text{Number of B}$$

$$\text{nuclides decayed} = N - \frac{N}{4} = \frac{3}{4}N$$

$\therefore$ Ratio of decayed numbers of A and B nuclei will be

$$\frac{(15/16)N}{(3/4)N} = \frac{5}{4}$$

- 34 Statement I** A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2 , but β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.
- Statement II** To conserve energy and momentum in β^- -decay, atleast three particles must take part in the transformation. [AIIEE 2011]

- (a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Ans. (c)

In particle situation, atleast three particles take place in transformation, so

Energy of β^- -particle + Energy of third particle

$$= E_1 - E_2$$

Hence, energy of β^- -particle $\leq E_1 - E_2$

- 35** The half-life of a radioactive substance is 20 min. The approximate time interval ($t_2 - t_1$) between the time t_2 when $2/3$ of it has decayed and time t_1 when $1/3$ of it had decayed is [AIIEE 2011]

- (a) 14 min (b) 20 min
(c) 28 min (d) 7 min

Ans. (b)

$$N_1 = N_0 - \frac{1}{3}N_0 = \frac{2}{3}N_0$$

$$N_2 = N_0 - \frac{2}{3}N_0 = \frac{1}{3}N_0$$

$$\frac{N_1}{N_2} = \left(\frac{1}{2} \right)^n \Rightarrow n = 1$$

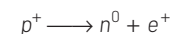
$\therefore t_2 - t_1 = \text{one half-life} = 20 \text{ min}$

- 36** A radioactive nucleus (initial mass number A and atomic number Z) emits 3α -particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be [AIIEE 2010]

- (a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$
(c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-4}{Z-2}$

Ans. (b)

In positive beta decay, a proton is transformed into a neutron and a positron is emitted.



Number of neutrons initially was $A - Z$.

Number of neutrons after decay
($A - Z$) - 3×2 (due to alpha particles)
+ 2×1 (due to positive beta decay)

The number of protons will reduce by 8.

[as 3×2 (due to alpha particles) + 2 (due to positive beta decay)]

Hence, atomic number reduces by 8.

So, the ratio of number of neutrons to

$$\text{that of protons} = \frac{A-Z-4}{Z-8}$$

- 37** The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then,

[AIEEE 2007]

- (a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

Ans. (b)

$$T_{1/2}(X) = \tau(Y)$$

$$\Rightarrow \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y} \text{ or } \lambda_Y = \frac{\lambda_X}{0.693} \Rightarrow \lambda_Y > \lambda_X$$

So, Y will decay faster than X.

- 38** In gamma ray emission from a nucleus,

[AIEEE 2007]

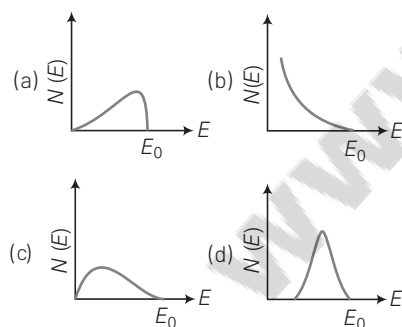
- (a) both the neutron number and the proton number change
(b) there is no change in the proton number and the neutron number
(c) only the neutron number changes
(d) only the proton number changes

Ans. (b)

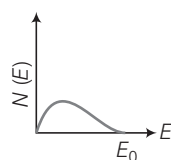
In gamma ray emission, the energy is released from nucleus, so that nucleus get stabilised (to get the minimum energy state).

- 39** The energy spectrum of β -particles [number $N(E)$ as a function of β -energy E] emitted from a radioactive source is

[AIEEE 2006]

**Ans. (c)**

Energy spectrum of emitted β -particles from radioactive source is drawn as



- 40** Starting with a sample of pure ^{66}Cu , $7/8$ of it decays into Zn in 15 min. The corresponding half-life is

[AIEEE 2005]

- (a) 10 min (b) 15 min
(c) 5 min (d) $7\frac{1}{2}$ min

Ans. (c)

$$N = N_0(1 - e^{-\lambda t}) \Rightarrow \frac{N_0 - N}{N_0} = e^{-\lambda t}$$

$$\therefore 1/8 = e^{-\lambda t} \text{ or } 8 = e^{\lambda t}$$

$$\text{or } 3 \ln 2 = \lambda t \quad [\because \log_e m^n = n \log_e m]$$

$$\text{or } \lambda = \frac{3 \times 0.693}{15}$$

$$\text{Half-life period, } t_{1/2} = \frac{0.693}{3 \times 0.693} \times 15$$

$$\left[\because t_{1/2} = \frac{0.693}{\lambda} \right]$$

$$= 5 \text{ min}$$

- 41** If radius of the $^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 fm, then the radius of $^{125}_{52}\text{Te}$ nucleus be nearly

[AIEEE 2005]

- (a) 6 fm (b) 8 fm (c) 4 fm (d) 5 fm

Ans. (a)

$$R = R_0(A)^{1/3}$$

$$\frac{R_{\text{Al}}}{R_{\text{Te}}} = \frac{R_0(A_{\text{Al}})^{1/3}}{R_0(A_{\text{Te}})^{1/3}}$$

$$\text{or } \frac{R_{\text{Al}}}{R_{\text{Te}}} = \frac{(A_{\text{Al}})^{1/3}}{(A_{\text{Te}})^{1/3}}$$

$$\text{or } \frac{R_{\text{Al}}}{R_{\text{Te}}} = \frac{(27)^{1/3}}{(125)^{1/3}} = \frac{3}{5}$$

$$\text{or } R_{\text{Te}} = \frac{5}{3} \times 3.6 \text{ or } R_{\text{Te}} = 6 \text{ fm}$$

- 42** A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be

[AIEEE 2004]

- (a) $2^{1/3} : 1$ (b) $1 : 3^{1/2}$
(c) $3^{1/2} : 1$ (d) $1 : 2^{1/3}$

Ans. (d)

Law of conservation of momentum gives

$$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$$

$$\text{But } m = \frac{4}{3} \pi r^3 \rho \text{ or } m \propto r^3$$

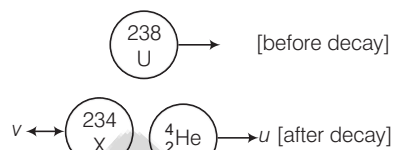
$$\therefore \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1} \Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{2} \right)^{1/3}$$

$$\text{or } r_1 : r_2 = 1 : 2^{1/3}$$

- 43** When ^{238}U nucleus originally at rest, decays by emitting an alpha particle having a speed u , the recoil speed of the residual nucleus is

[AIEEE 2003]

- (a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$ (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

Ans. (c)

Apply conservation of linear momentum.

$$0 = 4u - 234v \quad [\because Mu = m_1 v_1 + m_2 v_2]$$

$$\Rightarrow v = \frac{4u}{234}$$

The residual nucleus will recoil with a velocity of $\frac{4u}{234}$ unit.

Note If they will ask the recoil velocity, then answer remains same i.e., $\frac{4u}{234}$ and not

$-\frac{4u}{234}$ as the word 'recoil' itself is signifying the direction of motion of residual nucleus.

- 44** A radioactive sample at any instant has its disintegration rate 5000 disintegrations/min. After 5 min, the rate is 1250 disintegrations/min. Then, the decay constant (per minute) is

[AIEEE 2003]

- (a) $0.4 \ln 2$ (b) $0.2 \ln 2$
(c) $0.1 \ln 2$ (d) $0.8 \ln 2$

Ans. (a)

$$\text{Given, } N_0 \lambda = 5000, N \lambda = 1250$$

$$N = N_0 e^{-\lambda t} = N_0 e^{-5\lambda}$$

where, λ is decay constant per minute.

$$N \lambda = N_0 \lambda e^{-5\lambda} \Rightarrow 1250 = N_0 \lambda e^{-5\lambda}$$

$$\therefore \frac{N_0 \lambda}{N_0 \lambda e^{-5\lambda}} = \frac{5000}{1250} = 4$$

$$\Rightarrow e^{5\lambda} = 4$$

$$\text{or } 5\lambda = 2 \log_e 2$$

$$\Rightarrow \lambda = 0.4 \ln 2$$

- 45** A nucleus with $Z = 92$ emits the following in a sequence $\alpha, \alpha, \beta^-, \alpha, \alpha, \alpha, \alpha; \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$.

The Z of the resulting nucleus is

[AIEEE 2003]

- (a) 76 (b) 78
(c) 82 (d) 74

Ans. (b)

Since, 8 α -particles $4\beta^-$ -particles and $2\beta^+$ -particles are emitted, so new atomic number $[\alpha \rightarrow {}^4_2\text{He}$ and $\beta^- \rightarrow -{}_1^0n]$.

$$\begin{aligned} Z' &= Z - 8 \times 2 + 4 \times 1 - 2 \times 1 \\ &= 92 - 16 + 4 - 2 \\ &= 92 - 14 = 78 \end{aligned}$$

- 46** Which of the following cannot be emitted by radioactive substances during their decay? [AIEEE 2003]

- (a) Protons (b) Neutrinos
(c) Helium nuclei (d) Electrons

Ans. (a)

Protons cannot be emitted by radioactive substances during their decay.

- 47** If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ yr, then the amount of substance left after 15 yr, is [AIEEE 2002]

- (a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$
(c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$

Ans. (a)

N_0 is the initial amount of substance and N is the amount left after decay.

$$\text{Thus, } N = N_0 \left(\frac{1}{2} \right)^n$$

$$n = \text{number of half-lives} = \frac{t}{t_{1/2}} = \frac{15}{5} = 3$$

$$\text{Therefore, } N = N_0 \left(\frac{1}{2} \right)^3 = \frac{N_0}{8}$$

- 48** At a specific instant, emission of radioactive compound is deflected in a magnetic field. The compound can emit [AIEEE 2002]

- (i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons

The emission at the instant can be

- (a) (i), (ii), (iii)
(b) (i), (ii), (iii), (iv)
(c) (iv)
(d) (ii), (iii)

Ans. (a)

Electrons, protons and helium atoms are deflected in magnetic field, so the compound can emit electrons, protons and He^{2+} .

TOPIC 2

Nuclear Fission and Fusion and Binding Energy

- 49** From the given data, the amount of energy required to break the nucleus of aluminium ${}^{27}_{13}\text{Al}$ is $x \times 10^{-3}$ J.

Mass of neutron = 1.00866 u

Mass of proton = 1.00726 u

Mass of aluminium nucleus

= 27.18846 u

(Assume 1 u corresponds to x joule of energy)

(Round off to the nearest integer)

[2021, 25 July Shift-II]

Ans. (27.16)

Given,

Mass of neutron $m_n = 1.00866$ u

Mass of proton, $m_p = 1.00726$ u

Mass of aluminium nucleus,

$$m_{\text{Al}} = 27.18846 \text{ u}$$

As we know that,

$$\text{Binding energy (BE)} = \Delta mc^2$$

$$\text{Mass defect } (\Delta m) = Zm_p + (A - Z)m_n - m$$

where, Z is atomic number or number of proton

and $A - Z$ is number of neutron.

$$\begin{aligned} \therefore \Delta m &= 13 \times 1.00726 + 14 \times 1.00866 \\ &\quad - 27.18846 \\ &= 13.09438 + 14.12124 - 27.18846 \\ &= 0.02716 \text{ u} \end{aligned}$$

$$\text{Now, BE} = 27.16 \times 10^{-3} \text{ J}$$

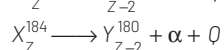
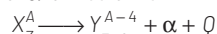
- 50** A nucleus with mass number 184 initially at rest emits an α -particle. If the Q -value of the reaction is 5.5 MeV, calculate the kinetic energy of the α -particle. [2021, 22 July Shift-II]

- (a) 5.0 MeV (b) 5.5 MeV
(c) 0.12 MeV (d) 5.38 MeV

Ans. (d)

Given, initial mass number be A is 184 and Q value is 5.5 MeV.

Equation of α -emission is



$p_Y, p_\alpha, m_Y, m_\alpha, KE_Y$ and KE_α be the momentum, mass, kinetic energy of Y and α , respectively.

By using conservation of momentum,

Momentum of α = Momentum of Y

$$\Rightarrow p_\alpha = p_Y$$

$$\therefore \sqrt{2m_\alpha E_\alpha} = \sqrt{2m_Y E_Y}$$

$$[\because \text{Momentum, } p = \sqrt{2mE}]$$

$$\Rightarrow E_\alpha = \frac{180}{4} E_Y = 45 E_Y$$

$$\Rightarrow E_Y = \frac{E_\alpha}{45}$$

By using conservation of energy,

$$Q = KE_Y + KE_\alpha = \frac{KE_\alpha}{45} + KE_\alpha$$

$$\Rightarrow 5.5 = KE_\alpha \left(\frac{46}{45} \right)$$

$$\Rightarrow KE_\alpha = \frac{5.5 \times 45}{46} = 5.38 \text{ MeV}$$

- 51** A nucleus of mass M emits γ -ray photon of frequency ν . The loss of internal energy by the nucleus is

[Take, c is the speed of electromagnetic wave.]

[2021, 20 July Shift-I]

- (a) $h\nu$ (b) zero
(c) $h\nu \left[1 - \frac{h\nu}{2Mc^2} \right]$ (d) $h\nu \left[1 + \frac{h\nu}{2Mc^2} \right]$

Ans. (d)

Energy of γ -ray, $E_\gamma = h\nu$

and momentum of γ -ray, $p_\gamma = \frac{h\nu}{\lambda}$... (i)

$$\text{As, } p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu}{c} \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$p_\gamma = \frac{h\nu}{c} = \frac{h}{\lambda} \left[\because \lambda = \frac{c}{\nu} \right]$$

Since, during the emission of γ -ray photon, momentum is conserved.

$$\therefore p_\gamma + p_{\text{decayed nuclei}} = 0$$

$$\Rightarrow p_\gamma = p_{\text{decayed nuclei}}$$

$$\Rightarrow \frac{h\nu}{c} = p_{\text{decayed nuclei}} \dots \text{(iii)}$$

Kinetic energy of decayed nuclei,

$$KE = \frac{1}{2} Mv^2 = \frac{(p_{\text{decayed nuclei}})^2}{2M} \dots \text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$\Rightarrow KE = \frac{1}{2M} \left[\frac{h\nu}{c} \right]^2$$

$\therefore$ Loss in internal energy

$$= E_\gamma + KE_{\text{decayed nuclei}}$$

$$= h\nu + \frac{1}{2M} \left[\frac{h\nu}{c} \right]^2 = h\nu \left[1 + \frac{h\nu}{2Mc^2} \right]$$

- 52** Find the binding energy per nucleon for $^{120}_{50}\text{Sn}$. Mass of proton $m_p = 1.00783 \text{ u}$, mass of neutron $m_n = 1.00867 \text{ u}$ and mass of tin nucleus $m_{\text{Sn}} = 119.902199 \text{ u}$. (Take, $1 \text{ u} = 931 \text{ MeV}$)

[2020, 4 Sep Shift-II]

- (a) 9.0 MeV (b) 7.5 MeV
(c) 8.0 MeV (d) 8.5 MeV

Ans. (d)

$m_{\text{theoretical}} = zm_p + (A - z)m_n$
 $= 50(1.00783 \text{ u}) + (120 - 50)(1.00867 \text{ u})$
 $= 50.391500 \text{ u} + 70.606900 \text{ u}$
 $= 120.998400 \text{ u}$
 $m_{\text{experimental}} = 119.902199 \text{ u}$
 So, mass defect,
 $\Delta m = m_{\text{theoretical}} - m_{\text{experimental}}$
 $= 120.998400 \text{ u} - 119.902199 \text{ u} = 1.096201 \text{ u}$
 Now, total binding energy, $BE = \Delta m \times 931 \text{ MeV}$
 $= 1.096201 \times 931 \text{ MeV}$
 $= 1020.563131 \text{ MeV}$

Therefore, binding energy per nucleon,

$$(BE)_{\text{per nucleon}} = \frac{BE}{\text{Total number of nucleons}}$$

$$= \frac{1020.563131}{120}$$

$$= 8.50469275833 \text{ MeV}$$

$$\approx 8.5 \text{ MeV}$$

Hence, option (d) is correct.

- 53** In a reactor, 2 kg of $^{235}_{92}\text{U}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that, the Avogadro number, $N = 6.023 \times 10^{26} \text{ K}^{-1} \text{ mol}^{-1}$ and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The power output of the reactor is close to

[2020, 2 Sep Shift-I]

- (a) 35 MW (b) 125 MW
(c) 60 MW (d) 54 MW

Ans. (c)

Number of moles of $^{235}_{92}\text{U}$ in given sample,
 $n = \frac{\text{Sample mass}}{\text{Molar mass}} = \frac{2}{235} \text{ (kilomoles)}$

Number of atoms of $^{235}_{92}\text{U}$ in sample
 $= \text{number of moles} \times N$
 $= \frac{2}{235} \times 6.023 \times 10^{26}$

Total energy obtained
 $= \text{number of atoms} \times \text{energy from one atom}$
 $= \frac{2 \times 6.023 \times 10^{26}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$

Power = $\frac{\text{Energy (J)}}{\text{Time (s)}}$
 $= \frac{2 \times 6.023 \times 10^{26} \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{235 \times 30 \times 24 \times 3600}$
 $= 63.2 \times 10^6 \text{ W}$
 $= 63.2 \text{ MW}$
 It is close to 60 MW.
 Hence, correct option is (c).

- 54** You are given that mass of

 $^7_3\text{Li} = 7.0160 \text{ u}$, mass of $^4_2\text{He} = 4.0026 \text{ u}$ and mass of

$^1_1\text{H} = 1.0079 \text{ u}$. When 20 g of ^7_3Li is converted into ^4_2He by proton capture, the energy liberated (in kWh), is

[Mass of nucleon = $1 \text{ GeV}/c^2$]

[2020, 6 Sep Shift-I]

- (a) 4.5×10^5 (b) 8×10^6
(c) 6.82×10^5 (d) 1.33×10^6

Ans. (d)Given that, $m(^7_3\text{Li}) = 7.0160 \text{ u}$ $m(^4_2\text{He}) = 4.0026 \text{ u}$ $m(^1_1\text{H}) = 1.0079 \text{ u}$

Nuclear reaction :



Energy released in one nuclear reaction,

$$Q = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$$

$$= (7.0160 + 1.0079 - 2 \times 4.0026) \times 931$$

$$= 17.41 \text{ MeV}$$

Number of atoms in 20 g of Li

$$= \frac{20}{7} \times 6.023 \times 10^{23} = 1.72 \times 10^{24}$$

 $\therefore$ Total energy liberated

$$= Q \times \text{Total number of Li atoms}$$

$$= (17.41 \text{ MeV}) \times (1.72 \times 10^{24})$$

$$= 4.79 \times 10^{12} \text{ J}$$

$$\text{Energy (in kWh)} = \frac{4.79 \times 10^{12}}{3.6 \times 10^6}$$

$$(\because 1 \text{ kWh} = 3.6 \times 10^6 \text{ J})$$

$$= 1.33 \times 10^6 \text{ kWh}$$

- 55** Given the masses of various atomic particles $m_p = 1.0072 \text{ u}$, $m_n = 1.0087 \text{ u}$, $m_e = 0.000548 \text{ u}$, $m_{\bar{\nu}} = 0$, $m_d = 2.0141 \text{ u}$, where p = proton, n = neutron, e = electron $\bar{\nu}$ = antineutrino and d = deuteron. Which of the following process is allowed by momentum and energy conservation? [2020, 6 Sep Shift-II]

(a) $n + n \rightarrow$ deuterium atom (electron bound to the nucleus)(b) $p \rightarrow n + e^+ + \bar{\nu}$ (c) $n + p \rightarrow d + \bar{\nu}$ (d) $e^+ + e^- \rightarrow \gamma$ **Ans. (c)**

In nuclear reaction, momentum and energy conservation laws are valid only, if total mass on reactant side is greater than that of the product side.

i.e. $\Sigma m_{\text{reactant}} > \Sigma m_{\text{product}}$
 because liberated energy appears due to mass defect.

If we check the above condition from given data, then it is found that among all options only option (c) satisfy the condition.

Mass of n + mass of

$$p = (1.0087 + 1.0072) \text{ u} = 2.0159 \text{ u}$$

$$\text{Mass of } d + \text{mass of } \bar{\nu} = (2.0141 + 0) \text{ u}$$

$$= 2.0141 \text{ u}$$

Hence, only statement in option (c) is true.

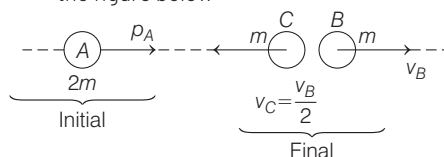
- 56** A nucleus A, with a finite de-Broglie wavelength λ_A , undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same directions as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C respectively

[2019, 8 April Shift-II]

- (a) $2\lambda_A, \lambda_A$ (b) $\frac{\lambda_A}{2}, \lambda_A$
(c) $\lambda_A, 2\lambda_A$ (d) $\lambda_A, \frac{\lambda_A}{2}$

Ans. (b)Let m be the mass of nuclei B and C.

So, the given situation can be shown in the figure below



Now, according to the conservation of linear momentum,

Initial momentum = Final momentum

$$\Rightarrow p_A = p_B + p_C \text{ or } 2mv_A = mv_B + mv_C$$

$$2mv_A = mv_B - \frac{mv_B}{2}$$

$$\Rightarrow 2v_A = \frac{1}{2}v_B \Rightarrow v_B = 4v_A \quad \dots(i)$$

$$\text{and } v_C = \frac{v_B}{2} = 2v_A \quad \dots(ii)$$

So, momentum of B and C respectively, can now be given as

$$p_B = m_B v_B = m v_A = 2(2m v_A) \quad [\text{using Eq. (i)}]$$

$$\text{or } p_B = 2p_A \quad \dots(\text{iii})$$

$$\text{and } p_C = m_C v_C = m v_A \quad [\text{using Eq. (ii)}]$$

$$\text{or } p_C = p_A \quad \dots(\text{iv})$$

From the relation of de-Broglie wavelength,

$$\text{i.e.} \quad \lambda = \frac{h}{p}$$

where, p is momentum and h is Planck's constant.

$$\text{So, for A, } \lambda_A = \frac{h}{p_A} \text{ or } p_A = \frac{h}{\lambda_A} \quad \dots(\text{v})$$

$$\text{For B, } \lambda_B = \frac{h}{p_B} = \frac{h}{2p_A} \quad [\text{using Eq. (iii)}]$$

From Eq. (v), λ_B can be written as

$$\lambda_B = \frac{h}{2 \times \frac{h}{\lambda_A}} = \frac{\lambda_A}{2}$$

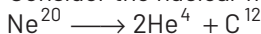
$$\text{Similarly, for C, } \lambda_C = \frac{h}{p_C} = \frac{h}{p_A}$$

[using Eq. (iv)]

Similarly, from Eq. (v), λ_C can be written as

$$\lambda_C = \frac{h}{\frac{h}{\lambda_A}} = \lambda_A$$

57 Consider the nuclear fission



Given that the binding energy/nucleon of Ne^{20} , He^4 and C^{12} are respectively,

8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement.

[2019, 10 Jan Shift-I]

- (a) Energy of 3.6 MeV will be released.
- (b) Energy of 12.4 MeV will be supplied.
- (c) 8.3 MeV energy will be released.
- (d) Energy of 11.9 MeV has to be supplied.

Ans. (*)

Energy absorbed or released in a nuclear reaction is given by;

$$\Delta Q = \text{Binding energy of products} - \text{Binding energy of reactants.}$$

If energy is absorbed, ΔQ is negative and if it is positive then energy is released.

Also, Binding energy = Binding energy per nucleon $\times$ Number of nucleons.

Here, binding energy of products

$$= 2 \times (\text{B.E. of He}^4) + (\text{B.E. of C}^{12})$$

$$= 2(4 \times 7.07) + (12 \times 7.86) = 150.88 \text{ MeV}$$

and binding energy of reactants

$$= 20 \times 8.03 = 160.6 \text{ MeV}$$

$$\text{So, } \Delta Q = (\text{B.E.})_{\text{Products}} - (\text{B.E.})_{\text{Reactants}} = 150.88 - 160.6 = -9.72 \text{ MeV.}$$

As ΔQ is negative

$\therefore$ energy of 9.72 MeV is absorbed in the reaction.

$\therefore$ No option is correct.

58 Assume that a neutron breaks into a proton and an electron. The energy released during this process is (mass of neutron

$= 1.6725 \times 10^{-27} \text{ kg}$, mass of proton

$= 1.6725 \times 10^{-27} \text{ kg}$, mass of

electron $= 9 \times 10^{-31} \text{ kg}$) [AIEEE 2012]

(a) 0.73 MeV (b) 7.10 MeV

(c) 6.30 MeV (d) 5.4 MeV

Ans. (a)

According to given data, mass of neutron and proton are equal which do not permit the breaking up of neutron and proton.

But if we take standard mass of neutron as $1.6750 \times 10^{-27} \text{ kg}$, then

$$\begin{aligned} \text{Energy released} &= \text{Mass defect} \times c^2 \\ &= (m_n - m_p - m_e) \times c^2 \\ [\therefore \text{energy released mass of reactant} & - \text{mass of product} \times c^2] \\ &= \frac{(1.6750 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31})}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV} \end{aligned}$$

$$= \frac{16 \times 10^{-31}}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV}$$

$$= \frac{1.6 \times 10^{-31}}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV}$$

$$= \frac{1.6 \times 0.9315}{1.66} \text{ MeV} = 0.73 \text{ MeV}$$

Directions (Q. Nos. 59 to 60) are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is c .

59 The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2 . Then,

(a) $E_2 = 2E_1$

(b) $E_1 > E_2$

(c) $E_2 > E_1$

(d) $E_1 = 2E_2$

Ans. (c)

After decay, the daughter nuclei will be more stable, hence binding energy per nucleon will be more than that of their parent nucleus.

60 The speed of daughter nuclei is

[AIEEE 2010]

$$(a) c \frac{\Delta m}{M + \Delta m}$$

$$(b) c \sqrt{\frac{2\Delta m}{M}}$$

$$(c) c \sqrt{\frac{\Delta m}{M}}$$

$$(d) c \sqrt{\frac{\Delta m}{M + \Delta m}}$$

Ans. (b)

By conservation of momentum,

$$0 = \frac{M}{2} v_1 - \frac{M}{2} v_2$$

$$v_1 = v_2 \quad \dots(\text{i})$$

$$\Delta mc^2 = \frac{1}{2} M v_1^2 + \frac{1}{2} M v_2^2 \quad \dots(\text{ii})$$

$$\Delta mc^2 = \frac{M}{2} v_1^2$$

$$\frac{2\Delta mc^2}{M} = v_1^2$$

$$\Rightarrow v_1 = c \sqrt{\frac{2\Delta m}{M}}$$

61 The below is a plot of binding energy per nucleon E_b , against the nuclear mass M ; A, B, C, D, E, F correspond to different nuclei.

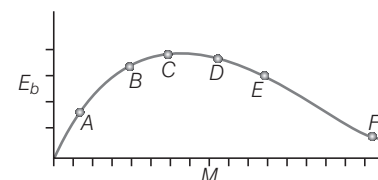
Consider four reactions

(i) $A + B \rightarrow C + \epsilon$

(ii) $C \rightarrow A + B + \epsilon$

(iii) $D + E \rightarrow F + \epsilon$ and

(iv) $F \rightarrow D + E + \epsilon$



where, ϵ is the energy released. In which reactions ϵ is positive?

[AIEEE 2009]

(a) (i) and (iv)

(b) (i) and (iii)

(c) (ii) and (iv)

(d) (ii) and (iii)

Ans. (a)

1st reaction is fusion and 4th reaction is fission.

Directions Q. No. 62 is Assertion-Reason type question. The question contains two statements : Statement I (Assertion) and Statement II (Reason). The question also has four alternative choices, only one of which is correct answer. You have to select the correct choice.

- 62 Statement I** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

Statement II For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei, it decreases with increasing Z .

[AIEEE 2008]

- (a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
 (b) Statement I is true, Statement II is false
 (c) Statement I is false, Statement II is true
 (d) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

Ans. (b)

Statement I is correct and Statement II is wrong which can be directly concluded from binding energy/ nucleon curve.

- 63** If M_o is the mass of an oxygen isotope ${}^8_{16}\text{O}$, M_p and M_n are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is

[AIEEE 2007]

- (a) $(M_o - 8M_p)c^2$
 (b) $(M_o - 8M_p - 9M_n)c^2$
 (c) $M_o c^2$
 (d) $(M_o - 17M_n)c^2$

Ans. (b)

$$\begin{aligned} \text{Binding energy,} \\ \text{BE} &= (M_{\text{nucleus}} - M_{\text{nucleons}})c^2 \\ &= (M_o - 8M_p - 9M_n)c^2 \end{aligned}$$

- 64** If the binding energy per nucleon in ${}^7_3\text{Li}$ and ${}^4_2\text{He}$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction
 $p + {}^7_3\text{Li} \longrightarrow 2 {}^4_2\text{He}$
 energy of proton must be

[AIEEE 2006]

- (a) 28.24 MeV
 (b) 17.28 MeV
 (c) 1.46 MeV
 (d) 39.2 MeV

Ans. (b)

From the equation,

$$\begin{aligned} \text{Energy of proton} \\ + (7 \times 5.60) &= 2 \times [4 \times 7.06] \end{aligned}$$

$$\therefore \text{Energy of proton} = 17.28 \text{ MeV}$$

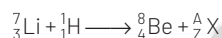
- 65** When ${}^7_3\text{Li}$ nuclei are bombarded by protons, and the resultant nuclei are ${}^8_4\text{Be}$, the emitted particles will be

[AIEEE 2006]

- (a) alpha particles
 (b) beta particles
 (c) gamma photons (d) neutrons

Ans. (c)

The nuclear reaction can be represented as



Applying conservation of atomic number (charge),

$$3 + 1 = 4 + Z$$

$$\Rightarrow Z = 0$$

Applying conservation of atomic mass,

$$7 + 1 = 8 + A$$

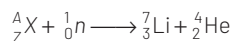
$$\Rightarrow A = 0$$

Thus, the emitted particles are γ -photons (${}^0_0\text{X}$).

- 66** A nuclear transformation is denoted by $X(n, \alpha) \longrightarrow {}^7_3\text{Li}$. Which of the following is the nucleus of element X?

[AIEEE 2005]

- (a) ${}^{12}_6\text{C}$ (b) ${}^{10}_5\text{B}$
 (c) ${}^9_5\text{B}$ (d) ${}^{11}_4\text{Be}$

Ans. (b)

It implies that, $A + 1 = 7 + 4$

$$\Rightarrow A = 10$$

$$\text{and } Z + 0 = 3 + 2$$

$$\Rightarrow Z = 5$$

Thus, it is Boron ${}^{10}_5\text{B}$.

- 67** The binding energy per nucleon of deuteron (${}^2_1\text{H}$) and helium nucleus (${}^4_2\text{He}$) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[AIEEE 2004]

- (a) 13.9 MeV (b) 26.9 MeV
 (c) 23.6 MeV (d) 19.2 MeV

Ans. (c)

As given ${}^2_1\text{H} + {}^2_1\text{H} \longrightarrow {}^4_2\text{He} + \text{energy}$

The binding energy per nucleon of a deuteron (${}^2_1\text{H}$)

$$= 1.1 \text{ MeV}$$

$\therefore$ Total binding energy of one deuteron nucleus

$$= 2 \times 1.1 = 2.2 \text{ MeV}$$

The binding energy per nucleon of helium (${}^4_2\text{He}$)

$$= 7 \text{ MeV}$$

$\therefore$ Total binding energy $= 4 \times 7 = 28 \text{ MeV}$
 Hence, energy released in the above process

$$= 28 - 2 \times 2.2$$

$$= 28 - 4.4$$

$$= 23.6 \text{ MeV}$$

- 68** In the nuclear fusion reaction,
 ${}^2_1\text{H} + {}^3_1\text{H} \longrightarrow {}^4_2\text{He} + n$

given that the repulsive potential energy between the two nuclei is $7.7 \times 10^{-14} \text{ J}$, the temperature at which the gases must be heated to initiate the reaction is nearly

[Boltzmann's constant,

$$k = 1.38 \times 10^{-23} \text{ J/K}] \quad \text{[AIEEE 2003]}$$

- (a) 10^7 K (b) 10^5 K
 (c) 10^3 K (d) 10^9 K

Ans. (d)

$$\frac{3}{2}kT = 7.7 \times 10^{-14} \text{ J}$$

$$\begin{aligned} \text{or } T &= \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} \\ &= 3.7 \times 10^9 \text{ K} \end{aligned}$$

28

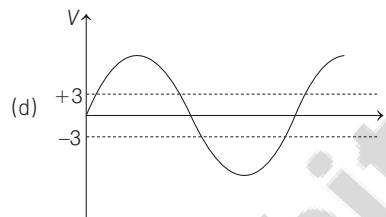
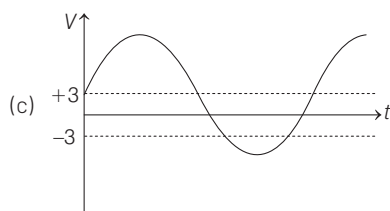
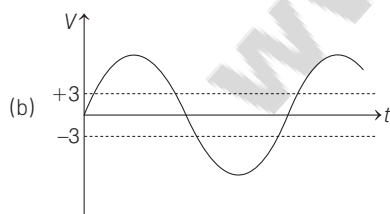
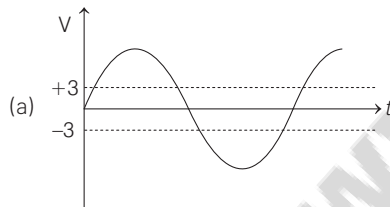
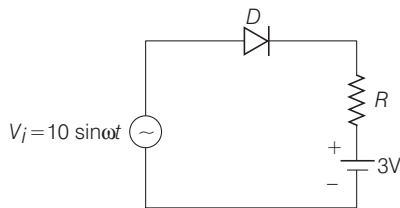
Semiconductor Electronics

TOPIC 1

Semiconductor and p - n Junction Diode

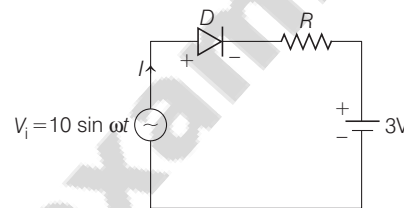
- 01** Choose the correct wave form that can represent the voltage across R of the following circuit, assuming the diode is ideal one.

[2021, 31 Aug Shift-I]



Ans. (*)

According to given circuit diagram,



Let I current is flowing through the circuit.

∴ Kirchhoff's voltage loop equation will be $10 \sin \omega t - IR - 3 = 0$

$$\Rightarrow IR = V = 10 \sin \omega t - 3$$

∴ At time, $t = 0$, $V = 0 - 3 = -3V$... (i)

$$t = \frac{T}{4}, V = 10 \sin\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right) - 3 = 10 - 3 = 7V$$

$t = T/2$ to T , diode is in reverse bias.

∴ $V = 0$... (iii)

We see that at $t = 0$, $V = -3$ and $T/2 \leq t \leq T$, $V = 0$

Therefore, no waveforms exist in option (a), (b), (c) and (d), which satisfies above condition. Hence, no option is correct.

- 02 Statement I** To get a steady DC output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load R_L .

Statement II To get a steady DC output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with R_L .

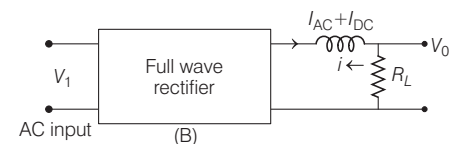
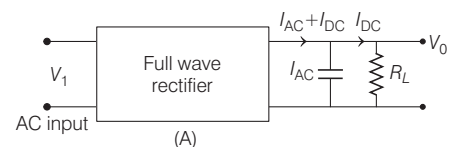
In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 31 Aug Shift-II]

- (a) Statement I is true but statement II is false.
- (b) Statement I is false but statement II is true.
- (c) Both statement I and statement II are false.
- (d) Both statement I and statement II are true.

Ans. (d)

According to statement I and II, full wave rectifier with capacitor and inductor are as shown in figure.



In figure (A), the capacitor is connected in parallel to load resistance (R_L), as it can block DC current.

∴ DC current can pass only through R_L which gives DC output.

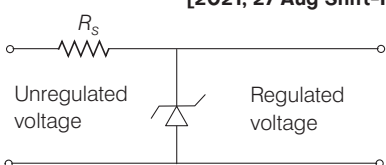
In figure (B), as the AC current is time varying current which can produce an

opposition in current due to inductor that cancel the effect of AC in the load resistance (R_L). Hence, pure DC output is obtained across R_L .

Hence, statement I and statement II are true.

- 03** A Zener diode of power rating 2W is to be used as a voltage regulator. If the Zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_S for safe operation should be Ω

[2021, 27 Aug Shift-II]



Ans. (20)

Given, power, $P = 2 \text{ W}$

Zener breakdown voltage, $V_Z = 10 \text{ V}$

Voltage range, (V_1 to V_2) = 6 V and 14 V

Let resistance be R .

Since, $P = VI_Z$

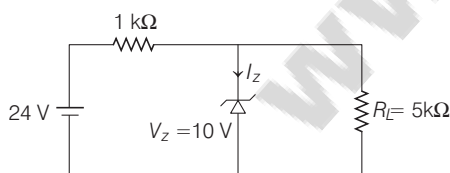
Zener current, $I_Z = \frac{P}{V_Z} = \frac{2}{10} = 0.2 \text{ A}$

For safe operation of Zener diode,

$$\begin{aligned} I_Z &= I_S = 0.2 \text{ A} \\ \therefore V_2 - V_Z &= I_S R_S \\ \Rightarrow 14 - 10 &= I_S R_S \\ \Rightarrow R_S &= \frac{4}{0.2} \\ &= \frac{40}{2} = 20 \Omega \end{aligned}$$

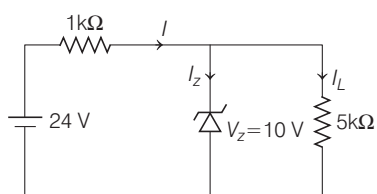
- 04** For the given circuit, the power across Zener diode is mW.

[2021, 26 Aug Shift-II]



Ans. (120)

The circuit diagram is shown below



The supply voltage, $V_S = 24 \text{ V}$

Zener voltage, $V_Z = 10 \text{ V}$

The value of supply voltage is greater than the Zener voltage. Hence, the voltage across the load resistance of 5 kΩ will be equal to Zener voltage as they are connected in parallel combination.

Current I_L flowing through the load resistance can be calculated as

$$I_L = \frac{10}{5 \times 10^3} = 2 \times 10^{-3} \text{ A} = 2 \text{ mA}$$

Since, the voltage drop across 5 kΩ is 10 V, then the voltage drop across 1 kΩ resistance, $V' = 24 - 10 = 14 \text{ V}$

Current flowing through 1 kΩ resistance can be calculated as

$$I = \frac{V'}{1 \times 10^3} = \frac{14}{1 \times 10^3} = 14 \times 10^{-3} \text{ A} = 14 \text{ mA}$$

From circuit diagram, $I = I_Z + I_L$
 $I_Z = I - I_L = 14 - 2 = 12 \text{ mA}$

Power across Zener diode is

$$P = V_Z \times I_Z = 10 \times 12 = 120 \text{ mW}$$

Thus, the power across the Zener diode is 120 mW.

- 05 Statement I** By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement II The n -type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 26 Aug Shift-I]

- Statement I is true but statement II is false.
- Statement I is false but statement II is true.
- Both statement I and statement II are true.
- Both statement I and statement II are false.

Ans. (a)

We know that, the pentavalent impurities have excess free electrons. So, when a silicon semiconductor is doped with pentavalent impurities, the electron density increases. But on the other hand, the whole semiconductor is electrically neutral, therefore net charge on n -type semiconductor is zero.

Hence, statement I true but statement II is false.

- 06** In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16} / \text{m}^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22} / \text{m}^3$. The electron density in the doped semiconductor is $\times 10^9 / \text{m}^3$.

[2021, 25 July Shift-II]

Ans. (5)

Given, number density of intrinsic charge carrier, $n_i = 1.5 \times 10^{16} / \text{m}^3$

Number density of hole,

$$n_h = 4.5 \times 10^{22} \text{ m}^{-3}$$

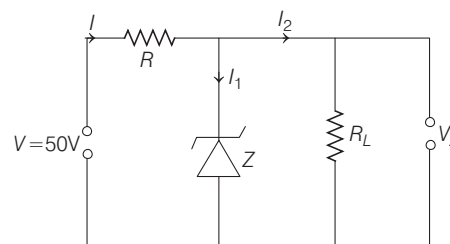
Let number density of electron be n_e .

As we know that,

$$\begin{aligned} n_i^2 &= n_e n_h \\ n_e &= \frac{n_i^2}{n_h} \\ &= \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} \\ &= \frac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}} \\ &= 0.5 \times 10^{10} = 5 \times 10^9 \text{ m}^{-3} \end{aligned}$$

- 07** In a given circuit diagram, a 5 V Zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum Zener current is 90 mA will be Ω .

[2021, 22 July Shift-II]

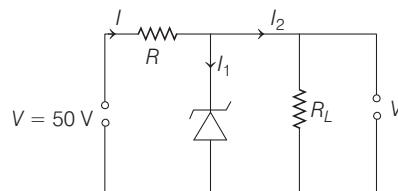


Ans. (500)

Given, Zener voltage, $V_Z = 5 \text{ V}$

Supply voltage, $V = 50 \text{ V}$

Zener current, $I_1 = 90 \text{ mA} = 90 \times 10^{-3} \text{ A}$



$$\therefore R = \frac{V - V_Z}{I}$$

$$\therefore R = \frac{50-5}{90 \times 10^{-3}} = \frac{45}{9} \times 10^2 = 500 \Omega$$

As maximum current is I_1 , so resistance of Zener diode will be minimum.

By using Kirchhoff's current law, $I = I_1 + I_2$

$\therefore I_1$ is maximum,

$\therefore I_2$ will be minimum

Hence, minimum value of load resistance R_L will be 500Ω .

- 08** Consider a situation in which reverse biased current of a particular p - n junction increases when it is exposed to a light of wavelength ≤ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly

[2021, 22 July Shift-II]

- (a) 2 eV
(b) 4 eV
(c) 1 eV
(d) 0.5 eV

Ans. (a)

Given, $\lambda = 621 \times 10^{-9}$ m

As we know that,

$$E = \frac{hc}{\lambda} \text{ J}$$

$$\Rightarrow E = \frac{hc}{\lambda e} \text{ eV} \quad \dots(i)$$

where, E = energy,

h = Planck's constant

i.e. 6.63×10^{-34} J-s,

c = speed of light in vacuum

i.e. 3×10^8 m s⁻¹

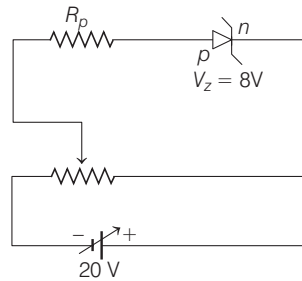
and e = charge of electron i.e. 1.6×10^{-19} C.

On substituting above values in Eq. (i), we get

$$\begin{aligned} E &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{621 \times 10^{-9} \times 1.6 \times 10^{-19}} \\ &= 0.02014 \times 10^2 \\ &= 2.014 \text{ eV} \\ &\approx 2 \text{ eV} \end{aligned}$$

- 09** A Zener diode having Zener voltage 8 V and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across Zener diode is as shown in the diagram. The value of protective resistance R_p is Ω .

[2021, 20 July Shift-II]



Ans. (192)

Given, $V_z = 8\text{V}$, $P = 0.5\text{W}$

$\therefore$ Power, $P = VI$

$$\Rightarrow 0.5 = 8 \times I \Rightarrow I = \frac{0.5}{8} = \frac{5}{80} = \frac{1}{16} \text{ A}$$

$$\therefore E = V + IR$$

where, E is emf and V is terminal voltage.

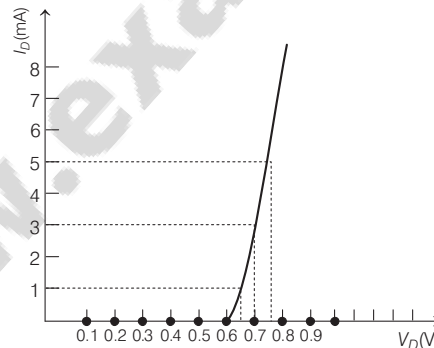
$$\Rightarrow 20 = 8 + \frac{1}{16} \times R \Rightarrow 20 - 8 = \frac{R}{16}$$

$$\Rightarrow 12 = \frac{R}{16} \Rightarrow R = 12 \times 16$$

$$\Rightarrow R = 192 \Omega$$

- 10** For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D = 3$ mA will be Ω .

[2021, 20 July Shift-II]



Ans. (25)

We know that,

$$\text{Dynamic resistance} = \frac{\Delta V}{\Delta I}$$

$$R_d = \frac{0.75 - 0.65}{(5 - 1) \times 10^{-3}}$$

(at $I_D = 3$ mA in the graph)

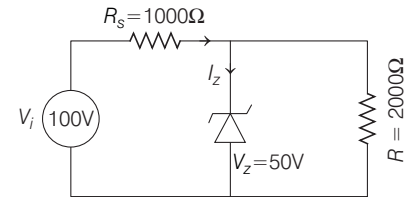
$$R_d = \frac{0.10}{4 \times 10^{-3}} \Rightarrow R_d = \frac{0.1 \times 10^3}{4}$$

$$R_d = \frac{100}{4} = 25 \Omega$$

$\therefore 25 \Omega$ is the required value of dynamic resistance at $I_D = 3$ mA.

- 11** For the circuit shown below, calculate the value of I_z .

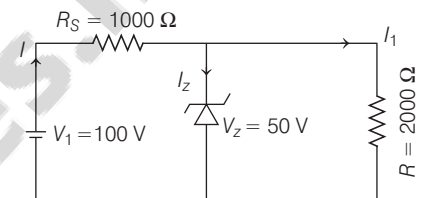
[2021, 20 July Shift-I]



- (a) 25 mA
(c) 0.1 A
(b) 0.15 A
(d) 0.05 A

Ans. (a)

Consider the given figure and draw the direction of current as follows



Here, current I is flowing through resistance R_s and I_1 is the current flowing through $R = 2000 \Omega$.

$$\therefore \text{Current, } I = \frac{V_i - V_z}{R_s} \Rightarrow I = \frac{100 - 50}{1000}$$

$$= 50 \text{ mA} \quad \dots(i)$$

[$\therefore$ Resistance and Zener diode are in parallel, therefore voltages across them are same]

$$\text{and current, } I_1 = \frac{50}{2000} = 25 \text{ mA} \quad \dots(ii)$$

By using Kirchhoff's current law,

$$\therefore I = I_1 + I_z$$

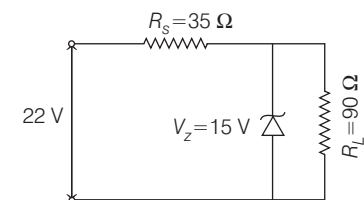
$$\Rightarrow I_z = I - I_1 \quad \dots(iii)$$

From Eq. (i) and (ii) and, we get

$$\Rightarrow I_z = (50 - 25) \text{ mA}$$

$$I_z = 25 \text{ mA}$$

- 12** The value of power dissipated across the Zener diode ($V_z = 15\text{V}$) connected in the circuit as shown in the figure is $x \times 10^{-1}$ W.

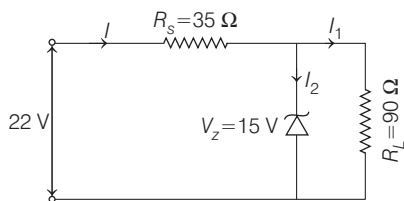


The value of x , to the nearest integer, is

[2021, 16 March Shift-I]

Ans. (5)

Consider the figure with current in the directions shown below.



Voltage across R_s , $V_s = 22 - 15 = 7\text{ V}$

Current through R_s , $I = \frac{V_s}{R_s}$

$$\Rightarrow I = \frac{7}{35} = \frac{1}{5}\text{ A}$$

Current through 90Ω resistance,

$$I_1 = \frac{V_z}{R_L} = \frac{15}{90} = \frac{1}{6}\text{ A}$$

$\therefore$ Current through Zener diode,

$$I_2 = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}\text{ A}$$

$\therefore$ Power through Zener diode,

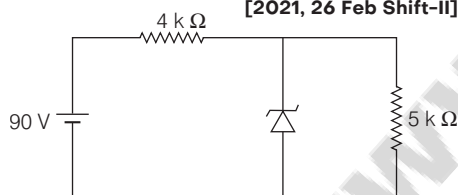
$$\begin{aligned} P &= V_z I_2 \\ &= 15 \times \frac{1}{30} = \frac{1}{2} = 0.5\text{ W} \\ &= 5 \times 10^{-1}\text{ W} \end{aligned}$$

Comparing with the given value in question i.e., $x \times 10^{-1}$, the value of $x = 5$.

13 The Zener diode has a $V_z = 30\text{ V}$.

The current passing through the diode for the following circuit is mA.

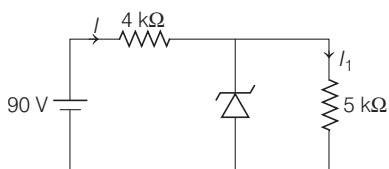
[2021, 26 Feb Shift-II]

**Ans. (9)**

Given, Zener diode voltage, $V_z = 30\text{ V}$

Supply voltage, $V = 90\text{ V}$

According to figure,



Let I be supply current, then

I_z be Zener current ($= I - I_1$)

and I_1 be current through $5\text{ k}\Omega$.

$$\therefore V_1 = V_z = 30\text{ V}$$

By Ohm's law,

$$V_1 = V_z = I_1 R$$

$$\Rightarrow I_1 = \frac{V_z}{R} = \frac{30}{5000} = 6\text{ mA}$$

If V' be the voltage across $4\text{ k}\Omega$ resistor.

$$\therefore V' = 90 - 30 = 60\text{ V}$$

$$\text{and } I = \frac{60}{4000} = 15\text{ mA}$$

$$\text{Therefore, } I_z = I - I_1 = 15 - 6 = 9\text{ mA}$$

14 LED is constructed from GaAsP semiconducting material. The energy gap of this LED is 1.9 eV . Calculate the wavelength of light emitted and its colour. [$h = 6.63 \times 10^{-34}\text{ Js}$ and $c = 3 \times 10^8\text{ ms}^{-1}$]

[2021, 26 Feb Shift-I]

- (a) 1046 nm and red colour
- (b) 654 nm and orange colour
- (c) 1046 nm and blue colour
- (d) 654 nm and red colour

Ans. (d)

Given, energy gap of LED, $E = 1.9\text{ eV}$

Speed of light in free space,

$$c = 3 \times 10^8\text{ ms}^{-1}$$

Planck's constant, $h = 6.63 \times 10^{-34}\text{ J-s}$

As we know that, $E = \frac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E}$$

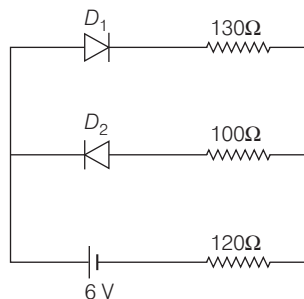
$$\begin{aligned} \Rightarrow \lambda &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}} \\ &= 654 \times 10^{-9}\text{ m} \\ &= 654\text{ nm} \end{aligned}$$

As, wavelength of red light is 600 nm .

$\therefore$ Required wavelength will be of red colour.

15 The circuit contains two diodes each with a forward resistance of 50Ω and with infinite reverse resistance. If the battery voltage is 6 V , the current through the 120Ω resistance is mA.

[2021, 26 Feb Shift-I]

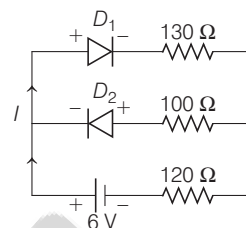
**Ans. (20)**

Given, forward resistance, $R_1 = 50\Omega$

Reverse resistance, $R_2 = \text{infinity}$

Battery voltage $= 6\text{ V}$

According to circuit diagram,



In this case, diode D_1 is forward biased, whereas diode D_2 is reverse biased.

So, D_2 will act as open circuit.

By using Kirchhoff's voltage law,

$$6 - 50I - 130I - 120I = 0$$

$$\Rightarrow 6 = 300I$$

$$\Rightarrow I = \frac{6}{300} = \frac{1}{50}$$

$$= \frac{2}{100} = 0.02\text{ A}$$

$$= 20\text{ mA}$$

Hence, current through $120\Omega = 20\text{ mA}$

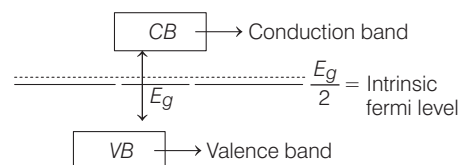
16 For extrinsic semiconductors when doping level is increased,

[2021, 25 Feb Shift-II]

- (a) Fermi level of p -type semiconductor will go upward and Fermi level of n -type semiconductors will go downward
- (b) Fermi level of p -type semiconductors will go downward and Fermi level of n -type semiconductor will go upward
- (c) Fermi level of p and n -type semiconductors will not be affected
- (d) Fermi level of both p -type and n -type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature

Ans. (b)

In case of n -type semiconductor, the energy level diagram will be

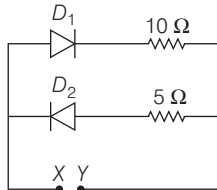


In case of n -type semiconductor $n > p$, so the fermi level will go upward.

Similarly, in case of p -type semiconductor $p > n$, so the fermi level will go downward.

- 17** A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery, if the positive terminal of the battery is connected to point X.

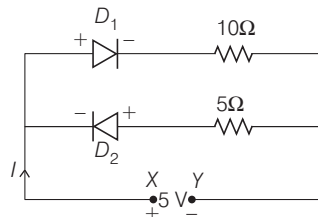
[2021, 25 Feb Shift-I]



- (a) ~ 0.5 A
(b) ~ 1.5 A
(c) ~ 0.86 A
(d) ~ 0.43 A

Ans. (d)Given, supply voltage, $V = 5$ V

The circuit diagram, when positive terminal of the battery is connected to X is as shown below



Let I current is coming from battery.

$\therefore D_1$ will act as closed circuit as forward biased and D_2 will act as open circuit as reverse biased.

Now, by using Kirchhoff's voltage law,

$$\begin{aligned} 5 - V_{D_1} - 10I &= 0 \\ \Rightarrow 5 - 0.7 - 10I &= 0 \quad (\because V_{D_1} = 0.7 \text{ V}) \\ \Rightarrow 4.3 &= 10I \\ \Rightarrow I &= 0.43 \text{ A} \end{aligned}$$

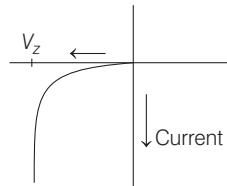
- 18** Zener breakdown occurs in a p - n junction having p and n both [2021, 24 Feb Shift-II]

- (a) lightly doped and have wide depletion layer
(b) heavily doped and have narrow depletion layer
(c) lightly doped and have narrow depletion layer
(d) heavily doped and have wide depletion layer

Ans. (b)

As we know that, Zener breakdown takes place, when we supply reverse bias voltage to Zener diode. Due to heavily

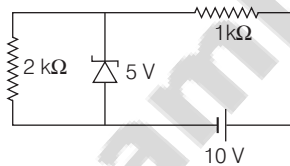
doping, the electrons in the valence band of p -type region can jump easily to the conduction band of n -type region, hence due to high electric field, zener breakdown occur. Thus, there is very high sudden increase in Zener current (I_z) that is caused by reverse breakdown voltage (V_z).



Hence, Zener breakdown is easily observed in Zener diode which is heavily doped and having narrow depletion region.

- 19** In connection with the circuit drawn below, the value of current flowing through $2\text{k}\Omega$ resistor is $\times 10^{-4}$ A.

[2021, 24 Feb Shift-II]

**Ans. (25)**

Given, resistance,

$$R = 2\text{k}\Omega = 2 \times 10^3 \Omega$$

In Zener breakdown,

$$\begin{aligned} i &= \frac{V}{R} = \frac{5}{2 \times 10^3} \\ &= 2.5 \times 10^{-3} \\ \therefore x \times 10^{-4} &= 25 \times 10^{-4} \\ \therefore x &= 25 \end{aligned}$$

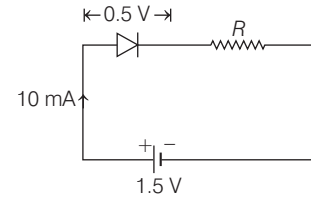
- 20** When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode, so that the current does not exceed the safe limit is

[2020, 3 Sep Shift-I]

- (a) 300 Ω
(b) 50 Ω
(c) 100 Ω
(d) 200 Ω

Ans. (c)

Given circuit is



Now, by Kirchhoff's loop rule,

$$1.5 - 0.5 - R \times 10 \times 10^{-3} = 0 \Rightarrow R = 100 \Omega$$

Hence, option (c) is correct.

- 21** If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is (Take, Planck's constant, $h = 6.63 \times 10^{-34}$ J-s and speed of light, $c = 3 \times 10^8$ m/s)

[2020, 3 Sep Shift-II]

- (a) 1.5 eV
(b) 1.1 eV
(c) 3.1 eV
(d) 2.0 eV

Ans. (c)

Photons are detected by photodiode, so band gap energy is given by

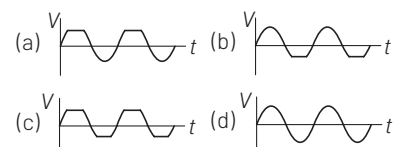
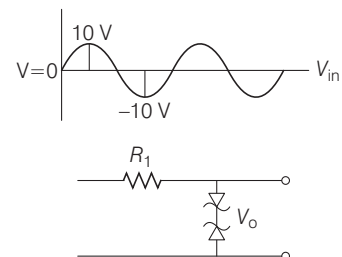
$$\begin{aligned} \Delta E &= \frac{hc}{\lambda} \\ \Rightarrow \Delta E &= \frac{1240 \text{ eV-nm}}{400 \text{ nm}} = 3.1 \text{ eV} \end{aligned}$$

(Note $hc \approx 1240 \text{ eV-nm}$)

Hence, correct option is (c).

- 22** Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is (Graphs are drawn schematically and on not to scale)

[2020, 4 Sep Shift-I]



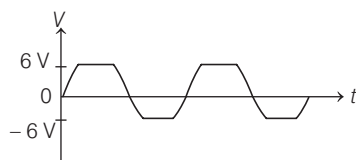
Ans. (c)

There are two zener diodes in reverse polarity. So, if the input voltage changes its polarity, then one zener diode will be in forward bias and other zener diode will be in reverse bias. This happens at each kind of polarity of input voltage.

Now, if the input voltage is less than 6 V, then output will follow the input voltage, which means it will remain sinusoidal.

And if the input voltage is more than 6 V, the output voltage will remain constant equal to 6 V (as forward biased zener diode will not be active and reverse biased zener diode will keep the potential across it constant).

So, we can conclude from available options, that the correct graph is



Hence, correct option is (c).

23 With increasing biasing voltage of a photodiode, the photocurrent magnitude [2020, 5 Sep Shift-I]

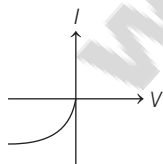
- (a) increases initially and after attaining certain value, it decreases
(b) increases initially and saturates finally
(c) remains constant
(d) increases linearly

Ans. (b)

The IV- characteristic curve for a photodiode is shown below,

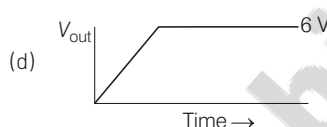
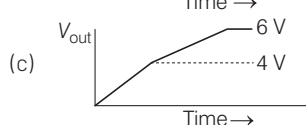
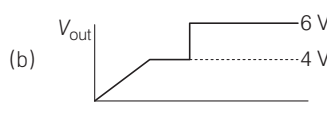
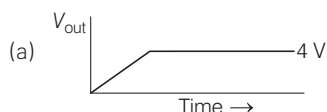
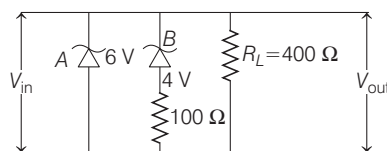
With increasing biasing voltage of a photodiode, the photocurrent first increases and then becomes saturated, i.e., after certain potential, electron-hole pair production will be nearly constant.

Hence, correct option is (b).



24 Two zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit below. The output voltage V_{out} variation with input voltage linearly increasing

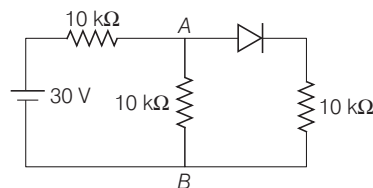
with time, is given by ($V_{in} = 0$ V at $t = 0$) (figures are qualitative)

[2020, 5 Sep Shift-II]**Ans. (c)**

For $V < 4$ V, both diodes are reverse biased but not working in zener region. So, output voltage (V_{out}) will be equal to input voltage. For voltages higher than 4 V, diode A will not conduct current but diode B will work in zener region and large current will flow through it. So, potential will increase but some voltage drop will occur in connecting wires. So, slope of V_{out} versus t graph will get reduced. Once both the diodes start working in zener region, the output voltage will become 6 V and after that, it will not change.

Hence, correct option is (c).

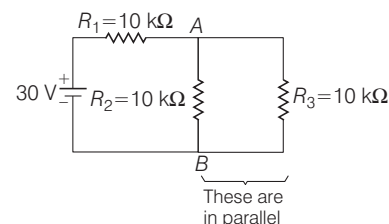
25 In the figure, potential difference between A and B is

[2020, 7 Jan Shift-II]

- (a) zero (b) 10 V (c) 5 V (d) 15 V

Ans. (b)

In ideal condition, a forward biased $p-n$ junction acts like a conductive path of zero resistance. So, given circuit is equivalent to circuit shown below



Total resistance of circuit,
 $R = (R_3 \text{ in parallel with } R_2) \text{ and these are in series with } R_1$
 $= \frac{10 \times 10}{10 + 10} + 10 = 15 \text{ k}\Omega$

So, total current in the circuit,

$$I = \frac{V}{R} = \frac{30}{15 \text{ k}\Omega} = 2 \times 10^{-3} \text{ A}$$

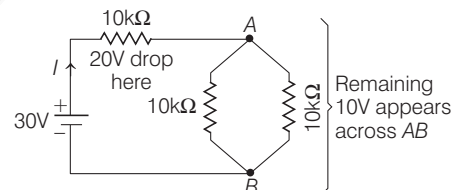
Now, potential drop across R_1 , (10 k Ω),

$$V_{R_1} = IR_1 = 2 \times 10^{-3} \times 10 \times 10^3 = 20 \text{ V}$$

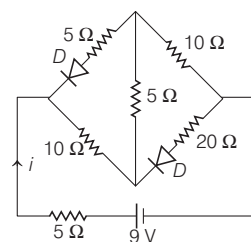
Hence, remaining potential appears across parallel combination of R_2 and R_3 ,

$$V_{AB} = 30 - 20 = 10 \text{ V}$$

This can be seen in the figure below.



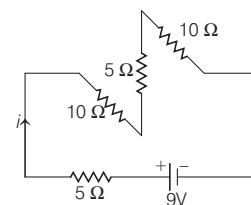
26 The current i in the network is

[2020, 9 Jan Shift-II]

- (a) 0.2 A (b) 0 A
(c) 0.6 A (d) 0.3 A

Ans. (d)

As both diodes are in reverse bias, so given circuit is equivalent to the circuit as shown in the figure.



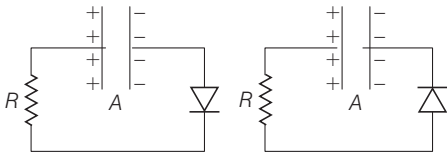
Equivalent resistance of circuit,
 $R = 5 + 10 + 5 + 10 = 30\Omega$

∴ Circuit current,

$$I = \frac{V}{R} = \frac{9}{30} = 0.3 \text{ A}$$

- 27** Two identical capacitors A and B, charged to the same potential 5V are connected in two different circuits as shown below at time $t=0$. If the charge on capacitors A and B at time $t=CR$ is Q_A and Q_B respectively, then (here e is the base of natural logarithm)

[2020, 9 Jan Shift-II]



(a) $Q_A = VC$, $Q_B = \frac{VC}{e}$

(b) $Q_A = \frac{VC}{e}$, $Q_B = \frac{CV}{2}$

(c) $Q_A = \frac{CV}{2}$, $Q_B = \frac{VC}{e}$

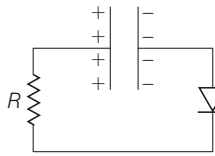
(d) $Q_A = VC$, $Q_B = CV$

Ans. (a)

Charge on capacitor after charging with a battery of potential V ,

$$Q = CV$$

Now, capacitor A is connected as shown in the figure.



As diode is in reverse bias, so capacitor charge remains same after time $t=CR$.

∴ $Q_A = CV$

In case B, capacitor is connected as shown in the figure.



Here, diode is in forward bias, so capacitor discharges and its charge decays as

$$Q_B = Q \cdot e^{-\frac{t}{RC}}$$

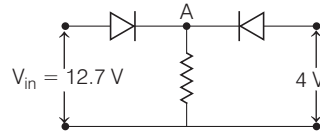
where,

$$t = RC$$

∴ $Q_B = CV \cdot e^{-1} = \frac{CV}{e}$

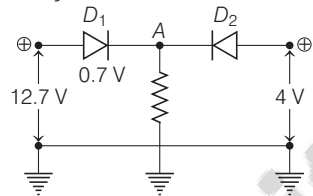
- 28** Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7V. For the input voltages shown in the figure, the voltage (in volts) at point A is

[2020, 9 Jan Shift-I]



Ans. (12)

We have given circuit,

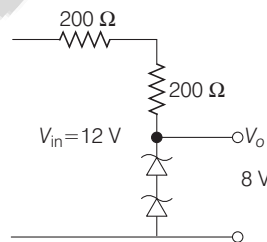


Diode D_1 is in forward bias and D_2 is in reverse bias, so D_1 conducts and D_2 resists.

∴ Potential at A = $12.7 - 0.7 = 12 \text{ V}$

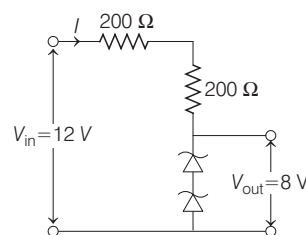
- 29** The circuit shown below is working as a 8 V DC regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both Zener diodes are identical)

[2020, 9 Jan Shift-II]



Ans. (40)

As input = 12V and output = 8V, this means 4V potential drop occurs in external resistance of 400 Ω .



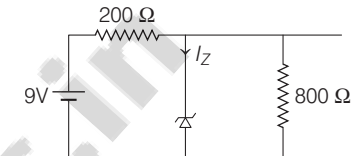
So, current I across resistors,

$$I = V/R = 4/400 = 10^{-2} \text{ A}$$

As, there are two identical diodes and potential drop across diodes 8V, i.e. potential drop across each diode is 4V. Hence, power dissipation across each diode,

$$P = VI = 4 \times 10^{-2} \text{ W} = 40 \text{ mW}$$

- 30** The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



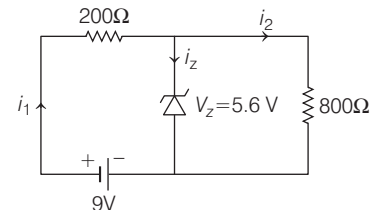
The current I_z through the Zener is

(a) 10 mA (b) 17 mA (c) 15 mA (d) 7 mA

[2019, 8 April Shift-I]

Ans. (a)

Given circuit is Zener diode circuit



where, potential drop across 800 Ω resistance = potential drop across Zener diode = 5.6 V

So, current, $i_2 = \frac{V}{R} = \frac{5.6}{800} = 7 \text{ mA}$

Now, potential drop across 200 Ω

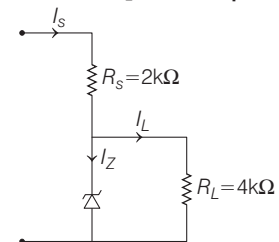
$$\text{resistance} = 9 - 5.6 = 3.4 \text{ V}$$

Current, $i_1 = \frac{V}{R} = \frac{3.4}{200} = 17 \text{ mA}$

So, current, $i_z = i_1 - i_2 = 17 - 7 = 10 \text{ mA}$

- 31** Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?

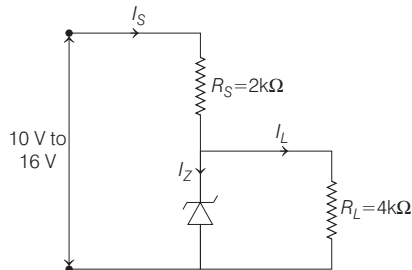
[2019, 12 April Shift-II]



- (a) 2.5 mA (b) 1.5 mA
(c) 7.5 mA (d) 3.5 mA

Ans. (d)

In given voltage regulator circuit breakdown of Zener occurs at 6 V.



After breakdown, voltage across load resistance ($R_L = 4k\Omega$) is,

$$V = V_Z = 6V$$

$\therefore$ Load current after breakdown,

$$I_L = \frac{V_Z}{R_L} = \frac{6}{4000} = 1.5 \times 10^{-3} A$$

When unregulated supply is of 16 V, potential drop occurring across series resistance ($R_S = 2k\Omega$) is;

$$V_S = V - V_Z = 16 - 6 = 10V$$

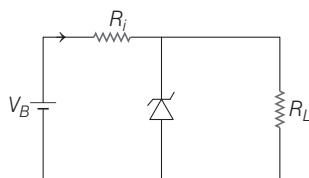
So, current across series resistance is

$$I_S = \frac{V_S}{R_S} = \frac{10}{2 \times 10^3} = 5 \times 10^{-3} A$$

So, current across Zener diode is

$$I_Z = I_S - I_L = 5 \times 10^{-3} - 1.5 \times 10^{-3} = 3.5 \times 10^{-3} A = 3.5 mA$$

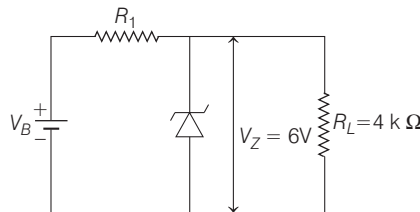
- 32** The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is $R_L = 4k\Omega$. The series resistance of the circuit is $R_i = 1k\Omega$. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode?
[2019, 10 April Shift-II]



- (a) 1.5 mA, 8.5 mA (b) 1 mA, 8.5 mA
(c) 0.5 mA, 8.5 mA (d) 0.5 mA, 6 mA

Ans. (c)

In given voltage regulator circuit,



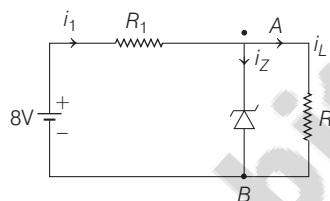
Zener breakdown voltage, $V_Z = 6V$

So, across R_L , potential drop is always 6 V.

So, current through load resistance is

$$I_L = \frac{V_Z}{R_L} = \frac{6}{4 \times 10^3} = 1.5 \times 10^{-3} A$$

Now, when $V_B = 8V$



Potential drop across $R_1 = 8 - 6 = 2V$

$$\text{So, current through } R_1 \text{ is } i_1 = \frac{V}{R_1} = \frac{2}{1 \times 10^3} = 2 \times 10^{-3} A$$

So, current through Zener diode is

$$i_Z = i_1 - I_L = 2 \times 10^{-3} - 1.5 \times 10^{-3} = 0.5 \times 10^{-3} A = 0.5 mA$$

Similarly, when $V_B = 16V$

$$V_{R_1} = 16 - 6 = 10V$$

$$\therefore i_1 = \frac{10}{1 \times 10^3} = 10 \times 10^{-3} A$$

Hence,

$$i_Z = i_1 - I_L = 10 \times 10^{-3} - 1.5 \times 10^{-3} = 8.5 \times 10^{-3} A = 8.5 mA$$

- 33** Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If for an n -type semiconductor, the density of electrons is $10^{19} m^{-3}$ and their mobility is $16 m^2 (V-s)$, then the resistivity of the semiconductor (since, it is an n -type semiconductor contribution of holes is ignored) is close to
[2019, 9 Jan Shift-I]

- (a) $2 \Omega\text{-m}$ (b) $0.2 \Omega\text{-m}$
(c) $0.4 \Omega\text{-m}$ (d) $4 \Omega\text{-m}$

Ans. (c)

Since, it is an n -type semiconductor and concentration of the holes has been ignored. So, its conductivity is given as $\sigma = n_e e \mu_e$

where, n_e is the number density of electron, e is the charge on electron and μ_e is its mobility.

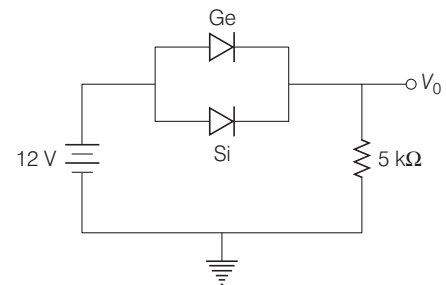
Substituting the given values, we get

$$\sigma = 10^{19} \times 1.6 \times 10^{-19} \times 1.6 = 2.56$$

$$\text{As, resistivity, } \rho = \frac{1}{\sigma} = \frac{1}{2.56}$$

$$\text{or } \rho = 0.39 \approx 0.4 \Omega\text{-m}$$

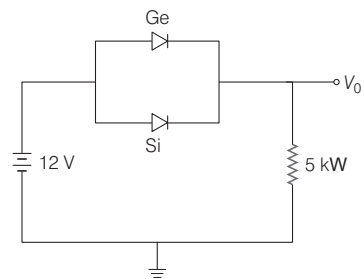
- 34** At 0.3 V and 0.7 V, the diodes Ge and Si become conductor respectively. In given figure, if ends of diode Ge overturned, the change in potential V_0 will be
[2019, 9 Jan Shift-II]



- (a) 0.2 V (b) 0.6 V
(c) 0.4 V (d) 0.8 V

Ans. (c)

Initially Ge and Si are both forward biased. So, current will effectively pass through Ge diode with a voltage drop of 0.3 V.

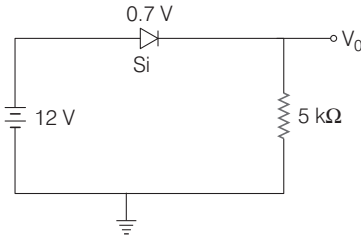


$\therefore$ Initial output voltage,

$$V_0 = 12 - 0.3 = 11.7 V$$

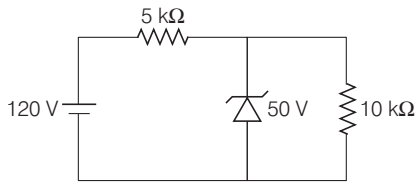
If Ge is reversed biased, then only Si diode will work. In this condition, output voltage

$$V_0 = 12 - 0.7 = 11.3 V$$



∴ Change in output voltage
 $= 11.7 - 11.3$
 $= 0.4 \text{ V}$

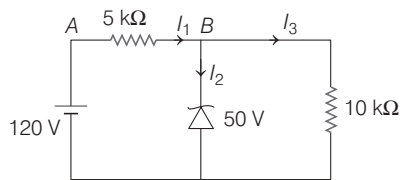
- 35** For the circuit shown below, the current through the Zener diode is
 [2019, 10 Jan Shift-II]



- (a) 14 mA
 (b) zero
 (c) 5 mA
 (d) 9 mA

Ans. (d)

In the circuit, let the current in branches is as shown in figure below



By Kirchhoff's node law,

$$I_1 = I_2 + I_3 \quad \dots (i)$$

Now, when diode conducts, voltage difference between points A and B will be

$$V_{AB} = 120 - 50 = 70 \text{ V}$$

So, current $I_1 = \frac{V_{AB}}{5 \text{ k}\Omega} = \frac{70}{5 \times 10^3}$

$$I_1 = 14 \text{ mA} \quad \dots (ii)$$

Since, diode and $10 \text{ k}\Omega$ resistor are in parallel combination, so voltage across $10 \text{ k}\Omega$ resistor will be 50 V only.

$$\Rightarrow I_3 = \frac{50}{10 \text{ k}\Omega} = \frac{50}{10 \times 10^3}$$

$$\Rightarrow I_3 = 5 \text{ mA} \quad \dots (iii)$$

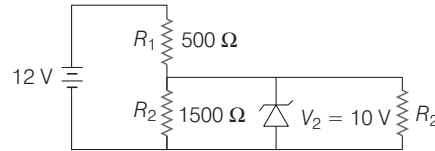
∴ From Eqs. (i), (ii) and (iii), we get

$$14 \text{ mA} = I_2 + 5 \text{ mA}$$

or current through diode,

$$I_2 = 14 \text{ mA} - 5 \text{ mA} \\ = 9 \text{ mA}$$

- 36** In the given circuit, the current through zener diode is close to



[2019, 11 Jan Shift-I]

- (a) 6.0 mA (b) 6.7 mA (c) 0 (d) 4.0 mA

Ans. (c)

Key Idea When the applied reverse voltage (V) reaches the breakdown voltage of the Zener diode, then only a large amount of current is flown through it, otherwise it is approximately zero.

In the given situation, if we consider that Zener diode is at breakdown. Then, potential drop across 1500Ω resistances will be 10 V. So potential drop at 500Ω resistor will be 2 V.

$$\therefore \text{Current in } R_1 = \frac{2}{500} = 4 \text{ mA} = I_1 \text{ (say)}$$

Current in each

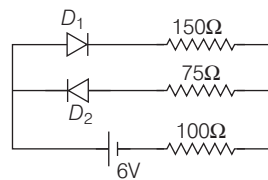
$$R_2 = \frac{10}{750} = \frac{2}{150} = 13.33 \text{ mA} = I_2 \text{ (say)}$$

$$\Rightarrow I_1 < I_2 \text{ which is not possible.}$$

So, Zener diode will never reach to its breakdown.

∴ Current flowing through a reverse biased Zener diode = 0.

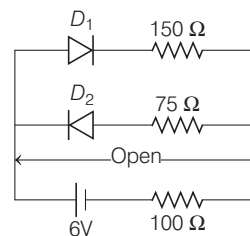
- 37** The circuit shown below contains two ideal diodes, each with a forward resistance of 50Ω . If the battery voltage is 6 V, the current through the 100Ω resistance (in ampere) is
 [2019, 11 Jan Shift-II]



- (a) 0.027 (b) 0.020 (c) 0.030 (d) 0.036

Ans. (b)

In this circuit, D_1 is forward biased and D_2 is reversed biased.



Resistance of D_1 is 50Ω .

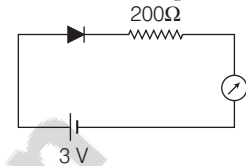
∴ Net resistance of the circuit,

$$R_{\text{net}} = 50 + 150 + 100 = 300 \Omega$$

∴ Current through the 100Ω resistance

$$= \frac{V}{R_{\text{net}}} = \frac{6}{300} = 0.020 \text{ A}$$

- 38** The reading of the ammeter for a silicon diode in the given circuit is
 [JEE Main 2018]



- (a) 0 (b) 15 mA
 (c) 11.5 mA (d) 13.5 mA

Ans. (c)

Key Idea Potential drop in a silicon diode in forward bias is around 0.7 V.

In given circuit, potential drop across 200Ω resistor is

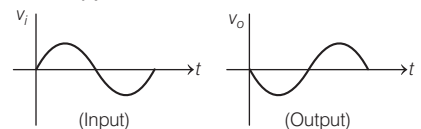
$$I = \Delta V_{\text{net}} / R = 3 - 0.7 / 200$$

$$\Rightarrow I = 0.0115 \text{ A} \Rightarrow I = 11.5 \text{ mA}$$

- 39** In a common emitter amplifier circuit using an $n-p-n$ transistor, the phase difference between the input and the output voltages will be
 [JEE Main 2017]

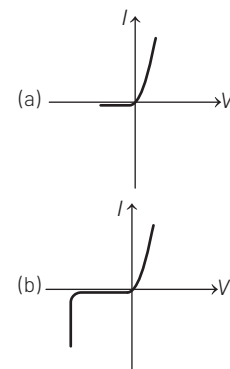
- (a) 90° (b) 135° (c) 180° (d) 45°

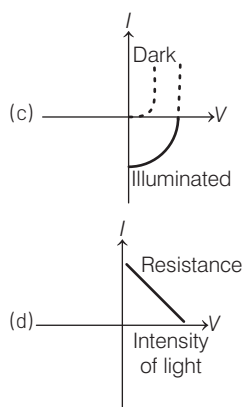
Ans. (c)



In a CE $n-p-n$ transistor amplifier output is 180° out of phase with input.

- 40** Identify the semiconductor devices whose characteristics are as given below, in the order (a), (b), (c), (d).
 [JEE Main 2016]





- (a) Simple diode, Zener diode, Solar cell, Light dependent resistance
 (b) Zener diode, Simple diode, Light dependent resistance, Solar cell
 (c) Solar cell, Light dependent resistance, Zener diode, Simple diode
 (d) Zener diode, Solar cell, Simple diode, Light dependent resistance

Ans. (a)

Zener diode works in breakdown region.

So, Simple diode $\rightarrow$ (a)

Zener diode $\rightarrow$ (b)

Solar cell $\rightarrow$ (c)

Light dependent resistance $\rightarrow$ (d)

- 41** A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

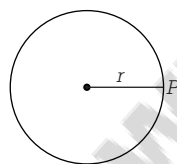
[JEE Main 2015]

- (a) 1.73 V/m (b) 2.45 V/m
 (c) 5.48 V/m (d) 7.75 V/m

Ans. (b)

Consider the LED as a point source of light.

Let power of the LED is P .



Intensity at r from the source

$$I = \frac{P}{4\pi r^2} \quad \dots(i)$$

$$\text{As we know that } I = \frac{1}{2} \epsilon_0 E_0^2 c \quad \dots(ii)$$

From Eqs. (i) and (ii), we can write

$$\frac{P}{4\pi r^2} = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\text{or } E_0^2 = \frac{2P}{4\pi \epsilon_0 r^2 c} = \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8}$$

$$\text{or } E_0^2 = 6 \Rightarrow E_0 = \sqrt{6} = 2.45 \text{ V/m}$$

- 42** The forward biased diode connection is

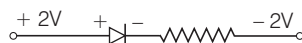
[JEE Main 2014]

- (a) $+2\text{ V}$ -2 V
 (b) -3 V -3 V
 (c) 2 V 4 V
 (d) -2 V $+2\text{ V}$

Ans. (a)

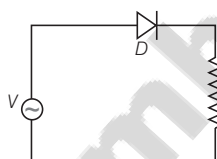
For forward bias, p -side must be a higher potential than n -side.

So,



is forward biased.

- 43** A p - n junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit.



[AIEEE 2009]

- (a)
 (b)
 (c)
 (d)

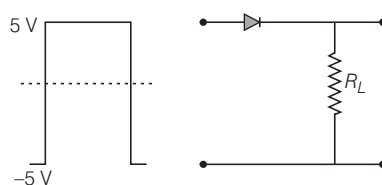
Ans. (c)

Given figure is half wave rectifier.

[since, diode allows the signal to pass during positive half only i.e., forward bias]

- 44** If in a p - n junction diode, a square input signal of 10 V is applied as shown.

[AIEEE 2007]



Then, the output signal across R_L will be

- (a)
 (b)
 (c)
 (d)

Ans. (d)

During -ve cycle, diode will not allow the signal to pass.

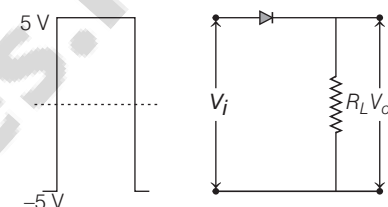


Fig. (i)

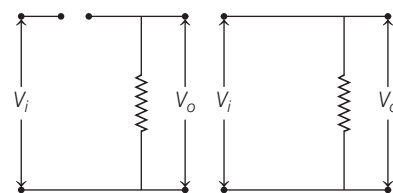


Fig. (ii)

Fig. (iii)

For $V_i < 0$, the diode is reverse biased and hence offer infinite resistance, so circuit would be like as shown in Fig. (ii) and $V_o = 0$.

For $V_i > 0$, the diode is forward biased and circuit would be as shown in Fig. (iii) and $V_o = V_i$.

Hence, the option (d) is correct.

- 45** Carbon, silicon and germanium have four valence electrons each. At room temperature, which one of the following statements is most appropriate?

[AIEEE 2007]

- (a) The number of free conduction electrons is significant in C but small in Si and Ge
 (b) The number of free conduction electrons is negligibly small in all the three
 (c) The number of free electrons for conduction is significant in all the three
 (d) The number of free electrons for conduction is significant only in Si and Ge but small in C

Ans. (d)

The number of free electrons for conduction is significant only in Si and Ge but small in C, as C is an impurity.

- 46** In a common-base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be [AIEEE 2006]

- (a) 49 (b) 50
(c) 51 (d) 48

Ans. (a)

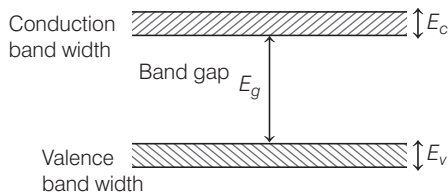
$$\beta = I_C / I_B$$

and $I_E = I_C + I_B$

$$\therefore \beta = \frac{I_C}{I_E - I_C}$$

$$= \frac{5.488}{5.60 - 5.488} = 49$$

- 47** If the lattice constant of this semiconductor is decreased, then which of the following is correct? [AIEEE 2006]



- (a) All E_c, E_g, E_v increase
(b) E_c and E_v increase but E_g decreases
(c) E_c and E_v decrease but E_g increases
(d) All E_c, E_g, E_v decrease

Ans. (c)

If lattice constant of semi-conductor is decreased, then E_c and E_v decrease but E_g increases.

- 48** If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities? [AIEEE 2006]

- (a) $\frac{5}{8}$ (b) $\frac{4}{5}$
(c) $\frac{5}{4}$ (d) $\frac{4}{7}$

Ans. (c)

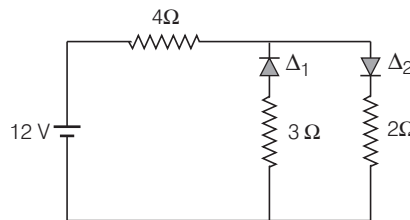
$$I = n_e A v_d \Rightarrow \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

$$\text{Here, } \frac{n_e}{n_h} = \frac{7}{5}, \frac{I_e}{I_h} = \frac{7}{4}$$

$$\therefore \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h}$$

$$\text{or } \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}$$

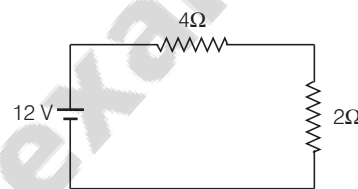
- 49** The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? [AIEEE 2006]



- (a) 1.71 A
(b) 2.00 A
(c) 2.31 A
(d) 1.33 A

Ans. (b)

In the given, circuit diode D_1 is reverse biased while D_2 is forward biased, so the circuit can be redrawn as



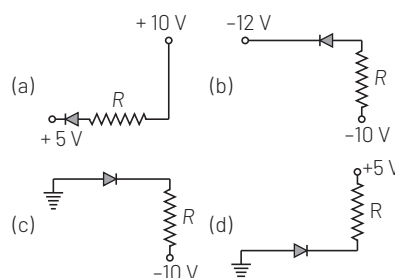
[$\therefore$ for ideal diodes, reverse biased means open and forward biased means short]

Apply KVL to get current following through the circuit

$$-12 + 4I + 2I = 0$$

$$\text{or } I = \frac{12}{6} = 2 \text{ A}$$

- 50** In the following, which one of the diodes is reverse biased? [AIEEE 2006]

**Ans. (d)**

For forward bias, p is $-ve$, n is $+ve$.



For reverse biasing of an ideal diode, the potential of n -side should be higher than potential of p -side. Only option (d) is satisfying the criterion for reverse biasing.

- 51** The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm, is incident on it. The band gap in (eV) for the semiconductor is [AIEEE 2005]

- (a) 1.1 eV (b) 2.5 eV
(c) 0.5 eV (d) 0.7 eV

Ans. (c)

$$E_g = h\nu$$

$$= \frac{hc}{\lambda} = \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2480 \times 10^{-9} \times 1.6 \times 10^{-19}} \right) \text{ eV}$$

$$[\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$= 0.5 \text{ eV}$$

- 52** In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be [AIEEE 2005]

- (a) 50 Hz (b) 25 Hz
(c) 100 Hz (d) 70.7 Hz

Ans. (c)

Given, $f = 50 \text{ Hz}$

$$\Rightarrow T = \frac{1}{50}$$

For full wave rectifier,

$$T_1 = \frac{T}{2} = \frac{1}{100}$$

$$\therefore f_1 = 100 \text{ Hz} \quad \left[\because f = \frac{1}{T} \right]$$

- 53** A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of [AIEEE 2004, 03]

- (a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases

Ans. (c)

We know that resistance of conductor is directly proportional to temperature (i.e., $R \propto \Delta t$), while resistance of semiconductor is inversely proportional to temperature (i.e., $R \propto \frac{1}{\Delta t}$).

Therefore, it is clear that resistance of conductor decreases with decrease in temperature or *vice-versa*, while in case of semi-conductor, resistance increases with decrease in temperature or *vice-versa*.

Since, copper is pure conductor and germanium is a semiconductor, hence due to decrease in temperature, resistance of conductor decreases while that of semi-conductor increases.

54 When p - n junction diode is forward biased, then

[AIEEE 2004]

- (a) the depletion region is reduced and barrier height is increased
- (b) the depletion region is widened and barrier height is reduced
- (c) both the depletion region and barrier height are reduced
- (d) both the depletion region and barrier height are increased

Ans. (c)

When p -end of p - n junction is connected to positive terminal of battery and n -end to negative terminal of battery, then p - n junction is said to be in forward bias.

In forward bias, the more numbers of electrons go from n -region to p -region and more numbers of holes go from p -region to n -region.

Therefore, major current due to both types of carriers takes place through the junction, causing more recombination of electron hole pairs, thus causing reduction in height of depletion region and barrier potential.

55 The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the

[AIEEE 2003]

- (a) crystal structure
- (b) variation of the number of charge carriers with temperature
- (c) type of bonding
- (d) variation of scattering mechanism with temperature

Ans. (b)

Metal has number of free electrons while semiconductor has not at room temperature.

The difference in the variation of resistance with temperature in metal and semiconductor is caused due to difference in the variation of the number of charge carriers with temperature.

56 In the middle of the depletion layer of reverse biased p - n junction, the

[AIEEE 2003]

- (a) electric field is zero
- (b) potential is maximum
- (c) electric field is maximum
- (d) potential is zero

Ans. (a)

Due to the reverse biasing, the width of depletion region increases and current flowing through the diode is almost zero. In this case, electric field is almost zero at the middle of the depletion region.

57 The energy band gap is maximum in

[AIEEE 2002]

- (a) metals
- (b) superconductors
- (c) insulators
- (d) semiconductors

Ans. (c)

Metals have minimum energy gap and insulators have maximum energy gap and semi-conductors have energy gap lying between insulators and metals.

58 By increasing the temperature, the specific resistance of a conductor and a semiconductor

[AIEEE 2002]

- (a) increases for both
- (b) decreases for both
- (c) increases, decreases respectively
- (d) decreases, increases respectively

Ans. (c)

Increases, decreases respectively.

If we increase the temperature, the specific resistance (resistivity) of conductor increases while that of semiconductor decreases.

59 At absolute zero, Si acts as

[AIEEE 2002]

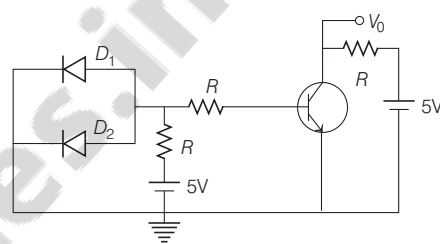
- (a) non-metal
- (b) metal
- (c) insulator
- (d) None of the above

Ans. (c)

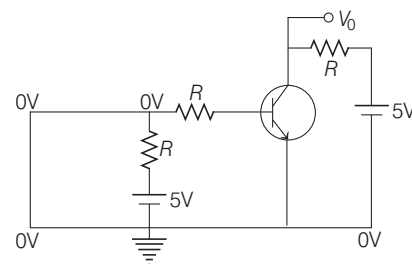
At absolute zero, the energy band is large (> 3 eV) and hence Si acts as insulator or in other words, we can say that there is no free charge carriers.

TOPIC 2 Transistors

60 A circuit is arranged as shown in figure. The output voltage V_0 is equal to V.

[2021, 27 Aug Shift-I]**Ans. (5)**

In the given circuit diagram shown in question, we can observe that diode D_1 and diode D_2 are in forward biasing. So, in forward biasing the diodes will offer zero resistance. Hence, given circuit diagram is redrawn as



The input voltage will become 0V and thus, the input current will be 0A.

The output current will also be zero.

Thus, the output voltage should be equal to 5 V.

61 For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then, the correct relation between α and β will be

[2021, 27 Aug Shift-II]

- (a) $\alpha = \frac{1-\beta}{\beta}$
- (b) $\beta = \frac{\alpha}{1-\alpha}$
- (c) $\alpha\beta = 1$
- (d) $\alpha = \frac{\beta}{1-\beta}$

Ans. (b)

Given, $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

where, α = current gain in common-base amplifier

and β = current gain in common-emitter amplifire.

Since, $I_E = I_C + I_B$

where, I_E = emitter current,

I_C = collector current.

and I_B = base current.

$$\therefore \alpha = \frac{I_C}{I_E} = \frac{I_C}{I_C + I_B} = \frac{1}{1 + \frac{I_B}{I_C}} = \frac{1}{1 + \frac{1}{\beta}}$$

$$\Rightarrow 1 + \frac{1}{\beta} = \frac{1}{\alpha}$$

$$\Rightarrow \frac{1}{\beta} = \frac{1}{\alpha} - 1$$

$$\Rightarrow \frac{1}{\beta} = \frac{1 - \alpha}{\alpha}$$

$$\therefore \beta = \frac{\alpha}{1 - \alpha}$$

- 62** A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is μA . (Round off to the nearest integer)

[2021, 27 July Shift-I]

Ans. (25)

Given, resistance across collector, $R_C = 1000 \Omega$

Voltage drop across collector resistance, $\Delta V = 0.6$

Current gain factor, $\beta = 24$

$\therefore$ We know that,

Current gain factor, $\beta = \frac{I_C}{I_B}$

$$\Rightarrow 24 = \frac{I_C}{I_B} \quad \dots(i)$$

$$\therefore \text{Collector current, } I_C = \frac{\Delta V}{R_C}$$

$$\Rightarrow I_C = \frac{0.6}{1000} = 6 \times 10^{-4} \quad \dots(ii)$$

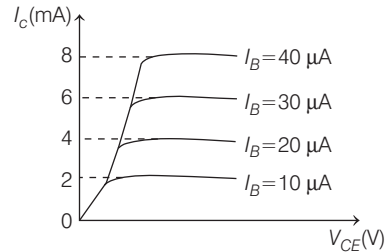
From Eqs. (i) and (ii), we get

$$\Rightarrow 24 = \frac{6 \times 10^{-4}}{I_B}$$

$$\Rightarrow I_B = \frac{6 \times 10^{-4}}{24}$$

$$\Rightarrow I_B = 25 \mu\text{A}$$

- 63** The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is

[2021, 18 March Shift-II]

Ans. (200)

For common emitter configuration,

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

$$\Rightarrow \beta = \frac{(4 - 2) \text{ mA}}{(20 - 10) \mu\text{A}} = 200$$

Hence, the estimated current gain is 200.

- 64** The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is

[2021, 18 March Shift-II]

(a) $\beta = \frac{\alpha}{1 + \alpha}$ (b) $\alpha = \frac{\beta}{1 - \alpha}$

(c) $\beta = \frac{1}{1 - \alpha}$ (d) $\alpha = \frac{\beta}{1 + \beta}$

Ans. (d)

Current gain in common base,

$$\alpha = \frac{\Delta I_C}{\Delta I_E}$$

Current gain in common emitter,

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

Here, ΔI_C = change in collector current,

ΔI_E = change in emitter current

and ΔI_B = change in base current.

We know that,

$$\Delta I_E = \Delta I_B + \Delta I_C$$

Divide by the ΔI_C on both sides, we get

$$\frac{\Delta I_E}{\Delta I_C} = \frac{\Delta I_B}{\Delta I_C} + \frac{\Delta I_C}{\Delta I_C}$$

$$\Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1 \Rightarrow \frac{1}{\alpha} = \frac{\beta + 1}{\beta}$$

$$\Rightarrow \alpha = \frac{\beta}{\beta + 1}$$

- 65** An n - p - n transistor operates as a common emitter amplifier with a power gain of 10^6 . The input circuit resistance is 100 Ω and the output load resistance is 10 k Ω . The common emitter current gain β will be (Round off to the nearest integer)

[2021, 18 March Shift-I]

Ans. (100)

Given,

The power gain of an n - p - n transistor,

$$P = 10^6$$

The input circuit of the resistance,

$$r = 100 \Omega$$

The output load resistance of the circuit,

$$R = 10 \text{ k}\Omega = 10000 \Omega$$

We know that,

Power gain of common emitter amplifier,

$$P = \frac{\beta^2 R}{r}$$

$$\text{Here, } 10^6 = \frac{\beta^2 \times 10000}{100}$$

$$\Rightarrow \beta = 100$$

- 66** Given below are two statements:

Statement I p - n junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

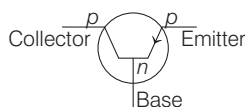
[2021, 24 Feb Shift-II]

- (a) Statement I is false but Statement II is true.
 (b) Both Statement I and Statement II are true.
 (c) Both Statement I and Statement II are false.
 (d) Statement I is true but Statement II is false.

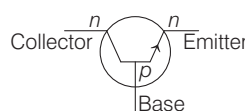
Ans. (a)

Transistor is a device used as switch or amplifier.

It is made by sandwiching three semiconductors, i.e. p - n - p and n - p - n .



p-n-p-transistor



n-p-n-transistor

Hence, Statement I is false, because we cannot make transistor from diode. As we know that, amplification factor (β) is the ratio of collector current to base current.

$$\therefore \beta = \frac{I_C}{I_B}$$

Hence, Statement II is true.

Therefore, option (a) is correct.

- 67** If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be [2021, 24 Feb Shift-I]

- (a) 7 (b) 0.875
(c) 0.5 (d) 3.5

Ans. (a)

Given, emitter current, $I_E = 4$ mA

Collector current, $I_C = 3.5$ mA

Current gain in common base amplifier,

$$\alpha = \frac{I_C}{I_E}$$

$$\Rightarrow \alpha = \frac{3.5}{4} = \frac{7}{8}$$

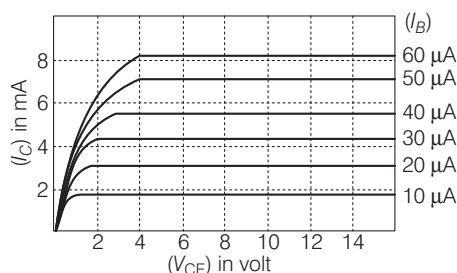
Also, current gain in common emitter amplifier,

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow \beta = \frac{7/8}{1-7/8}$$

$$\beta = 7$$

- 68** The output characteristics of a transistor is shown in the figure. When V_{CE} is 10V and $I_C = 4.0$ mA, then value of β_{ac} is [2020, 6 Sep Shift-II]



Ans. (150)

For a transistor, $\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE} = \text{constant}}$

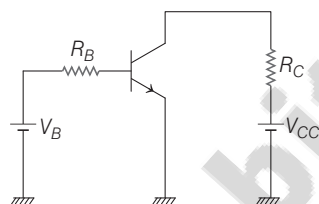
From given figure,

$$\Delta I_B = 30 - 20 = 10 \mu\text{A};$$

$$\Delta I_C = 4.5 - 3 = 1.5 \text{ mA}$$

$$\therefore \beta = \frac{1.5 \times 10^{-3}}{10 \times 10^{-6}} = 150$$

- 69** A common emitter amplifier circuit, built using an *n-p-n* transistor, is shown in the figure. Its DC current gain is 250, $R_C = 1 \text{ k}\Omega$ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation? [2019, 8 April Shift-II]



- (a) $40 \mu\text{A}$ (b) $10 \mu\text{A}$
(c) $100 \mu\text{A}$ (d) $7 \mu\text{A}$

Ans. (a)

For a common emitter *n-p-n* transistor, DC current gain is

$$\beta_{DC} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero.

$$\therefore V_{CC} - I_C R_C = 0$$

$$I_C \approx \frac{V_{CC}}{R_C} = \frac{10}{1000} = 10^{-2} \text{ A}$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{10^{-2}}{250} = 40 \mu\text{A}$$

- 70** An *n-p-n* transistor is used in common emitter configuration as an amplifier with $1 \text{ k}\Omega$ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and $15 \mu\text{A}$ change in the base current of the amplifier. The input resistance and voltage gain are [2019, 9 April Shift-I]

- (a) $0.67 \text{ k}\Omega$, 200 (b) $0.33 \text{ k}\Omega$, 1.5
(c) $0.67 \text{ k}\Omega$, 300 (d) $0.33 \text{ k}\Omega$, 300

Ans. (c)

Given, load resistance, $R_L = 1 \text{ k}\Omega$

Input voltage, $V_{in} = 10 \text{ mV} = 10 \times 10^{-3} \text{ V}$

Base current, $\Delta I_B = 15 \mu\text{A} = 15 \times 10^{-6} \text{ A}$

Collector current, $\Delta I_C = 3 \text{ mA}$

Input resistance,

$$R_{in} = \frac{V_{in}}{\Delta I_B} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67 \text{ k}\Omega$$

and voltage gain $= \beta \times \frac{R_L}{R_{in}}$

$$= \frac{\Delta I_C \times R_L}{\Delta I_B \times R_{in}} = \left(\frac{3 \text{ mA}}{15 \mu\text{A}} \right) \times \left(\frac{1 \text{ k}\Omega}{0.67 \text{ k}\Omega} \right)$$

$$= \left(\frac{3 \times 10^{-3}}{15 \times 10^{-6}} \right) \left(\frac{1 \times 10^3}{0.67 \times 10^3} \right)$$

$$= \frac{1000 \times 3 \times 3}{15 \times 2} = 300 \quad (\because 0.67 \approx 2/3)$$

Alternate Solution

$$\therefore \text{Voltage gain} = \frac{V_{\text{output}}}{V_{\text{input}}} = \frac{R_L \times \Delta I_C}{V_{in}}$$

$$= \frac{1 \times 10^3 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

- 71** An *n-p-n* transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is $10 \text{ k}\Omega$. The common emitter current gain β is [2019, 10 April Shift-I]

- (a) 10^2 (b) 6×10^2 (c) 10^4 (d) 60

Ans. (a)

Given, $A_P = 60$ dB (in decibel)

Power gain in decibel can be given as

$$A_P = 10 \log_{10} \left(\frac{\text{Output power}}{\text{Input power}} \right)$$

$$\Rightarrow 60 = 10 \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right) \Rightarrow \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right) = 6$$

$$\Rightarrow \frac{P_{\text{out}}}{P_{\text{in}}} = 10^6 = A_P \quad \dots (i)$$

Also, given $R_{\text{out}} = 10 \text{ k}\Omega$, $R_{\text{in}} = 100 \Omega$

$\therefore$ Power gain of a transistor is given by

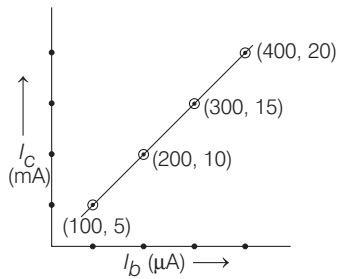
$$A_P = \beta^2 \left(\frac{R_{\text{out}}}{R_{\text{in}}} \right)$$

where, β is current gain.

$$\Rightarrow \beta^2 = A_P \times \frac{R_{\text{in}}}{R_{\text{out}}} = 10^6 \times \frac{100}{10 \times 10^3}$$

$$\Rightarrow \beta^2 = 10^4 \text{ or } \beta = 10^2$$

- 72** The transfer characteristic curve of a transistor, having input and output resistance 100Ω and $100 \text{ k}\Omega$ respectively, is shown in the figure. The voltage and power gain, are respectively [2019, 12 April Shift-I]



- (a) $2.5 \times 10^4, 2.5 \times 10^6$
 (b) $5 \times 10^4, 5 \times 10^6$
 (c) $5 \times 10^4, 5 \times 10^5$
 (d) $5 \times 10^4, 2.5 \times 10^6$

Ans. (d)

Given curve is between I_C and I_B as output and input currents, respectively. So, it is transfer characteristics curve of a common emitter (CE) configuration.

In CE configuration,

$$\text{Current gain, } \beta = \frac{I_{out}}{I_{in}} = \frac{I_C}{I_B} \quad \dots(i)$$

Voltage gain,

$$A_V = \frac{V_{out}}{V_{in}} = \frac{I_C \times R_{out}}{I_B \times R_{in}} = \beta \times \frac{R_{out}}{R_{in}} \quad \dots(ii)$$

and power gain,

$$A_P = \frac{P_{out}}{P_{in}} = \frac{I_C^2 \times R_{out}}{I_B^2 \times R_{in}} = \beta^2 \times \frac{R_{out}}{R_{in}} \quad \dots(iii)$$

Given, $R_{in} = 100 \Omega$ and $R_{out} = 100 \times 10^3 \Omega$

From Eq. (i), we get

$$\beta = \frac{5 \text{ mA}}{100 \mu\text{A}} \left(\text{or } \frac{10 \text{ mA}}{200 \mu\text{A}} \text{ or } \frac{15 \text{ mA}}{300 \mu\text{A}} \text{ or } \frac{20 \text{ mA}}{400 \mu\text{A}} \right)$$

$$\Rightarrow \beta = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} = 50 \quad \dots(iv)$$

From Eqs. (ii) and (iv), we get

$$\text{Voltage gain, } A_V = \beta \times \frac{R_{out}}{R_{in}} = 50 \times \frac{100 \times 10^3}{100} = 50000 = 5 \times 10^4 \quad \dots(v)$$

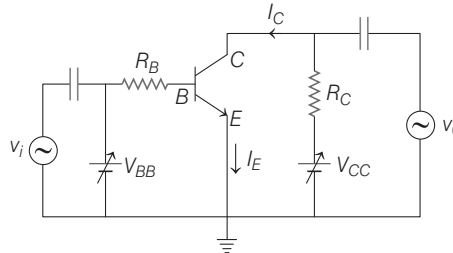
From Eqs. (iii) and (iv), we get

$$\begin{aligned} \text{Power gain, } A_P &= \beta^2 \times \frac{R_{out}}{R_{in}} \\ &= (50)^2 \times \frac{100 \times 10^3}{100} \\ &= 2500 \times 1000 \\ \Rightarrow A_P &= 2.5 \times 10^6 \end{aligned}$$

- 73** In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5 \text{ V}$, $\beta_{DC} = 200$, $R_B = 100 \text{ k}\Omega$, $R_C = 1 \text{ k}\Omega$ and $V_{BE} = 1.0 \text{ V}$. The minimum base current and the input voltage at

which the transistor will go to saturation, will be, respectively

[2019, 12 Jan Shift-II]

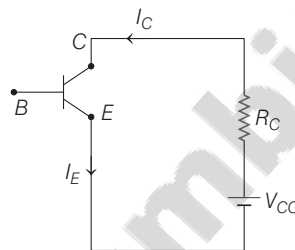


- (a) $25 \mu\text{A}$ and 2.8 V (b) $25 \mu\text{A}$ and 3.5 V
 (c) $20 \mu\text{A}$ and 3.5 V (d) $20 \mu\text{A}$ and 2.8 V

Ans. (b)

Transistor saturation occurs when $V_{CE} = 0$.

Now, for closed loop of collector and emitter by Kirchhoff's voltage rule, we have



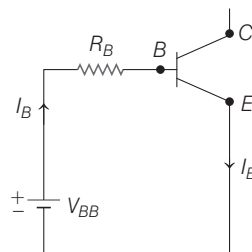
$$\begin{aligned} V_{CE} &= V_{CC} - I_C R_C \Rightarrow 0 = V_{CC} - I_C R_C \\ \Rightarrow I_C &= \frac{V_{CC}}{R_C} = \frac{5}{1 \times 10^3} = 5 \times 10^{-3} \text{ A} \end{aligned}$$

Now, $\beta_{DC} = 200$ (given)

$$\Rightarrow \frac{I_C}{I_B} = \beta_{DC} = 200 \Rightarrow I_B = \frac{I_C}{200} = \frac{5 \times 10^{-3}}{200}$$

$$\Rightarrow I_B = 2.5 \times 10^{-5} = 25 \mu\text{A}$$

Now, we apply Kirchhoff's voltage rule in base-emitter closed loop, we get



$$\Rightarrow V_{BB} = (25 \times 10^{-6} \times 100 \times 1000) + 1 = 3.5 \text{ V}$$

- 74** For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is [JEE Main 2016]

$$\begin{aligned} (a) \frac{1}{\alpha} &= \frac{1}{\beta} + 1 & (b) \alpha &= \frac{\beta}{1-\beta} \\ (c) \alpha &= \frac{\beta}{1+\beta} & (d) \alpha &= \frac{\beta^2}{1+\beta^2} \end{aligned}$$

Ans. (a, c)

As, we know, In case of a common-emitter configuration, DC current gain, $\alpha = \frac{I_C}{I_E}$.

Where, I_C is collector current and I_E is emitter current

$$\text{and AC current gain, } \beta = \frac{I_C}{I_B}$$

where, I_B is base current.

Also, $I_E = I_B + I_C$

Dividing whole equation by I_C , we get

$$\Rightarrow \frac{I_E}{I_C} = \frac{I_B}{I_C} + 1 \Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\Rightarrow \alpha = \frac{\beta}{1+\beta}$$

- 75** In a common base amplifier, the phase difference between the input signal voltage and output voltage is [AIEEE 2005]

- (a) $\pi/4$ (b) π (c) zero (d) $\pi/2$

Ans. (c)

In common base amplifier, the input signal is amplified but remain in phase with output signal.

- 76** When n - p - n transistor is used as an amplifier, [AIEEE 2004]

- (a) electrons move from base to collector
 (b) holes move from emitter to base
 (c) electrons move from collector to base
 (d) holes move from base to emitter

Ans. (d)

When forward bias is applied on n - p - n transistor, then it works as an amplifier. In forward biased n - p - n transistor, electrons move from emitter to base and holes move from base to emitter.

- 77** For a transistor amplifier in common emitter configuration for load impedance of $1 \text{ k}\Omega$ ($h_{fe} = 50$ and $h_{oe} = 25 \mu\text{A/V}$), the current gain is [AIEEE 2004]

- (a) - 5.2 (b) - 15.7
 (c) - 24.8 (d) - 48.78

Ans. (d)

For a transistor amplifier in common emitter configuration, current gain

$$A_i = -\frac{h_{fe}}{1 + h_{oe} R_L}$$

where, h_{fe} and h_{oe} are hybrid parameters of a transistor.

$$\therefore A_i = -\frac{50}{1 + 25 \times 10^{-6} \times 1 \times 10^3} = -48.78$$

78 The part of a transistor which is most heavily doped to produce large number of majority carriers is [AIEEE 2002]

- (a) emitter
- (b) base
- (c) collector
- (d) can be any of the above three

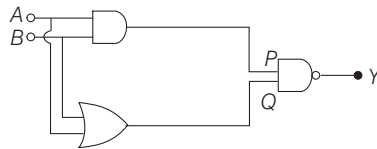
Ans. (a)

Because emitter has largest size and maximum number of charge carriers.

TOPIC 3

Digital Circuits

79 In the following logic circuit, the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be [2021, 31 Aug Shift-I]



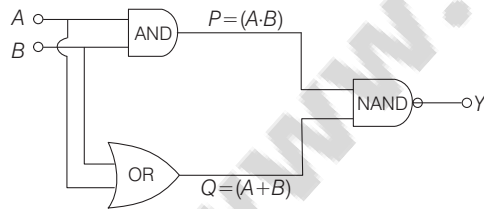
- (a) 1, 0, 1, 0
- (b) 0, 1, 0, 1
- (c) 1, 1, 1, 0
- (d) 0, 0, 1, 1

Ans. (c)

Given, logic inputs are given as

$(A, B) = (0, 0), (0, 1), (1, 0) \text{ and } (1, 1)$

And the logic circuit is drawn as



Output of the given logic circuit is given as

$$Y = \overline{AB \cdot (A + B)} = \overline{AB} + \overline{(A + B)}$$

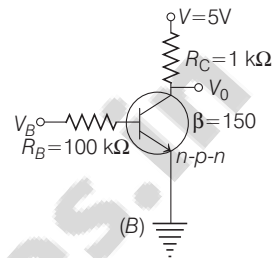
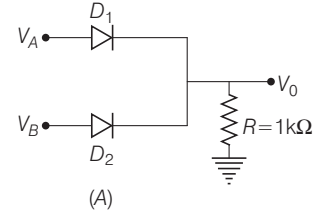
[using de-Morgan's theorem]

$$= (\overline{A} + \overline{B}) + \overline{A} \cdot \overline{B} = \overline{A} + \overline{B} + \overline{A}\overline{B}$$

Now, by using truth table for above logic circuit

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\overline{B}$	P	Q	Y
0	0	1	1	1	0	0	1
0	1	1	0	0	0	1	1
1	0	0	1	0	0	1	1
1	1	0	0	0	1	1	0

80 If V_A and V_B are the input voltages (either 5V or 0V) and V_0 is the output voltage then the two gates represented in the following circuit (A) and (B) are [2021, 31 Aug Shift-II]



- (a) AND and OR gate
- (b) OR and NOT gate
- (c) NAND and NOR gate
- (d) AND and NOT gate

Ans. (b)

In figure A, V_A and V_B are two input voltages (either 5 V or 0 V)

When $V_A = 0, V_B = 0$

Both D_1 and D_2 are OFF, so $V_0 = 0$,

When $V_A = 1$ i.e. 5 V, $V_B = 0$ i.e. 0 V

D_1 is ON but D_2 is OFF

So, $V_0 = 1$ i.e. 5 V

When $V_A = 0$ i.e. 0 V, $V_B = 1$ i.e. 5 V

D_1 is OFF and D_2 is ON

So, $V_0 = 1$ i.e. 5 V.

Similarly, when $V_A = V_B = 1$

Both D_1 and D_2 are ON

So, $V_0 = 1$ i.e. 5 V.

We get the final truth table on the basis of inputs and their corresponding output.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

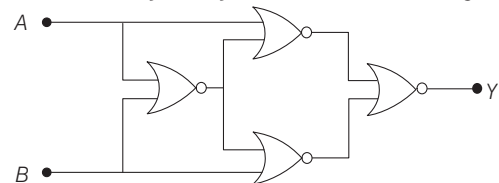
$\therefore$ The given circuit (A) is an OR gate.

In figure B, it is a transistor in common emitter configuration which gives high output at low input and low output at high input.

Hence, this circuit behaves as NOT Gate.

81 Four NOR gates are connected as shown in figure.

The truth table for the given figure is [2021, 26 Aug Shift-II]

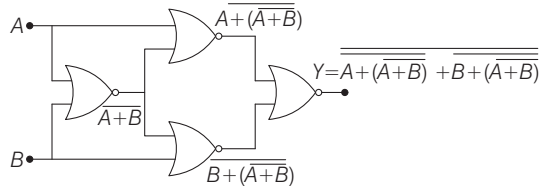


(a)			(b)		
A	B	Y	A	B	Y
0	0	1	0	0	0
0	1	0	0	1	1
1	0	1	1	0	1
1	1	0	1	1	0

(c)			(d)		
A	B	Y	A	B	Y
0	0	0	0	0	1
0	1	1	0	1	0
1	0	0	1	0	0
1	1	1	1	1	1

Ans. (d)

Given, logic circuit can be drawn as shown below.



The expression for output is

$$Y = A + (A + B) + B + (A + B) = A + (A + B) \cdot B + (A + B)$$

(Using de-Morgan's theorem,

$$\overline{x+y} = \overline{x} \cdot \overline{y}$$

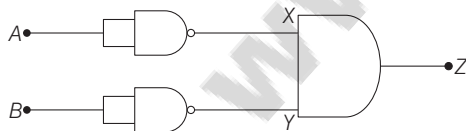
$$Y = (A + A + B) \cdot (B + A + B)$$

Draw the truth table for the logic circuit is shown below

A	B	A+B	$\overline{A+B}$	A+(A+B)	B+(A+B)	Y
0	0	0	1	1	1	1
0	1	1	0	0	1	0
1	0	1	0	1	0	0
1	1	1	0	1	1	1

82 Identify the logic operation carried out by the given circuit.

[2021, 26 Aug Shift-I]

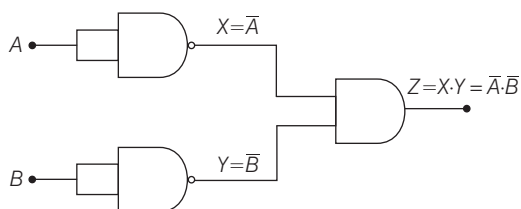


- (a) OR
(c) NOR

- (b) AND
(d) NAND

Ans. (c)

The given circuit can be drawn as



$$\begin{aligned} \text{The output, } Z &= \overline{A} \cdot \overline{B} \\ &= \overline{A + B} \end{aligned}$$

(Using de-Morgan theorem)

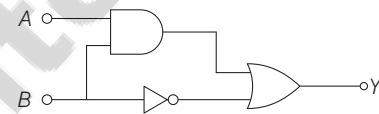
This is the expression of a NOR gate. So, the logic operation carried out by the given circuit is NOR.

Alternate Method

The given question can be solved using truth table, which is as given below

A	B	X	Y	Z
0	0	1	1	1
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0

This truth table is similar to that of a NOR gate.

83 Find the truth table for the function Y of A and B represented in the following figure. [2021, 27 July Shift-II]

(a)	A	B	Y
	0	0	0
	0	1	1
	1	0	0
	1	1	0

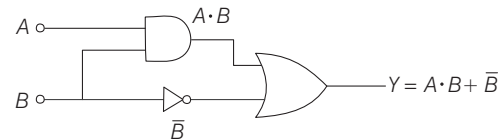
(b)	A	B	Y
	0	0	1
	0	1	0
	1	0	1
	1	1	1

(c)	A	B	Y
	0	0	0
	0	1	0
	1	0	0
	1	1	1

(d)	A	B	Y
	0	0	0
	0	1	1
	1	0	1
	1	1	1

Ans. (b)

The given logic diagram can be shown as



∴ The Boolean expression for the above figure is

$$Y = A \cdot B + \overline{B}$$

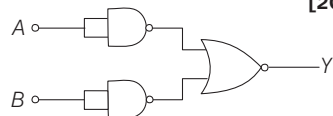
Now, let us draw the truth table for this expression as follows

A	B	A·B	$\overline{B}$	$Y = A \cdot B + \overline{B}$
0	0	0	1	1
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

On comparing the above truth table with that given in option (b), we find that both are same.

84 Identify the logic operation carried out.

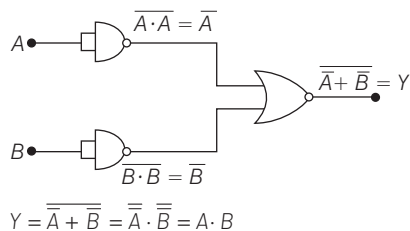
[2021, 25 July Shift-I]



- (a) OR
(b) AND
(c) NOR
(d) NAND

Ans. (b)

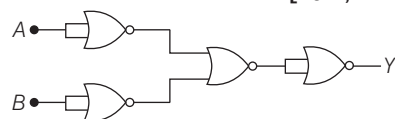
By using de-Morgan's theorem,



So, the given circuit perform AND operation.

85 The output of the given combination gates represents

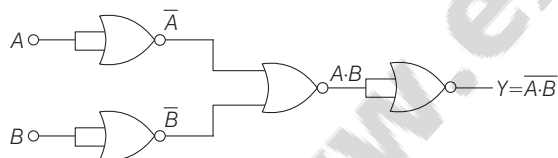
[2021, 17 March Shift-I]



- (a) XOR gate
(b) NAND gate
(c) AND gate
(d) NOR gate

Ans. (b)

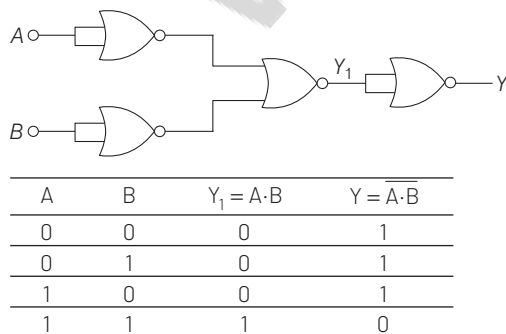
We can find the output of given combination of gates using Boolean algebra. Consider the given figure



The output $Y = \overline{A} \cdot \overline{B}$ which is the expression for NAND gate.

Alternate method

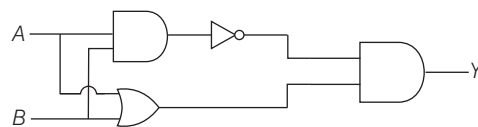
We can find the output of given combination of gates using truth table. Consider the given figure



The above truth table shows the output of NAND gate.

86 Which one of the following will be the output of the given circuit ?

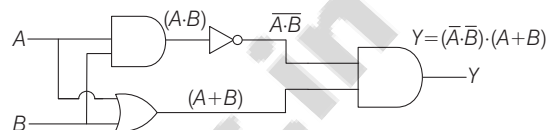
[2021, 17 March Shift-II]



- (a) NOR Gate
(b) NAND Gate
(c) AND Gate
(d) XOR Gate

Ans. (d)

The given circuit, with the output of the respective gates is as given below.



The Boolean expression of the output is

$$Y = (\overline{A \cdot B}) \cdot (A + B) \Rightarrow Y = (\overline{A} + \overline{B}) \cdot (A + B)$$

(using de-Morgan's theorem, $\overline{x \cdot y} = \overline{x} + \overline{y}$)

$$\Rightarrow Y = \overline{A} \cdot B + \overline{B} \cdot A$$

This represents the Boolean expression for XOR gate.

Alternate solution

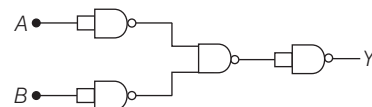
This question can also be verified from the following truth table.

A	B	$P = \overline{A \cdot B}$	$Q = A + B$	$Y = P \cdot Q$
0	0	1	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	0

Hence, the output of this circuit represents the output of a XOR gate.

87 The following logic gate is equivalent to

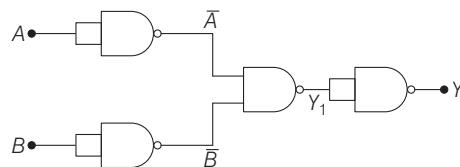
[2021, 16 March Shift-II]



- (a) NOR Gate
(b) OR Gate
(c) AND Gate
(d) NAND Gate

Ans. (a)

Let us consider the given logic circuit and solve it using Boolean algebra.

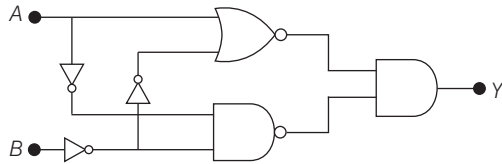


$$\text{Here, } Y_1 = \overline{A} \cdot \overline{B} = \overline{A + B}$$

$$\therefore Y = \overline{Y_1} = A + B$$

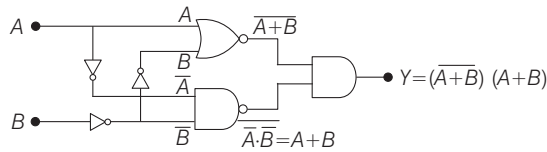
which is an expression for NOR Gate.

- 88** In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be x . The value of x is
[2021, 16 March Shift-I]



Ans. (0)

Consider the figure given in the question and solve it using Boolean identities.

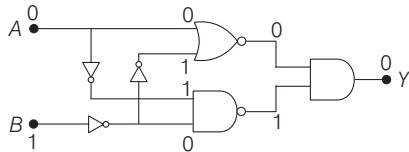


Now, put the value of A and B in the output, we get

$$Y = (\overline{0} + 1)(0 + 1) \Rightarrow Y = \overline{1} \cdot 1 = 0$$

Alternate method

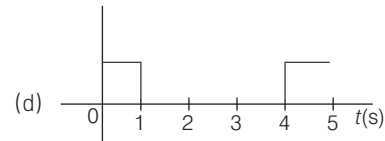
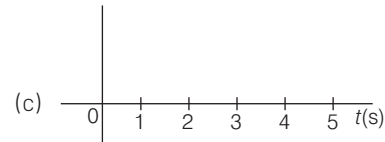
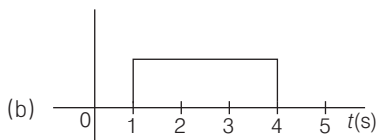
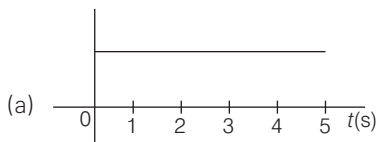
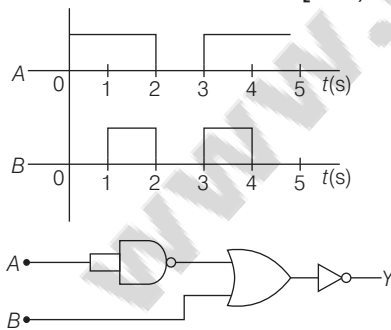
We can directly put the given values in the logic circuit given in the question and can find its output. Let us consider the given logic circuit,



The output is $Y = 0 \cdot 1 = 0$

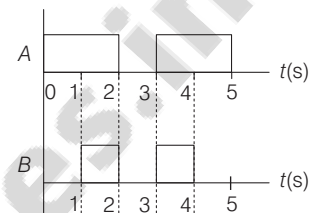
$\therefore$ The value of x is 0.

- 89** Draw the output signal Y in the given combination of gates
[2021, 26 Feb Shift-II]

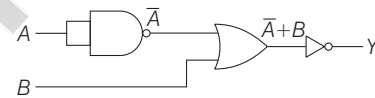


Ans. (d)

According to the given figure signals are



The output of given circuit diagram is



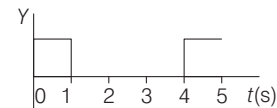
By using concept of de-Morgan's law,

$$Y = \overline{A + B} = A \cdot B$$

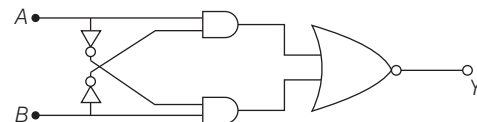
Truth table of given signals is

Time interval	A	B	$\overline{A}$	$\overline{B}$
(0-1)s	1	0	0	1
(1-2)s	1	1	0	0
(2-3)s	0	0	1	1
(3-4)s	1	1	0	0
(4-5)s	1	0	0	1

Hence, output signal is,



- 90** The truth table for the following logic circuit is
[2021, 25 Feb Shift-II]



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(c)

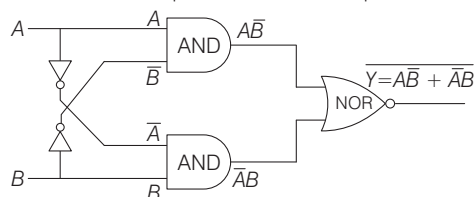
A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(d)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Ans. (b)

Here A and B be the input and Y be the output.



$$\therefore Y = \overline{A\bar{B} + \bar{A}B} = \overline{A\bar{B}} \cdot \overline{\bar{A}B} = (\bar{A} + B) \cdot (A + \bar{B})$$

$$= (\bar{A} + B)(A + \bar{B}) = \bar{A}A + \bar{A}\bar{B} + AB + B\bar{B}$$

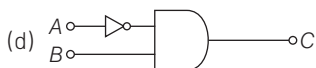
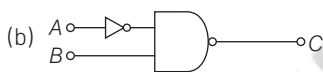
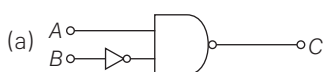
$$= 0 + \bar{A}\bar{B} + AB + 0 = AB + \bar{A}\bar{B}$$

According to the truth table

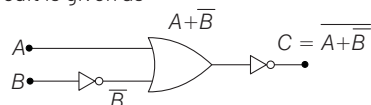
A	B	$\bar{A}$	$\bar{B}$	AB	$\bar{A}\bar{B}$	$Y = AB + \bar{A}\bar{B}$
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

91

The logic circuit shown above is equivalent to
[2021, 24 Feb Shift-II]

**Ans. (d)**

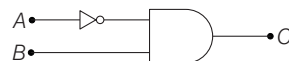
The logic circuit is given as



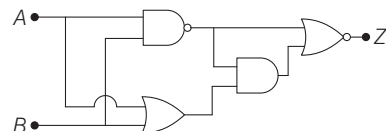
By using De-morgan's theorem,

$$C = \overline{A + \bar{B}} = \bar{A} \cdot \bar{\bar{B}} = \bar{A} \cdot B$$

This relation can be shown by the circuit drawn below



92 In the following digital circuit, what will be the output at Z, when the input (A,B) are (1, 0), (0, 0), (1, 1), (0, 1)?
[2020, 2 Sep Shift-II]



(a) 0, 0, 1, 0

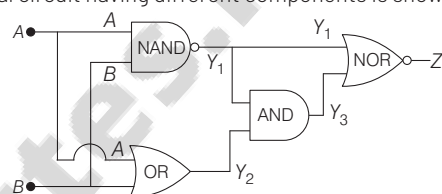
(b) 1, 0, 1, 1

(c) 1, 1, 0, 1

(d) 0, 1, 0, 0

Ans. (a)

The digital circuit having different components is shown below,

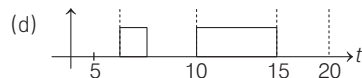
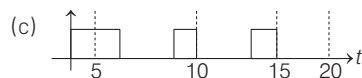
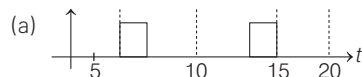
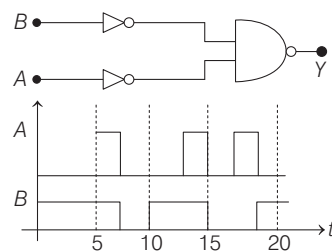


Truth table of above circuit is

A	B	$Y_1 = \overline{A \cdot B}$	$Y_2 = A + B$	$Y_3 = Y_1 \cdot Y_2$	$Z = \overline{Y_1 + Y_3}$
1	0	1	1	1	0
0	0	1	0	0	0
1	1	0	1	0	1
0	1	1	1	1	0

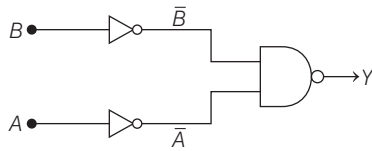
Hence, correct option is (a).

93 Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.
[2020, 6 Sep Shift-I]



Ans. (b)

Given circuit,

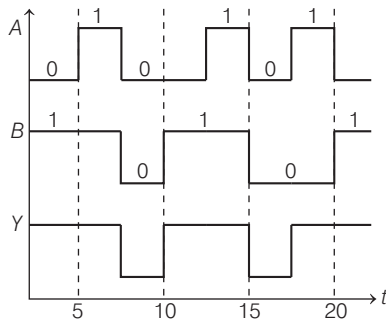


Output,

$$Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} \\ = A + B$$

Truth Table

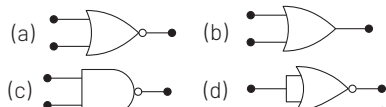
A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1



So, most appropriate option is (b).

94 Which of the following gives a reversible operation?

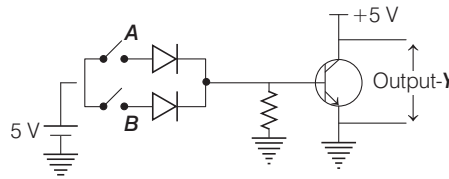
[2020, 7 Jan Shift-I]

**Ans. (d)**

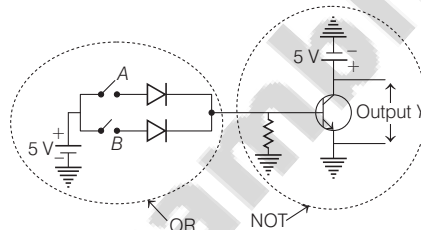
Given gates and their truth tables are as below.

1.		NOR gate	<table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	Y	0	0	1	1	0	0	0	1	0	1	1	0
A	B	Y																
0	0	1																
1	0	0																
0	1	0																
1	1	0																
2.		NAND gate	<table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	Y	0	0	1	1	0	1	0	1	1	1	1	0
A	B	Y																
0	0	1																
1	0	1																
0	1	1																
1	1	0																
3.		OR gate	<table><tr><th>A</th><th>B</th><th>Y</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	Y	0	0	0	1	0	1	0	1	1	1	1	1
A	B	Y																
0	0	0																
1	0	1																
0	1	1																
1	1	1																
4.		One input NOR gate	<table><tr><th>X</th><th>Y</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	X	Y	0	1	1	0									
X	Y																	
0	1																	
1	0																	

Clearly, 4th gate converts a low voltage (0) into a high voltage (1) and vice-versa. So, 4th gate or one input NOR gate gives a reversible operation.

95 Boolean relation at the output stage Y for the following circuit is [2020, 8 Jan Shift-I](a) $A \cdot B$ (b) $A + B$ (c) $\overline{A} + \overline{B}$ (d) $\overline{A} \cdot \overline{B}$ **Ans. (d)**

Given circuit is a combination of OR gate and NOT gate.



So, given combination is a NOR gate, its truth table is

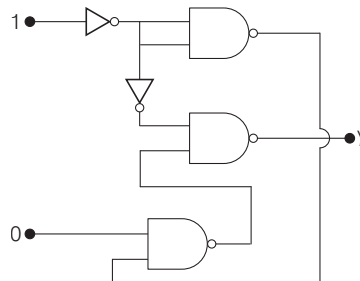
A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

Its boolean relation,

$$Y = \overline{A + B} = \overline{A} \cdot \overline{B} \quad (\text{by de Morgan's rule})$$

96 In the given circuit, value of Y is

[2020, 8 Jan Shift-II]



(a) toggles between 0 and 1

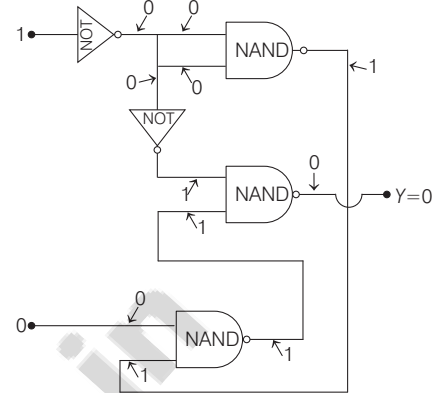
(b) 1

(c) 0

(d) will not execute

Ans. (c)

We are showing intermediate outputs on the circuit as given below



Output of above circuit at given inputs is zero.

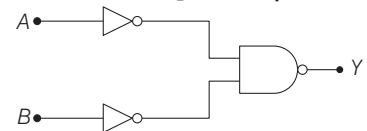
To calculate output, we use truth tables of NAND and NOT gates.

NAND gate		
A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

NOT gate	
Y _{in}	Y _{out}
0	1
1	0

97 The logic gate equivalent to the given logic circuit is

[2019, 9 April Shift-II]



(a) NOR (b) NAND (c) OR (d) AND

Ans. (c)

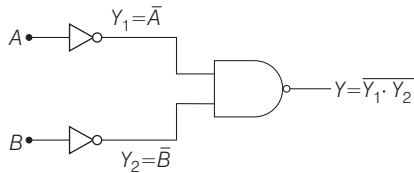
Truth table for given combination of logic gates is

A	B	Y ₁ = $\overline{A}$	Y ₂ = $\overline{B}$	Y = Y ₁ · Y ₂
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

Output Y resembles output of an OR gate. So, given combination acts like an OR gate.

Alternate Solution

The given logic gate circuit can be drawn as shown below



$$\text{Here, } Y = Y_1 \cdot Y_2 = \overline{A} \cdot \overline{B}$$

Using de-Morgan's theorem, i.e.

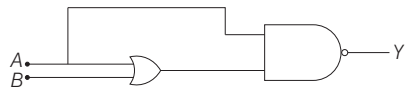
$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$$\therefore Y = \overline{\overline{A} \cdot \overline{B}} = A + B \quad [\because \overline{\overline{x}} = x]$$

This represents the boolean expression for OR gate.

- 98** The truth table for the circuit given in the figure is

[2019, 12 April Shift-I]

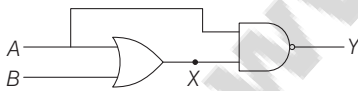


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Ans. (c)

Given circuit is



Let the intermediate state X of OR gate is shown in figure.

$$\text{Clearly, } Y = \overline{A} \overline{X} \quad \dots(i)$$

$$\text{Here, } X = A + B \quad \dots(ii)$$

$$\therefore Y = \overline{A} \cdot \overline{(A + B)} = \overline{A} \cdot \overline{A + B}$$

$$= \overline{A} \cdot \overline{A + B} \quad (\because \overline{AA} = A)$$

$$= \overline{A} \cdot \overline{(1 + B)}$$

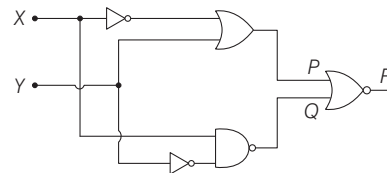
$$\Rightarrow Y = \overline{A} \quad (\because 1 + B = 1)$$

So, truth table shown in option (c) is correct.

Alternate Solution We can solve it using truth table

A	B	X = A + B	Y = $\overline{A} \overline{X}$
0	0	0	1
0	1	1	1
1	0	1	0
1	1	1	0

- 99** To get output '1' at R, for the given logic gate circuit, the input values must be [2019, 10 Jan Shift-I]



$$(a) X=0, Y=0$$

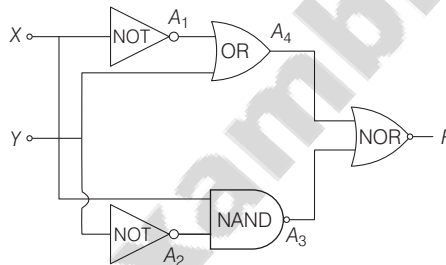
$$(b) X=1, Y=0$$

$$(c) X=1, Y=1$$

$$(d) X=0, Y=1$$

Ans. (b)

The given circuit can be drawn as shown in the figure given below

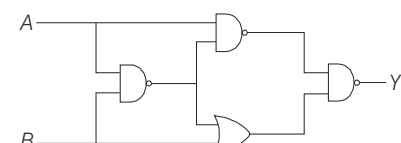


Truth table for this given logic gate is

Given inputs in the options	A ₁	A ₂	A ₃	A ₄	R
X	Y				
0	0	1	1	1	0
1	0	0	1	0	1
1	1	0	0	1	0
0	1	1	0	1	0

So to get output R = 1, inputs must be X = 1 and Y = 0.

- 100** The output of the given logic circuit is [2019, 12 Jan Shift-I]



$$(a) \overline{A} \overline{B}$$

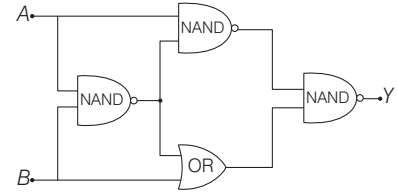
$$(b) \overline{A} B$$

$$(c) A \overline{B} + \overline{A} B$$

$$(d) \overline{A} B + \overline{A} \overline{B}$$

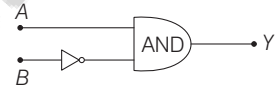
Ans. (a)

Truth table for given circuit is



A	B	Y ₁	Y ₂	Y ₃	Y
0	0	1	1	1	0
1	0	1	1	0	1
0	1	1	1	1	0
1	1	0	1	1	0

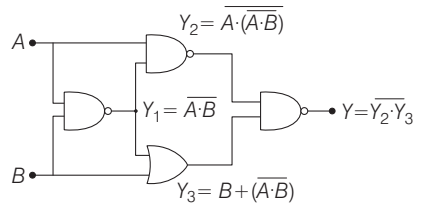
This is the same output produced by $A \cdot \overline{B}$ gate or



So, given circuit is equivalent to Boolean expression $A \cdot \overline{B}$.

Alternate Solution

Using the Boolean algebra, output of the given logic circuit can be given as



$$\text{Here } Y_2 = \overline{A \cdot (\overline{A \cdot B})}$$

Using de-Morgan's principle,

$$\overline{x \cdot y} = \overline{x} + \overline{y} \text{ and } \overline{x + y} = \overline{x} \cdot \overline{y}$$

$$\Rightarrow Y_2 = \overline{A} + \overline{(\overline{A \cdot B})} \quad [\because \overline{\overline{x}} = x]$$

$$= \overline{A} + (A \cdot B) \quad \dots(i)$$

$$\text{Similarly, } Y_3 = B + \overline{A} + \overline{B} = 1 + \overline{A}$$

$$[\because x + \overline{x} = 1]$$

$$\Rightarrow Y_3 = 1 \quad \dots(ii) [\because \overline{x} + 1 = 1]$$

$$\text{As, } Y = Y_2 \cdot Y_3$$

Using Eqs. (i) and (ii), we get

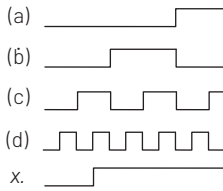
$$Y = (\overline{A} + A \cdot B) \cdot 1$$

$$= (\overline{A} + A \cdot B) + \overline{1} = \overline{A} + (A \cdot B) + 0$$

$$= \overline{A} + (A \cdot B) \quad [\because x + 0 = x]$$

$$= \overline{A} \cdot \overline{A} + A \cdot \overline{B} \quad [\because x \cdot \overline{x} = 0]$$

- 101** If a, b, c and d are inputs to a gate and x is its output, then, as per the following time graph, the gate is [JEE Main 2016]



- (a) NOT
(b) AND
(c) OR
(d) NAND

Ans. (c)

Output of OR gate is 0 when all inputs are 0 and output is 1 when atleast one of the inputs is 1. Observing output x It is 0 when all inputs are 0 and it is 1 when atleast one of the inputs is 1.

∴ The gate is OR.

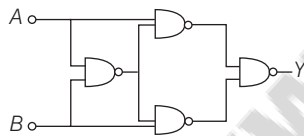
Alternative Solution

OR Gate

a	b	c	d	x
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1

102 Truth table for system of four NAND gates as shown in figure is

[AIEEE 2012]



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

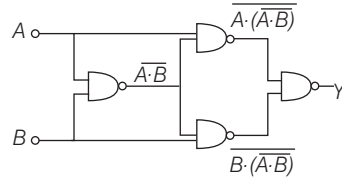
(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Ans. (a)



$$Y = [A \cdot (A \cdot B)] \cdot [B \cdot (A \cdot B)]$$

[using De-Morgan's rules]

$$= (\bar{A} + A \cdot B) \cdot (\bar{B} + A \cdot B)$$

[break the line, change the sign]

$$= (\bar{A} + A \cdot B) + (\bar{B} + A \cdot B)$$

$$= A \cdot (\bar{A} \cdot \bar{B}) + B \cdot (\bar{A} \cdot \bar{B})$$

$$= A \cdot (\bar{A} + \bar{B}) + B \cdot (\bar{A} + \bar{B})$$

$$= A \cdot \bar{B} + B \cdot \bar{A} \quad [\because A \cdot \bar{A} = B \cdot \bar{B} = 0]$$

A	B	$\bar{A}$	$\bar{B}$	$A \cdot \bar{B}$	$B \cdot \bar{A}$	Y
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

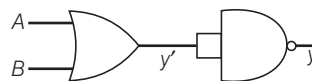
It is the truth table of XOR gate.

103 The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a

[AIEEE 2011]

- (a) OR gate
(b) NOT gate
(c) NOR gate
(d) AND gate

Ans. (c)



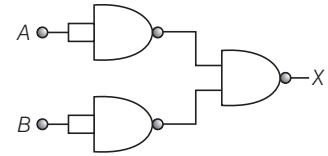
$$y' = A + B$$

$$y = y' \cdot y' = \bar{A} + \bar{B}$$

and

i.e., output of a NOR gate.

104 The combination of gates shown below yields [AIEEE 2010]



- (a) OR gate
(b) NOT gate
(c) XOR gate
(d) NAND gate

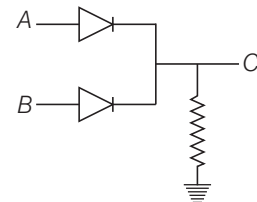
Ans. (a)

Truth table for given combination is

A	B	X
0	0	0
1	0	1
0	1	1
1	1	1

This comes out to be truth table of OR gate.

105 In the adjacent circuit, A and B represent two inputs and C represents the output.



The circuit represents

[AIEEE 2008]

- (a) NOR gate
(b) AND gate
(c) NAND gate
(d) OR gate

Ans. (d)

If we give the following inputs to A and B, then corresponding output is shown in truth table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

The above table is similar to OR gate.

29

Communication System

TOPIC 1

Elements of Communication System and Propagation of EM Wave

- 01** If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is km. (Take, radius of Earth = 6400 km) [2021, 31 Aug Shift-I]

Ans. (64)

Given,

Height of transmitting antenna (H_T) + height of receiving antenna (H_R) = 160 m

Radius of earth, $R_e = 6400$ km

Let, range = d

If height of transmitting antenna be x then, height of receiving antenna = $160 - x$

$$\begin{aligned} \text{We know that, } d &= \sqrt{2Rh_T} + \sqrt{2Rh_R} \\ &= \sqrt{2R} [\sqrt{h_T} + \sqrt{h_R}] \\ &= \sqrt{2 \times R} [\sqrt{x} + \sqrt{160 - x}] \\ &= \sqrt{2R} [\sqrt{x} + \sqrt{160 - x}] \end{aligned}$$

For maximum range

$$\frac{d}{dx}(d) = \frac{d}{dx} \sqrt{2R} [\sqrt{x} + \sqrt{160 - x}] = 0$$

$$\Rightarrow \frac{d}{dx} [\sqrt{x} + \sqrt{160 - x}] = 0$$

$$\Rightarrow \frac{1}{2} x^{-1/2} - \frac{1}{2} (160 - x)^{-1/2} = 0$$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{160 - x}}$$

$$\Rightarrow x = 160 - x$$

$$\begin{aligned} \Rightarrow 2x &= 160 \\ \Rightarrow x &= 80 \\ \therefore \text{Maximum range} \\ (d_{\max}) &= \sqrt{2R} [\sqrt{x} + \sqrt{160 - x}] \\ &= \sqrt{2 \times 6400} [\sqrt{80} + \sqrt{160 - 80}] \\ &= 80\sqrt{2} \left[\sqrt{\frac{80}{1000}} + \sqrt{\frac{80}{1000}} \right] \\ &= 80\sqrt{2} \times 2 \sqrt{\frac{80}{1000}} \\ &= 80\sqrt{2} \times \frac{2\sqrt{80}}{10\sqrt{10}} \\ &= 16 \sqrt{\frac{2 \times 80}{10}} \\ &= 16 \times 4 = 64 \text{ km} \end{aligned}$$

- 02** An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km? [2021, 27 Aug Shift-II]

- (a) 37.8 m (b) 605 m
(c) 75.6 m (d) 302 m

Ans. (b)

Given, height of tower, $H = 400$ m

Range of tower,

$$d = 44 \text{ km} = 44 \times 10^3 \text{ m}$$

Since, $d = \sqrt{2hR_e}$

where, R_e is radius of earth = 6400 km = 6400×10^3 m

$$\Rightarrow h = \frac{d^2}{2R_e} = \frac{44 \times 10^3 \times 44 \times 10^3}{2 \times 6400 \times 10^3}$$

$$h = 15125 \text{ m}$$

$$\text{and wavelength } (\lambda) = 4h = 15125 \times 4 = 605 \text{ m}$$

- 03** A transmitting antenna at top of a tower has a height of 50 m and the height of receiving antenna is 80 m. What is range of communication for line of sight (LOS) mode?

[Use radius of Earth = 6400 km]

[2021, 26 Aug Shift-II]

- (a) 45.5 km (b) 80.2 km
(c) 144.1 km (d) 57.28 km

Ans. (d)

Given, height of transmitting antenna, $H_T = 50$ m

Height of receiving antenna, $H_R = 80$ m

Radius of earth,

$$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$$

For maximum range of communication for LOS,

$$\begin{aligned} d_{\max} &= \sqrt{2H_T R} + \sqrt{2H_R R} \\ \Rightarrow d_{\max} &= \sqrt{2 \times 50 \times 6.4 \times 10^6} + \sqrt{2 \times 80 \times 6.4 \times 10^6} \\ \Rightarrow d_{\max} &= 10^3 [\sqrt{640} + \sqrt{1024}] \\ &= 57.29 \times 10^3 \text{ m} \\ d_{\max} &= 57.28 \text{ km} \end{aligned}$$

Hence, maximum range of communication for LOS is 57.28 km.

- 04** What should be the height of transmitting antenna and the population covered, if the television telecast is to cover a radius of 150 km? The average population density around the tower is $2000/\text{km}^2$ and the value of $R_e = 6.5 \times 10^6$ m. [2021, 22 July Shift-II]

- (a) Height = 1731 m
Population covered = 1413×10^5
- (b) Height = 1241 m
Population covered = 7×10^5
- (c) Height = 1600 m
Population covered = 2×10^5
- (d) Height = 1800 m
Population covered = 1413×10^8

Ans. (a)

Given, radius of region covered,

$$R = 150 \text{ km} \\ = 150 \times 10^3 \text{ m}$$

Population density around tower,
 $\rho = 2000/\text{km}^2 = 2000 \times 10^{-6} / \text{m}^2$

Let h be the height of antenna.

As we know that, $h = R^2 / 2R_e^2$

$$h = \frac{R^2}{2R_e} = \frac{150 \times 150 \times 10^6}{2 \times 6.5 \times 10^6}$$

$$= 1730.7 \text{ m} \approx 1731 \text{ m}$$

and population covered = $\pi R^2 \times \rho$
 $= 3.14 \times 150 \times 150 \times 10^6 \times 20000 \times 10^{-6}$
 $= 1413 \times 10^5$

- 05** A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at
- (i) ground level
- (ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is $n\%$.

The value of n , to the nearest integer, is

[2021, 18 March Shift-II]

Ans. (50)

Given, height of the TV transmission antenna, $H = 20 \text{ m}$

Initial range of the antenna at the ground level,

$$R = \sqrt{2RH}$$

Final range of the antenna at the height of 5 m,

$$R' = \sqrt{2RH} + \sqrt{2RH'}$$

Percentage increase in the antenna range,

$$\begin{aligned} \% \text{ range} &= \frac{R' - R}{R} \times 100 \\ &= \frac{\sqrt{2RH'} + \sqrt{2RH} - \sqrt{2RH}}{\sqrt{2RH}} \times 100 = \frac{\sqrt{H'}}{\sqrt{H}} \times 100 \\ &= \frac{\sqrt{5}}{\sqrt{20}} \times 100 = 50\% \end{aligned}$$

Hence, the value of n to the nearest integer is 50.

06 Match List-I with List-II.

[2021, 18 March Shift-I]

List-I	List-II
A. 10 km height over Earth's surface	(i) Thermosphere
B. 70 km height over Earth's surface	(ii) Mesosphere
C. 180 km height over Earth's surface	(iii) Stratosphere
D. 270 km height over Earth's surface	(iv) Troposphere

A	B	C	D
(a) (iv)	(iii)	(ii)	(i)
(b) (i)	(iv)	(iii)	(ii)
(c) (iii)	(ii)	(i)	(iv)
(d) (ii)	(i)	(iv)	(iii)

Ans. (a)

Troposphere is the lowest layer of all the mentioned layer. Most of the mass like clouds, rains, etc occurs in troposphere. Its height is upto the 10 km over the earth surface. Stratosphere is the layer after troposphere. Its height is upto the 70 km over the earth surface.

Mesosphere is a layer that is situated above the stratosphere and below the thermosphere. The temperature at the top of this layer is very low as compared to the atmosphere found near the earth surface. Its height is upto the 180 km over the earth surface.

Thermosphere constitutes the larger part of ionosphere. Its height is upto 270 km over the earth surface.

Hence, the correct option is (a).

07 For VHF signal broadcasting,

..... km^2 of maximum service area will be covered by an antenna tower of height 30 m, if the receiving antenna is placed at ground. Let radius of the Earth be 6400 km.

(Round off to the nearest integer).
(Take π as 3.14)

[2021, 17 March Shift-I]

Ans. (1206)

Given, height of antenna tower, $h = 30 \text{ m}$
 $= 30 \times 10^{-3} \text{ km}$

Radius of earth, $R = 6400 \text{ km}$

We know that, $d = \sqrt{2Rh}$... (i)

where, d = distance up to which signals can be transmitted

Area, $A = \pi d^2$... (ii)

From Eqs. (i) and (ii), we get

$$A = \pi(\sqrt{2Rh})^2 = 2\pi Rh$$

$$= 2 \times 3.14 \times 6400 \times 30 \times 10^{-3} = 1205.76 \text{ km}^2 \\ = 1206 \text{ km}^2$$

- 08** Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight?

(Assume, radius of earth is

6400 km.) [2021, 16 March Shift-II]

(a) 19.77 m (b) 39.55 m

(c) 79.1 m (d) 158.2 m

Ans. (b)

Given, distance between two antennas,
 $D = 45 \text{ km}$

Radius of Earth, $R = 6400 \text{ km}$

Height of the receiving antenna, $h = ?$

We know that,

$$D = 2\sqrt{2Rh}$$

$$\Rightarrow h = \frac{D^2}{8R} \quad \dots (i)$$

Substituting the given values in Eq. (i), we get

$$h = \frac{(45)^2}{8 \times 6400} = \frac{(45)^2}{8 \times 6400} \\ = 39.55 \text{ m}$$

- 09** A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in m) of the signal transmitted by this antenna would be
- [2021, 16 March Shift-I]

(a) 300 (b) 400 (c) 200 (d) 100

Ans. (d)

Since, height of antenna = Wavelength of the signal transmitted by antenna (λ)

$$\Rightarrow 25 = \frac{\lambda}{4} \Rightarrow \lambda = 25 \times 4$$

$$\Rightarrow \lambda = 100 \text{ m}$$

- 10** The wavelength of the carrier waves in a modern optical fibre communication network is close to

(a) 2400 nm (b) 1500 nm

(c) 600 nm (d) 900 nm

[2019, 8 April Shift-I]

Ans. (b)

In optical fibre communication network, the signals are transmitted by laser light operating in range of 1310 nm–1550 nm. So, the closest value is 1500 nm.

- 11** In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be (Radius of the earth = 6.4×10^6 m)

[2019, 8 April Shift-II]

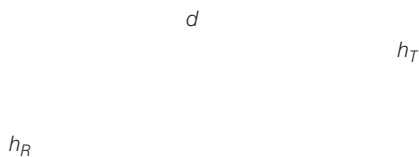
(a) 20 m (b) 32 m (c) 40 m (d) 51 m

Ans. (b)

Key Idea In line of sight communication, distance d between transmitting antenna and receiving antenna is given by

$$d = 2Rh_T + 2Rh_R$$

Here in figure, h_R and h_T is the height of receiving and transmitting antenna, respectively.



Given, $d = 50 \text{ km} = 50 \times 10^3 \text{ m}$,

$h_R = 70 \text{ m}$, $R = 6.4 \times 10^6 \text{ m}$

Then, distance between transmitting and receiving antenna, i.e.

$$\begin{aligned} d &= 2Rh_T + 2Rh_R \\ 50 \times 10^3 &= 2R(h_T + h_R) \\ &= 2 \times 6.4 \times 10^6 (h_T + 70) \end{aligned}$$

$$\Rightarrow h_T \approx \frac{50 \times 10^3}{3577.7} - 8.37 = 13.98 - 8.37 = 5.61$$

$$\text{or } h_T = 31.5 \approx 32 \text{ m}$$

- 12** Given below in the left column are different modes of communication using the kinds of waves given in the right column.

[2019, 10 April Shift-I]

A. Optical fibre communication	P. Ultrasound
B. Radar	Q. Infrared light
C. Sonar	R. Microwaves
D. Mobile phones	S. Radio waves

From the options given below, find the most appropriate match between entries in the left and the right column.

- (a) A-Q, B-S, C-R, D-P
(b) A-S, B-Q, C-R, D-P
(c) A-Q, B-S, C-P, D-R
(d) A-R, B-P, C-S, D-Q

Ans. (c)

In optical fibre communication, infrared light is used to transmit information from one point to another.

RADAR (Radio detection and ranging) is a detection system that uses radio waves to determine range, angle or velocity of objects.

SONAR (Sound navigation and ranging) is also a detection system that uses ultrasound to detect under water objects, submarines, etc.

Mobile phone is a portable telephone that can make and receive calls, which make use of microwave.

$\therefore$ Correct sequence is

A $\rightarrow$ Q, B $\rightarrow$ S, C $\rightarrow$ P and D $\rightarrow$ R

- 13** In communication system, only one per cent frequency of signal of wavelength 800 nm can be used as bandwidth. How many channel of 6 MHz bandwidth can be broadcast this? ($c = 3 \times 10^8 \text{ m/s}$, $h = 6.6 \times 10^{-34} \text{ J-s}$)

[2019, 9 Jan Shift-II]

- (a) 3.75×10^6 (b) 3.86×10^6
(c) 6.25×10^5 (d) 4.87×10^5

Ans. (c)

Here, Signal wavelength, $\lambda = 800 \text{ nm} = 8 \times 10^{-7} \text{ m}$

Frequency of source is

$$\text{As, } f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-7}} = 3.75 \times 10^{14} \text{ Hz}$$

$$\therefore \text{Total bandwidth used for communication} = 1\% \text{ of } 3.75 \times 10^{14} = 3.75 \times 10^{12} \text{ Hz} \quad \dots(i)$$

So, number of channel for signals

$$\begin{aligned} &= \frac{\text{total bandwidth available}}{\text{bandwidth of TV signal}} \\ &= \frac{3.75 \times 10^{12}}{6 \times 10^6} = 0.625 \times 10^6 = 6.25 \times 10^5 \end{aligned}$$

- 14** A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W . The value of x is

$$[\text{Gain in dB} = 10 \log_{10} (P_0 / P_i)]$$

[2021, 24 Feb Shift-II]

Ans. (8)

Given, power of transmitted signal, $P_i = 0.1 \text{ kW} = 0.1 \times 10^3 \text{ W} = 10^2 \text{ W}$

Rate of attenuation, $R = -5 \text{ dB/km}$

Length of cable, $l = 20 \text{ km}$

Power received at receiver, $P_x = 10^{-x} \text{ W}$

Total loss, $\beta = R \times l = -5 \times 20 = -100 \text{ dB}$

$$\therefore \text{Gain } (\beta) = 10 \log_{10} \frac{P_0}{P_i}$$

$$\therefore \beta = -100 = -10 \log_{10} \frac{P_0}{P_i}$$

$$\Rightarrow -10 = \log_{10} \frac{P_0}{P_i} \Rightarrow 10^{-10} = \frac{P_0}{P_i}$$

$$\Rightarrow \frac{P_0}{P_i} = 10^{-10} \Rightarrow P_i = 10^{-10} \times 10^2 = 10^{-8}$$

$$\Rightarrow \frac{P_0}{P_i} = 10^{-8} \text{ W}$$

Hence, $x = 8$

- 15** A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Take, radius of earth = $6.4 \times 10^6 \text{ m}$).

[2019, 10 Jan Shift-I]

- (a) 65 km (b) 80 km
(c) 40 km (d) 48 km

Ans. (a)

Maximum distance of transmission is given by

$$d = 2Rh_T + 2Rh_R$$

where, h_T = height of transmitter, = 140 m,

h_R = height of receiver = 40 m and

R = radius of earth

$= 6.4 \times 10^6 \text{ m}$.

Substituting values, we get

$$d = 2 \times 6.4 \times 10^6 (140 + 40)$$

$$= 65 \text{ km}$$

- 16** To double the covering range of a TV transmission tower, its height should be multiplied by

[2019, 12 Jan Shift-II]

- (a) 2 (b) 4
(c) 2 (d) 1
2

Ans. (b)

Range of TV transmitting tower $d = 2hR$ where, h is the height of the transmission tower. when range is doubled,

$$\therefore 2d = 2 \cdot 2hR$$

$$= 2(4h)R$$

So, height must be multiplied with 4.

- 17** A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilised for transmission. How many telephonic channels can be transmitted simultaneously, if each channel requires a bandwidth of 5 kHz? [JEE Main 2018]

- (a) 2×10^3 (b) 2×10^4
(c) 2×10^5 (d) 2×10^6

Ans. (c)

Only 10% of 10 GHz is utilised for transmission.

$\therefore$ Band available for transmission

$$= \frac{10}{100} \times 10 \times 10^9 \text{ Hz} = 10^9 \text{ Hz}$$

Now, if there are n channels each using 5 kHz then,

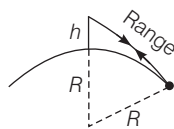
$$n \times 5 \times 10^3 = 10^9 \Rightarrow n = 2 \times 10^5$$

- 18** A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is [AIEEE 2012]

- (a) 80 km (b) 16 km
(c) 40 km (d) 64 km

Ans. (a)

Range of radar on earth surface (optical distance, for space wave, i.e., line of view).



$$\begin{aligned} \text{Range} &= \sqrt{(R+h)^2 - R^2} \\ &= \sqrt{2Rh + h^2} \approx \sqrt{2Rh} \\ &= \sqrt{2 \times 6400 \times \frac{1}{2}} \text{ km} = 80 \text{ km} \end{aligned}$$

- 19** This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement II The state of ionosphere varies from hour to hour, day to day and season to season. [AIEEE 2011]

- (a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

Ans. (b)

In radio communication, sky wave refers to the propagation of radio waves reflected or refracted back towards earth from the ionosphere.

Since, it is not limited by the curvature of the earth, sky wave propagation can be used to communicate beyond horizon. Ionosphere is a region of upper atmosphere and induces the thermosphere and parts of mesosphere and exosphere. It is distinguished because it is ionised by solar radiation. It plays an important part in atmospheric electricity.

- 20** Consider telecommunication through optical fibres. Which of the following statements is not true?

- (a) Optical fibres can be of graded refractive index [AIEEE 2003]
(b) Optical fibres are subjected to electromagnetic interference from outside
(c) Optical fibres have extremely low transmission loss
(d) Optical fibres may have homogeneous core with a suitable cladding

Ans. (b)

Some of the characteristics of an optical fibre are as follows

- (i) It works on the principle of total internal reflection.
(ii) It consists of core made up of glass/silica/plastic with refractive index n_1 , which is surrounded by a glass or plastic cladding with refractive index n_2 , ($n_2 > n_1$). The refractive index of cladding can be either changing abruptly or gradually changing (graded index fibre).
(iii) There is a very little transmission loss through optical fibres.
(iv) There is no interference from stray electric and magnetic fields to the signals through optical fibres.

TOPIC 2 Modulation and Demodulation

- 21** A bandwidth of 6 MHz is available for AM transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this band simultaneously without interfering with each other will be [2021, 26 Aug Shift-II]

Ans. (500)

Given, maximum frequency of audio signal = 6 kHz

Available bandwidth = 6 MHz

Bandwidth of each station

$$= 2 \times \text{Maximum frequency of audio signal}$$

$$= 2 \times 6 \times 10^3 \text{ Hz} = 12 \times 10^3 \text{ Hz}$$

Number of stations that can be broadcasted within the bandwidth of

$$6 \text{ MHz} = \frac{6 \times 10^6}{12 \times 10^3} = 500$$

Thus, the correct answer is 500.

- 22** An amplitude modulated wave is represented by $C_m(t) = 10(1 + 0.2 \cos 12560t) \sin(111 \times 10^4 t)$ V. The modulating frequency in kHz will be [2021, 26 Aug Shift-I]

Ans. (2)

The amplitude modulated wave is represented by

$$C_m(t) = 10(1 + 0.2 \cos 12560t) \sin(111 \times 10^4 t) \text{ V}$$

Comparing with standard equation,

$$C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$\omega_m = 12560 \text{ rad/s}$$

$$\Rightarrow 2\pi f_m = 12560$$

$$\Rightarrow f_m = \frac{12560}{2\pi} = \frac{12560 \times 7}{2 \times 22} \approx 2000 \text{ Hz}$$

$$\approx 2 \text{ kHz}$$

- 23** A message signal of frequency 20 kHz and peak voltage of 20 V is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 V. The modulation index will be [2021, 27 July Shift-II]

Ans. (1)

Given, message signal of frequency, $f_m = 20$ kHz, Peak voltage of message signal, $V_m = 20$ V
 Carrier signal of frequency, $f_c = 1$ MHz
 Peak voltage of carrier signal, $V_c = 20$ V
 Modulation,

$$m = \frac{\text{Peak voltage of modulating signal}}{\text{Peak voltage of carrier signal}}$$

$$= \frac{20}{20} = 1$$

- 24** The maximum amplitude for an amplitude modulated wave is found to be 12 V, while the minimum amplitude is found to be 3 V. The modulation index is 0.6x, where x is [2021, 27 July Shift-II]

Ans. (1)

Given,
 Maximum amplitude for modulated waves, $A_{\max} = 12$ V
 and minimum amplitude, $A_{\min} = 3$ V
 $\therefore$ We know that,

$$A_{\max} = A_c + A_m$$

[Here, A_c is the maximum value of carrier signal & A_m is the maximum value of message signal]

$$\Rightarrow 12 = A_c + A_m \quad \dots(i)$$

$$\text{and } A_{\min} = A_c - A_m$$

$$\Rightarrow 3 = A_c - A_m \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$A_c = \frac{15V}{2} \text{ and } A_m = \frac{9V}{2}$$

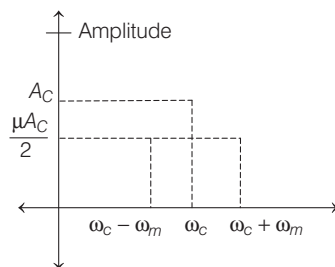
$$\therefore \text{Modulation index} = \frac{A_c}{A_m} = \frac{9/2}{15/2} = \frac{9}{15} = 3/5 = 0.6$$

$\therefore$ As per question, the value of modulation index is 0.6x. So, on comparing we can write $x = 1$

- 25** The amplitude of upper and lower side bands of AM wave, where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5 V amplitude are $\frac{a}{10}$ V and $\frac{b}{10}$ V, respectively. Then, the value of $\frac{a}{b}$ is [2021, 27 July Shift-I]

Ans. (1)

The variation of amplitude with side band frequencies can be shown as follows.



As we can see that, the amplitudes of side band for both left and right are equal.

$$\therefore \frac{a}{10} = \frac{b}{10} = \frac{\mu A_c}{2} \Rightarrow \frac{a}{b} = 1$$

- 26** In amplitude modulation, the message signal

$$V_m(t) = 10 \sin(2\pi \times 10^5 t) \text{ volts}$$

and carrier signal

$$V_c(t) = 20 \sin(2\pi \times 10^7 t) \text{ volts}$$

The modulated signal now contains the message signal with lower side band and upper side band frequency.

Therefore, the bandwidth of modulated signal is α kHz. The value of α is [2021, 25 July Shift-I]

- (a) 200 kHz (b) 50 kHz
 (c) 100 kHz (d) zero

Ans. (a)

Given,

Message signal $V_m(t) = 10 \sin(2\pi \times 10^5 t)$ V
 and carrier signal,

$$V_c(t) = 20 \sin(2\pi \times 10^7 t) \text{ V}$$

As we know that,

$$\text{Bandwidth} = 2f_m \quad \dots(i)$$

where, f_m = frequency of message signal
 and ω_m = angular frequency of message signal.

$$\text{Since, } f_m = \frac{\omega_m}{2\pi}$$

Substituting in Eq. (i), we get

$$\begin{aligned} \text{Bandwidth} &= 2 \cdot \frac{\omega_m}{2\pi} = 2 \cdot \frac{(2\pi \times 10^5)}{2\pi} \\ &= 200 \text{ kHz} \end{aligned}$$

- 27** A carrier wave $V_c(t) = 160 \sin(2\pi \times 10^6 t)$ V is made to vary between $V_{\max} = 200$ V and $V_{\min} = 120$ V by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ V. The peak voltage A_m of the modulating signal is [2021, 20 July Shift-I]

Ans. (40)

Given, $V_c(t) = 160 \sin(2\pi \times 10^6 t)$ V
 $V_{\max} = 200$ V,
 $V_{\min} = 120$ V and $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ V.

Since, maximum amplitude,

$$A_{\max} = A_m + A_c$$

$$\Rightarrow V_{\max} = V_m + V_c$$

$$\Rightarrow 200 = V_m + 160$$

$$\Rightarrow V_m = 200 - 160$$

$$\Rightarrow V_m = 40$$

$\therefore$ Peak voltage, $A_m = 40$

- 28** A carrier signal $C(t) = 25 \sin(2.512 \times 10^{10} t)$ is amplitude modulated by a message signal $m(t) = 5 \sin(1.57 \times 10^8 t)$ and transmitted through an antenna. What will be the bandwidth of the modulated signal? [2021, 17 March Shift-II]

- (a) 8 GHz (b) 2.01 GHz
 (c) 1987.5 MHz (d) 50 MHz

Ans. (d)

Given, carrier signal,

$$C(t) = 25 \sin(2.512 \times 10^{10} t)$$

Message signal,

$$m(t) = 5 \sin(1.57 \times 10^8 t)$$

Angular frequency of the message signal,

$$\omega_m = 1.57 \times 10^8 \text{ rads}^{-1}$$

Frequency of the message signal,

$$f_m = \frac{\omega_m}{2\pi}$$

Substituting the values in the above equation, we get

$$f_m = \frac{1.57 \times 10^8}{2\pi} \text{ Hz}$$

Bandwidth of the modulated signal is

$$\beta = 2f_m$$

Substituting the values in the above equation, we get

$$\beta = 2 \left(\frac{1.57 \times 10^8}{2\pi} \right)$$

$$\beta = 50 \text{ MHz}$$

Hence, the bandwidth of the modulated signal is 50 MHz.

- 29** If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are [2021, 26 Feb Shift-II]

Ans. (9)

Given, highest modulating frequency,
 $f = 5 \text{ kHz}$

Bandwidth = 90 kHz

Since, number of station

$$= \frac{\text{bandwidth}}{\text{bandwidth/station } (2f)}$$

$$= \frac{\text{Bandwidth}}{2f} = \frac{90}{2 \times 5} = 9$$

- 30** The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V, respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is

[2021, 26 Feb Shift-I]

Ans. (33.33)

Given, maximum amplitude $A_{\max} = 16$

Minimum amplitude, $A_{\min} = 8$

Since, modulation index = $\frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$

$$= \frac{16 - 8}{16 + 8} = \frac{8}{24} = \frac{1}{3} = 0.3333$$

$$= 33.33 \times 10^{-2} = x \times 10^{-2}$$

Hence, $x = 33.33$

- 31** If a message signal of frequency f_m is amplitude modulated with a carrier signal of frequency f_c and radiated through an antenna, the wavelength of the corresponding signal in air is [2021, 25 Feb Shift-II]

(a) $\frac{c}{f_c - f_m}$ (b) $\frac{c}{f_c + f_m}$ (c) $\frac{c}{f_c}$ (d) $\frac{c}{f_m}$

Ans. (c)

Let λ be the wavelength of carrier signal.

Since, frequency (f_c) = $\frac{\text{speed } (c)}{\text{wavelength } (\lambda)}$

$$\therefore \lambda = \frac{c}{f_c}$$

- 32** Given, below are two statements
Statement I A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.
Statement II The side band frequencies are 1002 kHz and 998 kHz. In the light of the above statements, choose the correct answer from the options given below [2021, 25 Feb Shift-I]

- (a) Statement I is true but Statement II is false.
 (b) Statement I is false but Statement II is true.
 (c) Both Statement I and Statement II are true.
 (d) Both Statement I and Statement II are false.

Ans. (c)

Given, frequency of modulated signal,

$$f_m = 2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$$

Frequency of carrier signal, $f_c = 1 \text{ MHz}$

$$= 1 \times 10^6 \text{ Hz}$$

$$= 1000 \text{ kHz}$$

Then, bandwidth = $2f_m = 4 \text{ kHz}$

and side band frequency = $f_c \pm f_m$

$$= (1000 \pm 2) \text{ kHz}$$

$$= 1002 \text{ kHz and } 998 \text{ kHz}$$

Hence, option (c) is the correct.

- 33** An audio signal $v_m = 20 \sin 2\pi(1500t)$ amplitude modulates a carrier $v_c = 80 \sin 2\pi(100000t)$. The value of per cent modulation is [2021, 24 Feb Shift-I]

Ans. (25)

Given, audio signal,

$$V_m = 20 \sin 2\pi(1500t) \quad \dots (i)$$

Carrier signal,

$$V_c = 80 \sin 2\pi(100000t) \quad \dots (ii)$$

We know that, modulation index,

$$m_f = \frac{A_m}{A_c}$$

From Eqs. (i) and (ii), we get

$$A_m = 20, A_c = 80$$

Percentage of modulation index,

$$m_f = \frac{A_m}{A_c} \times 100 = \frac{20}{80} \times 100 = 25\%$$

- 34** An amplitude modulated wave is represented by the expression $V_m = 5(1 + 0.6 \cos 6280t) \sin(211 \times 10^4 t)$ volt. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively [2020, 2 Sep Shift-I]

- (a) 3 V, 5 V (b) $\frac{5}{2}$ V, 8 V
 (c) $\frac{3}{2}$ V, 5 V (d) 5 V, 8 V

Ans. (b)

Equation of amplitude modulated wave given as

$$V = 5(1 + 0.6 \cos 6280t) \sin(211 \times 10^4 t)$$

$$= (5 + 3 \cos 6280t) \sin(211 \times 10^4 t)$$

Comparing above equation with standard equation,

$$V = (A_c + A_m \cos \omega_m t) \cdot \sin \omega_c t$$

We get, Amplitude of carrier signal

$$(A_c) = 5 \text{ V}$$

Amplitude of message signal (A_m) = 3 V

Maximum amplitude of modulated signal

$$= A_c + A_m = 5 + 3 = 8 \text{ V}$$

Minimum amplitude of modulated signal

$$= A_c - A_m = 5 - 3 = 2 \text{ V}$$

The answer is close to option (b).

Note Due to phase shift minimum/maximum amplitudes may differ slightly.

- 35** A signal $A \cos \omega t$ is transmitted using $v_0 \sin \omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is [2019, 9 April Shift-I]

(a) $(v_0 \sin \omega_0 t + A \cos \omega t)$

(b) $(v_0 + A) \cos \omega t \sin \omega_0 t$

(c) $v_0 \sin[\omega_0(1 + 0.01A \sin \omega t)t]$

(d) $v_0 \sin \omega_0 t + \frac{A}{2} \sin(\omega_0 - \omega)t + \frac{A}{2} \sin(\omega_0 + \omega)t$

Ans. (d)

Given, modulating signal, $A_m = A \cos \omega t$

Carrier wave, $A_c = v_0 \sin \omega_0 t$

In amplitude modulation, modulated wave is given by

$$Y_m = [A_0 + A_m] \sin \omega_0 t$$

where, A_0 is amplitude of the carrier wave (given as v_0)

$$\therefore Y_m = [v_0 + A \cos \omega t] \sin \omega_0 t$$

$$= v_0 \sin \omega_0 t + A \sin \omega_0 t \cos \omega t$$

$$= v_0 \sin \omega_0 t + \frac{A}{2}$$

$$[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$$

$$= v_0 \sin \omega_0 t + \frac{A}{2} \sin(\omega_0 - \omega)t + \frac{A}{2} \sin(\omega_0 + \omega)t$$

- 36** The physical sizes of the transmitter and receiver antenna in a communication system are [2019, 9 April Shift-II]

- (a) proportional to carrier frequency
 (b) inversely proportional to modulation frequency
 (c) independent of both carrier and modulation frequency
 (d) inversely proportional to carrier frequency

Ans. (d)

Size of antenna is directly proportional to the wavelength of the signal.

Also, the speed at which signal moves = carrier frequency $\times$ wavelength

$$\Rightarrow f\lambda = c \Rightarrow \lambda \propto \frac{1}{f}$$

$\therefore$ Size of antenna $\propto \frac{1}{f}$

Note Minimum size of the antenna is $\lambda/4$.

- 37** In an amplitude modulator circuit, the carrier wave is given by $C(t) = 4 \sin(20000 \pi t)$ while modulating signal is given by, $m(t) = 2 \sin(2000 \pi t)$. The values of modulation index and lower side band frequency are

[2019, 12 April Shift-II]

- (a) 0.5 and 10 kHz (b) 0.4 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

Ans. (d)

Given, carrier wave, $C(t) = 4 \sin(20000 \pi t)$

Modulating signal, $m(t) = 2 \sin(2000 \pi t)$

So, carrier wave's amplitude and frequency are

$$A_c = 4 \text{ V}, \omega_c = 20000 \pi = 2\pi \times 10^4 \text{ rad/s}$$

$$\Rightarrow f_c = \frac{\omega_c}{2\pi} = 10^4 \text{ Hz} = 10 \text{ kHz}$$

and modulating signal's amplitude and frequency are

$$A_m = 2 \text{ V}, \omega_m = 2000 \pi = 2\pi \times 10^3 \text{ rad/s}$$

$$\Rightarrow f_m = \frac{\omega_m}{2\pi} = 10^3 \text{ Hz} = 1 \text{ kHz}$$

$$\text{So, modulation index is } m = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$$

and lower side band frequency is,

$$f_{\text{LSB}} = f_c - f_m = 10 - 1 = 9 \text{ kHz}$$

- 38** The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot? [2019, 10 Jan Shift-II]

- (a) 2000 kHz (b) 2250 kHz
(c) 2900 kHz (d) 2750 kHz

Ans. (a)

For a given carrier wave of frequency f_c with modulation frequency f_m , the bandwidth is calculated by $f_{\text{upper}} = f_c + f_m$

$$f_{\text{lower}} = f_c - f_m \quad \dots (i)$$

To avoid overlapping of bandwidths, next broadcast frequencies can be

$$f_1 = f_c \pm 2f_m, f_2 = f_c \pm 3f_m$$

So, next immediate available broadcast frequency is $f_1 = f_c + 2f_m$

$$\text{and } f'_1 = f_c - 2f_m$$

$$\text{Given, } f_m = 250 \text{ kHz}$$

and also that f_m is 10% of f_c ,

$$\text{i.e. } f_c = 2500 \text{ kHz}$$

$$\text{So, } f_1 = 2500 + (2 \times 250) = 3000 \text{ kHz}$$

$$\text{and } f'_1 = 2500 - (2 \times 250) = 2000 \text{ kHz}$$

- 39** An amplitude modulates signal is given by $v(t) = 10[1 + 0.3 \cos(2.2 \times 10^4 t)] \sin(5.5 \times 10^5 t)$.

Here, t is in seconds. The sideband frequencies (in kHz) are

$$\left(\text{Take, } \pi = \frac{22}{7} \right)$$

[2019, 11 Jan Shift-I]

- (a) 892.5 and 857.5 (b) 89.25 and 85.75
(c) 178.5 and 171.5 (d) 1785 and 1715

Ans. (b)

$$v(t) = 10[1 + 0.3 \cos(2.2 \times 10^4 t)] [\sin(5.5 \times 10^5 t)]$$

Upper band angular frequency

$$\omega_u = (2.2 \times 10^4 + 5.5 \times 10^5) \text{ rad/s} \\ = 572 \times 10^3 \text{ rad/s}$$

Similarly, lower band angular frequency.

$$\omega_L = (5.5 \times 10^5 - 2.2 \times 10^4) \text{ rad/s} \\ = 528 \times 10^3 \text{ rad/s}$$

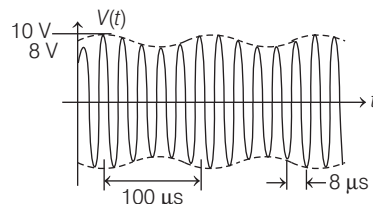
$\therefore$ Side band frequency are,

$$f_u = \frac{\omega_u}{2\pi} = \frac{572}{2\pi} \text{ kHz} \approx 91 \text{ kHz}$$

$$\text{and } f_L = \frac{\omega_L}{2\pi} = \frac{528}{2\pi} \text{ kHz} \approx 84 \text{ kHz}$$

Thus, the approximate side band frequencies are 89.25 and 85.75.

- 40** An amplitude modulated signal is plotted below



Which one of the following best describes the above signal?

[2019, 11 Jan Shift-II]

- (a) $[1 + 9 \sin(2\pi \times 10^4 t)] \sin(2.5\pi \times 10^5 t) \text{ V}$
(b) $[9 + \sin(2\pi \times 10^4 t)] \sin(2.5\pi \times 10^5 t) \text{ V}$
(c) $[9 + \sin(4\pi \times 10^4 t)] \sin(5\pi \times 10^5 t) \text{ V}$
(d) $[9 + \sin(2.5\pi \times 10^5 t)] \sin(2\pi \times 10^4 t) \text{ V}$

Ans. (b)

Equation of an amplitude modulated wave is given by the relation,

$$C_m = (A_c + A_m \sin \omega_m t) \cdot \sin \omega_c t \quad \dots (i)$$

For the given graph,

maximum amplitude,

$$A_c + A_m = 10 \quad \dots (ii)$$

and minimum amplitude, $A_c - A_m = 8$ $\dots (iii)$

From Eqs. (ii) and (iii), we get

$$A_c = 9 \text{ V and } A_m = 1 \text{ V} \quad \dots (iv)$$

$\therefore$ For angular frequency of message signal and carrier wave, we use a relation

$$\omega_c = \frac{2\pi}{T_c} = \frac{2\pi}{8 \times 10^{-6}}$$

(as from given graph, $T_c = 8 \times 10^{-6} \text{ s}$)

$$= 2.5\pi \times 10^5 \text{ s}^{-1} \quad \dots (v)$$

and

$$\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{100 \times 10^{-6}}$$

$$= 2\pi \times 10^4 \text{ s}^{-1}$$

(as from given graph, $T_m = 100 \times 10^{-6} \text{ s}$)

When we put values of A_c , A_m , ω_c and ω_m in Eq. (i), we get

$$C_m = [9 + \sin(2\pi \times 10^4 t)] \sin(2.5\pi \times 10^5 t) \text{ V}$$

- 41** A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?

[2019, 12 Jan Shift-I]

- (a) 0.4 (b) 0.5
(c) 0.6 (d) 0.3

Ans. (c)

Modulation index is given by

$$\mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}} = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}} \\ = \frac{160 - 40}{160 + 40} \\ = \frac{120}{200} = 0.6$$

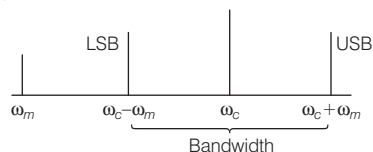
- 42** In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m . The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?

[JEE Main 2017]

- (a) ω_c
(b) $\omega_m + \omega_c$
(c) $\omega_c - \omega_m$
(d) ω_m

Ans. (d)

Frequency spectrum of modulated wave is



Clearly, ω_m is not included in the spectrum.

43 Choose the correct statement:
[JEE Main 2016]

- (a) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
- (b) In amplitude modulation, the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
- (c) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
- (d) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

Ans. (b)

As, we know, an amplitude modulated wave, the bandwidth is twice the frequency of modulating signal. Therefore, amplitude modulation (AM), the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

44 A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are
[JEE Main 2015]

- (a) 2 MHz only
- (b) 2005 kHz and 1995 kHz
- (c) 2005 kHz 2000 kHz and 1995 kHz
- (d) 2000 kHz and 1995 kHz

Ans. (c)

Key Idea Frequency associated with AM are

$$f_c - f_m, f, f_c + f_m$$

According to the question

$$\begin{aligned} f_c &= 2 \text{ MHz} \\ &= 2000 \text{ kHz} \end{aligned}$$

$$f_m = 5 \text{ kHz}$$

Thus, frequency of the resultant signal is/are carrier frequency $f_c = 2000 \text{ kHz}$, LSB frequency

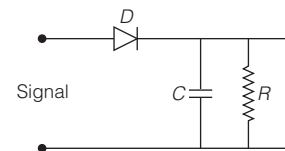
$$f_c - f_m = 2000 \text{ kHz} - 5 \text{ kHz}$$

$$= 1995 \text{ kHz and USB frequency}$$

$$f_c + f_m = 2005 \text{ kHz}$$

45 A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pF in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it. [JEE Main 2013]

- (a) 10.62 MHz
- (b) 10.62 kHz
- (c) 5.31 MHz
- (d) 5.31 kHz

Ans. (b)

$$\begin{aligned} \tau &= RC = 100 \times 10^3 \times 250 \times 10^{-12} \text{ s} \\ &= 2.5 \times 10^7 \times 10^{12} \text{ s} = 2.5 \times 10^{-5} \text{ s} \end{aligned}$$

The highest frequency which can be detected with tolerable distortion is

$$\begin{aligned} f &= \frac{1}{2\pi m_a RC} \text{ [where, } m_a \text{ is modulation]} \\ &= \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}} \text{ Hz} \\ &= \frac{100 \times 10^4}{25 \times 1.2 \pi} \text{ Hz} = \frac{4}{1.2 \pi} \times 10^4 \text{ Hz} \\ &= 10.61 \text{ kHz} \end{aligned}$$

This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated signal voltage for proper detection of modulated signal.

46 Which of the following four alternatives is not correct? We need modulation

- (a) to increase the selectivity
- (b) to reduce the time lag between transmission and reception of the information signal
- (c) to reduce the size of antenna
- (d) to reduce the fractional band width, i.e., the ratio of the signal band width to the centre frequency

[AIEEE 2011]**Ans. (b)**

Modulation does not change time lag between transmission and reception.

30

Experimental Physics

TOPIC 1

Experiments Related to Units and Measurements

- 01** In a screw gauge, 5th division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.

[2021, 26 Aug Shift-I]

- (a) 5.00 mm (b) 5.25 mm
(c) 5.15 mm (d) 5.20 mm

Ans. (c)

Least count of a screw gauge is given by
Least count (LC) = $\frac{\text{Pitch}}{\text{Number of circular scale divisions}}$

$$\Rightarrow LC = \frac{0.5}{50} \dots (i)$$

The actual reading (true reading)

= Main scale reading + Least count $\times$
Circular division - Zero error

$$= 5 + \left(\frac{0.5}{50}\right) \times 20 - \frac{0.5}{50} \times 5$$

[$\because$ Zero error = Least count $\times$ Coinciding division]

$$= 5 + \frac{0.5}{50} (20 - 5)$$

$$= 5 + \frac{0.5}{50} (15) = 5 + \frac{15}{100}$$

$$= \frac{515}{100} \text{ mm} = 5.15 \text{ mm}$$

- 02 Assertion (A)** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason (R) Least count = $\frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In the light of the above statements, choose the most appropriate answer from the options given below.

[2021, 27 July Shift-I]

- (a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct and R is not the correct explanation of A.
(c) A is correct but R is not correct.
(d) A is not correct but R is correct.

Ans. (d)

According to question,

In 5 complete rotations of circular scale, distance travelled on the main scale of screw gauge is 5mm, so in one complete rotation of circular scale, distance travelled on the main scale of screw gauge will be

$$= \frac{5\text{mm}}{5} = 1\text{mm}$$

This is the required value of pitch.

$\therefore$ Least count

$$= \frac{\text{Pitch}}{\text{Total division on circular scale}}$$

$$\Rightarrow \text{Least count} = \frac{1}{50} = 0.02$$

It means, Assertion is not correct, but Reason is correct.

- 03** Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes 0.01 cm^3 of oleic acid per cm^3 of the solution. Then, you make a thin film of this solution

(monomolecular thickness) of area 4 cm^2 by considering 100 spherical

drops of radius $\left(\frac{3}{40\pi}\right)^{1/3} \times 10^{-3} \text{ cm}^2$.

Then, the thickness of oleic acid layer will be $x \times 10^{-14} \text{ m}$,

where x is

[2021, 17 March Shift-II]

Ans. (25)

Given, the radius of the spherical drop

$$r = \left(\frac{3}{40\pi}\right)^{1/3} \times 10^{-3} \text{ cm}$$

The volume of 100 spherical drops,

$$V = 100 \times \frac{4\pi r^3}{3}$$

Substituting the value of the radius in the above equation, we get

$$V = 100 \times \frac{4\pi}{3} \times \frac{3}{40\pi} \times 10^{-9}$$

$$V = 10^{-8} \text{ cm}^3$$

Now, Volume of the film = Area of the film $\times$ Thickness of the film

$\Rightarrow$ Volume of the film = $4 \times t$

The film is made up of 100 spherical drops.

$\therefore$ The volume of the film = Volume of 100 spherical drops

$$4t = 10^{-8} \Rightarrow t = 25 \times 10^{-12} \text{ m}$$

The thickness of the oleic layer,

$$t_0 = 0.01 \times t = 25 \times 10^{-14} \text{ m}$$

Comparing with $x \times 10^{-14}$, we get $x = 25$

- 04** The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is

[2021, 25 Feb Shift-I]

- (a) 1.64 mm (b) 0.82 mm
(c) 1.80 mm (d) 0.90 mm

Ans. (b)Given, pitch of screw gauge, $P = 1$ mmNumber of division, $n = 100$

$$\therefore \text{Least count (LC)} = \frac{P}{n} = 1/100 = 0.01 \text{ mm}$$

As, zero of circular division lies 8 divisions below.

$$\therefore \text{Zero error} = 8 \times \text{LC} = 8 \times 0.01 = 0.08 \text{ mm}$$

Since, 1st linear scale division coincide with 72nd circular scale division.

$$\therefore \text{Radius (r)} = \frac{[P + (72 \times \text{LC}) - \text{Zero error}]}{2}$$

$$= [1 + (72 \times 0.01) - 0.08] / 2$$

$$= (1.72 - 0.08) / 2 = 1.64 / 2$$

$$= 0.82 \text{ mm}$$

- 05** The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10th division and coincide with 9th division of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of a cylinder the zero of the vernier scale lies between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division)

[2020, 2 Sep Shift-I]

- (a) 3.2 cm (b) 2.99 cm
(c) 3.07 cm (d) 3.21 cm

Ans. (c)

Given, least count of main scale = 1 main scale division (MSD) = 1 mm

Also, 10 vernier scale divisions = 9 main scale divisions

$$\Rightarrow 1 \text{ vernier scale division (VSD)}$$

$$= \frac{9}{10} \text{ MSD} = 0.9 \text{ MSD} = 0.9 \text{ mm}$$

Vernier constant or least count of instrument is,

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = 1 \text{ MSD} - \frac{9}{10} \text{ MSD}$$

$$= \left(\frac{10 - 9}{10} \right) \text{ MSD} = \frac{1}{10} \text{ MSD}$$

$$= 1 \text{ mm} / 10 = 0.1 \text{ mm or } 0.01 \text{ cm}$$

The zero of vernier scale lies to the right of main scale. So, error is positive.

$$\text{Zero error} = 7 \times \text{LC} = 7 \times 0.01 \text{ cm} = 0.07 \text{ cm}$$

Given, main scale reading = 3.1 cm

Vernier scale reading = 4 divisions

 $\therefore$ Observed value

$$= \text{MSR} + \text{VSR} \times \text{LC} - \text{Zero error}$$

$$= 3.1 + 4 \times 0.01 - 0.07$$

$$= 3.14 \text{ cm} - 0.07 \text{ cm} = 3.07 \text{ cm}$$

Hence, correct option is (c).

- 06** Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as

[2020, 3 Sep Shift-I]

- (a) 2.121 cm (b) 2.124 cm
(c) 2.125 cm (d) 2.123 cm

Ans. (b)

Given, pitch = 0.1 cm

Number of divisions on circular scale

$$= 50$$

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$= 0.1 / 50 = 0.002 \text{ cm}$$

Now, correctly recorded measurement is an integral multiple of LC.

So, amongst the given options, only 2.124 cm is the integral multiple of LC.

Thus, it is the correctly recorded value.

Hence, option (b) is correct.

- 07** A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved and the least count of the screw gauge, are respectively

[2020, 6 Sep Shift-I]

- (a) negative, $2 \mu\text{m}$
(b) positive, $10 \mu\text{m}$
(c) positive, 0.1 mm
(d) positive, $0.1 \mu\text{m}$

Ans. (b)

Given that, number of divisions on circular scale = 50

$$\text{Pitch} = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$$

Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$= \frac{5 \times 10^{-4}}{50} = 1 \times 10^{-5} \text{ m} = 10 \mu\text{m}$$

Since, circular scale is 4 units ahead of the pitch scale marking which means screw gauge has positive zero error.

- 08** A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings; 5.50 mm, 5.55 mm, 5.45 mm and 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should be therefore recorded as

[2020, 6 Sep Shift-II]

- (a) $(5.5375 \pm 0.0739) \text{ mm}$
(b) $(5.5375 \pm 0.0740) \text{ mm}$
(c) $(5.538 \pm 0.074) \text{ mm}$
(d) $(5.54 \pm 0.07) \text{ mm}$

Ans. (d)

There are three significant figures in the measured value (or readings) whereas there are 5 and 4 significant figures in average readings and standard deviation of the data, respectively. So, answer must be upto three significant figures.

After rounding off the uncertain digits and considering least count of vernier scale, i.e. 0.01 mm, average diameter of pencil is recorded as $(5.54 \pm 0.07) \text{ mm}$.

Hence, option (d) is correct.

- 09** If the screw on a screw gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale, the least count of the screw gauge is

[2020, 9 Jan Shift-I]

- (a) 0.001 cm
(b) 0.01 cm
(c) 0.02 cm
(d) 0.001 cm

Ans. (a)

Pitch of screw of a screw gauge,

$$\text{pitch} = \frac{\text{distance moved by screw}}{\text{number of rotations}}$$

$$= \frac{3 \text{ mm}}{6} = 0.5 \text{ mm}$$

and least count of screw gauge,

least count

$$= \frac{\text{pitch}}{\text{number of circular scale divisions}}$$

$$= \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm} = 0.001 \text{ cm}$$

- 10** A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible.

What is the thickness of the sheet, if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

[JEE Main 2016]

- (a) 0.75 mm (b) 0.80 mm
(c) 0.70 mm (d) 0.50 mm

Ans. (a)

Given that the screw gauge has zero error.

So, least count of a screw gauge

$$= \frac{0.5}{50} \text{ mm.}$$

Thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line, we have

$$= 0.50 \text{ mm} + (25) \times \frac{0.5}{50} \text{ mm}$$

$$= 0.50 \text{ mm} + 0.25 \text{ mm}$$

$$= 0.75 \text{ mm}$$

- 11** A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

[JEE Main 2014]

- (a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm

(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm

(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Ans. (a)

If student measures 3.50 cm, it means that there is an uncertainty of order 0.01 cm.

For vernier scale with 1 MSD = 1 mm and 9 MSD = 10 VSD

LC of vernier calliper = 1MSD – 1VSD

$$= \frac{1}{10} \left(1 - \frac{9}{10} \right) = \frac{1}{100} \text{ cm}$$

- 12** A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading 0 mm Circular scale reading 52 divisions Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is

[AIEEE 2011]

- (a) 0.052 cm (b) 0.026 cm
(c) 0.005 cm (d) 0.52 cm

Ans. (a)

Diameter of wire,

$$d = \text{MSR} + (\text{CSR} \times \text{LC}) \quad \left[\because \text{LC} = \frac{1}{100} \right]$$

$$= 0 + 52 \times \frac{1}{100} = 0.52 \text{ mm}$$

$$= 0.052 \text{ cm}$$

- 13** Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

[AIEEE 2008]

- (a) 3.32 mm (b) 3.72 mm
(c) 3.67 mm (d) 3.38 mm

Ans. (d)

Diameter = Main scale reading

+ Circular scale reading $\times$ LC + Zero error

$$z = 3 + 35 \times \frac{1}{2 \times 50} + 0.03 = 3.38 \text{ mm}$$

TOPIC 2

Experiments Related to Oscillations

- 14** A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is

[2021, 25 Feb Shift-I]

- (a) 13 cm (b) 16.6 cm
(c) 18.4 cm (d) 14.8 cm

Ans. (d)

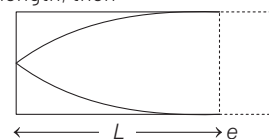
Given, diameter of the column tube, $d = 6 \text{ cm} = 6 \times 10^{-2} \text{ m}$

Frequency of tuning fork, $f = 504 \text{ Hz}$

Speed of sound at given temperature, $v = 336 \text{ ms}^{-1}$

As, this is a closed organ pipe.

Let L be the length of tube and λ be the wavelength, then



$$L + e = \lambda / 4 = v / 4f \quad \left(\because \lambda = \frac{v}{f} \right)$$

and $e = 0.6 \times d / 2$

$$= \frac{6}{10} \times \frac{6 \times 10^{-2}}{2} = 0.018$$

$$\Rightarrow L + e = \frac{v}{4f} = \frac{336}{4 \times 504}$$

$$\Rightarrow L = \frac{336}{2016} - 0.018 = 0.1667 - 0.018$$

$$= 0.1487 \text{ m} = 14.87 \text{ cm} \approx 14.8 \text{ cm}$$

- 15** A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is

[2020, 8 Jan Shift-II]

- (a) 2.40% (b) 5.40%
(c) 4.40% (d) 3.40%

Ans. (c)

Given, length of pendulum, $l = 25.0$ cm
So, there is an uncertainty of 0.1 cm in measurement of length.

Resolution of stopwatch is 1s, so, uncertainty in measurement of time is 1s.

$$\text{Now using, } T = 2\pi\sqrt{\frac{l}{g}} \text{ or } g = \frac{4\pi^2 l}{T^2}$$

$$\text{We have, } \frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \Rightarrow \frac{\Delta g}{g} \times 100\% = \left(\frac{\Delta l}{l} + \frac{2\Delta T}{T} \right) \times 100\%$$

Accuracy in measurement of g is

$$\begin{aligned} \frac{\Delta g}{g} \times 100\% &= \left(\frac{0.1}{25} + \frac{2 \times 1}{50} \right) \times 100\% \\ &= (0.004 + 0.04) \times 100\% \\ &= 4.40\% \end{aligned}$$

TOPIC 3

Experiments Related to Current Electricity

- 16** In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is

[2021, 18 March Shift-I]

- (a) 3.9 (b) 8.4 (c) 7.5 (d) 3.0

Ans. (a)

Given, the potential difference applied across the ends of the conductor, $V = 5$ V

The length of the conductor, $L = 10$ cm

The measured value of the current in the conductor, $I = 2$ A

The diameter of the conductor, $d = 5$ mm

As we know that, $R = \frac{\rho l}{A}$

Using Ohm's law,

$$V = IR \Rightarrow R = V/I$$

$$\frac{\rho l}{A} = \frac{V}{I} \Rightarrow \rho = \frac{V}{Il} \left(\frac{\pi d^2}{4} \right)$$

In error form,

$$\begin{aligned} \frac{\Delta \rho}{\rho} &= \frac{\Delta V}{V} + \frac{\Delta l}{l} + \frac{\Delta I}{I} + 2 \frac{\Delta d}{d} \\ \Rightarrow \frac{\Delta \rho}{\rho} &= \frac{0.1}{5} + \frac{0.01}{2} + \frac{0.1}{10} + 2 \left(\frac{0.01}{5} \right) \\ \Rightarrow \frac{\Delta \rho}{\rho} &= 0.039 \Rightarrow \frac{\Delta \rho}{\rho} \times 100 = 3.9\% \end{aligned}$$

The maximum permissible percentage error in the resistivity of the conductor is 3.9%.

- 17** Which of the following will not be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?

[2020, 3 Sep Shift-II]

- Multimeter shows no deflection in both cases, i.e. before and after reversing the probes if the chosen component is metal wire.
- Multimeter shows an equal deflection in both cases, i.e. before and after reversing the probes if the chosen component is resistor.
- Multimeter shows no deflection in both cases, i.e. before and after reversing the probes if the chosen component is capacitor.
- Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and no deflection on reversing the probes if the chosen component is LED.

Ans. (c)

If a capacitor is put between probes of a multimeter, then during charging, a current flows in circuit, so there is a deflection which becomes soon zero.

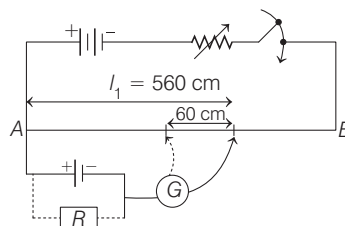
∴ Statement in option (c) is not correct.

- 18** The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer, then value of N is

[2020, 7 Jan Shift-II]

Ans. (12)

In experiment of finding internal resistance,



Balancing length with only cell,

$$l_1 = 560 \text{ cm.}$$

Balancing length with cell and

resistance, $l_2 = 560 \text{ cm} - 60 \text{ cm} = 500 \text{ cm}$

External resistance, $R = 10\Omega$

Now using

$$r = R \left(\frac{E}{V} - 1 \right) = R \left(\frac{l_1}{l_2} - 1 \right), \text{ we get}$$

$$r = 10 \left(\frac{560}{500} - 1 \right) = \frac{6}{5}$$

Now given,

$$r = \frac{N}{10} \Rightarrow \frac{6}{5} = \frac{N}{10}$$

$$\Rightarrow N = 60/5 = 12$$

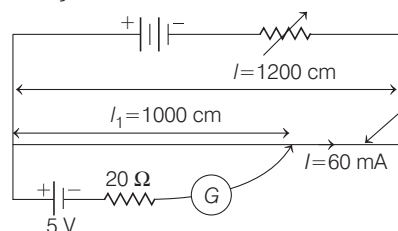
- 19** The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of 20Ω , the null point on it is found to be at 1000 cm. The resistance of whole wire is

[2020, 9 Jan Shift-II]

- (a) 80Ω (b) 100Ω
(c) 60Ω (d) 120Ω

Ans. (b)

The given situation is as shown below.



At null point, a length of 1000 cm balances emf of 5 V.

So, potential drop across 1000 cm of wire is equal to emf of cell,

$$E = I_1 \cdot x \cdot l$$

$$\Rightarrow 5 = 1000 \times x \times 60 \times 10^{-3}$$

where, x = resistance per centimetre length of potentiometer wire.

$$\Rightarrow x = \frac{5}{1000 \times 60 \times 10^{-3}}$$

$$\text{or } x = \frac{1}{12} \Omega \text{ cm}^{-1}$$

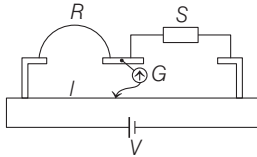
So, resistance of 1200 cm length of wire,

$$R = l \cdot x = 1200 \times \frac{1}{12} = 100 \Omega$$

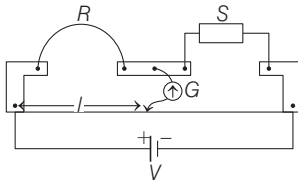
- 20** In a meter bridge experiment, S is a standard resistance, R is a resistance wire. It is found that balancing length is $l = 25$ cm. If R is replaced by a wire of half length

and half diameter that of R of same material, then the balancing distance l' (in cm) will now be

..... [2020, 9 Jan Shift-II]



Ans. (40)



Given, balance length, $l = 25$ cm

At balance point, $\frac{R}{l} = \frac{S}{100-l}$

$$\Rightarrow \frac{R}{25} = \frac{S}{75}$$

$$\Rightarrow R = \frac{S}{3} \Omega \quad \dots(i)$$

When R is replaced by some other resistance of half length and half diameter, then resistance in left gap,

$$R' = \frac{\rho \frac{l}{2}}{\frac{\pi \left(\frac{d}{2}\right)^2} = 2 \left\{ \frac{\rho l}{\left(\frac{\pi d^2}{4}\right)} \right\}}$$

$$\Rightarrow R' = 2R$$

Now, if balance is obtained at length l' , then

$$\frac{R'}{l'} = \frac{S}{(100-l')}$$

$$\Rightarrow \frac{2R}{l'} = \frac{S}{100-l'}$$

Substituting for S from Eq. (i), we get

$$\frac{2R}{l'} = \frac{3R}{100-l'}$$

$$\Rightarrow 200 - 2l' = 3l'$$

$$\text{or } l' = 40 \text{ cm}$$

TOPIC 4

Experiments Related to Electronics Devices

21 A common transistor radio set requires 12 V (DC) for its operation. The DC source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (AC) on standard domestic AC supply. The number of turns of secondary coil are 24, then the number of turns of primary are

[2021, 24 Feb Shift-I]

Ans. (440)

In a transformer,

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

where, N_p = number of turns in primary circuit,

N_s = number of turns in secondary circuit = 24,

V_p = potential of primary circuit = 220 V

and V_s = potential of secondary circuit = 12 V

$$\Rightarrow \frac{N_p}{24} = \frac{220}{12}$$

$$\Rightarrow N_p = 440$$

TOPIC 5

Experiment Related to Optics

22 An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by [AIEEE 2008]

- (a) a vernier scale provided on the microscope
- (b) a standard laboratory scale
- (c) a meter scale provided on the microscope
- (d) a screw gauge provided on the microscope

Ans. (a)

If an experiment is performed to find the refractive index of glass using a travelling microscope, distances are measured by a vernier scale which is provided on the microscope.



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